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On Both Sides of the Strait of Gibraltar

*Studies in the History of Medieval Astronomy in
the Iberian Peninsula and the Maghrib*

By

Julio Samsó



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Cover illustration: The royal codex of the Alfonsine *Libro del saber de astrología* (Biblioteca Histórica de la Universidad Complutense de Madrid, BH MSS 156, fol. 72r).

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*To David King
with gratitude for all these years of friendship and help*



Contents

Preface XIII

List of Figures XVII

1 Historical Outline

A Brief Chronological Survey 1

1.0 Foreword 1

1.1 Al-Andalus (711–1085) 2

1.2 The Maghrib (700–1050) 4

1.3 Almoravids (ca. 1050–ca. 1147) and Almohads (ca. 1147–1276) 7

1.4 Al-Maghrib and al-Andalus between the Thirteenth and the
Fifteenth Centuries 10

1.4.1 *Introduction* 10

1.4.2 *Zijes* 11

1.4.3 *Astrology* 12

1.4.4 *Miqāt* 15

1.5 Astronomy in the Christian Kingdoms of the Iberian Peninsula 18

1.5.1 *The Transmission of Arabic Astronomy into Latin* 18

1.5.2 *The Kingdom of Castile in the Thirteenth Century: the Alfonsine
Astronomical Production* 33

1.5.3 *Astronomy in Aragon and Castile in the Fourteenth Century* 36

1.5.4 *The Fifteenth Century in Castile* 39

1.6 A Brief Conclusion 42

2 *Miqāt*: Timekeeping and *Qibla* 44

2.0 Introduction 44

2.1 Calendars and Years 48

2.2 Eras 52

2.3 The Beginning of the Lunar Month 55

2.3.1 *Introduction* 55

2.3.2 *The Sources* 56

2.3.3 *The Astronomical Problem* 58

2.3.4 *The Computation* 60

2.4 The Hour 70

2.4.1 *Computational Methods for Telling the Time* 70

2.4.2 *Instruments for Telling the Time* 100

2.5	The <i>qibla</i>	128
2.5.1	<i>The qibla between the Seventh and the Ninth Centuries</i>	128
2.5.2	<i>The Astronomers' Solutions</i>	135
2.5.3	<i>Folk Astronomy and Legal Scholars</i>	144
2.5.4	<i>Qibla Indicators and the Actual Orientations of the Buildings</i>	147
3	Astrology	152
3.0	Introduction	152
3.1	Patronage and the Practice of Astrology	153
3.1.1	<i>Al-Andalus and the Maghrib</i>	153
3.1.2	<i>The Practice of Astrology in the Christian Kingdoms of the Iberian Peninsula</i>	180
3.2	Thematic Surveys	214
3.2.1	<i>Simple Systems of Astrological Prediction</i>	214
3.2.2	<i>Standard Astrology Based on Classical and Eastern Arabic Sources: 'Alī ibn Abī l-Rijāl</i>	227
3.2.3	<i>Mathematical Astrology in al-Andalus and the Maghrib</i>	257
3.2.4	<i>The Animodar</i>	291
3.2.5	<i>World Astrology: Planetary Conjunctions</i>	300
3.2.6	<i>Meteorological Astrology: al-Baqqār's Kitāb al-amṭār wa l-as'ār</i>	319
4	Astronomical Instruments	324
4.0	Introduction	324
4.1	Spherical Instruments	326
4.1.1	<i>Celestial Globe</i>	326
4.1.2	<i>Spherical Astrolabe</i>	341
4.1.3	<i>The Armillary Sphere</i>	352
4.2	The Astrolabe	373
4.2.1	<i>The Introduction of the Standard Astrolabe in al-Andalus</i>	373
4.2.2	<i>The Introduction of the Astrolabe in Catalonia (Tenth Century)</i>	378
4.2.3	<i>On the Standard Astrolabe in al-Andalus and the Maghrib from the Eleventh Century Onwards</i>	399
4.2.4	<i>The Astrolabe in the Christian Kingdoms of the Iberian Peninsula</i>	410
4.3	Universal Astrolabes	440
4.3.0	Introduction	440
4.3.1	<i>'Alī ibn Khalaf's Universal Plate</i>	443
4.3.2	<i>Ibn al-Zarqālluh's ṣafiḥa ("azafea")</i>	450

4.3.3	<i>On the Relative Chronology of the Three Universal Astrolabes</i>	460
4.3.4	<i>On the Transmission of Ibn al-Zarqālluh's ṣafiḥa to the Mashriq and the Maghrib</i>	462
4.3.5	<i>Ibn Bāṣoh's Plate for All Latitudes</i>	466
4.3.6	<i>al-Jazzār's Astrolabe</i>	469
4.4	Quadrants	470
4.5	Equatoria	475
4.5.1	<i>Introduction</i>	475
4.5.2	<i>Ibn al-Samḥ's Equatorium</i>	476
4.5.3	<i>The Equatorium of Ibn al-Zarqālluh</i>	479
4.5.4	<i>The Equatorium of Abū l-Ṣalt Umayya of Denia</i>	487
4.5.5	<i>Other equatoria</i>	489
4.6	Jābir ibn Aflaḥ's Observational Instrument	492
5	<i>Hay'a (Cosmology)</i>	496
5.1	<i>Introduction: on the Meaning of Hay'a in Western Islam</i>	496
5.2	<i>The Treatises on hay'a by Dūnash ibn Tamīm, Qāsim ibn Muṭarrif and an Anonymous Eleventh-Century Toledan Astronomer</i>	499
5.2.1	<i>Dūnash ibn Tamīm</i>	499
5.2.2	<i>Qāsim ibn Muṭarrif al-Qaṭṭān</i>	502
5.2.3	<i>An Anonymous Toledan (?) Astronomer of the Eleventh Century</i>	506
5.3	Jābir ibn Aflaḥ's Mathematical Criticism of the <i>Almagest</i>	508
5.3.1	<i>Jābir's Chronology and Scientific Works</i>	508
5.3.2	<i>The Aim of the Iṣlāḥ</i>	510
5.3.3	<i>Jābir's Criticisms of the Almagest</i>	511
5.3.4	<i>Conclusions</i>	516
5.4	<i>The Twelfth-Century Andalusī Revolt against Ptolemy</i>	516
5.4.1	<i>Introduction</i>	516
5.4.2	<i>Were These Andalusī Philosophers Competent in Astronomy?</i>	517
5.4.3	<i>Andalusī Criticisms of Ptolemy: Ibn Bājja and Ibn Rushd</i>	525
5.4.4	<i>Al-Bīṭrīj's Astronomical System</i>	530
5.5	<i>Hay'a in Castile during the Reign of Alfonso X (1252–1284)</i>	545
5.5.1	<i>Petrus Gallecus' Summa de astronomia</i>	545
5.5.2	<i>The Alfonsine Translation of Ibn al-Haytham's Fī hay'at al-ʿālam</i>	553
5.6	<i>Other hay'a Sources between the Twelfth and the Fifteenth Centuries</i>	567
5.6.1	<i>Jewish Sources</i>	567
5.6.2	<i>Pseudo-Enrique de Villena's Treatise on Astrology</i>	572
5.7	<i>Conclusions</i>	575

6 Astronomical Theory 577

- 6.1 Introduction 577
- 6.2 The Motion of Accession and Recession of the Equinoctial Points
(*al-iqbāl wa l-idbār*, Trepidation Theory) 579
 - 6.2.1 *On the Introduction of Trepidation Theory in al-Andalus: the Liber de motu octaue spere* 579
 - 6.2.2 *Ibn al-Zarqālluh's "Treatise on the Motion of the Fixed Stars"*
(*Maqāla fi ḥarakat al-iqbāl wa l-idbār*) 586
 - 6.2.3 *Ibn al-Kammād on Trepidation* 610
 - 6.2.4 *al-Istijī and the Muntakhab Zīj* 617
 - 6.2.5 *Ibn al-Hā'im on Trepidation* 619
 - 6.2.6 *Trepidation in Maghribī zījes* 622
 - 6.2.7 *Trepidation in the Christian Kingdoms of the Iberian Peninsula* 642
- 6.3 Ibn al-Zarqālluh's Solar Model 654
 - 6.3.1 *Introduction* 654
 - 6.3.2 *The Motion of the Solar Apogee* 657
 - 6.3.3 *Ibn al-Zarqālluh's Solar Model* 661
- 6.4 The Lunar Model 676
- 6.5 Conclusions 681

7 Astronomical Tables (zījes) 683

- 7.1 Introduction: the Eastern Input in al-Andalus 683
- 7.2 A General Survey of Andalusī and Maghribī zījes 688
 - 7.2.1 *Al-Khwārizmī – Maslama* 688
 - 7.2.2 *Ibn Mu'ādh's Tabulae Jahen* 708
 - 7.2.3 *The Versified zīj of Abū l-Ḥasan 'Alī b. Abī 'Alī al-Qusantīnī*
(ca. 1359–61) 714
 - 7.2.4 *The Toledan Tables* 719
 - 7.2.5 *The zījes of Ibn al-Kammād* 735
 - 7.2.6 *Ibn al-Hā'im's al-Zīj al-kāmil fi l-ta'ālīm* 751
- 7.3 Maghribī zījes: the School of Ibn Ishāq 764
 - 7.3.1 *Introduction: Ibn Ishāq's Lost zīj* 764
 - 7.3.2 *The Five zījes of Ibn Ishāq's School* 767
 - 7.3.3 *An Attempt to Recover Some Materials from Ibn Ishāq's Original Tables* 771
- 7.4 Other Maghribī zījes 795
 - 7.4.1 *Ibn 'Azzūz and His Muwāfiq zīj* 795

7.5	The Introduction of Eastern <i>zīj</i> es in the Maghrib	814
7.5.1	<i>Ibn Abī l-Shukr al-Maghribī</i>	814
7.5.2	<i>Ibn al-Shāṭir</i>	816
7.5.3	<i>Ulugh Beg</i>	817
7.6	<i>Zīj</i> es in the Christian Kingdoms of the Iberian Peninsula	820
7.6.1	<i>Abraham bar Hiyya and Abraham ibn Ezra</i>	820
7.6.2	<i>The Alfonsine Tables</i>	827
7.6.3	<i>Pedro IV of Aragon and the Tables of Barcelona</i>	858
7.6.4	<i>Other Iberian <i>zīj</i>es of the Fourteenth and Fifteenth Centuries</i>	867
7.7	Almanacs and Ephemerides	876
7.7.1	<i>Introduction</i>	876
7.7.2	<i>Yearly Ephemerides</i>	878
7.7.3	<i>Perpetual Almanacs</i>	880
7.8	Conclusions	902
	Bibliography	907
	Index of Parameters and Numerical Values	976
	Index of Names and Subjects	986

Preface

Around 1977, I attended the Second Symposium of the History of Arabic Science in Aleppo. At this conference I realised that my senior international colleagues were mainly interested in Eastern Islamic astronomy; nobody seemed to be examining Andalusī and Maghribī astronomy, a field in which my masters at the University of Barcelona, José María Millás-Vallicrosa and Juan Vernet, had carried out important research. As a result, I decided to concentrate on the latter area. Obviously, working alone, my capacity to obtain results was limited. Fortunately, in 1982, I obtained a chair in Arabic and Islamic Studies at the University of Barcelona, where there was a department of Arabic Studies offering an undergraduate degree. This meant that there were postgraduate students interested in doing research for their Ph.D. theses, who were looking for somebody to guide them. Consequently, I began assigning specific topics to these students within a general programme which aimed to review all the extant Andalusī astronomical sources. The resulting doctoral theses helped me to acquire a clear idea of the general development of Andalusī astronomy. Soon afterwards, I became aware of the existence of a set of Maghribī astronomical tables (*zīj*es), from the thirteenth and fourteenth centuries, strongly influenced by the theories of the Toledan astronomer Abū Ishāq Ibrāhīm ibn Yahyā al-Naqqāsh, known as Ibn al-Zarqālluh (d. 1100). As there was very little information about Maghribī astronomy before the thirteenth century, the idea of the existence of an Andalusī-Maghribī astronomical tradition began to take shape in my mind.

I had also been reading systematically the Alfonsine *Libro del saber de astrología*, which contains the Castilian translations of many Andalusī astronomical sources not preserved in their original Arabic form. This study introduced me to the field of Latin and Castilian translations and, as a result of my reading the works of Millás, Goldstein and Chabás on Hebrew sources, I finally reached the conclusion that there was a shared astronomical tradition written in Arabic, Latin, Castilian and Hebrew by Muslims, Jews and Christians in the Iberian Peninsula and in the Maghrib, on both sides of the Strait of Gibraltar.

I had had the idea of writing a book that describes this tradition many years ago, but it was difficult to carry out the project while I was still active at the University of Barcelona. The undertaking was delayed until I retired and it has now kept me occupied for about seven years, during which my wife Carmen has patiently endured my absences locked away in my office. She deserves my gratitude.

The book begins with a brief chronological survey which can be considered a summary or a chapter of conclusions of the whole book, although it appears at the beginning and not at the end. Besides, in this introductory chapter, I explain my own interpretation of the role of al-Andalus as the bridge through which Arabic science reached Europe and the reason why, in my opinion, Arabic sources published in the Mashriq from the eleventh century onwards only rarely arrived in al-Andalus and from there continued northwards.

Chapters 2 and 3 deal with the applications of astronomy and try to establish why astronomy was cultivated in the Middle Ages. Astronomy was useful for telling the time, determining the correct orientation of mosques and religious buildings, and for predicting the future. In any serious analysis of the history of astronomy, astrology cannot be ignored because it is one of the main reasons why astronomical research was patronised by political rulers.

Chapter 4 deals with astronomical instruments, a topic that interested Andalusī and, later, Maghribī astronomers, who introduced new developments in their design. These instruments were mainly used as analogical computers, which avoided the need for long computations in order to obtain approximate results in the problems related to the motion of the Sun and of the fixed stars, and, in the case of equatoria, to calculate planetary positions. These instruments made it possible to perform standard astrological operations. Their users were, probably, astrologers or astronomers interested in problems of timekeeping. We have very little information about observational instruments; we know that there were observations but we are not aware of any detailed reports.

Chapter 5 is an attempt to describe *hay'a* (cosmology), a topic that was relatively underdeveloped in the Iberian Peninsula and the Maghrib; there, this term is usually applied to elementary treatises which describe a three-dimensional Ptolemaic system of the world, without any technical details. Only al-Biṭrūjī, in the twelfth century, tried to replace Ptolemy with a description of the cosmos that would agree with the Aristotelian or Neo-Platonic physics known at the time, but his results were unsuccessful and only the *Nūr al-ālam* (ca. 1400) by Joseph ibn Naḥmias, recently published by Robert Morrison, can be considered a serious attempt to overcome al-Biṭrūjī's shortcomings.

Chapters 6 (astronomical theory) and 7 (astronomical tables) are closely related and in fact they form the core of the book. Chapter 6 describes the innovations introduced in Ptolemaic astronomy by Ibn al-Zarqālluh (trepidation theory, motion of the apogees, solar model with variable eccentricity and the slight modification of Ptolemy's lunar model), later adopted by Andalusī and Maghribī astronomers in their *zīj*es. The echoes of Zarqāllian theories also reached the astronomical works undertaken in the Christian kingdoms of the Iberian Peninsula, mainly by Jews, and they are the main justification for the

existence of a common astronomical tradition on both sides of the Strait of Gibraltar. Finally, Chapter 7 is a general survey of the Iberian and Maghribī *zīj*es between the tenth and the fifteenth centuries and it expands on a topic which has already appeared in chapters 3 (astrology) and 6 (astronomical theory): namely, the fact that Western Islamic astronomy and astrology was basically Khwārizmian and therefore computed sidereal planetary positions. However, from the twelfth century onwards we see a tendency to replace sidereal astronomy with tropical astronomy, following the school of al-Battānī, whose *zīj* had been known in al-Andalus since the tenth century. In the Iberian Peninsula, this tendency is represented by Jewish astronomers, beginning with Abraham bar Hiyya and Abraham ibn ‘Ezra, followed by the Jewish authors of the *Alfonsine Tables* and of the *Tables of Barcelona*, and culminating in Zacut’s almanacs in the fifteenth century, although this is not to say that all Jewish Iberian astronomers followed this school. The same evolution took place independently in the Maghrib, for different reasons: new observations made there led Maghribī astronomers to conclude that the Zarqāllian astronomical theories had lost their predictive capacity and that there were major differences between the computed and observed positions of the celestial bodies. As a result, the common Andalusī-Maghribī tradition was abandoned and replaced by Eastern Islamic *zīj*es that computed tropical positions.

This book reproduces the results of doctoral theses presented at the University of Barcelona since 1982. Some of them (Mercè Comes, Mercè Viladrich, Roser Puig, Miquel Forcada, Emilia Calvo, Josep Casulleras, Marc Oliveras) have been published, while other more recent ones (Montse Díaz Fajardo, María José Parra, Rachid Saidi) are accessible at the website <http://www.tdx.cat>. Unfortunately three of these theses (Angel Mestres, Muḥammad ‘Abd al-Raḥmān, and José Bellver) have not been published. This absence is particularly noticeable in the case of Mestres and ‘Abd al-Raḥmān, as Bellver’s main results have appeared in the form of articles. Readers of this book will soon realise my indebtedness to ‘Abd al-Raḥmān’s study of Ibn al-Raqqām’s *Shāmil zīj* and to Mestres’ analysis of the anonymous Tunisian recension of the *zīj* of Ibn Ishāq.

In this book I have also reproduced the results of my own research published since the 1960s. Sometimes the reproduction is almost literal, although I have revised the earlier texts, in many cases checking the primary sources, and correcting errors. Many calculations have been made using computer programmes designed by E.S. Kennedy, Honorino Mielgo, Josep Casulleras, John D. North and Benno van Dalen. My deepest gratitude to all these scholars.

One cannot write this kind of book without the help of friends and colleagues to whom I also wish to express my gratitude here, especially to José Chabás who has revised its seven chapters, and to Michael Maudsley who

has corrected my English. Specific chapters or sections have also been read by David A. King, Ahmed Djebbar, Bernard R. Goldstein, Shlomo Sela, Ron B. Thomson, Gerold Hilty, Benno van Dalen, David Juste, José Martínez Gázquez, Roser Puig, Taro Mimura, Eric Mercier, Mònica Rius, Josep Casulleras, Montse Díaz-Fajardo, Cristina Álvarez-Millán and Juan Estadella. Both Goldstein and Sela have done their best to overcome one of my greatest shortcomings: my ignorance of Hebrew. Fortunately, Hebrew astronomy and astrology has been the object of serious research by Goldstein, Chabás and Sela and, in this field, I have often limited myself to summarising their works. My gratitude also to Father José Luís del Valle, curator of the Escorial Library, who sent me free copies of all the manuscripts I needed, as well as to Rosa Comes, who provided me with copies of other manuscripts which were in the possession of her late sister Mercè. Finally I want to thank Montse Díaz-Fajardo, as well as Armando and Rodrigo Huapaya for their help with the geometrical drawings and to Roser Puig who did things I am unable to do.¹

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Figures

- 2.1 Crescent visibility according to D.A. King 59
- 2.2 The *quadrans vetustissimus* in ms. Vatican Reg. lat. 1661, fol. 86v
(© Biblioteca Apostolica Vaticana) 102
- 2.3 The declination scale in the *quadrans vetustissimus* 102
- 2.4 Determination of the hour with the *quadrans vetustissimus* 103
- 2.5 Ibn al-Zarqālluh's quadrant according to al-Marrākushī (*Jāmi' al-mabādī*,
Ms. Istanbul Ahmet III 3343) 104
- 2.6 The *quadrans vetus* in the Alfonsine *Libro del Saber de Astrología*.
© Biblioteca Histórica de la Universidad Complutense de Madrid.
[BH MSS 156 fol. 173v] 107
- 2.7 Use of the *quadrans vetus* (borrowed from Richard Lorch) 108
- 2.8 Sundial made, according to the inscription, by Aḥmad ibn al-Ṣaffār.
© Museo Arqueológico de Córdoba 111
- 2.9 Ana Labarta's tracings of the remnants of a sundial found in the Camposanto
de los Mártires together with a reconstruction of the whole instrument 112
- 2.10 Sundial made by Abū l-Qāsim ibn Ḥasan al-Shaddād in Tunis (746/1345–46)
preserved in the Carthage National Museum (Tunis) 114
- 2.11 a) The illustration in Qāsim b. Muṭarrif's *Kitāb al-hay'a* (above, ms. Istanbul
Carullah 1279 fol. 319r) and b) in Ibn Khalaf's *Kitāb al-asrār* (below, ms. Or. 152
fol. 47v © Biblioteca Medicea Laurenziana, Florence) 117
- 2.12 The orientation of the Ka'ba and of the Great Mosque of Córdoba 133
- 2.13 The qibla (borrowed from D.A. King) 135
- 2.14 The "standard approximate method" to calculate the *qibla* (borrowed from
D.A. King) 137
- 2.15 Plan of Madīnat al-Zahrā' by Antonio Vallejo (*La ciudad califal de Madinat al-
Zahra. Arqueología de su excavación*. Córdoba, 2010) 140
- 3.1 The horoscope of the crosses in ms. Escorial 918 fol. 190v (© Patrimonio
Nacional, Spain) 216
- 3.2 The horoscope of the crosses in ms. Madrid Biblioteca Nacional 9294, fol. 5v
(© Biblioteca Nacional de España) 217
- 3.3 Diagram of the horoscope of the crosses 217
- 3.4 Llull's instrument in ms. British Library Add. 16434, fol. 1v
(© British Library) 226
- 3.5 Llull's instrument in Frances Yeats' reconstruction 226
- 3.6 Standard method for the division of the houses borrowed from
E.S. Kennedy 260
- 3.7 Prime vertical (fixed boundaries) method 263

- 3.8 Al-Bīrūnī's design of an astrolabe plate for casting the houses according to the prime vertical method 265
- 3.9 Plate for the projection of rays, according to the prime vertical method, for the latitude of Valencia in the astrolabe of Muḥammad al-Ṣabbān (Inv. 52473 © History of Science Museum, Oxford University) 266
- 3.10 A plate for the division of the houses for latitude 37;30° extant in the Vienna Technical Museum (©Technisches Museum Wien) 267
- 3.11 Equatorial method for the division of the houses borrowed from E.S. Kennedy 268
- 3.12 Design of an astrolabe plate for the division of the houses (equatorial method) 270
- 3.13 Parallel of declination (borrowed from M. Díaz-Fajardo) 278
- 3.14 Position semicircle method borrowed from Casulleras & Hogendijk 282
- 3.15 Al-Battānī's method for casting the rays directly on the ecliptic when the planet has latitude (borrowed from Casulleras & Hogendijk) 284
- 3.16 Four position circles method for casting the rays (borrowed from J.P. Hogendijk) 289
- 4.1 al-Battānī's *bayḍa* in Nallino's reconstruction 327
- 4.2 A reconstruction of the Alfonsine celestial globe 334
- 4.3 Mūsā's spherical astrolabe. Inv. 49687 © History of Science Museum, Oxford University 343
- 4.4 Vertical circles, almucantars, hour diagram and position circles in the Alfonsine treatise on the spherical astrolabe (Biblioteca Histórica de la Universidad Complutense de Madrid, BH MSS 156, fol. 48v) 347
- 4.5 Alidade and sights in the Alfonsine spherical astrolabe (Biblioteca Histórica de la Universidad Complutense de Madrid, BH MSS 156, fol. 49r) 351
- 4.6 The spherical astrolabe in Biblioteca Histórica de la Universidad Complutense de Madrid, BH MSS 156, fol. 55v 352
- 4.7 Dūnash ibn Tamīm method to calculate the difference in geographical longitude between two localities 359
- 4.8 The Alfonsine representation of the armillary sphere. Biblioteca Histórica de la Universidad Complutense de Madrid, BH MSS 156, fol. 142r 372
- 4.9 Plate for the seventh climate in Khalaf ibn al-Mu'ādh's astrolabe. The image preserves the projection of the zenith and those of the almucantars (circles of altitude) but not the vertical circles. Ms. BNF Lat. 7412, fol. 20r (© Bibliothèque Nationale de France) 375
- 4.10 Part of the back of Khalaf ibn al-Mu'ādh's astrolabe. His name is clearly visible along the diagonal of the shadow square. The drawing also shows the zodiacal scale corresponding to the signs of Capricorn, Aquarius and Pisces and to the months of January, February, March (ms. BNF Lat. 7412 fol. 23v, © Bibliothèque Nationale de France) 375

- 4.11 Projection of a fixed star on the rete according to *h'* 383
- 4.12 Procedure for the computation of the *altitudo* 384
- 4.13 Ms. Bibliothèque de Chartres 214 390
- 4.14 Ms. British Library Royal 15 B, IX (© British Library) 391
- 4.15 The “Carolingian” astrolabe preserved in the Institut du Monde Arabe (Paris).
© Musée de l’Institut du monde arabe, Paris / Philippe Maillard 393
- 4.16 Plate for latitude 47;30° (MZL), in the “Carolingian” astrolabe, with the
projection of almucantars and hour lines but not the azimuthal curves.
© Musée de l’Institut du monde arabe, Paris / Philippe Maillard 394
- 4.17 The plate for latitude 41;30° (MAL) with the inscription “ROMA ET
FRANCIA”. © Musée de l’Institut du monde arabe, Paris /
Philippe Maillard 395
- 4.18 Detail of the inscription “ROMA ET FRANCIA”. © Musée de l’Institut du
monde arabe, Paris / Philippe Maillard 395
- 4.19 The horary alidade according to Ron B. Thomson, 2019 420
- 4.20 Maslama’s method to divide the ecliptic traced on an astrolabe 422
- 4.21 Maslama’s auxiliary theorem applied to the division of the ecliptic traced
on an astrolabe 422
- 4.22 Maslama’s method to divide the horizon on an astrolabe 428
- 4.23 Procedure to project azimuth circles on the astrolabe 430
- 4.24 Pseudo Māshā’allāh’s division of the horizon (Thomson’s fig. 14) 435
- 4.25 Alfonsine division of the horizon. Biblioteca Histórica de la Universidad
Complutense de Madrid. BH MSS 156, fol. 72r) 435
- 4.26 Meridian stereographic projection. Borrowed from Roser Puig 442
- 4.27 Mater of the universal plate in Biblioteca Histórica de la Universidad
Complutense de Madrid. (BH MSS 156 fol. 84v) 445
- 4.28 Mater of the universal plate reconstructed by E. Calvo and R. Puig, with the
collaboration of X. Crous 446
- 4.29 Rete of the universal plate in Biblioteca Histórica de la Universidad
Complutense de Madrid. (BH MSS 156 fol. 86v) 447
- 4.30 Rete of the universal plate reconstructed by E. Calvo & R. Puig, with the
collaboration of X. Crous 448
- 4.31 Rete of the universal plate superimposed on the mater (E. Calvo, R. Puig &
X. Crous) 448
- 4.32 Face of the *shakkāziyya* (back of an anonymous and undated Maghribī
astrolabe (Inv. 41122 © History of Science Museum, Oxford University) 452
- 4.33 Back of the *shakkāziyya* in R. Puig’s reconstruction 453
- 4.34 Face of the *ṣafiḥa zarqāliyya*. Biblioteca Histórica de la Universidad
Complutense de Madrid (BH MSS 156 fol. 112v) 454
- 4.35 Back of the *ṣafiḥa zarqāliyya*. Biblioteca Histórica de la Universidad Com-
plutense de Madrid (BH MSS 156 fol. 113r) 454
- 4.36 Back of the *ṣafiḥa zarqāliyya* in Roser Puig’s reconstruction 455

- 4.37 Alidade and transversal ruler, borrowed from Roser Puig 456
- 4.38 Drawing the ellipses of the orthographic projection. Borrowed from R. Puig 457
- 4.39 a), b) and c) Use of the alidade and of the orthographic projection to determine the hour of the day. Borrowed from R. Puig 459
- 4.40 Ibn al-Zarqālluh's quadrant in al-Marrākushī's *Jāmi' al-mabādi'* (Ms. Istanbul Ahmet III 3343) 464
- 4.41 Ibn Bāṣoh's universal plate, borrowed from Emilia Calvo and Honorino Mielgo 467
- 4.42 Face of al-Jazzār's astrolabe borrowed from E. Calvo 470
- 4.43 Determining the true planetary longitude with Ibn al-Samḥ's equatorium 477
- 4.44 Circles for Venus, Mars, Jupiter and Saturn, according to M. Comes and H. Mielgo 481
- 4.45 Circles for Mercury, the Moon and the Sun according to M. Comes and H. Mielgo 482
- 4.46 Construction of Mercury's deferent in Ibn al-Zarqālluh's equatorium 483
- 4.47 The Sun and the planets in Abū l-Ṣalt's equatorium: deferents in broken lines, equants in continuous lines (drawing by M. Comes and H. Mielgo) 488
- 4.48 Use of threads to determine the true longitude of a planet: Z centre of the equant; K centre of the deferent; D centre of the Earth (drawing by M. Comes and H. Mielgo) 489
- 4.49 Jābir's instrument in Richard Lorch's reconstruction 493
- 4.50 Jābir's instrument in the Latin translation, according to Richard Lorch's reconstruction 495
- 5.1 Al-Bīṭrūjī's planetary model 533
- 5.2 Al-Bīṭrūjī's model for the motion of the fixed stars 535
- 5.3 Accession and recession of the fixed stars according to Mancha 537
- 6.1 Trepidation model in the *Liber de motu* 581
- 6.2 Secular variations of the obliquity of the ecliptic according to the trepidation model of the *Liber de motu* 584
- 6.3 Ibn al-Zarqālluh's first trepidation model 593
- 6.4 Ibn al-Zarqālluh's model 1a 593
- 6.5 Ibn al-Zarqālluh's model 1b 594
- 6.6 Ibn al-Zarqālluh's second trepidation model 596
- 6.7 Ibn al-Zarqālluh's model 11a 596
- 6.8 Ibn al-Zarqālluh's model 11b 597
- 6.9 Computation of the value of precession in Ibn al-Zarqālluh's third model 599
- 6.10 Ibn al-Zarqālluh's secondary model for the variation of the obliquity of the ecliptic 600

- 6.11 Secular variations of the obliquity of the ecliptic according to Ibn al-Zarqālluh's third trepidation model 607
- 6.12 Ibn al-Kammād's trepidation model according to M. Comes 613
- 6.13 Ibn al-Zarqālluh's model to justify the variation of the solar eccentricity 663
- 6.14 Graph of Ibn al-Bannā's two functions used to compute the solar equation 670
- 6.15 Historical values of solar eccentricity according to Ibn al-Zarqālluh's solar model 674
- 6.16 Ibn al-Zarqālluh's lunar model, borrowed from R. Puig 677
- 7.1 The "second accession" in Ibn al-Hā'im's *zīj*, borrowed from M. Comes 763

Historical Outline

A Brief Chronological Survey

1.0 Foreword

This book has been conceived as a result of my belief in the existence of a common astronomical tradition on both sides of the straits of Gibraltar. Muslim, Jewish and Christian astronomers of the Iberian Peninsula and the Maghrib cultivated this science and its astrological applications on the same basis: a Greek and Indo-Persian heritage, assimilated, developed and criticised by astronomers of the eastern Islamic lands until approximately the end of the tenth century.¹ These materials were also appropriated by astronomers of al-Andalus who, in the eleventh century, had lost their contact with the new productions of the Mashriq. As a result of this, after the political union between al-Andalus and the Maghrib forged by Almoravids and Almohads, an astronomical tradition with a certain degree of originality was inherited by the Maghribī astronomers of the thirteenth and fourteenth centuries. On the other side of the strait, after the great period of Arabic-Latin translations in the Iberian Peninsula of the twelfth century, exactly the same tradition reached the Christian lands and was developed in the main by Jewish astronomers. It is fairly obvious to me that the *Alfonsine Tables* (thirteenth century) or the *Tables of Barcelona* of Pedro IV of Aragon (fourteenth c.) are standard Islamic *zīj*es, and the same could be said of the early tradition of European astronomical tables: a recent book by J. Chabás and B.R. Goldstein has clearly established that the development of European tables during the Late Middle Ages is the result of the assimilation of Arabic materials which circulated in al-Andalus and which were, therefore, available to medieval translators.²

Tracing the historical outline of the development of astronomy is fairly easy in the case of al-Andalus but much more difficult for the Maghrib. This is due, on the one hand, to the fact that Andalusian astronomy has been the object of systematic research during the last fifty or sixty years, while the same cannot be said of Maghribī astronomy; and on the other to the fact that al-Andalus was a political unit between 711 and 1492, with the sole exception of the period

¹ Sabra, 1987; Gutas, 1999.

² Chabás & Goldstein, 2012.

of the “petty kingdoms” (*al-ṭawāʾif*, 1031–1086). In spite of its vital historical importance, this period does not pose any serious difficulty to the study of astronomy, because we need only deal with two or three cities: Toledo, under the Dhū l-Nūn dynasty, and, to a lesser extent, Córdoba (governed by the Banū ‘Abbād) and Jaén (*Jayyān*), which belonged to the kingdom of Granada, under the Zīrī dynasty. The study of the Maghrib is an entirely different proposition, because it only became a united state (which also included al-Andalus) under the Almoravids and the Almohads, between the end of the eleventh century and the middle of the thirteenth century.

1.1 Al-Andalus (711–1085)³

There are no records of astronomical study in Andalusian civilisation until the reign of *amīr* ‘Abd al-Raḥmān II (821–852), who was the first to introduce astronomical tables in al-Andalus. Before that period we can only perceive the survival of a very modest Latin astrological tradition, and suppose that it probably coexisted with an Arabic tradition of folk-astronomy. This survival of a Latin astrological tradition – which was, no doubt, also present in the Maghrib – is the first of the distinctive characteristics of Andalusian astronomy.

The middle of the ninth century was marked by the beginning of a period of orientalisation of Andalusian culture during the reign of ‘Abd al-Raḥmān II (821–852). This process was favoured by the common practice of the *riḥla* towards the East, a journey which completed the standard education of any young man belonging to a family who could afford it, and also by the cultural policy of the Umayyad *amīrs* who encouraged Eastern scholars to settle in Córdoba, and who did their best to buy the new books published in the great cities of the Mashriq. There was, however, an inevitable delay in the arrival of scientific knowledge, and also a certain degree of selection, which meant that some books never reached al-Andalus or only arrived at a later date. This process of orientalisation went on until the fall of the Umayyad Caliphate (1031), and the delay in the spread of scientific knowledge continued: al-Khwārizmī’s *zīj*, compiled ca. 830, which had reached al-Andalus almost immediately, was not properly assimilated until the second half of the tenth century; and the same can be said of al-Battānī’s *zīj* or Ptolemy’s *Almagest*. This task of assimilation was undertaken by Maslama al-Majrīṭī (d. 1007) and the members of his school, who carried out serious research on stereographical projection and

³ See Vernet & Samsó, 1996; See also Samsó, 2011.

wrote books on the construction and use of astronomical instruments such as the astrolabe and the equatorium.

Besides mathematical astronomy, represented by Maslama, we should also remember the existence of folk astronomy represented by the *Kitāb fi l-nujūm* ("Book on stars") written by the polymath 'Abd al-Malik ibn Ḥabīb (d. 853),⁴ a Mālikī *faqīh* who rejected astrology and reacted against both the Latin astrological tradition and the new Eastern Islamic astronomy (based on Indian and Greek sources) and embraced an astronomy depending exclusively on Arabic sources, mainly Mālik b. Anas (d. 795–96). The text presented some adaptations to local conditions, such as the ratio 15/9 for the maximum/minimum length of daylight at the solstices, which may correspond to the latitude of Córdoba (38;30°). Ibn Ḥabīb also described the procedure, which also appears in later sources, of determining the hour during the night by observing which of the lunar mansions is crossing the meridian at a given moment,⁵ and also gives other approximate rules. This is therefore the earliest known source of information on Andalusian timekeeping. Almost a century later we find, in the *Maghrib*, a reference to a non-extant *Kitāb al-mawāqīt wa ma'rifat al-nujūm wa-l-azmān* ("On timekeeping and knowledge of the stars and time") written by Ibn al-Ḥajjām al-Qayrawānī (876–957).⁶ This tradition is continued in the tenth century by Ibn 'Āṣim (d. 1013) and by 'Arīb b. Sa'īd (d. 980), the author of an *anwā'* book which was later summarised and combined with materials of a different source, probably by the Mozarabic bishop Rabī' b. Zayd, to form the famous *Calendar of Córdoba*.⁷

The loss of political unity ushered in a period of some fifty years (1031–1086) which saw the maturity of Andalusian astronomy and the beginning of the development of its most distinctive characteristics due to the research of Abū Ishāq ibn al-Zarqālluh (d. 1100), who began his activity as an instrument builder and worked together with *qāḍī* Ṣā'īd al-Andalusī on the compilation of the *Toledan Tables*. He seems to have been the instigator of the Andalusian tradition of universal astrolabes and the introducer of certain features of astronomical theory (such as trepidation theory, a new solar model with variable eccentricity and a slight modification of the Ptolemaic lunar model) which characterise Andalusian and Maghribī *zīj*es. To this one should add the fact that Ibn al-Zarqālluh followed Ibn al-Samḥ (d. 1035) in writing on the *equatorium*, an astronomical computing device which also seems to

4 Kunitzsch, 1994 & 1997.

5 Forcada, 1990.

6 Lamrabet, 2013 no. M17, p. 128.

7 Forcada, 2000.

have developed in al-Andalus, and that Ibn al-Zarqālluh recovered the old Hellenistic tradition of perpetual almanacs which had an obvious interest, much later, for navigation.

In the eleventh century a literary description also ascribes to Ibn al-Zarqālluh the construction of a water-clock in Toledo, which marked the date of the lunar month. During the same period, and in the same city, we find the first western treatise on applied mechanics: the *Kitāb al-asrār fī natā'ij al-afkār* ("Book of the secrets on the results of thoughts") written by a certain Aḥmad or Muḥammad ibn Khalaf al-Murādī,⁸ which describes mechanical toys, water-clocks and war machines. This western tradition reappears in the clocks described in the Alfonsine *Libro del Saber de Astrología*.

Isolation seems to be the clue to understanding the existence of a particular kind of astronomy in al-Andalus, which is different from its Eastern counterpart. Even though the kings reigning in Zaragoza, Toledo or Seville encouraged the development of science, they do not seem to have been able to maintain contacts with the Mashriq, and the study of astronomy evolved on the basis of the Eastern astronomical materials which had reached al-Andalus before the end of the tenth century. On the one hand, Andalusian scientists of this period seem to have considered that an adequate scientific education could be obtained without the traditional *riḥla*; on the other, none of the sovereigns of the "petty kingdoms" seems to have been able to afford the expense involved in receiving information on the new books published in the Mashriq and, subsequently, buying them. There are, of course, exceptions, such as the mathematician and astronomer Ibn Mu'ādh al-Jayyānī (d. 1093) whose knowledge of the new Eastern developments in spherical trigonometry allowed him to introduce very important new ideas in Western Islam. Nevertheless, isolation was the rule, and the loss of contact with a cultural area which was producing new developments in the field of astronomy, especially from the thirteenth century onwards, was one of the main reasons for the decline of Andalusian science.

1.2 The Maghrib (700–1050)⁹

The history of astronomy in the Maghrib is less well known, although we can guess that it shares some common characteristics with al-Andalus. One of

8 A facsimile of the manuscript and a (not very good) edition and English translation have been published by Lisa, Tadei & Zanon, 2008. See the critical review by J. Samsó in *Suhayl* 9 (2009–2010), pp. 234–238.

9 My main sources of information are Djebbar & Moyon, 2011, pp. 59–81; Djebbar, 1988; Lamrabet, 2013.

these shared features is the relative delay in its scientific development in comparison with the Mashriq. There is no information whatsoever about any astronomical activity during the eighth century and there are only symptoms of the beginning of intellectual activity, centred in Qayrawān, at the beginning of the Aghlabī period (800–910). During the reign of Ibrāhīm II (875–902) the first scientific institution was founded in Raqqāda, the new administrative capital of the Aghlabī kingdom: its name was *Dār al-ḥikma* ("House of Wisdom"), and it was clearly an imitation of the famous Baghdad institution of the same name. There is some evidence of astronomical activity in the Tunisian institution: Ibrāhīm II was interested in mathematics and astronomy, and his astrologer Ismā'īl al-Ṭallā' accompanied the emir on his military campaigns in order to advise him with his predictions. Ismā'īl al-Ṭallā' had been associated with the House of Wisdom, and had studied astronomy and astrology in Baghdad; later he was forced to leave Qayrawān and settled in Córdoba, where he died at the beginning of the tenth century. 'Uthmān al-Sayqal (d. 941) was in charge of the building of astronomical instruments in the *Dār al-ḥikma*.

The Fāṭimids (910–972) were also interested in astrology. In 912 they began the building of the city of Mahdiyya which was officially inaugurated on the 8th Shawwāl 308/ 19th February 921. According to Ibn al-Khaṭīb,¹⁰ the city was built in a propitious moment established on astrological grounds, with the ascendent in Leo, because it is a fixed sign and because it is the domicile of the Sun, which is the indicator of kings. Mahdiyya became a pole of attraction for scientists. The *Dār al-ḥikma* of the Aghlabīs survived under the Fāṭimid Caliphs, under the direction of the Baghdadi scholar Abū l-Yusr al-Shaybānī (d. 911), although its library and astronomical instruments were transferred from Raqqāda to the newly built royal palace of al-Manṣūriyya at the beginning of the reign of Caliph al-Manṣūr (946–953). During this period we know the name of Muḥammad b. 'Abd Allāh b. Muḥammad al-'Utaqī al-Furriyānī al-Ifriqī, Abū 'Abd al-Raḥmān (d. 955) who worked as an astrologer at the court of Caliph al-'Azīz (975–996) and who wrote a book entitled *Kitāb fī l-nujūm wa aḥkāmi-hā*. He may have left the Maghrib and moved to Egypt together with the Fāṭimids. Nevertheless, in spite of all these data, it seems clear that the development of astronomy in Ifriqiya under the Fatimids cannot be compared to the achievements accomplished in Fāṭimid Egypt.

Al-Ṭallā's aforementioned migration to Córdoba is symptomatic of a trend which seems clear in the early development of Andalusian astronomy during the tenth century: namely, that Eastern astronomical materials often reached al-Andalus through Qayrawān. The Córdoba author Qāsim ibn Muṭarrif al-Qaṭṭān (fl. ca. 950) explained the procedure for calculating the length of an

10 'Abbādī & Kattānī, 1964, p. 47.

unequal hour using a table of oblique ascensions, and gave an example for the latitude of Ifrīqiya; he also described a candle clock¹¹ – of which we have an almost identical description attributed to the famous astronomer Ibn Yūnus (d. 1009)¹² – in which the amount of oil needed to keep the instrument working during the night throughout the whole year was calculated for a latitude of 33°, a value documented in early Islamic sources for both Tunis and Baghdad.¹³ My impression is that Qāsim used Iraqi materials which reached him through a Tunisian channel. A more obvious example of this kind of transmission is found in the figure of the Jewish astronomer of Qayrawān Abū Sahl Dūnash ibn Tamīm (fl. 900–950) who corresponded with Abū Yūsuf Ḥasdāy ibn Shaprūt, the famous Jewish physician of the Umayyad Caliph ‘Abd al-Raḥmān III (912–961), to whom he sent a book, in reply to questions formulated by Ḥasdāy, composed of three parts: the science of the structure of the spheres, mathematical astronomy, and astrology.¹⁴

Astronomy seems to have reached a certain level of development in the Maghrib under the Fatimids and the Zīrid dynasty (972–1152) in the first half of the tenth century, at a time when its importance in al-Andalus was still minimal. Unfortunately, with the exception of Dūnash ibn Tamīm and the astrological works of Ibn Abī-l-Rijāl al-Qayrawānī (d. after 427/1035–36),¹⁵ hardly any Maghribī astronomical works written before the beginning of the thirteenth century are extant. Only the information gathered in biographical dictionaries bears witness to the existence of some astronomical and astrological activity between the tenth and the twelfth century.

All the information available at present points to a clear superiority of Qayrawān and Ifrīqiya over the rest of the Maghrib. In spite of this, there is some information about the practice of astrology in al-Maghrib al-Aqṣā, where the Berber tribal confederation of the Barghawāṭa ruled the Atlantic coast of Morocco between the 8th and the twelfth centuries. This community followed a new kind of religion, with elements derived from shi‘ism and khārijism, and had a keen interest in astrology. One of its leaders, Yūnus, who reigned between 842 and 884, had studied astrology and divination (*kihāna*) and accurately predicted events by casting the corresponding horoscopes. He had travelled to the East, together with ‘Abbās ibn Nāṣiḥ (d. after 844) and several other companions.¹⁶ This information has a certain interest because ‘Abbās b. Nāṣiḥ

11 Comes, 1993.

12 See King, 1999 (see p. 509).

13 Casulleras, 1998.

14 Stern, 1956.

15 Samsó, 2009, pp. 10–12.

16 Colin & Lévi-Provençal, 1, pp. 225–227. On ‘Abbās see Terés, 1962.

was one of the astrologers who served the Andalusī emirs al-Ḥakam I (796–822) and ‘Abd al-Raḥmān II (822–852) and was the first to introduce in al-Andalus several collections of astronomical tables (al-Khwārizmī’s *Sindhind* among them) after one of his travels to the East.¹⁷ Much later, Ibn Marāna al-Sabtī (fl. 1069) wrote an astrological poem for Sukūt al-Barghawātī of Tangiers in which he predicted the arrival of the Almoravids and their conquest of Ceuta.

As for al-Maghrib al-Awsaṭ, more or less equivalent to modern-day Algeria, there is some information about the khārijite ibādī dynasty of the Banū Rustum (778–909) who founded the city of Tāhart/ Tihart in 778. The imāms of that dynasty were educated people, interested in religious and profane sciences. Several of them were proficient in astronomy: for example, ‘Abd al-Raḥmān b. Rustum (778–788, who used to send money to ‘Irāq to stock his library with books), ‘Abd al-Wahhāb (788–824) and Aflaḥ (824–872). As a result of their efforts, Tāhart had a library which was purported to contain 300,000 books, according to historical sources. This number is clearly exaggerated and it reminds me of the totally unreal 400,000 books claimed for the library of al-Ḥakam II (961–976) in Córdoba. In any case the library of Tāhart was destroyed by the Fāṭimids who kept the books on mathematics, astronomy and medicine for themselves.¹⁸

1.3 Almoravids (ca. 1050–ca. 1147) and Almohads (ca. 1147–1276)

When the Almoravid leader Yūsuf b. Tashfin died in 500/1106–7, he had created a Maghribī empire which included al-Maghrib al-Aqṣā, al-Maghrib al-Awsaṭ and a part of Ifrīqiya, as well as the whole of al-Andalus after 1086. This empire grew even larger under the Almohad dynasty as the whole of Ifrīqiya fell under its control. This political unity had immediate consequences for the development of science: as the centres of power were clearly in the Maghrib, Andalusian scientists were often keen to cross the sea in search of patronage and Maghribī science began to receive a strong Andalusian influence. It is fairly clear that this is an Andalusian period for the history of astronomy, as we cannot mention a single name of a Maghribī astronomer or astrologer. The situation changed entirely after the fall of the Almohads, especially under the dynasties of the Marinids (ca. 1269–1465) in Morocco and the Ḥafṣids (1206–1574) in Tunis, during which Maghribī astronomy experienced spectacular development. There seems to be no doubt about the efforts made by Almohad

17 Samsó, 2011, pp. 458–459.

18 Sālim, 1981, pp. 574–576.

caliphs to encourage the cultural promotion of the Maghrib: the first madrasas were built in Marrakesh by Ya'qūb al-Manṣūr (1184–1199) and the same policy was followed by the Marinids, among them Abū l-Ḥasan 'Alī (1331–1351) who founded several madrasas in Fez and in other cities in Morocco.¹⁹ Although the main purpose of a madrasa was to educate legal scholars, other subjects were also taught; we know that the mathematician and physician Ibn Mun'im (d. 1228) taught at one of the madrasas of Marrakesh.²⁰

Astronomy during the Almoravid period in al-Andalus seems to be a weak continuation of the kind cultivated during the period of the *ṭawā'if*. The great Toledan astronomer Ibn al-Zarqālluh was active in Córdoba until ca. 1090 and his research was continued by his disciple Ibn al-Kammād (fl. 1116). In spite of this, nothing new was produced and, as the circles of power tended to treat the duet of astronomy and astrology with suspicion, the scientists dedicated to these disciplines appear to have concealed their activities as far as they could. A good example is Mālik ibn Wuhayb (1061–62 – 1130–31),²¹ who had cordial relations with 'Alī b. Yūsuf b. Tashfin (r. 1106–1143) and was deeply interested in astronomy and astrology (something he tried to conceal). He had a copy of an Arabic translation of Ptolemy's *Almagest*, with notes in his own hand, and had himself copied pseudo-Ptolemy's *Kitāb al-Thamara* (*Karpós* or *Centiloquium*). It is interesting to see how this hidden practice of astronomy and astrology produces the development of more esoteric, less scientific disciplines: Ibn Wuhayb seems to have undertaken other divinatory practices like the *zā'irja*, which uses numerological techniques.

The Almoravid period also sees the beginning of a new astronomical tendency, strongly influenced by Aristotelian philosophy, and critical of Ptolemaic astronomy. Its instigator is Ibn Bājja (1070?–1138),²² who knew the *Almagest* well and had read Ibn al-Haytham's *Shukūk 'alā Baṭlamīyūs*. The development of this trend clearly increased during the Almohad period. Towards the beginning of this period we find Jābir b. Aflaḥ, whose *Iṣlāḥ al-Majisṭī* was a serious effort to rewrite the *Almagest*, simplifying the mathematical demonstrations by using the new trigonometry developed in the Mashriq towards the end of the tenth and beginning of the eleventh centuries and introduced in al-Andalus by Ibn Mu'adh al-Jayyānī in the second half of the eleventh century. To this Jābir b. Aflaḥ added a serious criticism of the mathematical inconsistencies

19 Viguera, 1981, pp. 405–407; Viguera, 1977, pp. 335–336.

20 Djebbar & Moyon, 2011, p. 66.

21 See D. Serrano and M. Forcada in *BA* 5, pp. 603–608.

22 Forcada, 2011.

of Ptolemy's work and proposed alternative solutions.²³ Unlike Ibn Aflah, the philosophers who followed Ibn Bājja, like Ibn Rushd (1126–1198) and al-Biṭrūjī (fl. before 1200) criticised Ptolemy on the grounds that the Ptolemaic models were purely mathematical and could not have a real physical existence. Ibn Rushd declared the need to conceive a new cosmology, as he considered the Ptolemaic one to be unsatisfactory; Al-Biṭrūjī went further by attempting to create a new world system which was clearly unsuccessful but had the obvious interest of posing, for the first time in a western context, important problems of celestial dynamics such as the transmission of motion from the prime mover to the inner planetary spheres.

The Almohad caliphs were interested in using the intellectual capacities of Andalusian philosophers (who were also physicians) in order to defend the principles of their own creed. Caliphs like Abū Ya'qūb Yūsuf became interested in the "Sciences of the Ancients" (*ʿulum al-awā'il*) and indeed Abū Ya'qūb gathered together a good library in Marrakesh in which he stored the books on astronomy and astrology he had seized from private collectors. In spite of this, astronomers never reached the centres of power, because of the enmity of both religious and philosophical circles. This is why we have practically no solid references to astrological practices in this period, although there are some indications of their existence: one year after the death of Abū Ya'qūb, there was panic in al-Andalus because of the prediction of a conjunction of the five planets, as well as the Sun and the Moon, in the sign of Libra on sixteenth September 1186, which was believed to foreshadow a global catastrophe.²⁴

As a result of the suspicion in which astronomy and astrology were held, astronomers often tended to cultivate folk astronomy (considered acceptable by the *fuqahā'*), and *mīqāt*. Thus, already at the beginning of the Almohad period, Aḥmad b. Muḥammad al-Yaḥṣubī al-Qurṭubī wrote a treatise on folk astronomy which he dedicated to Caliph 'Abd al-Mu'min (1133–1163).²⁵ Much later, al-Ḥasan b. 'Alī b. Khalaf al-Umawī (d. 1205–1206) wrote a *Kitāb al-anwā'* and a *Kitāb al-lu'lu' al-manẓūm fī ma'rifat al-awqāt bi l-nujūm* ("The versified pearl on the knowledge of times by means of the stars"), in which he dealt with a topic which was not very common in al-Andalus. Interestingly, we sometimes find that the authors of such books had a good knowledge of mathematical astronomy. Aḥmad b. Jumhūr al-Judāmī (d. 1229), for instance, wrote a *Qaṣīda fī ma'rifat al-mutawassiṭ min al-manāzil fī waqt al-fajr* ("Poem for the knowledge of the lunar mansion which crosses midheaven at the beginning of dawn").

23 Bellver, 2009.

24 See an analysis of this event, as well as a bibliography, in Samsó, 2011, pp. 518–520.

25 Kaddouri, 2005.

This poem gives information about the lunar mansion which crosses the meridian at the moment of dawn, which marks the moment of the *fajr* prayer, throughout the solar year, in a location whose latitude is $37;30^\circ$ (Seville). The analysis of the information contained in the poem shows that it is the result of a careful computation made by a competent astronomer.²⁶

On the other hand, during the caliphate of Abū ‘Abd Allāh al-Nāṣir (1199–1213), the situation seems to have changed. In this period, we find a mathematician (Ibn Mun‘im) and an astronomer (Ibn al-Hā‘im) dedicating their books (*Fiqh al-ḥisāb* and *al-Zīj al-Kāmil fī l-ta‘ālīm* respectively) to the caliph himself. The second work (written ca. 1205) is particularly interesting for us because it is not a standard *zīj*; it contains an extremely elaborate set of canons, but no numerical tables. These canons are mainly concerned with computational procedures for solving astronomical problems and they include careful geometrical proofs. This work contains a large amount of historical information on the work carried out in the eleventh century by the Toledan school of Ibn al-Zarqālluh and it should be considered as the last important Andalusian astronomical source.

1.4 Al-Maghrib and al-Andalus between the Thirteenth and the Fifteenth Centuries

1.4.1 Introduction

The Almohad Caliphate came to an end with the conquest of Marrakesh by the Merinids (1269–1465) whose reign brought a period of cultural splendour to the al-Maghrib al-Aqṣā, in cities like Fes, Marrakesh, Ceuta and others. Half a century earlier, the Ḥafṣid dynasty (1206–1574) had gained control of Ifrīqiya; Tunis, their capital, developed rapidly as a centre of culture and displaced Qayrawān, which never recovered its previous splendour after its destruction by the Banū Hilāl in 1057. In central Maghrib the Banū ‘Abd al-Wād (1236–1550) of Tilimsān/Tlemcen and the city of Constantine also bear witness to some astronomical activity. These dynasties were contemporary to the Naṣrides (1232–1492) of the kingdom of Granada, who reigned over what was left of al-Andalus during a painfully long period of political and cultural decline.

Between the tenth and the twelfth centuries Andalusian astronomy was clearly superior to that of the Maghrib, but the situation was reversed from the thirteenth century onwards. During the twelfth century the scientists of the Maghrib seemed to have learnt from their Andalusian colleagues but the

²⁶ Samsó, 2008.

situation changed in the following periods, during which the masters became students and the students masters. A good example is found in the *riḥla fī ṭalab al-ʿilm* (journey in the search of knowledge) made by a man who was probably the last important Andalusian mathematician: al-Qalaṣādī (ca. 1412–1486).²⁷ His round trip lasted fifteen years (1436–1451) and took him from Baza to Tilimsān, Oran, Tunis, Alexandria, Cairo, Jidda, Mecca, Medina, Cairo, Barqa, Tripoli, Tunis, Oran, Tilimsān/Tlemcen, Almería, Baza and Granada. He spent eight years and seven months in Tilimsān, three and a half years in Tunis, and one year and almost three months in Cairo, during which time he was both learning and teaching. As he merely passed through the rest of the cities, it is clear that his teachers were in the Maghrib.

1.4.2 Zījēs

The thirteenth and fourteenth centuries saw an astronomical renaissance strongly influenced by the Andalusian tradition; it shared many of its characteristics, and was its obvious continuation at a time in which Islamic astronomy in the Iberian Peninsula entered a period of decline. This can be seen from the appearance of an Andalusian-Maghribī group of *zījēs* which begins with the unfinished *zīj* by Abū l-ʿAbbās ibn Ishāq al-Tamīmī al-Tūnisī (fl. Tunis and Marrakush ca. 1193–1222). According to Ibn Khaldūn, Ibn Ishāq used the results of the observations made by an unnamed Sicilian Jew. The analysis of the extant materials does not seem to confirm Ibn Khaldūn's information and Mestres²⁸ suggests that his Sicilian correspondent provided Ibn Ishāq not with the results of his observations, but rather with Andalusian astronomical literature. It seems clear that Ibn Ishāq compiled a set of tables for the computation of planetary longitudes, eclipses, equation of time, parallax and, probably, solar and lunar velocity. These tables were not accompanied by an elaborate collection of canons, although they contained instructions of some kind for the use of a few tables. Such a collection had to be completed and “edited” in order to be used, and this is the origin of a set of five “recensions” of Ibn Ishāq's *zīj* made in the Maghrib in the second half of the thirteenth and beginning of the fourteenth century. All of them contain the same sidereal mean motion and equation tables for the computation of planetary longitudes and follow the ideas of Ibn al-Zarqālluh on trepidation, cyclical variation of the obliquity of the ecliptic, motion of the solar apogee, and correction of the Ptolemaic lunar model. These recensions were prepared by three Maghribī astronomers:

²⁷ Abū l-Ajfan, 1978; Marín, 2004.

²⁸ Mestres, 1996.

- 1) An anonymous Tunisian astronomer who prepared the recension extant in ms. Hyderabad Andra Pradesh State Library 298, ca. 1266–1281,
- 2) Ibn al-Bannā' of Marrakesh (1256–1321) in his extremely popular *Minhāj*,
- 3), 4) and 5): three recensions prepared by the Andalus-Maghribī astronomer Ibn al-Raqqām (d. 1315): *al-Zīj al-Mustawfī* (Tunis, after 1280–81), *al-Zīj al-Shāmil* (Bougie, ca. 1290) and *al-Zīj al-Qawīm* (Tunis, after 1280–81, revised in Granada).

Ibn al-Raqqām was an astronomer of Andalusī origin who worked for most of his life in the Maghrib but who settled in Granada some time after 1288–89. The *Qawīm zīj*, which was used in that city, marked the return to al-Andalus of astronomical materials of a clear Andalusian origin.

Apart from this group of *zīj*es closely related to Ibn Ishāq's lost *zīj*, three other sets of astronomical tables, also with Andalusian connections, were prepared in the fourteenth century by two astronomers born in Constantine in the central Maghrib, who were active in Fes: Abū l-Qāsim ibn 'Azzūz al-Qusanṭīnī (d. 1354) compiled two *zīj*es (*al-Zīj al-Muwāfiq* and *al-Zīj al-Kāmil*) of which only the first one is extant;²⁹ and Abū l-Ḥasan 'Alī ibn Abī 'Alī al-Qusanṭīnī compiled a small *zīj* with canons written in verse so that they could easily be learnt by heart. This work has the obvious interest of being the only known document extant in Arabic in which the planetary theory is Indian rather than Ptolemaic.³⁰

This Andalusian tradition of *zīj*es was progressively abandoned during the second half of the fourteenth century and in the fifteenth century due to disagreements between computation and observed planetary positions, especially in relation to trepidation tables and the computation of the obliquity of the ecliptic according to Ibn al-Zarqālluh's model. As a result, Eastern *zīj*es, by Ibn Abī l-Shukr al-Maghribī (d. 1283), Ibn al-Shāṭir (d. 1375) and Ulugh Beg (1393–1449), were introduced and used in the Maghrib.

1.4.3 *Astrology*

The main use of astronomical tables was to cast horoscopes, and there is no doubt that astrologers were active both in al-Andalus and in the Maghrib. In the case of Naṣrid Granada, we have some information from Ibn al-Khaṭīb's *Iḥāṭa*.³¹ Under the reign of Ismā'īl II (1359–1360), a horoscope was cast for the moment when Muḥammad al-Fihri took up his position as minister. This horoscope predicted that his time in office would end badly. Aḥmad b. Muḥammad

²⁹ Samsó, 1997.

³⁰ Kennedy & King, 1982.

³¹ 'Inān, 1973–77, cf. I, 205–206; II, 91.

b. Yūsuf al-Anṣārī was an astrologer in the service of Muḥammad VI (1360–62) who told the king when the time was propitious to begin a revolt against Muḥammad V (1354–1359, 1362–1391). He predicted, later, that Muḥammad V would recover the throne in 1362. We also have information about the nativity horoscope of Muḥammad V himself.³²

In the case of the Maghrib of the Merinids, we know for sure that astrology was practised either openly or secretly. Ibn Marzūq (b. between 1310–1312, d. 1379) wrote a hagiography of sultan Abū l-Ḥasan ‘Alī (1331–1351) including a whole chapter on Abū l-Ḥasan’s rejection of astrology, which, nonetheless, does not appear to have been followed by a serious persecution of astrologers and astrological beliefs.³³ This text mentions Ibn al-Bannā’ al-Marrākushī (1256–1321)’s reputation as an astrologer, and it is clear that the famous mathematician of Marrakesh was interested in astrology at least at an early stage of his life. In fact, the short astrological texts edited by Jabbār (= Djebbar) and Aballāgh³⁴ from an Escorial manuscript are rather elementary; some of them seem to be a set of student’s notes copied by Ibn al-Bannā’ in his youth from different sources.

During Abū l-Ḥasan’s reign, at the battle of El Salado (*Faḥṣ Ṭarīf*) in 1340 a coalition of the Merinid army of Abū l-Ḥasan and the Naṣrid army of Yūsuf I (1333–1354), fought the armies of Alfonso XI of Castile and Alfonso IV of Portugal. The issue of the battle was the object of a mistaken prediction by the astronomer Ibn ‘Azzūz al-Qusanṭīnī (d. 1354) who ascribed the responsibility for his error to the astronomical tables of Ibn Isḥāq which he had used to cast his horoscope. To correct his mistake, he made astronomical observations in Fez ca. 1344 with which he calculated new tables of mean planetary motions and recast two new horoscopes that allowed him to make new predictions of the known results of the battle. All this poses the problem of whether the first horoscopes (not extant) were the result of an official request from the circles of power (not necessarily the sultan himself) or whether Ibn ‘Azzūz cast them on his own initiative.³⁵

Some twenty years later, the aforementioned Abū l-Ḥasan ‘Alī b. Abī ‘Alī al-Qusanṭīnī dedicated his *zīj*, with canons written in *rajaz* verse, to the Merinid sultan Abū Sālim Ibrāhīm al-Musta‘īn (1359–1361). As the main purpose of a *zīj* is to compute the planetary positions needed to cast a horoscope, the dedication to Abū Sālim is clearly significant. Astrology interested men

32 Samsó, 2011, pp. 409–412, 562.

33 Viguera, 1981, pp. 438–44; Viguera, 1977, pp. 361–366.

34 Jabbār & Aballāgh, 2001, pp. 160–184.

35 Samsó, 1999.

in power at the time, as is witnessed by the work of another astrologer from Constantine, Ibn Qunfudh (1339–1407), who wrote a commentary on Ibn Abī l-Rijāl's astrological *urjūza* and dedicated his work to Abū Bakr ibn Ghāzī ibn al-Kās, minister of the Merinid sultans Abū Fāris (1366–1372) and Abū Zayyān (1372–1374). The book was probably written during the brief reign of Abū Zayyān and it contains a set of eleven horoscopes as examples: only two of them are dated and all are unidentified, but they correspond to an astrological history of the dynastic crisis (1348–1372) which followed the murder of sultan Abū 'Inān (1348–1358). As the minister Ibn Ghāzī was profoundly interested in astrology, my guess is that the collection of horoscopes was presented to him as a kind of challenge, in order to see whether he could guess the subjects of the horoscopes or the events alluded to in them.³⁶

Out of the 11 horoscopes in Ibn Qunfudh's commentary, five deal with the prediction of the duration of the reign of five Merinid sultans. This topic also interested Ibn Mas'ūd ibn Farmīja (possibly Firmījuh = Bermejo) (fl. 1372–1394), who was *muwaqqit* in Fes, Tunis, Jerusalem and Damascus and had a role in the introduction of Ibn Abī l-Shukr's *Tāj al-azyāj* in the Maghrib. He wrote a short work entitled *Ḥuṣūl al-maqāṣid wa l-āmāl min al-ṭuruq wa l-fawā'id allatī tu'lamu min-hā mudad al-wulāt wa l-'ummāl* ("How to reach your purposes and hopes, using the procedures and useful information with which you can establish how long governors and authorities will stay in power"). This text is not particularly original, for it contains many passages copied from works by Ibn Abī l-Rijāl, Abū Ma'shar and a certain Abū Yūsuf (al-Kindī?) but it reveals to us a *muwaqqit* interested in astrology and, especially, in political astrology, who was able to use *tasyīr* techniques.³⁷

Far more interesting are the theoretical works written by two astrologers who worked in Fes: the aforementioned Ibn 'Azzūz (d. 1354), the author of the *Kitāb al-fuṣūl fī jam' al-uṣūl* ("Chapters which assemble all the principles"), and Abū 'Abd Allāh al-Baqqār (fl. 1418) who wrote the *Kitāb al-adwār fī tasyīr al-anwār* ("Cycles for the prorogation of celestial elements") and the *Kitāb al-amṭār wa l-as'ār* ("On rains and prices"). M. Díaz-Fajardo has edited the astronomical part of the *Kitāb al-adwār*, an extremely interesting text that will be analysed in 7.5 (the introduction of Eastern *zīj*es) and in 6.2.6.6 (the problem of trepidation).³⁸ As for the astrological part, a large section of it has been edited in Díaz-Fajardo's PhD thesis, together with many passages from the *Kitāb*

36 Samsó, 2004; Samsó, 2009; Oliveras, 2012.

37 Herrera, 2001.

38 Díaz-Fajardo, 2001.

al-fuṣūl and Ibn Qunfudh's commentary on Ibn Abī l-Rijāl's *urjūza*.³⁹ All these texts deal with the techniques of *tasyīr* (prorrogation) and the projection of rays; they use the theory of the conjunctions of Saturn and Jupiter (as well as Saturn and Mars) and four kinds of cycles (*dawr al-āḥād*, *al-dawr al-akbar*, *al-awsaṭ* and *al-aṣghar*), and discuss the different techniques (on the ecliptic or on the equator) used for the computation of both the prorrogation and the projection of rays. All this shows the development of a Maghribī astrology with roots in the works of Eastern astrologers like Ibn Hibintā and al-Qabiṣī, but which attains a certain level of originality which harks back to Ibn Abī l-Rijāl.

Al-Baqqār's *Kitāb al-amṭār wa l-as'ār*⁴⁰ deals with an entirely different topic: it is a treatise on astrological meteorology entirely independent of the Eastern tradition represented by al-Kindī.⁴¹ Its starting point is Andalusian: the "method of the crosses" (*ṭarīqat aḥkām al-ṣulub*), which seems to have a late-Latin origin assimilated by astrologers of al-Andalus towards the end of the 8th century (see 3.2.1.1). This method is extremely simple and primitive and the same /is true of the *Lāmiyya* (a poem rhymed with the letter *lām* = *l*) by the eleventh century Andalusian astrologer Ibn al-Khayyāt which predicts meteorological changes from the passage of Saturn through the four triplicities and the twelve zodiacal signs. To these, al-Baqqār adds more sophisticated procedures for forecasting the weather and establishes a relation between rainfall and the evolution of prices, in which his point of reference is clearly that of farmers and merchants: prices fall when the malefic planets are in a position of power, but rise when the benefic elements are favoured.

1.4.4 Mīqāt

The characteristic Islamic astronomy applied to the problems of religious worship (*mīqāt*) is developed in sources of two kinds: those related to folk astronomy, and those which apply mathematical techniques. Among the former we should mention the *anwā'* book by Ibn al-Bannā' (1256–1321) in which the author follows Andalusian sources and makes no effort to adapt them to the coordinates of Fez or Marrakesh, the two cities in which his activity is well documented.⁴² This kind of literature often contains monthly determinations of the length of day and night, dawn and dusk, solar meridian altitudes, the entrance of the Sun in the zodiacal signs according to the different astronomical systems in use, and times for prayers using crude shadow schemes which

39 Díaz-Fajardo, 2008.

40 Chedli & Samsó, 2018.

41 Bos & Burnett, 2000; Burnett, 2003.

42 Forcada, 1992.

sometimes have a Classical origin.⁴³ Similar materials appear in other sources such as the writings of professional astronomers who try to explain simple approximate rules – very often valid only for one latitude – to solve the astronomical problems posed by worship. This kind of literature does not appear in al-Andalus but is quite common in the Maghrib⁴⁴ where Ibn al-Bannā' (1256–1321) wrote on the *qibla* and timekeeping,⁴⁵ and the Moroccan *muwaqqit* (timekeeper) at the Qarawiyyīn mosque in Fez, Abū Muqrī' or Miqrā' (first half of the fourteenth century) composed a mnemotechnical poem on the same subject which gave rise to many commentaries.⁴⁶ Another *muwaqqit* of the same mosque, Abū Zayd 'Abd al-Raḥmān b. Muḥammad al-Jādīrī (b. Meknès ca. 1375, d. Fez 1416) wrote several works on *mīqāt*, among them another very famous *urjūza* whose title is *Rawḍat al-azhār fī 'ilm waqt al-layl wa l-nahār* ("Garden of flowers on the science of timekeeping during night and day"), which was also the object of several commentaries, among them a very interesting one written by the astronomer Abū 'Abd Allāh Muḥammad b. Aḥmad b. Abī Yaḥyā ibn al-Ḥabbāk al-Tilimsānī (d. after 1514).⁴⁷ Interestingly, both al-Jādīrī and al-Ḥabbāk combine approximate methods derived from the folk astronomy tradition with others which are extremely precise and bear witness to the use of reliable astronomical sources.

The end of the thirteenth and beginning of the fourteenth century is the period in which the new profession of *muwaqqit* appeared in the Maghrib, al-Andalus and Egypt; it provided astronomers with alternative to astrology as a way of earning a living.⁴⁸ Timekeeping problems also attracted the interest of other professionals such as jurists (*fuqahā'*) who dealt with problems such as *qibla* (the orientation towards Mecca). In the twelfth century a Maghribī jurist called Abū 'Alī al-Mattījī strongly criticised the southern orientation of most of the mosques in the Maghrib and offered simple rules which, sometimes, were fairly accurate.⁴⁹

Besides folk astronomy, timekeeping based on mathematical methods often appeared in the form of tables. Sources of this kind are extremely rare in Naṣrid Granada: only Ibn al-Raqqām (d. 1315) included a lunar visibility table

43 King, 1990 and 2004, pp. 461–527; Forcada, 1994.

44 Khaṭṭābī, 1986.

45 Jabbār and Aballāgh, 2001, pp. 185–190; Khaṭṭābī, 1986, pp. 86–99.

46 Lamrabet, 2013, no. M141, pp. 181–2.

47 The traditional date we find in the standard bibliography is 867/1462–1463; see Lamrabet, 2013, no. 219, p. 207. Saidi, 2013 has established that 1514 appears repeatedly as the *annus praesens* of the book.

48 King, 2004, pp. 623–677.

49 Rius, 1996.

in his *Qawīm zīj*, based on a method which yields good results only for the latitude of Granada.⁵⁰ No Andalusian tables for the computation of prayer times are known to me but a fourteenth-century set of such tables for Tunis (latitude 37°), and a similar set calculated for the latitudes of Sijilmasa, Fez and Marrakesh are extant, as well as other sets of tables which are adaptations of well-known Eastern tables for timekeeping to the latitude of a Maghribī city.⁵¹ The most important collection of materials related to spherical astronomy and astronomical timekeeping is the *Kitāb al-mabādī' wa l-ghāyāt* written by Abū 'Alī al-Marrākushī in the late thirteenth century: this work, which deals extensively with astronomical instruments, shows that its author was very familiar with the Andalusian-Maghribī astronomical tradition but, as he emigrated to Egypt, he also used many Eastern astronomical sources and computed the majority of his tables for the latitude of Cairo.

A certain number of poorly made Andalusian horizontal sundials are extant, although none of them seem to correspond to the Naṣrid period.⁵² In the Maghrib only one instrument of this kind from this period has been studied: a sundial made by a certain Abū-l-Qāsim ibn Shaddād (probably a *muwaqqit*) in 1345–46. This instrument is as crude as its Andalusian predecessors and it was not intended to indicate the hours of daylight; the only markings present correspond to the times of prayers.⁵³ To the best of my knowledge, there does not seem to have been an Andalusian-Maghribī sundial tradition of any importance – somewhat surprisingly, in view of the demonstrable skill that craftsmen of Western Islam showed in the construction of astrolabes, and the existence of a number of manuscripts with tables for the construction of sundials for the latitudes of Córdoba, Fez, Marrakesh and Tunis. The maximum expression of Western gnomonics appears at the end of the thirteenth century in the *Risāla fī 'ilm al-ẓilāl* ("On the science of shadows") in which the Tunisian-Andalusian astronomer Ibn al-Raqqām (d. 1315) describes how to construct all kinds of sundials using the Hellenistic mathematical tool known as the *analemma*.⁵⁴

To end this survey of timekeeping I should add a brief summary of what we know about other kinds of time-measuring devices such as clocks. The Andalusian tradition of applied mechanics was probably introduced in the Maghrib in the twelfth century, when the influence of Eastern sources was

⁵⁰ Kennedy, 1997.

⁵¹ King, 1998, see p. 189.

⁵² Barceló & Labarta, 1988; King, 1992.

⁵³ King, 1977; Jarray, 2015.

⁵⁴ Carandell, 1988.

also present. In this way, while mechanical technology suffered a decline in al-Andalus in the fourteenth century (the candle clock built for Muḥammad v of Granada in 1362⁵⁵ bears no comparison with the devices described by Ibn Khalaf al-Murādī), it flourished in Fez, mainly as a result of the work of *mu-waqqits* who were active in that city in the thirteenth and fourteenth centuries and who were influenced by Eastern Islamic mechanics. The first known clepsydra, of an elementary type, was made for a room in the minaret of the Qarawiyyīn mosque by Abū ‘Abd Allāh Muḥammad al-Ḥabbāk al-Tilimsānī in 1271–72;⁵⁶ a new one was built, for the same place, by Abū ‘Abd Allāh Muḥammad al-Ṣinhājī in 1317; the final version (parts of which survive today) was made in 1346–48 by Abū ‘Abd Allāh Muḥammad al-‘Arabī and restored, in 1362, by Abū Zayd ‘Abd al-Raḥmān al-Lujā’ī al-Fāsī (d. 1371). In 1346 this latter instrument was fitted with an astrolabic dial and with a system of twenty-four doors corresponding to the twenty-four hours of the day: every hour, the clock would eject a metal ball through the correct door, thus marking the time with a noise. A similar technique was used in the clepsydra built in 1357 by Abū l-Ḥasan ‘Alī al-Tilimsānī in the *madrasa* Bū ‘Ināniyya in Fez: the instrument’s dial (still extant, and formed by a set of twelve doors each one of which would stay open for an hour) was placed in the street and passers-by could tell the time by noting which door was open.⁵⁷

1.5 Astronomy in the Christian Kingdoms of the Iberian Peninsula

1.5.1 *The Transmission of Arabic Astronomy into Latin*

1.5.1.1 Catalonia, Tenth Century

The process of transmission began in the Barcelona area towards the end of the tenth century. As a result of this we have a collection of texts of a clearly Arabic origin, dealing with the astrolabe and other astronomical instruments like the horary quadrant which Millàs called *quadrans vetustissimus*,⁵⁸ an instrument similar to al-Battānī’s *bayḍa*, and various types of sundials which seem to be based on Latin models. Most of the corpus deals with the construction and use of the astrolabe, and the corresponding texts seem to have clear connections with the treatises on the astrolabe written by members of

55 Samsó, 2011, pp. 443–444, 71–72.

56 Lamrabet, 2013, M105, p. 158.

57 Price, 1964.

58 Millàs, 1932.

the school of Maslama al-Majrīṭī (d. 1007),⁵⁹ although Paul Kunitzsch has established the existence of another source: one of the texts of the collection, the *Sententiae astrolabii*, is a partial translation of a treatise by al-Khwārizmī (fl. 830).⁶⁰ Seniofredus, also called Lupitus Barchinonensis, archdeacon of Barcelona Cathedral (975–995), seems to have been involved in this process of transmission.

I insist on the word “transmission” instead of “translation” because I believe that only a small part of the corpus is the result of the translations of a very limited collection of Arabic sources which included an Arabic translation of Ptolemy’s *Planisphaerium*. Most of the texts seem to be notes taken during or after an oral explanation in which drawings and actual instruments were used.⁶¹ Only Arabic astrolabes were available at an early stage, although we also have the famous “Carolingian astrolabe”, which seems to be the only case of an astrolabe “translated” from an Arabic original.⁶² A minimal knowledge of Arabic was necessary to read the inscriptions (mainly the *abjad* notation, star names, etc.) if one had to use an astrolabe. This is why the *Astrolabii sententiae* or *De utilitatibus astrolabii* contain Arabic terms (in some cases even a full sentence) which are in fact totally unnecessary because they are immediately followed by a Latin translation. A good example of the kind of instruments used can be found in the bilingual drawings of an Arabic astrolabe made by Khalaf ibn al-Mu‘ādh, extant in ms. Paris BnF lat. 7412 and studied by Kunitzsch.⁶³ Here all the inscriptions of the instrument are copied carefully in Kufic script and are perfectly legible in an eleventh century manuscript. An evolution of this kind of bilingual illustration can be seen in a photograph of the (no longer extant) twelfth century ms. Chartres 214 fol. 30r, where we find a clumsy attempt to reproduce the Arabic inscriptions on the instrument which here are barely legible; a third example, which is of even poorer quality, can be found in the ms. London, British Library Old Royal 5 B, fol. 71r. The date of the drawing is doubtful; the author has made an unsuccessful attempt to reproduce the Arabic inscriptions but he seems to have tired of the effort, leaving most of the spaces empty. It is obvious that the author of the drawings of the BnF manuscript knew Arabic, but this is not the case of the other two; the successive copies

59 Samsó, 2000.

60 Kunitzsch, 1987.

61 Samsó, 2004a, especially pp. 132–141; Borelli, 2008, pp. 118–129.

62 Destombes, 1962; see the Proceedings of the symposium on the Carolingian astrolabe held in Zaragoza in 1993 and published in *Physis* 22 (1995), 450 pp. My friend David King disagrees with me on my interpretation of the Carolingian astrolabe as an instrument “translated” from an Arabic original. He may be right, but I still stick to this hypothesis.

63 Kunitzsch, 1998.

of Arabic inscriptions by illustrators who did not know the Arabic alphabet became progressively more corrupt and illegible.

The reader of the old corpus will possibly be puzzled by the fact that there are very few astrological materials in it. The situation has become slightly clearer with David Juste's publication of the *Alchandreana*, a collection of Latin astrological texts that show clear signs of an Arabic influence.⁶⁴ This series of eight texts share common techniques of prediction: the data of the horoscope are calculated using numerological procedures (numerical values of the letters forming the name of the subject) and the prediction is based on isolated elements (the onomatomantic ascendent, the planetary hours, the position of the planets in the triplicities or in the lunar mansions, etc.). This kind of very simplified astrology is also found in two other works written in the Iberian Peninsula during the thirteenth century: the Alfonsine *Libro de las Cruces* and Raimundus Lullius' *Tractatus de nova astronomia*. Both books represent the same tendency towards simplification as the *Alchandreana* collection, although they have very little in common with it. As for the origin of the name *Alchandreana*, I can only offer a hypothesis: bearing in mind that one of the derivations of the corpus is the *Liber Arcandam*, very popular from the sixteenth century onwards, I believe that Alchandreus maybe a corruption of the *Arkand*, an Indian astronomical book known under this name in Arabic: the name, at least, circulated in al-Andalus towards the middle of the ninth century.

1.5.1.2 The Ebro Valley and Toledo (Twelfth–Thirteenth Centuries)

1.5.1.2.1 *The Sources Selected for Translation*

The early transmission of texts on astronomical instruments which took place in the tenth century did not mark the beginning of a continuous process of translations or of other ways of transmission, as it came to a halt in the eleventh century. Marie Thérèse d'Alverny is one of the many scholars intrigued by this interruption, as she says: "Why this promising prelude was not followed immediately by an increasing stream of translations during the eleventh century is a question still unsolved."⁶⁵

In the following pages, I will try to give a hypothetical answer to this question as well as to another problem posed by Dimitri Gutas, in relation to the translations of the twelfth century: "Everything – that is the entire Arabic corpus of writings until the twelfth century – was theoretically available to all who would have wanted to translate it. But this is certainly not what happened, ...". This author tries to attribute the selection of the translated texts to

64 Juste, 2007.

65 D'Alverny, 1982, p. 140.

the “andalusocentrism” which appears in Ibn Ḥazm’s *Risāla fī faḍl al-Andalus* and in Ṣāʿid’s *Ṭabaqāt al-Umam*. According to him, “the works actually selected were those that were appreciated and cultivated by the Arabic-writing Andalusians of the eleventh century.”⁶⁶

In my opinion, the two problems are interrelated if we accept the following two principles:

1. Only Eastern books which had reached al-Andalus could be the object of translations. Gutas agrees with this point: “The translations done in Spain in the twelfth century were done on the basis of Arabic manuscripts available in Spain at that time, and upon recommendation, apparently, of such local experts, all of whom, naturally must have shared the Andalusocentric bias we see in Ibn Ḥazm and Ṣāʿid.”⁶⁷
2. A translator needed two things to be able to undertake his work: a) access to libraries to consult the raw materials he needed for his translations; b) patronage, which allowed him to cover his basic needs.

My second principle explains the interruption of the process of transmission in the eleventh century: the work in Catalonia towards the end of the tenth century had focused on a very limited topic, the astrolabe, and, to a lesser extent, the horary quadrant and al-Battānī’s *bayḍa*. The bibliography used – as I have explained in 1.5.1.1 – was also a small number of Arabic sources and instruments. No large library was needed. Patronage was also unnecessary: Seniofredus/Lupitus was an archdeacon of the Cathedral of Barcelona, and one of the presumed authors of the Alchandreana collection was Miró Bonfill (d. 984), Bishop of Gerona since 971. The process could not continue during the eleventh century because the scholars did not have access to libraries; not a single large Muslim city had been conquered by the Christian kings and, consequently, there was no need for any patronage. The situation was transformed towards the end of the century when Alfonso VI of Castile conquered Toledo in 1085, and was transformed still further in 1110 with Alfonso I of Aragon’s conquest of Zaragoza: these two cities were important centres of research in the fields of astronomy and mathematics, and their occupation by the Christians marked the starting point of the translation process.

Once this point is established, we can proceed with the problem of the selection of Arabic sources to be translated. My impression is that this selection has a chronological basis and is the result of the fact that scientific works written in the Mashriq after 950 only very rarely reached al-Andalus; therefore, they were not available to the Latin translators of the twelfth century. Let us

66 Gutas, 2007. The two literal quotations appear in pp. 6 and 8. I owe my knowledge of this paper to the generosity of Cristina Álvarez Millán.

67 Gutas, 2007, p. 9.

begin by putting the authors translated by Gerard of Cremona into chronological order (I omit Greek authors translated into Arabic). I use the list of his translations established by his disciples (*socii*):⁶⁸

1. Gerard of Cremona

Eastern sources:

VIII-2: Māshā'allāh, Jābir b. Ḥayyān

IX-1: Banū Mūsā, al-Khwārizmī, al-Farghānī, al-Kindī, Ibn Māsawayh

IX-2: al-Rāzī, Thābit b. Qurra, Yaḥyā b. Sarāfyūn, al-Nayrizī

X-1: al-Fārābī, Aḥmad b. Yūsuf, Abū Kāmil

XI-1: Ibn Sīnā, Ibn al-Haytham (*Optics*)

Andalusian and Maghribī Sources:

X-1: Ishāq Isrā'īlī, al-Zahrāwī, 'Arīb b. Sa'īd

XI: *De motu octaue spere*, Ibn al-Zarqālluh, Ibn Mu'ādh, Ibn Wāfid

XII: Jābir b. Aflaḥ

With the exceptions of Ibn Sīnā and Ibn al-Haytham, the list of Eastern authors translated by Gerard of Cremona ends in around 950. Works translated after that date are Andalusian. We can reach the same conclusion if we analyse works translated into Hebrew during the thirteenth and fourteenth centuries by Jewish translators in Languedoc and Provence, most of whom came from the other side of the Pyrenees.⁶⁹ In this case, another exception from the first half of the eleventh century appears: 'Alī b. Riḍwān.

2. Hebrew translations (thirteenth–fourteenth century)

Eastern sources:

VIII-2 and IX: Māshā'allāh, Jābir b. Ḥayyān, Sahl b. Bishr, al-Kindī, Abū Ma'shar, al-Farghānī, Ḥunayn b. Ishāq, Thābit b. Qurra, Qusṭā b. Lūqā, al-Rāzī.

X-1: al-Fārābī

XI-1: 'Alī b. Riḍwān, Ibn al-Haytham, Ibn Sīnā

Andalusian sources:

X-1: al-Zahrāwī

X-2–XI-1: Ibn al-Ṣaffār, Ibn al-Samḥ

XI-2: Ibn al-Zarqālluh, Ibn Mu'ādh

XII: Jābir b. Aflaḥ, Maimonides, Ibn Rushd, al-Bitrūjī

68 See Charles Burnett's critical edition of the *Vita* and *Commemoratio librorum* in Burnett, 2001, pp. 273–287. Repr. Burnett, 2009, no. VII.

69 Romano, 1991.

The same results could be obtained with the authors translated into Castilian at the court of Alfonso X (see below 1.5.2), with the list of translations of Arabic medical sources established by Danielle Jacquart,⁷⁰ or the similar list of Arabic mathematical works given by Richard Lorch.⁷¹

It seems, then, that scientific works written in the Mashriq after ca. 950 only very rarely reached al-Andalus and were available for translation into Latin in the twelfth century. This is confirmed by the analysis of the sources accessible to the Toledan astronomer and patron Ṣāʿid al-Andalusī (1029–1070) and mentioned in his *Ṭabaqāt al-Umam*. An analysis of this kind was made years ago by Richter-Bernburg,⁷² who remarked that, in the fields of medicine and astronomy, Ṣāʿid's information on the Mashriq fell off rapidly at the end of the tenth century and that the two Mashriqī scholars close to his own time and mentioned in the *Ṭabaqāt* are Ibn Yūnus (d. 1009) and Ibn al-Haytham (965–1041).⁷³

1.5.1.2.2 *The Libraries*

1.5.1.2.2.1 The Royal Library in Córdoba

We should now turn to the problem of the libraries in al-Andalus, beginning with the royal library in Córdoba.⁷⁴ The origins of this library should be dated around 800. The publication, some years ago, of volume 11-1 of Ibn Ḥayyān's *Muqtabis* has shed light on the books brought to Córdoba from Baghdad by the astrologer ʿAbbās b. Nāṣiḥ (fl. ca. 800–850) which includes a list of presumably astronomical tables called *al-Zīj*, *al-Qānūn*, *al-Sindhind*, and *al-Arkand*.⁷⁵ There seems to be no doubt about the identification of the *Sindhind* (probably al-Khwārizmī's *Zīj*, whose use in al-Andalus is well documented in the tenth century); *al-Qānūn* refers probably to the *Handy Tables*, and *al-Arkand* is another set of Indian astronomical tables introduced in Baghdad in the eighth century. The reference to *al-Zīj* seems too vague to allow an identification.

Although there is little information on the fate of the library in the ninth century, it seems clear that it existed and was accessible to scholars who frequented the royal court. One of them, the poet and astrologer ʿAbbās b. Firnās (d. 887) borrowed al-Khalil's *Kitāb al-ʿArūḍ* from the library, and became well known for being able to understand this difficult book on Arabic metrics.⁷⁶

70 Jacquart, 1996, pp. 981–984.

71 Lorch, 2001, pp. 317–318.

72 Richter-Bernburg, 1987.

73 Richter-Bernburg, 1987, p. 379.

74 Ribera, 1928; Wasserstein, 1990–91; Maribel Fierro provided me with a digitalised copy of this second paper. See also M. F. al-Wasif, “Al-Mustanṣir al-Ḥakam” in *BA* 6, pp. 590–598.

75 Makki, 2003, pp. 278, 525–527; Makki & Corriente, 2001, pp. 169–170. Forcada, 2004–5, pp. 20–22.

76 Terés, 1960.

Ibn Ḥayyān also reproduced a poem dedicated to the emir Muḥammad (852–886) by ‘Abbās b. Firnās in which the latter complained that Ibn al-Shamir (or Shimr), another court astrologer, had had a book described as *al-daftar al-muḥkam* out on loan for a very long time, and ‘Abbās wanted to use it.⁷⁷ This book probably contained another set of astronomical tables.⁷⁸

The library reached its summit under the caliphate of al-Ḥakam II al-Mustaṣṣir (961–976). Al-Ḥakam became caliph when he was 47 years old and, before that age, he had spent his life collecting books, bought in the Eastern cities by his correspondents, which were assembled in his private residence (*Dār al-Mulk*). In 951, his father ‘Abd al-Raḥmān III (912–961) ordered the execution of his son, al-Ḥakam’s brother, ‘Abd Allāh, and al-Ḥakam inherited the latter’s private library. When al-Ḥakam acceded to the throne, his library was assembled with the royal library and the sources state that the process of transferring the books took six months. We do not know for sure where the library was housed: it may have been the royal palace (al-Qaṣr), in front of the great mosque, or the royal residence, Madīnat al-Zahrā’, outside the city. Whatever the case, the library must have been enormous: historical sources say that it contained 400,000 volumes, even though this number is clearly exaggerated (and in fact is also the number attributed to the Library of Alexandria and to the library gathered by Abū Ja‘far ibn ‘Abbās, vizier of Zuhayr, king of the *ta’ifa* of Almería in the eleventh century).⁷⁹ In any case it seems that the catalogue, which contained only the titles of the books, was registered in 44 volumes, each one of them with 20 or 50 folios according to different sources. We do not know the exact contents of the library but a study made in 1994 of the amount of books circulating in Córdoba around 975 concluded that the sum total was 897, among which 44 dealt with medicine, 32 with astronomy, astrology and mathematics, eight with philosophy and five with alchemy and agronomy.⁸⁰

At an unknown date between 981 and 989 the *ḥājib* al-Manṣūr b. Abī ‘Āmir (governor between 981 and 1002), ordered a selective burning of Al-Ḥakam’s library in order to please the orthodox *fuqahā’*. The enmity of the *fuqahā’* was directed mainly towards the Sciences of the Ancients (*‘ulūm al-awā’il*) although disciplines such as arithmetic, medicine, and *mīqāt* were excluded.

During the siege of Córdoba by Berber troops in 1010, part of the library was auctioned off and, in this way, its books reached Toledo and other capitals of the *tā’ifa* period. The rest of the library was destroyed by the Berbers.

77 Makki, 1973, pp. 281–282.

78 Vernet, 1977. Repr. in Vernet, 1979, pp. 233–234.

79 See above (1.2) on the 300,000 volumes of the library of Tāhart.

80 See Maribel Fierro in *al-Qanṭara* 19 (1998), p. 490.

Today, a single book from al-Ḥakam's library survives; it was identified by E. Lévi-Provençal and is now preserved in the library of the Qarawiyyīn mosque-university in Fez. The explicit states "copied by Ḥusayn ibn Yūsuf, servant of the *imām* al-Mustanṣir bi-llāh" and the date is "month of Sha'bān of year 359 H." / 9th June–7th July 970.⁸¹

1.5.1.2.2.2 The Library of King al-Mu'taman in Zaragoza

Al-Ḥakam's library is the last case in al-Andalus of a general library which received a large part of the important books published in the Mashriq. During the *ṭa'ifa* period (1031–1086) none of the rulers of the small kingdoms which emerged from the fall of the caliphate had the interest or the financial capacity to continue with this policy. In spite of this, there is no doubt that there were smaller, more specialised libraries, although they rarely contained new stocks from the East. The library of king Yūsuf ibn Hūd al-Mu'taman (r. 1081–1085) of Zaragoza is especially significant for our purpose, because he was a highly competent mathematician and the author of the *Istikmāl*, a great treatise on geometry and number theory which reached a large readership in the Islamic world from Morocco to Marāgha in Iran. It is also possible that al-Mu'taman's father al-Muqtadir (1046–1081), who was also interested in mathematics, began to collect the books of the library. The analysis by J.P. Hogendijk⁸² has produced a list of the mathematical sources quoted in the *Istikmāl*. They include Arabic translations of Greek classics: Euclid's *Elements*, *Data* and *Porisms*, Ptolemy's *Almagest*, Apollonius' *Conics* and *Plane loci*, Archimedes' *Sphere and Cylinder*, *Measurement of the circle*, and the commentaries by Eutocius, as well as Theodosius' and Menelaus' *Spherics*. Among the Arabic works we find *Measurement of plane and spherical surfaces* by the Banū Mūsā (fl. ca. 830), *On the sector figure* and *On amicable numbers* by Thābit b. Qurra (d. 901), *On the quadrature of the parabola* by Ibrāhīm ibn Sinān (d. 946) and, finally, *Analysis and synthesis*, *Optics* and *On known data* by Ibn al-Haytham (d. ca. 1040) who, once more, is an exception.

What we do not find in al-Mu'taman's *Istikmāl* are the works of the great Eastern mathematicians who, besides Ibn al-Haytham, were active between ca. 950 and ca. 1050: Abū Ja'far al-Khāzin (d. ca. 965), Abū l-Wafā' al-Būzjānī (940–997), Abū Sahl al-Kūhī (fl. ca. 988), Abū Maḥmūd al-Khujandī (d. ca. 1000), Abū Naṣr Maṣṣūr ibn 'Irāq (d. before 1036) and al-Bīrūnī (973–1048).

81 The book in question is the *Mukhtaṣar* of Abū Muṣ'ab (d. 856–7), a compendium of the legal doctrines of Mālik b. Anas: see Schacht, 1965. A good photograph of the explicit page in Dodds, 1992, p. 177.

82 Hogendijk, 1986.

This agrees with the hypothesis I presented in 1.5.1.2.1: namely, that the new sources published in the East reached al-Andalus only very rarely. There are, of course, authors who must have had privileged information: one of them is Ibn Mu'adh al-Jayyānī (d. 1093) whose treatise on spherical trigonometry (*Kitāb majhūlāt qisī al-kura*) was the first Western book to deal with the new trigonometrical theorems (sine law, cosine law, tangent law, rule of four, Geber's theorem ...) discovered precisely by some of the Eastern mathematicians and astronomers mentioned above. Ibn Mu'adh's book was never translated into Latin, but the new trigonometry was reintroduced in the *Iṣlāḥ al-Majisī* of Jābir b. Aflaḥ (twelfth c.) which was translated into Latin and Hebrew.

In 1110 the Almoravid emir conquered Zaragoza and the king 'Imād al-Dawla (1110–1130), grandson of al-Mu'taman, established himself in the fortress of *Rūṭa* (Rueda del Jalón) where he resisted even after the conquest of Zaragoza in 1118 by Alfonso I, King of Aragon. It is logical to imagine that al-Mu'taman's library was, during all this period, in Rueda. The problem is to establish what happened to the library when King al-Mustanṣir (1130–1146) exchanged Rueda for lands near Toledo, as a result of an agreement in 1140 with Alfonso VII of Castile. It is quite possible, as Burnett suggests, that the library, or what was left of it, ended up in Toledo.⁸³

1.5.1.2.2.3 The Use of al-Mu'taman's Library by the Translators of the Ebro Valley
Al-Mu'taman's library was, no doubt, accessible to Bishop Michael (1119–1151) of Tarazona, a town near Rueda, who patronized the translations of Hugo Sanctelliensis. Hugo translated for him Ibn al-Muthannā's commentary of the astronomical tables of al-Khwārizmī, a book which was also translated to Hebrew by Abraham b. 'Ezra (ca. 1092–ca. 1167).⁸⁴ Hugo's dedication contains the following significant passage:

Quia ergo, mi domine Tyrassonensis antistes, ego Sanctalliensis, tue petitioni ex me ipso satisfacere non possum huius commenti translationem, quod ... in Rotensi armario et inter secretiora bibliotece penetralia tua insaciabilis filosofhandi aviditas meruit repperiri, tue dignitati offerre presumo.⁸⁵

83 Burnett, 2001, p. 251.

84 Goldstein, 1967.

85 Millás Vendrell, 1963, pp. 95–96.

Which we can translate as:

My lord Bishop of Tarazona, as I, Sanctalliensis, cannot satisfy by myself your demand, I offer to your dignity the translation of this commentary ... which your insatiable philosophical avidity deserved to find in a cupboard in Rota [= Rueda] and in the most secret part of the library.

This translation is not dated, but one might wonder whether Bishop Michael had reached some kind of agreement with ʿImād al-Dawla or with his son al-Mustanşir. Had he bought all or part of the library? Was the library really transferred to Toledo?

In any case, it seems that the library was the source of the books translated not only by Hugo Sanctelliensis, but also by Hermann of Carinthia (fl. 1138–1143) and Robert of Ketton (fl. 1141–1157) who were, at that time, working in the nearby town of Tudela.⁸⁶ I even think that the library might have been used by Petrus Alfonsi of Huesca (ca. 1062–after 1110), who collaborated with Adelard of Bath (fl. 1100–1150) and provided him with Andalusian manuscripts, among them al-Khwārizmī's *zīj*, in the revision by Maslama al-Majrīṭī. I would also suggest that the library was used by Abraham b. ʿEzra (fl. 1140–1160, born in Tudela), Abraham bar Ḥiyya (fl. Barcelona, 1133–1145) and by his collaborator Plato of Tivoli, some of whose translations are carefully dated between 1133 and 1145. All these authors seem to have had a common interest in mathematics, astronomy, astrology and other forms of divination.

1.5.1.2.2.4 The Libraries of Toledo

We have no information about the Arabic libraries of Toledo but it is obvious that these libraries existed given the high level of scientific activity, especially in the field of astronomy, attained in the eleventh century by Toledan scholars. There were, no doubt, important private libraries in the city, as we know from Marín's study on the Toledan *ʿulamāʾ*, most of whom were dedicated to religious sciences, although some of them were also interested in the Sciences of the Ancients.⁸⁷ Some of these libraries were still extant at the beginning of the thirteenth century, since Mark of Toledo mentions the *armaria arabum*.⁸⁸

86 Burnett, 2001, p. 251: "whose library had been used by the translators of the Valley of the Ebro".

87 Marín, 2000.

88 González, 1997, p. 58.

To these one should add the possible existence in Toledo of parts of al-Ḥakam II's library, sold in Córdoba in 1010: in support of this idea we have Ṣā'id al-Andalusī's report that he had seen a book with notes written in al-Ḥakam's own hand, in which he established that al-Ḥasan ibn Aḥmad ibn Ya'qūb al-Hamdānī, one of the sources used by Ṣā'id, had died in prison in Ṣan'ā' in 334/946.⁸⁹ It is also possible that what was left of al-Mu'taman's library also reached Toledo: Charles Burnett argues that "the texts on geometry that Gerard of Cremona chose to translate correspond to those used by one of the kings of the dynasty in the late eleventh century",⁹⁰ and this is true, since Gerard translated Euclid's *Elements* and *Data*, Ptolemy's *Almagest*, Archimedes' *On the Measurement of the Circle*, Theodosius' and Menelaus' treatises *On the Sphere*, a treatise *On Geometry* by the Banū Mūsā and Thābit ibn Qurra's *On the Sector-Figure*. Furthermore, the manuscripts of Theodosius' and Menelaus' *Spherics* used by al-Mu'taman belong to the same family as those translated by Gerard of Cremona.⁹¹

Reading Ṣā'id's *Ṭabaqāt* can be a good guide to establish the astronomical sources available in Toledo towards the middle of the eleventh century. Nevertheless some caution is necessary because, in some cases, Ṣā'id seems to give information gathered from other informants.⁹² This is probably the case of his references to Ibn al-Haytham and also to Ibn Yūnus.⁹³ In some instances, however, there is clear evidence that he had direct access to the sources because he gives details of their contents or because these details are given in works written by other members of the Toledan group – for example, some of Aristotle's works which he analyses in detail.⁹⁴ He also knew Ptolemy's works of which he mentions the *Almagest*, the *Geography*, the *Optics*, the *Tetrabiblos* and the *Kitāb al-Qānūn*;⁹⁵ of these, there is no doubt that he had direct knowledge of, at least, the *Almagest*. He also refers (probably through secondary sources) to Theon of Alexandria's version of the *Qānūn* as well as a *Kitāb al-aflāk* in which, according to Ṣā'id, Theon describes the structure (*hay'ā*) of the celestial spheres and the planetary models, explained in a simple way, without Ptolemy's geometrical proofs: he is probably referring to Theon's *Commentary of the Almagest*. In the *Qānūn* (he must be referring to Theon's commentary on the *Handy Tables*) Ṣā'id finds a description of the trepidation model, according

89 Bū 'Alwān, 1980, p. 149; Blachère, 1935, p. 116.

90 Burnett, 2001, p. 251.

91 Lorch, 1996; Hogendijk, 1996.

92 A thorough analysis of Ṣā'id's sources in Richter-Bernburg, 1987, pp. 377–385.

93 Bū 'Alwān, 1980, pp. 149–150; Blachère, 1935, p. 116.

94 Bū 'Alwān, 1980 pp. 76–82; Blachère, 1935, pp. 62–69.

95 Bū 'Alwān, 1980, pp. 88–91; Blachère, 1935, pp. 72–73.

to the *aṣḥāb al-ṭillasmāt*.⁹⁶ It is also clear that Ṣāʿid had access to the *Kitāb fī l-ḥarakāt al-samāwīyya* by al-Farghānī (d. 861)⁹⁷ and to al-Battānī's *zīj*:⁹⁸ he considers both works to be useful summaries of the *Almagest*.

Ṣāʿid also knows al-Khwārizmī's *zīj*,⁹⁹ its adaptation to the Islamic calendar and to the geographical coordinates of Córdoba by Maslama al-Majrīṭī and the versions prepared by Maslama's disciples.¹⁰⁰ His reference to al-Ḥasan b. al-Ṣabbāḥ's *zīj* (9th century) is interesting because he states that the mean motions followed the *Sindhind* system, which means that they were sidereal; for his part, he used Ptolemaic equations and the table of the solar declination according to recent observations.¹⁰¹ This description is in keeping with the main characteristics of Andalusī-Maghribī *zīj*es from the *Toledan Tables* to Ibn al-Raqqām (d. 1320).

Within the Khwārizmian tradition, an important source that Ṣāʿid undoubtedly knew well is Ibn al-Ādamī's *Naẓm al-ʿiqd*, a *zīj*, left unfinished by its author and completed by his disciple al-Qāsim b. Muḥammad ibn Hāshim al-Madāʾinī, known as al-ʿAlawī, who published the work in 949. It is in this book that Ṣāʿid found information on the introduction of the *Sindhind* in Baghdad under the caliphate of al-Manṣūr (754–775) and the version of this *zīj* prepared by al-Fazārī. In the *Naẓm* Ṣāʿid found the first accurate description of trepidation theory which allowed him to understand the problem, study it, and reach unprecedented solutions, the results of which he described in his non-extant *Iṣlāḥ ḥarakāt al-nujūm*.¹⁰²

Two more sources that were clearly available to Ṣāʿid and to his collaborators are two books by al-Ḥasan ibn Aḥmad ibn Yaʿqūb al-Hamdānī (d. 946), *Kitāb fī sarāʾir al-ḥikma* and *Kitāb al-iklīl*. The *Saraʾir al-ḥikma* contained a systematic treatment of astronomy and astrology and it was quoted by al-Istijī,¹⁰³ one of Ṣāʿid's disciples. The *Kitāb al-iklīl* was a historical book on the genealogies of the Ḥimyār which also contained astronomical and astrological materials.¹⁰⁴

In spite of the information that Ṣāʿid gives on the observations made ca. 830, under the caliphate of al-Maʾmūn (813–833) and the *zīj*es based on these

96 Bū ʿAlwān, 1980, p. 109; Blachère, 1935, p. 86.

97 Bū ʿAlwān, 1980, p. 141; Blachère, 1935, p. 110.

98 Bū ʿAlwān, 1980, p. 91; Blachère, 1935 p. 73. Another reference to al-Battānī's *zīj* as well as to his commentary on the *Tetrabiblos* in Bū ʿAlwān, 1980, pp. 142–143; Blachère, 1935, pp. 111–112.

99 Bū ʿAlwān, 1980, p. 132; Blachère, 1935, pp. 102–103.

100 Bū ʿAlwān, 1980, pp. 168–177; Blachère, 1935, pp. 129–136.

101 Bū ʿAlwān, 1980, pp. 143–144; Blachère, 1935, p. 112.

102 Bū ʿAlwān, 1980, pp. 130–132, 146–147; Blachère, 1935, pp. 102, 114.

103 Samsó & Berrani, 2005, pp. 194–195.

104 Bū ʿAlwān, 1980 pp. 66, 113, 147–149; Blachère, 1935, pp. 53, 89–90, 114–116.

observations by Yahyā b. Abī Manṣūr, Khālīd b. ‘Abd al-Malik al-Marwazī, Sanad b. ‘Alī and ‘Abbās b. Sa‘īd al-Jawharī (which, according to Ṣā‘īd, were available to all scholars of his time)¹⁰⁵ the presence of these sources in Toledo during Ṣā‘īd’s life is most doubtful.¹⁰⁶ The same could be said of the three *zīj*es authored by Ḥabash al-Ḥāsib, although it seems that Ḥabash’s treatise on the use of the astrolabe¹⁰⁷ had reached al-Andalus in the eleventh century, given that Ibn al-Samḥ (d. 1035) used it.¹⁰⁸ Some evidence of the Ma’mūnī solar observations was, however, accessible to Ṣā‘īd in the form of the book on the solar year, attributed to Thābit ibn Qurra (836–901), which our author,¹⁰⁹ like Ibn al-Zarqālluh,¹¹⁰ had clearly read: both authors ascribed observations to Thābit which were, in fact, made by al-Ma’mūn’s astronomers in 830–831.

In the field of astrology, besides Ptolemy’s *Tetrabiblos*, Ṣā‘īd had access to several works by Abū Ma’shar (d. 886): he gives a long list of his books¹¹¹ but only adds some details about his *al-Zīj al-Kabīr* and the small *zīj* entitled *Zīj al-qirānāt*. He must have been able to read the great introduction to astrology (*al-Madkhal al-Kabīr*), which was quoted by his disciple al-Istijī,¹¹² who also referred to the *Kitāb al-milal wa l-duwal*,¹¹³ possibly the source used by Ṣā‘īd for his description of the Persian cycles that govern the history of the world.¹¹⁴ Ṣā‘īd also had access to the *Kitāb al-mudhākarāt*¹¹⁵ and to the *Kitāb al-Ulūf*,¹¹⁶ from which he gathered his information about the Hermetic legend. To end this list, I will only add al-Ḥusayn/al-Ḥasan b. al-Khaṣīb (fl. 844), author of a *Kitāb fī l-mawālīd*,¹¹⁷ quoted also by al-Istijī.¹¹⁸

105 Bū ‘Alwān, 1980, pp. 132–133; Blachère, 1935, pp. 103–104.

106 However, Chabás & Goldstein, 1994 (pp. 2 and 32) have found evidence of the influence of Ibn Abī Manṣūr’s *zīj* in the work by Ibn al-Zarqālluh’s disciple Ibn al-Kammād (fl. 1116). The allusions to the collection of Ma’mūnī *zīj*es in Abraham ibn ‘Ezra’s *De rationibus tabularum* are probably less significant, as Ibn ‘Ezra had access to Eastern sources which never reached the Iberian Peninsula: see Samsó, 2012.

107 Bū ‘Alwān, 1980, pp. 140–141; Blachère, 1935, pp. 109–110.

108 Viladrich, 1986, pp. 70–77.

109 Bū ‘Alwān, 1980, pp. 103–104; Blachère, 1935, pp. 81–82.

110 Samsó, 1994b, p. 8.

111 Bū ‘Alwān, 1980, pp. 144–145; Blachère, 1935, pp. 112–113.

112 Samsó & Berrani, 2005, pp. 213–214.

113 Samsó & Berrani, 2005, pp. 187–188.

114 Bū ‘Alwān, 1980, pp. 62–63; Blachère, 1935, pp. 50–51.

115 Bū ‘Alwān, 1980, pp. 102, 142; Blachère, 1935, pp. 81, 111.

116 Bū ‘Alwān, 1980, pp. 68–69; Blachère, 1935, p. 55. On the Hermetic legend see also Bū ‘Alwān, 1980, pp. 106–108; Blachère, 1935, pp. 84–86.

117 Bū ‘Alwān, 1980, pp. 145–146; Blachère, 1935, p. 113. The full title is *al-Kitāb al-Muqni‘ fī l-mawālīd* and it is extant, at least, in two Escorial manuscripts (940 and 978).

118 Samsó & Berrani, 2005, pp. 193–195.

1.5.1.2.2.5 Conclusions

The limited list of astronomical sources available to an astronomer like Şā'id is clearly significant because it shows, once more, that the arrival of Eastern books in al-Andalus was interrupted towards the middle of the tenth century, and because it indicates to us the kind of Eastern sources used by Andalusian astronomers in the following centuries. Besides, from the eleventh century onwards, Andalusian scholars seemed to believe that a student did not need to complete his education by travelling to the great capitals of the East, and that the cultural level of al-Andalus was equivalent to that of Baghdad, Damascus or Cairo. A statistical survey based on the recently published *Enciclopedia de al-Ándalus*¹¹⁹ shows a major reduction in the number of “journeys in search of knowledge” (*riḥla fī ṭalab al-‘ilm*) to the East undertaken by Andalusian scholars between the eleventh and early thirteenth centuries and, as I have shown in 1.4.1, an increase in their travels to the Maghrib. Indeed, first-rate scholars such as Ibn Ḥazm, Ibn al-Zarqālluh, Ibn Rushd or Ibn Zuhr do not seem to have travelled eastwards.

The lack of contact with Eastern culture and science affected not only the world of translators, but the history of Andalusian science as a whole. The golden half-century of the *ṭawā'if* (ca. 1035–1085) saw a splendid flourishing of science in al-Andalus (especially in the fields of astronomy, mathematics and agronomy). From this period onwards, Andalusian science would develop on the basis of its own resources. On the one hand, this produced a certain originality, but, on the other, it led to a steady decline from the twelfth century onwards, due among other things to the almost total lack of contact with Eastern Islamic science, which continued to be creative until the fifteenth century at least.

	Number of biographies	Travellers to the East	%	Travellers to the Maghrib	%
Emirate & Caliphate (8th–10th c.)	456	101	22.2%	12	2.6%
<i>Ṭawā'if</i> (11th c.)	428	58	13.6%	24	5.6%
Almoravids & Almohads (1085–1232)	995	126	12.7%	249	25%

119 Lirola, 2004–12.

1.5.1.2.3 *Patronage*

Until the middle of the thirteenth century, the Church seems to have been responsible for the patronage of translations, at which point this task was taken on by King Alfonso X of Castile (r. 1252–1284).¹²⁰ At the beginning of the process (in Catalonia, end of the tenth century) we find that the only figures related to the old corpus of Latin texts on astronomical instruments and to the *Alchandreana* collection were Seniofredus, the archdeacon of the Cathedral of Barcelona and Miró Bonfill, Bishop of Gerona (971–984), both mentioned above. When the translation period really began in the Ebro Valley (1119–1157), Michael Bishop of Tarazona (1119–1151) acted as the patron of Hugo Sanctelliensis, as is clearly shown by the latter's dedications to his sponsor.¹²¹ Although it is clear that there was a relation between Hugo Sanctelliensis and Hermann of Carinthia and Robert of Ketton, the translators working in nearby Tudela, there is no evidence that Bishop Michael also sponsored their translations and we have no information about how they supported themselves until the period 1141–1143, when both Hermann and Robert worked for Peter the Venerable, Abbot of Cluny, on the translation of the *Qur'ān* and other religious texts. Robert's collaboration was probably more important than Hermann's, and his reward was the archdeaconship of Pamplona (1143–1157).¹²²

In Toledo, some archbishops were interested in translations, such as Raymond of La Sauvetat (1125–1152)¹²³ and his successor John (1152–1166).¹²⁴ This interest continued during the thirteenth century: Sancho of Aragón (1266–1275) had eleven translations from Arabic in his library¹²⁵ and Gonzalo Pétrez (= Gonzalo García Gudiel) (1280–1299) kept a *scriptorium* where books were copied; he ordered new translations by Juan González of Burgos and the Jew Solomon, and two inventories of his possessions, dated 1273 and 1280, contain some thirty books translated from the Arabic, including autograph copies handwritten by Michael Scott and Hermann the German.¹²⁶

More important than this is to realise that the majority of the important Toledan translators occupied positions related to the cathedral of Toledo. Dominicus Gundissalinus was archdeacon of Cuéllar, which was dependent

120 There are precedents to this royal patronage: Johannes Hispalensis dedicates his translation of the pseudo-Aristotelic *Secret of Secrets* to Queen Teresa of Portugal (1112–1128). See Burnett, 1995.

121 Haskins, 1924, pp. 67–81.

122 Burnett, 1977.

123 Burnett, 2001, p. 250.

124 Burnett, 2001, pp. 251–252.

125 González, 1997, pp. 272–274, 280–293.

126 González, 1997, pp. 426–444, 467–512.

on the Toledan see, and his name appears in documents of the cathedral until 1181.¹²⁷ A Mozarab (d. 1215), who could be identified with Johannes Hispanus, was dean of Toledo and archdeacon of Cuéllar after Gundissalinus.¹²⁸ Gerard of Cremona (mentioned in cathedral documents dated in 1157, 1174 and 1176), Mark of Toledo, Michael Scott¹²⁹ and Hermann the German (d. 1272) were canons of Toledo. The latter appears as canon of the cathedral in 1263, and became bishop of Astorga between 1266 and 1272.¹³⁰ There seems to be no doubt that this was how the archbishops of Toledo exercised their patronage of translations.

1.5.2 *The Kingdom of Castile in the Thirteenth Century: the Alfonsine Astronomical Production*

Alfonso X of Castile (r. 1252–1284) represents the first clear instance of the royal patronage of a serious programme of transmission of Arabic astronomical materials into a language which is not Latin, but Castilian. This preceded the appearance, in French, of the *Pratique de astralabe* by Pèlerin de Prusse (1362)¹³¹ by almost a century, and Geoffrey Chaucer's treatise on the same subject for the education of his son Lewis (1391) by even longer.¹³² The king's collaborators translated many new sources which had not been previously rendered into Latin¹³³ and which had probably been found in the libraries of Cordoba and Seville, conquered in 1236 and 1248 respectively by Fernando III (r. 1217–1252), Alfonso's father. On the other hand, the pattern described in 1.5.1.2.1 for the chronology of the sources translated is still the same: the Eastern sources dated after the end of the tenth century are limited to Ibn al-Haytham – who also appeared in the previous lists – and the physician-astrologer 'Alī b. Riḍwān:

Eastern sources

IX-2: al-Battānī, Qusṭā b. Lūqā

X-2: al-Ṣūfī

XI-1: Ibn al-Haytham (*Cosmology*), 'Alī b. Riḍwān

127 Burnett, 2001, p. 264.

128 Burnett, 2001, p. 252.

129 Burnett, 2001, pp. 252–253.

130 González, 1997, pp. 588–600.

131 Laird & Fischer, 1994.

132 Skeat, 1872.

133 The exceptions are al-Battānī's canons, Ptolemy's *Tetrabiblos* and Ibn al-Haytham's *Cosmology*.

Andalusian and Maghribi sources

X-1: *Picatrix*

X-2: Maslama, Ibn al-Samḥ

X1-1: Ibn Abī l-Rijāl, ‘Abd Allāh al-Ṭulayṭulī (*Libro de las Cruces*)

X1-2: Ibn al-Zarqālluh, ‘Alī b. Khalaf, Ibn Wāfid, Ibn Baṣṣāl

It is important to stress that the Alfonsine corpus reflects the king’s interest in both astrology and magic and it is structured in two great miscellaneous collections.¹³⁴ The first of them is astronomical and astrological, and we find in it the famous *Libro del Saber de Astrología*.¹³⁵ This collection contains:

1. The four *Libros de la Ochava Espera* or *Libro de las Estrellas Fijas* (“Books of the Eighth Sphere” or “Book of the Fixed Stars”), an adaptation of the Uranography of ‘Abd al-Raḥmān al-Ṣūfī (903–998) containing a detailed description of the 48 Ptolemaic constellations (46 in the Alfonsine treatise) and the 1022 stars of the star-catalogue of the *Almagest*. Together with the *Almagest* material we also find specific information on the astrological characteristics of each star.¹³⁶
2. A series of treatises on the construction and use of astronomical instruments, which are analogue computers (armillary spheres, celestial spheres, spherical and plane astrolabes and universal astrolabes – *azafea* and *lámina universal*). Only the armillary sphere can be used for observations, while the others are extremely useful to solve (without any kind of computation) problems of spherical astronomy, among which we find the division of the ecliptic into the twelve astrological houses. The so-called *cuadrante sennero*, not included in the *Libro del Saber*, has similar applications.¹³⁷ These instruments are, therefore, very useful tools for the practising astrologer. We should recall here that when the Alfonsine collaborators were able to find an Arabic source on the construction and use of a particular instrument, this source was then translated into Castilian;

¹³⁴ I am adapting here the classification proposed by Procter, 1951, p. 5, who suggests three collections: astronomical, astrological and magical. It seems artificial to me to separate astronomy and astrology in Alfonso’s works; the *Lapidario*, which Procter considered to belong to the astrological collection, seems to fit the magical one better.

¹³⁵ The whole collection was edited uncritically by Rico, 1863–67, who introduced a title (*Libros del saber de astronomía*) that does not appear in any of the manuscripts. A facsimile edition of the royal codex (ms. Villa Amil 156 preserved in the Universidad Complutense de Madrid) has been published by Ebrisa, Planeta de Agostini, Barcelona, 1999.

¹³⁶ On the number of stars in Ptolemy’s catalogue see Kunitzsch, 2002. See also the exhaustive study by Comes, 1990 and Samsó & Comes, 1988.

¹³⁷ Millás, 1956.

- otherwise, one of the collaborators (usually Rabbi Ishāq b. Šīd) wrote an original treatise on the topic.
3. A third group of texts, included in the collection, are related to the determination of the hour which, together with the local latitude, are essential data for casting a horoscope. Instruments of this kind are the horary quadrant, usually called *quadrans vetus*, and the collection of Alfonsine clocks which includes two types of sundials, a clepsydra, the mercury clock and a candle clock.
 4. The collection ends with two treatises on the construction and use of the equatorium, an astronomical instrument which contains a series of Ptolemaic planetary models, drawn to scale, which are used to obtain (with minimum computation) the planetary longitudes for a given date and hour, which are essential when casting a horoscope.¹³⁸ This is also the purpose of other Alfonsine astronomical works which are not included in the *Libro del Saber*: I am referring to the translation of al-Battānī's *zīj*,¹³⁹ that of Ibn al-Zarqālluh's *Almanach* and, of course, the famous *Alfonsine Tables*, which are quite unusual in the context of Andalusian astronomy because they compute tropical longitudes.¹⁴⁰

With the *Libro del Saber* and the series of astronomical tables we have all the instruments necessary for casting a horoscope but, in order to interpret it, we need information on astrological theory. This is the purpose of *El Libro de las Cruces*, an astrological handbook based, probably, on a late Latin source but which was used in al-Andalus at least until the eleventh century. A more elaborate astrological theory can be found in the translation of the *Cuadripartito* (*Tetrabiblos*) and in Aly Aben Ragel's (= 'Alī ibn Abī l-Rijāl) *Libro conplido en los iudizios de las estrellas* (*al-Kitāb al-Bārī' fī aḥkām al-nujūm*)¹⁴¹ a source which, curiously enough, was unknown to Šā'id of Toledo.

Besides the books just mentioned we have another Alfonsine translation (extant only in a Latin version) which is purely astronomical: Ibn al-Haytham's *Cosmology* (*Kitāb fī hay'at al-'ālam*).¹⁴² This seems to be the only concession of the Alfonsine collection to the theoretical problems of cosmology.

The second group of works has a magical character and their purpose is not to predict the future but to shape it by applying the principles of talismanic

¹³⁸ Comes, 1991.

¹³⁹ Bossong, 1978.

¹⁴⁰ See 7.6.2 on the two extant versions of the *Alfonsine Tables*: a Castilian text of the canons, edited and commented by Chabás & Goldstein, 2003, which describe a set of tables which compute sidereal longitudes, and the Latin numerical tropical tables.

¹⁴¹ Hilty, 1954; Hilty, 2005.

¹⁴² Mancha, 1990.

magic in an astrologically propitious moment, using adequate materials, oral invocations to the planetary gods, fumigations, or ointments. This is the topic dealt with by other Alfonsine works like the *Picatrix*,¹⁴³ the series of *Lapidarios* (four are extant in Castilian, and we also have the indexes of another ten)¹⁴⁴ and the book entitled *Libro de la magia de los signos*.¹⁴⁵

Before ending this presentation, something should be said about Alfonso X's collaborators, given the fact that the king's personal intervention in the task was kept to a minimum. Alfonso was surrounded by a team formed by a Muslim convert (Bernardo el Arábigo), four "Spanish" Christians (Fernando de Toledo, Garci Pérez, Guillén Arremón d'Aspa and Juan d'Aspa), four "Italians" (Juan de Cremona, Juan de Mesina, Pedro de Regio and Egidio Tebaldi de Parma) and five Jews (Yehudah b. Mosheh, Ishāq b. Šīd called Rabiçag, Abraham Alfaquín, Samuel ha-Leví and Mosheh).¹⁴⁶ The extent of the participation of these four groups varied: Bernardo el Arábigo and three of the Spanish Christians only worked in collaboration with a Jew: only Fernando de Toledo produced an independent translation of the treatise on Ibn al-Zarqālluh's *azafea* in 1255 or 1256, but this translation was considered unsatisfactory and underwent a revision in 1277. As for the Italians, Juan de Cremona and Juan de Messina participated in the revision of the treatise on the *azafea*, while the two others were in charge of the retranslations into Latin of the Castilian texts in the Alfonsine collection. The Jews were the most productive group, as they participated in the 23 works, counting both translations and original texts. Their relative importance is shown in the following list:

Mosheh: 1 work

Abraham Alfaquín and Samuel ha-Levi: 2 works each

Yehudah b. Mosheh: 7 works

Ishāq b. Šīd: 11 works.

1.5.3 *Astronomy in Aragon and Castile in the Fourteenth Century*

The celebrated developments at the Castilian court of Alfonso X did not have an equivalent in the Crown of Aragon. Throughout the thirteenth century we can only mention the figure of Ramon Llull (Raymundus Lullius) (1232–1316) who, like the King of Castile, began to use the vernacular language in scientific

143 Pingree, 1986.

144 Rodríguez, 1981.

145 D'Agostino, 1979.

146 Procter, 1945; Romano, 1991a; Roth, 1990.

works; he wrote a *Tractatus novus de astronomia* in both Latin and Catalan,¹⁴⁷ a very elementary astrological treatise aimed to provide very simple rules to allow the astrologer to make a prediction. We have already seen that similar characteristics can be found in the Alfonsine *Libro de las Cruces* and in the old corpus of the *Alchandreana*.

This situation did not change until the reign of King Pedro IV “el Ceremonioso” (r. 1336–1387) who, following the Alfonsine model, patronised Catalan translations and original works related to astronomy and astrology. One should, however, bear in mind that although the aforementioned translations derived ultimately from an Arabic source, they were actually retranslations from Latin or from Castilian. Most of the available Arabic astronomical literature had already been translated during the two previous centuries and the conquests of the cities of Mallorca (1229) and Valencia (1238) by Jaime I (1213–1276) did not provide translators with important libraries containing new Arabic sources. An example of these retranslations is the *Tractat d'Astrologia* by Bartomeu de Tresbéns,¹⁴⁸ a physician at the service of King Pedro IV between 1361 and 1374. This book is strongly influenced by the Alfonsine *Libro conplido de los iudizios de las estrellas* by ‘Alī ibn Abī l-Rijāl.

King Pedro IV commissioned the local astronomers Petrus Gileberth (d. 1362) and Dalmacius Planes (d. 1383) to undertake a programme of planetary observations over a seven-year period (1360–1366), after building a set of thirteen observational instruments, some of which seem to have been quite large. The first result of their work was the compilation of an almanac with tables, of which only the Latin text of the prologue and part of the canons are extant. From these scarce materials we can conclude that the almanac in question was probably a set of ephemerides which gave solar, lunar and planetary tropical longitudes for the years 1361–1433: so it was not a standard perpetual almanac like those of Ibn al-Zarqālluh or, later, the one compiled by Jacob b. Makhir b. Tibbon (Profatius Judaeus, d. 1304) in Montpellier, or the Almanac of 1307, which seems to be a translation from the Arabic, and is extant in Latin, Catalan, Castilian and Portuguese versions.¹⁴⁹

Pedro IV does not seem to have been satisfied with these tables and asked the Sevillian Jew Jacob Corsuno, whose presence in Barcelona is documented between 1378 and 1380, to compile a set of real astronomical tables that would

147 Critical edition of the Latin text by Pereira, 1989; the Catalan text has been edited, also critically, by Badia, 2002.

148 Vernet & Romano, 1957–58.

149 Millás, 1949a. The same contents in Millás, 1943–50, pp. 395–405. See also Chabás, 1996, pp. 263–265.

give sidereal positions and could be used to compute horoscopes. The result of Corsuno's work was the *Tables of Barcelona*, many parts of which seem to be mere copies from one of the *zīj*es of Ibn al-Kammād (fl. 1116); on the whole, the tables were little short of disastrous and could not be compared to the Alfonsine Tables.¹⁵⁰

Far more interesting are the astronomical tables of Jacob b. David Bonjorn (Yomṭob) (fl. 1361), another astronomer at the service of Pedro IV, who lived in Perpignan, a city which, at that time, belonged to the Crown of Aragon. These tables¹⁵¹ exerted an important influence on two later works: the *Lunari* of Bernat de Granollachs (1485) and Zacut's *ha-Ḥibbur ha-gadol* and *Almanach Perpetuum*. For the first time, these tables used a lunar cycle of 31 Egyptian years, 9 days, 23 hours, 34 minutes and 11 seconds, equivalent to 383.5 synodic months (767 consecutive syzygies) to compute a table of true syzygies and facilitate the calculation of eclipses.

During the fourteenth century in Castile¹⁵² some translations from the Arabic were still being made: Pero Fernández translated into Castilian al-Qabīṣī's introduction to astrology, while Alfonso Dinis, astrologer and physician of King Alfonso IV of Portugal (1326–1367), translated an unidentified Arabic astrological text into Latin.¹⁵³ Besides, the almanac tradition continued, towards the end of the century, with the perpetual almanac, compiled by Ferrand Martínez in Castilian in 1391. The tables of this book, which calculate sidereal positions, are mere adaptations of the almanac of 1307 to a new radix date (1391) and the coordinates of Seville, although some tables, such as those of the division of the houses of the horoscope, were calculated for the latitude of Toledo. The canons seem to be original.¹⁵⁴

Apart from this, the practice of astronomy during this century seems to be restricted to Jewish astronomers, who are the true heirs of the Andalusian tradition. Their role has been clarified by the research of Bernard R. Goldstein and José Chabás. The Jewish astronomers of this century seem to have been totally unaware of the Alfonsine Tables, compiled by two Jewish collaborators of King Alfonso (Yehudah b. Mosheh and Ishāq b. Šīd); most of them continue the school of Ibn al-Zarqālluh (d. 1100), represented mainly by Ibn al-Kammād (fl. 1116) (see 7.6.4.2). We have already seen that this was the case of the *Tables of Barcelona*, the work of another Jew, Jacob Corsuno. In 1310, the Toledan Ishāq

150 Millás, 1962; Chabás, 1996; Chabás, 2004; Vernet & Samsó, 2004.

151 Chabás, Roca & Rodríguez, 1992; Chabás, 1988; Chabás, 1991.

152 On the development of astronomy in Castile see Chabás, 2002.

153 Beaujouan, 1969, pp. 7–8.

154 Chabás, 1996.

Israeli wrote the *Yesod 'Olam*, a book containing interesting information about the astronomical research of Ibn al-Zarqālluh and Ishāq b. Šīd as well as a set of astronomical tables which, again, had no relation to the *Alfonsine Tables*. Throughout the fourteenth century, the Jewish family of the Banū Waqār were active as physicians and astronomers. Towards the beginning of the century (ca. 1312), the anonymous author of *al-Ṭibb al-Qaṣṣṭālī al-Malūkī* ("The Royal Castilian Medicine"), probably a member of this family, included in a medical treatise an astronomical digression which showed his interest in cosmology (*hay'a*).¹⁵⁵ A medical miscellaneous manuscript, parts of which are dated in Toledo and Guadalajara between 1388 and 1425, seems to have been copied by several members of this family. It contains astronomical and astrological notes with materials deriving from al-Khwārizmī's *zīj*, the *Toledan Tables* and Ibn al-Zarqālluh's *Almanac*.¹⁵⁶ One of the Banū Waqqār, Yosef b. Yiṣḥāq b. Mosheh b. Waqār, compiled a set of astronomical tables (*Sefer Luḥot*) with canons in Arabic (1357–58), which he later translated into Hebrew (1395–96). Ibn Waqār used materials derived from Ibn al-Kammād and, surprisingly, from the Tunisian astronomer Ibn Ishāq (fl. 1193–1222).¹⁵⁷ Echoes of Ibn al-Kammād reappear in the Hebrew tables of Solomon Franco (fl. 1375),¹⁵⁸ Juan Gil de Castiello or de Burgos (fl. 1350–1352)¹⁵⁹ and Judah b. Asher II of Burgos (d. 1391), whose set of tables is dated in 1364.¹⁶⁰

1.5.4 *The Fifteenth Century in Castile*

The first half of the fifteenth century shows clear symptoms of decline among Christian scientists, although one should stress that Castilian continued to be used as the language of science; this much is shown by the *Tratado de Astrología* attributed to Enrique de Villena (1384–1434),¹⁶¹ which was in fact probably written in 1438 or 1439 by Andrés Rodríguez, from Zamora, who was at the service of the Marquis of Santillana. The author was, apparently, an educated man interested in astronomy but not a professional astronomer. The treatise is a handbook containing all the materials a fifteenth century reader expected to find in a book with such a title: in it we find astrology, but also astronomy, ecclesiastical computus, cosmography and geography. Its astronomy is basically Ptolemaic although we also find echoes of the Indian tradition

¹⁵⁵ Samsó, 2011, pp. 559–560.

¹⁵⁶ Castells, 1991.

¹⁵⁷ Castells, 1996.

¹⁵⁸ Goldstein, 2013.

¹⁵⁹ Goldstein, 1985, p. 237; Beaujouan, 1969, p. 11.

¹⁶⁰ Goldstein, 1998, pp. 179ff; Chabás & Goldstein, 2000, pp. 49–50.

¹⁶¹ Cátedra & Samsó, 1983.

represented by al-Khwārizmī's *zīj*. One of its most interesting aspects is the author's attempt to enlarge the size of the universe, following a tendency we find both in professional astronomers like Levi b. Gerson (1288–1344)¹⁶² and in amateurs like Bishop Pedro Gallego (ca. 1200–1267).¹⁶³ While Ptolemy's radius of the sphere of fixed stars equals 20,000 terrestrial radii, according to the pseudo-Enrique de Villenait reaches 10^{11} t.r.

An interesting event during the second half of the century was the reintroduction in the Iberian Peninsula of the *Alfonsine Tables*.¹⁶⁴ These tables had been composed ca. 1272 but the extant Castilian canons do not fit them, with headings in Latin, known in the version prepared (?) between 1320 and 1330 by Parisian astronomers like John of Lignères and John of Murs. In spite of the allusions to them that we find in the canons of the tables of Jacob b. David Bonjorn (1361), in those of the *Tables of Barcelona* (ca. 1380), and in pseudo-Enrique de Villena's *Tratado de Astrología* (ca. 1438), there is no evidence of their use in the Iberian Peninsula before c.1400;¹⁶⁵ later, in 1460, they were introduced by Nicholas Polonius in the recension known as *Tabulae Resolutae*, (quite popular in Central Europe, especially in Poland).¹⁶⁶ This was a period of effervescence of astronomical activity in the city of Salamanca: Polonius was the first scholar to occupy the chair of Astrology of the University (ca. 1460–1464), followed by Juan de Salaya or Selaya (1464–1469), Diego Ortiz de Calzadilla (1469–1476), Fernando de Fontiveros (1476–ca. 1480), Diego de Torres (ca. 1480–after 1487), and Rodrigo de Vasurto (after 1487–1504).¹⁶⁷

Polonius made a version of the *Tabulae Resolutae* calculated for Salamanca with year 1460 as epoch. This is not the only set of tables computed for the longitude of this city and based on the *Alfonsine Tables*.¹⁶⁸ there are others that include Hebrew and Castilian versions. This is the backdrop to the works of Abraham Zacut (1452–1515) of Salamanca who was, no doubt, in contact with Christian astronomers of that city, mainly with those who occupied the chair of Astrology. Zacut was the main Jewish astronomer who followed the Alfonsine tradition. His main work was the *ha-Hibbur ha-gadol* ("The Great Composition"), a great almanac in the tradition of Ibn al-Zarqālluh, Jacob b. Makhir b. Tibbon and the Almanac of Tortosa, but which introduced important novelties: the computations were based on the *Alfonsine Tables*, and so the

162 Goldstein, 1986.

163 Samsó, 2000 a.

164 Chabás & Goldstein, 2003.

165 Chabás, 2000.

166 Dobrzycki, 1987; Chabás, 1998.

167 Beaujouan, 1969, pp. 12–15. Cantera, 1931, pp. 371–382.

168 Chabás and Goldstein, 2000, pp. 18–47.

calculated positions were tropical and not sidereal; it used the cycle of 767 consecutive syzygies, introduced by Jacob b. David Bonjorn (Yomṭob, fl. 1361), for the computation of lunar longitudes and eclipses. The radix date of the *Hibbur* was 1473 and the tables were calculated for the coordinates of Salamanca. It is interesting that both 1473, as a radix, and the meridian of Salamanca were used in the Hebrew translation of the *Tabulae Resolutae* extant in a manuscript of St. Petersburg.¹⁶⁹ The interest of the Christian astronomers who occupied the University chair is obvious: Juan de Salaya translated the *Hibbur* into Castilian,¹⁷⁰ while Diego de Torres compiled a Latin summary of the canons.

Somewhat later, probably after Zacut's migration to Portugal in 1492, a Portuguese scholar, José Vizinho, prepared (possibly without any help from Zacut) a new set of canons of the *Hibbur*, in Latin and Castilian, to which he added a set of tables derived from the Hebrew original work. A double edition of the Zacut-Vizinho work (with canons in Latin and Castilian) was printed in Leiria in 1496, under the title of *Almanach Perpetuum*. It was an enormous success and was reprinted in Venice in 1498, 1502, 1525 and 1528.¹⁷¹ The *Almanach* was translated into Arabic at least twice:¹⁷² a summary of the canons was prepared by the Jewish physician Mūsā Jalīnūs in Istanbul in 912H/ 1506–1507 and a full translation was made in Morocco by the Morisco exile Aḥmad b. Qāsim al-Ḥajarī ca. 1624. The latter work spread very widely, and was used from Morocco to Yemen until the nineteenth century, thus indirectly introducing Alfonsine astronomy in the Arab world.¹⁷³

Another Jewish astronomical tradition was independent from the *Alfonsine Tables* and is represented here by Judah ben Verga (fl. 1455–1480), a contemporary of Zacut who mentions him in the *Hibbur*. Judah lived in Lisbon where he made astronomical observations and composed several astronomical works, among which we find a set of tables entitled *Ḥuqot Shamayim* ("Ordinances of the Heavens"); it is difficult to establish specific sources, although it seems clear that Judah used Ibn al-Kammād's *zīj* and was mostly unaware of the Alfonsine tradition.¹⁷⁴ Another instance of the same tradition can be found in an anonymous and undated Hebrew *zīj* closely related to the *Ḥuqot*: the mean motion tropical parameters are the same (with the exception of those for the Sun and lunar longitude), but the radix positions are different. The positions

169 Chabás & Goldstein, 2000, pp. 22–23.

170 Edited by Cantera, 1931, pp. 151–236.

171 Chabás & Goldstein, 2000, pp. 161–163.

172 Parra, 2013 states that Mūsā Jalīnūs' version, extant only in ms. Escorial 966, is followed by a few chapters of an independent translation by an anonymous author.

173 See Parra, 2013 and Samsó, 2002–3, Samsó, 2004 and Samsó 2007 a.

174 Langermann, 1999 a, pp. 19–25; Goldstein, 2001; Goldstein, 2004.

of the apogees are sidereal and the apogees have the same proper motion as the solar apogee, in addition to precession. The *zīj* includes a trepidation table derived from Ibn al-Kammād's *zīj*, with a maximum equation of 10° , and the same origin can be ascribed to the maximum solar equation of $1;52^\circ$.¹⁷⁵

The century ends with the figure of Alfonso de Córdoba (born 1458 and active until at least 1502), who is probably the last competent astronomer of Christian origin known to us. Mentioned as *Hispalensis* in Copernicus' *Commentariolus*, and editor of Zacut & Vizinho's *Almanach Perpetuum*, printed in Venice in 1502, he lived in Rome where he wrote a Latin treatise on the equatorium entitled *Lumen caeli* (printed in Rome in 1498). Other treatises on the same instrument had been written by two Jewish astronomers of Iberian origin and resident in Italy: al-Ḥadib (ca. 1396, Sicily) and Yosef ha-Parsi (Seville, 1439, Bologna, 1444).¹⁷⁶ Alfonso de Córdoba also prepared a set of astronomical tables entitled *Tabule astronomice Elisabeth Regine* for Queen Isabel I of Castile (1451–1504) using as epoch 24 December 1474, the date of Queen Isabel's accession to the throne. These tables, which were printed twice in Venice (1503 and 1524), depended on the 1483 edition of the *Alfonsine Tables* but introduced several modifications and additions which show that Alfonso was extremely competent and capable of performing difficult computations.¹⁷⁷ Alfonso de Córdoba's tables represent the end of medieval astronomy in the Iberian Peninsula.

1.6 A Brief Conclusion

The purpose of this introductory chapter has been twofold. On the one side it has been an attempt to summarize the contents of the whole book with emphasis in the chronology of the events and this implies the repetition of many data which will be developed in the following chapters. On the other, I have tried to present my own ideas on the role of al-Andalus as a bridge between Arabo-Islamic and European science.¹⁷⁸ Only books actually reaching al-Andalus could be translated into Latin (or into another languages) and one of the hypotheses presented here is that the arrival of Eastern books was interrupted with the fall of the Cordoban Caliphate. Only exceptionally did books from the Mashriq, produced after ca. 950, reach Córdoba or the main *tā'ifa* cities. This explains why the great works of Eastern Islamic science produced

¹⁷⁵ Goldstein, 2003.

¹⁷⁶ Goldstein, 1987.

¹⁷⁷ Chabás, 2004.

¹⁷⁸ Samsó, 2015.

from the tenth century onwards were rarely known in Medieval Europe: they simply never reached the bridge across which transmission took place.

The transmission process began in Catalonia towards the end of the tenth century, but it was interrupted for over a hundred years and did not resume until the beginning of the twelfth century. I contend, and this is my second hypothesis, that transmission recommenced only when great Arabic libraries were once more accessible to Latin scholars, that is after the fall of Toledo (1085), Zaragoza (1118), Córdoba (1236) and Seville (1248). With each conquest of one of these cities, new libraries became available and had a bearing on the selection of sources to be translated. At the same time, translators were dependent on patrons, who provided them with a living but whose personal tastes also influenced the choice of texts. We know of two clear instances of this: Bishop Michael of Tarazona, whose personal interests were taken into account by translator Hugo Sanctelliensis, and Alfonso X, surrounded by a team of (mainly) Jews, but also "Spaniards" and "Italians", who oversaw the process of translation and the revision of translated texts. Here it should be pointed out that Alfonso X constitutes the first true instance of royal patronage; before him, only high-ranking members of the Church seem to have been interested in patronising translations.

Mīqāt: Timekeeping and *Qibla*

2.0 Introduction

Very little is known about *mīqāt* (astronomy applied to the problems of religious worship) in al-Andalus. We know that some literature on this topic did exist: for example, biographical dictionaries mention a certain Abū Bakr ‘Ubayd Allāh al-Qurashī (d. 1052), who was a *faqīh* and wrote a *Ta’līf fī awqāt al-ṣalawāt ‘alā madhāhib al-‘ulamā’* (“On the times for prayers according to the opinions of scholars”) which seems not to be extant.¹ Besides, in his *Tabṣīrat al-mubtadī’ wa-tadhkirat al-muntahī fī ma’rifat al-awqāt bi l-ḥisāb min ghayr āla wa-lā kitāb*,² a fourteenth-century Maghribī source, Abū l-Ḥasan al-Muqri’ (fl. Bejaia, 1384) mentions another Andalusian author, al-Ḥasan b. ‘Alī al-Qurṭubī (1120–1205), who wrote an unexplored *Kitāb al-lu’lu’ al-manẓūm fī ma’rifat al-awqāt bi l-nujūm*;³ and when al-Manṣūr ibn Abī ‘Āmir burned a part of the library of al-Ḥakam II, he spared the books related to the *‘ilm awqāt al-ṣalawāt* (times of the canonical prayers), as well as those dealing with medicine and arithmetic.⁴ There is no secondary literature on this topic or on the visibility of the lunar crescent, with the exception of the analysis of some scattered materials which will be studied in this chapter. The *qibla* problem seems clearer even though, as Rius stresses,⁵ the richness of Maghribī sources on this topic shows a clear contrast with the lack of precise information about al-Andalus.

In spite of the fact that the first *muwaqqits* (astronomers in the service of mosques) attested in al-Andalus appear in the Naṣrid kingdom of Granada towards the end of the thirteenth century, more or less at the same time as the references to this kind of profession in Egypt, it is quite probable that their role and duties were fulfilled by other officers of the staff of the mosque: D.A. King⁶ has drawn our attention to the role of the muezzin in this respect in the Mashriq, and probably the same practices were in place in al-Andalus. I have no information on muezzins dealing with the problem of the determination

1 Balty Guesdon, 1992, pp. 429–430.

2 On this work see Baklī, ‘Aysānī & Ilhām, 2014, pp. 10–12.

3 Baklī, ‘Aysānī & Ilhām, 2014, pp. 10–11; Lamrabet, 2013, A252, p. 95.

4 Samsó, 2011, p. 71.

5 Rius, 2000.

6 King, 2004, pp. 623–677.

of the times of prayer, but Balty-Guesdon⁷ has brought to light the fact that two astronomers who flourished towards the end of the tenth century and in the eleventh century occupied posts in mosques and were interested in *mīqāt* problems. One of them is ‘Alī b. Sulaymān al-Zahrāwī al-Ḥāsib, a disciple of Maslama, who was *imām* and *khaṭīb* in the mosque of Granada and wrote a *Risāla fī ma’rifat sa’at* [Balty Guesdon reads and translates *sā’at*] *al-mashriq* (on the knowledge of orive amplitude), which implies that he was interested in astronomy. The second is Ziyād ibn ‘Abd Allāh al-Anṣārī, Abū ‘Abd Allāh (d. 1085) who was *khaṭīb* and *ṣāhib al-ṣalāt* in the Great Mosque of Córdoba. He knew astronomy and determined a new qibla for Córdoba “along its great river” (*‘alā nahri-hā al-a’zam*),⁸ which probably implies an Eastern orientation of the *qibla*, since the Guadalquivir flows approximately East-West.

The situation changed radically towards the end of the thirteenth century with the appearance of the first Andalusī *muwaqqits*, who were also called *mu’adhdhin* (?), *mu’addil*, or *amīn al-awqāt*: Ḥasan/Ḥusayn b. Muḥammad b. Bāṣo (d. 1316) and his son Aḥmad b. Ḥasan/Ḥusayn (d. 1310) were both *muwaqqits* (as well as instrument makers) in the Great Mosque of Granada and the former was, according to Ibn al-Khaṭīb, “chief of time-keepers” (*ra’īs al-muwaqqitīn*) in the same mosque.⁹ The profession was still alive in the fifteenth century for Abū l-Ḥasan ‘Alī b. Mūsā al-Lakhmī, known as al-Qarabāqī (d. 1440), one of the teachers of the mathematician al-Qalaṣādī, was a *muwaqqit* (probably in Baza) and had a long discussion with Abū l-Qāsim b. Sirāj, *imām* and *muftī* in Granada, on the problem of the orientation of Andalusī mosques.¹⁰ We will see, later in this chapter, that the development of *mīqāt* might be the cause of the improvement in the orientation of mosques built in the fourteenth century.

The situation is different in the Maghrib. We have already seen (1.3) that al-Ḥasan b. ‘Alī b. Khalaf al-Umawī (d. 1205–1206) wrote a *Kitāb al-lu’lu’ al-manẓūm fī ma’rifat al-awqāt bi l-nujūm* (“The versified pearl on the knowledge of times by means of the stars”). Towards the end of the thirteenth century the information on professional *muwaqqits* serving in mosques, mainly in Morocco and, especially in Fez, begins to appear. Driss Lamrabet¹¹ has collected information on eight *muwaqqits* active in Fez, one in Marrakech, one in the city of Tunis

7 Balty Guesdon, 1992, pp. 430, 637–638; 295, 432, 654.

8 Ibn Bashkuwāl, I no. 431, pp. 189–190.

9 Calvo, 1993.

10 Samsó, 2011, p. 412.

11 Lamrabet, 2013, M105 (p. 158), M141 (pp. 181–182), M162 (p. 188), M183 (p. 193), M189 (p. 194), M200 (p. 203), M234 (p. 212), M235 (p. 212), M242 (p. 214), M262 (p. 222), M286 and 286bis (p. 225) and M296 (p. 226).

and another one in Sūsa (Tunis), between the thirteenth and the sixteenth centuries. The earliest of his references concerns Abū ‘Abd Allāh Muḥammad b. al-Ḥabbāk al-Tilimsānī (fl. 1286).¹² Some of these characters had been involved in the building of clocks for the Qarawiyyīn mosque or for the *madrasa* Bū ‘Ināniyya in Fez (see 1.4.4 and below 2.4.2.1.6). Lamrabet’s book contains also information on written works on timekeeping,¹³ most of which remain entirely unexplored. I will try here to begin an analysis of the two *urjūzas* which seem to have reached the maximum level of popularity and were the object of multiple commentaries:

- 1) The *rajaz* compiled by Abū ‘Abd Allāh Muḥammad b. ‘Abd al-Ḥaqq b. ‘Alī b. Muḥammad al-Baṭṭīwī, called Abū Miqrā’/Muqrī’/Maqrā’¹⁴ (fl. 1320). I have read this poem in ms. Escorial 954 (fols. 101v–106r), which contains 154 verses. In spite of its reputation, the *urjūza* is somehow disappointing: it deals with the lunar and solar calendar and contains quite a lot of *anwā’* materials. In it we find a discussion on the date of the birth of both the Prophet and Jesus, rules to calculate the day of the week corresponding to the beginning of each month of the Julian calendar, the lunar mansion occupied by the Sun throughout the whole year, the solar shadow, for a gnomon of seven feet, corresponding to the moments of the prayers of the *zuhr* and the *‘aṣr*, an Indian rule (see below 2.4.1.2.2) for the computation of the seasonal hour of the day from sunrise or before sunset, and an explanation of the use of the lunar mansions for calculating the hour during the night (see 2.4.1.2.4). Surprisingly, the poem ends with an elementary introduction to astrology and with a rule to calculate the illuminated part of the lunar disk for every day of the lunar month.

12 There seem to have been at least three al-Ḥabbāks in Tlemcen, all of them astronomers or mathematicians: Abū ‘Abd Allāh Muḥammad b. al-Ḥabbāk al-Tilimsānī (fl. 1286), one of the builders of the Qarawiyyīn clepsydra (Lamrabet, 2013, M105, p. 158); Muḥammad b. Aḥmad b. Abī Yahyā al-Ḥabbāk al-Tilimsānī (d. 1463) (Lamrabet, 2013, M219, p. 207), and Abū ‘Abd Allāh b. ‘Abd Allāh b. Aḥmad al-Ḥabbāk (fl. 1514), probably his grandson (Lamrabet, 2013, M248, p. 220).

13 Lamrabet, 2013, M112 (pp. 159–160), M134 (pp. 164–177), M170 (p. 190), M205 (pp. 204–205), M219 (p. 207), M225 (p. 208), M229 (pp. 208–211), M242 (p. 214), M248 (p. 220), M262bis (p. 222), M263 (p. 222), M298 (p. 226).

14 Since Renaud & Colin, 1938, he has been known as Abū Miqrā’. The seventeenth-century Moroccan astronomer Muḥammad ibn Sa’id al-Sūsī al-Marghīthī summarised his teachings in a poem called *al-Muqrī’ fī ikhtīṣār ‘ilm Abī Muqrī’* and the rhyme in the title seems to require us to read Abū Muqrī’ instead of Abū Miqrā’. In spite of this al-Marghīthī himself, in his commentary on *al-Muqrī’* states that the name should be read as Maqrā’ (lithographic edition published in Fez, 1313, p. 5). See Carl Brockelmann, *GAL* II, 255/331; id. *GALS* II, 364; Lamrabet, 2013, no. M141, pp. 181–182; King, 2004, pp. 495–496.

As we know, Abū Miqra's urjūza was the object of many commentaries¹⁵ and I have used two of them. The first is the *sharḥ* composed by Abū 'Abd al-Raḥmān al-Jādirī (written after the *Rawḍa*, see below)¹⁶ which I have been able to read in ms. Escorial 361 (fols. 115v–159v). Second, Abū 'Abd Allāh Sayyidī Muḥammad b. Sa'īd b. Muḥammad b. Aḥmad al-Sūsī al-Marghīthī (or al-Mīraghtī) (d. 1678) summarised Abū Miqra's urjūza in his didactic poem *al-Muqni' fī ikhtiṣār 'ilm Abī Muqri'* (99 vv), and wrote a commentary entitled *al-Mumti' fī sharḥ al-Muqni'*.¹⁷ Both works were printed, together with a supercommentary by Abū 'Abd Allāh Muḥammad b. Muḥammad b. 'Abd Allāh al-Warzīzī (d. ca. 1170),¹⁸ in a lithographical edition published in Fez, 1313 H/ 1895–96.

- 2) The second source is more interesting: it is the well-known *Rawḍat al-azhār fī 'ilm waqt al-layl wa l-nahār*, written in 794/1391–92 by Abū Zayd 'Abd al-Raḥmān b. Muḥammad al-Jādirī (1375–1416),¹⁹ on whose work several commentaries were written.²⁰ One of them is the *Natā'ij al-afkār fī sharḥ Rawḍat al-azhār*,²¹ written by an astronomer named al-Ḥabbāk who is not to be confused with the well-known Abū 'Abd Allāh Muḥammad b. Aḥmad b. Abī Yaḥyā al-Ḥabbāk al-Tilimsānī (d. 1463), author of another commentary on al-Jādirī's poem entitled *Fajr al-anhār*.²² The author of the *Natā'ij* mentions year 920/1514–15 and calculates a table of solar longitude for the year 1514. This work also contains a table calculating the mediation of the 28 lunar mansions for two years: 794/1391–92 (the date of the composition of the *Rawḍa*) and 970/1562–63, which must correspond to the author's time. Lamrabet has offered a solution to this chronological problem by proposing that the author is al-Ḥabbāk al-Tilimsānī al-Ḥafid, the grandson of Abū 'Abd Allāh al-Ḥabbāk.²³

Al-Jādirī also wrote a treatise on timekeeping explaining, in prose, the contents of the *Rawḍa*, under the title *Iqtīṭāf al-anwār min Rawḍat al-azhār*. This work has been edited by Muḥammad al-Khaṭṭābī²⁴ and

15 See Baklī, 'Aysānī & Ilhām, 2014, p. 10, who mention Ibn al-Bannā' (d. 1321) and al-Qalaṣādī (d. ca. 1486) among the commentators.

16 See ms. Escorial 361 fol. 150r in which we find a reference to the *Rawḍa*.

17 Brockelmann, *GALS* II, 707–708. Lamrabet, 2013, M340, pp. 234–235.

18 Brockelmann, *GALS* II, 707.

19 Lamrabet, 2013, no. M200, pp. 203–204; King, 2004, p. 501.

20 See Baklī, 'Aysānī & Ilhām, 2014, p. 9.

21 Both the *Rawḍa* and the *Natā'ij* have been edited and analysed (in Arabic) by Saidi, 2013.

22 Lamrabet, 2013, no. M219, p. 207.

23 Lamrabet, 2013, no. M248, p. 220.

24 Khaṭṭābī, 1986, pp. 100–134.

commented by Emilia Calvo.²⁵ Finally al-Khaṭṭābī edited, in the same volume, Ibn al-Bannā's *ʿIlm al-awqāt bi l-ḥisāb*,²⁶ also analysed by E. Calvo.

2.1 Calendars and Years

In the study of timekeeping in the Iberian Peninsula and the Maghrib during the Middle Ages, one has to bear in mind the coexistence of two different kinds of calendars:

- 1) a solar calendar, based on a civil year of 365.25 days, which was used by both Christians and Muslims, since the yearly cycle of agriculture requires the use of a solar year.
- 2) a lunar year, based on mean synodic months of 29.5 days in which the year length has $354 \frac{11}{30}$ days.

Obviously it should borne in mind that the year is a unit of time directly related to the solar calendar, while its twelve months are mere arbitrary divisions of the year with no relation to any natural cycle. In contrast, in the lunar calendar, the natural unit of time is the month, based on the lunar synodic cycle, while the lunar year is as artificial as the solar month.

To these two calendars we should add a third one: the Jewish calendar,²⁷ which aims to combine two incompatible units of time: a lunar month beginning with a new Moon and a year which respects the cycle of the seasons. With this purpose in mind, the Jewish luni-solar calendar uses the Metonic cycle of 19 solar years which contains 235 lunar months. If we consider that a mean lunar month contains 29;31,50,8,20 days, an ordinary year of 12 months will have:²⁸

$$29;31,50,8,20 \text{ days} \cdot 12 = 354;22,1,40 \text{ days}$$

and an intercalary year of 13 months will have:

$$29;31,50,8,20 \text{ days} \cdot 13 = 383;53,51,48,20 \text{ days}$$

²⁵ Calvo, 2004.

²⁶ Khaṭṭābī, 1986, pp. 86–99.

²⁷ See the collection of papers edited by Stern & Burnett, 2014.

²⁸ See Gandz, Obermann & Neugebauer, 1956, pp. 113–116.

As a cycle of 19 years contains 12 ordinary years and 7 intercalary years, we have:

$$\begin{aligned} 354;22,1,40 \cdot 12 &= 4252;24,20 \text{ days} \\ 383;53,51,48,20 \cdot 7 &= 2687;17,2,38,20 \text{ days} \\ 4252;24,20 + 2687;17,2,38,20 &= 6939;41,22,38,20 \text{ days} \end{aligned}$$

and

$$6939;41,22,38,20 : 19 = 365;14,48,33,35,47,22 \dots \text{ days}$$

which is very near 365;15 days, the standard value of the calendar's solar year.

The three calendars share a common unit: the day. This explains why the *Alfonsine Tables*, in the "Parisian" version, use the day as the basis of their tables of planetary mean motions. So we have a set of tables that can be used with any one of the three calendars.

Pre-Islamic Arabs knew a rudimentary luni-solar calendar. They were aware of the existence of a solar year by observing the last star (*raqīb*) which crossed the Eastern horizon at the same point through which the Sun was going to rise on a specific day (heliacal rising). Simultaneously, at a distance of 180°, another star (*naw'*) crossed the Western horizon at a point opposite that of sunrise (achronical setting). The pairs of stars (*raqīb* and *naw'*) changed over the course of the year as the rising Sun also changed its position on the horizon from its extreme Northern rising point (summer solstice) to its extreme Southern position (winter solstice), passing through a midpoint, which corresponded to the Eastern point of the horizon, in the equinoxes. The system divided the year into 27 periods of 13 days and one period of 14 days, so that:

$$27 \cdot 13 + 14 = 365 \text{ days}$$

As pre-Islamic Arabs never abandoned the strict lunar month, it makes sense to accept the testimony of the sources which allude to the system used to determine when to introduce an intercalary month, and have a 13-month year which allowed the *naw'-raqīb* stars to return to their traditional date. The custom of intercalation was rejected by Prophet Muḥammad who considered that the references to the *naw'* had astrological connotations, as the popular tradition related the *naw'* to meteorological cycles of rain and drought. This prohibition did not have important consequences because of the spectacular expansion of Islam shortly after the death of the Prophet, which meant

that the Arabs quite quickly came into contact with pre-Julian or Julian solar calendars. In spite of this, they never abandoned the *anwā'* system which was kept alive in the following centuries and was even accepted by the religious authorities, especially when the first translations of Greek scientific texts, considered more heterodox, appeared. The *anwā'* system was more acceptable due to its Arab origin, and this explains why a conservative *faqīh* like 'Abd al-Malik ibn Ḥabīb (Córdoba, ca. 790–ca. 852), a fanatical champion of Arabic scientific culture who fiercely opposed the invasion by a foreign one, wrote a *Kitāb fī l-nujūm* (Book on the stars), which is the earliest Andalusian version of an *anwā'* book.²⁹ This tradition endured in a series of popular calendars in which it was mixed with information gathered from Hellenistic sources, appropriated and developed by Arab scientists in the fields of both astronomy and medicine, and with data deriving from the Christian tradition, using the feasts of the saints to mark particular dates in the year. This is the case of a series of *anwā'* books like the *Mukhtaṣar fī l-anwā'* written ca. 982 by Aḥmad b. Fāris al-Munajjim, astrologer of al-Ḥakam II (961–976) and of al-Manṣūr ibn Abī 'Āmir (981–1002): in this peculiar source, Aḥmad b. Fāris used an *anwā'* book as a pretext to defend astrology at a time when astrological practices were under threat, due to the policy of religious orthodoxy applied by al-Manṣūr.³⁰ Other examples of the *anwā'* literature during the second half of the tenth century in al-Andalus are the works of Ibn 'Āṣim (d. 1013),³¹ a semi-anonymous al-Kātib al-Andalusī, identified with 'Arīb b. Sa'īd (d. 980),³² and a Christian calendar of saints' days attributed to Bishop Rabī' b. Zayd/ Recemundus. A synthesis of 'Arīb's and Rabī's calendars appears in the famous *Calendar of Córdoba*,³³ a work which, together with 'Arīb's calendar, had a strong influence in later calendars produced in al-Andalus and the Maghrib between the twelfth and the fourteenth centuries: the anonymous and undated *Risāla fī awqāt al-sana*,³⁴ the *anwā'* book by al-Ḥasan ibn 'Alī ibn Khalaf al-Umawī al-Qurṭubī (d. 1205–1206), and the *Risāla fī l-anwā'* by Ibn al-Bannā' al-Marrākushī (1256–1321).³⁵ Interestingly, the aforementioned late sources were written in al-Andalus during the Almohad era, or in Merinid Maghrib – that is, during periods when astrology was practised only surreptitiously, while *anwā'* literature underwent notable development.

29 Kunitzsch, 1994 & 1997.

30 Forcada, 1996; Forcada, 2000a.

31 Forcada, 1992a; see also Forcada, 1992b and 1998.

32 Forcada, 2000.

33 Dozy & Pellat, 1961; Martínez Gázquez & Samsó, 1981.

34 Navarro, 1990.

35 Renaud, 1948; Forcada, 1992c.

These sources use different criteria in order to divide the length of the year into four seasons. The *Calendar of Córdoba*, for example, gives the date corresponding to the entrance of the Sun in the twelve zodiacal signs according to two different astronomical systems: the *Madhhab aṣḥāb al-mumtaḥan*, which should be interpreted as solar tropical longitudes, and the *Sindhind*, which gives sidereal positions (7.1). The dates of the beginning of the four seasons are, in this source:³⁶

	<i>Mumtaḥan</i>	<i>Season</i>	<i>Sindhind</i>	<i>Season</i>
Spring	16 March	94 days	20 March	95 d.
Summer	18 June	92 d.	23 June	92 d.
Autumn	18 Sept.	90 d.	23 September	89 d.
Winter	17 Dec.	89 d.	21 December	89 d.

There is no doubt that these dates respect the unequal length of the seasons. The same cannot be said of the dates given by Abū Miqrā³⁷ which imply approximately equal seasons³⁸ and do not correspond to the values expected in the fourteenth century;³⁹ they were probably copied from an earlier source:

	Abū Miqrā ³⁷	Length of the season
Spring	16 March	92 d.
Summer	16 June	92 d.
Autumn	16 September	91 d.
Winter	16 December	90 d.

The *Calendar of Córdoba* also uses a different division of the year ascribed to the *ahl al-ḥisāb wa l-ta'dīl wa madhhab Buqrāṭ wa-Jalīnūs* (mathematicians, astronomers and members of the school of Hippocrates and Galen), according to whom summer and winter last for four months each, while spring and autumn have a duration of only two months. We do not know the identity of these mathematicians and astronomers called *ahl al-ḥisāb wa l-ta'dīl*, but the reference to Hippocrates and Galen is clear because the Hippocratic treatise

36 Viladrich, 1996.

37 Ms. Escorial 954, fol. 102v; Escorial 361, fol. 128v–130r. al-Jādirī (ms. Escorial 361, fol. 138v) gives almost the same dates (15.3, 15.6, 16.9, 15.12) according to physicians and astronomers (*ahl al-raṣad wa l-imtihān*).

38 Al-Jādirī (ms. Escorial 361, fol. 137r) states that the length of each season is 91 and 1/4 days.

39 In the *Muqni'*, Muḥammad al-Marghīthī says that these dates for the beginning of the seasons correspond to Abū Miqrā's time and gives another series of dates (10.3, 10.6, 11.9 and 10.12) for his own epoch (seventeenth century). See the printing Fez, 1313 pp. 47–48.

On diet includes an exact description of a structure of this kind.⁴⁰ The dates in the *Calendar of Córdoba*, which we can also find in Ibn al-Bannā's *Kitāb al-anwā'*, are:

Spring	17 March
Summer	16 May
Autumn	16 September
Winter	14 November

In his commentary on Abū Miqra's *urjūza*, Al-Jādirī ascribes this distribution of the seasons to "other physicians" (*wa min al-aṭibbā' man qasama al-fuṣūl taqsīman aḡar ...*). He does not give precise dates but says that spring lasts from March and April, summer from May to August, autumn from September and October, and winter from November to February.⁴¹

Finally, Abū Miqra' also gives a third distribution of the seasons, attributed to peasants, which also advances the beginning of each season by one month:⁴²

Spring	15 February
Summer	17 May
Autumn	17 August
Winter	15 November

2.2 Eras

The coexistence of two or three different calendars complicates things even more if and when the same calendar uses different eras as a starting point of its chronologies. This topic was always of interest to astronomers because they needed to use observations made by other astronomers in the past, and which were dated according to different chronological systems. This is why the canons of astronomical tables usually begin by explaining how to transform dates from one system to another.

The first set of tables used in al-Andalus were those by al-Khwārizmī (fl. 830) which, in its primitive version, used the Persian calendar with years containing 365 days (without fractions) and the era of the last Sassanian king, Yazdijird III (16.6.632). These tables were revised towards the end of the tenth century by

⁴⁰ Samsó, 1978.

⁴¹ Ms. Escorial 361 fol. 138v.

⁴² Ms. Escorial 954 fols 102v–103r; Escorial 361, fol. 138r.

the Andalusian astronomer Maslama al-Majrīṭī (d. 1007), as well as by his two disciples Ibn al-Ṣaffār and Ibn al-Samḥ. Maslama's recension adapted the tables to the Muslim lunar calendar and the Hijra era (midday of 14.7.622). In this version, the canons give instructions to transform dates between three calendars: Persian, Muslim and *Rūmī*. *Rūmī* here means Byzantine and it uses Julian years of 365.25 days and the era of Alexander *Dhū l-Qarnayn* which begins on 1.10.-311 (JDN 1607739). This selection of eras and calendars leaves out the Egyptian calendar which is quite meaningful for an astronomer. The reason is probably that a mention of this calendar is unnecessary, as it is based, like the Persian one, on years of 365 days without fraction, and on the era of Nabonassar, which appears in the list below. In fact the tables of al-Khwārizmī- Maslama, which we only know in the Latin translations of Adelard of Bath and Petrus Alfonsi,⁴³ give the following list of eras:

1. Adam: – 5380
2. Flood: 17.2.-3101 (JDN 588465)
3. Nabonassar: 26.2.-746 (JDN 1448638)
4. Philippus: 12.11.-323 (JDN 1603398)
5. Alexander: 1.10.-311 (JDN 1607739)
6. *Azophar* (*ta'rīkh al-ṣufr*) or Spanish Era: 1.1.-37 (JDN 1707544)
7. Jesus Christ: 1.1.1 (DJ 1721424)
8. Diocletian: 29.8.284 (JDN 1825030)
9. Arabs: 15.7.622 (JDN 1948439)
10. Yazdijird: 16.6.632 (1952063)

This list is the starting point of an Andalusian and Maghribī tradition and it reappears in many *zīj*es. Most of them describe the Muslim, Julian (in its Byzantine, Syriac, Western *Rūmī* and late coptic version) and Persian calendars. The same ten eras can be found in the *Toledan Tables*,⁴⁴ Ibn al-Hā'im's *al-Zīj al-Kāmil fī l-Ta'ālīm* (ca. 1205),⁴⁵ the Hyderabad recension (ca. 1266–1281) of the *zīj* of Ibn Ishāq (fl. 1193–1222),⁴⁶ Ibn al-Bannā⁴⁷ and Ibn al-Raqqām (d. 1315)

43 Suter, 1914, p. 109; Neugebauer, 1962, pp. 82–84; van Dalen, 1996.

44 Pedersen, 2002, 111, pp. 216–232. Ibn al-Kammād, who was Ibn al-Zarqālluh's disciple, only mentions the Hijra, Alexander, Yazdajird, Nabonassar and the Spanish era. See ms. Madrid BN Lat. 10023, fols. 1r–5v. In Ibn Mu'ādh's *Tabulae Jahan* we find the eras of Alexander, Spanish era, Jesus Christ, Caesar, Diocletian, Hijra and Yazdijird. See *Scriptum antiquum Sarraceni cuiusdam de diuersarum gentium eris, annis ac mensibus et de reliquis astronomiae principijs*, printed in Nuremberg, 1549, chapter 1.

45 Ms. Oxford Bodleian 11.2 no. 285 (Marsh 618), pp. 20–42. In pp. 24–25 Ibn al-Hā'im says that the ten aforementioned eras were well known in the Maghrib, especially in its northern part, although some of them were not used frequently.

46 Mestres, 1999, pp. 11–26.

47 Vernet, 1952, p. 69.

in his *al-Zīj al-Shāmil*,⁴⁸ and *Qawīm zīj*.⁴⁹ Ibn al-Raqqām's *al-Zīj al-Mustawfī* uses the same calendars and eight out of the ten eras, excluding those of Adam and the Flood.⁵⁰ This list reaches the Latin *Alfonsine Tables* and the *Alfonsine Partidas*: the former omits Adam's era and adds the eras of Augustus and King Alfonso (1.1.1252).⁵¹ This set of eras became more or less standard in medieval European tables.⁵²

The use of certain eras responds to an astronomical logic: the era of Nabonassar is the most often mentioned in the *Almagest*, in which we also find the eras of Philippus, Augustus, Adrianus and Antoninus;⁵³ Theon's *Handy Tables* use mainly Philippus' era and the Egyptian calendar, but also mention that of Diocletian;⁵⁴ al-Battānī, whose *zīj* was well known in al-Andalus and in the Christian kingdoms of the Iberian Peninsula, employs the eras of Nabonassar, Philippus, Alexander, Hijra and Yazdijird.⁵⁵ The Flood era is fully justified in the Indo-iranian context of al-Khwārizmī's *zīj* because it corresponds to the beginning of an Indian cycle, the *kalīyuga*, which started at midnight between 17 and 18 February, or at 6 o'clock in the morning of 18 February -3101.⁵⁶ Surprisingly, the Flood era is mentioned in al-Jādirī's commentary on Abū Miqrā's *urjūza*, in a quotation of Abū Marwān al-Istijī (fl. Toledo 1068) who states that the Prophet was born 3675 years after the Flood, which corresponds to 574 AD. The Persian astronomical tradition considers that the beginning of the *kalīyuga* coincided with a great planetary conjunction which announced the Flood.⁵⁷ It is more difficult, however, to justify the use of Adam's era, which does not correspond to the creation era of the Jewish calendar (7.10.-3760). Neugebauer⁵⁸ accounts for its presence by referring to a quotation of al-Bīrūnī's *Chronology*⁵⁹ who states that some Christians considered that 5069 years had passed between the eras of Adam and Alexander (as in our text), while others believed that the figure was 5180. The era of the birth

48 Ms. Istanbul Kandilli 249, fols. 3v-10r.

49 Ms. National Library of Rabat 260, pp. 1-9.

50 Samsó, 2014, pp. 300-301.

51 Poulle, 1984, pp. 107-108; Craddock, 1974. Adam's era is preserved in the Castilian *Alfonsine Tables* (Chabás & Goldstein, 2003, pp. 20-21) as well as in the *Partidas*.

52 Chabás & Goldstein, 2012, pp. 14-15.

53 Pedersen, 1974, p. 127.

54 Neugebauer, 1975, pp. 970-971. See also Mercier, 2011, pp. 51-78.

55 Nallino, 1899-1907, vol. 1, pp. 66-71.

56 Pingree, 1978, p. 555. This is the Flood date used by Abū Ma'shar: see Pingree, 1968, p. 37.

57 Kennedy, 1983a, pp. 352-354.

58 Neugebauer, 1962, p. 84.

59 Sachau, 1878, pp. 15 and 302.

of Christ and the Spanish⁶⁰ era cannot be ascribed to al-Khwārizmī: they are probably the result of an interpolation made by Maslama which survived in the later *zīj*es.

Although the AD era is mentioned in late Maghribī *mīqāt* treatises,⁶¹ its use in the Maghrib was not very common. This is why the manuscripts of al-Ḥajarī's Arabic translation of the *Almanach Perpetuum* – with tables compiled by Abraham Zacut and canons added by José Vizinho towards the end of the fifteenth century – often bear notes with the equivalence between the AD dates, used by Zacut and Vizinho, and those based on Alexander's era.⁶²

2.3 The Beginning of the Lunar Month

2.3.1 Introduction

The beginning of each month should, in principle, be established by the actual observation of the new Moon, although, since the very beginnings of their activity Muslim astronomers had been interested, in determining the conditions of visibility according to which the Moon will be seen. A very early example of this interest appears within a set of anecdotes, given by different sources, which stress the reputation of Mūsā ibn Nuṣayr, the Muslim conqueror of the Maghrib and al-Andalus, as an astrologer and astronomer. One of them, which we can read in the pseudo-Ibn Qutayba's *Kitāb al-Imāma wa l-Siyāsa*,⁶³ attributes to Mūsā the capacity to predict the visibility of the lunar crescent at the beginning of the month:

Mūsā went to see [the caliph] Sulaymān the last day of the month of Sha'bān at the time of sunset. He [Sulaymān] was, together with other people, trying to observe [the new Moon] on the terrace. When Sulaymān saw him [Mūsā], he exclaimed: "Here, by God, you have a man who, if you ask him whether he has seen the new Moon [*al-hilāl*], will tell you that he has already seen it". For the Moon, at that moment, could not be seen by Sulaymān or by his companions. Once Mūsā had approached and greeted him, Sulaymān asked: "Mūsā, have you already seen the new Moon?". Mūsā, then, answered: "Yes, Commander of the Faithful, there it is". And he pointed with his finger towards one side, while he was facing

60 Levi della Vida, 1943; Neugebauer, 1981.

61 See for example, Saidi, 2013, p. 32.

62 Cf. Samsó, 2002–03, pp. 74–75.

63 Ribera, 1926, pp. 176–177 (Arabic text) and 153 (Spanish translation).

Sulaymān. The people, then, strained their eyes in the direction Mūsā had pointed at and they saw [it]. Mūsā, then, sat down and said: “My eyesight is not as sharp as yours but I know much better than you its [the Moon’s] rising points [*maṭāli*] and its symmetrical [setting] points [*manāsiq*].”

The story is, obviously, difficult to accept because one cannot believe that a military leader at the beginning of the eighth century like Mūsā would have had the astronomical competence to deal with such a difficult problem. But the anecdote is interesting because it shows that Muslims were concerned with this question at a very early point in their history.

2.3.2 *The Sources*

In fact, the problem of crescent visibility has attracted the attention of modern scholars who have dealt with the solutions given by both Eastern and Western astronomers.⁶⁴ I have not been able to find any references to this problem in the *mūqāt* sources mentioned above in 2.0, and I will only give a list of Andalusī and Maghribī astronomical sources which contain relevant information:

1. King (1987a, pp. 197–207) has studied what he considers to be “an early Andalusian crescent visibility table”. We cannot be sure of the Andalusian origin of the table but it is clear that it was used in the Maghrib. Among the 14 sources used by King, we find ms. Hyderabad 298 which contains a recension of the *zīj* of Ibn Ishāq, ms. Escorial 909 (*zīj* with a collection of versified canons by Abū l-Ḥasan ‘Alī al-Qusanṭīnī) (cf. 1.4.2 and 7.2.3) and ms. Milan Ambrosiana 338 (translation by al-Ḥajārī of Zacut and Vizinho’s *Almanach Perpetuum*). To the list of sources mentioned by King, one should add the *Toledan Tables*.⁶⁵
2. In Adelard of Bath’s Latin translation of al-Khwārizmī-Maslama’s *zīj*⁶⁶ we find a visibility table which does not belong to al-Khwārizmī’s original *zīj* or to Maslama’s revision. This table has been studied by Kennedy (1965), King (1987, pp. 192–197) and finally by Hogendijk (1988 a, pp. 32–35). Part of this table is also extant in ms. Hyderabad 298, where it is ascribed to an otherwise unknown al-Qallās. The same table appears in the *Toledan Tables* (Pedersen 2002, IV, pp. 1482–1483) and the corresponding canons (Pedersen, 2002, I, 311) contain the same text as al-Khwārizmī-Maslama’s

64 Kennedy, 1960; Kennedy & Janjanian, 1965; Kennedy, 1968; Kennedy, 1997; Neugebauer, 1949; Gandz, Obermann and Neugebauer, 1956; King, 1987a; King, 1988; King, 1991; Hogendijk, 1988a; Hogendijk, 1988b; Giahī, 2002–03; Giahī, 2009–10.

65 Pedersen, 2002, vol. IV, pp. 1484–1485.

66 Suter, 1914, pp. 16–17, 168; Neugebauer, 1962, pp. 42–44, 102.

- zīj*. Hogendijk has established that the table was computed for a latitude 41;35°, which corresponds to Zaragoza, and implies that it was a later addition, unrelated to al-Khwārizmī's *zīj*.
3. In the eleventh century, two different procedures for calculating lunar visibility seem to be related to Ibn al-Zarqālluh (d. 1100): a very crude one extant in ms. Hyderabad 298;⁶⁷ the other, much more elaborate, can be found in Ibn al-Zarqālluh's *Almanac*.⁶⁸ The second procedure has been studied by King (1987, pp. 210, 212) but it still needs clarification.
 4. The *Toledan Tables* (Pedersen, 2002, II, pp. 536–539) explain a procedure which will reappear in Ibn al-Raqqām's *Shāmil* and *Qawīm zīj*es (see below). It probably derives from an earlier source, as it introduces a correction that is valid for the latitude of Córdoba. The collection also contains (Pedersen, 2002, IV, 1486) a table of horizontal parallax for lunar visibility calculated for sunset or soon after, at the moment of conjunction and for the latitude of Córdoba.
 5. Ms Hyderabad 298 ascribes to Ibn Mu'ādh al-Jayyānī (d. 1093) and to an unidentified al-Sabtī⁶⁹ another procedure whose canons correspond to chapter 19 of Ibn Mu'ādh's *Tabulae Jahen*.⁷⁰ These canons describe a table calculated for all climates and another one for the latitude of Tunis. The latter cannot be the work of Ibn Mu'ādh himself, as the *Tabulae Jahen* refer to a table calculated for the latitude of Jaén.⁷¹
 6. In his book on the "Sanctification of the New Moon", Maimonides (1135–1204) provides a very detailed description of the procedure for predicting the visibility of the new Moon.⁷² His interest in the topic is quite surprising, as Jewish religious authorities had accepted the use of a continuous fixed calendar many centuries before his time.
 7. Ms Hyderabad 298 contains an elaborate procedure, considered to be "exact", due to Ibn Ishāq (fl. 1193–1222).⁷³
 8. Ibn al-Bannā's *Minhāj*⁷⁴ explains the standard procedure and uses a *Jadwal al-musattah al-maqsūm 'alā maqām al-ru'ya* which also appears in

67 Mestres, 1999: Arabic text in pp. 149–150 and commentary in p. 86.

68 Millás, 1943–50, pp. 97–99 (Arabic text), 133–135 (Spanish translation) and 228 (table).

69 Muḥammad ibn Hilāl al-Sabtī (fl. 1300–1350)? Lamrabet, 2013, M142, p. 182.

70 Mestres, 1999, pp. 150–151 of the Arabic edition.

71 Mestres, 1999, pp. 87–88 (commentary) and 287 (tables).

72 Neugebauer, 1949, pp. 349–360; Gandz, Obermann and Neugebauer, 1956, pp. 136–146.

73 Mestres, 1999, pp. 151–155 (Arabic edition), 88–91 (commentary), 275, 283, 287 and 289 (tables).

74 Vernet, 1952, pp. 62–64 (Arabic edition), 131–132 (Spanish translation).

- ms. Hyderabad 298 (table 156: Mestres, 1999, pp. 287 and 289) related to Ibn Ishāq's method.⁷⁵
9. Ibn al-Raqqām's *Mustawfī* (Ms. Rabat National Library 2461, chapter 58, pp. 208–210, 323–324) explains two methods for determining crescent visibility: the first one derives from al-Battānī⁷⁶ while the second is an approximation which uses a table giving, to varying degrees of accuracy, $\tan \varphi$ for values between 1° and 45° .
 10. Ibn al-Raqqām's *Shāmil* has been studied by E.S. Kennedy (1997, p. 53): the method is standard and it is followed by approximate rules for calculating the amount of light on the visible face of the Moon.
 11. Ibn al-Raqqām's *Qawīm*: the procedure has also been clarified by Kennedy (1997, pp. 38–41). It is different from the ones described in Ibn al-Raqqām's two other *zīj*es and it uses a table calculated for $\varphi = 37;10^\circ$, the value for Granada established by Ibn al-Raqqām himself.
 12. In his *Muwāfiq zīj*⁷⁷ Ibn 'Azzūz describes three different methods for the computation of lunar visibility.⁷⁸ The first one uses the table of *maqām al-ru'ya* calculated for the latitude of Fez. A very similar table appears in ms. Hyderabad 298 (Mestres, 1999, p. 288). It is a double entry table in which the vertical argument is the difference in longitude between the Moon and the Sun (between 9° and 22°), and the horizontal argument is the lunar latitude (from 1° to 5°). The second one uses another table which has the same structure as the one extant in the Latin version of al-Khwārizmī's *zīj* (see above no. 2) and it seems to have been calculated for the latitude of a city in the south of the Iberian Peninsula. The third method uses a third table, that is not dependent on the local latitude either, calculated using $90^\circ +$ the lunar true anomaly as an argument. In this chapter I will deal only with the second procedure.

2.3.3 *The Astronomical Problem*

All the aforementioned sources use the well-known Indian criterion according to which the new Moon will be visible if the difference in setting times between the Sun and the Moon is, at least, equal to 12 equatorial degrees. Al-Battānī justifies this limit of visibility on the grounds of the mean difference between solar and lunar velocity per day and uses a limit of $12;11^\circ$ instead.

75 Ibn al-Bannā' also wrote a work entitled *al-Manākh fī ru'yat al-ahilla*, which was probably the basis of Ibn al-Raqqām's *Ta'dil manākh al-ahilla*: see Bakli, 'Aysānī & Ilhām, 2014, p. 10.

76 Nallino, 1899–1907, vol. 1. pp. 85–92, 265–272; vol. 111, pp. 129–136; Samsó, 2014, pp. 309–310, 323–324.

77 Samsó, 1997, p. 79.

78 Ms. Rabat Hassaniyya 8772 p. 50; Ms. Rabat National Library 2461 p. 392.

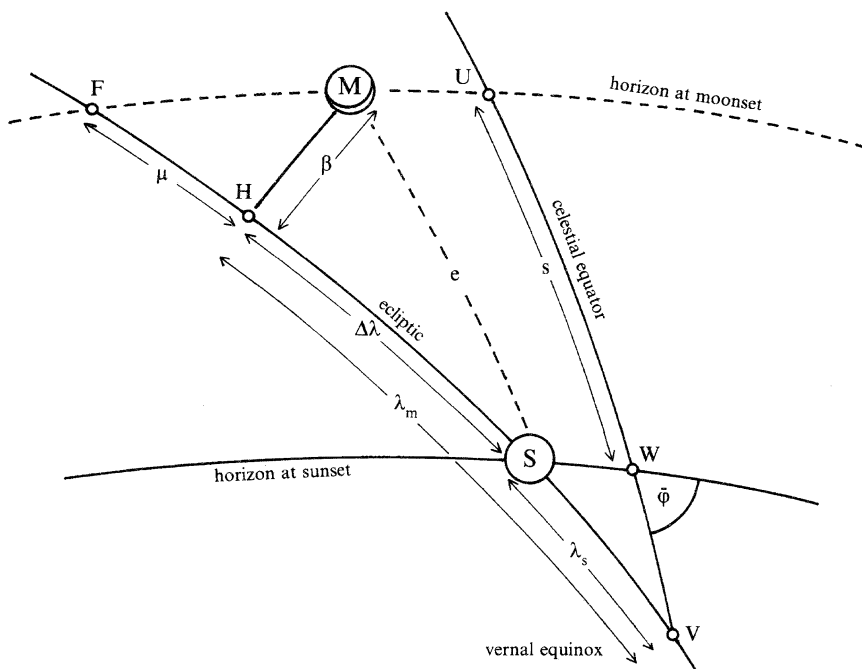


FIGURE 2.1 Crescent visibility according to D.A. King

Ibn al-Raqqām, in the *Mustawfī*, uses Ibn Ishāq's mean motion tables in which we find:

mean daily solar motion: $0;59,8,11^{\circ}$

mean daily lunar motion: $13;10,34,53^{\circ}$

The difference between these two values is:

$$12;11,26,42^{\circ} \approx 12;11,27^{\circ}$$

very near the limit of visibility used by Ibn al-Raqqām ($12;11,28^{\circ}$).

In Fig. 2.1, which I borrow from King, 1988 (p. 156),⁷⁹ S and M are the positions of the Sun and the Moon at sunset and arc MS is an arc of a great circle that passes through the bodies of the Sun and the Moon. SH ($\Delta\lambda$) is the difference in longitude between the two celestial bodies, MH (β) is the lunar latitude, UW (s) is an arc of the equator which corresponds to the difference in setting times between the Sun and the Moon. It is clear that this difference

79 See also King, 1987, pp. 186–189.

cannot be obtained by directly calculating the difference in oblique descensions between the ecliptic position of the Sun and the Moon, and that we need a correction which takes into account the lunar latitude. This correction is arc $FH (\mu)$ which can be calculated using the expression:

$$\sin \mu \approx \tan \beta \tan \varphi$$

if we assume that angle $\angle MFH = 90^\circ - \varphi$

Other approximations which we will find in the sources analysed are:

$$\begin{aligned} \mu &\approx \beta \tan \varphi \\ \text{or } \mu &\approx \beta \end{aligned}$$

We will, therefore, obtain a corrected lunar longitude (λ_{cM}), which will be arc VF , and we can calculate the difference in oblique descensions between the Sun and the corrected Moon:

$$\Delta\alpha_\varphi = \alpha_\varphi (\lambda_{cM} + 180^\circ) - \alpha_\varphi (\lambda_S + 180^\circ) = UW$$

The rules usually establish that the Moon will be visible if

$$\Delta\alpha_\varphi > 12^\circ$$

2.3.4 The Computation

2.3.4.1 The “Standard” Procedure

All the sources state that one must begin by computing the solar and lunar longitude for sunset of the *laylat al-shakk* (the night of doubt, which corresponds to the evening of the 29th day of the month). Some of them (Ibn al-Raqqām’s *Shāmil*, Ibn al-Bannā’ and Ibn ‘Azzūz) say that the computation should be made for half an hour after sunset.

Four sources (*Toledan Tables*, Maimonides, Ibn al-Raqqām’s *Shāmil*, Ibn al-Bannā’) begin their analysis of the problem by giving limits of visibility based only on $\Delta\lambda$. This criterion allows the user to avoid unnecessary computations. Thus, if the Moon is in one of the long time setting signs (between Capricorn and Gemini, that is, in the ecliptic semicircle containing the spring equinox):

- If $\Delta\lambda \leq 9^\circ$ the Moon will not be seen
- If $\Delta\lambda \geq 15^\circ$ the Moon will be seen
- If $9^\circ < \Delta\lambda < 15^\circ$ computation is necessary

If the Moon is in the ecliptic semicircle containing the autumn equinox (between Cancer and Capricorn):

If $\Delta\lambda \leq 10^\circ$ the Moon is not visible

If $\Delta\lambda \geq 24^\circ$ the Moon will be seen

If $10^\circ < \Delta\lambda < 24^\circ$ computation is necessary

This kind of criterion was applied by Ibn al-Bannā⁸⁰ on the occasion of the discussion on the visibility of the new Moon for the beginning of Ramaḍān of year 700H/ 1301. News reached the people of Marrakesh that the Muslims of Fez had begun their fasting on Wednesday 1st Ramaḍān of that year (10 May 1301), while in Marrakesh the new Moon had not been seen in spite of the good conditions of visibility. The sultan and his court, who were in Tlemcen, began their fasting the following day (Thursday) like the people of Marrakesh. Ibn al-Bannā calculated the solar and lunar longitude for the longitude of Fez and remarked that the difference of longitudes was less than 9° , both luminaries being in Taurus, which is one of the long time setting signs.⁸¹ He concluded from this that the Moon could not possibly have been seen on Wednesday evening (beginning at sunset of 9 May) in Fez.

To this, Ibn al-Raqqām's *Shāmil* adds a second simple criterion: the difference between the time of conjunction and sunset must be, at least, 15 equal hours.

If computation is necessary, we should calculate the lunar parallax in longitude and latitude and obtain λ'_M and β'_M , corrected in parallax. Interestingly, as we have already seen (cf. 2.3.2, source 4), the *Toledan Tables* contain (Pedersen, 2002, IV, 1486) a table of horizontal parallax for lunar visibility calculated for sunset, or soon after, at the moment of conjunction and for the latitude of Córdoba.

The *Toledan Tables*, Ibn Mu'ādh and Ibn al-Raqqām's *Qawīm* all introduce a second correction to the lunar longitude which begins by entering, with λ'_M as argument, a table⁸² which calculates $0;24 \cos \lambda'_M$ and returns a value m . Then:

80 Jabbār & Aballāgh, 2001, pp. 138–140.

81 This can easily be checked by computing the solar ($45;26,22^\circ$) and lunar ($53;46,27^\circ$) sidereal longitudes for six hours after midday of 9 May 1301. with the parameters of Ibn Ishāq which were those used by Ibn al-Bannā: the difference of longitudes is $8;20,5^\circ$. Ibn Ishāq's radices are calculated for the longitude of Toledo (28°), while the longitude of Fez used by Ibn al-Bannā is 25° . The time difference between these two cities is only 12 minutes.

82 *Tabula reflexionis Lunae* in the *Toledan Tables* (Pedersen, 2002, IV, 1486); Mestres, 1999, p. 287; ms. 260 of the Rabat National Library p. 105.

$$\beta'_M \cdot m$$

and $\lambda'_M + \beta'_M \cdot m = \lambda''_M$

Maimonides uses the same correction although he works with the elongation ($\Delta\lambda'$, corrected in parallax) and obtains:

$$\Delta\lambda'' = \Delta\lambda' + \beta'_M \cdot m$$

Obviously, the value of λ'_M will rise if the Moon is between Capricorn and Cancer and fall in the other half of the ecliptic if the lunar latitude is positive. For a negative latitude, the situation will be reversed.⁸³

A third standard correction is based on $\tan \varphi$, and modifies the lunar latitude component. Some sources (Ibn Mu'ādh, Ibn al-Raqqām's *Qawīm*) apply the correction to λ''_M and obtain:

$$\lambda'''_M = \lambda''_M + \tan \varphi \cdot \beta'_M$$

Others (e.g., the *Toledan Tables*) apply the third correction to the difference in oblique descensions between the corrected position of the Moon and the position of the Sun:

$$\Delta \alpha_\varphi = \alpha_\varphi (\lambda''_M + 180^\circ) - \alpha_\varphi (\lambda_S + 180^\circ) + 4/5 \beta'_M$$

where $4/5$ corresponds, approximately, to $\tan 38;40^\circ$, the latitude of Córdoba.

Maimonides follows the same technique as the *Toledan Tables*, although he works with the elongation ($\Delta\lambda''$) to which he applies a correction (c) corresponding to the difference between 30° and the setting time of the corresponding sign. In this way his $\Delta\lambda'''$ becomes the difference in setting times between the Moon and the Sun, which will be less than 30° :

$$\Delta\lambda''' = \Delta\lambda'' + c$$

Then, Maimonides applies the third correction in order to obtain b , the “arc of vision”:

$$b = \Delta\lambda''' + \tan \varphi \cdot \beta'_M$$

83 See Kennedy, 1997, pp. 38–39.

TABLE 2.1

Latitude (φ)	$\tan \varphi$
22°	0.40
27°	0.5
30°	0.57
33°	0.66

where he uses $\tan \varphi \approx 2/3$, which corresponds approximately to the latitude of Jerusalem ($\approx 33^\circ$).

We find approximate values of this kind for the tangents of the local latitude in other sources. We have already seen, in the *Toledan Tables*, the use of $4/5$ for the latitude of Córdoba ($\varphi \approx 38;40^\circ$). The same source gives the following set of approximations, which correspond roughly to the beginning and the end of climates II and III (see Table 2.1).

In Ibn al-Zarqālluh’s *Almanac* we find a description of an approximate procedure with only one correction of the lunar longitude, which corresponds to:

$$\lambda'_M = \lambda_M + 5/6 \cdot \beta_M$$

where $5/6 \approx \tan 40^\circ$ (the latitude of Toledo). In Ibn al-Raqqām’s *Qawīm zīj* we find a correction of

$$5 \tan \varphi = 3;48,45$$

in which 5 is the maximum lunar latitude and the φ involved is $37;10^\circ$, the latitude of Granada.

In some *zīj*es we find tables that make the computation easier. This is what happens in ms. Hyderabad 298 where, in the procedure attributed to Ibn Mu‘ādh and al-Sabtī, there is a reference to a table (see Mestres, 1999, pp. 87–88 and 287) in which the argument is the argument of latitude and the values (only the first two are extant, the others being blank) should correspond to $\tan \varphi \cdot \beta'_M$.

Finally, in Ibn al-Raqqām’s *Mustawfī* we find a description of an approximate method with only one correction (c), as it uses:

$$\begin{aligned} \Delta\alpha_\varphi &= \alpha_\varphi(\lambda_M + 180^\circ) - \alpha_\varphi(\lambda_S + 180^\circ) \\ c &= \Delta\alpha_\varphi + n(\varphi) \end{aligned}$$

TABLE 2.2

Latitude	Ms. values	Recomputation
1	0;1	0;1,3
5	0;5	0;5,15
10	0;11	0;10,35
15	0;16	0;16,5
20	0;22	0;21,50
25	0;27	0;27,59
30	0;34	0;34,38
35	0;41	0;42,1
40	0;51	0;50,21
45	0;57	1;0,0
50	1;0	1;11,30

where $n(\varphi)$ should be found in the *Jadwal nisbat al-'arḍ fi ru'yat al-ahilla* (table of the ratio of latitude for lunar visibility).⁸⁴ It is a table in which the argument is the local latitude, from 1° to 90°, with an interval in the argument of 1°. The computed values correspond, approximately, to $\tan \varphi$, calculated for a radius = 1, from 1° to 45° of the argument. From 50° onwards the values are always 1;0°. It seems that the author of the table interrupted his calculation after 45°, as it does not make much sense to bear in mind latitudes greater than 45°. I reproduce an excerpt of this table here (see Table 2.2).

2.3.4.2 Other Criteria

2.3.4.2.1 *Al-Khwārizmī-Maslama and Ibn 'Azzūz*

Some sources use different criteria and computational techniques. One of them is the Latin translation of the *zīj* of al-Khwārizmī-Maslama where the procedure and the table used has been studied by Kennedy (1965), King (1987, pp. 192–197) and Hogendijk (1988 a, pp. 32–35). It seems clear that the table has no relation to al-Khwārizmī's original *zīj* and it is probably a later interpolation in Maslama's version; it has been calculated for an obliquity of the ecliptic of 23;35° and a local latitude of 41;35°, which corresponds to Zaragoza, a city where there was significant scientific activity during the eleventh century. The canons⁸⁵ state that one must find the arc SF (fig. 2.1) with the approximation:

84 Ms. 2461 Rabat National Library, p. 286

85 Suter, 1914, pp. 16–17; Neugebauer, 1962, pp. 42–44.

$$SF \approx \lambda_M - \lambda_S + \beta_M$$

and then compare the result with the value obtained in the table ($T(\lambda_M)$). The Moon will be visible if

$$SF \geq T(\lambda_M)$$

The argument of the table is the lunar longitude and it has been calculated with a ten degree interval. The table has a strict symmetry, which does not correspond to the actual values obtained in the computation, although the difference is small. Hogendijk has established that the table was calculated using the following expression:

$$\mu = \alpha_\varphi^{-1}(\lambda_M + 180^\circ + 12^\circ)$$

$$T(\lambda_M) = \mu - \lambda_M - 180^\circ$$

A very similar table appears in the *Muwāfiq Zīj* of Ibn ‘Azzūz (d. 1354). I have made several attempts to recompute it, without much success, although it seems clear to me that the table was not calculated for a latitude of 33;40°, the value used by Ibn ‘Azzūz for the latitude of Fez.⁸⁶ Given the fact that Ibn ‘Azzūz borrows materials derived from Andalusian astronomers like Ibn Mu‘ādh al-Jayyānī (d. 1093), Ibn al-Raqqām (d. 1315) and others, my guess is that the table was calculated for an Andalusian city and, as an example, I edit the first eighteen values of the table together with a recomputation for three different latitudes:

- a) 37;10°: the latitude of Granada, established by Ibn al-Raqqām, and for an obliquity of the ecliptic of 23;32,40° used by Ibn al-Raqqām in the *Shāmīl* and in the *Qawīm zīj*es.
- b) 38;30°: the most frequent value for the latitude of Córdoba, with $\epsilon = 23;33^\circ$.
- c) 38°: Ibn Mu‘ādh’s value for the latitude of Jaén. As we do not know which value of ϵ this astronomer used, I have recomputed the table for $\epsilon = 23;33^\circ$ and for $\epsilon = 23;51^\circ$.

The table (Mss. Rabat General Library D2461 (p. 393) and Rabat Hassaniyya Library 8772 (p. 50).

86 Samsó, 1997. Interestingly, in the *Muwāfiq* Ibn ‘Azzūz copies a table of oblique ascensions and the length of the seasonal hours for the latitude of Granada which we also find in Ibn al-Raqqām’s *Qawīm zīj*.

TABLE 2.3

Arg.	Ibn ‘Azzūz	Recomp.	Recomp.	Recomp.	Recomp.
		$\varphi = 37;10^\circ$ $\varepsilon = 23;32,40^\circ$	$\varphi = 38;30^\circ$ $\varepsilon = 23;33^\circ$	$\varphi = 38^\circ$ $\varepsilon = 23;33^\circ$	$\varphi = 38^\circ$ $\varepsilon = 23;51^\circ$
0°	9;45°	9;50°	9;43°	9;45°	9;45°
10°	9;45°	9;48°	9;41°	9;43°	9;43°
20°	9;38°	9;44°	9;37°	9;40°	9;39°
30°	9;38°	9;40°	9;34°	9;36°	9;35°
40°	9;38°	9;38°	9;32°	9;35°	9;33°
50°	9;38°	9;41°	9;36°	9;38°	9;36°
60°	9;58°	9;50°	9;37°	9;20°	9;45°
70°	10°	10;36°	10;7°	10;8°	10;5°
80°	10;17/57°	10;42°	10;40°	10;32°	10;39°
90°	11;5°	11;27°	11;29°	11;29°	11;27°
100°	12;0°	12;29°	12;34°	12;32°	12;32°
110°	12;52°	12;30°	13;52°	13;48°	13;50°
120°	15;18°	15;4°	15;19°	15;13°	15;17°
130°	16;0°	16;25°	16;45°	16;38°	16;45°
140°	17;8°	17;40°	18;4°	17;55°	18;3°
150°	18;57°	18;39°	19;6°	18;56°	19;7°
160°	19;31°	19;16°	19;45°	19;34°	19;45°
170°	20;0°	19;28°	19;57°	19;47°	19;57°

2.3.4.2.2 *Arc of Light*

Ibn Ishāq⁸⁷ begins by calculating the arc MS (fig. 2.1), the arc of a great circle which passes through the centres of the solar and lunar bodies, by using Pythagoras' theorem:

$$e \approx \sqrt{(\sin^2 \Delta\lambda + \sin^2 \beta_M)} \text{ qaws } al-nūr \text{ ("arc of light")}$$

This computation can be made directly using table 154 of ms. Hyderabad 298 (Mestres, 1999, p. 288): it is a double entry table in which the vertical argument is e (tabulated between 8° and 25°, with an interval of 1°) and the horizontal argument is the lunar latitude (from 1° to 5°).

87 See Mestres, 1999, pp. 151–155 (Ar. text) and 88–91 (comm.)

The same arc is calculated by Ibn al-Zarqālluh⁸⁸ and Ibn al-Bannā' (who calls it *maqām al-ru'ya*) using an even cruder approximation:

$$e \approx \sqrt{(\Delta\lambda^2 + \beta_M^2)}$$

Ibn al-Raqqām, in the *Mustawfī*,⁸⁹ calls *e qaws al-bu'd* and calculates it using the cosine law:

$$e = \cos^{-1}(\cos \Delta\lambda \cdot \cos \beta_M)$$

The value of *e* determines the width of the crescent,⁹⁰ but the aforementioned sources do not establish a criterion for lunar visibility on the basis of the illuminated part of the lunar disk. In spite of this, the last column of table 154 of the Hyderabad ms. tabulates the *aṣābi' al-nūr* ("digits of light") which seem to be a function of $\Delta\lambda$ and not of *e*, as one should expect: the minimum tabular value is 0;32° (for $\Delta\lambda = 8^\circ$) and its maximum is 1;40° (for $\Delta\lambda = 25^\circ$). A crude rule for the computation of the digits of light (*D*) appears in Ibn al-Raqqām's *Shāmil Zīj*:⁹¹

$$D = 0;4 \cdot \Delta\lambda$$

The value will be more precise with the following correction:

$$D' = D + 5 / \beta_M \cdot 0;4$$

The first part of the rule seems to be quite old, as it appears in a more refined formulation which uses *e* instead of $\Delta\lambda$, in a quotation of Sind b. 'Alī (fl. Baghdad ca. 830):⁹²

$$D = 0;4 \cdot e$$

To which Sind b. 'Alī adds that the Moon will be visible if $40' \leq D \leq 50'$.

88 Millás, 1943–50, p. 99.

89 Ms. Rabat National Library 2461, pp. 208–209.

90 On the use of the digits of light as a visibility criterion, see Hogendijk, 1988b.

91 Ms. Kandilli 249, chapter 137, fol. 55r. See Kennedy, 1997, p. 52.

92 Sezgin, *GAS* VI, p. 138; Hogendijk, 1988b, p. 96 fn. 8.

2.3.4.2.3 Lunar Velocity

This criterion is used by Ibn al-Zarqālluh, Ibn Ishāq and Ibn al-Bannā':

After computing the value of arc e , Ibn Ishāq enters table 156, *Jadwal al-musaṭṭah al-maqsūm 'alā qaws al-nūr*,⁹³ with the true lunar anomaly (α_v) as the argument. This table can also be found in Ibn al-Bannā's *Minhāj*⁹⁴ and it yields:

$$a = 144;30 + 4 \cos \alpha_v$$

144;30 seems to be the difference, expressed in minutes, between maximum and minimum lunar velocity per day, which is a function of the true lunar anomaly.⁹⁵ However I have not been able to find the source of this value, which does not agree with Ibn Ishāq's maximum and minimum per hour (0;36,1° and 0;30,21°)⁹⁶ which give a difference of 2;16° = 136' per day.

The table is calculated for arguments between 1° and 180°, with an interval of 1°, and diminishes between 148;30 (for $\alpha_v = 1$, near the apogee of the epicycle, when the lunar velocity reaches its minimum) and 140;30 (for $\alpha_v = 180^\circ$, in the perigee, where the velocity attains its maximum). In Ibn al-Zarqālluh's *Almanac* there is no table for the computation of a , but the Toledan astronomer gives instructions which are equivalent to the following formula:

$$a = 134' + 6/5 \text{ vers } \alpha_v$$

Ibn al-Zarqālluh, Ibn Ishāq and Ibn al-Bannā' calculate the *qaws al-ru'ya* ("arc of vision", q_r) by dividing e (the *qaws al-nūr* also called *maqām al-ru'ya* or *qaws al-bu'd*, see 2.3.4.2.2) by a :

$$q_r = e / a$$

In the simplified procedure described by Ibn al-Bannā', q_r will be compared to $\Delta\alpha_\varphi$, the difference in oblique descensions between the lunar (corrected in parallax) and the solar longitudes. Then:

If $\Delta\alpha_\varphi > q_r$ the Moon will be visible

If $\Delta\alpha_\varphi < q_r$ the Moon will not be visible

If $\Delta\alpha_\varphi = q_r$ the Moon will be in the limit of visibility

93 Mestres, 1999, p. 289.

94 Unnumbered ms. of the Museo Naval of Madrid fol. 13v.

95 Goldstein, 1996.

96 Goldstein, 1996, p. 190; Mestres, 1996, p. 420.

Ibn Ishāq operates in the same way but introduces a series of corrections in the value of $\Delta\alpha_\varphi$ which differ somewhat from what I have called “the standard procedure” (see 2.3.4.1).⁹⁷ As in Ibn al-Bannā’s procedure, he corrects λ_M in lunar parallax and obtains λ'_M . Then he obtains β'_M , also corrected in parallax, and adds:

$$\lambda'_M + \beta'_M = \lambda''_M$$

The following step is the difference in oblique descensions:

$$\Delta\alpha_\varphi = \alpha_\varphi (\lambda''_M + 180^\circ) - \alpha_\varphi (\lambda_S + 180^\circ)$$

$\Delta\alpha_\varphi$ will again be corrected, in equatorial degrees, by calculating the difference between the half day arc (D) of λ'_M and λ''_M , which he calls *ikhtilāf al-ufq (i)*:

$$i = D (\lambda''_M) - D (\lambda'_M)$$

i will be added to (if β'_M is positive) or subtracted from (if β'_M is negative) $\Delta\alpha_\varphi$:

$$\Delta'\alpha_\varphi = \Delta\alpha_\varphi \pm i$$

The next correction is difficult to understand: the canons give the instruction to enter, with the distance of the Moon from the nearest solstice⁹⁸ as argument, table 150 (*al-Jadwal al-thānī min ikhtilāf nisbat mamarrāt al-kawākib al-thābita fī dawā'ir intiṣāf nahār kull kawkab*, “Second table of the divergence in the ratio of passage through the meridian of the celestial bodies”).⁹⁹ This table is calculated from $0;30^\circ$ to 90° with an interval in the argument of $0;30^\circ$. It is in a very bad condition, with many corrupt values, and it seems to calculate $0;26 \cdot \tan d$ (d being the argument), where $0;26$ could be $\tan 23;33^\circ$. This, obviously, reminds us of the correction $0;24 \cos \lambda'_M$ we found in 2.3.4.1. The value obtained from the table will be the *ḥāṣil al-mamarr (h)*. Then he multiplies:

$$h \cdot \beta'_M$$

97 The method described by Ibn al-Zarqālluh is really difficult to understand and I will not try to explain it. It involves the computation of the mediation (*tawassuṭ*) of the lunar disk at sunset using a procedure which seems corrupt. See Samsó, 2008b, pp. 401–404.

98 If the distance is $< 90^\circ$; if it is $> 90^\circ$, one has to subtract the value from 180° and operate with the remainder.

99 Mestres, 1999, pp. 275 and 283.

and adds or subtracts the result to or from $\Delta'\alpha_\varphi$:

$$\Delta''\alpha_\varphi = \Delta'\alpha_\varphi \pm h$$

curiously enough, Ibn Ishāq does not seem to introduce any correction dependent on the geographical latitude.

The Moon will be visible if

$$\Delta''\alpha_\varphi > q_r$$

2.4 The Hour

Being able to tell the time during both day and night was a major concern in the Middle Ages. Muslims needed to establish daily the moment of the five canonical prayers¹⁰⁰ and much the same can be said about Christian monks who had communal prayers at fixed times of the day or of the night.¹⁰¹ Therefore, in the old tenth-century collection of Latin texts on the astrolabe and other astronomical instruments it is not surprising to find a defence of astronomy (always viewed with suspicion because of its association with astrology) based on its applications for timekeeping which allow the computation of the date of Easter and the determination of the time of prayers.¹⁰² The instruments described in this corpus will mainly be used to determine the hour: a set of chapters on the use of the astrolabe begin with the mention « Incipiunt capitula *orologii* regis Ptolomei »; a treatise on the celestial sphere opens with an « Incipit de *horologio* secundum alkoram id est speram rotundam », and one of the texts related to the quadrant called *vetustissimus* begins with « componitur *orologium* cum astrolabii quarta parte ». Finally, when stars are mentioned, the standard justification of the need to know them is that they serve to tell the time during the night.

2.4.1 Computational Methods for Telling the Time

2.4.1.1 Introduction

Instruments were used for telling the time, but there also was a rich variety of computation methods to fix a given moment of the day based on the solar

¹⁰⁰ King, 2004.

¹⁰¹ See McCluskey, 1998.

¹⁰² The texts of this old collection were edited by Millàs, 1931. A detailed analysis of its contents can be found in Samsó, 2004b. See also Borelli, 2008.

altitude, the shadow projected by the Sun throughout the day, taking into consideration the local latitude and the solar longitude on the ecliptic or, else, the date of the year. We know of Abbasid tables giving the hour as a function of the shadow length in each day and hour of the year, which were calculated for the latitude of Baghdad, but we do not have sources of this kind in al-Andalus. I only know, in a late source (the fifteenth-century *Tratado de astrología* attributed to Enrique de Villena), a table which allows the determination of the hour as a function of the solar altitude throughout the months of the year, for a latitude of 38° (Córdoba) and an obliquity of the ecliptic of 24°. I believe this table was copied from a very old Arabic source which uses an approximate method of computation.¹⁰³ As for the Maghrib, I am only aware of a table giving the solar altitude and the length of the shadow ($g = 12$) for hours 1–6 of the days in which the Sun enters the zodiacal signs and for the latitude of Fez, extant in Ibn ‘Azzūz al-Qusantīnī’s *Muwāfiq zīj* (see below Table 2.9 in 2.4.1.3).

As a result of this lack of Andalusian sources specifically related to the problems of *mīqāt* (timekeeping), my analysis will be based, on the one hand, on books on folk-astronomy and, on the other, on *zīj*es.

The situation in the Maghrib is entirely different. As a result of the research made by David King throughout his scholarly life we have a great deal of information on Maghribī tables for timekeeping which seem to be independent from its Eastern counterparts. We know of the existence of an anonymous corpus of tables for timekeeping calculated for the latitude of Tunis (37°) and dedicated to the Ḥafsid ruler Abū Fāris al-Mutawakkil (r. 1394–1434), as well as another late (seventeenth- or eighteenth-century) set of tables for Sfax. To this one should add collections of prayer tables for Tunis (lat. 36;40°), Tlemcen (lat. 35°), Algiers (lat. 36;40°), and probably Fez (lat. 34;10°),¹⁰⁴ and the *urjūzas* of Abū Miqrā’ and al-Jādirī and the corresponding commentaries mentioned above in 2.0.

Muslim scholars were mainly interested in the determination of prayer times, and I will summarise here the periods during which the canonical prayers should be performed:¹⁰⁵

Maghrib: between sunset and nightfall.

Ishā’: between nightfall and daybreak.

Fajr: between daybreak and sunrise.

103 Catedra & Samsó, 1983, pp. 67–69, 222–227; King, 2004, p. 185.

104 King, 2004, pp. 427–436.

105 King, 2004, pp. 203–205, 428, 547–549.

Zuhr (in the Andalusī-Maghribī tradition): between the moment in which the increase of the shadow over its midday minimum is one fourth of the length of the gnomon, and the beginning of the ‘*aṣr*’ prayer. If S_m is the meridian shadow, S_z the shadow at the beginning of the *zuhr* prayer and g the length of the gnomon:

$$S_z = S_m + g / 4$$

Aṣr: when the shadow of the gnomon equals its meridian shadow increased by the length of the gnomon. If S_a is the shadow at the beginning of the ‘*aṣr*’:

$$S_a = S_m + g$$

According to al-Jādirī,¹⁰⁶ the shadow for the end of the ‘*aṣr*’ prayer (S_{a2}) should be:

$$S_{a2} = S_m + 2g$$

As well as these five prayers which are considered canonical, Andalusī and Maghribī scholars are interested in two other times during the day:

Ḍuhā: a prayer performed at a time before midday equal to the time after midday of the beginning of the ‘*aṣr*’ prayer. Thus, if S_d is the shadow at the moment of *ḍuhā*:

$$S_d = S_m - g$$

Ta’hib: a prayer performed one equinoctial hour before midday.¹⁰⁷

The aforementioned criteria allow us to understand the importance of:

1. The duration of dawn and twilight, which determines the period of time during which the *fajr* and *maghrib* prayers should be performed.
2. The length of the shadow projected by a gnomon which allows Muslims to establish the moment of the beginning of the *zuhr*, ‘*aṣr*’ and *ḍuhā* prayers. The meridian shadow is the starting point for all three.

¹⁰⁶ Calvo, 2004, p. 191.

¹⁰⁷ The anonymous Tunisian set of tables for timekeeping (late fourteenth-century) contains tables which display the solar altitude at the moment of *ta’hib*. See King, 2004, pp. 579–580.

The gnomon, in medieval shadow tables, is usually considered to measure 12 digits, 7 feet or $6\frac{2}{3}$ feet. There are also instances in which we find the gnomon divided into 6, $6\frac{1}{3}$, $6\frac{2}{3}$, or $6\frac{2}{5}$ parts.¹⁰⁸

2.4.1.2 Folk-astronomy Sources

2.4.1.2.1 *Shadow Schemes*

In sources of this kind we often find the values of the length of the solar meridian shadow through the year.¹⁰⁹ In 2.4.1.2.2 we will see that the meridian shadow is needed in one of the expressions used to calculate the hour using shadows. One of the aforementioned sources is the *Kitāb al-Hay'a* of Qāsim b. Muṭarrif al-Qaṭṭān (ca. 950).¹¹⁰ The values which appear in the text correspond to a gnomon of 12 digits, and they oscillate between a minimum of 3.5 digits and a maximum of 23.5 digits, which correspond to a latitude of Córdoba somewhere between $38;22^\circ$ and 39° , depending on the value of the obliquity of the ecliptic used. The *Calendar of Córdoba* also gives the value of the meridian solar altitude¹¹¹ and the meridian shadow twice a month: the dates correspond to the entry of the Sun in each zodiacal sign and its passage through the middle of the sign. The local latitude implied is $37;30^\circ$ (a relatively unusual value for Córdoba, although it appears in one of the manuscripts of the *Toledan Tables*¹¹²) and the obliquity of the ecliptic is $23;50^\circ$ (a rounded value of the Ptolemaic $23;51,20^\circ$). Interestingly, the gnomon used for the computation of the meridian shadows is $g = 1$, which probably implies that these shadows were intended for the computation of prayer hours using the height of the observer as a gnomon. These values, however, seem to derive from another table computed for $g = 12$, as 19 of the 25 values which the *Calendar* gives using fractions become entire numbers when multiplied by 12. On the other hand, the shadows in the *Calendar* have not been computed using a shadow table like the one appearing in al-Khwārizmī-Maslama's *zīj* and they seem to derive from a table, similar to the Greek shadow tables studied by Neugebauer.¹¹³ Below is

108 King, 2004, pp. 495–496, 501–502. See also al-Jādirī's commentary to Abū Miqra's *Urjūza*, ms. Escorial 361 fol. 147v.

109 King, 1990.

110 See Casulleras, 1998.

111 A much less precise set of values for the solar meridian altitude for a latitude $38;30^\circ$ (Córdoba) and an obliquity of the ecliptic of ca. $23;30^\circ$ appears in an appendix to the treatise on mechanics entitled *Kitāb al-asrār fī natā'ij al-aḥkār* written by Ibn Khalaf al-Murādi, presumably a Toledan scientist of the eleventh century. See King, 1978, pp. 367 fn. 22, 388–389; Casulleras, 1996, p. 617.

112 Toomer, 1968 p. 136 fn 3.

113 Neugebauer, 1975, II, pp. 736–748.

an attempt to reconstruct the original values corresponding to the beginning of each sign:

I	II	III	IV	V	VI
XII	XI	X	IX	VIII	VII
21	16	11 ¹¹⁴	7	4;30	3 ¹¹⁵

Shadow schemes often appear in *anwā'* books, such as the one written by al-Umawī al-Qurṭubī (d. 1205–1206)¹¹⁶ which contains two different collections of values. The first is a derivation of a scheme, appearing often in Latin and Arabic texts, which was originally calculated for Athens whose latitude (ca. 38°) can also be used for Córdoba. This scheme is more complete than the one we have seen in the *Calendar of Córdoba*, for it gives the length of the shadow for each unequal hour of the day, using the corresponding symmetries:

I/XI	II/X	III/IX	IV/VIII	V/VII	VI

Unfortunately, the sole surviving manuscript contains only these values for the first three months of the Julian year (January–March).¹¹⁷

The second collection gives the meridian shadow every five days of the year, for a gnomon of 12 digits.

Parts of the first scheme reappear in the *Kitāb al-anwā'* of Ibn al-Bannā' (d. 1321),¹¹⁸ although he only gives the values of the shadow for the prayers of the *ḡuhr* and the *ʿaṣr*:

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
<i>ḡuhr</i>	7	6	5	4	3	3	3	4	5	7	9 1/5	11
<i>ʿaṣr</i>	14	13	11	10	9	8	8	10	10	11	15	15

This scheme was revised by Neugebauer¹¹⁹ in the following way:

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
<i>ḡuhr</i>	7	6	5	4	3	[2]	3	4	5	[6]	[7]	[8]
<i>ʿaṣr</i>	14	13	11	10	9	8	[9]	10	[12]	[13]	[14]	15

¹¹⁴ 11;15 in the text.

¹¹⁵ 3;18 in the text.

¹¹⁶ Forcada, 1992, pp. 187–189.

¹¹⁷ Forcada, 1994; King, 2004, p. 492.

¹¹⁸ There are actually three different sets of values, found in different manuscripts used by Renaud, 1948. In the following lines I reproduce the third one.

¹¹⁹ Neugebauer, 1975, II, pp. 743–744.

A similar set of values can also be found in the anonymous and undated (probably thirteenth century) *Risāla fī awqāt al-sana*.¹²⁰ In another undated source (although the extant manuscript was copied ca. 1400), a treatise on folk-astronomy written by an otherwise unknown Abū ‘Abd Allāh al-Mu’addib gives the shadow lengths in digits ($g = 12$) for each seasonal hour and for the *zuhr* and the ‘*aṣr*’ prayers for each day when the shadow is a whole number. The latitude implied is about 35° or 36° and $\varepsilon = 23;30^\circ$.¹²¹

Finally Abū Miqra¹²² gives, in a verse, the length of the meridian shadow for each month of the solar year, beginning with January and, apparently, for a gnomon of 7 feet and for the latitude of Marrakesh ($\varphi = 31;30^\circ$):

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
9	7	5	3	2	1	1	2	4	5	8	10

And al-Jādirī¹²³ gives another set of values calculated for the latitude of Fez ($\varphi = 34;10^\circ$):

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
10	8	5	3	1	1	2	3	4	5	8	10

2.4.1.2.2 General Formulas for Time Reckoning

Folk-astronomy sources often use or explain computational techniques for time reckoning. The *Calendar of Córdoba*,¹²⁴ for example, offers fairly accurate values of the length of daylight, expressed in equinoctial hours, twice a month. The recomputation gives adequate results when using a formula which is implicit in al-Battānī’s *zīj*.¹²⁵

$$\text{Length of the day} = [90^\circ + \sin^{-1}(\tan \delta \cdot \tan \varphi)] \cdot 2/15$$

where, again, the local latitude (φ) is $37;30^\circ$ and the solar declination (δ) is based on an obliquity of the ecliptic of $23;50^\circ$.

¹²⁰ Navarro, 1990.

¹²¹ King, 2004, pp. 498–500.

¹²² Ms. Escorial 954 fol. 104r; Escorial 361 fol. 148r. Muḥammad al-Marghīthī, *al-Mumti‘* ed. Fez, 1313, p. 94.

¹²³ Ms. Escorial 361 fol. 148v. The values which follow correspond to what I read in the manuscript. King, 2004, pp. 495–496 ascribes to al-Jādirī the following series: 10, 8, 5, 3, 2, 1, 1, 2, 4, 6, 8, 10.

¹²⁴ Samsó, 1983. Reprinted in Samsó, 1994 a (no. v), and in Samsó, 2008a (no. VII).

¹²⁵ Nallino, 1899–1907, I, pp. 22 and 180.

This expression is simplified by Ibn al-Bannā' in his *Kitāb fī 'ilm al-awqāt bi l-ḥisāb*¹²⁶ and by al-Jādirī in the *Rawḍat al-azhār*¹²⁷ as well as in his *Iqtīṭāf al-anwār min Rawḍat al-azhār*.¹²⁸ It is interesting that al-Jādirī gives, in the *Iqtīṭāf*, an exact solution for the computation of the length of daylight,¹²⁹ in the form:

$$\text{Length of the day} = \alpha_{\varphi}(\lambda + 180^{\circ}) - \alpha_{\varphi}(\lambda)$$

λ being the solar longitude for a particular date and α_{φ} the oblique ascension corresponding to a given value of λ .

However, the expression used in the *Rawḍa* is mistaken:¹³⁰

$$\begin{aligned} \text{Length of the day} &= \alpha_{\varphi}(\lambda) - \alpha_{\circ}(\lambda) \\ \text{as } e &= \alpha_{\varphi}(\lambda) - \alpha_{\circ}(\lambda) = \tan \delta \cdot \tan \varphi \end{aligned}$$

In the aforementioned simplifications the length of daylight, expressed in time-degrees, can be calculated by:

$$180^{\circ} + [(11 \cdot 12 \tan \varphi \cdot \delta) / 60]$$

This simplification avoids the use of sine and tangent tables, and the computation of the day arc is based on a few simple arithmetical operations in which the *muwaqqit* is only required to know the tangent of the local latitude. The procedure assumes that:

$$\begin{aligned} \sin^{-1}(\tan \delta \cdot \tan \varphi) &\approx \tan \delta \cdot \tan \varphi \\ 12 \tan \delta &\approx 0;11 \cdot \delta^{131} \end{aligned}$$

Al-Jādirī adds two more approximate procedures. The first one appears in the *Qatf*:¹³²

¹²⁶ Calvo, 2004, p. 184.

¹²⁷ Saidi, 2013, pp. 179–182.

¹²⁸ Calvo, 2004, pp. 186–187.

¹²⁹ Calvo, 2004, p. 187.

¹³⁰ Saidi, 2013, pp. 187–189.

¹³¹ $0;11 \cdot \delta$ appears also in a recension of the 13th century Egyptian *Muṣṭalaḥ Zij*: see King, 2004, p. 155.

¹³² Calvo, 2004, p. 187. The procedure is also explained by al-Ḥabbāk in his commentary to the *Rawḍa* (Saidi, 2013, pp. 189–191), as well as by Abū l-Ḥasan al-Muqri' (fl. 1384): see Baklī, 'Aysānī & Ilhām, 2014, p. 12.

Length of the day = $180^{\circ} + \delta \cdot \varphi / \varepsilon$

The second one, valid only for the latitude of Fez, can be found in both the *Rawḍa* and the *Qatf*:¹³³

Length of the day = $180^{\circ} + \delta + \delta / 2 - 1$

As it is difficult to find a rationale for these two procedures, I will limit myself to calculating the value, in degrees, of the length of daylight for every 5° of solar longitude between 0° and 90°, with $\varepsilon = 23;32,40^{\circ}$ and $\varphi = 33^{\circ}$, using an exact solution and the three approximate methods mentioned above (see Table 2.4).

TABLE 2.4

λ	$180^{\circ} + \sin^{-1}(\tan \delta \cdot \tan \varphi)$	$180^{\circ} + [(11 \cdot 12 \tan \varphi \cdot \delta) / 60]$	$180^{\circ} + \delta \cdot \varphi / \varepsilon$	$180^{\circ} + \delta + \delta / 2 - 1$
0°	180°	180°	180°	179;0°
5	182;30	182;51	182;48	182;0
10	185;15	185;41	185;35	184;59
15	187;45	188;29	188;19	187;54
20	190;15	191;13	191;0	190;47
25	192;45	193;53	193;37	193;35
30	195;15	196;27	196;9	196;17
35	197;30	198;56	198;34	198;53
40	199;45	201;16	200;52	201;20
45	202;0	203;26	202;59	203;36
50	204;0	205;27	204;58	205;44
55	206;0	207;17	206;46	207;39
60	207;45	208;54	208;22	209;21
65	209;15	210;20	209;46	210;51
70	210;30	211;30	210;54	212;5
75	211;30	212;26	211;49	213;3
80	212;15	213;6	212;28	213;45
85	212;45	213;30	212;52	214;11
90	213;0	213;39	213;0	214;20

133 Saidi, 2013, pp. 183–187.

Qāsim b. Muṭarrif's *Kitāb al-hay'a* also offers explanations which allow the computation of the number of hours (t) elapsed from sunrise, in the morning, or before sunset, in the afternoon:

$$t = (S / 12 - S_m) / 72$$

where S is the length of the shadow projected by a gnomon of 12 digits at a given moment and S_m is the meridian solar shadow of the corresponding day. The aforementioned expression does not seem to have any meaning and I believe it is a corruption of another formula appearing in an eighth-century Arabic source¹³⁴ and which is also used by Ibn al-Bannā,¹³⁵ Abū Miqrā¹³⁶ (for a gnomon of 7 feet) and by al-Jādirī in the *Rawḍa*¹³⁷ and in the *Iqtīṭāf*.¹³⁸

$$t = 72 / (S - S_m + 12)$$

in which $72 = 6 \cdot 12$, 12 being the length of the gnomon (g). This expression can, therefore, be formulated in a more general way:

$$t = 6 g / (\Delta S + g)$$

where $\Delta S = S - S_m$.

In both his works, al-Jādirī also uses another approximate expression that seems to have been widely applied during the Middle Ages, which is the basis of the procedure used with the sine quadrant for reckoning the hour:

$$\sin t = \sin h / \sin h_m$$

In which t is the hour angle from sunrise or sunset,¹³⁹ h the instantaneous solar altitude and h_m the meridian solar altitude.¹⁴⁰

Al-Ḥabbāk (d. after 1514),¹⁴¹ who was the author of the commentary to al-Jādirī's *Rawḍat al-azhār* and a competent astronomer, adds to this commentary

¹³⁴ Kennedy, 1976, II, 118–119.

¹³⁵ Edited by Khaṭṭābī, 1986, pp. 86–99; see Calvo, 2004, pp. 184–185.

¹³⁶ Ms. Escorial 954, fol. 104v; Ms. Escorial 361 fols. 148v–149r; al-Marghīthī, *Mumtī'* pp. 98–99.

¹³⁷ Saidi, 2013, pp. 206–209.

¹³⁸ Edited by Khaṭṭābī, 1986, pp. 100–134; cf. Calvo, 2004, p. 188.

¹³⁹ The same expression appears in Abū l-Ḥasan al-Muqrī': Baklī, 'Aysānī & Ilhām, 2014, p. 13.

¹⁴⁰ E. Calvo, 2004, p. 189; Saidi, 2013, pp. 209–211. In his commentary, Al-Ḥabbāk states that this procedure is exact only when the Sun is in one of the equinoxes.

¹⁴¹ See 2.0 for the identification of this author.

a set of tables, calculated for the latitude of Tlemcen (35°) and for an obliquity of the ecliptic of $23;35^\circ$, which allow an exact computation of the time arc t , based on the expression:

$$t = \text{arc vers} [(\sin h_m - \sin h) / \cos \delta \cdot \cos \varphi]^{142}$$

This formula can also be found in Ibn al-Raqqām's *al-Zīj al-mustawfī*¹⁴³ and it corresponds to a standard procedure appearing in many Eastern sources¹⁴⁴ as well as in the anonymous Tunisian collection of tables for timekeeping.¹⁴⁵

2.4.1.2.3 *The Times of the zuhr and the 'aṣr Prayers*

With regard to the determination of these two moments of the day, my sources are mainly the *Iqtiṭāf al-anwār* and the *Rawḍat al-azhār* by al-Jādirī, and the commentary on this latter source by al-Ḥabbāk. Obviously the treatment of the problem expressed in shadows is merely an application of the rules explained in 2.4.1.1.

However, towards the end of the thirteenth century Abū 'Alī al-Ḥusayn ibn Bāṣo (d. 1316), who was a *muwaqqit* at the Great Mosque of Granada, deals with the determination of the moment of the two prayers as a function of the meridian solar altitude (h_m) and gives the following rules for the beginning of the *zuhr* prayer (h_z):

$$\begin{aligned} h_z &= (h_m - 10) - 1 / 10 (h_m - 30) & \text{if } h_m > 30^\circ \\ h_z &= (h_m - 10) & \text{if } h_m < 30^\circ \end{aligned}$$

For the beginning of the *'aṣr* (h_a), the rules are:

$$\begin{aligned} h_a &= h_m / 2 + 1 / 10 (80 - h_m) & \text{if } h_m < 80^\circ \\ h_a &= h_m / 2 & \text{if } h_m > 80^\circ \end{aligned}$$

Emilia Calvo has shown that these expressions are fairly accurate in the case of the determination of the *zuhr* prayer but the same cannot be said of the formulas used for the beginning of the *'aṣr*.¹⁴⁶

142 Saidi, 2013, pp. 211–224.

143 Chapter 39 of the canons: see Samsó, 2014, pp. 305–306.

144 King, 2004, pp. 26, 33–36, 114–115, 140, 157, 432–432.

145 King, 2004, pp. 174–175.

146 Calvo, 1992, pp. 78–80.

About a century later, Al-Jādirī (1375–ca. 1416)¹⁴⁷ insists on the same kind of approach; in the *Qatf* and the *Rawḍa*, he states that:

$$h_z = 5 / 6 h_m$$

or the equivalent rule:

$$h_z = h_m - [(h_m / 10) + (h_m / 20)]$$

We can see in table 2.5 that the rule is fit for purpose: it has been calculated for $g = 12$ and for each digit of the meridian shadow between 1 and 12.

To his commentary on al-Jādirī’s rule, Al-Ḥabbāk adds a table that yields the exact solar altitude for the beginning of the *ẓuhr* for each degree of the solar longitude and for the latitude of Tlemcen (35°).¹⁴⁸

TABLE 2.5

S_m	h_m	$S_m + g / 4$	$\cot^{-1} (S_m + g / 4)$	$5 / 6 h_m$
1 ^d	85;14°	4 ^d	71;34°	71;2°
2 ^d	80;32°	5 ^d	67;23°	67;7°
3 ^d	75;58°	6 ^d	63;26°	63;18°
4 ^d	71;34°	7 ^d	59;45°	59;38°
5 ^d	67;23°	8 ^d	56;19°	56;9°
6 ^d	63;26°	9 ^d	53;8°	52;52°
7 ^d	59;45°	10 ^d	50;12°	49;47°
8 ^d	56;19°	11 ^d	47;29°	46;47°
9 ^d	53;8°	12 ^d	45°	44;17°
10 ^d	50;12°	13 ^d	42;43°	41;50°
11 ^d	47;29°	14 ^d	40;36°	39;34°
12 ^d	45°	15 ^d	38;40°	37;30°

147 E. Calvo, 2004, p. 190; Saidi, 2013, pp. 228–231.

148 Saidi, 2013, pp. 232–232.

TABLE 2.6

S_m	h_m	$S_m + g$	$\cot^{-1}(S_m + g)$	$\frac{h_m}{2} + (80;30^\circ - \frac{2}{5}h_m)$ $h_m / 10$	$2 / 5 h_m + 9^\circ$
1 ^d	85;14°	13 ^d	42;43°	42;9°	43;6°
3 ^d	75;58°	15 ^d	38;40°	38;26°	39;23°
4 ^d	71;34°	16 ^d	36;52°	36;41°	37;38°
5 ^d	67;23°	17 ^d	35;13°	35;0°	35;57°
6 ^d	63;26°	18 ^d	33;41°	33;25°	34;22°
7 ^d	59;45°	19 ^d	32;17°	31;57°	32;54°
8 ^d	56;19°	20 ^d	30;58°	30;35°	31;32°
9 ^d	53;8°	21 ^d	29;45°	29;18°	30;15°
10 ^d	50;12°	22 ^d	28;37°	28;8°	29;5°
11 ^d	47;29°	23 ^d	27;33°	27;3°	28;0°
12 ^d	45°	24 ^d	26;34°	26;3°	27;0°

As for the solar altitude for the beginning of the *‘aṣr* prayer, al-Jādirī¹⁴⁹ gives three approximate rules in the *Rawḍat al-azhār*:

- 1) $h_a = h_m / 2 + (h_m \max - h_m)$
 where $h_m \max = 90^\circ - \varphi + \varepsilon$
 and, for Tlemcen ($\varphi = 35^\circ$, $\varepsilon = 23;35^\circ$), $h_m \max = 78;35^\circ$
 for Fez ($\varphi = 33^\circ$, $\varepsilon = 23;30^\circ$), $h_m \max = 80;30^\circ$
- 2) $h_a = 2 / 5 h_m + 9^{0150}$
- 3) $h_a = 45^\circ - 2 / 5 (90^\circ - h_m)$

Expressions 2) and 3) are equivalent and we can check the precision of formulas 1) and 2) in table 2.6, in which formula 1) has been calculated for the latitude of Fez.

To these expressions al-Ḥabbāk adds a set of three tables, calculated for the latitude of Tlemcen ($\varphi = 35^\circ$) and for each degree of solar longitude, in which he gives:

1. The solar altitude in the moment of the *‘aṣr* prayer.
2. The hour angle of the beginning of the *zuhr* computed from midday.
3. The hour angle of the beginning of the *‘aṣr* computed from midday.¹⁵¹

¹⁴⁹ Saidi, 2013, pp. 234–236.

¹⁵⁰ See also Calvo, 2004, p. 191.

¹⁵¹ Saidi, 2013, pp. 237–242.

TABLE 2.7

S_m	h_m	$S_m + 2g$	$\cot^{-1}$ ($S_m + 2g$)	$h_m / 4 + 5$	$h_a - h_a /$ $6 - h_a / 12$	$h_a - h_a /$ $6 - h_a / 30$
1 ^d	85;14°	25 ^d	25;38°	26;18°	32;2°	34;10°
2 ^d	80;32°	26 ^d	24;46°	25;8°	30;27°	32;29°
3 ^d	75;58°	27 ^d	23;58°	23;59°	29;0°	30;56°
4 ^d	71;34°	28 ^d	23;12°	22;53°	27;39°	29;30°
5 ^d	67;23°	29 ^d	22;29°	21;51°	26;17°	28;10°
6 ^d	63;26°	30 ^d	21;48°	20;51°	25;16°	26;57°
7 ^d	59;45°	31 ^d	21;10°	19;56°	24;13°	25;50°
8 ^d	56;19°	32 ^d	20;33°	19;5°	23;13°	24;46°
9 ^d	53;8°	33 ^d	19;59°	18;17°	22;19°	23;48°
10 ^d	50;12°	34 ^d	19;26°	17;33°	21;28°	22;54°
11 ^d	47;29°	35 ^d	18;55°	16;52°	20;40°	22;2
12 ^d	45°	36 ^d	18;26°	16;15°	19;56°	21;15°

Finally, al-Jādirī states – as we have seen in 2.4.1.1 – that the end of the ‘aṣr prayer can be determined by

$$S_{a2} = S_m + 2g$$

and, in both the *Rawḍa* and the *Iqtitāf*, he offers three approximate expressions to calculate the solar altitude at the end of the ‘aṣr. Formula 1 appears in both sources, Formula 2 in the *Iqtitāf* and Formula 3 in the *Rawḍa*.¹⁵²

1. $h_{a2} = h_m 4 + 5$
2. $h_{a2} = h_a - h_a / 6 - h_a / 12$
3. $h_{a2} = h_a - h_a / 6 - h_a / 30$

In table 2.7 above it is obvious that only method 1 gives more or less acceptable results. Al-Jādirī is right when he says that operating with shadows is more convenient and more precise (*awlā wa aḥaqq*).¹⁵³

¹⁵² Saidi, 2013, pp. 244–245; Calvo, 2004, p. 191.

¹⁵³ Khaṭṭābī, 1986, p. 126.

2.4.1.2.4 *Determining the Hour during the Night: the Time of the fajr and of the maghrib Prayers*

Timekeeping by night requires the observation of the stars and, in the anonymous Tunisian corpus of tables for timekeeping, we find a table that yields the hour angle calculated from the altitudes of 21 stars in which t is tabulated for each degree of stellar altitude up to the maximum of the star in question.¹⁵⁴ One of the problems of timekeeping is calculating the duration of dawn and twilight, for they are the periods of time for the *fajr* and the *maghrib* prayers. The sources used employ approximate procedures which, usually, apply general expressions for timekeeping to the specific problem considered. Thus, the *Calendar of Córdoba* applies a general expression that allows calculation of the hour angle (t) as a function of the shadow (cotangent of the solar altitude, h) at a given moment and half the day arc (D) corresponding to a particular date. The procedure used is approximate and has an Indian origin:

$$t = D / (\cotan h + 1)$$

This formula is applied to solve the problem of establishing the duration of dawn and twilight (*mudda*). The calendar gives this value 28 times in the text and it calculates the time passed between two moments: when the Sun attains a negative altitude of -17° and sunrise, or between sunset and an altitude of -17° .¹⁵⁵

¹⁵⁴ King, 2004, p. 65.

¹⁵⁵ See Samsó, 1983, pp. 131–135. The *maghrib* prayer should be performed in the period of time between sunset and the end of twilight, and the *fajr* between the beginning of dawn and sunrise. The end of twilight and the beginning of dawn is defined, in astronomical terms, by a given value of the negative altitude of the Sun but the sources give different values for this altitude. In this chapter we will find cases in which -17° and 18° are used. Besides this, a set of tables for timekeeping, calculated probably for Fez, use -18° for the morning and -19° for the evening (King, 2004, pp. 434–436); another anonymous collection of prayer tables for Tunis use -20° for the morning and -18° for the evening (King, 2004, p. 429). Several commentaries on al-Jādir's poem on timekeeping (*Rawḍat al-azhār*) give different values: -18° for both morning and evening; -20° for dawn and -16° for twilight; -20° and -19° ; -19° and -17° ; -16° and -18° ; -18° except when the Sun is in Taurus, Gemini and Cancer, when it reaches -21° . See Saidi, 2013, pp. 252–253 and Samsó, 2001, pp. 174–175; Aguiar, 2003.

In both the *Rawḍa*¹⁵⁶ and the *Iqtiṭāf al-anwār*,¹⁵⁷ Al-Jādirī applies to this problem the two approximate formulas explained in 2.4.1.2.2. The first one is:

$$t = 6g / (\Delta S + g)$$

where t is the number of seasonal hours since sunrise or before sunset, g the length of the gnomon and ΔS the difference between the meridian shadow (S_m) and the shadow corresponding to an instantaneous solar altitude (S). The formulation given in the *Rawḍa* for the computation of the duration (*mudda*) of dawn and twilight is:

$$t = 6g / [48 - 12 \cotan h_m (\lambda_S + 180^\circ)]$$

in which $\lambda_S + 180^\circ$ is the longitude of the nadir of the solar degree and

$$48 = 36 + 12 \approx 12 \cotan 18^\circ + 12$$

The cotangent of 18° being $36;56$ for a gnomon of 12 digits.

The second formula used by al-Jādirī is:

$$\sin t = \sin h / \sin h_m$$

and it is used to calculate the *mudda* expressed in equinoctial hours:

$$t = 1 / 15 \arcsin (1112 / \sin h_m)$$

in which $1112 \approx 60 \cdot \sin 18^\circ$, expressed in minutes, assuming that $\sin 18^\circ = 18;32$.

Finally al-Jādirī gives another approximate rule for the computation of the *mudda*, expressed in time-degrees, which is valid only for Fez and other cities with the same latitude ($33;40^\circ$). The rule is:

If the Sun is south of the equator: $t = \delta / 4 + 22^\circ$

If the Sun is in the beginning of Capricorn: $t = \delta / 4 + 23^\circ$

If the Sun is north of the equator: $t = \delta / 4 + 21^\circ$ ¹⁵⁸

¹⁵⁶ Saidi, 2013, pp. 248, 253–261.

¹⁵⁷ Calvo, 2004, pp. 192–192.

¹⁵⁸ In his commentary on the *Rawḍa*, al-Ḥabbāk ends this chapter by mentioning two other approximate methods ascribed to an Eastern author named Ibn al-Sā'iḥ and to an otherwise unknown Ibn Jandūz. See Saidi, 2013, pp. 260–261.

Other folk-astronomy sources use the lunar mansions (*manāzil al-qamar*): 28 asterisms which mark the position of the Moon during a lunar sidereal month of, approximately, 28 days. The idea is that the Moon spends one night in each of these mansions.

In the first half of the ninth century the Andalusī polymath ‘Abd al-Malik ibn Ḥabīb (d. ca. 238/ 852–53) transmitted from Mālīk ibn Anas (d. 179/ 795–96) an untitled short text on folk astronomy which is often referred to as *Kitāb fī l-nujūm*.¹⁵⁹ In it, Ibn Ḥabīb describes the procedure to determine the hour during the night by observing the lunar mansion that crosses the meridian at a given hour and calculating the number of mansions that have culminated since sunset. As the total number of mansions is 28, he considers that there are always 14 mansions above the horizon and other 14 below it. Therefore, each new mansion that crosses the meridian is assumed to imply that half a seventh of the night has elapsed. The same technique is used, about one century later (ca. 950), by Qāsim ibn Muṭarrif al-Qaṭṭān in his *Kitāb al-hay’a*; he mentions the use of an instrument (called *al-dā’ira*, the circle),¹⁶⁰ also described, towards the end of the twelfth century, by Abū ‘Alī al-Ḥasan ibn ‘Alī ibn Ḥasan al-Umawī al-Qurṭubī (d. 1205–6): it consists of a circle divided into 12 hours and a second, movable, circle where the lunar mansions and the zodiacal signs are represented. One places the mansion or the sign of the Sun at the end of the semicircle, in the western point of the horizon. The opposite mansion will be the one rising at sunset and it will indicate the hour through its motion across the celestial vault.¹⁶¹ Another twelfth-century author who describes a similar method to determine the hour during the night using the lunar mansions is the philologist Ibn Hishām al-Lakhmī (d. 577/ 1181–82) who wrote a commentary (*sharḥ*) on a *Qaṣīda fī tarḥīl al-nayyirayn*, also entitled *Qaṣīda fī l-hay’a*, which Ibn Hishām ascribes to the famous scientist Ibn al-Haytham (d. after 432/1040).¹⁶²

The apparent rotation of the lunar mansions is also used by Ibn ‘Āṣim (d. 403/1013) to determine the moment of the optional night prayer (*al-tahajjud*) and the moment of the last meal before dawn during the month of Ramaḍān (*al-saḥūr*).¹⁶³ His contemporary, Aḥmad ibn Fāris al-Munajjim

159 Kunitzsch, 1994 & 1997.

160 Casulleras, 1998.

161 Forcada, 1990. See also Forcada, 1995.

162 I have not been able to find any reference to this work by Ibn al-Haytham in the lists of his authentic works. See King, 1986b, nos B77 and F10. With his customary generosity, my friend José Pérez Lázaro sent me his provisional edition of Ibn Hishām's commentary. On this author see Pérez Lázaro, 1990.

163 Miquel Forcada, 1992 a.

(fl. ca. 371/982) is the author of a *Mukhtaṣar min al-anwāʾ* where we find (chapter 15) the dates for each month in which every lunar mansion rises at dawn (*fajr*), although I am not sure of the precise meaning of the word *fajr* in this context.¹⁶⁴ A twelfth-century source, *Al-Muʿrib ʿan baʿḍ ʿajāʾib al-Maghrib*, written by Abū Ḥāmid al-Gharnāṭī (d. 565/ 1169) assumes that since 14 mansions are above the horizon during the night, 12 of them correspond to the period of complete darkness while the rising of the thirteenth mansion coincides with the beginning of dawn.¹⁶⁵ The aforementioned Ibn Hishām states, in his commentary to the *Qaṣīda* attributed to Ibn al-Haytham, that, in order to determine the lunar mansion which mediates in the moment of the rising of dawn (*fajr*), one should count six mansions from the one rising at sunset.¹⁶⁶ In Abū Miqra's *urjūza* we read that, to determine the time during the night, one has to consider the mansion rising at sunset. Every new mansion which rises from this moment implies that 1 hour – 1/7 of an hour has elapsed.¹⁶⁷ This expression is corrected by al-Jādirī¹⁶⁸ (1 hour + 1/6 of an hour) who considers that 14 mansions are always above the horizon and that

$$14 / 12 = 1 + 1 / 6$$

A more elaborate procedure is applied by al-Jādirī (d. 1435) in his *Qatf al-anwār* to determine the number of seasonal hours elapsed since sunset by using the lunar mansion crossing the meridian line at a given moment:¹⁶⁹ one has to calculate the mediation of the lunar mansion, and take the difference between this mediation and the degree of the ecliptic crossing the meridian at sunset; then one should calculate the difference in right ascension between the two

164 Forcada, 2000 a: chapter 15 appears in pp. 175–191. On this work see also Forcada, 1996.

165 Bejarano, 1991, pp. 134–135 and 65–66 (of the Arabic edition). On this work see also Ducène, 2001.

166 David King reminds me of the existence of a treatise on timekeeping by the lunar mansions authored by a certain Ibn Hārūn al-Ṣiqillī (MS Dublin CB 4538, copied ca. 1300). The tables in this text, compiled for use in Egypt, “display the configuration of the lunar mansions with respect to the horizon and meridian at each of thirteen times during the night”. See King, 2004, p. 512.

167 Ms. Escorial 954 fol. 104v.

168 Ms. Escorial 361 fol. 149v. The same rule appears in al-Marghithī, *Mumtiʿ* p. 101.

169 Al-Jādirī, in the *Rawḍa*, gives the values of the mediation of the 28 lunar mansions for year 794/1375 and the author of the commentary, al-Ḥabbāk, gives the corresponding values for 970/1562–62. Al-Ḥabbāk's mediations are not the result of a new computation, for they are only the result of adding 3° to the values of al-Jādirī. This difference seems to be a mere correction for precession: 3° in 187 solar years implies a displacement of 1° in 62+1/3 years. See Samsó, 2008, pp. 133–136; Saidi, 2013, pp. 264–266.

aforementioned points of the ecliptic and divide the result by the time-degrees of one nocturnal hour. The result will be the number of hours since sunset.¹⁷⁰

The lunar mansions were used in order to determine the beginning of dawn. Abū Bakr Muḥammad b. Yūsuf al-Laythī al-Ishbilī al-Sabtī (thirteenth century) wrote an *Urjūza fī tawassuṭ al-manāzil ‘inda ṭulū‘ al-shams*.¹⁷¹ His contemporary Abū Ja‘far Aḥmad b. Jumhūr al-Judhāmī (d. Seville, 14 December 1229)¹⁷² is the author of a similar poem¹⁷³ describing the lunar mansion crossing the horizon at the beginning of dawn, which marks the beginning of the period in which one can perform the *fajr* prayer, for a place the latitude of which is 37;30° (Seville) and throughout the solar year. A century later, Abū Ja‘far al-Sulamī (d. 1346), from Granada although settled in Bejaia, wrote another *urjūza* on the same topic (*Tawassuṭ al-manāzil fī l-shuhūr bi-ma‘rifat waqt al-fajr wa l-ṣuḥūr*).¹⁷⁴

The first fifteen verses of al-Judhāmī’s *qaṣīda* have a certain interest and I reproduce a translation here:

I saw many people offering, to those who were interested, some rules,
based on the [lunar] mansions, so that they may know [the moment] of
dawn (*fajr*)

They said things which were not true and which gave no profit to those
who listened to them¹⁷⁵

Your eyes will see that things do not agree with what they showed and,
so, those having knowledge seem to be ignorant

How often they forced a mansion to rise when it was not rising or to set
when it was not setting

And how often the meridian was crossed by a mansion which you
could not see mediating, something that was considered as a clear indi-
cation of dawn

170 E. Calvo, 2004, pp. 194–195.

171 Lamrabet, 2013, M112, pp. 159–160. This work is mentioned by al-Jādirī in his commentary on Abū Miqrā’s *urjūza*: see ms. Escorial 361 fol. 145v.

172 The title of the poem is *Qaṣīda fī ma‘rifat al-mutawassiṭ min al-manāzil waqt al-fajr* and it has been preserved by Ibn ‘Abd al-Malik, *al-Dhayl wa’l Takmila li-kitābay al-Mawṣūl wa l-Ṣila* (Ibn Sharīfa, 1971, pp. 209–212, no. 290). See Samsó, 2008.

173 Also mentioned by al-Jādirī: ms. Escorial 361 fols. 145v–146r. He states that he used this poem in his *Rawḍat al-azhār*.

174 Baklī, ‘Aysānī & Ilhām, 2014, p. 9, who do not mention their source. I wonder whether this author might be Aḥmad b. Ḥasan b. Bāšo, Abū Ja‘far al-Aslāmī (d. 1309): see Calvo, 1993, p. 24, and Lamrabet, 2013, A296, p. 102.

175 *Al-sāmi‘ūn* in the ms. The editor makes an emendation and reads *al-sābiqūn* (the predecessors), a correction that does not make much sense.

You have here those [mansions] which crossed the meridian without any need of relying on those traversing the horizon treading on it as a *nāzil*¹⁷⁶

In agreement with the school of those who make observations and study these problems (*‘alā madhhab al-arṣād wa l-naẓar*) and following the opinions of the best scholars

I have followed the bright ones (*al-nayyirāt*) and not taken into consideration a star of reduced light (*manqūṣ*) when it is together with a perfect one (*kāmil*)

I have used the brightest [star] of each mansion (*dhā l-ishrāq min kull manzil*) and omitted a star which is more obscure and has a weak light

When they were equivalent in their light I caught one without allowing it to escape the net of the hunter

With what I have gathered you will obtain the truth adding to it what I said about the rising (*ṭālī‘āt*) mansions

And what I also said previously about the setting ones (*ghāribāt*). This will become obvious for those who make an effort

It is necessary to know the latitude of the city for which the computation has been made, without any deviation

It amounts to thirty degrees plus seven and a half, based on a secure calculation without any uncertainty

I am showing here the truth, as I know it and I pray the Merciful to preserve me of vain things.

The rest of the poem gives the date in which the lunar mansions cross the meridian at dawn.

In order to check the approximate validity of the method, the first problem is to establish to which one of the stars of each mansion the author refers when he states that this mansion is mediating at the moment of the beginning of dawn on a particular date in the solar year. Al-Judhāmī gives the theoretical solution when he says, in the aforementioned verses that he has used the brightest star of each mansion. The problem is, however, that, out of the 28 lunar mansions, we can apply this criterion only to around half of them: seven lunar mansions have only one star (Dabarān, Haq’a, Nathra, Ṣarfa, al-Simāk, al-Qalb and al-Rishā’) while in other eight (Buṭayn, Thurayyā, Han’a, Jabha,

¹⁷⁶ The Arabic word for a lunar mansion is *manzil*, derived from the root NZL which is also the root of the active participle *nāzil*. The *nāzil* is obviously the Moon which spends each night of the month in a *manzil* (mansion).

Shawla, Balda, Sa'd Bula' and Sa'd al-Su'ūd) there is at least an additional star greater than the relatively faint star which in all strictness marks the beginning of the mansion. This is not the case of the thirteen remaining mansions (Naḥ, Dhirā', Ṣarfa, Zubra, 'Awwā', Ghafr, Zubānā, Iklīl, Na'ā'im, Sa'd al-Dhābiḥ, Sa'd al-Akhbiya, al-Fargh al-Muqaddam and al-Fargh al-Mu'akhkhar).

For this reason, I have decided (see Table 2.8) to calculate the hour of the beginning of dawn (H₁), counted from midnight when each star of the 69 mentioned in Ibn al-Raqqām's table of lunar mansions crosses midheaven. I have used Ibn al-Raqqām's table as a starting point, even though it is later (probably 687/1288–89) than al-Judhāmī's poem.

In Table 2.8 we have:

- (1) The date of the solar year given by al-Judhāmī
- (2) The number and name of the lunar mansion
- (3) The identification and magnitude of each of the 69 stars: *n* indicates a nebula.
- (4) The ecliptic longitude of the star. I have used Ibn al-Raqqām's values in his table of lunar mansions extant in ms. Istanbul Kandilli 249,¹⁷⁷ dated in 1288–89. As al-Judhāmī died in 1229, I have assumed a vaguely approximate date of 1200–1201, some 87 years before the date of Ibn al-Raqqām's table. Using a precession of 1° per 66 years, the corresponding value will be 1;19°, which I have subtracted from Ibn al-Raqqām's longitudes.¹⁷⁸
- (5) The latitude of the star given by Ibn al-Raqqām's table of lunar mansions.
- (6) *Med* (m) corresponds to the degree of mediation (*mediatio coeli*, *tawassuṭ al-samā'*) of each star. It has been calculated using the following expressions:

$$\delta = \arcsin(\sin \beta \cos \epsilon + \cos \beta \sin \epsilon \sin \lambda)$$

$$\alpha_o = \arccos(\cos \beta \cos \lambda / \cos \delta)$$

$$m = \arctan(\tan \alpha_o / \cos \epsilon)$$

¹⁷⁷ I have a copy of this MS thanks to the generosity of Dr. Benno van Dalen. I edited the full table in Samsó, 2008, pp. 139–150.

¹⁷⁸ I published table 2.8 in Samsó, 2008, pp. 151–154. During a visit to Barcelona in the summer of 2012, Dr. Hamid-Reza Giahī Yazdī noted that there were many errors in the column of the star longitudes. In fact, 21 values were wrong in the previous edition of the table. I have checked all the values and recomputed column 5 (mediation). Surprisingly, in most cases, my errors in column 4 did not affect the results of column 5. It seems obvious that the errors were introduced when I copied column 4 in my computer.

where

δ is the star declination

β is the star latitude (column 5 of table 2.7)

ε is $23;32,40^\circ$, the value used by Ibn al-Raqqām

λ is the star longitude in column (4) of table 2.7

α_0 is the right ascension of the star

m is the mediation of the star.¹⁷⁹

- (7) The solar tropical longitudes for the corresponding dates of years 1200–1201 have been calculated with a computer programme based on the parameters of the *Toledan Tables*,¹⁸⁰ which is a *zīj* likely to have been used towards the beginning of the thirteenth century
- (8) H_1 is the hour (from midnight) at which the corresponding star of columns (3) and (4) is crossing the meridian. One of the alternatives in John North's programme HOROSC¹⁸¹ has been used for the computation of this hour. The calculation uses the longitude of midheaven (5), the local latitude ($37;30^\circ$) and the longitude of the Sun (6).
- (9) H_2 is the hour (from midnight) of the beginning of dawn (assuming it corresponds to the moment in which the Sun reaches an altitude of -17°).
- (10) $\Delta H = H_1 - H_2$

H_2 has been calculated with Benno van Dalen's computer programme *Table Analysis* (specifically with the subprogramme *Table calculator*). I have used the following expressions:

$$N/2 = 180^\circ - D/2 = 180^\circ - (90^\circ + \text{asin}(\tan \delta \tan \phi) \times 2)$$

$$R = \text{acos}(-\tan \phi \tan \delta) - \text{acos}((\sin h - \sin \phi \sin \delta) / (\cos \phi \cos \delta))$$

$$H = N/2 - R$$

for

$N/2$ = half the night arc

$D/2$ = half the day arc

R = rotation of the sphere between the beginning of dawn and sunrise

¹⁷⁹ For this computation I have used the extremely useful programme Stars.exe, prepared by my colleague Dr. Josep Casulleras, to whom I would like to express here all my gratitude.

¹⁸⁰ The skeleton of the programme was prepared by Prof. E.S. Kennedy during one of his stays in Barcelona. It was later revised by Dr. Honorino Mielgo. My gratitude to both of them.

¹⁸¹ See North, 1986.

$$\phi = 37;30^\circ$$

δ = declination values corresponding to the solar longitudes calculated with the *Toledan Tables*.

$$h = -17^\circ \text{ (for H}_2\text{)}$$

considering the kind of source used, the results obtained are surprisingly good and I have marked with an asterisk (*), within each mansion, the star that gives the best results, which was probably the one used by al-Judhāmī for his computations. In most of the cases (with the exception of mansions al-Šarfa, al-Simāk al-A'zal, al-Zubānā, al-Iklīl, al-Qalb, al-Shawla and Sa'd al-Dhābiḥ) the differences between the time of the beginning of dawn and that of the mediation of a star belonging to a particular lunar mansion amount to between 0 and 8 minutes.

TABLE 2.8

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Date	Mansion	Star	λ	β	Med	$\lambda(\text{Sun})$	H ₁	H ₂	ΔH
1.8	28 Rishā'	[69] * β And (2)	19;21 ^o	26;20 ^o	6;59 ^o	135;14 ^o	3;12h	3;14h	-0;2h
10.8	1 Naḥḥ	[1] γ Ari (3)	22;11	7;20	19;9	143;53	3;26	3;30	-0;4
		[2] * β Ari (3)	23;11	8;20	19;44		3;28		-0;2
22.8	2 Buṭayn	[3] δ Ari (4)	36;50	4;50	36;16	155;29	3;47	3;42	0;5
		[4] ε Ari (5)	39;21	1;40	38;47		3;56		0;14
		[5] * ρ Ari (5)	35;11	1;10	34;46		3;39		-0;3
30.8	3 Thurayyā	[6] *BSc1188 (4)	47;41	4;30	46;20	163;15	3;57	3;59	-0;2
		[7] ν Tau (5)	48;1	3;40	46;55		3;57		-0;2
		[8] 23 Tau (5)	49;11	3;20	48;13		4;5		-0;6
		[9] 27 Tau (5)	49;11	5;0	47;43		4;3		0;4
9.9	4 Dabarān	[10] * α Tau (1)	58;11	5;10	56;57	173	4;4	4;10	-0;6
22.9	5 Haq'a	[11] * λ Ori (n)	72;31	-13;50	74;11	185;47	4;30	4;24	0;6
6.10	6 Han'a	[12] * γ Gem (3)	87;31	-7;30	87;39	199;39	4;37	4;41	-0;4
		[13] ξ Gem (4)	90;11	-10;30	90;10		4;48		0;7
17.10	7 Dhirā'	[14] α Gem (2)	98;51	9;40	99;33	210;37	4;48	4;58	-0;10
		[15] * β Gem 2	102;11	6;15	102;47		5;2		0;4
28.10	8 Nathra	[16] * ε Cnc (n)	115;41	0;40	115;49	221;39	5;15	5;10	0;5
9.11	9 Ṭarfa	[17] * λ Leo (4)	126;41	7;30	128;44	233;46	5;19	5;21	-0;2
		[18] * κ Cnc (4)	127;11	5;40	128;44		5;19		-0;2
16.11	10 Jabha	[19] * α Leo (1)	138;1	11;0	141;50	240;52	5;32	5;31	0;1
		[20] ζ Leo (3)	137;41	8;30	140;34		5;37		0;6
		[21] γ Leo (2)	136;11	4;30	137;38		5;26		-0;5
		[22] η Leo (3)	138;1	0;10	138;4		5;27		-0;4
30.11	11 Zubra	[23] δ Leo (3)	149;41	13;40	155;12	255;6	5;33	5;35	-0;2
		[24] * θ Leo (3)	151;41	9;40	155;33		5;33		0;0

TABLE 2.8 (*cont.*)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Date	Mansion	Star	λ	β	Med	$\lambda(\text{Sun})$	H ₁	H ₂	ΔH
8.12	12 Šarfa	[25] $^*\beta$ <i>Leo</i> (1)	160;1	11;50	165;5	263;15	5;35	5;42	-0;23
18.12	13 'Awwā'	[26] β <i>Vrg</i> (3)	164;31	0;10	164;35	273;28	4;48	5;44	-0;56
		[27] η <i>Vrg</i> (3)	173;46	1;10	174;16		5;24		-0;22
		[28] $^*\gamma$ <i>Vrg</i> (3)	178;41	2;50	179;55		5;44		0;0
		[29] δ <i>Vrg</i> (3)	179;51	8;30	176;25		5;32		-0;12
		[30] ϵ <i>Vrg</i> (3)	177;41	15;10	175;33		5;29		-0;15
29.12	14 A'zal	[31] $^*\alpha$ <i>Vrg</i> (1)	192;11	-2;0	191;20	284;42	5;38	5;44	-0;19
10.1	15 Ghafr	[32] ι <i>Vrg</i> (4)	202;11	7;30	205;10	296;56	5;37	5;42	-0;5
		[33] κ <i>Vrg</i> (4)	202;51	2;40	203;55		5;32		-0;10
		[34] $^*\lambda$ <i>Vrg</i> (4)	205;31	0;30	205;43		5;39		-0;3
21.1	16 Zubānā	[35] α <i>Lbr</i> (3)	213;31	0;30	213;42	308;7	5;23	5;37	-0;14
		[36] $^*\beta$ <i>Lbr</i> (3)	217;41	8;30	220;31		5;50		0;13
14.2	17 Iklil	[37] $^*\alpha$ <i>Lbr</i> (4)	228;31	3;0	229;22	332;21	4;50	5;29	-0;39
		[38] θ <i>Lbr</i> (4)	226;41	-1;0	226;23		4;38		-0;51
22.2	18 Qalb	[39] $^*\alpha$ <i>Sco</i> (2)	238;11	-4;30	237;7	340;22	4;52	5;7	-0;15
19.3	19 Shawla	[40] $^*\lambda$ <i>Sco</i> (3)	253;1	-13;20	251;6	5;12	4;19	4;58	-0;39
		[41] υ <i>Sco</i> (4)	252;31	-13;30	250;31		4;16,30		-0,41,30
14.4	20 Na'ā'im	[42] γ <i>Sgr</i> (3)	260;1	-6;20	259;31	30;27	3;21	4;27	-1;6
		[43] δ <i>Sgr</i> (3)	263;11	-6;30	262;50		3;35		-0;52
		[44] ϵ <i>Sgr</i> (3)	263;31	-10;50	262;56		3;36		-0;51
		[45] η <i>Sgr</i> (3)	262;11	-13;0	261;19		3;29		-0;58
		[46] σ <i>Sgr</i> (3)	270;51	-3;10	270;52		4;8		-0;21
		[47] ϕ <i>Sgr</i> (3)	268;31	-3;50	268;28		4;0		-0;27
		[48] $^*\tau$ <i>Sgr</i> (4)	273;11	-4;30	273;18		4;21		-0;6
		[49] κ <i>Sgr</i> (3)	271;51	-6;45	271;57		4;15		-0;12
2.5	21 Balda	[50] 61(g) <i>Sgr</i> (5)	281;11	5;30	280;44	48;2	3;45	3;51	-0;6
		[51] 55(e) <i>Sgr</i> (6)	285;1	5;50	284;23		4;0		0;9
		[52] $^*\gamma$ <i>Sgr</i> (6)	283;11	2;0	282;59		3;54		0;3
28.5	22 S. Dhābiḥ	[53] α <i>cap</i> (3)	292;51	7;20	291;40	72;57	2;48	3;25	-0;37
		[54] $^*\beta$ <i>cap</i> (3)	292;51	5;0	292;2		2;49		-0;36
6.6	23 S. Bula'	[55] $^*\epsilon$ <i>Aqu</i> (4)	303;11	5;30	301;55	81;32	2;54	2;57	-0;3
		[56] ν <i>Aqu</i> (6)	301;41	8;0	299;56		2;45		-0;12
		[57] μ <i>Aqu</i> (5)	300;11	8;40	298;22		2;39		-0;18
17.6	24 Sa'd Su'ūd	[58] β <i>Aqu</i> (3)	312;1	8;50	309;33	92;1	2;39	2;52	-0;13
		[59] ξ <i>Aqu</i> (5)	312;51	6;15	311;3		2;45		0;7
		[60] $^*\zeta$ <i>Aqu</i> (5)	314;11	4;20	312;54		2;53		0;1
1.7	25 Sa'd Akhbiya	[61] γ <i>Aqu</i> (3)	325;1	8;45	321;59	105;21	2;31	2;50	-0;19
		[62] π <i>Aqu</i> (4)	327;11	10;45	323;23		2;36		-0;14
		[63] ξ <i>Aqu</i> (3)	327;31	9;0	324;18		2;40		-0;10
		[64] $^*\eta$ <i>Aqu</i> (3)	328;51	8;30	325;46		2;47		-0;3

TABLE 2.8 (*cont.*)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Date	Mansion	Star	λ	β	Med	$\lambda(\text{Sun})$	H ₁	H ₂	ΔH
7.7	26 Fargh Muqaddam	[65] * α Peg (2)	347;41	31;0	334;4	111;5	2;53	2;45	-0;3
		[66] * β Peg (2)	342;11	19;40	334;8		2;53		-0;3
20.7	27 Fargh Mu'akhkhar	[67] γ Peg (2)	3;21	26;0	8;46	123;30	4;8	3;1	1;7
		[68] * α And (2)	357;41	12;30	352;11		3;9		0;8

2.4.1.3 *Zījēs*

Zījēs do not seem to have been interested in *mīqāt* problems, for example, regarding the times of prayer. Only Ibn al-Raqqām, in the *Shāmīl zīj*, deals with the problem of the determination of the beginning of dawn and the end of twilight as a function of the negative altitude of the Sun.¹⁸² However, *zījēs* treat the problem of the calculation of the length of the day and the determination of the hour as a function of the solar altitude in some detail.

2.4.1.3.1 *Day Arc*

All *zījēs* analysed in this survey deal with the problems of timekeeping in the same standard way. They all deal with the computation of the day arc (*D*) for which they calculate the equation of daylight (*e*), also called the ascensional difference, using:

$$e = \text{Sin}^{-1} (R \tan \delta \tan \varphi)$$

and $D = 180^\circ \pm 2e$ ¹⁸³

182 Ms. Kandilli 249 fol. 58r. See Kennedy, 1997, p. 54.
183 Pedersen, 2002, 111, pp. 494–497, with an obvious error, according to which: $D = 180^\circ \pm e$; the same mistake appears in the edition of Ibn al-Bannā's *Minhāj*: see Vernet, 1952, pp. 46–48 (Ar. txt) and 109–111 (Sp. translation); Millás, 1943–50, pp. 91 and 127; Ibn al-Kammād's *Muqtabas*, ms. Madrid NL Lat. 10023, fols. 22r and v; Hyderabad recension of Ibn Ishāq's *zīj*, cf. Mestres, 1999, pp. 119–121 (Ar. txt) and 70–71 (commentary); Ibn al-Raqqām in his *al-Zīj al-Mustawfī*, ms. Rabat National Library 2461, p. 184; Ibn 'Azzūz in his *al-Zīj al-Muwāfiq* (ms. Rabat NL D2461, p. 382; ms. Rabat Ḥasaniyya Library 8772, p. 32).

The equation of daylight can also be calculated with

$$e = \alpha_{\varphi}(\lambda_S) - \alpha_o(\lambda_S)^{184}$$

and the arc of daylight with

$$D = \alpha_{\varphi}(\lambda_S + 180^\circ) - \alpha_{\varphi}(\lambda_S)^{185}$$

The computation of $e = \sin^{-1}(R \tan \delta \tan \varphi)$ involves the troublesome difficulty of using a shadow table calculated usually for $g=12$ and a sine table for $R=60$. This is why the analysed *zīj*es often contain an auxiliary table giving directly $5 \tan \delta$, which is the result of a conversion of factor 12 into factor 60:

$$5 \tan \delta = 60 \tan \delta / 12$$

This kind of table¹⁸⁶ is used to calculate the ascensional difference for any latitude and to avoid the need to employ a shadow table, as its user only needs to know the tangent of the local latitude. Obviously the conversion factor 5 should be changed if a gnomon other than 12 or a radius other than 60 is used. Thus, the *Toledan Tables* contain two tables of this kind: one of them is computed for $\varepsilon = 23;51^\circ$ and $R = 60$, while the other is calculated for the same obliquity of the ecliptic but for $R = 150$. Therefore, the latter table uses a conversion factor of $150 / 12 = 12;30$.¹⁸⁷

This kind of table is usually called *Jadwal fuḍūl al-maṭālī* (Table of ascensional differences), *Jadwal nisbat al-tafāḍul*, *Jadwal nisbat al-ikhtilāf* (Table of the proportion of differences), or *Jadwal juyūb al-tafāḍul* (Table of the sines of the difference) and we can find examples in Ibn al-Zarqālluh's *Almanac*,¹⁸⁸

184 Ibn al-Kammād, *Muqtabas*, ms. Madrid NL 10023 fols. 11v–12r; Hyderabad recension of the *zīj* of Ibn Ishāq; see Mestres, 1999, pp. 110–114 (Ar. txt) and 66–67 (commentary).

185 In chapter 5 of his *Jaén zīj*, Ibn Mu'adh in gives this expression with a mistake: $D/2 = \alpha_{\varphi}(\lambda_S + 180^\circ) - \alpha_{\varphi}(\lambda_S)$. The correct formulation is given in Ibn al-Kammād's *Muqtabas*, ms. Madrid NL 10023 fols. 22r and v; *al-Zīj al-Kāmil fī l-Ta'ālīm* by Ibn al-Hā'im al-Ishbīlī (fl. ca. 1205); see ms. Bodleian II, 2 no. 285 (Marsh 618), fols. 75r and v. Ibn al-Raqqām in his *al-Zīj al-Shāmil* (Abd al-Raḥmān, 1996, p. 103), as well as in his *al-Zīj al-Qawīm* (ms. Rabat NL 260, pp. 29–30).

186 See King, 2004, pp. 146–152.

187 Pedersen, 2002, III, pp. 986–990. This second table was edited and analysed by Neugebauer & Schmidt, 1952.

188 Millás, 1943–50, p. 225.

the Hyderabad recension of Ibn Ishāq's *zīj*,¹⁸⁹ Ibn al-Bannā's *Minhāj*,¹⁹⁰ Ibn al-Raqqām's *Mustawfī*,¹⁹¹ *Shāmīl*¹⁹² and *Qawīm*¹⁹³ and in the *Muwāfiq zīj* of Ibn 'Azzūz.¹⁹⁴

2.4.1.3.2 *The Hour*

Once the day arc (D) is known, one can calculate the hour of the day using the instantaneous solar altitude (h), its meridian altitude (h_m) and half the day arc ($D/2$):

$$t = \text{vers}^{-1} (\text{vers } D/2 - [\sin h \cdot \text{vers } D/2 / \sin h_m])$$

t being the hour angle counted from the meridian.¹⁹⁵ From this value one can obtain the value in degrees of the earth's rotation (*al-dā'ir min al-falak*) since sunrise (d) with:

$$d = D/2 \pm t$$

This value can be expressed in equinoctial or seasonal hours by dividing d by 15° or by the number of degrees corresponding to one day or night hour.

This procedure can be found in many sources: the *Toledan Tables*,¹⁹⁶ Ibn al-Zarqālluh's *Almanac*,¹⁹⁷ Ibn al-Kammād's *Muqtabas*,¹⁹⁸ Ibn Mu'adh's Jaén *zīj*,¹⁹⁹ the Hyderabad recension of the *zīj* of Ibn Ishāq,²⁰⁰ Ibn al-Bannā's *Minhāj*²⁰¹ and Ibn al-Raqqām's *Mustawfī*.²⁰² Most of them apply the same expression for the computation of the night hour using h , h_m and $D/2$ of the Moon or a star.

189 Mestres, 1999, pp. 279–281.

190 Vernet, 1952, pp. 46–48 (Ar. txt) and 109–111 (Sp. translation).

191 Ms. Rabat NL 2461, p. 285–286.

192 Ms. Kandilli 249, fol. 80v.

193 Ms. Rabat NL 260, p. 99.

194 Ms. Rabat NL D2461 p. 439 (with a maximum value of 2;10,44) and ms. Rabat HL 8772, p. 58 (max. 2;10,40).

195 This expression seems to have a Khwārizmian origin and it can be found in Ibn al-Muthannā's commentary: see Goldstein, 1967, pp. 83, 208–209. A demonstration of the validity of this rule in Nadir, 1960.

196 Pedersen, 2002, I, 242–245; II, 422–423 and 624–625.

197 Millás, 1943–50, pp. 100, 136 where we actually find the inverse procedure: calculating the instantaneous altitude (h) as a function of t .

198 Ms. Madrid BN 10023 fols. 12r and v.

199 Ibn Mu'adh's Jaén *zīj*, chapter 15.

200 Mestres, 1999, pp. 124–125 (Ar. text) and 73–74 (comm.).

201 Baklī, 'Aysānī & Ilhām, 2014, p. 11.

202 Ms. Rabat NL 2461, pp. 189–190.

Ibn al-Raqqām's *Mustawfī* also uses it for calculating the hour of the beginning of dawn and the end of twilight, considering $h = -19^\circ$.²⁰³

In Ibn al-Zarqālluh's *Almanac*,²⁰⁴ the Hyderabad recension of Ibn Ishāq's *zīj*,²⁰⁵ Ibn al-Raqqām's *Mustawfī*²⁰⁶ and Ibn 'Azzūz' *Muwāfiq*²⁰⁷ we are given a second set of instructions for calculating the hour angle measured from the meridian line (t):²⁰⁸

$$t = \text{vers}^{-1} ([\sin h_m - \sin h] / \cos \varphi \cos \delta)$$

In all the aforementioned sources $1/\cos \varphi$ and $1/\cos \delta$ are calculated using a table of secants for $R = 1$ which receives a variety of titles: *Jadwal fuḍūl khaṭṭ al-madār 'alā l-irtifā'* in the *Almanac*,²⁰⁹ *Jadwal nisbat al-fuḍūl wa l-mamarrāt li l-shams wa l-kawākib* in the Hyderabad ms.,²¹⁰ *Jadwal nisbat juyūb al-tafāḍul wa l-madārāt li l-shams wa l-kawākib* in the *Mustawfī*²¹¹ and *Jadwal nisbat al-madārāt* in the *Muwāfiq*.²¹²

Finally, two sources²¹³ use the standard approximate formula, fairly accurate for the latitudes of the countries surrounding the Mediterranean, to calculate the hour angle since sunrise (t):²¹⁴

$$t = \sin^{-1} (\sin h / \sin h_m)$$

2.4.1.3.3 Tables

The *zīj*es analysed contain tables (or give references to them) of the length of daylight or half daylight (LD), the corresponding number of equal hours (EH) and the time degrees of a seasonal hour (TD), as a function of the solar longitude and the local latitude. I will give a list of these tables according to

203 Ms. Rabat NL 2461, pp. 201–202.

204 Millás, *Azarquiel*, pp. 108–109, 142–142.

205 Mestres, 1999, pp. 128–129 (Ar. text) and 76 (comm.).

206 Ms. Rabat NL 2461, pp. 189–190.

207 Ms. Rabat NL D2461, p. 381; Rabat HL 8772, p. 32.

208 For a proof of this formula see Nallino, *Battānī* 1, p. 191.

209 Millás, *Azarquiel* p. 226.

210 Mestres, 1999, pp. 279–281.

211 Ms. Rabat NL 2461, p. 285.

212 Ms. Rabat NL D2461, p. 382; Rabat HL 8772, p. 32.

213 The *Toledan Tables* (see Pedersen, 2002, II, pp. 422–423 and 624–625) and Ibn al-Kammād's *Muqtabas* (Ms. Madrid BN 10023 fols. 12r–12v).

214 This well-known formula and its use in Eastern Islam have been exhaustively analysed by King, 2005, pp. 111–206. See other procedures, given by Abū l-Ḥasan al-Muqri' (fl. 1384), to calculate the hour during the day and the night in Baklī, 'Aysānī & Ilhām, 2014, p. 14.

the locality and the corresponding latitude (when mentioned or calculated) for which they were computed:

- **Toledo** (*Toledan Tables*): LD expressed in hours,²¹⁵ for $\varphi = 39;54^\circ$ and $\varepsilon = 23;33^\circ$.
- **Córdoba** (Ibn al-Kammād): LD in hours; TD for $\varphi = 38;30^\circ$ and $\varepsilon = 23;33^\circ$.
- **Jaén**: Ibn Mu'ādh refers to tables of EH and TD.²¹⁶
- **Sevilla (?)**: Ibn al-Hā'im states that his table of oblique ascensions contains a column indicating TD and refers to another table of EH.²¹⁷
- **Granada** (Ibn al-Raqqām, *Qawīm*): tables of LD in degrees and hours, and of TD for $\varphi = 37;10^\circ$ and $\varepsilon = 23;32,40^\circ$.²¹⁸ This table of TD for Granada reappears in Ibn 'Azzūz's *Muwāfiq*.²¹⁹
- **Salé** (Ibn al-Kammād, *Muqtabas*): TD for Córdoba $\varphi = 38;30^\circ$.²²⁰
- **Tunis** (Hyderabad recension of Ibn Ishāq): EH and TD.²²¹ As established by Mestres,²²² these tables seem to have been computed for $\varphi = 36;53^\circ$ and $\varepsilon = 23;33^\circ$, although the standard values used in this *zīj* are $\varphi = 36;40^\circ$ and $\varepsilon = 23;32,30^\circ$.
- **Tunis** (Ibn al-Raqqām, *Mustawfi*): LD, TD and EH for $\varphi = 36;40^\circ$.²²³ Ibn al-Raqqām uses $\varepsilon = 23;32,30^\circ$ in this *zīj*.
- **Tunis** (Ibn al-Raqqām, *Qawīm*): LD, EH and TD for $\varphi = 36;47^\circ$.²²⁴ According to Kennedy,²²⁵ the embedded ε is about $23;36^\circ$ (LD and EH), or $23;35^\circ$ (TD).
- **Bijāya** (Ibn al-Raqqām, *Shāmil*): TD and EH for $\varphi = 36^\circ$ and $\varepsilon = 23;34^\circ$.
- **Fez** (Ibn 'Azzūz, *Muwāfiq*): 12 tables (one for each zodiacal sign) giving the time degrees of 1 seasonal hour for $\varphi = 33;40^\circ$.²²⁶ Ibn 'Azzūz uses $\varepsilon = 23;33^\circ$ in this *zīj*.

As an example of these tables, below is an edition of the tables of TD and EH calculated by Ibn al-Raqqām for the latitude of Granada (Tables 2.9 and 2.10).

²¹⁵ Pedersen, 2002, III, 1125–1127.

²¹⁶ Ibn Mu'ādh's Jaén *zīj*, chapter 6.

²¹⁷ Ms. Bodleian II, 2 no. 285 (Marsh 618), fols. 75r and v.

²¹⁸ Ms Rabat NL 2461 pp. 87–88.

²¹⁹ Ms Rabat NL D2461 p. 439.

²²⁰ Ms. Madrid NL 10023 fols. 49v–51r and 59v–61r.

²²¹ Mestres, 1999, p. 285.

²²² Mestres, 1999, p. 276.

²²³ Ms. Rabat NL 2461, pp. 278–279.

²²⁴ Ms Rabat NL 260, pp. 161–162.

²²⁵ Kennedy, 1997, pp. 70–71.

²²⁶ Ms. Rabat NL D2461, pp. 428–437; Rabat HL 8772, pp. 20–27.

TABLE 2.9 Time degrees of one unequal hour for the latitude of Granada (Ms. Rabat NL 260, p. 87). In parenthesis the difference between the table and the recomputed values based on $\varepsilon = 23;32,40^\circ$ and $\varphi = 37;10^\circ$

Degrees	Capricorn	Aquarius	Pisces	Aries	Taurus	Gemini
5	11;51 ⁰ (+3)	12;29 ⁰ (+1)	13;46 ⁰ (+1)	15;15 ⁰	16;42 ⁰ (−1)	17;50 ⁰ (−1)
10	11;52 (+1)	12;41 (+2)	14;0	15;30	16;55 (−1)	17;57 (−2)
15	11;57 (+2)	12;52 (+1)	14;15	15;45	17;8 (−1)	18;3 (−2)
20	12;3 (+2)	13;5 (+1)	14;30	15;59 (−1)	17;19 (−2)	18;8 (−1)
25	12;10 (+1)	13;18 (+1)	14;45	16;14 (−1)	17;[3]1 (−1)	18;10 (−2)
30	12;19 (+1)	13;32 (+1)	15;0	16;28 (−1)	17;41 (−1)	18;11 (−2)

TABLE 2.10 Equal hours of half the day arc for the latitude of Granada (Ms. Rabat NL 260, p. 88). In parenthesis the difference between the table and the recomputed values

Degrees	Capricorn	Aquarius	Pisces	Aries	Taurus	Gemini
5	4;44 ^h (−1)	5;0 ^h	5;30 ^h (−1)	6;6 ^h	6;41 ^h	7;8 ^h (−1)
10	4;45 (−1)	5;4 (−1)	5;36	6;12	6;46	7;11 (−1)
15	4;47	5;9	5;42	6;18	6;51 (−1)	7;13 (−1)
20	4;49 (−1)	5;14	5;48	6;24	6;56	7;15 (−1)
25	4;52 (−1)	5;19 (−1)	5;54	6;30	7;0 (−1)	7;16 (−1)
30	4;56	5;25	6;0	6;35 (−1)	7;4 (−1)	7;16 (−1)

A few more tables that are also useful for timekeeping can be found in our sources. The Hyderabad recension contains a table (no. 158) giving $90^\circ + \delta_s$, as a function of the solar longitude, which can be used to calculate the solar meridian altitude. In another table (no. 159) the solar meridian altitude is calculated for the latitude of Tunis ($36;40^\circ$).²²⁷ Quite exceptionally, Ibn ‘Azzūz’s *Muwāfiq zīj* contains a table (Table 2.11) giving the solar altitude and the shadow ($g = 12$) for hours 1–6 and for the days of the entrance of the Sun in the twelve zodiacal signs. The tabular values are only approximated to the nearest degree or digit. The values corresponding to the solar meridian altitude of Aries 0° (56°),

227 See Mestres (1999) pp. 66–67, who does not edit these tables.

Cancer 0° (80°) and Capricorn 0° (32°) seem to imply a latitude for Fez of 34° and an obliquity of the ecliptic of 24°.228

TABLE 2.11 Solar altitude and digits of the shadow (g = 12) for hours 1–6 corresponding to the entrance of the Sun in the zodiacal signs, for the latitude of Fez (Ibn ‘Azzūz, *al-Zīj al-Muwāfiq*)

Aries/ Libra			Taurus-Virgo		Gemini-Leo		Cancer	
Hours	Altitude	Shadow	Altitude	Shadow	Altitude	Shadow	Altitude	Shadow
1st	13°	52 ^d	10°	[6]8 ^{da}	15°	45 ^d	15°	45 ^d
2nd	24°	26 ^d (−1)	20°	33 ^d	29°	21 ^d (−1)	29°	21 ^d (−1)
3rd	36°	16 ^d	29°	21 ^d (−1)	53°	9 ^{db}	44°	12 ^d
4th	46 ^{oc}	11 ^d (−1)	37°	16 ^d	57°	8 ^d	58°	7 ^d
5th	54°	9 ^d	42°	13 ^d	71°	4 ^d	72°	4 ^d
6th	56°	8 ^d	44°	1[2] ^{dd}	77°	3 ^d	83°	2 ^d (+1)

a 48 in both mss.
b 13 in both mss.
c 47 in ms. RHL
d 14 in both mss.

Scorpio–Pisces			Sagitt.–Aquar.		Capricorn	
Hours	Altitude	Shadow	Altitude	Shadow	Altitude	Shadow
1st	10°	[6]8 ^a	8°	85 ^{db}	8°	85 ^d
2nd	20°	33	16°	42 ^d	15°	45 ^{dc}
3rd	29°	21 (−1)	24°	27 ^d	22°	30 ^d
4th	37°	16	30°	20 ^d (−1)	27°	23 ^d
5th	42°	13	34°	18 ^{dd}	31°	20 ^d
6th	44°	1[2]	35°	17 ^d	32°	19 ^d

a 48 in both mss.
b 45 in both mss.
c 55 in ms. RNL
d 13^d in ms. RHL

228 Ms. Rabat NL D2461, p. 366; Rabat HL 8772, p. 13.

2.4.2 *Instruments for Telling the Time*

2.4.2.1 Astrolabes and Quadrants

2.4.2.1.1 *Astrolabes*

The instrument most frequently used by astronomers to tell the time was probably the astrolabe. Instruments of this kind of an Andalusian and Maghribī origin had, on their back, a zodiacal scale which made it possible to establish the solar longitude corresponding to a specific day of the Julian year, without making any kind of computation. For many years I believed that this kind of device had a Western origin because it does not appear in Eastern astrolabes until the twelfth century, but, some years ago, David King published new evidence showing that this calendrical scale is a part of an instrument made by Naṣṭūlus, a significant astronomer and leading instrument-maker in Baghdad ca. 900.²²⁹

When the solar longitude had been established, the astronomer determined the solar altitude using the alidade and the graduated quadrant on the back of the astrolabe. The same operation could be performed during the night by an observation of the altitude of one of the stars projected in the astrolabe's rete or spider. Immediately afterwards, the solar degree (obtained with the calendrical scale), or the star projected in the spider, was placed on the almucantar (a circle parallel to the horizon) whose graduation corresponded to the observed altitude of the Sun or the star. At that precise moment one could say that the watch was set correctly and, using the horary diagram placed below the horizon, on the face of the astrolabe, one could tell the time by looking at the seasonal hour on which the degree opposite to the solar or stellar longitude was placed. This procedure is described in all the treatises, written in Arabic, Latin or Hebrew, on the use of the astrolabe. In addition to this, many astrolabes, made after ca. 1000, had specific curves which allowed the calculation of the hour of the beginning of the *ḡuhr* and the *ʿaṣr* prayers.

2.4.2.1.2 *Horary Quadrants*

2.4.2.1.2.1 The *quadrans vetustissimus*

Two kinds of horary quadrants circulated in the Iberian Peninsula during the Middle Ages. In the old corpus of Latin texts on the astrolabe and other astronomical instruments made in Catalonia towards the end of the tenth century, we find descriptions of the instrument, named *quadrans vetustissimus* (fig. 2.2)²³⁰ by Millás in order to distinguish it from the standard *quadrans vetus*,

²²⁹ King, 2008.

²³⁰ The texts describing this kind of quadrant are the *Incipiunt regule de quarta parte astrolabii*, some quotations in the *Geometria incerti auctoris* (attributed to Gerbert of Aurillac)

incorrectly attributed to Robert the Englishman.²³¹ The former is a variety of the sine quadrant (a graphic scale of sines and cosines) with a movable cursor (a scale of solar declinations as a function of the date of the Julian year); using it, one can obtain, without any kind of computation, the meridian solar altitude (h_m) for a given date. The quadrant also has a plumb line and two sights, with which we can obtain the solar altitude for any moment of the day in question. The purpose of the instrument, according to the sources, was to tell the time, for any day of the year and for any latitude.

The cursor²³² (see Figs. 2.2, 2.5 and 2.6) is a movable part that can be displaced along the rim of the instrument. The arc of altitude is graduated from 0° to 90° . In order to graduate the cursor one has to begin by fixing it so that the radius of the quadrant passing through the centre of the cursor coincides with graduation 45° of the arc of altitude. From that point we will divide the cursor, using the graduation of the arc of altitude, in order to obtain a scale of declinations (δ) on the right and left of 45° from $45^\circ - 24^\circ$ until $45^\circ + 24^\circ$.

Besides the declination scale, in the Alfonsine description the cursor also has two unequally divided diagrams: one is a scale of solar longitudes while the other contains the days and the months of the Julian year. In this way the cursor has:

- a) A zodiacal scale, similar to those appearing in astrolabes, which relates the date to the solar longitude.
- b) A declination scale for which the entry can be the date or the solar longitude.

In the case of the tenth-century Catalan texts on the quadrant, only the calendar scale seems to appear in the instrument described (see fig. 2.2).

In order to adjust the cursor to the local latitude (φ), it must be placed so that its midpoint (Aries 0°) coincides with the graduation of the altitude quadrant which corresponds to the colatitude ($90^\circ - \varphi$) and it should be fixed in this position with a nail or something similar (see Fig. 2.3). Then, if we place the plumb-line on the day of the year, the graduation of the altitude quadrant will correspond to:

and the *Liber secundus de utilitatibus astrolabii* (attributed to Hermann Contractus). See the edition of these texts in Millàs Vallicrosa, 1931, pp. 304–308.

231 David A. King has proved that the *vetustissimus* and the *vetus* quadrants were instruments known in Baghdad in the ninth century Cf. King, 2002, and King, 2005, pp. 213–220, 240–246.

232 This description is based on the Alfonsine text of the *Libro del cuadrante con el que rectifican*, which deals with the so-called *quadrans vetus*, although the cursor was probably similar to that of the *quadrans vetustissimus*. See Rico, 1863–67, III, pp. 295–300.

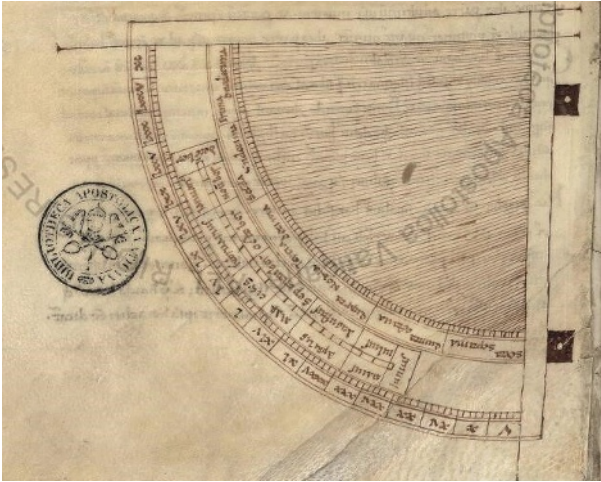


FIGURE 2.2
The *quadrans vetustissimus* in ms. Vatican Reg. lat. 1661, fol. 86v
© BIBLIOTECA APOSTOLICA VATICANA

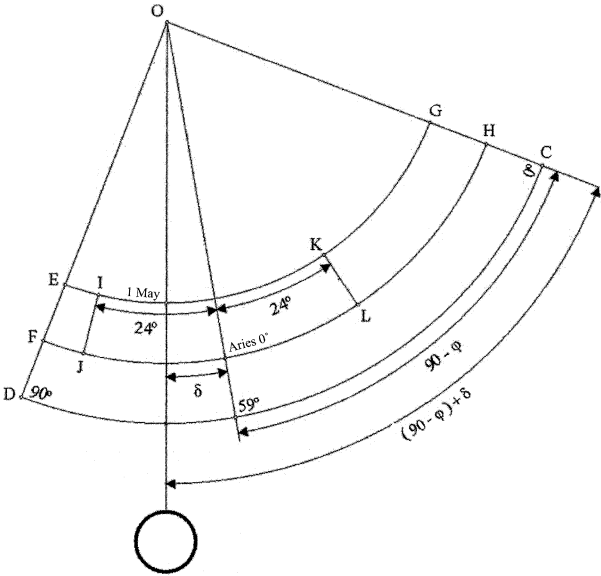


FIGURE 2.3
The declination scale in the *quadrans vetustissimus*

$$h_m = (90^\circ - \varphi) + \delta$$

If we want to use the quadrant to determine the time, the procedure to be used is the following:

1. Having obtained the solar meridian altitude (h_m) with the cursor, we will place the plumb line (OF in Fig. 2.4) on the graduation of the quadrant corresponding to this meridian altitude ($DF = h_m$).

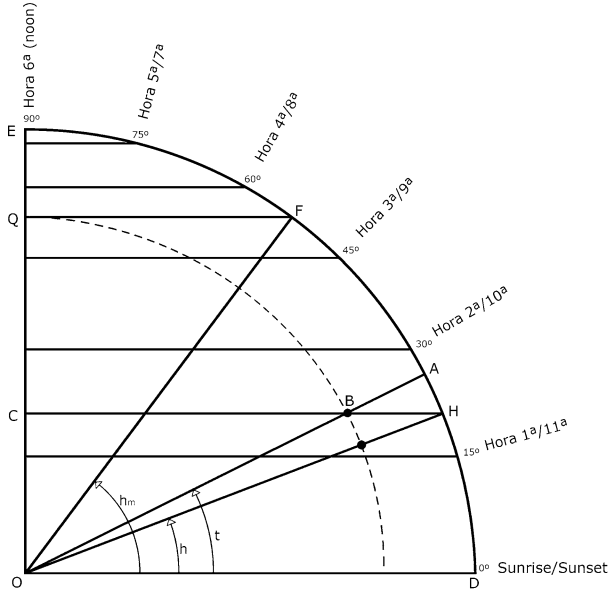


FIGURE 2.4
Determination of the
hour with the *quadrans*
vetustissimus

2. We will observe which perpendicular line of the sine scale passes by point F and we will see that it is FQ, which reaches the side of the quadrant OE at point Q. We will place the plumb-line on side OE and, using a movable bead, we will mark on it the distance OQ (= sin h_m).
3. Using the sights of the instrument, we will observe the instantaneous solar altitude (h). Then, the plumb-line will be in the position OH and $OC = \sin h$.
4. We will observe the perpendicular HC which corresponds to the instantaneous altitude DH and we will rotate the plumb-line until the bead is placed on the perpendicular HC at point B. Then arc DA (= t) will correspond to the hour angle counted from sunrise or before sunset, depending on whether we have made the observation before or after midday. In figure 2.4 we can easily see that:

$$\sin t (=DA) = OC / OB = \sin h / \sin h_m$$

Therefore the whole procedure is a mere graphical application of the old Indian rule we have seen in 2.4.1.2.2 and 2.4.1.3.2.²³³

²³³ On the use of the *quadrans vetustissimus* see García Franco, 1945, pp. 385–386; Millás, 1932; Lorch, 1981; Lorch, 2000. See also Samsó, 2004 b, pp. 142–150; King, 2005, p. 172.

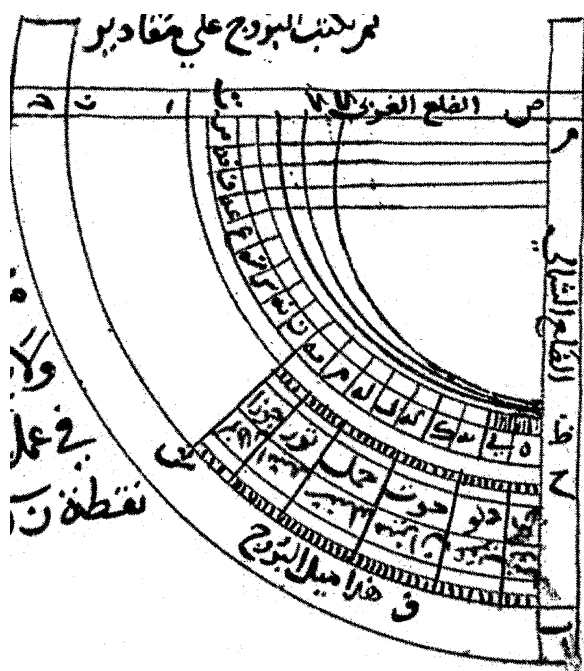


FIGURE 2.5
Ibn al-Zarqālluh's quadrant
according to al-Marrākushī
(*Jāmi' al-mabādī* Ms.
Istanbul Ahmet III 3343)

The sine quadrant²³⁴ is not a common instrument in the Andalusian tradition. Apart from the aforementioned Latin sources, it only appears in the lower right quadrant on the back of Ibn al-Zarqālluh's *ṣafīḥa zarqāliyya* where it is used to solve trigonometrical problems graphically,²³⁵ and also on the back of an astrolabe by Muḥammad ibn al-Fattūḥ al-Khamā'irī of Seville (634/1236–37).²³⁶ To this one should add that Abū 'Alī al-Marrākushī, in his *Jāmi' al-mabādī wa l-ghāyāt*,²³⁷ ascribes to Ibn al-Zarqālluh the design of a quadrant which combines elements of the tradition of the sine quadrant²³⁸ with the orthographic projection of the celestial sphere on the plane of the solstitial colure (see fig. 2.5), which also appears on the back of the *ṣafīḥa zarqāliyya*:²³⁹ in it, the

234 On the sine quadrant see Schmalzl, 1929, pp. 83 ff.; D.A. King, "Rub". *EI2*, VIII, pp. 574–575; King, 2005, pp. 162 ff.

235 Puig, 1991. In spite of its name, I do not believe that the Alfonsine "cuadrante sennero" is a sine quadrant. This source was edited by Millás, 1956.

236 Pingree, 2009, pp. 2–5.

237 Abū l-Ḥasan, 1984, pp. 374–377; Sédillot, 1844, pp. 104–106.

238 Abū l-Ḥasan al-Marrākushī begins the passage with *wa ammā l-wajh al-jaybī min al-rub' al-Zarqālī* ("Concerning the sine face of Ibn al-Zarqālluh's quadrant"): Abū l-Ḥasan, 1984, p. 375; Sédillot, 1844, p. 104.

239 Puig, 1996.

parallels are straight lines, perpendicular to one of the sides of the quadrant, which can be used as a graphical scale of sines and cosines, while the meridians, in Sédillot's interpretation, are arcs of ellipses. The instrument also has a zodiacal scale which gives the solar declination as a function of the solar longitude.

Much later, we have the instrument called *sexagenarium*, used in Cairo and introduced in Valencia in 1450 by a *faqīh* from Paterna, who also brought with him a non-extant Arabic treatise on the use of the instrument. This treatise was translated into Catalan in 1456 by an unknown translator and retranslated into Latin in 1464 by Johannes de Bonie, a Valencian physician who seems to have been interested in astronomy.²⁴⁰ Both translations contain a collection of chapters dealing with the use of the sine quadrant for the solution of standard problems of timekeeping including the determination of the *qibla*.²⁴¹ Ms. 7416 A of the Bibliothèque National de France, which contains these Catalan and Latin translations, also contains a much longer text which describes an instrument with two sides, one of them being a standard sine quadrant, while the other has a graphical scale giving the mean motions of all planets. The purpose of the instrument is to use the sine quadrant for the graphical computation of planetary equations, and so it becomes an instrument belonging to the family of equatoria. Besides, the latter description fits perfectly the only extant instrument of this kind, preserved in the Oxford Museum of the History of Science.²⁴² We have, therefore, the unsolved problem of determining whether this planetary use of the instrument corresponds to the original Arabic treatise or is a later European addition.

As we have seen, this sine quadrant seems to have been little used in al-Andalus; however, there are frequent references to its use in the Maghrib,²⁴³ and it seems to have become the favourite instrument of the *muwaqqitūn*. Unfortunately only one of these texts has been edited: the *Risāla kāfiyat al-sayb fī l-'amal bi l-jayb* written by 'Izz al-Dīn 'Abd al-'Azīz b. Mas'ūd (fl. 1372–1392), known as ibn Farmīja (possibly Firmījūh = Bermejo)²⁴⁴ probably born in

240 Aguiar & González, 2005.

241 Aguiar & González, 2003, pp. 98–103, 115–162; Aguiar & González, 1996.

242 Thorndike, 1951; Poulle, 1966; Poulle, 1980, pp. 417–444. Good photographs of the Oxford Museum's *sexagenarium* can be seen in Vernet, 1985, pp. 108–109, and in Vernet & Samsó, 1992, pp. 216–217.

243 See, for example, Lamrabet, 2013, M195 (pp. 198–199), M214 (p. 206), M219 (p. 208), M282 (p. 224) and M310 (p. 228) for authors of treatises on the sine quadrant until the sixteenth century.

244 Edition and Spanish translation by Maravillas Aguiar in her unpublished Ph.D. thesis presented in the University of La Laguna in 1995. I have a copy of that thesis, thanks to the generosity of its author.

Tlemcen, who was *imām* and *muwaqqit* in Fez, Tunis, Jerusalem and Damascus, an instrument maker²⁴⁵ and author of an astrological treatise,²⁴⁶ who had a role in the introduction of Ibn Abī l-Shukr's *Tāj al-azyāj* in the Maghrib.²⁴⁷ The treatise on the sine quadrant was written in Cairo in 795/1392–3 and it describes a trigonometrical quadrant with a double set of perpendiculars traced from the arc to both sides of the instrument, which resemble modern-day graph paper. The treatise describes in detail the uses of the sine quadrant for solving the standard problems of a *muwaqqit* as well as for topography, measuring distances and surfaces (*misāḥa*)²⁴⁸ and for trivial arithmetical operations. It shows Western (quotations of Andalusian authors like Ibn al-Zarqālluh and Ibn al-Ṣaffār) and Eastern influences (a precession table copied from Ibn Abī l-Shukr al-Maghribī's *Tāj al-azyāj*) and it was definitely used in the Maghrib: the only extant manuscript (Escorial 918) was copied somewhere in the Islamic West in 888/1483 and the work is quoted in the *Maḥṣalat al-maṭlūb fī l-ʿamal bi-rubʿ al-juḡyūb* of the Tunisian astronomer ʿUmar b. ʿAbd al-Raḥmān al-Tawzarī (d. 1454).²⁴⁹

Besides texts, we should also bear in mind the number of sine quadrants built in the Maghrib: on one face we usually find a double grid of perpendiculars, forming a double scale of sines and cosines, and, on the other, a set of astrolabic markings (*rubʿ al-muqanṭarāt, quadrans novus*). This is the case of a quadrant made by Aḥmad b. ʿAbd al-Raḥmān al-Dahmānī, dated in 854/1450–51) in which the astrolabic markings correspond to a latitude of 36;40° (Tunis).²⁵⁰ Later instruments of the same kind are the quadrants made by Muḥammad b. ʿAlī Ḥajjāj (1006/1597–98) for latitude 31;30° (Marrakesh)²⁵¹ and by Muḥammad b. Aḥmad al-Baṭṭūṭī (1142/1729–30) for latitude 34° (Meknès?).²⁵²

245 See King, 2005, pp. 835–914, especially pp. 902–902.

246 Herrera, 2001.

247 Samsó, 1998, p. 96.

248 Aguiar, 1996.

249 King, 1986b, p. 141 (F39); Lamrabet, 2013, M214, p. 206.

250 Millás, 1947, pp. 61–64; King, 2007, p. 192.

251 Arribas, 1965.

252 Millás, 1947, pp. 56–61. Good photographs of another undated Maghribī sine quadrant with the (apparently unfinished) other face calculated for Marrakesh in Pingree, 2009, p. 204. See a list of extant Maghribī quadrants in King, 2007, p. 205.

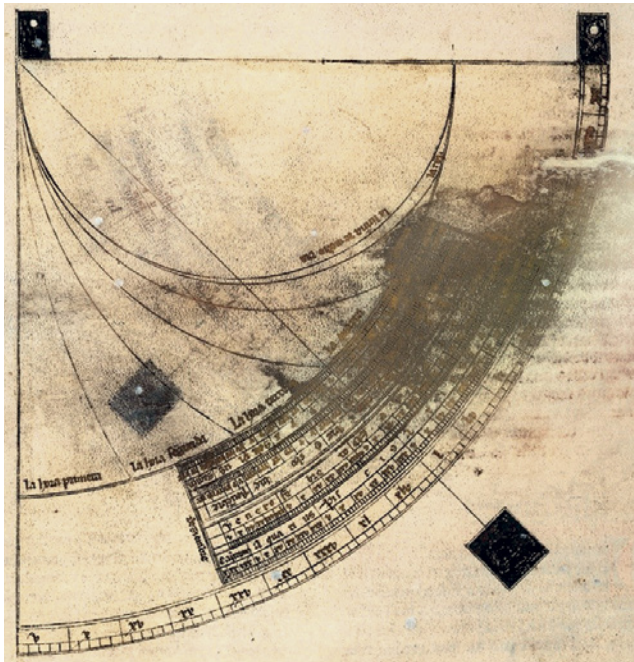


FIGURE 2.6
The *quadrans vetus*
in the Alfonsine
*Libro del Saber
de Astrología*.
Biblioteca Histórica
de la Universidad
Complutense de
Madrid. [BH MSS 156
fol. 173v]

2.4.2.1.2.2 The *quadrans vetus*

The *quadrans vetus*, described in the Alfonsine *Libro del Saber de Astrología* (see fig. 2.6)²⁵³ seems to be closely related to the *quadrans vetustissimus*, although the graphical scale of sines is replaced by the diagram of the hours, from 1 to 6, each one of them being an arc of a circle whose centre lies on side AC (fig. 2.7) of the quadrant (or AC produced, if necessary). These arcs of a circle must pass by the centre of the quadrant A and by the divisions 15°, 30°, 45°, 60°, 75° and 90° of the graduated arc of the quadrant. In this way the arc corresponding to the sixth hour (midday) will be a semicircle with radius equal to half the side of the quadrant.

After determining the solar meridian altitude (H) corresponding to the day in question, one should place the plumb-line in the graduation equal to H ($= \angle BAM'$) and displace the bead of the plumb-line (M) to the point of its intersection with the semicircle of the sixth hour (M'). Then, an observation of the instantaneous solar altitude h ($= \angle BAM$) will be made using the sights of the instrument: the plumb-line will be in the position AM in fig. 2.7. The bead,

²⁵³ On the *quadrans vetus* see García Franco, 1945, pp. 374–377, 384–388; Lorch, 1981; King, 2005, pp. 169–173.

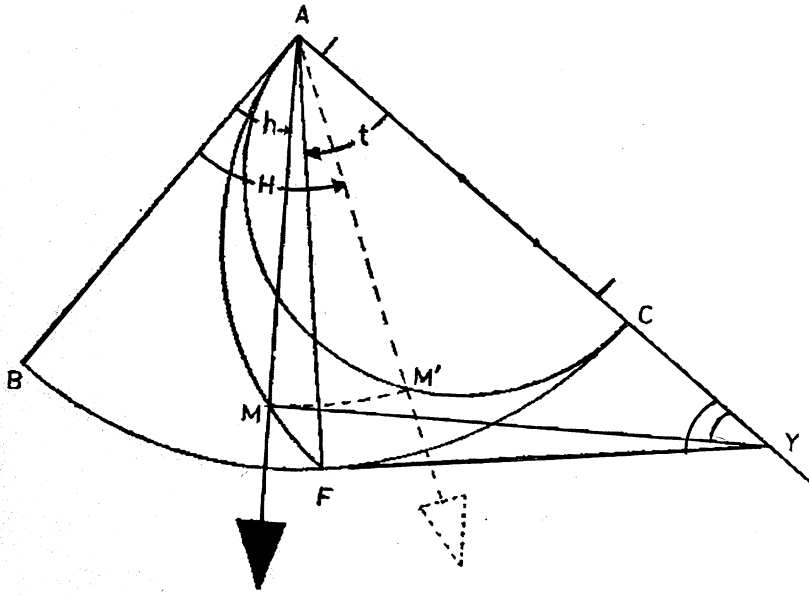


FIGURE 2.7 Use of the *quadrans vetus* (borrowed from Richard Lorch)

then, will coincide with an hour arc (as in the figure, hour arc AMF) or will be placed between two hour arcs. In fig. 2.7, the angle $\angle BAF$ will be $90^\circ - t$, and it will also be the hour angle measured from sunrise.

In our figure, angles $\angle YFA$ and $\angle YMA$ are right angles. We can easily see that:

$$\begin{aligned}\sin H &= AM / AF = AM' / AC \\ \sin h &= \sin \angle AYM = AM / AY \\ \sin \angle BAF &= \sin \angle AYF = AF / AY\end{aligned}$$

Therefore,

$$\begin{aligned}\sin H &= AM / AF = \sin h / \cos t \\ \text{and } \cos t &= \sin h / \sin H\end{aligned}$$

which is the standard approximate formula to obtain the time since sunrise in seasonal hours, for any latitude.²⁵⁴

²⁵⁴ On the use of this formula in Islamic astronomy see King, 2005, section XI (pp. 115–197).

2.4.2.1.3 *Sundials*

2.4.2.1.3.1 Ibn al-Raqqām's Treatise on Sundials

The Andalusian astronomer Ibn al-Raqqām (d. 1315) is the author of the *Risāla fī 'ilm al-ẓilāl*, an extremely important treatise on sundials and the only extant work of this kind that has been properly edited and studied.²⁵⁵ In the introduction to this book²⁵⁶ the author states that, while still a young man, a simple method occurred to him for the projection of the sphere that was different from the one invented by Ptolemy. It seems clear that the Ptolemaic method alluded to is the stereographical projection, used in astrolabes and similar instruments, and described for the first time in Ptolemy's *Planisphere*; surprisingly, though, Ibn al-Raqqām's procedure corresponds to the *analemma*, an old Greek technique consisting in representing on a plane certain elements of the sphere in order to find some arcs and angles which determine a point in space, avoiding the use of trigonometry. Clearly Ibn al-Raqqām is unaware of Ptolemy's treatise entitled precisely *Analemma*, which had not been translated into Arabic, although analemmatic techniques had often been used by Eastern Arab scientists for a variety of purposes, above all for the building of sundials.²⁵⁷

Ibn al-Raqqām did not invent analemmas but he wrote an important book on sundials using this technique, of which we know no precedent in al-Andalus or the Maghrib before him. He wrote the book during his youth, but it was borrowed and then lost. Later, other scholars from Ifrīqiya asked him to write the book again. All this makes it likely that the extant book was written, at an unknown date, in Tunis or Bijāya, the two cities where Ibn al-Raqqām is known to have lived before his arrival in Granada.

In this book, Ibn al-Raqqām describes the following types of sundials:

1. *Horizontal*
2. *Vertical*: the plane of the sundial is parallel to the prime vertical and the gnomon is placed on the meridian line.
3. *Vertical declining diptych*: two perpendicular planes; the line that bisects the right angle formed by the two planes is placed in the direction of the meridian line.
4. *Equatorial*: the plane of the dial is parallel to that of the equator, and the perpendicular gnomon points to the North Pole.
5. *Inclined*, forming a specific angle with the plane of the horizon and oriented southwards.

²⁵⁵ Carandell, 1988.

²⁵⁶ Carandell, 1988, pp. 55 and 256.

²⁵⁷ Sezgin, *GAS* V, pp. 156–7, 263–4; King, 1986b, B24 (p. 36).

6. *Inclined and declining*: inclined in respect to the horizon and oriented in any direction.
7. *Scaphe*: a sundial traced on the inner surface of a hollow hemisphere whose pole is the basis of the gnomon, the end of which will coincide with the centre of the sphere.
8. *Portable*, with a magnetic needle which allows a correct orientation of the instrument. This is, in fact, one of the earliest reliable references to the magnetic compass in Western Islam.

Besides this, Ibn al-Raqqām poses and solves the problems of the transformation of solar quadrants: that is, given a sundial built for a particular latitude, how to build another one, belonging to a different type, for the same latitude. He also explains the procedure to trace other lines on the sundial (projections of the declination parallels corresponding to the zodiacal signs, and those of the almucantars or altitude circles) which allow the use of the instrument for applications that normally corresponded to astrolabes or similar computing devices. Following the standard tradition, Ibn al-Raqqām explains in detail how to trace the curves corresponding to the prayer hours²⁵⁸ and also deals with the determination of the azimuth of the *qibla*, for which he uses an analemma similar to the one used by Ḥabash al-Ḥāsib in the ninth century and by al-Bīrūnī, in the eleventh, in his *al-Qānūn al-Masʿūdī*.²⁵⁹

2.4.2.1.3.2 Standard Horizontal Sundials

The wealth of information on sundials we find in Ibn al-Raqqām's book is not reflected in the actual instruments found in archaeological excavations. Only eight Andalusian sundials are extant and they all correspond to the horizontal type. The oldest ones were probably made in the tenth century.²⁶⁰ This is probably the date corresponding to the Cordoban sundial ascribed, explicitly, to Ibn al-Ṣaffār (d. 1035) (see fig. 2.8), as well as to the three instruments found in the "sundial courtyard" in Madīnat al-Zahrā', which should be dated before the destruction of the royal palace, which began in 1010 and was completed during the caliphate of Muḥammad III al-Mustakfī (r. 1024–1025). From this period are also the two fragments of another sundial discovered by Labarta and Barceló (fig. 2.9), probably made at the end of the ninth century or the beginning of the tenth; I say this because the Sun is shown entering Scorpio on 18 October and Sagittarius on 16 November, which agree with the values we can obtain with al-Battānī's *zīj* for ca. 900.

258 Carandell, 1984b.

259 Carandell, 1984a.

260 Barceló & Labarta, 1988; Carandell, 1989; King, 1992; Labarta & Barceló, 1995.



FIGURE 2.8
Sundial made, according to
the inscription, by Aḥmad ibn
al-Ṣaffār

In a horizontal sundial, the shadow projected by the Sun at the solstices describes two arcs of a hyperbola, while in the equinoxes it follows a straight line. The instruments are poorly made and the arcs of the hyperbola are replaced by crude approximations made by broken straight lines. Indeed, these Caliphal instruments are not of high quality; the only exception are the two fragments discovered by Labarta and Barceló which seem to correspond to a more elaborate kind of instrument. The low calibre of Andalusian sundials is rather surprising when seen them alongside the excellent astrolabes produced by the instrument makers of al-Andalus. Among these artisans we find Ḥasan b. Muḥammad b. Bāšo (d. 1316) and his son Aḥmad b. Ḥasan, who built astrolabes as well as sundials which, unfortunately, have not been preserved.²⁶¹

²⁶¹ Inān, 1973–77, I, pp. 204 and 468.

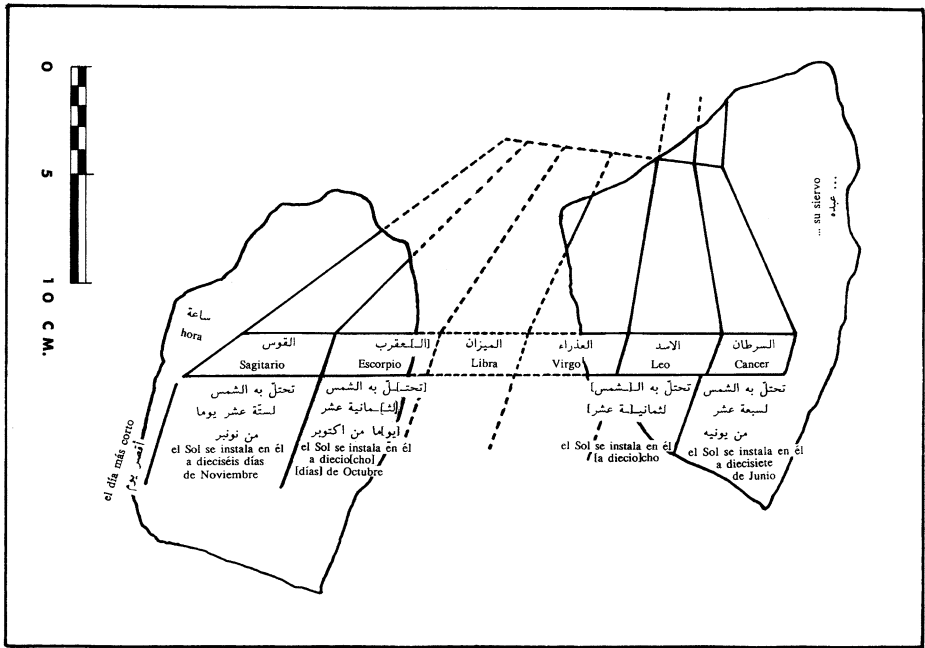


FIGURE 2.9 Ana Labarta's tracings of the remnants of a sundial found in the Camposanto de los Mártires together with a reconstruction of the whole instrument

We have less information about Maghribī sundials.²⁶² I am aware of only two that have been studied properly, although brief published descriptions of the Merinid sundials of the mosques of Sīdī Ḥalwī (1347) and al-Manṣūra, in Tlemcen, show that they were basically designed to determine the time of the canonical prayers.²⁶³ One of the well-studied sundials is a late spectacular instrument extant in the courtyard of the Great Mosque of Sīdī ‘Uqba in Qayrawān. Its author was Aḥmad b. Qāsim b. ‘Ammār al-Sūsī who built it in 1258/ 1842, probably copying an older undated instrument which can still be seen on top of a pillar of the arcade in the same courtyard.²⁶⁴ The second sundial was made by Abū l-Qāsim ibn Ḥasan al-Shaddād in Tunis, 746/1345–46,²⁶⁵ an unusual instrument that marks not the standard hours but the times of the canonical and non-canonical prayers.²⁶⁶ As we see in fig. 2.10, this sundial shows the curves of the solar shadow corresponding to the two solstices (two arcs of a circle instead of arcs of a hyperbola) and the two equinoxes, as well as curves or straight lines for four prayer hours and the midday shadow, which are (from left to right, see 2.4.1.1):

1. *Ḍuhā* (midmorning):²⁶⁷ the curve is the mirror image of the ‘*aṣr*’ curve. If S_d is the shadow at the moment of *ḍuhā* and S_m is the midday shadow:

$$S_d = S_m - g$$

262 See a list of extant Maghribī sundials in King, 2007, p. 206. E. Mercier, 2014, Jarray & Mercier, 2015 and Jarray, 2015, describe an important collection of late Tunisian sundials (16th–19th centuries), exceeding the period of time studied in this book. See also E. Mercier 2019. As for Moroccan sundials, more than a hundred instruments have been listed (with extremely brief non-technical descriptions) by Kharbūsh, 2019, in a book whose main interest relies on the information given about Moroccan sundial makers. There is written evidence about the existence of a tenth-century sundial in the Qarawiyyīn great mosque of Fez (Kharbūsh, 2019, p. 134). The extant medieval instruments are a standard horizontal sundial dated 537/1142–3 in the museum Dār Ba-l-Ghāzī, of a small locality between Salā (Salé) and Qunayṭira (Kharbūsh, 2019, p. 36), another fourteenth century instrument of the same type in the Qarawiyyīn mosque (Kharbūsh, 2019, pp. 23–24), a vertical dial in the Mosque of al-Andalus (dated 786/1384–5, Kharbūsh, 2019, pp. 33–34) and, finally another vertical dial in the Great Mosque of Tāza, probably made before ca. 1250 (Kharbūsh, 2015, p. 69). Most of the extant Moroccan dials correspond to the 17th century onwards and Kharbūsh (2015, p. 123) states that the interest in this kind of instrument in mosques developed mainly under the ‘Alawī dynasty.

263 Baklī, ‘Aysānī & Ilhām, 2014, pp. 23–25. The cylindrical sundial, traced on a column of the mosque of Sīdī Ḥalwī is particularly interesting. I owe this information to Muḥammad Riḍā Baklī.

264 Janin, 1977; E. Mercier, 2014, pp. 56–57.

265 King, 1977; E. Mercier, 2014, pp. 57–58; Jarray, 2015, pp. 30–31.

266 As shown by Jarray, 2015, this kind of design reappears in several dials made in the seventeenth and eighteenth centuries.

267 King, 2004, pp. 571–3.



FIGURE 2.10 Sundial made by Abū l-Qāsim ibn Ḥasan al-Shaddād in Tunis (746/1345–46) preserved in the Carthage National Museum (Tunis)

2. *Ta'hīb* (late morning): represented in the sundial by a straight line. It corresponds to a non-canonical religious obligation performed one equinoctial hour before midday.
3. *Zawāl* (midday): another straight line.
4. *Ẓuhr*: represented by a curve showing the time of the midday prayer.
5. *ʿAṣr*: another curve for the afternoon prayer.

The sundial also shows, to the lower right of the main markings, a segment corresponding to the length of the gnomon and a diagram for the *qibla* which marks the direction 45° south of east (see below 2.5.4).²⁶⁸

Andalusian texts describing how to make a sundial are scarce. It is clear that al-Battānī's *zīj* circulated in al-Andalus in the second half of the tenth century, which shows how to build this kind of instrument. Al-Battānī's text was the source used by Alfonso X's collaborators to write the *Libro del Relogio dicho*

268 Jarray, 2015, pp. 28–29 describes a standard horizontal Tunisian sundial attributed to Ibn Abī l-Rijāl (see also Jarray, 2013), as well as (pp. 32–35) other two horizontal sundials from the Zaytūna mosque in Qayrawān and the Great Ribāt in Monastir. The other sundials in Jarray's catalogue date from the 17th century onwards.

de la piedra de la sombra.²⁶⁹ Besides, D.A. King has discovered a sundial table calculated for the latitude of Córdoba (38°) and a brief undated Andalusian treatise describing several varieties of primitive sundials which do not seem to appear in other Arabic sources.²⁷⁰ As for the Maghrib, al-Ḥasan b. Khalīl b. ‘Alī al-Karādīsī al-Ṭubnī (1420–1482) wrote a treatise on the construction of sundials entitled *Ashkāl al-wasā’iṭ fī rasm al-munḥarīfāt wa l-basā’iṭ*, extant in several manuscripts but still unpublished,²⁷¹ and we know of the existence of tables for marking sundials for the latitude of Marrakesh and Fez, as well as a treatise on the construction of horizontal sundials for the latitude of Tunis.²⁷²

2.4.2.1.3.3 Other Types of Sundials: the *balāṭa*

Other texts give information about other types of sundials used in al-Andalus. In the eleventh century, the Toledan astronomer Ibn al-Zarqālluh (d. 1100) mentions the kinds of instruments he knows, among which we find horizontal as well as vertical, cylindrical and conical sundials.²⁷³ I know of no other references to a conical dial until the twelfth century when al-Umawī al-Qurṭubī’s *Kitāb al-anwā’* refers to a sundial of this kind called a *mukḥula*.²⁷⁴ The cylindrical type was, no doubt, known in al-Andalus towards the end of the tenth century, because Hermann Contractus, one of the transmitters of Arabic astronomy in the early eleventh century, describes it in the treatise *De horologio viatorum*. This kind of dial was still used at the beginning of the twentieth century in the Catalan Pyrenees, where it was known as “rellotge de pastor” (shepherd’s dial). The instrument was also used in the Maghrib: two examples with markings for the prayers of the *zuhr* and the *‘aṣr* have been described, one (1346) in Tlemcen and an earlier one in al-Manṣūra.²⁷⁵

Besides this, Arabic and Latin sources describe a type of “horizontal” dial which makes sense only if it is interpreted as a deformation of an equatorial sundial or, perhaps, as a dial placed on the plane of the first vertical. It is interesting that all the rudimentary sundials described in these sources receive the name of *balāṭa*. Thus, the *Kitāb al-asrār fī natā’ij al-afkār* by Aḥmad or Muḥammad b. Khalaf al-Murādī (fl. probably in Toledo ca. 1050) ascribes to

269 Dorce, 1999.

270 King, 1977; King, 1986b, p. 135 (F5 and F6).

271 Lamrabet, 2013, no. M552, p. 270.

272 King, 1977 p. 201.

273 Samsó, 2011, p. 172.

274 Forcada, 1990, pp. 59–69.

275 King, 2007, p. 192.

Ibn al-Šaffār (d. 1035) a small text describing an atypical dial.²⁷⁶ If this attribution to Ibn al-Šaffār is correct, it is clear that the Cordoban astronomer is using an earlier source because the same text, with small variants, is to be found in Qāsim b. Muṭarrif's *Kitāb al-hay'a* (mid-tenth century),²⁷⁷ and a third description appears in a later source: Maimonides' commentary of the *Mishnah*.²⁷⁸ None of these descriptions is particularly clear and we can only say that the instrument has hour lines in two sectors, one of which corresponds to the hours before midday and the second to those after midday. These hour lines are, probably, circle radii and the texts state explicitly that in each one of the "points where the hour lines meet" (centres) there is a gnomon. The two sources do not state that the aforementioned radii should correspond to equal intervals of 15°, and we can hypothesise that the intervals were unequal. The only thing that is clear in the texts is that the face of the quadrant bearing the hour lines should be oriented towards the south, and they describe carefully how to determine the meridian line by the use of the "Indian circle".

The information contained in the written descriptions is complemented in a way by the figures appearing in the two manuscripts (see fig. 2.11): the figure of the *Kitāb al-hay'a* is too rough and does not fit the description; the other drawing, which appears at the end of the *Kitāb al-asrār*, is more interesting. David King believes that it corresponds to a crude horizontal sundial with a gnomon perpendicular to the plane of the instrument and that the radii should mark 15° intervals. He is probably right, due to the evidence furnished by the illustration of the *Kitāb al-asrār* where the meridian line is traced on the dial, something which makes one think of a horizontal sundial. In this case it is suggestive to consider this (following Casulleras) as the survival of a sundial of Roman tradition which was probably the result of the degeneration of an equatorial dial.²⁷⁹

One might also relate dials of this kind to the very rudimentary vertical sundials on the southern walls of old Romanesque churches in the north of the

276 There is a recent facsimile edition of the unique manuscript of Ibn Khalaf's work (Florence, Medicea Laurenziana 152), accompanied by an edition of the Arabic text, an English translation, a commentary (often not very accurate) and an interactive CD published by Lisa *et al.* in Milan, 2008. The volume with the English translation (pp. 182–187) contains a poor analysis of the passages on the *balāṭa* and the Indian circle: the authors do not seem to know the previous bibliography on the topic. On this publication see my review in *Suḥayl* 9 (2009–10) pp. 234–238.

277 Casulleras, 1993.

278 King, 1978.

279 See Jarray, 2015, pp. 24–25 for an instrument of this kind found in the Dāmūs al-Karriṭa in Carthage.

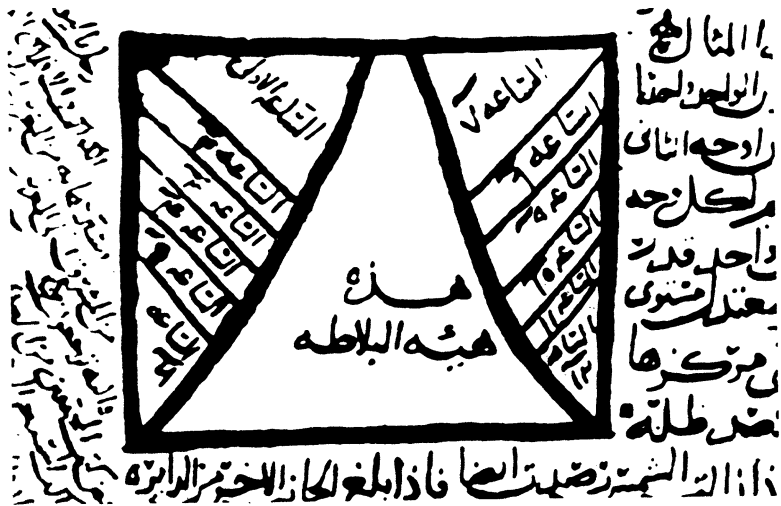


FIGURE 2.11 The illustration in Qāsim b. Muṭarrif's *Kitāb al-hay'a* (above) and in Ibn Khalaf's *Kitāb al-asrār* BELOW, MS. OR. 152 FOL. 47V © BIBLIOTECA MEDICEA LAURENZIANA, FLORENCE

Iberian Peninsula.²⁸⁰ They usually appear as semicircles with radii placed at 15° intervals and the gnomon in their centre. They are similar to the Saxon

²⁸⁰ Valdés, 1996 contains an enormous amount of information on Spanish dials of this kind. I thank Miquel Dorca and Tura Clarà from Besalú (Girona) for a photocopy of this book.

“mass clocks” or “incised dials” found in England,²⁸¹ and seem to have been used in many other cultural contexts.

Other crude sundials are described in passages of the “old corpus” of Latin texts connected with the early introduction of Arabic astronomical materials in Europe, although these descriptions seem to be free of any Arabic influence.²⁸² In them we find the “Indian circle” technique for the determination of the four cardinal points. This instrument is also used as a sundial, after dividing it into 24 equal parts and observing the part of the circle intersected by the solar shadow at sunrise and sunset. Obviously, this description could only work if the “Indian circle” was placed on the plane of the equator with the gnomon pointing to the North Pole.²⁸³ Another rudimentary horizontal sundial appears in the Tunisian recension of Ibn Ishāq’s *zīj* in a passage attributed to Ibn Mu’adh al-Jayyānī (d. 1093) although the references in the text to its use in Ifrīqiya make this attribution doubtful: it describes a semicircle divided into twelve equal parts, with a (perpendicular?) gnomon in its centre, whose shadow marks the seasonal hours.²⁸⁴

A reference to an equatorial or first vertical dial appears in an appendix to the *Astrolabii sententiae*, where we find confusing allusions to the two-faced Janus Bifrons and Apollo: Janus is placed in front of Apollo and his double face, illuminated by Apollo, “reflects” the solar light and marks the hours and the parts of the hour.²⁸⁵ It seems, therefore, that Janus is a sundial with two faces oriented towards the north and the south, as one would expect in an equatorial sundial or an instrument situated on the plane of the first vertical.

Finally, manuscript Ripoll 225 contains a fairly complete description (without any Arabic influence) of another sundial in which we find that the longest daylight, in the summer solstice, lasts fifteen hours, which corresponds to the beginning of the fifth climate and to a latitude of 41° (Barcelona?), according to the Ptolemaic table of climates. This text has been analysed in depth by Casulleras²⁸⁶ who showed that a surprisingly similar description appears in the *Kitāb al-asrār* by Ibn Khalaf al-Murādī.²⁸⁷ Ibn Khalaf’s sundial (also called *balāṭa*) uses implicitly a maximum ortive amplitude of 30°, corresponding to a latitude of about 37°, which could correspond to Córdoba. The Cordoban

281 Green, 1978.

282 Samsó, 2004 b, pp. 150–154.

283 Millàs, 1931, pp. 326–327.

284 Casulleras, 1993, pp. 69–71, 81–84.

285 Millàs, 1931, p. 288.

286 Casulleras, 1996.

287 Cf. Lisa *et al.*, 2008, pp. 178–181. The commentary is confusing and inadequate; it does not take account of either the Latin source or Casulleras’ paper.

dial is, apparently, a horizontal instrument, as its design is based precisely on the maximum rising or setting amplitude, but the same cannot be said of the Ripoll dial, for which the text describes how to orient it so that the sight line passing through the length of the gnomon should reach the Pole Star. This implies that the plane of the sundial forms with the horizon an angle equal to the colatitude because the text states explicitly that the gnomon is perpendicular to the plane of the dial. The description corresponds, therefore, to an equatorial sundial even though the Latin source does not mention that the instrument should have two opposite faces which will be used in the two halves of the year, depending on whether the Sun is north or south of the equator. Besides, Casulleras' paper shows that the dial described in the Ripoll instrument has minimal errors if it is used as an equatorial instrument, but has major errors if it is placed on a horizontal plane. We have, therefore, a clear example of an equatorial sundial, which corresponds to a Latin tradition introduced in the Arab Andalusian world, where it has been interpreted as a horizontal dial.

Finally we have a description of a monumental sundial: the "Religio del Palacio de las Horas", designed by Rabbi Ishāq b. Sīd, known as Rabiçag, for King Alfonso x and included in the *Libro del Saber de Astrología*. This instrument has been studied by J.D. Morales²⁸⁸ who has shown that the Alfonsine texts refer to two different varieties of the sundial corresponding to a cylindrical and a square tower. In both cases the observer sees, from inside the tower, how the rays of solar light enter the palace during the hours of daylight. The cylindrical tower has twelve windows placed on the wall and a door in the northern part of the building. The square tower is covered by a hemispherical dome which has a set of twelve windows; in both cases the twelve windows correspond to the twelve seasonal hours of the day. The text, edited by Morales, describes the procedure for building both structures but does not completely overcome the technical problems involved which, according to Rabiçag, must be resolved by the artisan's ability and competence. It seems, therefore, that the "Palacio de las Horas" was probably never built.

2.4.2.1.4 *The Nocturnal*

The hour during the night can be determined with the nocturnal, an instrument based on the rotation of two circumpolar stars which describe a whole circle every 24 hours around the North Pole, approximately identified with the Pole Star. If we know the position of the star at sunset or midnight and observe its position at any particular moment using the nocturnal, the difference between the two positions (expressed in degrees) will give us the equinoctial

²⁸⁸ Morales, 2010.

hour, using the equivalence $1\text{ h} = 15^\circ$. The stars commonly used in the nocturnal are the two Guards of the Little Bear (β and γ *Ursae Minoris*) and their function is similar to that of the hands on a watch.

There is little evidence that Arab astronomers knew of the nocturnal²⁸⁹ in spite of the similarity of the method just mentioned and the use of the rotation of the lunar mansions for determining the hour during the night.²⁹⁰ The instrument seems, however, to have been known to Latin authors towards the end of the tenth century as, according to Thietmar of Magdeburg (d. 1018), Gerbert of Aurillac built an *orologium* during his stay in Magdeburg which allowed him to determine the hour at night by sighting the Pole star with a kind of optical tube. This comment was interpreted by Poulle²⁹¹ as a reference to the nocturnal. Also according to Poulle,²⁹² Walcher of Malvern used a nocturnal in 1092 to establish the hour of a lunar eclipse. Another obscure allusion to this instrument is to be found in one of the texts of the “old corpus”, extant only in ms. Ripoll 225 (Inc. « cum hominum habitationes equales sibi fore »).²⁹³ This text describes the *clavus* and the *equus* (sic), the *clavus* being the pin which holds together the mother, the plates and the spider of the astrolabe, while the *equus* is the wedge-shaped piece of metal which secures the *clavus* and was often in the form of a horse. Interestingly, this description states that the *equus* has two holes which correspond to two *stellas horarias*, one of which, placed on the neck, is smaller, while the star on the tail of the horse is bigger. Both stars are driven by *arcturus* (clearly a confusion with *arctos*, the Bear) in its perpetual motion around the Pole. The passage is surprising, as I do not know of any horse-piece of an astrolabe with holes corresponding to the projection of stars. We might conceive (although the hypothesis may be rather bold) of a horse-piece which, extracted from the astrolabe, could be used as a nocturnal. In any case, the two *stellas horarias* (probably β and γ *Ursae Minoris*) on the horse, which can rotate around the centre of the astrolabe (the projection of the Pole), seem to contain an allusion to a nocturnal.

Two other references to our instrument are to be found in Catalan sources: Ramon Llull (1232–1316) describes the nocturnal in the *Liber Principiorum Medicine* and in *De Nova Geometria* (1299), as a copper disc with a hole in its centre (through which the observer will sight the pole star) and two concentric circles, one of which is divided into the twelve months of the solar year, while

289 A late Maghribī nocturnal is preserved in the Oxford Museum for the History of Science (inv. 48046). See King, 2007, p. 206; Bakli, ‘Aysānī & Ilhām, 2014, pp. 21–22.

290 See 2.4.1.2.4 and Forcada, 1995.

291 Poulle, 1985.

292 Poulle, 1980a.

293 Millàs, 1931, p. 310.

the other has 24 apertures corresponding to the 24 hours of the day. These holes are used to sight β *Ursae Minoris*, whose position with respect to its position at sunset or midnight will give the hour. The *Liber Principiorum Medicine* has an illustration of the instrument indicating the number of hours of daylight in each month of the year which attains a maximum of 15 hours in July and August (sic). If these 15 hours correspond to the length of the day at the summer solstice, the corresponding latitude is $41;21^\circ$, which fits Barcelona or another site to the north of the city on the Catalan coast. Another Catalan (?) source, the *Almanach of Tortosa* (1307) (see 7.7.3.2) contains a description of the nocturnal similar to Ramon Llull's.²⁹⁴

The second source, written in Catalan, containing a reference to the nocturnal is the *Catalan Atlas* (1375) attributed to Cresques Abraham and preserved in the Paris National Library. Here we do not find a description of the instrument but an analysis of the principles of its use, with information on the position of the two Guards at sunset in the equinoxes and solstices and also on the solar rising and setting amplitude in these dates. The data contained in the text are not always consistent: from the rising and setting amplitude in the solstices we can calculate a latitude of $44;8^\circ$, while the maximum length of daylight given in the text (220°) yields a latitude of $38;11^\circ$, while the Sun's meridian altitude in the winter (25° , instead of 15° in the text) and the summer solstice (50°) correspond to two different latitudes: $43;30^\circ$ (summer solstice) and $41;30^\circ$ (winter solstice).²⁹⁵

2.4.2.1.5 *Candle Clocks*

Qāsim b. Muṭarrif's *Kitāb al-hay'a* (ca. 950) deals with the problem of determining the hour during the night using an instrument called a *thurayyā*, which is simply a candle clock with twelve glass lamps, each one with a cotton wick soaked in oil. The first lamp contains the amount of oil necessary to keep the lamp alight during one seasonal hour of a particular night. The oil of the second lamp will last two hours, the third lamp three hours, and so on until the last lamp, which will remain lit the whole night. The author gives details about the amount of oil needed for one night hour throughout the year, which allows us to calculate the maximum length of the night in the winter solstice (14;15 h.). This corresponds to a latitude comprised between 33° and $33;30^\circ$, which does not fit any city in al-Andalus but may correspond to the latitude of Tunis. This is clearly significant, because Qāsim's text gives an example of the procedure for calculating the length of a night hour for Ifrīqiya. The same latitude also

294 Vernet, 1978, pp. 361–163; Samsó, 1981–83; Farré, 1996.

295 Samsó, 2005.

corresponds to Baghdad, and this is why we hypothesise that the candle clock of the *Kitāb al-hay'a* came to al-Andalus from Baghdad, via Tunis.²⁹⁶ An almost identical description of this candle clock has been attributed to Ibn Yūnus (fl. 980) but, in this case, the implicit latitude is about 35°. ²⁹⁷

Two other candle clocks are documented in the medieval Iberian Peninsula. The more sophisticated one is described in the *Libro del Relogio de las Candelas*, in the Alfonsine *Libro del Saber de Astrología*. A wax candle, inside a tube (only the wick is visible) lies on a platform which may rise, pulled by two weights hanging from a pulley. As the candle burns, the platform ascends together with a moving table which shows the hour, the ascendant and midheaven.²⁹⁸

The second candle clock is an instrument (*minkān*) built in Granada for the celebration of the *al-mawlid al-nabawī* (birth of the Prophet) on 30 December 1362. This year had a special significance in the history of Naşrid Granada because King Muḥammad v (r. 1354–1359, 1362–1391) recovered the throne of Granada on 16 March 1362 after an exile in Fez of more than two years. During his stay in Merinid Morocco he would, no doubt, have been aware of the custom of celebrating the *mawlid* with a *minkān* built for its use for one night or, perhaps, for a few more days. We know that the Merinid sultan Abū 'Inān (1348–1358) had an instrument of this kind built for the *mawlid* of 1350: it consisted of a big wooden box with twelve small windows, and, on top, a rotating crescent Moon and a flying metallic bird. At the moment of the beginning of an hour of the night, the bird threw a weight onto a small saucer placed in the window corresponding to that hour and the figure of a girl, with a piece of paper in her right hand, appeared in the window. We also have a reference to a similar celebration in Tlemcen for the *mawlid* of 1359, during the reign of Abū Ḥammū Mūsā II (1359–1389).²⁹⁹ In this case we know that the corresponding *minkān* was built by 'Alī b. Aḥmad al-Şanhājī ibn al-Faḥḥām, who in 1357 had

296 Comes, 1993.

297 Kennedy & Ukashah, 1969. King, 1999 (p. 509) believes that the author of this text on the candle clock is Yūnus ibn al-Ḥusayn al-Aşṭurlābī (fl. 1144): see King, 1987, no. xx, p. 286.

298 Farré, 1997. This paper contains photographs of Farré's reconstruction of the instrument for the museum of the Institut für Geschichte der Arabisch-Islamischen Wissenschaften (Frankfurt), now in Istanbul. Besides, a doctoral student of the University of Barcelona, Juan David Morales, is preparing a PhD thesis consisting in the edition and study of the Alfonsine clocks described in the *Libro del Saber de Astrología*; here I am using materials from his research.

299 Baklī, 'Aysānī & Ilhām, 2014, pp. 34–36 refers to the existence of an earlier clepsydra with automats, in Tlemcen, during the reign of Abū Tāshufīn I (r. 1318–1337) and suggests that the clepsydra used in the celebration of the *mawlid* of 1359 had been made during the reign of the Merinid Abū 'Inān (r. 1348–1358).

also made the clepsydra of the *madrasa* Bū 'Ināniyya in Fez.³⁰⁰ The instrument was, again, a box with a rectangular base and the corresponding twelve windows and twelve saucers. At the beginning of the hour two eagles, placed in two more windows at both ends of the line of the twelve hour-windows, threw a brass weight, a girl was seen in the corresponding window with a poem in her hand, and a snake appeared whistling from behind a tree and tried to swallow a chicken who was hiding under the wings of its father. All the imagery of the instrument (eagles, girl, snake, tree, chicken and cock) were made in silver.³⁰¹

The Fez *minkān* was, no doubt, the model for Muḥammad V's instrument, although I do not believe that the two aforementioned clocks were candle clocks. Their description suggests a more sophisticated technology. The Granada instrument was used for the celebration in the Alhambra which lasted the whole night beginning with the *maghrib* prayer after sunset (at 17:16 h GMT) of 29 December and ending with the *fajr* prayer (sunrise was at 7:32 h GMT) of the following day. The night hours were marked by a *minkān* described by Ibn al-Khaṭīb: the instrument had the shape of a dodecagonal prism, made in wood and hollow, with a height of one *qāma* (about 1.70 m.). On the upper part of each side there was a concave opening (*miḥrāb*) with a door closed by a bolt. This bolt was connected with an iron bar which blocked a hole placed inside the *miḥrāb* and prevented the fall of a copper ball of the size of a hazelnut. On top of the *minkān* there was a candle with twelve divisions corresponding to the twelve hours of the night.³⁰² A string came out of each division and ended in the bolt of the corresponding *miḥrāb*. Once the candle was lit, it began to burn until the end of the first hour: then the fire reached the string which also burnt, and the bolt unlocked. The door of the first *miḥrāb* opened and it dragged the iron bar. The corresponding hole allowed, then, the fall of the copper ball which fell into a metal saucer placed underneath, producing a sound. At the opening of the *miḥrāb* door, one could see a piece of paper with a poem relating to the hour that had gone by, and the keeper of the instrument delivered the paper to a reciter (*musmi'*) who read the poem aloud.³⁰³

300 Lamrabet, 2013, M162, p. 188.

301 García Gómez, 1988, pp. 88–89.

302 The use of calibrated candles in order to measure time during the night was common in the Middle Ages and is attested in Naṣrid Granada: García Gómez, 1988, p. 95.

303 García Gómez, 1988, pp. 94–98, 131–132, 154–155. An attempt to reconstruct this instrument has been made by Fernández Puertas, 2006, pp. 171–185, and Fernández Puertas, 2010. Another reconstruction of the same instrument was made by Eduard Farré for the exhibition “El legado científico andalusí” (Palacio de Mondragón, Ronda, 1995). See photographs in Farré, 1998.

2.4.2.1.6 *Clepsydras and Other Clocks*

The oldest reference we have to a clepsydra seems to be ascribed to the poet, astrologer and, perhaps, engineer ‘Abbās b. Firnās (m. 887) who, according to Ibn Ḥayyān³⁰⁴ “invented peculiar objects which deserved to be owned by kings, marvellous and astonishing devices, with beautiful images and fantastic motions, due to their changes of colour and to the fact that they emptied the waters of basins and other places. In order to place his statues in the royal palace and take care of their mechanical structures, he asked the help of Aṣḥab, the master craftsman of the carpenters, whom he taught how to act [...] according to the techniques of his craft”. One of these devices must have been the clock he designed as a present for the emir Muḥammad (852–886). The clock was accompanied by a poem in which ‘Abbās b. Firnās indicated that the instrument was useful to know the hours of the canonical prayers when neither the Sun or the stars could be used as an indicator. If the previous reference to the filling and emptying of the basins and to moving figures is related to this clock, this is the earliest Western Arabic mention of a clepsydra with automata.

This tradition was also alive in Latin Europe. Towards 1125 William of Malmesbury says, in his *De gestis regum Anglorum* that Gerbert of Aurillac left, in Reims, a *horologium arte mechanica compositum*. If this reference is correct, the object is likely to have been a clepsydra. Besides, Ms. Ripoll 225 contains the description of a clepsydra with a float and an alarm, which does not show any Arabic influence.³⁰⁵ In this instrument the water of an upper container falls in drops through a hole in another container placed underneath which has the float. As the water level goes up in this second container, the float rises. The float is attached to a string which hangs in a pulley and at whose end there is a weight. The pulley will rotate and an index will mark the hour. The clepsydra is accompanied by an alarm mechanism and the bells will sound at the hour previously established in the hour disk.

Clepsydras with automata are well documented in Toledo in the eleventh century, because this is probably the place and the time in which Ibn Khalaf al-Murādī wrote his *Kitāb al-asrār fī natā’ij al-afkār*³⁰⁶ which contains the description of 18 clocks moved by water, many of which are ingenious toys rather than practical instruments (machines no. 6–13, 17–20, 27–30).³⁰⁷

304 Makkī, 1973, pp. 282–284; Vernet, 1980; Samsó, 2011, pp. 55–56.

305 Millàs, 1931, pp. 316–318; Vernet, 1980; Maddison, Scott & Kent, 1962; Farré, 1989; Farré & Segura, 1989.

306 Described in 1975 by King; see King, 1987, no. xx, pp. 288–9.

307 Machine no. 1 of this collection is not a clock (although, as it is moved by water, it is a clepsydra). It was studied by Vernet, Casals & Villuendas, 1982–3. On the basis of this study, it was reconstructed by Eduard Farré for the exhibition “El legado científico

Another machine with the same origin (no. 14) is a clock and a yearly astronomical calendar which gives the ascendant and the descendant for each moment. The source contains a second astronomical calendar (no. 15) with the same applications.³⁰⁸ As the only extant manuscript of this work (Florence, Medicea-Laurenziana 152) was copied in Toledo in 1266 during the reign of Alfonso X, and as we find in it, after the *Kitāb al-asrār*, a note in Arabic with Hebrew script written by Rabbī Ishāq b. Sīd (Rabiçag), one of the king's two great scientific collaborators, we might imagine that the *Kitāb al-asrār* was one of the sources used by this author for his description of the Alfonsine clepsydras. This is a problem that has not been studied to date, but it is interesting that machines 14 and 15 can be used to establish the ascendant and the descendant for a particular moment, a feature often found in the Alfonsine clocks.

It was probably during the eleventh century that a certain Abū l-Qāsim b. 'Abd al-Raḥmān known as al-Zarqāl (not the famous Ibn al-Zarqālluh) built a clepsydra in Toledo used to mark the date of the lunar month, but not the hour.³⁰⁹ According to the description made by the geographer al-Zuhri (d. 1137), al-Zarqāl built two large basins in a house outside Toledo, on the banks of the river Tajo, near the *Bāb al-dabbāghīn* ("Door of the tanners"), which were filled and emptied in consonance with the lunar phases. When the new Moon appeared, the water began to flow, and the water level rose by 1/14 of the total volume of the basin per day. On the fourteenth day of the lunar month the basin was full and, on the fifteenth the level of water began to lower at the same rate, and by the end of the month the basin was empty. If, during the process, somebody removed or added water to the basin, the system automatically compensated the excess or lack of water.³¹⁰

To this we should add the two clepsydras (*Libro del relogio dell agua*, *Libro del relogio del argent uiuo*) preserved in the Alfonsine *Libro del Saber de Astrología*.³¹¹ The *relogio dell agua* is an inflow clepsydra based on the same principle as the instrument described in ms. Ripoll 225 although, obviously, it

andalusí" (Madrid, 1992). See photographs of the instrument in Farré, 1997a. Two copies of this reconstruction are preserved in the Museo Nacional de la Ciencia y de la Técnica (La Coruña, Spain) and in the museum of the Institut für Geschichte der Arabisch-Islamischen Wissenschaften (Frankfurt, now in Istanbul).

308 See the facsimile of the manuscript, edition and translation published by Lisa *et al.*, 2008; Hill, 1981; Hill, 1992; Casals, 1982; Vernet, 1978.

309 Millás, 1943–50, pp. 7–9; Vernet, 1980.

310 In a conversation in 1990, Donald Hill offered to reconstruct this clepsydra, using mechanisms known in the eleventh century, for the exhibition "El legado científico andalusí" which I was preparing for Madrid, 1992.

311 Photographs of the reconstructions of these two instruments can be seen in Farré, 1997, pp. 7, 9–10. See also Hill, 2013, p. 240. I am also using a draft of J.D. Morales' PhD thesis.

is far more sophisticated. The water passes from a large tank into a smaller one which keeps a fixed level of water with a pneumatic feedback control. This is an important technological innovation which guarantees a stable water pressure and a constant flow of water in agreement with the prologue which states that clepsydras that allow the water to flow, without regulating its pressure, transform equal hours into unequal ones. From the small tank the water drops into a third tank in which there is a float which rises with the increase in the water level, pushing a board which indicates, in its ascension, the hour, the ascendant and the midheaven of the ecliptic.

The *Relogio del argent uiuo* is a particularly interesting kind of clepsydra because it uses mercury instead of water. The mercury is enclosed in a ring-shaped tank, divided into twelve separate compartments. Each separation has a small hole allowing the mercury to move from one compartment to another. As the mercury moves, the tank is unbalanced and produces a rotation. The axis of the tank is also the axis of a pulley which rotates together with the tank. A rope placed around the pulley, at whose end there is a weight, receives the motion, and a system of gears connects the pulley with an astrolabe dial, which rotates once every 24 hours. One can read the hour and the position and altitude of the Sun and the stars on the astrolabe dial. Farré describes this instrument as a “closed circuit clepsydra”. As this clepsydra was described at the time in which the first mechanical clock appeared (ca. 1271), something of which the Alfonsine collaborators were not aware, it has been considered to be an immediate predecessor of this instrument, as it has all the elements of a mechanical clock with the sole exception of the escape mechanism.³¹² It is also interesting that the use of mercury also appears in machine no. 1 (not a clock) in Ibn Khalaf al-Murādī’s *Kitāb al-asrār*, which uses mercury to regulate the oscillation of a balance beam, as it is contained in a tube placed on top of the aforementioned beam.

The Alfonsine model was clearly imitated by Pedro III of Catalonia and IV of Aragon (r. 1319–1387). We find an example of this in relation to clocks: in 1376 the king sent a small (?) clock to his daughter with an alarm system, consisting of three bells. The dial is replaced by an astrolabe whose rete will give a complete rotation every 24 hours. The instrument contained “three” plates (the document only describes two) one of which is designed for latitudes 38°/39° (on both sides of the plate) and the other for 42°/43°. It is obvious that these latitudes intend to cover the southern and northern limits of the king’s reign: from the southern part of the kingdom of Valencia up to Montpellier. The text

³¹² Mills, 1988, “The Mercury Clock of the *Libros del Saber*”. *Annals of Science* 45 (1988), pp. 329–344. I am also using information from the draft of J.D. Morales’ PhD thesis.

of the document goes on to say that the hour can be established with the astrolabe using the Sun during the day and the stars in night time, as well as the ascendant and other things which one can obtain using the instrument and the art of astrology.³¹³ The text does not give any other details but the description of the clock reminds us of the Alfonsine mercury clock. Besides this document, which is the only one that gives a fairly complete description, we find many other references to clocks owned and bought by Pedro III, or by his son Juan, as well as references to clockmakers from different cities in the kingdom.³¹⁴

The decline of mechanical engineering in fourteenth-century Granada, exemplified by the Alhambra candle clock, contrasts with the contemporary flourishing of clepsydras built in Merinid Fez, about which we have descriptions and some remains.³¹⁵ The first known clepsydra, of an elementary type, was made in a room in the minaret of the Qarawiyyīn mosque by Abū ‘Abd Allāh Muḥammad b. al-Ḥabbāk al-Tilimsānī in 685/1286–7.³¹⁶ It soon fell into disuse due to the difficulty of refilling the tank with water, which had to be brought up a steep staircase of 101 steps.³¹⁷ A new one was built on the ground floor of the building by Muḥammad b. ‘Alī al-Ṣiddīnī³¹⁸ and Muḥammad b. ‘Alī b. Yaḥya al-Ṣanḥājī al-Naṭṭāḥ in 1317;³¹⁹ the final version (parts of which survive today) was made in 1346–48 by Abū ‘Abd Allāh Muḥammad al-‘Arabī and restored, in 1362, by Abū Zayd ‘Abd al-Raḥmān al-Lujā’ī al-Fāsī, known as Ibn al-Faḥḥām (d. 1371). In 1346 this latter instrument was fitted with an astrolabic dial and a system of 24 doors corresponding to the 24 hours of the day: each hour the clock would eject a metal ball through the correct door, thus marking the time with a noise. A similar technique (a metal ball falling into a copper cup at the beginning of each hour), the problem being that the procedure was repeated thirteen times every day [?], was used in the clepsydra built in 1357 by Abū l-Ḥasan ‘Alī b. Aḥmad al-Tilimsānī in the *madrasa* Bū ‘Ināniyya of Fez:

313 Rubió, 1908–1921, doc. no. 284, I, pp. 265–266.

314 Rubió, 1908–1921, I, pp. 144–145 (doc. 141, dated in 1349); Rubió, 1908–1921, II, pp. 79–80 (doc. 79, dated 1345), 84 (doc. 85, dated in 1347), 86 (doc. 88, dated in 1349), 96–97 (doc. 102 dated in 1352), 172 (doc. 181, dated in 1374), 183–184 (doc. 197, dated in 1377), 247–248 (doc. 253, dated in 1381), 260 (doc. 268, dated in 1382), 279–280 (doc. 287, dated in 1385), 281 (doc. 290, dated in 1385). The last document of this collection is dated in 1405 during the reign of Martin I (1396–1410): see Rubió, 1908–1921, II, p. 376, doc. 392.

315 Sources contain evidence of the existence of a great clepsydra clock (height 50 cubits) in the door of the Kutubiyya mosque of Marrakesh, apparently built during the Almohad period. See Baklī, ‘Aysānī & Ilhām, 2014, p. 36.

316 Baklī, ‘Aysānī & Ilhām, 2014, pp. 27–28; Lamrabet, 2013, M105, p. 158.

317 García Gómez, 1988, p. 89.

318 Lamrabet, 2013, M103, p. 158.

319 Lamrabet, 2013, M104 and 104 bis, p. 158.

the instrument's dial (still extant, and formed by a set of twelve doors each one of which would stay open for an hour) was placed in the street and passers-by could tell the time by noting which door was open.³²⁰

2.5 The *qibla*

2.5.1 *The qibla between the Seventh and the Ninth Centuries*

2.5.1.1 The Mosque of Sīdī 'Uqba in Qayrawān

The mosque of Sīdī 'Uqba in Qayrawān was the first mosque built in the Maghrib.³²¹ 'Uqba b. Nafi' began its construction in year 50/ 670–671 or (variant) 51/671. He delimited its layout and hoisted his flag in the place in which the *miḥrāb* was to be built. The foundation of the city and the orientation of the mosque has been the object of legends, and here I will summarise the information gathered by Ibn 'Idhārī al-Marrākushī in his *al-Bayān al-Mughrib*:³²²

When 'Uqba began the building of Qayrawān, his companions feared that the place might be full of wild animals, snakes, and other vermin. In his army there were eighteen companions of the Prophet (*ṣaḥāba*), while the rest were *tābi'yyūn*.³²³ 'Uqba prayed with his companions and declaimed: "Snakes and wild animals, we are the companions of God's Messenger. Get away from us. We are going to settle here. From this moment, if we find any wild animal, we will kill it". The snakes and vermin left immediately and took with them their young. During forty years not a single snake, scorpion, or any wild animal was seen there.

'Uqba determined the limits of the great mosque. His companions disagreed on the direction of the *qibla* and they remarked that all the inhabitants of the Maghrib would orient their mosques in agreement with the *qibla* of Qayrawān. This was why 'Uqba was required to make a serious effort (*juhd*) to determine it with precision. For a few days they made observations (*yanẓurūna*) of winter and summer risings of the Sun, as well as of risings of stars (*nujūm*). When 'Uqba saw their disagreement, he went to sleep worried, after praying and asking for God's help. He dreamt that somebody was telling him: "At sunrise, take a

320 Solla Price, 1964; Baklī, 'Aysānī & Ilhām, 2014, pp. 30–33. The first description of the instrument and its identification as a clock seems to have been due to Ricard, 1924. I owe a copy of this latter paper to the kindness of Muḥammad Riḍā Baklī.

321 Eric Mercier tells me that the mosque of Sīdī Abū l-Bāba in Gabes (Qābis), another companion of the Prophet, built ca. 661, is considered to be older.

322 Colin & Lévi-Provençal, 1980, pp. 20–21. See, for other sources of the same story, Rius, 2000, pp. 142–148.

323 Muslims belonging to the second generation.

flag and place it over your neck. You will hear, in front of you, a *takbīr*,³²⁴ which no other Muslim will be able to hear. Consider the place in which you will no longer hear the *takbīr*: this will be your *qibla* and your *miḥrāb*". When he woke up, he felt nervous. He washed for his prayers and began to pray in the mosque with his most important companions. At sunrise, he prayed two *rak'as*³²⁵ together with the Muslims and began to hear the *takbīr* in front of him. He asked his comrades whether they could hear it and the answer was no. He then took the flag, put it on top of his neck and followed the *takbīr* until he reached the place of the *miḥrāb*, where the sound was interrupted. He planted his flag in the ground and said: "This is your *miḥrāb*". The other mosques of the city followed this model.

According to the sources, the *qibla* orientation of the mosque was never changed and 'Uqba's *miḥrāb* was not destroyed, in spite of the later alterations and extensions of the building. The mosque was demolished, except for the *miḥrāb*, in year 84/ 703 by Ḥassān b. al-Nu'mān al-Gassānī, who later rebuilt it. According to al-Bakrī, Ziyādat Allāh b. Ibrāhīm, emir of Ifrīqiya, rebuilt the mosque again, after demolishing a good part of it in 221/836, and he planned to destroy the *miḥrāb* so that his name would be the only one associated with the building; finally, however, he limited himself to covering 'Uqba's *miḥrāb* under a new marble structure, and the old *miḥrāb* is still visible.³²⁶

We have no reliable information about the orientation of the *qibla* in 'Uqba's mosque although there seems to be an agreement about values between 144° and 147°, measured from the north point of the horizon.³²⁷ The correct orientation is about 110;30°. Muslim scholars seem to have been aware of this error and of the difficulty of explaining it in a mosque whose orientation was either the result of a divine revelation or of the consensus (*ijmā'*) of the companions of the Prophet. This is why al-Mittijī (d. ca. 1150) argues that 'Uqba's orientation was correct and that the error arose during the building process.³²⁸ Whatever the case, the standard orientation in Qayrawān, according to the legal sources, is the rising point of the Sun in the winter solstice (about 119°), and this

324 The expression *Allāhu akbar* ("God is great").

325 A *rak'a* is composed by a series of gestures and religious formulas uttered by a Muslim during the prayer.

326 Sālim, 1981, pp. 422–424.

327 Philibert, 1972, pp. 57–58; Rius, 2000, pp. 101–102; Belmonte & Hoskin, 2002, pp. 385–388; Bonine, 2008, p. 153–161. This orientation also reappears in the Great Mosques of Sfax, Monastir, Mahdia, Hammamet and Tunis. The *qibla* of these four cities agrees with the street-plan and, especially in the case of Tunis, Sfax and Hammamet, Bonine (2008, pp. 162–176) suggest an influence of the orientation established by the earlier Roman centuriation system. There is no evidence, however, for the case of Qayrawān

328 Rius, 1996, p. 798; Rius, 2000, p. 148.

criterion seems to have been applied by ‘Uqba himself in several mosques he built in southern Morocco (see below 2.5.4). On the other hand, a common criterion used in the Maghrib is the rising point of Suhayl (Canopus) (166°), towards which the major axis of the Ka’ba in Mecca is oriented.³²⁹ This supports the hypothesis formulated by Belmonte and Hoskin³³⁰ who stated that the mean value between 119° and 166° is 142;30°, which is near the orientation of the building (estimated between 144° and 146°).³³¹ If there was disagreement between ‘Uqba’s comrades, with some favouring the rising Sun in the winter solstice, and others the orientation of the Ka’ba, a consensus could have been reached by accepting the mean value between the two.

2.5.1.2 The Great Mosques of Zaragoza and Córdoba

One of the *tābi’ūn* who accompanied ‘Uqba in the process of establishing the orientation of the Great Mosque of Qayrawān was Ḥanash al-Ṣan’ānī who, later, came to al-Andalus³³² and who, according to the tradition, was responsible for the orientation and building of the *miḥrāb* of the great mosques of Córdoba³³³ and Zaragoza.³³⁴ Ḥanash, whose totemic name means “snake”, was buried in the cemetery placed near the door of the *qibla* in Zaragoza, and, according to tradition, his grave protected the city from snakes: those who entered it died immediately.³³⁵ There is, therefore, a similarity between the legends of ‘Uqba in Qayrawān and Ḥanash in Zaragoza. In any case it seems that, in spite of the tradition, Ḥanash has a clear relation to the orientation of the Zaragoza *miḥrāb*, not to that of Córdoba, and the historian Ibn Abī l-Fayyād (986–1066) insists that the *miḥrāb* built by Ḥanash was carefully transported to a new site when the mosque was extended in year 400/1009–10.³³⁶ Some information on the orientation of the *qibla* in Zaragoza can be obtained from al-‘Udhri (1003–1085) who describes the structure of the Muslim city based on two main streets forming a cross in the middle (the *cardus* and *decumanus* of a Roman city) and which ended in four gates: two of them were oriented to the rising and setting points of the Sun in the summer solstice, while the other two faced the winter solstice risings and settings. The gate corresponding to the winter sunrise was

329 Rius, 2000, p. 102.

330 Belmonte & Hoskins, 2002, pp. 385–388.

331 Both Eric Mercier and Mònica Rius have obtained 150° using Google Earth.

332 Marín, 1981.

333 Rius, 2000, p. 177.

334 On the parallel legends of the foundations of the Qayrawān and the Cordoba mosques see Rius, 2000, pp. 184–186, and Calvo Capilla, 2007.

335 Ahwānī, 1965, pp. 22–23; Bramon & Souto, 1987.

336 Álvarez de Morales, 1978–9, pp. 46, 79–80; Rius, 2000, p. 187.

called *Bāb al-qibla*.³³⁷ The winter sunrise ca. 750 was approximately 122° and the eighth century *Jāmi'* mosque (now the Seo of the city) has an orientation of 134;6°, which may correspond to the criterion just mentioned, the modern-day value for the *qibla* of Zaragoza being 107;50°. ³³⁸

As for the Great Mosque of Córdoba we find, again, the legend of Muslims and Christians sharing the church of St. Vincent³³⁹ for their prayers – probably inspired by the similar story told about the church of St. John in Damascus – until 'Abd al-Raḥmān I (756–788) bought the church from the Christians, demolished it and built the first phase of what would become, after several enlargements, one of the greatest mosques in the Islamic lands.³⁴⁰ In order to confirm or disprove this information, Félix Hernández made excavations in the oldest part of the mosque between 1931 and 1936 and found only very humble remains which do not seem to correspond to the primitive Christian great church (*kanīsa kabīra*) mentioned by some Arabic sources.³⁴¹ Recent research seems, however, to confirm the existence of a primitive building complex which might (or not) correspond to this church.³⁴²

Historical sources state that the extensions of the mosque did not imply a change in the orientation of the *miḥrāb* which, according to the careful measurements of Alfonso Jiménez,³⁴³ is 152° from the north point of the horizon. This value corresponds, approximately, to the estimations of Philibert (154°),³⁴⁴ Antonio Almagro (152.4°),³⁴⁵ David King (150°), as well as to the value we find in medieval Andalusian sources like the treatises on the astrolabe by Ibn al-Ṣaffār and Ibn al-Naṭṭāḥ (150°).³⁴⁶ The excessive southern orientation of the mosque (the modern *qibla* value is 100;18°) was already noticed in the second half of the ninth century: in his *Ṭabaqāt al-umam*, Ṣā'id al-Andalusī mentions Abū 'Ubayda Muslim b. Aḥmad al-Laythī (d. 907), a traditionist expert in

337 Ahwānī, 1965, p. 21. In fact the edited text says that the door of the *qibla* faced the rising Sun at the end of winter (*ākhir al-maṭālī' fi l-shitā'*) which seems an obvious error.

338 Rius, 2000, pp. 187–189.

339 Rius, 2000, p. 178.

340 Ocaña, 1942. See also Calvo Capilla, 2007, pp. 166–177.

341 Gómez Moreno, 1951, pp. 19–20.

342 Marfil, 2000, pp. 123–130; Bermúdez, 2010; Arce, 2015.

343 Jiménez, 1991 contains accurate measurements of the orientation of all the extant mosques in Spain and it is the main source of the archaeological evidence used by Rius, 2000.

344 Philibert, 1972, p. 57.

345 Private communication during a visit to Granada in 1989.

346 King, 1978. Ibn al-Naṭṭāḥ gives two other values: 157° and 135°. According to Gibson (2017, pp. 96–97) the orientation is 157;12°, in parallel to a line drawn between Petra and Mecca (?), something which seems difficult to justify.

astronomy and astrology. Abū ‘Ubayda received the nickname *ṣāhib al-qibla*, which I interpret as meaning “the fellow of the *qibla*”, because he faced the East in his prayers,³⁴⁷ something which probably made him stand out from the lines of Muslims in the communal prayers in the mosque. The results of these criticisms will appear under the reign of al-Ḥakam II, in the second half of the tenth century.

David King and Mònica Rius³⁴⁸ have suggested that the orientation was determined in “parallel” to the minor axis of the Ka’ba (sunrise in the summer solstice), which corresponded to the axis passing through the midpoints of the walls perpendicular to the *qibla* wall. As the Sun rises in Córdoba in the summer solstice at 60°, the *qibla* direction should be $60^\circ + 90^\circ = 150^\circ$, which agrees with the 152° of the actual orientation of the building. This has been confirmed by archaeological research done during the last 20 years, which has established that the same orientation had been adopted by the Roman suburb of Colonia Patricia (*urbs nova*), built in the first century CE, inclined at 30° to the earlier city (*urbs vetus*). The Great Mosque was built within the Colonia Patricia and its orientation was predetermined by the street-plan of this suburb,³⁴⁹ like many other mosques in Tunis and Morocco, as Michael Bonine has established.³⁵⁰

This orientation probably needed to be explained and I will try to propose a hypothesis based on the work of David King. Some years ago,³⁵¹ King established that the axis of the Córdoba Mosque is “parallel” to the major axis of the Ka’ba, which points to the rising of Suhayl.³⁵² Thus, the Córdoba *qibla* wall of the Mosque would also be “parallel” to the northwestern wall of the Ka’ba (Fig. 2.12). This is an explanation which fits many other mosques with an anomalous orientation, and it corresponds to a different conception of the *qibla*. Muslims were conscious of the astronomical orientation of the Ka’ba and of the fact that, if they were in Mecca, in front of the Ka’ba, they had to pray in the direction of one of the walls or of the corners of the building, and its major axis

347 Bū ‘Alwān, 1985, pp. 159–161. The editor reads *yusharriqu fi ṣalawāti-hi* instead of the *yus-rifu fi ṣalawāti-hi* (he was too exaggerated in his prayers) appearing in some manuscripts.

348 King, 1995, p. 266. A revised version of this paper is published in King, 2004, pp. 749–771 (see p. 768); Rius, 2000, p. 115.

349 King, 2018.

350 Bonine, 1990 and 2008. The fundamental study by González, 2015 (see pp. 454–463) and González, 2018 (p. 9) of the neighbourhood mosques in Córdoba shows that, in many cases, their orientation also depends on the city plan. Two of them (Puerta de Gallegos, built during the emirate, and Santa Rosa, post-caliphal) show orientations (ca. 155° and ca. 153° respectively) near that of the Great Mosque (152°).

351 King, 1991a; King, 1982; Hawkins & King, 1982.

352 The rising point of Suhayl in the eighth-ninth century in Mecca was 147,6° according to the computation of Eric Mercier.

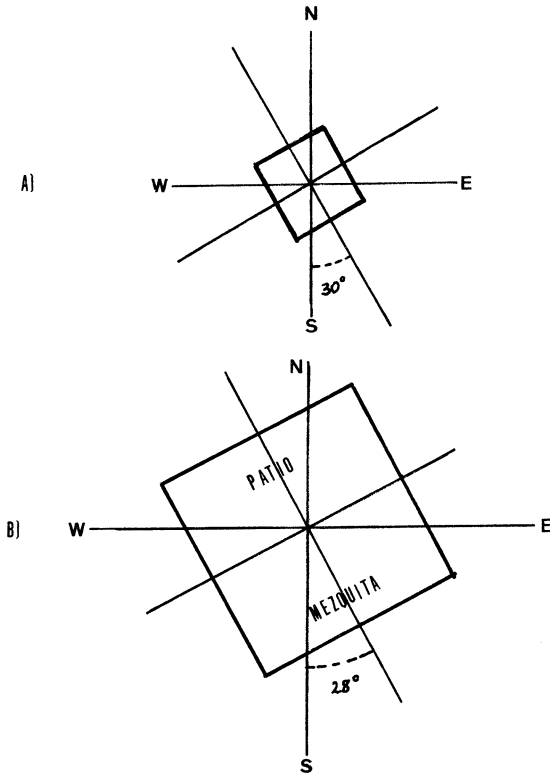


FIGURE 2.12
The orientation of the
Ka'ba (A) and of the Great
Mosque of Córdoba (B)

was the direction of the Syrian community to which the Umayyad emir 'Abd al-Raḥmān I belonged. Eighth- and ninth-century Andalusians were probably aware of this fact, which allowed them to justify the orientation of the building forced upon them by the street plan of Colonia Patricia. To this one should add that, according to Félix Hernández,³⁵³ the primitive mosque built in the eighth century was a perfect square if one includes the mosque proper and the courtyard. Therefore, it seems that the Umayyads probably thought that they were building a mosque with a structure that reproduced the Ka'ba itself, which is not a perfect square but has a rectangular base very close to a square, with sides in the proportion 8:7.³⁵⁴

In fact this hypothesis seems to have received some confirmation in the publication of volume II-1 of Ibn Ḥayyān's *Muqtabis*.³⁵⁵ In it we find the fragment

353 Hernández, 1961, p. 30.

354 See Samsó, 2011, pp. 66–67.

355 Makkī, 2003, p. 282; Makkī & Corriente, 2001, p. 173.

of a poem of ʿUthmān b. al-Muthannā in praise of ʿAbd al-Raḥmān II (822–852), who had enlarged the Cordoban mosque, with the following words:

You have built, for God, the best possible house
in front of which people fall silent, unable to describe it.
It is the objective of pilgrims coming from everywhere
as if it was the Great Mosque of Mecca
and its *miḥrāb*, when they get near it,
was the corner (*rukn*) and the place (*maqām*) [on which Abraham stood]

The parallel between the Córdoba Mosque and the Kaʿba seems clear: the first sentence (*banayta li-llāhi khayra baytin*) contains the two key-words: *bayt* (house) and *Allāh*, and the Kaʿba is considered to be *bayt Allāh* (the house of God); the *rukn* refers to the south-eastern corner of the Kaʿba where the Black Stone is, while the *maqām* is a small isolated building, placed in front of the north-eastern wall, which contains a stone on which Abraham stood while he was building the Kaʿba.

The orientation of the Great Mosque of Córdoba was followed by at least two other Andalusian mosques of the ninth century: the mosques of the Alcazaba (*al-Qaṣba*) of Badajoz and of Almonaster la Real (Huelva), both oriented towards 152°,³⁵⁶ as well as by the Jāmiʿ al-Andalus in Fez (151°), while the Qarawiyyīn mosque of the same city faces the rising point of Suhayl (163°).³⁵⁷ Similar orientations are found in Tunisian mosques built between the seventh and the ninth centuries: 148° for the mosque of Sidi Abū l-Bāba in Gabes (built ca. 661), about 145° in the Great Mosque of Qayrawān (ca. 670), while the Great Mosques of Tunis (704), Sfax (850), Monastir (ninth century), and the Three Door Mosque of Qayrawān (866) have orientations between 145° and 150°. Other Tunisian mosques of the same period are oriented towards the rising of Suhayl (166°): the Great Mosques of Gafsa (Qafṣa), Susa (Sūsa) and Nabeul (Nābil) have *qiblas* between 160° and 166°.³⁵⁸

Other Andalusian mosques of the eighth and ninth centuries have very different orientations, for which some hypothetical justifications can be given: thus, the 139° and 141° of the churches of San Juan (Córdoba) and Santa Catalina (Seville) respectively, both built in the ninth century,³⁵⁹ seem to cor-

356 Rius, 2000, p. 117.

357 Rius, 2000, p. 128.

358 E. Mercier, 2014 a. Jaray & Mercier, 2019, pp. 155–156, have studied the orientation of the *qibla* in 18 Tunisian sundials made between the beginning of the 11th and the end of the 15th century: the earliest one (ca. 1000) gives an exceptional *qibla* of 110° from the north point, while the other 17 sundials have orientations between 120° and 160°.

359 Rius, 2000, p. 112.

respond to an attempt to fix the *qibla* at the midpoint between East and South (135°). As for the 174° of the church of El Salvador (ninth century, Seville),³⁶⁰ the 187° of the church of Santiago (ninth century, Córdoba) or the 188° of El Gatillo (eighth century, Cáceres)³⁶¹ point to a southern orientation which was probably intended to agree with the *qibla* of the Prophet in Medina and with a literal interpretation of the *ḥadīth*: *Mā bayna l-mashriq wa l-maghrib qibla* (The *qibla* is between East and West).

2.5.2 The Astronomers' Solutions

2.5.2.1 Computational Procedures

The azimuth of the *qibla*³⁶² is angle q in fig. 2.13 in which P corresponds to the position, on the earth's surface, of the locality for which the *qibla* is going to be calculated, M is the position of Mecca, AB is the equator, φ is the latitude of P , φ_M the latitude of Mecca and Δ_L the difference of longitudes between P and M . A modern expression for calculating q is:

$$q = \cot^{-1} ([\sin \varphi \cos \Delta_L - \cos \varphi \tan \varphi_M] / \sin \Delta_L)$$

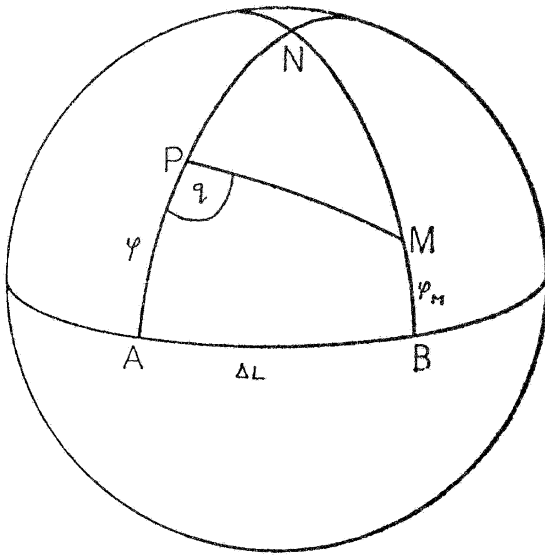


FIGURE 2.13
The *qibla*, borrowed from
D.A. King

³⁶⁰ Eric Mercier remarks (private communication) that this orientation coincides approximately with the azimuth of the rising point of Suhayl in the ninth century for latitudes around 38° (175.7°).

³⁶¹ Rius, 2000, p. 120.

³⁶² Among many other publications by David King, see King, 1986c, and King, 1999a, pp. 47–127 for a general introduction to the subject.

Exact procedures equivalent to this one were developed in the Islamic East in the ninth and tenth century. They do not seem to have reached al-Andalus and the Maghrib until much later, although accurate solutions appear in the treatises on the astrolabe written by Ibn al-Ṣaffār³⁶³ and Ibn al-Samḥ³⁶⁴ towards the end of the tenth century: in both cases the procedure consists in determining the position of the zenith of Mecca on an astrolabe plate calculated for the local latitude and seeing which azimuthal circle of the plate passes through the zenith of Mecca. Similar solutions, adapted to the characteristics of the instrument, can later be found in treatises on the use of the universal astrolabes of Ibn al-Zarqālluh (d. 1100)³⁶⁵ and Ibn Bāṣo (d. 1316).³⁶⁶

To this one should add approximate methods which seem to be documented before the second half of the eleventh century. One of them, attested for the first time in al-Battānī's *zīj*,³⁶⁷ is called by King "the standard approximate method" (fig. 2.14)³⁶⁸ and seems to have been known to Ibn al-Samḥ, although the source is not very clear.³⁶⁹ In the figure, point O represents the locality for which the *qibla* should be calculated. Using O as the centre we trace a circle of arbitrary radius (the local horizon) and mark the points NESW, which correspond to the four cardinal points. We mark ΔL (the difference of longitudes between the locality and Mecca) from the south point eastwards and draw a parallel to the vertical axis NS which passes through the end of arc ΔL. Then we mark Δφ (the difference of latitudes between the locality and Mecca) from point E southwards and trace a parallel line to the horizontal axis WE passing through the end of arc Δφ. Finally, we draw a radius of the circle passing through the intersection of the two parallel lines. This radius will mark the direction of the *qibla*. Obviously, this graphical method is equivalent to the expression:

$$q = \tan^{-1} (\sin \Delta L / \sin \Delta \phi)$$

where q is measured from the south point of the circle (S).

363 See the Arabic edition, based on ms. Escorial 964, in Millás, 1955, pp. 64–65 of the Arabic part of the journal; Catalan translation of the same text in Millás, 1931, pp. 37–38.

364 Viladrich, 1986, pp. 62–65, 131–132.

365 Puig, 1986, pp. 160–161 (translation), 73–74 (Arabic section of the volume); Puig, 1987, pp. 72–72.

366 Calvo, 1993, pp. 81–85, 134–136, 138, and 43–47, 52 (Arabic edition).

367 Nallino, 1899–1907, vol. 1, pp. 137 (Latin translation) and 318–319 (commentary); Vol. III, pp. 206–207 (Arabic text).

368 See for example King, 1999a, pp. 58–60.

369 Viladrich, 1986, pp. 65 and 132.

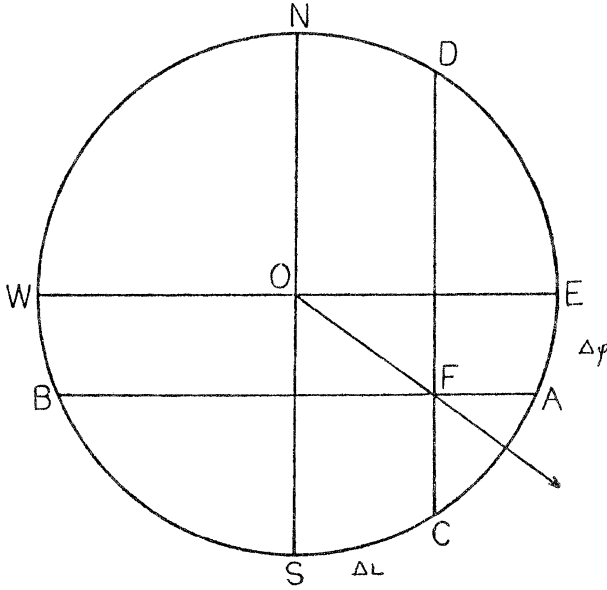


FIGURE 2.14
The “standard
approximate method”
to calculate the *qibla*,
borrowed from D.A. King

A second approximate method has been attributed to al-Khwārizmī or to an even earlier Abbasid source³⁷⁰ and it reappears in the Alfonsine *Libro del astrolabio llano*,³⁷¹ which means that it was probably known in al-Andalus. The expression implied is:

$$\begin{aligned} &\text{for } x = \Delta L \cos \varphi_M / R \\ &\text{and } y = \Delta \varphi \\ &q = \sin^{-1} [(x \cdot R) / \sqrt{(x^2 + y^2)}] \end{aligned}$$

The first precise computational solutions appear in the second half of the eleventh century in the *zīj* calculated for the coordinates of Jaén (Jayyān) by the mathematician and astronomer Ibn Mu‘ādh al-Jayyānī (d. 1093). The canons of this *zīj* are extant in a Latin translation by Gerard of Cremona, and the Arabic original of chapter 18, on the computation of the *qibla*, has also been preserved by the anonymous Tunisian recension of the *zīj* of Ibn Ishāq extant in ms. Hyderabad Andra Pradesh 298.³⁷² This chapter explains the so-called “method of the *zīj*es”, well known in the Eastern astronomical tradition at least

370 King, 1986c, pp. 84–85, 102.

371 Rico, 1863–67, vol. II, pp. 288–289; Samsó, 1980, pp. 174–175; Martí & Viladrich, 1983, pp. 30–32.

372 Samsó & Mielgo, 1994; Schwartz, 2010.

since the time of Ḥabash al-Ḥāsib (fl. 850). The anonymous Tunisian compiler adds a worked example which, after the correction of obvious mistakes in the manuscript, gives a good result for the *qibla* in Tunis of 106;[2],2[6]°, if one compares it with the modern-day value of 112;9°. ³⁷³

The computation has five steps for which I keep the names given by Ibn Mu‘ādh, while I use David King’s symbols: ³⁷⁴

1. *Al-faḍla al-ṭūliyya* or *al-‘amūd* (“difference of longitudes” or “the perpendicular”) (ΔL):

$$\sin \Delta L' = \cos \varphi_M \sin \Delta L / 60$$

2. *Al-bu‘d min mu‘addil al-nahār* (“The distance from the equator”) (φ_M'):

$$\sin \varphi_M' = 60 \sin \varphi_M / \cos \Delta L'$$

3. *Bu‘d al-balad* (“Distance of the location”) ($\Delta \varphi'$):

$$\Delta \varphi' = \varphi - \varphi_M'$$

4. *Masāfa mā bayna baladi-ka wa Makka* (“Distance between your locality and Mecca”) (d):

$$\cos d = \cos \Delta \varphi' \cos \Delta L' / 60$$

5. *Samt al-qibla* (azimuth of the *qibla*) (q):

$$\sin q = 60 \sin \Delta L' / \sin d$$

The method of the *zīj*es, as presented by Ibn Mu‘ādh, was fairly successful in the Maghrib and we find it not only in the aforementioned Tunisian recension but also in the *Shāmil*,³⁷⁵ *Qawīm*³⁷⁶ and *Mustawfī zīj*es of Ibn al-Raqqām (d. 1315), in which the four auxiliary arcs receive the same name as in Ibn Mu‘ādh’s *zīj*. In chapter 54 of the *Mustawfī*, the author explains three methods for determining the *qibla*: the first one is an almost literal copy of Ibn

373 Philibert, 1872, p. 55.

374 King, 1999a, p. 62.

375 Ms. Kandilli 249 fol. 42r and v. ‘Abd al-Raḥmān, 1996, pp. 160–162.

376 Ms. Rabat National Library 260 pp. 44–45.

al-Haytham's *Qawl fī samt al-qibla bi l-ḥisāb*,³⁷⁷ which is also a variant of the same method; the second is, again, a copy of Ibn Mu'adh's version; and the third seems to be another variant of the same method in which the computation is made in six steps. The whole procedure is almost the same as the one that appears in chapter 52 of the same *zīj*, which deals with the computation of the distance between two stars or planets on a great circle passing through their centres.³⁷⁸ Ibn al-Raqqām's *Mustawfī* seems to be the source used by al-Jādirī (1375–1416) for his presentation of the method of the *zīj*es in the *Rawḍat al-azhār* as well as in the *Natā'ij al-afkār*, the commentary written by al-Ḥabbāk (fl. 1514–1562):³⁷⁹ this is quite surprising, because his presentation of this long and cumbersome computation is not the kind of information one expects to find in a popular didactic poem on *mīqāt*. On the other hand, the method of the *zīj*es, as explained in the *Mustawfī zīj*, is mentioned by the Fez *muwaqqit* al-Māwāsī (d. 1505),³⁸⁰ who also refers to Ibn al-Haytham on the *qibla* problem.³⁸¹ Finally, al-Wansharīshī³⁸² refers to a text (*juz'*) authored by Ibn Mu'adh on the determination of the *qibla* (*Fī istikhraj al-qibla*) in which he states that its correct orientation (in Córdoba?) is 113°.

Ibn al-Raqqām dealt with the *qibla* problem not only in his *Mustawfī zīj* but in the *Risāla fī 'ilm al-zilāl* as well, where he explains a perfectly accurate graphical construction for the *qibla* using an analemma method.³⁸³ The procedure is very similar to that of al-Bīrūnī in his *al-Qānūn al-Mas'ūdī*, which derives from another analemma by Ḥabash al-Ḥāsib.³⁸⁴ We cannot be sure about the source used by our author, and, as shown by Berggren,³⁸⁵ al-Bīrūnī's analemma has an error which has been corrected in Ibn al-Raqqām's book.

2.5.2.2 The Intervention of Astronomers in the Orientation of Mosques

There are a few cases in which astronomers seem to have played a role in the orientation of the *miḥrāb* of some mosques. The earliest information available on this topic comes from Córdoba in the first half of the tenth century, coinciding with 'Abd al-Raḥmān III's building of the palace complex of Madīnat al-Zahrā', near the city, which began in 936. The complex included a Great

377 Dallal, 1995.

378 Samsó, 2014, p. 308.

379 Saidi, 2013, pp. 320–329.

380 Khaṭṭābī, 1986, pp. 59–60, 70.

381 Khaṭṭābī, 1986, pp. 73–75.

382 Wansharīshī, 1981, vol. 1, p. 118.

383 Carandell, 1984a.

384 Kennedy, 1974.

385 Berggren, 1980.

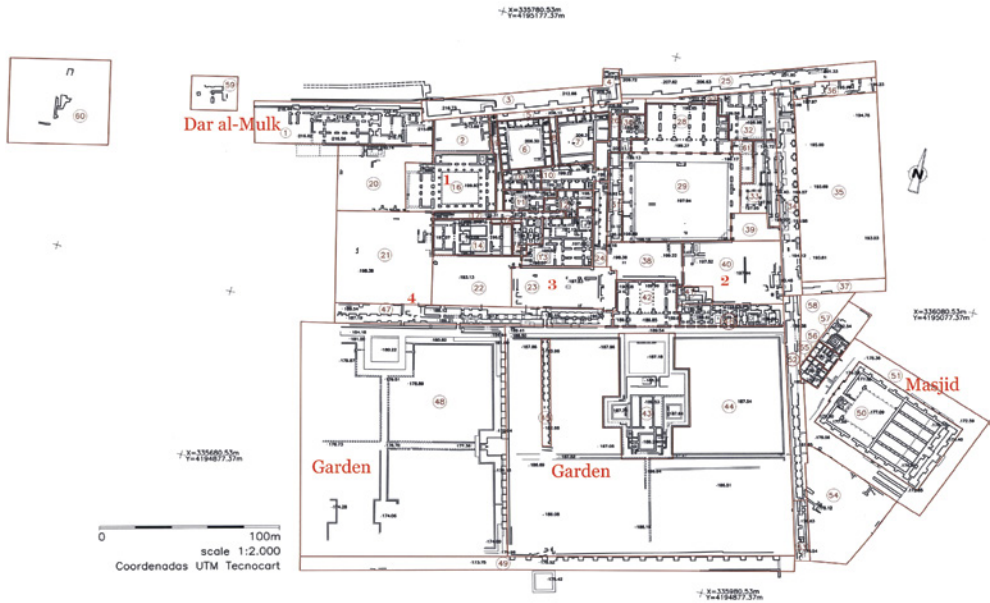


FIGURE 2.15 Plan of Madinat al-Zahrā' by Antonio Vallejo (*La ciudad califal de Madinat al-Zahra. Arqueología de su excavación*. Córdoba, 2010) taken from Susana Calvo Capilla, "The reuse of Classical Antiquity in the Palace of Madinar al-Zahra' and its role in the construction of Caliphal legitimacy," *Muqarnas* 31 (2014), 1–33. The Great Mosque appears on the right with an orientation which is clearly different from that of the other buildings

Mosque, oriented towards 107.8° ³⁸⁶ or 109° ,³⁸⁷ a value which implies a significant improvement in the orientation if one compares it to the Great Mosque of Córdoba (152°), although it does not correspond exactly to the modern-day value of around 100° . Of course, the builders of the mosque worked in an empty space and were not constricted by a previously existing street plan. It is important to stress that the orientation of the mosque clashes with the rest of the buildings in the complex: in fact, the mosque is an architect's nightmare. Clearly, astronomy was beginning to reach a certain level of maturity at the time and, consequently, the Caliph probably requested the help of astronomers to establish the correct orientation of the mosque.³⁸⁸

386 According to Antonio Almagro (private communication).

387 According to A. Jiménez: see Rius, *La Alquibla* p. 110.

388 A new mosque, discovered in 2007 about 1 km. from the centre of Madinat al-Zahrā' has the same orientation: 109° according to González, 2015, p. 456. Besides, one of the neighbourhood mosques studied by González, 2015, pp. 459–50, the post-Caliphal mosque of Ollerías, has an orientation of 116° , not very different from those of Madinat al-Zahrā', and wholly independent of the nearby buildings and street plan (ca. 143 – 150°).

The problems related to the orientation of mosques in Córdoba continued during the reign of caliph al-Ḥakam II (961–976), the son of ‘Abd al-Raḥmān III (912–961), when the growth of the population in the city created the need for the enlargement of the Great Mosque. I reproduce here al-Maqqarī’s text³⁸⁹ in which he explains the situation and the problems involved:

[al-Ḥakam II] asked the advice of the experts (*al-‘ulamā’*) on the question of changing the orientation of the *qibla* towards the East, in agreement with what had been done by his father [‘Abd al-Raḥmān III] al-Nāṣir with the *qibla* of the Great Mosque of [Madīnat] al-Zahrā’, because astronomers (*ahl al-ta’dīl*) said that the old *qibla* of the mosque was deviated towards the West. The *faqīh* Abū Ibrāhīm told him: “Amīr of the Believers, the best people of this community, including the *imāms* that were your ancestors and the righteous Muslims and scholars, have prayed facing this *qibla* since the conquest of al-Andalus until this moment, in agreement with the principles of those who, for the first time, established it, who were Muslims belonging to the second generation [after the Prophet] (*tābi‘yyūn*) like Mūsā ibn Nuṣayr, Ḥanash al-Ṣan‘ānī and others ... [God] favours those who prefer to follow traditions (*al-ittibā’*) and dislikes those who let them to be destroyed by innovations (*al-ibtidā’*)”. The caliph, then, adopted his point of view and said: “You have spoken well and our belief is that we should follow tradition”.

A summary of this information, with some significant changes, can be found in al-Wansharīsī:³⁹⁰

When al-Ḥakam [II], son of ‘Abd al-Raḥmān [III] wanted to change the *qibla* of the Great Mosque of Córdoba – the astronomers (*ahl al-ḥisāb*) who advised him, among whom there were leading figures deserving to be followed, had agreed on the fact that it was strongly deviated towards the West – he had to give up the idea due to the strong opposition of all the people (*‘āmmat al-nās*) who did not want to abandon the custom of their ancestors. Therefore, he desisted of his pretension.

These texts show that both the tenth-century Cordoban astronomers and the caliph al-Ḥakam II were interested in the problem of the *qibla*. When the caliph faced this question he asked the opinion of “experts” (*‘ulamā’*), which

389 ‘Abbās, 1968, vol. I, pp. 561–562.

390 Wansharīsī, 1981, vol. I, p. 118; Rius, 2000, p. 174.

would have included men of law (*fuqahā'*) as well as astronomers (*ahl al-ta'dīl*, *ahl al-ḥisāb*). According to al-Wansharīsī there was strong popular opposition to this change, which forced the Caliph to abandon the project. In al-Maqqarī's text the cause was the opinion of the *fuqahā'*, which prevailed. In any case the final decision probably coincided with the interest of the caliph who had a serious problem if he had to change the *qibla*: the solution was either to demolish the building in order to change its orientation, or to build a new *miḥrāb*; this would improve the orientation, but would seriously affect the whole structure of the mosque.

Al-Ḥakam's concern with the *qibla* problem is confirmed by another anecdote explained in the *anwā'* book of al-Umawī al-Qurṭubī (d. 1205–1206). This author explains that, in year 358/ 968–69, the caliph sent his astrologer, the Baṣrian Aḥmad b. Fāris al-Munajjim,³⁹¹ to the Fortress of Suhayl (*ḥiṣn Suhayl*), whose modern name is Fuengirola, near Málaga, in order to establish whether the star Suhayl (Canopus) could be seen from that locality.³⁹² Fuengirola received the name of Suhayl because the local tradition claimed that it was the only place in the Iberian Peninsula in which the star was visible.³⁹³ When Aḥmad b. Fāris arrived in Fuengirola he met Sahl b. 'Uthmān b. Abī Ḥabīb, the *ṣāḥib al-ṣalāt*, and Ḥusayn b. Muḥammad al-Hamdānī, who belonged to Aḥmad's own family, and the two local authorities explained to him the cycle of visibility of an unidentified star which might have been Suhayl, although Ibn 'Āṣim believed that it should be identified with Ḥaḍari (either β Centauri or α Columbae). Aḥmad did not reach any conclusion about the identification of the star although the text does not clarify whether he actually made an observation³⁹⁴ or based himself on the description given by his informants.³⁹⁵ Whatever the case, and setting aside the problem of whether Suhayl was

391 On Aḥmad b. Fāris see Forcada, 1996 and Forcada, 2000a.

392 Eric Mercier (private communication) has calculated the azimuth of the rising point of Suhayl for all latitudes between 20° and 39°. He concluded that this angle was about 175° in ninth century Córdoba and, consequently, the star could be seen from this city, whose latitude is 37:53°. This visibility must have been difficult, given the evidence provided by historical sources.

393 Al-Wansharīsī (1981, 1, p. 123) discusses the visibility of Suhayl in al-Andalus and states that the star is only visible from some localities such as Shalawbāniya (= Salobreña, also in the province of Málaga).

394 If there was an actual observation, it was not made from Fuengirola itself but, more probably, from the top of the nearby mountain to north of the village, at the site of the modern-day village of Mijas. Ibn Rushd, who tried to observe Suhayl in Marrakesh in 1153–54, and explains the popular belief that the star was visible, refers not to the village of Suhayl (Fuengirola) but to *Jabal Suhayl* (the mountain of Suhayl). Rius, 2000, p. 242.

395 Forcada, 1992a, pp. 81–85.

visible or not from Fuengirola, it seems clear that the reason why al-Ḥakam II sent Aḥmad b. Fāris there was related to his interest in the possible use of the rising point of Suhayl as an indicator of the *qibla*.

Similar cases of the intervention of astronomers in the orientation of religious buildings and the scandal caused by the abandonment of tradition can be found in the Maghrib. In one such instance, the Almoravid Caliph ‘Alī b. Yūsuf (1106–1143) asked the advice of forty legal scholars to determine the orientation of the Great Mosque of Marrākush (*Jāmi‘ al-Siqāya*), among them the *qāḍī* Abū l-Walīd ibn Rushd al-Jadd (d. 1126) and Mālik ibn Wuhayb (alive in 1106), who was knowledgeable in astronomy.³⁹⁶ Their advice does not seem to have been very accurate, for the Almoravid mosque seems to have been oriented towards 110°, ³⁹⁷ while the modern-day *qibla* value for Marrakesh is 91;30°.

Far more significant is the story of the foundation of the *madrasa Ya‘qūbiyya* (now the *madrasat al-Ṣaffārīn*) in Fez.³⁹⁸ When the Merinid *amīr* Abū Yūsuf ordered the building of the madrasa in 1271–72,³⁹⁹ the *mu‘addil* Abū ‘Abd Allāh Muḥammad ibn al-Ḥabbāk was ordered to fix the *qibla* and he calculated an orientation different from that of the Great Mosque of al-Qarawiyyīn.⁴⁰⁰ The *fuqahā’* of the city rejected his decision because of its disagreement with the orientation of the old mosque (see 2.5.1.2), which had been established by Idrīs,⁴⁰¹ the founder of Fez. When the news reached Abū Yūsuf, he called a meeting of the legal scholars belonging to the Berber tribe of the Zanāta, to which the emir himself belonged. This assembly decided that the *qibla* of the madrasa was the same as that of the Qarawiyyīn mosque and, therefore, al-Ḥabbāk’s orientation was respected. Besides this, al-Ḥabbāk was also commissioned by Abū Yūsuf to choose the astrologically propitious moment for the building of the new city of Fez *jadīd* and of the orientation of its *Jāmi‘* mosque.⁴⁰²

396 Rius, 2000, pp. 148–151; Lamrabet, 2013, M46, pp. 133–134.

397 Rius, 2000, p. 122.

398 Schatzmiller, 1976, pp. 116–117; Schatzmiller, 1982, p. 32.

399 Rius (2000, p. 155, fn. 363) remarks that, according to the edition of the *Zahrat al-As* by ‘Abd al-Wahhāb b. Maṣṣūr (Rabat, 1967) the date should be 675/1276–77.

400 The *Madrasat al-Ṣaffārīn* has an orientation of 120°, which corresponds, approximately, to the rising Sun in the winter solstice and to the rising point of *Qalb al-Aqrab* (α Scorpii, Antares), while the Qarawiyyīn Mosque faces 163° and is directed towards the rising point of Suhayl (α Carinae, Canopus). The modern *qibla* in Fez is 97°. See Rius, 2000, pp. 127–128, 255.

401 See Schatzmiller, 1982, pp. 138 ff. on the role of the worship of Idrīs in the opposition of the inhabitants of Fez to the Merinids.

402 Rius, 2000, pp. 155–156. For a later example in the Maghrib cf. Elkhammar & Mercier, 2015: according to an inscription on a marble slab, a team of experts in *tawqīt* and in the *dalā’il*

Later, in fourteenth-century Naṣrid Granada we find, again, the echoes of the influence of astronomers in the orientation of religious buildings. We have already seen (cf. 2.0) that the first *muwaqqits* appear in Granada towards the end of the thirteenth century, and that the famous Tunisian astronomer Ibn al-Raqqām arrived in the city during the reign of Muḥammad II (1273–1302). These two events coincided with an interesting improvement in the orientation of mosques built in the city slightly later, during the reigns of Yūsuf I (1333–1354) and Muḥammad V (1354–1359, 1362–1391). If the correct orientation of the *qibla* in Granada is 100;21°, most of the mosques in that city are oriented to *qiblas* comprised between 169° (church of St. John, thirteenth century)⁴⁰³ and 124° (Great Mosque of the Alhambra, built ca. 1305).⁴⁰⁴ There are however two notable exceptions and a third one which is quite remarkable, all of them in the Alhambra: the small oratory of the Mashwār (Mexuar) (108°) and those of the *Rawḍa* (108;54°) and of the Palace of Comares (100;48°).⁴⁰⁵ The first two have an orientation very similar to that of the Mosque of Madīnat al-Zahrā', while the third one must be the result of the use of an exact procedure and the choice of a very lucky set of geographical coordinates for which we have no precedent: even though Ibn al-Raqqām's *Qawīm zīj* bears witness to a very careful determination of the latitude of Granada (37;10°) which is precisely the modern value, the geographical longitudes are less good and, using them, we obtain an azimuth of the *qibla* of 95;41°, which is far worse than that of the Palace of Comares. In any case it seems clear that the aforementioned improvement in the orientation bears witness to the intervention of astronomers in the orientation of small mosques located far away from the inhabitants of Granada: during the reign of Yūsuf I the *Madrasa Yūsufiyya* was also built in the centre of the city, right in front of the Great Mosque, and the *miḥrāb* of the *madrasa* has an orientation of 137;42°, probably in order not to discredit the Great Mosque (eleventh century) oriented towards 141;18°.⁴⁰⁶

2.5.3 Folk Astronomy and Legal Scholars

Two important Maghribī legal texts related to the *qibla* problem have been edited: one of them is the *Kitāb dalā'il al qibla* by Abū 'Alī al-Mittijī

al-qibla were commissioned by sultan Mawlāy Ismā'īl to establish the orientation of the al-Anwār mosque (now *Maṣjid sūq al-ṣabbāt* in Meknès in the beginning of Jumādā II 1122 H/1710. The result obtained (101°) was fairly good. See also (2.5.4) the interesting case of the *Zāwiyya Nāṣiriyya* in Safi.

403 Rius, 2000, p. 119.

404 Rius, 2000, p. 110.

405 Rius, 2000, pp. 106–107, 110.

406 Rius, 2000, p. 112.

(twelfth century),⁴⁰⁷ while the other is the *Kitāb al-qibla* by ‘Ubayd Allāh Abū ‘Alī Šāliḥ ibn Abī Šāliḥ al-Mašmūdī (alife in 1314).⁴⁰⁸ Mònica Rius has also studied a long list of legal Andalusian and Maghribī sources (twelfth–eighteenth centuries) containing passages on the determination of the *qibla*;⁴⁰⁹ more recently, Ahmed Dallal has reassessed the problem, insisting particularly on the *Tanbīh al-ghāfilīn ‘an qiblat al-ṣaḥāba wa l-tābi‘īn* by al-Tājūrī (d. 1590),⁴¹⁰ and established that the *qibla* of Fez has been debated for several centuries.⁴¹¹ To this one should add the excerpts of the commentary on al-Jādirī’s *Rawḍat al-azhār* written by the famous Fez *muwaqqit* Abū l-‘Abbās Aḥmad b. Muḥammad b. ‘Isā al-Māwāsī (d. 1505)⁴¹² edited by Khaṭṭābī.⁴¹³ Within this group of sources we should also include two short texts by the astronomer and mathematician Ibn al-Bannā’ al-Marrākushī (1256–1321)⁴¹⁴ who expresses his doubts about the use of astronomical methods for the determination of the *qibla*; he says that, on the one hand, the difference of longitudes used is never accurate⁴¹⁵ and, on the other, Muslims without specific scientific knowledge should not be required to apply these methods. Mosques built in different directions are always valid because all their builders made a serious intellectual effort (*ijtihād*) to orient them correctly.⁴¹⁶

A quick analysis of these lists shows that writings on the *qibla* are far more frequent in the Maghrib than in al-Andalus and that the Andalusian authors studied by Rius deal with the problem only incidentally in works related to other topics.⁴¹⁷ This is the case of Abū Ḥāmid al-Gharnāṭī (d. 1169), in his book

407 Edited by ‘Ammār Sa‘īd al-Shabīb in his unpublished PhD dissertation presented in the University of Barcelona in 1992. See a study of its contents in Rius, 1996.

408 Critical edition by Rius, 2000, pp. 1–61 of the Arabic part of the book; Spanish translation in pp. 265–304. Introduction with analysis of the known data about the author and his work in pp. 42–70.

409 See the list in Rius, 2000, pp. 29–41.

410 Lamrabet, 2013, no. M557, p. 271.

411 Dallal, 2010, chapter 1 “Beginnings and Beyond” (I have read the electronic edition of the book which is not paginated). See a list of Maghribī authors who wrote on the *qibla*, between the twelfth and the nineteenth century, in Lamrabet, 2013, M314 (p. 230), M354 (p. 240), M360 (p. 239), M361 (p. 240), M421 (p. 248), M434 (p. 248), M443 (p. 250), M557 (p. 271).

412 Lamrabet, 2013, M242, p. 214.

413 Khaṭṭābī, 1986, pp. 45–82.

414 Jabbār & Aballāgh, 2001, pp. 185–190. Sp. translation in Rius, 2000, pp. 225–230.

415 This problem is also raised by al-Māwāsī who gives lists of different values found in the sources for the latitude and the longitude of Mecca: cf. Khaṭṭābī, 1986, pp. 66–67.

416 Passages of these works by Ibn al-Bannā’ are quoted by al-Māwāsī: cf. Khaṭṭābī, 1986, pp. 52–57.

417 Al-Māwāsī mentions a certain Abū Muḥammad ‘Abd al-Majīd al-Mālaqī, author of a book entitled *Īdāḥ al-adilla fī ma‘rifat samt al-qibla* (Khaṭṭābī, 1986, p. 66), whom I have been unable to identify.

on *‘ajā’ib*, the result of his travels through the Eastern part of the Islamic world, of Ibn Rushd al-Jadd (d. 1126) and Ibn Juzayy (d. 1340), in their legal works, and of the *anwā’* author al-Umawī al-Qurṭubī (d. 1205).

The legal sources discuss the problem of establishing whether Muslims should, when praying, face the *samt* (the precise direction of the *qibla*) or only the *jiha*, which corresponds to an approximation with a tolerance of about 90° (*jiha ṣugrā*) or even 180° (*jiha kubrā*), although some authors consider that the orientation should be towards the mid-point of the *jiha* (45° or 90°).⁴¹⁸ As it was clear to inhabitants of al-Andalus and the Maghrib that their *qibla* was placed in the quadrant of the horizon between the East and the South points, one can easily see the logic of the mosques oriented towards 135°, which corresponds to the mid-point of the assumed *jiha ṣugrā*.

The orientation of a Western mosque and, consequently, that of the praying Muslim may be the result of the consensus of the *imāms* (*ijmā’*), as in the case of the ‘Uqba mosque in Qayrawān, or of the intellectual effort (*ijtihād*) based on the research done in the sources of law and in the *qibla* indicators (*dalā’il al-qibla*). If the two aforementioned procedures cannot be used, one can follow the *taqlīd*, which means using the orientation of another mosque with the sole condition that the orientation must have been determined by a competent scholar.⁴¹⁹

On the whole, it seems that legal scholars should be classified into two groups: those who favour the *taqlīd*, the *jiha kubrā* and the Southern orientation (as a result of an application of the *ḥadīth* “*Mā bayn al-mashriq wa l-maghrib qibla*”) and, on the other side, the *fuqahā’* who recommend the *ijtihād*, the *jiha ṣugrā* and an Eastern orientation in the Maghrib. The Eastern orientation seems to be predominant although, as we will see, this preference was not applied to the building of Maghribī mosques.⁴²⁰ This tendency is well represented by the Libyan *faqīh* al-Tājūrī (d. 1582),⁴²¹ in his *Tanbīh al-ghāfilīn ‘an qiblat al-ṣaḥāba wa l-ṭābi‘īn*, which includes the author’s request to the scholars of Cairo for a report about the *qiblas* of the mosques in Fez which, according to al-Tājūrī, did not face the legal *qibla* (*al-qibla al-shar‘iyya*), that is, the East. Al-Tājūrī’s book was criticised or defended by several scholars of Fez, among them al-‘Arabī b. ‘Abd al-Salām al-Fāsī (fl. end of the seventeenth century) who mentions the distinction between the *jiha* and the *samt* and considers that the two terms have the same meaning, and that a person who is capable of

418 See King, 2004, pp. 756–8, 817–9.

419 Rius, 2000, pp. 83–92.

420 Rius, 2000, pp. 256–259.

421 Lamrabet, 2013, M557, p. 271.

determining the exact *qibla* should not follow the *taqlīd* corresponding to the *miḥrāb* of a particular mosque. He states that many mosques own astrolabes or Ibn al-Zarqālluh's universal *ṣaḥīḥa*. One cannot say that the orientation of a mosque is based on *ijtihād* because the only valid *ijtihād* is the effort based on astronomical evidence. The real *mujtahids* are the astronomers and not the *fuqahā'*. As a consequence he states that 1) the *ijtihād* of the companions of the Prophet is certain; 2) the *ijtihād* of the *fuqahā'* is probable but uncertain; 3) the *ijtihād* of the astronomers relies on science, which means that it is certain.⁴²²

2.5.4 *Qibla Indicators and the Actual Orientations of the Buildings*

Reliable information about the orientation of mosques and other religious buildings in Al-Andalus and the Maghrib has been given by two scholars: Alfonso Jiménez⁴²³ for the Iberian Peninsula. and M. Bonine for some buildings in Morocco.⁴²⁴ This means that our information about the rest of the Maghrib is limited to the few data collected by Marcel Philibert⁴²⁵ deriving, in some cases, from the work of archaeologists and historians of architecture (who were not always careful with the *qibla* values they gave) and, in others, from his own measurements made with a compass. To this I should add the information on the orientations of early Tunisian mosques, given to me by Eric Mercier and based on the use of Google Earth.

The information gathered from these sources was classified by Mònica Rius⁴²⁶ who has succeeded in making sense of what seems, at first sight, a chaotic set of different orientations. I will follow her lead, trying to establish a relation between the known astronomical *qibla* indicators (*dalā'il al qibla*), leaving aside other criteria explained in the *anwā'* literature (direction of winds, for example) which are difficult to use. The main indicators used are:

1. The rising Sun in the winter solstice. This indicator is recommended for al-Andalus by Ibn Ḥabīb,⁴²⁷ in an appendix to Ibn Khalaf al-Murādī's *Kitāb al-asrār fī natā'ij al-aḥkām* (fl. Toledo?, eleventh century?), Ibn al-Ṣaffār (d. 1035), Ibn al-Naṭṭāḥ (early twelfth century?),⁴²⁸ and al-Mittijī,⁴²⁹ while other authors recommend the same indicator for the Maghrib and

⁴²² Dallal, 2010, chapter 1.

⁴²³ Jiménez, 1991.

⁴²⁴ Bonine, 1990.

⁴²⁵ Philibert, 1972, pp. 57–58.

⁴²⁶ Rius, 2000, pp. 97–135.

⁴²⁷ Rius, 2000, p. 162.

⁴²⁸ D.A. King, 1978, pp. 371, 388–389. Rius, 2000, p. 182.

⁴²⁹ Rius, 2000, pp. 183–184: al-Mittijī claims that the *qibla* in al-Andalus is 110°, applying the aforementioned criterion (?).

believe that this was the orientation chosen by ‘Uqba for the mosque of Qayrawān:⁴³⁰ a *qibla* facing 120° seems to have been used by ‘Uqba for several mosques built in southern Morocco (Nafīs, Agmāt Warīka and Agmāt Haylāna).⁴³¹ Other sources recommend the use of the rising point of Qalb al-‘Aqrab (α Scorpii, Antares),⁴³² an indicator that gives practically the same results for the latitudes involved, as can be seen in table 2.12, calculated for year 1050:

TABLE 2.12

Cities	Sunrise in the winter solstice	Rising point of <i>Qalb al-‘Aqrab</i>
Marrakesh	117.75°	119.14°
Fez	119.39°	119.95°
Córdoba	120°	121.57°
Toledo	120.92°	122.62°
Zaragoza	121.78°	123.56°

Five Andalusian mosques in which the orientation of the *qibla* is comprised between 115° and 125° (dated in the tenth, eleventh and fourteenth centuries) were probably built according to one of the aforementioned indicators.⁴³³ Seven Moroccan mosques, dated between the twelfth and the sixteenth centuries, have orientations comprised between 120° and 129°.⁴³⁴ Finally, two Algerian mosques seem to agree with this indicator: the Great Mosque of Constantina (1221?, 117°) and the Merinid Maṣṣūra of Tlemcen (1336, 118°).⁴³⁵

2. The rising Sun in the equinoxes which corresponds, obviously, to an orientation towards 90°. This is not very common and we only know of the case of the *Zāwīyya Nāṣiriyya* in Safī, oriented towards 92°, with the peculiarity of the existence of a hole in the *miḥrāb* through which the solar rays of the rising Sun in the spring equinox (9 March) enter and illuminate the door opposite to it. According to al-Asfī al-Andalusī (fl. 1701) the mosque was built with the cooperation of an unidentified ‘Abd Allāh b.

430 Rius, 2000, p. 236.

431 Rius, 2000, pp. 157–163, 292–295.

432 Rius, 2000, pp. 240–241.

433 Rius, 2000, p. 110.

434 Rius, 2000, p. 127.

435 Philibert, 1972, p. 57.

Sāsī, who was a *muwaqqit* and mathematician.⁴³⁶ Besides, the rising Sun in the equinoxes is used by al-Maṣmūdī to correct the previous indicator (rising Sun in the winter solstice): the *qibla*, in the Maghrib, should be between the rising Sun in the winter solstice and in the equinox.⁴³⁷

3. The rising point of Suhayl (α Carinae, Canopus): the use of this indicator implies an orientation of the building similar to that of the Ka'ba. Its azimuth is 166° in Qayrawān, 158° in Marrakesh and 163° in Fez, while the star is not easily visible in al-Andalus.⁴³⁸ We have already seen (2.5.1.2) that caliph al-Ḥakam II (2.5.2.2) was interested in the possibility that the star could be seen in Fuengirola (Málaga). The problem attracted the attention of the great philosopher Ibn Rushd who, in Marrakesh in 1153–54, was able to observe a star, invisible from al-Andalus, which the local people identified with Suhayl.⁴³⁹

The orientation of the Córdoba mosque (152°) was a model much imitated in al-Andalus, where we find 19 mosques whose *miḥrāb* has an azimuth comprised between 144° and 156° .⁴⁴⁰ In the Maghrib, a similar orientation to that of the Great Mosque of Córdoba model appears, in the ninth century, in the *Jāmi' al-Andalus* in Fez (151°), while the Qarawiyyīn mosque has a different orientation (163°) which also corresponds to the Suhayl criterion. The Almoravid Great Mosque of Algiers (1097) is oriented towards 163° ,⁴⁴¹ probably using the rising point of Suhayl (171°). In the twelfth century the Suhayl indicator was favoured by the Almohads who demolished the Kutubiyya of Marrakesh built in 1147 by the Almoravids (154°) and built the new mosque in 1162 oriented towards 159° .⁴⁴² Seven twelfth-century Maghribī mosques have an orientation comprised between 153° and 162° . It is rather surprising that this orientation was not followed in the Almohad Great Mosque of Seville (174°), which seems to follow the southern criterion. Another seven Moroccan mosques of the thirteenth and fourteenth centuries have their *miḥrāb* oriented between 161° and 163° .⁴⁴³ Interestingly, the mosque of Tinmal in the Upper Atlas

436 Rius, 2000, pp. 170–171.

437 Rius, 2000, pp. 336–338.

438 King remarks (private communication) that, in Mecca, the rising point of Canopus is perpendicular to the solstitial axis. If one uses the solstitial minor axis in al-Andalus, one still gets a *qibla* toward Canopus, even though the star cannot be seen there.

439 Rius, 2000, p. 242.

440 Rius, 2000, p. 117.

441 Philibert, 1972, p. 57.

442 Rius, 2000, pp. 123–126.

443 Rius, 2000, pp. 128–129.

is oriented towards 157° , while the rising of Suhayl cannot be observed there, because the south-eastern horizon is occupied by high mountains. This led Belmonte and Hoskins to hypothesise that the orientation of many of these mosques was based on computation rather than on the actual observation of the star,⁴⁴⁴ or, perhaps, on the use of the orientation of another mosque.

4. The mid-point of the South-Eastern quadrant of the horizon (135°), is the result of the use of the *jīha šuġrā*, which encompasses 90° , and the common knowledge that, in al-Andalus and the Maghrib, the *qibla* is placed in the South-Eastern quadrant. This is an orientation recommended by Ibn al-Naṭṭāḥ⁴⁴⁵ and, in 1546, the sultan Muḥammad al-Shaykh built the *muṣallā* placed outside the gate of al-Sharī'a in Fez with the *miḥrāb* oriented towards 135° .⁴⁴⁶ The same orientation was applied to the *muṣallā* of the gate of al-Futūḥ (1584). This criterion was often applied in al-Andalus in all periods, and fifteen Andalusian mosques have a *miḥrāb* oriented between 129° and 141° . A clear case is that of the small mosque in the palace of the Aljafería (*al-Ja'fariyya*) in Zaragoza: the whole building of the palace has an approximate N-S orientation and the mosque is placed on a diagonal, indicating an attempt to orient it towards 135° although its actual orientation is 141° .⁴⁴⁷ In Morocco we have nine mosques with *miḥrābs* oriented between 136° and 146° .⁴⁴⁸ As for Algeria, the mosques of Sīdī Ramaḍān (thirteenth century) in Algiers (137°), and of Sīdī al-Ḥalwī (1353, 135°) and Sīdī Bel Ḥusayn (1296, 143°) in Tlemcen correspond clearly to this orientation.⁴⁴⁹
5. A southern orientation as a result, probably, of the *qibla* in Medina, to the north of Mecca: eleven Andalusian mosques are oriented between 163° and 180° , while another six have orientations between 187° and 195° .⁴⁵⁰ In Morocco only two mosques facing 172° and 192° are documented in Bonine's report.⁴⁵¹ In Algeria we have the *Jāmi'* mosque of the Qal'a Banī Ḥammād (173°) and the mosque of Sīdī Ghānim in Mīla (188°).⁴⁵² The latter is considered to be the oldest mosque of Algeria.

444 Belmonte & Hoskins, 2002, p. 391.

445 Rius, 2000, p. 182.

446 Rius, 2000, p. 168.

447 Rius, 2000, pp. 112–112.

448 Rius, 2000, p. 127.

449 Philibert, 1972, p. 57.

450 Rius, 2000, pp. 118–120.

451 Rius, 2000, pp. 130–131.

452 Information provided by Muḥammad Riḍā Baklī.

6. Other criteria rarely reflected in actual buildings: al-Mittijī and al-Maṣmūdī recommend the use of the rising point of al-Nasr al-Ṭā'ir (α Aquilae, Altair), which is about 82° in Marrakesh, for the *qibla* in southern Morocco. The rising of al-Shi'ra al-'Abūr (α Canis Maior, Sirius) (110° in Marrakesh) is also considered a good indicator for the Maghrib and the same is said, in other sources, about the rising point of Rijlā al-Jawzā' (β and χ Orionis, Rigel) which corresponds to 101° in Marrakesh.⁴⁵³

Interestingly, we find a reference to an exact method in legal sources: al-Dāwūdī (d. 1011) says that the Sun crosses the zenith of Mecca at mid-day of the summer solstice. Al-Mittijī⁴⁵⁴ states that there is a mistake in this affirmation: as the latitude of Mecca is 21° , and assuming an obliquity of the ecliptic of 24° , the meridian altitude in the summer solstice will be:

$$h_m = 90^\circ - \varphi + \delta = 90^\circ - 21^\circ + 24^\circ = 93^\circ$$

As explained by al-Mittijī, the procedure is wrong but it clearly alludes to the fact that there are two days in the solar year in which the Sun will reach the zenith of Mecca. If the praying Muslim knows the hour difference between Mecca and his own location, by facing the Sun at the local time corresponding to the midday in Mecca, he will be facing the *qibla*. The method is rejected by Ibn al-Bannā' because it requires the knowledge of the difference of longitudes between the two cities.⁴⁵⁵

453 Rius, 2000, pp. 243–246.

454 Rius, 1996, p. 816.

455 Jabbār & Aballāgh, 2001, p. 54.

Astrology

3.0 Introduction

Astronomy in the Middle Ages was narrowly associated to astrology which, together with timekeeping, was the main cause of its development and cannot be omitted in any history of the discipline like the present one.¹ I must confess, however, that I am more interested in the techniques used for astrological predictions than in the predictions themselves, as a good part of the predictive process is clearly scientific, in the modern sense of this term. There is a section of this discipline, usually called “mathematical astrology”, which is simply mathematics applied to astrological problems like the division of the houses of the horoscope, the *tasyīr* or the casting of rays. This is a topic that attracted the interest of a great Andalusī mathematician of the 11th c, Ibn Mu‘ādh al-Jayyānī who will be mentioned many times in 3.2.3. Besides, it is also obvious that astrological horoscopes are computed by using a particular set of astronomical tables and their study allows the scholar to follow the diffusion of a specific *zīj*.

The present chapter is divided into two parts: the first one, which follows a chronological order, deals with patronage and the practice of astrology and, in it, I try to establish that most of the extant important astrological sources are somehow related to the existence of a patron, usually a monarch of some kind, who is interested in the subject. When emirs, caliphs or kings loose their interest in astrology, sources become scarce, the practice of astrology becomes more or less hidden and, sometimes, magic and occult sciences appear. On the other hand, I believe that the existence of an astrological practice is best shown by the existence of horoscopes and I have given, here, a good collection of them which I have tried to recompute using the software available to me.²

The purpose of the second part of this chapter (Thematic surveys) is the study of some specific topics on which we have a certain amount of

1 Neugebauer, 1951.

2 Mainly John North's programme HOROSC, Benno van Dalen's Table Analysis (TA), Sexagesimal Calculator (SCTR) and programme for date conversion (CALH). To this I should add that Prof. E.S. Kennedy designed, during one of his frequent visits to Barcelona, the skeleton of a programme, later improved by Honorino Mielgo and Josep Casulleras, which allows the computation of planetary longitudes using a medieval *zīj*: on this basis I have been able to use the *Mumtaḥan* tables, al-Khwārizmī-Maslama's *zīj*, al-Battānī's, Toledan and Alfonsine Tables, and Ibn Ishāq's *zīj*.

background research. Such is the case of what I have called “simple systems of astrological prediction”. These systems deal with predictions based on a very limited number of variables, like those we find in modern popular horoscopes, published in newspapers, which try to foretell the near future of a person on the basis of the zodiacal sign in which the Sun was placed at his birth (3.2.1). Another topic dealt with in this section is mathematical astrology (3.2.3), as it has been the object of important studies by E.S. Kennedy, John North, Jan Hogendijk, Josep Casulleras and many others. In other cases I was personally interested in one specific topic and what I have written here is the result of my own research. Examples of this second kind surveys are my analysis of Ibn Abī l-Rijāl’s *al-Kitāb al-Bārī fī aḥkām al-nujūm* (3.2.2), the technique of the animodar (3.2.4), or Abraham bar Ḥiyya’s astrological history (3.2.5.3).

3.1 Patronage and the Practice of Astrology

3.1.1 *Al-Andalus and the Maghrib*

3.1.1.1 Beginnings of Astrology in al-Andalus

Scientific patronage was mainly in the hands of *amīrs*, caliphs and kings, who were interested in the prediction of future and, therefore, in a serious kind of mathematical astrology, which gave guarantees of a scientific result. There was, obviously, another kind of more popular astrology, for treatises on the *ḥisbat al-sūq* (policing of the market), such as that of al-Saqaṭī (fl. ca. 1200–1225), forbade astrologers to practise in markets,³ but there is no information about these professionals, although we might guess that certain characteristic developments of Andalusī astronomy, designed to simplify the computation of planetary longitudes (equatoria, perpetual almanacs), were the result of the demands of popular astrologers who needed to sell a product (i.e. a horoscope) for a reasonable price. Besides this, I will deal, in this chapter, with astrological treatises designed for this kind of public. Concerning the social history of the professional astrologer we are, however, restricted to the circles of power in which patronage followed the evolution of political and religious history.

Astrology was, no doubt, practised on both sides of the Strait of Gibraltar from the end of the 8th c. although we have much more information about such practice in al-Andalus than in the Maghrib. We have already seen in 1.2 that the Aghlabī Ibrāhīm II (r. 875–902), who was interested in mathematics and astronomy, founded the *Dār al-Ḥikma* in Raqqāda, where astronomical instruments were built by ʿUthmān al-Ṣayqāl (d. 941). Besides this, Ismāʿīl

3 Chalmeta, 1967–68.

al-Ṭallāʿ was the official astrologer of Ibrāhīm II and accompanied him in his military campaigns.

As for al-Andalus, the origins of the practice of astrology are related, according to legend, to the period of the conquest. I will argue, later in this chapter (3.2.1.1), that the Muslim conquerors discovered the existence of a local Latin astrological tradition in the Iberian Peninsula and, as a result of this, historical sources tried to compensate this by attributing to Muslim leaders like Mūsā ibn Nuṣayr some knowledge of astronomy and the capacity to predict the future.⁴ Serious information appears at the end of the 8th c. where we find the first reference to the interest of the Umayyad *amīrs* in astrology in the time of Hishām I (r. 788–796) who, in spite of his religious beliefs, required the service of al-Ḍabbī (fl. ca. 788–ca. 852), the first known Andalusī astrologer,⁵ and asked him the dangerous question of calculating the length of his life and of his reign. Besides this, al-Ḍabbī was the author of a famous astrological *urjūza* (a mnemonic poem in meter *rajaz*, considered to be easy) which is the earliest extant astrological source (see below 3.2.1.1). The situation is clearer in the time of al-Ḥakam I (r. 796–822) and, especially, under ʿAbd al-Raḥmān II (r. 822–852). The former had his son ʿAbd al-Raḥmān adequately taught in the sciences of the ancients, and both *amīrs* had, in their court, an important group of poets-astrologers who are well attested in the sources precisely because they were poets. The recent publication of vol. II-1 of Ibn Ḥayyān's *Muqtabis*⁶ has brought to light new information on these characters (al-Ḍabbī, Yahyā al-Ghazāl, Ibn al-Shamir/ Shimr, ʿAbbās b. Firnās, Marwān b. Ghazwān etc.)⁷ for the book contains an important chapter entitled *Akhbār al-munajjimīn maʿa l-amīr ʿAbd al-Raḥmān b. al-Ḥakam* (= ʿAbd al-Raḥmān II).⁸ The materials contained in this chapter and in the rest of the text have been analysed critically by Miquel Forcada⁹ and Mònica Rius.¹⁰ The chapter in question is mainly concerned with an attempt by the *amīr* to control the veracity of the astrological predictions made by his astrologers in a set of anecdotes two of which, at least, are also attributed to Eastern astrologers such as Abū Maʿshar and

4 See, on this topic, Marín, 1986; Samsó, 1985–86.

5 Samsó, 2001.

6 Makkī, 2003; Makkī & Corriente, 2001.

7 On these characters see the corresponding biographies in *BA* (Almería, 2004–2013): M. Rius, “Al-Ḍabbī, ʿAbd al-Wāḥid”, vol. I (2012), pp. 303–304; M. Rius, “al-Gazāl, Yahyā b. Ḥakam” vol. I (2012), pp. 405–408; M. Rius, “Ibn al-Šamir, Abū Muḥammad” vol. 5 (2007), pp. 231–233; M. Aragón, “Ibn Firnās, ʿAbbās” vol. 3 (2004), pp. 168–172; M. Rius, “Ibn Gazwān, Marwān” vol. 3 (2004), pp. 215–216.

8 Makkī, 2003, pp. 389–406; Makkī & Corriente, 2001, pp. 258–270.

9 Forcada, 2002; Forcada, 2004–5.

10 Rius, 2003.

al-Bīrūnī.¹¹ The new information provided by this source shows the interest ‘Abd al-Raḥmān II and the members of his court felt in astrology, and also astronomy: the *amīr* himself is credited with the capacity to calculate planetary positions¹² and a musician like Ziryāb is presented as having a deep knowledge of both disciplines.¹³ Among the courtiers mentioned by Ibn Ḥayyān, al-Ḍabbī, Marwān b. Ghazwān and Muḥammad b. ‘Abd Allāh b. al-‘Adhrā’ were pure astrologers, while al-Ghazāl, Ibn al-Shamir and ‘Abbās ibn Firnās were also poets.¹⁴ It would be worthwhile to establish whether there was an official list (*dīwān*) of astrologers who received a royal salary in a similar way to the list of poets (*dīwān al-shu‘arā’*) or the list of physicians (*dīwān al-mutaṭabbibīn*) which existed in Córdoba under al-Ḥakam II. The only hint we have of such arrangement is the fact that Ibn Sa‘īd¹⁵ says that ‘Abd al-Raḥmān II assigned to Ibn al-Shamir a double salary (*rizq*) for his poetry and his astrology (*tanjīm*), while Ibn Ḥayyān asserts the same about ‘Abbās b. Firnās.¹⁶ Were they the only cases?

On the other hand, it seems that this introduction of astrology in the court is somehow related to the first manifestations of the *mu‘tazila* movement (Yaḥyā al-Ghazāl)¹⁷ and of [Ḥarrānian?] Magic (al-Ḍabbī,¹⁸ ‘Abbās b. Firnās¹⁹). One should remember that Yaḥyā al-Ghazāl had travelled to the East and that al-Ḥarrānī was the physician in the service of ‘Abd al-Raḥmān II.

11 In one of these anecdotes (Makkī, 2003, p. 391; Makkī & Corriente, 2001, p. 259) ‘Abd al-Raḥmān II, who is sitting in a room with four doors, asks Ibn al-Shamir which is the door he is going to use to get out of the room. The astrologer casts the corresponding horoscope and writes the result in a sheet of paper which, conveniently closed, is given to the *amīr*. Then ‘Abd al-Raḥmān orders his servants to open a fifth door behind his throne. Ibn al-Shamir had foreseen what the *amīr* was going to do. The same story is told by Nizāmī ‘Arūḍī Samarqandī in the *Chahār Maqāla* where the characters involved are al-Bīrūnī and Maḥmūd of Ghazna (see Samsó, 2011, p. 51). In a second anecdote (Makkī, 2003, pp. 391–392; Makkī & Corriente, 2001, p. 260), the *amīr* asks Ibn al-Shamir, Ibn Firnās and Ibn Ghazwān to describe the foetus of a pregnant cow. Their answers are correct, with minor differences. Ibn Tāwūs (12th c.) explains the same story about Abū Ma’shar and an unknown ruler: see Saliba, 1992, p. 55.

12 Forcada, 2004–5, p. 10.

13 Forcada, 2004–5, p. 24; Rius, 2003, pp. 524–527.

14 Forcada, 2004–5, p. 14.

15 Ḍayf, 1953, p. 125.

16 Makkī, 2003, p. 348; Makkī & Corriente, 2001, p. 226.

17 Forcada, 2004–5, pp. 40–44. The *mu‘tazila* was a rationalistic school of religious thought born in Iraq (Baṣra) in the first half of the 8th century.

18 See Samsó, 2001 a.

19 Forcada, 2004–5, pp. 25 and 39.

3.1.1.2 Yaḥyā al-Ghazāl Predicts the Death of Naṣr (851)

Our astrologers cast horoscopes and some of them are significant: Ibn al-Shamir predicted that ‘Abd al-Raḥmān II would attain the throne and, like al-Ḍabbī (and Mūsā ibn Nuṣayr), predicted the time of his own death. A poem of Ibn al-Shamir states that the end of the kings of Córdoba (i.e. the dynasty of the Umayyads) will take place when Saturn has rotated six times over them.²⁰ As the period of revolution of Saturn is, approximately, thirty years, this is equivalent to a hundred and eighty years which, assuming that the poem was written about 850, leads us to $850 + 180 = 1030$, which fits nicely 1031, the date of the end of the Umayyad Caliphate. These horoscopes, as well as many others, are given by Ibn Ḥayyān without technical details although in one case we have a poem by Yaḥyā al-Ghazāl containing information which deserves an analysis.²¹ The poem begins:²²

Tell al-fatā Naṣr, Abū l-Faṭḥ:
Saturn is in al-Naṭḥ.
I see that it retrogrades and then advances
going forwards and backwards reaching *al-rumḥ*

The poem consists of a prediction of the death of eunuch (*fatā*) Naṣr, the powerful minister of ‘Abd al-Raḥmān II, which took place immediately after the end of the month of *Sha’bān* of year 236 (the 29th of *Sha’bān* 236 = 7th March 851). As al-Naṭḥ is a lunar mansion, belonging to the sign of Aries, whose longitude in the first half of the 9th c. was about 20° , we can calculate the position of Saturn for the 7th March 851, using al-Khwārizmī’s *zīj*,²³ and we obtain $22;7,26^\circ$, which agrees with al-Naṭḥ. Besides, Saturn, according to the aforementioned tables, reached a station the 6th August 850 (its longitude was $23;29,23^\circ$) and its retrogradation lasted from the 8th of August until the 19th December of the same year (longitude $17;8,34^\circ$). The arc of retrogradation was, therefore, $6;20,50^\circ$ and this seems to explain the mention to *al-rumḥ*, an angular unit used by sailors whose values oscillate between $4;30^\circ$ and something more than 14° .²⁴

20 Makkī, 2003, p. 395; Makkī & Corriente, 2001, pp. 262; Forcada, 2004–5, pp. 33–34.

21 Samsó, 2004 a, pp. 482–488.

22 Makkī, 1973, pp. 11–12.

23 I am using in this chapter the programme KHWRLN designed by Kennedy & Mielgo.

24 Vernet, 1978, pp. 417–419.

The poem continues:

I see that the *nuḥūs* are helping it (= Saturn)
Take care of yourself and follow my advice.

The *nuḥūs* are the unfortunate indicators and, in the case of the planets, they are identified with Saturn and Mars and both planets were in conjunction (Saturn at 19;53,45°; Mars 19;58,34°) three weeks before, the 14th February 851. Besides this, the 7th March 851, Saturn was in a position of strength, having recovered its direct motion after retrogradation and it was placed to the east of the Sun (*tashrīq*), the solar longitude being 347;21,8° the 7th March. Mars, whose longitude the 7th March was 34;49,48° was also in *tashrīq* and it is obviously one of the *nuḥūs*. Besides this, at the moment of the Saturn-Mars conjunction of the 14th February the ascending node had a longitude of 20;59,18°, at 1° distance from the two malefic planets and al-Bīrūnī states that, according to the Babylonians, the ascending node increases the effect of both the beneficial and evil planets. Therefore, my impression is that the ascending node also belongs to the *nuḥūs*.

The next verse, although unrelated to the prediction, has also a certain interest:

I realised this when I calculated for him (*idh ḥasabtu la-hu*)
the indicators of the high prize (*ghalā'*) of wheat.

This seems to be related to the period, in al-Ghazāl's biography, in which al-Ghazāl was in charge of collecting the tithe of the crops.²⁵ We will see, in 3.2.1.1, the interest, shown in the texts related to the astrological "system of the crosses", about the prediction of periods of rain and draught and their influence in the prices of agricultural products. Besides this, it seems that Yaḥyā al-Ghazāl is doing this kind of astrological computations for the minister Naṣr which shows that even important poets like him were working for the state in their condition of astrologers.

The actual prediction of Naṣr's death appears two verses below the last one:

When you see the full Moon (*al-badr*) in *Bulā'*
the target will be met in the most disgraceful way.

25 Haykal, 1970, pp. 172–186; M. Rius in *EA* 1, pp. 405–408.

Here the text refers to the lunar mansion of Sa'd Bula' which, in the first half of the 9th c, was placed between 295° and 298° of longitude. *Al-Badr* being the full Moon, the Sun must be in opposition to it, with a longitude between 115° and 118°, something which, obviously, does not correspond to the month of March but to the month of July. The prediction, therefore, may correspond, approximately, to the 17th July 851 in which the positions of the Sun and the Moon were:

Sun	112;56,18°
Moon	296;42,2°

which implies that Yaḥyā al-Ghazāl must have been rather surprised with the death of Naṣr which took place on the 8th March, four months earlier than the date he had predicted.

3.1.1.3 The Decay of Astrology between ca. 850 and 1031

Astrology was strongly attacked by *fuqahā'* like 'Abd al-Malik ibn Ḥabīb (d. 853), who tried to defend the existence of a legitimate folk-astronomy tradition (*anwā'* and *manāzil*).²⁶ In any case the astrological tradition seems to have decayed under Muḥammad I (852–886)²⁷ and his followers, until the reign of 'Abd al-Raḥmān III (912–961). In the turn of the 9th/ 10th c. only one astrologer is known: Ibn al-Samīna (d. 927–928), who was also competent in literary and religious sciences, as well as in medicine.²⁸ In spite of this, it is clear that Cordoban astrologers must have continued practising their profession, for an anecdote, mentioned by the poet and expert in *adab* Ibn 'Abd Rabbihi (860–940), shows the astrologer Ibn 'Adhrā' and a group of colleagues trying to predict the end of a period of draught.²⁹ In the 10th c. astrology seems to be replaced by more esoteric disciplines like magic: in the first half of the century we see the publication of the *Ghāyat al-ḥakīm* (*Picatrix*), authored by Abū l-Qāsim Maslama b. al-Qāsim al-Qurṭubī (d. 964), who should not be confused with

26 Kunitzsch, 1994 & 1997.

27 Most of the astrologers of the court of 'Abd al-Raḥmān II seem to have continued their task during the reign of Muḥammad but little is known about their activity and, according to Ibn Ḥayyān's *Muqtabis II-1* (Makkī, 2003 and Makkī & Corriente, 2001), this emir condemned to death al-Ḍabbī (see Samsó, 2001a) and put into prison Marwān b. Ghazwān. In both cases the cause of the sentence was their incapacity to be discreet about the secrets of the palace.

28 Forcada, 1997, p. 215.

29 See Viguera, 1977, pp. 365–366; Viguera, 1981, p. 443; Samsó, 1979.

the mathematician and astronomer Maslama b. Aḥmad al-Majrīṭī (d. 1007).³⁰ More or less at the same time Abū ‘Alī b. al-Ḥasan b. al-Ḥātim, about whom we only know that he observed, somewhere in al-Andalus the solar eclipse of the 19th July 939, wrote a much shorter magical work, known in Europe through a Latin translation entitled *De imaginibus caelestibus*.³¹

Under the rule of ḥājib al-Manṣūr (981–1002), a return to orthodoxy takes place and leads to a selective burning of the library of al-Ḥakam II, including, obviously, books on astrology. According to Ibn al-Khaṭīb, al-Manṣūr arrested, tortured and executed sooth-sayers (*mukahhinūn*) and astrologers (*munajjimūn*) because of their predictions about the end of his regime.³² Ibn ‘Idhārī, on his side, states that Muḥammad ibn Abī Jum‘a published false rumours about the existence of a *qaṭ’*³³ which would imply the end of al-Manṣūr’s regime and, for this reason, the ḥājib cut Muḥammad’s tongue and crucified him.³⁴

In spite of this, there is some evidence that there were important exceptions to this policy. Thus, among the educated people surrounding al-Manṣūr’s son, ‘Abd al-Malik al-Muẓaffar (975–1008), for the benefit of his education, Ibn Bassām includes *mu’addils* (?). The same source, using information gathered in Ibn Ḥayyān’s *Matīn*, explains that Aḥmad b. Fāris al-Munajjim (see 2..5.2.2) had studied the nativity horoscope of al-Muẓaffar when he was a child. He had never seen a horoscope with happier prospects. Ibn Ḥayyān satyrises this prediction as a result of the unhappy outcome of the life of al-Muẓaffar.³⁵ Al-Muẓaffar’s horoscope is not the only one and there is evidence showing that the upper classes were also interested in astrology, in spite of its official rejection, and had horoscopes cast in the moment of the birth of their children. Such is the case of the famous writer Ibn Ḥazm of Córdoba, whose father occupied important posts in al-Manṣūr’s administration. The author himself explains that he was born in that city before sunrise but after the moment in which the *imām* saw off the praying Muslims, at the end of the morning prayer, the 7th November 994 and that, at the moment of his birth, the ascendant was

30 Fierro, 1996; Callatay, 2013.

31 Oliveras, 2009.

32 Lévi-Provençal, 1934, p. 77; Balty-Guesdon, 1992, p. 258.

33 The *tasyīr* (see 3.2.3.2) is an astrological technique which pretends to calculate the amount of time before a particular event takes place (the death of the subject of the horoscope, for example). One considers a planet or a significant point of the ecliptic like the ascendant which begins to rotate until a malefic element, called *qāṭi’* = cutter) interrupts the process. Ibn ‘Idhārī refers here to one of such cuts (*qaṭ’*).

34 Colin & Lévi-Provençal, 1980, II, p. 294. On this passage see also Balty-Guesdon, 1992, p. 258.

35 ‘Abbās, 1979, VII, p. 79.

Scorpio. Considering that the Sun rose in Córdoba, in that date, at about 7 in the morning, it is easy to check that the ascendant was in Scorpio at 6:30 h. if we calculate the solar position at midday with al-Battānī's (ascendant 225;33°) or al-Khwārizmī's parameters (219;40°).³⁶ The situation was not clear, however, for the astrologer Aḥmad b. Fāris felt the need to defend astrology by linking it to the religiously acceptable folk-astronomy.³⁷

With the arrival of the period of civil wars after the fall of the Caliphate (*fitna*) this hidden interest in astrology and astronomy reappears openly. Ibn al-Samḥ (d. 1035) leaves Córdoba and goes to Granada where he is protected by Ḥabūs ibn Mākzan (1019–1038); Ibn al-Ṣaffār (d. 1035), another of Maslama's disciples, fled to Denia under the patronage of Mujāhid (ca. 1010–1045). These migrations of important astronomers to Granada and Denia pose the problem of establishing to what extent the members of Maslama's school had to leave Córdoba because they had been protected by the circles of political power of the *ʿāmirī* dynasty, who could have exerted a hidden patronage: the case of Ibn al-Ṣaffār is particularly significant because Mujāhid of Denia was a *mawlā* of the *ʿāmirīs* and his kingdom was a standard place of asylum for those who had served al-Manṣūr's family.

3.1.1.4 The Conjunction of Saturn and Jupiter of 1007 and the Fall of the Caliphate

The predictions annoying al-Manṣūr were usually related to the Saturn-Jupiter great conjunction (with change of triplicity) of year 1007.³⁸ Here, we find, for the first time, a reference to astrological history, based on the recurrence of cyclical conjunctions of Saturn and Jupiter (3.2.5.1), which take place, approximately, every twenty years and affect the whole of mankind. Successive conjunctions of these two planets tend to stay in the same triplicity,³⁹ but, after a period of some 243 years, the conjunction moves to another triplicity and its effects are far more important.

At 10h PM of the 19th November 1007, according to al-Khwārizmī's *zīj*, the longitude of both planets was 158;43°. The phenomenon and the rumours associated with it must have produced, among the elites of Córdoba, a terror

36 'Abbās, 1968, II, pp. 78–79. The source of this information is Ibn Ḥazm himself according to Ṣā'id of Toledo: *Bū 'Alwān*, 1985, p. 184.

37 Forcada, 1996; Forcada, 2000.

38 Samsó, 2004 a, pp. 488–496.

39 The triplicities are four groups of three zodiacal signs associated to the classical four elements: Fire (Aries, Leo, Sagittarius), Earth (Taurus, Virgo, Capricorn), Air (Gemini, Libra, Aquarius) and Water (Cancer, Scorpio, Pisces).

similar to the panic which took place in Christian countries with the change of the millenium. Four sources give information on the predictions associated to that conjunction:

1. Ibn 'Idhārī⁴⁰ talks about a conjunction of the seven planets (!) which began in Leo and arrived later to Virgo⁴¹ thus, affecting in a special way, Córdoba, because Virgo was the zodiacal sign which ruled the city. In the conjunction Saturn (malefic) dominated over Jupiter (benefic) and this produced the ruin of the state. One of the astrologers who authored such predictions was Maslama b. Aḥmad al-Majrīṭī (d. 1007–8) according to whom the situation would be upturned, the state would fall in other hands, there would be killings and famine. The *fitna*⁴² arrived later and produced worse effects than those predicted by Maslama.
2. Ibn al-Khaṭīb's *A'māl al-a'lām*⁴³ who quotes Ibn Ḥayyān's *Matīn*: as Virgo is a sign having two bodies (*dhū l-jasadayn*),⁴⁴ those who occupied the throne during the *fitna* would reign twice and Ibn al-Khaṭīb gives the list of five monarchs who fulfilled the prediction.
3. The Alfonsine *Libro de las Cruces*,⁴⁵ which seems to be a translation of an Arabic book authored, in the 11th c., by 'Abd Allāh b. Aḥmad al-Ṭulayṭulī (see 3.2.1.1) insists in the fact that the true conjunction took place in the sign of Virgo (earth). This implied a change of triplicity which had as a consequence that the Western kingdoms (Christians and Berbers) overcame the Eastern ones (Arabs). This indication might be explained either as a result of the original Arabic book having, probably, been written in Toledo under the Berber dynasty of the Banū Dhī l-Nūn, or as an allusion to the arrival of the Almoravids, The text mentions another Saturn-Jupiter conjunction, also in the triplicity of earth. in 459/1066–67: the actual true conjunction took place, according to al-Khwārizmī, the 18th September 1067 at a longitude of 164;58° (Virgo again). This second conjunction produced a loss of power of the Arabs in the East and an increase of Christian power in the West.

40 Colin & Lévi-Provençal, 1980 vol. III, pp. 14–15.

41 Apparently the text means that the mean conjunction took place the 16th October 1007 in 143;26°, in the sign of Leo, while the true conjunction the 19th November in 158;43°, in the sign of Virgo.

42 Period of civil war after the fall of the Caliphate.

43 Ed. E. Levi-Provençal (Rabat, 1934), pp. 148–149, or Beirut, 1956, pp. 127–128.

44 Like Gemini, Sagittarius and Pisces: the third signs after the equinoxes and the solstices.

45 Kasten & Kiddle, 1961, pp. 9–10.

4. The fourth source is the commentary by Ibn Qunfudh al-Qusanṭīnī (1339–1407) of Ibn Abī l-Rijāl's astrological *urjūza*.⁴⁶ Ibn Qunfudh refers to three mean conjunctions of Saturn and Jupiter:
- Conjunction of the Religion (*qirān al-Milla*) in Scorpio (water) which announced the birth of the Prophet. Ibn Qunfudh casts the horoscope of the spring equinox of the year of the conjunction (19th March 571) See 3.2.5.2.
 - Conjunction in Sagittarius (fire) towards the beginning of the 2nd c. H: the true conjunction took place in 241;30° (Sagittarius) the 12th October 809 /28th Dhū l-Ḥijja 193H.
 - Mean conjunction in Leo (fire) in 1007, with true conjunction in Virgo (earth). He mentions the actual horoscope of that conjunction which probably circulated among the astrologers of Córdoba and says that the ascendant was in Aquarius and Saturn in the 7th house (which probably began between 120° and 150°). This horoscope was probably cast for the spring equinox of 1007 (22nd March). At midday of that day the planetary positions were:

Sun: 359;49°
 Moon: 8;34°
 Saturn: 141;7°
 Jupiter: 121°
 Mars: 29;41°
 Venus: 324;47°
 Mercury: 333;38°
 Asc. node: 239;6°

Saturn, therefore, could be in the 7th house. Jupiter (beneficient planet) was in the first decan of Leo, dominated by Saturn, and this explains Ibn Qunfudh's mention of Saturn being *ṣāhib ṭālī' al-qirān wa l-muqtaran*: lord of the ascendant of the conjunction (Aquarius is one of Saturn's domiciles) and of the planet in conjunction (Jupiter). Venus (also a beneficent planet) was in the ascendant, under the dominion of Saturn. All this explains the extremely malefic character of the situation which caused the ruin of the Caliphate.

46 Oliveras, 2012, p. 204 (Arabic) and 356 (Sp. translation).

3.1.1.5 The Period of the *ṭawāʾif* (ca. 1035–1085)

3.1.1.5.1 *Ṣāʿid of Toledo and Abū Marwān al-Istijī*

This period is the golden age of astronomy and astrology, probably as a result of the development of patronage, a fact emphasized in Ṣāʿid's *Ṭabaqāt* in comparison to the unhappy period of al-Manṣūr's dictatorship:⁴⁷ the Banū ʿAbbād of Seville, for instance, patronized the works of Ibn al-Zarqālluh when he was still living in Toledo⁴⁸ and al-Muʿtamid (1069–1091) had at his service during twenty years the Jewish astrologer Yishāq ibn Barukh ibn al-Baliyya (1034–1093), as well as Abū Bakr al-Munajjim al-Khulānī.⁴⁹

The great centre of astronomical learning was Toledo, where interest in astrology had already begun during the *fitna*, as we have some information about the activities of Yūsuf ibn ʿUmar ibn Abī Thalla (d. 1043) in that city. Patronage was not restricted to kings: the Banū Dhakwān family supported the studies of the physician and astronomer Muḥammad b. Sulaymān b. al-Hannāṭ (d. 1045).⁵⁰ The most important case is, however, that of *qāḍī* Ṣāʿid of Toledo (1029–1070), the author of the well known *Kitāb ṭabaqāt al-umam*, who gathered around him the team of astronomers authors of the *Toledan Tables*. One of them was Abū Marwān ʿAbd (or ʿUbayd) Allāh al-Istijī. We have no precise dates for his birth and death but his family must have originated in Ēcija (Lat. Astigi, Ar. Istija), near Córdoba. Ṣāʿid mentions him twice in his *Ṭabaqāt*:⁵¹ as one of the young astronomers who worked in Toledo at the time of the completion of the *Ṭabaqāt* (460/1068) and as an expert in astrology who had written an excellent *Risāla fī l-tasyīrāt wa-maṭāriḥ al-shuʿāʾāt wa taʿlīl baʿḍ uṣūl al-ṣināʿa* (“On progressions and projection of rays, with a verification of some foundations of this technique”) which he sent to him from Cuenca. We do not know the reasons for this change of residence, from Toledo to Cuenca, and, in the introduction to the *Risāla*,⁵² he complains that he has been separated from his master Ṣāʿid and is compelled, as a result, to change his field of research from computational astronomy to astrology. Whatever the case, the *risāla* is dedicated “to the honourable minister and judge Abū l-Qāsim” (*al-Wazīr al-jalīl al-qāḍī Abū l-Qāsim*), identified by Richter-Bernburg with the well-known Abū l-Qāsim Ṣāʿid al-Andalusī who, being a *qāḍī* of Toledo, had the right to be called *wazīr*.⁵³

47 Forcada, 1997, pp. 230–231.

48 Samsó, 2011, p. 198.

49 Balty-Guesdon, 1992, pp. 315, 667–8, 693.

50 Balty-Guesdon, 1992, pp. 283, 643–4.

51 Bū ʿAlwān, 1985, pp. 180, 199–200; Blachère, 1935, pp. 139, 153–154.

52 Samsó & Berrani, 2005, pp. 176–178, 225. See also Samsó & Berrani, 1999.

53 Richter-Bernburg, p. 377.

We will deal with the contents of this *risāla* quite frequently in this chapter. Apart from it, we have indirect information about three other works written by al-Istijī, two of them showing that he was a competent astronomer: 1) a *Risālat al-iqbāl wa l-idbār* ("On accession and recession"), dealing with the theory of trepidation of the equinoxes, apparently one of the main subjects of the research of the Toledan team; 2) a *zīj* without title which I will discuss in chapters 6 and 7; 3) a treatise on the animodar, one of the topics we find in our *risāla*.

Šā'id said that al-Istijī was "one of those who have a sound knowledge of astrology and who have read the books on this subject by both ancient and modern authors"⁵⁴ This remark is probably accurate, as we can see by an analysis of the sources quoted in the text, most of which are also mentioned by Šā'id himself. They include Ptolemy's *Tetrabiblos*, Hermes, the *Kitāb al-amthāl li l-Furs* (probably the *Kitāb al-amthāl* with several variants in the title quoted by Ibn Abī l-Rijāl, see 3.2.2.2.1). These are the ancient authorities and it is interesting to remark that he complains of the mistakes made by translators which might be the cause of the errors of practising astrologers.⁵⁵ Once in the Islamic period, we find Kankah al-Hindī (fl. Baghdad ca. 775–820), Abū Ma'shar (d. 886) (*Kitāb al-milal wa l-duwal*, *Kitāb al-qirānāt*, the *Mudhakarāt* and *al-Madkhal al-kabīr*), al-Khaṣībī (probably al-Ḥusayn/al-Ḥasan b. al-Khaṣīb, fl. 844), Aḥmad b. Yūsuf al-Kātib (d. 941), al-Ḥasan b. Aḥmad b. Ya'qūb al-Hamdānī (d. 946), al-Battānī (d. 929), and the *Rasā'il ikhwān al-ṣafā'*. Of the aforementioned list only the *Kitāb al-amthāl li l-Furs* was, apparently unknown to Šā'id, who seems also to be unaware of a book on astrological predictions by al-Kindī (d. ca. 870), entitled *al-Madkhal al-awsaṭ*.

3.1.1.5.2 *Ibn al-Khayyāt's Astrological Lāmiyya*

Royal patronage reached its apex under al-Ma'mūn (1043–1075) (whose *laqab* is probably the result of a mimesis of that of the famous Abbasid caliph patron of astronomy) served by Yaḥyā b. Aḥmad b. al-Khayyāt (ca. 977–1055), a disciple of Maslama al-Majrītī, who had been previously the astrologer of the Umayyad Sulaymān al-Musta'in (r. 1009, 1013–1016).⁵⁶ According to the emir 'Abd Allāh ibn Buluqqīn, this astrologer predicted the conquest of Denia in 468/1076 by al-Muqtadir b. Hūd, king of Zaragoza and father of the famous king-mathematician al-Mu'taman b. Hūd.⁵⁷ Ibn al-Khayyāt dedicated to al-Ma'mūn a *Risāla fi l-qirānāt al-nujūmiyya* (Epistle on planetary conjunctions)

54 Bū 'Alwān, 1985, pp. 199–200; Blachère, 1935, pp. 153–154.

55 Samsó & Berrani, 2005, pp. 214–215, 240.

56 Balty-Guesdon, 1992, pp. 278–279, 643–644.

57 Lévi Provençal, 1935–6, pp. 323 (1935) and 43 (1936); Lévi-Provençal, 1955, p. 78; Lévi-Provençal & García Gómez, 1980, p. 164. Tibi, 1986, p. 94.

dealing, among other things, with the events of al-Andalus, how long would the Muslims stay in this country and signs indicating that they would soon have to leave this place.⁵⁸ This epistle does not seem to be extant but the Moroccan astrologer al-Baqqār has preserved, in his *Kitāb al-amṭār wa l-as'ār* (Book on rains and prices) 91 verses of Ibn al-Khayyāt's *Lāmiyya* (poem rhyming in *lām = l*)⁵⁹ dealing with:

- The passage of Saturn through the four triplicities (15 verses). The three final verses of this group allow us to conjecture a possible date for the poem (around 1050):

Listen to what I am going to describe and do not ignore what I am saying today

I began in the moment in which I found it (=Saturn) stationary (*muqīm*) according to its position calculated with tables

And I found it in the end of Capricorn, getting out of the sign towards Aquarius in which earthquakes are frequent.

Using the tables of al-Khwārizmī, we can easily calculate that towards the 8th of Jumādā 1 of 442 (28 September 1050) Saturn reached a station, after retrogradation, in Capricorn 25°. He entered Aquarius 0° the 3rd Sha'bān 442 (21st December 1050). Saturn had also entered Aquarius about 30 years earlier (22 Shawwāl 411/7th February 1021) but, in this case, there was no retrogradation before or after this date and, therefore, Ibn al-Khayyāt could not find it stationary.

The rest of the *Lāmiyya* deals with the passage of Saturn through the twelve zodiacal signs. As we will see, a few of Ibn al-Khayyāt's predictions seem to be related to actual events which took place during his lifetime:

- Saturn in Aries (8 vv): the penultimate verse of the series is concerned with a war with the Berbers.
- Saturn in Taurus (3 vv), Gemini (4 vv) and Cancer (8 vv). The last verses (Cancer) refer to an earthquake in Córdoba and a solar eclipse at midday which allowed people to see the lunar mansions during the day. This may refer to the solar eclipse which took place the 29th June 1033, with the line of totality crossing the north part of the Iberian Peninsula. At the moment of the eclipse, Saturn was in Cancer. On the other hand the first verse of the

58 According to the Moroccan astronomer and astrologer al-Baqqār (fl. 1411–1418) in his *Kitāb al-amṭār wa l-as'ār* (Book on rains and prices) on which see below 3.2.1.1 and 3.2.6. See Guesmi & Samsó, 2018, no. [35], pp. 57–8 and 205–6.

59 Guesmi & Samsó, 2018, no. [35] (pp. 57–58, 205–6), [45] (pp. 67, 210), [51] (pp. 69, 211), [56] (pp. 70, 213), [62] (pp. 72, 214), [66] (pp. 74, 215), [70] (pp. 75, 216), [74] (pp. 76, 217), [78] (pp. 77, 218), [82] (pp. 78, 219), [86] (pp. 79, 220), [90] (pp. 80, 221), [95] (pp. 82, 222).

Cancer series mentions the presence, in the sign of Cancer, of *al-qahhārān* or *al-qāhirān* (the two powerful ones): this is probably a reference to Saturn and Mars which, according again to the tables of al-Khwārizmī, had a conjunction in Cancer (110°) the 15th September 1033.

- Saturn in Leo (6 vv), Virgo (8 vv) and Libra (6 vv): the passage of Saturn through Libra 21°, which is the degree of its exaltation, produces all kinds of disasters.
- Saturn in Scorpio (5 vv) and Sagittarius (8 vv): the sixth verse of the Sagittarius series seems to allude to an occultation of Saturn by the full Moon which was advancing towards the ascending node together with the planet. Saturn was actually covered by the Moon the 16th January 1045 towards 6 AM and the occultation was visible because both planets were over the horizon (their altitude was ca. 18° in Toledo) and the Sun rose at 7:37 AM.⁶⁰ Both planets were in Scorpio (243:30° according to al-Khwārizmī's *zīj*) but very near Sagittarius.
- Saturn in Capricorn (5 vv), Aquarius (10 vv) and Pisces (2 vv): the passage through Aquarius is the cause of great catastrophes, something justified by the fact that Aquarius is one of the domiciles of Saturn. It produces earthquakes and there was an earthquake in 415/1024–25,⁶¹ considered to be “the great Andalusian earthquake”, Saturn being in Aquarius or at the beginning of Pisces. Peasants will suffer hunger and all kinds of miseries which corresponds to the fact that the interval between 440/1048–49 and 450/1058–59 was a period of draught and famine.

Al-Baqqār ends with two other verses, dealing with different topics, and the whole gives the impression that he has not quoted in full Ibn al-Khayyāt's *Lāmiyya*.

3.1.1.6 The End of the *ṭawāʾif* Period and the Horoscope of ‘Abd Allāh b. Buluqqīn's Fourth Anniversary

The period ends with two tragical events: the conquest of Toledo in 1085 by Alfonso VI of Castile and the arrival of Yūsuf ibn Tāshufīn, leader of the Almoravids one year later. The last Muslim king of Toledo, al-Qādir b. Dhī l-Nūn left the city holding an astrolabe with which he was trying to establish the propitious moment to get out of Toledo. According to witnesses present there “he was surrounded by Christians and Muslims. The former were laughing at him, while the latter marvelled of his ignorance”.⁶²

60 These data have been obtained with the programme EZCosmos.

61 Ibn Abī Zarʿ, 1972, p. 118; Huici, 1964, I, p. 224. I owe this information to Jorge Lirola.

62 ‘Abbās, 1979, VII, p. 167.

In 1090 the Almoravids conquered Granada and deposed the last king of the Zirid dynasty ‘Abd Allāh b. Buluqqīn who was, then, exiled in Aghmāt, near Marrakech, where the king-poet of Seville al-Mu‘tamid was also brought one year later. During his exile, ‘Abd Allāh wrote an interesting book of memoirs, in which he tried to justify his own attitude in front of the Almoravids. Among his arguments he uses his own horoscope, cast on the occasion of his fourth anniversary.⁶³ The data he offers are the following ones:

Ascendant: Pisces 4° (334°)

Jupiter and Venus in the 11th house

Sun and Mercury in Aquarius

Saturn and Mars in Taurus, in the 3rd house

Moon in the 7th house

The only precise information is the longitude of the ascendant. With this and assuming that the latitude involved corresponds to Granada (37;30° in many sources), and that the obliquity of the ecliptic is 23;51°, we can establish the division of the houses⁶⁴ according to the standard method and with values rounded to the nearest degree:

I	334°
II	9°
III	44° (Saturn and Mars)
IV	75°
V	100°
VI	126°
VII	154° (Moon)
VIII	189°
IX	224°
X	255°
XI	280° (Jupiter and Venus)
XII	306° (Mercury and Sun)

63 Lévi-Provençal, 1955, p. 179; Lévi-Provençal & García Gómez, 1980, pp. 305 ff; Tibi, 1986, pp. 174 ff. An analysis of the horoscope in Samsó, 1990, although here I have introduced many corrections.

64 Using John North's programme HOROSC.

This allows us to give an estimation of the planetary longitudes:

Sun: 306°–334°
 Moon: 154°–189°
 Saturn: 44°–75°
 Jupiter: 280°–306°
 Mars: 44°–75°
 Venus: 280°–306°
 Mercury: 306°–334°

I obtain a reasonable fit for these estimations for the 26th January 1060 (JDN 2,108,248) at midday, although it is clear that the positions of Sun and Venus lie slightly out of the aforementioned limits. The longitudes have been calculated using the sidereal parameters of al-Khwārizmī's *zīj*.⁶⁵

Sun	304°
Moon	189°
Saturn	56°
Jupiter	288°
Mars	45°
Venus	277°
Mercury	320°

We can, now, calculate the approximate date of 'Abd Allāh's birth which, according to Ibn al-Khaṭīb, happened 35 years and 7 months (= 12,609 days) before his deposition on the 10th Rajab 483/ 8th September 1090 (JDN 2,119,431).⁶⁶ From this remark we can calculate a date of birth ca. 10 Dhū l-Ḥijja 447/ 1 March 1056 (JDN 2,106,822).⁶⁷ To calculate that date more precisely I use the excess of revolution in Maslama's revision of al-Khwārizmī's *zīj* which yields a sidereal year of 365;15,30,22,30 days:⁶⁸

$$365;15,30,22,30^d \times 4 = 1461;2,1,30^d$$

$$2,108,248 - 1461 = 2,106,787$$

65 In my previous paper on this horoscope (1990) I believed these values had been calculated with al-Battānī's *zīj*. I was not aware, at that time, of the importance of the tradition of sidereal astrology in al-Andalus and the Maghrib. The results obtained here are slightly different.

66 Lévi-Provençal, 1934, p. 270 (Rabat) or p. 236 (Beirut).

67 García Gómez and Lévi-Provençal deduced that he had been born towards the end of 447/1056.

68 Dalen, 1996, p. 210.

And JDN 2,106,787 corresponds to the 5 Dhū l-Qaʿda 447/26 January 1056. For this date we can find an ascendant of 334° at about 8;30^h AM.

3.1.1.7 The Maghrib and al-Andalus during the Almoravid and Almohad Empires: the Planetary Conjunction of 1186

In 1.2 and 1.3 I have given some scattered references to the small number of facts known about the practice of Astrology in the Maghrib between the 8th and the middle of the 11th centuries. This lack of references in historical sources and the fact that no written astrological works seem to be extant contrasts with the sudden apparition, in an unknown date after 1038, of Ibn Abī l-Rijāl's *Kitāb al-bāriʿ fī aḥkām al-nujūm* which is, without any doubt, the most important astrological handbook of the Islamic West, extremely influential in Europe through its Castilian and Latin translations. This book deserves an independent treatment and it will be dealt with extensively in 3.2.2.2. I would only like to advance here a couple of ideas: on the one hand, the fact that he quotes an extremely long list of Eastern sources and not a single Maghribī one may confirm that the previous Maghribī astrological tradition had not reached a level of maturity which deserved mentioning. On the other, the important role played by Ibn Abī l-Rijāl in the Zīrid court of Qayrawān, under Bādīs b. al-Manṣūr (r. 996–1016) and his son al-Muʿizz b. Bādīs (r. 1016–1062), and not only as an astrologer, enhances, once more, the importance of patronage for the development of science, in general, and especially of astrology.

The lack of patronage is probably the main cause of the decay of public astrological practices, both in al-Andalus and the Maghrib, during the Almoravid (ca. 1050–ca. 1147) and Almohad (ca. 1147–1276) periods.⁶⁹ In spite of this, Ibn al-Kammād (fl. 1116), Ibn al-Zarqālluh's disciple, wrote a book entitled *Kitāb maḥfātih al-asrār* ("Keys of secrets") of which only the chapters describing the animodar (an astrological technique which tries to calculate the beginning of the gestation of a new-born) are extant and were well known in the Maghrib (see 3.2.4).

Popular belief in astrology seemed, however, to remain. Pointing to this conclusion is an anecdote dated in 1185, one year after the death of Abū Yaʿqūb Yūsuf (r. 1163–1184), an Almohad Caliph who felt some interest in astronomy and astrology. According to al-Marrākushī⁷⁰ a document sent "by the people of Egypt", based on a source due to "the people of India", reached Málaga in 581/1185. It contained a prediction of a great catastrophe (*tūfān*), consisting

69 Forcada, 2015.

70 'Abbās, 1964, IV, pp. 210–214. On this passage see Forcada, 2000 b, p. 400, fn. 101; Comes, 2002, p. 129; Ibn Sharīfā, 1992, p. 185 fn.1 (where other sources referring the same anecdote are mentioned).

in cyclonic winds and earthquakes, which was going to take place in Monday 29th Jumādā 11 582/ 16.9.1186 and which would last between the 16th and the 18th of September. The cause of this event was a conjunction of the five planets in the sign of Libra.⁷¹ The people of India had already prepared themselves for the disaster by digging underground shelters. Such prediction produced a general panic and some adopted measures similar to those of the people of India. The *faqīh* Abū l-Ḥajjāj ibn al-Shaykh⁷² wrote a poem, in order to calm down the inhabitants of the city, using religious arguments against the belief in astrology and arguing, after the presumed date, that the predicted catastrophe had not taken place.

What is interesting in this anecdote is that it contains a partial truth. The aforementioned conjunction is referred in many Latin sources,⁷³ the earliest one due to an unknown “Magister Johannes David Toletanus”. The *Chronicle* written by Roger of Howden contains an horoscope of the conjunction, apparently calculated for the latitude of London and for the 16.9.1186 at six in the morning, in which the planetary longitudes seem to derive from the *Toledan Tables*. This event was felt as a serious menace to Christendom, as a 13th century source (Roger of Wendover) considered that the conjunction was related to Ṣalāḥ al-Dīn conquest of Jerusalem in 1187.

TABLE 3.1 Planetary longitudes for midday of the 16.9.1186

Planets	al-Khwārizmī	<i>Toledan tables</i>
Sun	169;35,23°	179;34,9°
Moon	188;19,17°	200;13,47°
Saturn	176;34,35°	184;27,12°
Jupiter	173;25,49°	182;1,16°
Mars	178;24,22°	190;7,41°
Venus	176;39,28°	184;28,52°
Mercury	181;18,42°	185;34,28°

71 This conjunction as well as the corresponding predictions (cyclonic winds, floods and, as a result, people digging underground shelters) is mentioned in many Eastern sources (ʿImād al-Dīn, al-Maqrīzī, Bar Hebraeus, Taghribirdī, al-Suyūṭī). See a translation and study of these passages in McPartlan, 1991, pp. 86–102.

72 See the entry by S. Peña in *BA* vol. 5, pp. 274–290.

73 Callatay, 2000.

Returning to al-Marrākushī's text, we do not know which were the Indian sources used but we can compute the planetary longitudes for midday of the 16.9.1186 using al-Khwārizmī's *zīj* (which has a clear Indian origin) and compare the results with those obtained with the *Toledan Tables*, keeping in mind that the former give sidereal positions, while the latter, corrected with a trepidation model, are tropical.

As the sign of Libra is comprised between 180° and 210°, it is obvious that not only the five planets but also the Sun and the Moon are in Libra, according to the *Toledan Tables*. The results obtained with al-Khwārizmī's *zīj* are not so tight-fitting but, in any case, all positions correspond to this sign or are near it. It seems, therefore, that the calculated positions were tropical and not sidereal, and this agrees with an Eastern Islamic source but not an Indian one, while it disagrees with the Andalusī-Maghribī tradition which prefers to cast sidereal horoscopes. This can be checked by using modern tables which give similar results,⁷⁴ and implies that, in the aforementioned date, all planets, seen from the Earth, seemed to be practically aligned, a phenomenon which is not very common but which sometimes occurs (a similar situation happened in May 2000 and was the object of comments in the media). The prediction of the catastrophe uses the term *tūfān*, commonly applied to the Flood, and seems to be related to the astrological belief that the Flood was announced by a conjunction of all planets in Aries 0°. According to Abū Ma'shar,⁷⁵ the Flood took place between the 17th and the 18th February of year – 3101, in a moment in which the five planets were placed between Pisces 27° and Aries 0°, in a situation similar to that of 1186.

3.1.1.8 Astrology in Naṣrid Granada and in the Maghrib between the Thirteenth and the Fifteenth Centuries: the Horoscopes of the Battle of El Salado (1340)

We have already seen in 1.4.3 a summary of the extant evidence about the practice of astrology in Naṣrid Granada, mainly related to the reign of Muḥammad V (1354–59 and 1362–91) and furnished by Ibn al-Khaṭīb's *Iḥāṭa*. One of these references – corresponding to the nativity horoscope of the sultan in 1339 – contains a small number of details which deserve a full translation of the Arabic text:⁷⁶

74 Tuckerman, 1964, p. 611.

75 Pingree, 1968, p. 35.

76 'Inān, 1973–77, vol. 11, p. 91; Puig, 1984, pp. 72 and 75.

His birth, in a propitious moment with a good prospect, took place approximately during the first third of the night of [Monday]⁷⁷ 22nd Jumādā II of year 739. This corresponds, in the Christian calendar, to the 4th of January⁷⁸ of year 1377 of the Spanish Era [1339 CE]. The computation of the astronomical equation, according to Claudius⁷⁹ Ptolemy, determined that the ascendant was in the sign of the Moon and that the Moon was governing the place of its opposition to the Sun, that took place before the birth. One can surmise that it occurred in the sixth hour and twenty three minutes of the night. The ascendant was in 15° 48' of Virgo.

In spite of its apparent precision, Ibn al-Khaṭīb's text is problematical beginning with the fact that we do not know whether the date is the 4th or the 5th of January. The ascendant (Virgo 15° 48') is in the second decan of Virgo, under the power of the Moon, and this explains, probably, Ibn al-Khaṭīb's remark that the ascendant was in the sign of the Moon. We can also understand the text when it says "that the Moon was governing the place of its opposition to the Sun" because, at this time of the year, the Sun is in Capricorn, in opposition to the Moon's domicile (Cancer). Besides, it seems clear that Ibn al-Khaṭīb is trying to calculate the exact moment of the birth of the sultan, using an horoscope – of which he transcribes only a small part – and establishes that it took place six hours and twenty three minutes after the beginning of the night. It is obvious that these hours and minutes are seasonal and not equal. 6;23 hours after the beginning of the night implies an hour near midnight and this contradicts Ibn al-Khaṭīb's previous reference to the birth taking place during the first third of the night. This forces us to correct the text.

The 4th January 1339 the Sun had a sidereal longitude of 281°.80 Operating with an obliquity of the ecliptic of 23;32,30° and a latitude of Granada of 37;30°, frequently quoted in medieval sources, we can easily compute that sunset took place that day at 4^h 43^m PM and that "six [seasonal] hours and twenty three minutes" after sunset corresponds to fifteen minutes after midnight.

77 The edited text has *laylat al-ithnayn wa l-ʿishrīn* (night of the 22nd), but, later in the text, we find a clear reference to the fact that it corresponded to a Monday. I understand, therefore, that one should supply an extra *al-ithnayn*.

78 As a matter of fact, the 22nd Jumādā II 739 corresponds to the 5th January 1339, which was a Thursday.

79 The consonantic ductus of the edition has QYMWḌ' which has no sense.

80 Calculated with Ibn Ishāq's parameters and the computer programme ISHAQSID prepared by Josep Casulleras. I am revising here my previous analysis of this horoscope in Samsó, 2011, pp. 410–412.

With these data we can calculate the ascendant⁸¹ obtaining 192;43,45°, which is very far from the 165;48° given by Ibn al-Khaṭīb. If we correct the text and use four (instead of six) hours and twenty three minutes after sunset, we will be near the “first third of the night” of our text and we will obtain an ascendant of 165;40° which fits nicely the 165;48° given by the source. It seems, therefore, that the nativity horoscope of Muḥammad v was well computed using a sideral *zīj*.

Also in 1.4.3 I have summarised what I know about the practice of astrology in Merinid Morocco during the 14th century, which is probably a result of a more or less hidden patronage of the activities of astrologers. Thus, in spite of the fact that the Merinid sultan Abū l-Ḥasan ‘Alī (1331–1351) rejected astrology,⁸² a serious astrological analysis of the outcome of the battle of El Salado (Faḥṣ Ṭarīf, 1340) was made by an astrologer of Constantine, Ibn ‘Azzūz (d. 1354), who was, perhaps, at the service of the sultan. Some twenty years later, a second astronomer with the same origin, Abū l-Ḥasan ‘Alī b. Abī ‘Alī, dedicated his *zīj* to another Merinid sultan, Abū Sālim Ibrāhīm al-Musta‘īn (1359–1361)⁸³ and a third astrologer, also from Constantine, Ibn Qunfudh (1339–1407) dedicated his commentary of Ibn Abī l-Rijāl’s astrological *urjūza* to Abū Bakr ibn Ghāzī ibn al-Kās, minister of the Merinid sultans Abū Fāris (1366–1372) and Abū Zayyān (1372–1374).

A certain number of astrological sources, produced in the 14th c. and in the beginning of the 15th, have been examined. I am referring to:

- Ibn ‘Azzūz’s *Kitāb al-fuṣūl fī jam‘ al-uṣūl* of which only the second *maqāla* is extant in two manuscripts of the Ḥasaniyya Library of Rabat (mss. 1110) and the National Library of Rabat (ms. D 2118). In this book, Ibn ‘Azzūz mentions two other astrological works of his which do not seem to be extant: the *Madkhal al-ṣinā’a ‘alā madhhab al-jamā’a* and the *Maqālat al-Zuḥal*.⁸⁴ Another lost work of Ibn ‘Azzūz is *al-Ḥurūz fī sharḥ al-rumūz*, which seemed to contain several horoscopes related to the fall of the Almohads as well as to the arrival to power of the Merinid dynasty. Finally, a *Risāla fī adwār al-nayyirayn*, not examined so far, has been preserved in the Šubayḥiyya Library of Salé (ms. 509/2).
- The aforementioned commentary by Ibn Qunfudh of Ibn Abī l-Rijāl’s astrological *urjūza*.

81 Using John North’s programme HOROSC.

82 Viguera, 1981, pp. 438–444; Viguera, 1977, pp. 361–366.

83 Kennedy & King, 1982.

84 Díaz-Fajardo, 2008; Díaz-Fajardo, 2010a; Díaz-Fajardo, 2012; Samsó, 1999 (p. 2 of the reprint). On Ibn ‘Azzūz see also Lamrabet, 2012, no. M159, pp. 187–8; Samsó, 1997.

- Ibn Mas‘ūd ibn Farmīja (possibly Firmījūh = Bermejo) (fl. 1372–1394), author of the *Ḥuṣūl al-maḳāsid wa l-āmāl min al-turuq wa l-fawā'id allatī tu'lamu min-hā mudad al-wulāt wa l-ummāl* (“How to reach your purposes and hopes, using the procedures and useful informations with which you can establish how long governors and authorities will stay in power”), already mentioned in 1.4.3.⁸⁵
- Abū ‘Abd Allāh al-Baqqār (fl. Fez 1411–1418), who wrote two astrological treatises: the *Kitāb al-amṭār wa l-as‘ār* (“On rains and prices”) dealing with meteorological astrology and the *Kitāb al-adwār fī tasyīr al-anwār*, which has an astronomical part and an analysis of astrological techniques (*tasyīr* and projection of rays).

The aforementioned books by Ibn ‘Azzūz and al-Baqqār have been partially or fully edited and studied by Díaz Fajardo,⁸⁶ Chedli Guesmi and Samsó,⁸⁷ while Ibn Qunfudh’s commentary has been the object of a critical edition of Marc Oliveras.⁸⁸ In the thematic part of this chapter these sources will be mentioned extensively when analysing the survival of the old “system of the crosses” as well as astrological techniques and meteorological astrology. Here I will limit myself to the study of the set of particularly interesting retrospective horoscopes related to the battle of El Salado, cast by Ibn ‘Azzūz and preserved in his *Kitāb al-fuṣūl fī jam‘ al-uṣūl*.⁸⁹

In his *Muwāfiq zīj*, Ibn ‘Azzūz states that modern astronomers have been able to establish that the positions of the planets calculated with the *zīj* of Ibn Ishāq (fl. 1193–1222) were in disagreement with observations. Ibn ‘Azzūz was himself able to establish the existence of such disagreements when he calculated the *tasyīr* of the significators of conjunctions, eclipses and their ascendants in order to determine the time of events of the past and, especially, the great battle of El Salado which took place in the days of the Merinid sultan Abū l-Ḥasan ‘Alī (1331–1351) in the month of Jumādā 1 of 741 H [7 Jumādā 1 741/ 30 October 1340]. The times calculated did not correspond to historical reality and Ibn ‘Azzūz blamed Ibn Ishāq’s tables for the mistake. Therefore, in order to correct the divergence, he made observations with an armillary sphere in 745/1344: as a result, he corrected Ibn Ishāq’s mean motion tables and cast

85 Herrera, 2001; On this author see also Lamrabet, 2012, no. 485 bis, pp. 160–161.

86 Díaz-Fajardo, 2008.

87 Guesmi & Samsó, 2018. On al-Baqqār see also Lamrabet, 2012, no. 505, pp. 171–172 and Díaz Fajardo, 2010; Díaz Fajardo, 2012–14. An edition and translation of the full text of the *Kitāb al-adwār* is now under preparation by Díaz-Fajardo.

88 Oliveras, 2012; Samsó, 2009. On Ibn Qunfudh see also Lamrabet, 2012, no. 494, pp. 162–163.

89 Samsó, 1999.

again the corresponding horoscopes. This time, he obtained adequate results, once he had made the necessary corrections to the mean motion parameters deducted from observations. He then compiled his new *Muwāfiq zīj*.

This is quite a peculiar example of an experimental method in which the data are obtained using astronomical and astrological criteria.⁹⁰ Unfortunately, the first set of unsuccessful horoscopes of the battle is not extant and it is not clear whether they were cast before or after the battle. We can only analyse the second collection of horoscopes which are clearly retrospective and have been preserved in ms. Ḥasaniyya 1110.

The *Kitāb al-fuṣūl* contains full details about two horoscopes (no. 1 and 2) and partial information about another two (no. 3 and 4):

Horoscope 1: solar-lunar opposition preceding the Saturn-Jupiter conjunction, with change to the triplicity of air (Gemini, Libra and Aquarius), in the sign of Libra, which took place in year 1305. He specifies that the place of the conjunction was Libra 16;4,31°. In my recomputation,⁹¹ the true sidereal conjunction took place on 2 Dec. 1305 at 9 PM. At that time we have (recomputation):

Saturn: 196;18,49°

Jupiter: 196;18,32°

Horoscope 2: spring equinox before the aforementioned conjunction (23 March 1305).

Horoscope 3: spring equinox of the year of the battle of El Salado (23 March 1340). The longitude of the ascendant (196;50°) and that of Mars (43°, recomp. 43;4,46°) are the only precise values given in the text.

Horoscope 4: spring equinox of year 1344 (24 March) preceding his observations with an armillary sphere. Here we have:

Ascendant: 207;45°

Saturn: 295° (recomp. 294;49,0°)

Mars: 73° (recomp. 72;48,24°)

Moon: 114° (recomp. 113;11,39°)

90 Mentionning experiment (*tajriba*) is quite common in sources related to Medicine, Astrology, Magic, interpretation of dreams and Alchemy. See Langermann, 2014.

91 I am using a computer programme in which the parameters of the *Muwāfiq zīj* had been introduced. This programme is the result of the successive efforts of E.S. Kennedy, Honorino Mielgo and Josep Casulleras.

The text adds to this horoscope a reference to the Saturn-Jupiter conjunction which took place in Aquarius 3° (recomputed $302;56,27^\circ$) the 25th February 1345.

Horoscopes 1 and 2 offer an unusual number of details about the computation of planetary longitudes. For each planet they give its sidereal true longitude but also the *wasaf* (mean longitude), the *markaz thānī* (second *markaz*, the distance of the centre of the epicycle from the corrected planetary apogee, as seen from the centre of the Earth) and the *ḥiṣṣa thāniya* or second anomaly (true anomaly measured from the true apogee of the epicycle).⁹² This has allowed me to ascertain details of the procedures used by our astrologer in his computations and has given us important information about the astronomical theories he follows. Thus, Ibn ‘Azzūz’ planetary apogees are sidereal and they are kept at a fixed distance from the solar sidereal apogee. This implies that their positions are displaced at a rate of $0;0,0,2,7,15^\circ$ per day, which corresponds, approximately, to the solar apogee’s own motion as established by Ibn al-Zarqālluh (d. 1100) (see 6.3.2). Besides, horoscope 2 gives the latitudes of the Moon, Saturn, Jupiter, Mars and Venus, though not Mercury’s.

These two horoscopes are carefully dated by Ibn ‘Azzūz in a peculiar way, by giving, in *abjad* notation, a series of numbers which correspond to the number of years, months, days, hours and minutes since epoch (beginning of the Hijra). Thus, in the case of horoscope 1, we find:

$$703 \text{ [years]} - 7 \text{ [months]} - 12 \text{ [days]} - 7 \text{ [hours]} - 58 \text{ [minutes]}$$

which corresponds to the 14th Sha‘bān 704 H./11th March 1305 (JDN 2197779). This agrees with the recomputation of the planetary positions with two corrections: the number of days should be 13, instead of 12 and the hour is apparently wrong: the best fit (in agreement with the lunar position) is 12:30 hours after midday.⁹³

The mean and true planetary longitudes in horoscope 1 appear in Table 3.2, where we can see that the positions have been very carefully computed with an approximation to seconds:

92 See the details of the computation in Samsó, 1999, pp. 24–25.

93 It seems that $7;58^h$ is a copying mistake. The copyist is giving here the hour and minute of horoscope 2 ($7;53^h$) as 58 and 53 are very easily confused in *abjad* notation.

TABLE 3.2 True planetary longitudes in horoscope 1 (11.3.1305)

Planet	Mean sidereal longitude		True sidereal longitude	
	Text	Recomp.	Text	Recomp.
Sun	346;31,45°	346;32,59°	348;23,45°	348;24,58°
Moon	173;10,53°	173;26,48°	168;23,45°	168;39,14°
Saturn	177;26,54°	177;31,58°	184;6,22°	185;8,35°
Jupiter	168;20,30°	168;20,32°	167;56,?°	167;56,16°
Mars	194;2,52°	194;3,55°	208;32,21°	208;36,2°
Venus	346;31,45°	346;32,59°	8;37,56°	8;39,53°
Mercury	346;31,45°	346;32,59°	340;31,57°	340;39,55°
Asc. node	2[3]2;50 ^{oa}	232;55,2°	–	–

a 222;50° in the manuscript. The correction is obvious, for the longitude of the descending node is 52;50°.

As for horoscope 2, the date and hour is given according to the Hijra and Alexander’s era (Seleucid calendar):

703 [years] – 7 [months] – 26 [days, read 25] – 7 [hours] – 10 [minutes]

1615 [years] – ? [months] – 23 [days, read 22] – 7 [hours] – 53 [minutes]

The first date corresponds to the 26th of Shaʿbān of 704 H., and the second to the 23rd Adhār (which implies that the number of complete months was 5) of 1616 of the Seleucid era and to the 23rd of March of 1305 CE (JDN 2197791). As in horoscope 1 the number of days has an error of 1 day. The two dates agree on the hour (7) though not on the minute, and 53^m seems a better reading. The agreement is fairly good with the recomputation for the aforementioned date at 8 hours after midday (Table 3.3).

In horoscopes 1 and 2 the houses are divided according to the standard method. The beginnings of houses I–VI in both horoscopes appear in Table 3.4, where the differences between the text and the recomputed values appear in parentheses:

TABLE 3.3 Planetary positions in horoscope 2 (23.3.1305)

Planet	Mean sidereal longitude		True sidereal longitude	
	Text	Recomp.	Text	Recomp.
Sun	358;9,59°	358;10,18°	0;0,1°	0;0,18°
Moon	328;45,16°	328;49,4°	334;14,58°	334;19,19°
Saturn	177;55,[3]8°	177;55,38°	184;6,22°	184;13,44°
Jupiter	169;19,23°	169;19,19°	166;27,9°	166;26,31°
Mars	200;14,30°	200;14,40°	208;26,11°	207;36,2°
Venus	358;9,59°	358;10,18°	23;7,23°	23;7,17°
Mercury	358;9,59°	358;10,18°	2;59,15°	2;54,53°
Asc. node	232;17°	232;17,32°	–	–

TABLE 3.4 Beginning of the houses in horoscopes 1 and 2

Houses	Horoscope 1	Horoscope 2
I	252;23°	210°
II	2[85];38 ^{oa} (0;0,53°)	242;32° (–0;31,8°)
III	321;38° (0;1,41°)	273;14° (–0;50°)
IV	358;56° (–0;4,43°)	305;17° (–0;2,27°)
V	25;22° (0;29,25°)	332;12° (–0;14,30°)
VI	51;[18] ^{ob} (1;51,46°)	1;52° (0;45,56°)

- a 291;38° in the manuscript. The longitude of house VIII is 105;17° which I suggest to replace by 105;38°.
- b 51;29° in the manuscript. The longitude of house XII is 231;18°.

By using John North’s HOROSC programme it is easy to see that horoscope 1 was calculated for a latitude comprised between 36;44,58° and 37;12,27°, the mean value being 37;3,40°. My guess is that this horoscope was cast for the latitude of Granada, 37;10° established by Ibn al-Raqqām, whose work was known to Ibn ‘Azzūz: he copies from Ibn al-Raqqām a table of oblique ascensions for this city calculated for the aforementioned latitude. As for horoscope 2, the latitude implied lies between 39;6,45° and 40;3,53° (the mean value being 39;46,50°). My guess is, here, that this second horoscope was calculated for the latitude of Toledo (39;54° in the canons of the *Toledan Tables*).

We have, therefore, two horoscopes related to the same battle between King Alfonso XI of Castile and Yūsuf I, the Naṣrid sultan of Granada, who had obtained the help of Abū l-Ḥasan ‘Alī, sultan of the Banū Marīn. There is a certain logic in realizing that Ibn ‘Azzūz has cast horoscope 1 for Granada, the capital of the Banū Naṣr, and horoscope 2 for Toledo, which he undoubtedly considers to be the capital of the Castilian king. These two are not, however, the only latitudes used by Ibn ‘Azzūz in his analysis of the situation. Using horoscope 2 as his radical horoscope, he progresses (*tasyīr*) its ascendant according to four different kinds of *tasyīr* (see 3.2.3.2), one of which is the “middle cycle” (*al-dawr al-awsaṭ*) which advances 5° per year and $0;25^\circ$ per month. On the basis of a difference in time of 35 solar years and 7 months between 23rd March 1305 and 30th October 1340 (the date of the battle) and an ascendant 210° in the radical horoscope, he operates:

$$5^\circ \times 35 + 0;25^\circ \times 7 = 177;55^\circ$$

which he rounds to 178° .

Ibn ‘Azzūz adds to this that we should progress (*tasyīr*) the degree of the ascendant expressed in ascensions: if we take the oblique ascension of 210° , add to it 178° (the middle cycle *tasyīr*) and subtract 360° from the result, we will obtain an arc of *tasyīr* of $34;6,19^\circ$. This implies that the oblique ascension of 210° is $216;6,19^\circ$. Using this value and an obliquity of the ecliptic of $23;33^\circ$, we can easily calculate that the latitude implied is $35;1,22^\circ$. This result can be checked because Ibn ‘Azzūz seems to obtain the inverse oblique ascension of $34;6,19^\circ$:

$$\alpha_\varphi^{-1}(34;6,19^\circ) = 49;6,28^\circ$$

which, again, can be used to obtain a latitude of $34;23,23^\circ$, not far from the 35° that we have previously found. It is obvious that this latitude does not correspond to Granada or Toledo and I suspect that he might be using the latitude of the place of the battle which took place near Tarifa, slightly to the south of Algeciras, for which Arabic sources give latitudes comprised between $35;50^\circ$ and $36;30^\circ$.⁹⁴

3.1.2 *The Practice of Astrology in the Christian Kingdoms of the Iberian Peninsula*

3.1.2.1 The Introduction of Eastern Islamic Astrological Materials

3.1.2.1.1 *The Latin Translators of the First Half of the Twelfth Century*⁹⁵

Most Latin translations of Arabic astrological works made in the Christian kingdoms of the Iberian Peninsula were the result of the activity of a few translators who were not resident in Toledo but in Barcelona, Tarazona, Tudela and other locations situated on the banks of the river Ebro as well as in the North of Portugal and other indeterminate places including, perhaps, Toledo. This implies that Toledo, which was to become the main centre of the Arabic-Latin translations in the second half of the 12th century, seems to be less important during this early period, during which most astrological sources were translated.⁹⁶

Our sources are only dated in a few cases. In this respect, the great number of dates given by Plato of Tivoli (active in Barcelona between 1133 and 1145) is absolutely exceptional:⁹⁷ he often gives the date for the completion of one of his translations using the Julian and Hijra calendars or, even, the Seleucid calendar and Alexander's era. The list of recorded dates of his works is the following one:

- ‘Alī b. Aḥmad al-‘Imrānī's *Kitāb al-ikhtiyārāt* (*Liber de electionibus horarum*): 1124, 1131, 1133 or 1134.⁹⁸ This translation is, however, ascribed to Abraham bar Ḥiyya without any mention of Plato of Tivoli. This poses the problem of ascertaining whether these two authors really collaborated, as has often been assumed.⁹⁹

95 On the importance of this period for the transmission of astrological sources see Juste, 2011, pp. 30–32.

96 Another contemporary of this group of translators is Adelard of Bath (fl. ca. 1100–1150) who will not be taken into consideration here, because he does not seem to have ever been in the Iberian Peninsula, in spite of the fact that he used Andalusian sources (such as Maslama's revision of al-Khwārizmī's *zīj*) which reached him probably through Petrus Alfonsi. See Burnett, 1987 and 1990; Mantas, 2014, pp. 195–208.

97 See Samsó, 2004c, pp. 271–275.

98 The manuscripts give different readings of the date of the translation and these readings seem to be corrupt: see Samsó, 2004c, pp. 271–272. David Juste tells me that the actual date is probably 25 September 1133, obtained from a horoscope extant in two manuscripts.

99 With the exception of al-‘Imrānī's *Liber de electionibus horarum*, all the other translations only mention the name of Plato of Tivoli. In the *Liber embadorum*, a treatise on practical geometry, the incipit states that Bar Ḥiyya was the author of the original Hebrew source and Plato of Tivoli its translator into Latin (“Incipit liber embadorum a Savasorda in hebraico compositus et a Platone Tiburtino in latinum sermonem translatus”). The explicit repeats the same information: cf. Curtze, 1902, pp. 10 and 182. The question is: did Plato

- The *Kitāb al-mawālīd*, on nativities, by Abū ‘Alī al-Khayyāt (fl. ca. 854),¹⁰⁰ translated in Barcelona with date “annis arabum 530 14^a die mensis scrul [sic] anno vero Alexandrino 1447 15 die mensis Iulius” (14th Shawwāl 530, 16th Tammūz 1447 = 16th July 1136).
 - *Iudicia Almansoris*.¹⁰¹ This work is dated “in civitate Barchinona anno arabum 530 18 die mensis dulkigida [var. dialkiada]”. The date is 18th of Dhū l-Qa‘da 530/ 18th August 1136.
 - The pseudo-Ptolemaic *Karpós/ Fructus/ Centiloquium* with the commentary of Aḥmad b. Yūsuf b. al-Dāya (*Centiloquium Ptolemei cum expositione Aly translatum a platone Tiburtino de arabico in latinum*): 1136.¹⁰²
 - Ptolemy’s *Quadripartitum* dated in Barcelona “die veneris hora tertia xx die mensis octobris anno Domini 1138, xv die mensis saphar anno arabum 533”. The 15th Šafar 533 (Saturday) corresponds to the 22nd October 1138.
- We also know the date of Plato’s translation of Abraham bar Ḥiyya’s *Liber embadorum* (a treatise on practical geometry): 15th Šafar 540H/ 7th August 1145 (see below 3.1.2.1.2.1).

As for the rest of the translators, we have the following information:

Hugo Sanctelliensis¹⁰³ was (alive in 1145) and most of his translations are dedicated to the bishop Michael of Tarazona who held this position between 1119 and 1151.

Hermann of Carinthia,¹⁰⁴ fl. 1138–1143 probably in Tudela and other places on the banks of the river Ebro and in Southern France (Béziers, Toulouse): three of his translations are dated, namely Sahl b. Bishr’s *Fatidica* (1138), Abū Ma‘shar’s *Introductorium maius* (1140) and Ptolemy’s *Planispherium* (1143).

Robert of Kettene/Ketton (fl. 1141–1157) worked in collaboration with Hermann of Carinthia during the period of his astrological translations.

Johannes Hispalensis (fl. ca. 1120–1135)¹⁰⁵ worked in the North-West of the Iberian Peninsula and, perhaps, in Toledo. His translation of al-Farghānī is dated 11th March 1135 in Limia/Luna, while Qusṭā b. Lūqā’s *De differentia spiritus et animae*, dedicated to the archbishop of Toledo Raymond of Sauvetat

of Tivoli know enough Arabic to be able to make independent translations? Did he also know Hebrew to undertake the translation of Bar Ḥiyya’s treatise on practical geometry?.

100 Carmody, 1956, pp. 49–51.

101 See the edition and French translation by Vadet, 1963.

102 The attribution of this translation to Plato has been discussed: see Haskins, 1924, pp. 68–70; Carmody, 1956, pp. 74–75; Lemay, 1978.

103 See the list of translations in Burnett & Pingree, 1997, pp. 1–2. Updated list in Burnett, 2006, pp. 114–116. See also Santoyo 2016.

104 List of works in Haskins, 1924, pp. 43–66; Burnett, 1978; Burnett, 1982, pp. 7–8. See also Burnett, 1977, pp. 62–108.

105 Burnett, 1995; Burnett, 2002; Lemay, 1996 IV, pp. 307–309; Robinson, 2007; Hasse, 2016.

(1125–1152), was made before 1143. Besides, al-Qabīṣī's *Introductorius* was made before 1135 and our author may be the Johannes Toletanus who translated Abū 'Alī's *De nativitatibus* in 1152–1153.

3.1.2.1.1.1 The Works Translated

It is interesting to give here a provisional list of the sources translated by our authors because it contains information about the kind of astrological books which circulated in al-Andalus and were available to the translators.¹⁰⁶ It shows that the date of the original works does not exceed ca. 950 in agreement with what I have suggested in 1.5.1.2.1. Besides, we can easily see that the list includes general introductions to astrology (the *Tetrabiblos*, Abū Ma'shar and al-Qabīṣī's great handbooks), treatises on world astrology (Abū Ma'shar, Māshā'allāh, Sahl b. Bishr), meteorological astrology (*Liber imbrium*), nativities (Abū Ma'shar, 'Umar ibn Farrukhān, Māshā'allāh, al-Khayyāt, Ibn al-Khaṣīb), anniversaries of nativities (Abū Ma'shar), interrogations ('Umar ibn Farrukhān, Sahl b. Bishr, al-Kindī), and elections (al-'Imrānī, Sahl b. Bishr). On the whole, the list shows that all branches of astrology have attracted the attention of this group of translators. Interestingly, not a single Andalusian source is translated during this period: these sources will not be translated until the reign of Alfonso x in the thirteen century.

A few of these scholars did not limit themselves to translate but assimilated the contents of the sources available to them and compiled works of synthesis and original books. The most relevant case is that of Hermann of Carinthia's *De essentiis* which shows, for example, that the author knew Abū Ma'shar's *Kitāb al-Ulūf*, which had not been translated. Hermann also wrote the *De indagacione cordis* or *De occultis*, a work summarising ideas of Arabic sources on how to find lost treasures.¹⁰⁷ This book, like the *Liber imbrium* also authored by Hermann, can be considered as being midway between translations and original works.¹⁰⁸ In this work, Hermann mentions *Jafar Indus*, who does not seem to be Abū Ma'shar Ja'far¹⁰⁹ and Hermann's book seems to be narrowly related to Hugo Sanctallensis' *Liber imbrium ab antiquo Indorum astrologo nomine Iafar editus*.¹¹⁰ Within this group we should introduce the *Epitome totius astrologie* wrongly attributed to Johannes Hispalensis (1142).

106 A general survey of astrological Latin translations with an evaluation of their influence (based on the number of extant manuscripts) in Juste, 2016.

107 Lemay, 1996, vii, 189; Burnett, 1987a, p. 17.

108 Burnett, 1987a, pp. 16–17.

109 Burnett, 2004, p. 64.

110 Haskins, 1924, p. 77; edition in Burnett, 2004, pp. 63–116.

3.1.2.1.1.1 *Ptolemy and Pseudo-Ptolemy*

- Ptolemy's *Tetrabiblos* or *Quadripartitum*: translated by Plato of Tivoli in 1138. Another translation was made for Alfonso X by Egidius de Thebaldis of Parma. There is a third anonymous translation dated 29 August 1206. There are other two medieval translations from Arabic: one made in Toledo in 1234¹¹¹ and the other sometime before ca. 1250.¹¹²
- The pseudo-Ptolemaic *Karpós*, known in Latin as *Fructus* (from the Arabic *Kitāb al-Thamara*) or *Centiloquium*, was translated by Plato of Tivoli in 1136, together with the commentary of Abū Ja'far Aḥmad b. Yūsuf b. al-Dāya (d. ca. 944) who, according to Lemay, was the real author of the original work.¹¹³ Plato's translation contains two appendices one of which explains the procedure of the animodar, which consists in determining the moment of conception of a newborn (see 3.2.4). Another translation was made by Hermann of Carinthia¹¹⁴ and an incomplete version of the same work is due to Adelard of Bath.¹¹⁵ A fourth translation, dedicated to bishop Michael of Tarazona, was made by Hugo Sanctelliensis.¹¹⁶ There is a fifth twelfth century translation which might be by Johannes Hispalensis.¹¹⁷

3.1.2.1.1.2 *Abū Ma'shar (787–886)*

- Abū Ma'shar's *Liber introductorii maioris ad scientiam iudiciorum astrorum*¹¹⁸ was translated ca. 1133 by Johannes Hispalensis¹¹⁹ and, again, in 1140, by Hermann of Carinthia (*Introductorium maius*).¹²⁰ According to Lemay,¹²¹ Hermann knew the translation of Johannes Hispalensis and considered it to be too literal.
- Abū Ma'shar's *De magnis coniunctionibus* probably translated by Johannes Hispalensis.¹²²
- Abū Ma'shar On nativities (not extant)¹²³ probably translated by Hugo Sanctelliensis.

¹¹¹ Carmody, 1956, p. 18 no. 10d; Thorndike, 1964.

¹¹² Haskins, 1924, p. 111 n. 163; Carmody, 1956, p. 18 no. 10c.

¹¹³ Lemay, 1978.

¹¹⁴ Haskins, 1924, p. 51.

¹¹⁵ Burnett, 1987b, p. 166; Burnett, Yamamoto & Yano, 1994, p. 2.

¹¹⁶ Haskins, 1924, pp. 69–71.

¹¹⁷ Lemay, 1996, vol. IV, pp. 90–92.

¹¹⁸ Critical edition by Lemay, 1996, vols. IV–VI.

¹¹⁹ Burnett, 2002, pp. 61–62.

¹²⁰ Lemay, 1996, vols. VII and VIII. See also Burnett, 1987a, pp. 14–16.

¹²¹ Lemay, 1996, VII, 177 and 192.

¹²² Burnett & Yamamoto, 2000.

¹²³ Haskins, 1924, p. 76. Not mentioned by Burnett.

- Abū Ma'shar's *De revolutione annorum nativitatum* (*Taḥāwīl sinī al-mawālīd*) was partially translated by Johannes Hispalensis.¹²⁴ It is mentioned in Hermann of Carinthia's *De essentiis*¹²⁵ in which we also find quotations of Abū Ma'shar's *Kitāb al-Ulūf*.
- Abū Ma'shar's *Liber experimentorum* or *Flores*¹²⁶ have been attributed to Johannes Hispalensis.¹²⁷
- *Kitāb taḥāwīl sinī al-'ālam* = *De revolutionibus annorum mundi* translated by Johannes Hispalensis.
- The *Mudhakarāt fī 'ilm al-nujūm* written by Abū Ma'shar's disciple Abū Sa'īd Shādhān b. Bahr circulated, in a lost Latin translation, in Spain in the 12th c. Abraham ibn 'Ezra quotes this work in the *Sefer ha-'Olam*.¹²⁸

3.1.2.1.1.1.3 'Umar ibn al-Farrukhān al-Ṭabarī (d. ca. 815)

- *Liber Aomaris abin Farchan Tyberiadis*, a translation of the *Kitāb mukhtaṣar al-mas'āl* by Hugo Sanctelliensis, included in the collection known as *Liber trium iudicum*. This work was also translated by Salio of Padua in 1217.¹²⁹
- *De nativitatibus* translated by Johannes Hispalensis. Most manuscripts of this work append five short chapters which are commonly known as *Liber universus*.¹³⁰ the original source of this appendix dates from the 10th century.

3.1.2.1.1.1.4 Māshā'allāh (d. ca. 815)

- *Liber Messehale de nativitatibus* (*Kitāb al-mawālīd al-kabīr*)¹³¹ translated by Hugo Sanctelliensis.
- *De rebus eclipsium* or *In radicibus revolutionum* translated by Johannes Hispalensis.
- *Liber receptionis*, also known as *De receptione planetarum* and *De interrogationibus*, translated by Johannes Hispalensis.
- *Liber Aristotilis De ducentis LV Indorum voluminibus universalium questionum tam geneticalium quam circularium summam continens*.¹³² The 255 books mentioned in the title are, actually, some 125 and the list derives from an astrological bibliography compiled by Māshā'allāh and extant in a Byzantine

¹²⁴ Carmody, 1956, p. 94 no. 4b; Burnett, 1978, pp. 127–129; and Burnett, 2009a, pp. 22–29.

¹²⁵ Burnett, 1982, pp. 166–167.

¹²⁶ Identified by Juan Vernet with Abū Ma'shar's *Kitāb al-nukat*: see Vernet, 1979.

¹²⁷ Burnett, 2002.

¹²⁸ Burnett, 2005.

¹²⁹ Recent discussion in Burnett, 2011.

¹³⁰ Pingree, 1977. Lemay, 1996, IV, pp. 205–206.

¹³¹ Haskins, 1924, p. 76.

¹³² Haskins, 1924, pp. 74–76; edited by Burnett and Pingree, 1997. See also Burnett, 1987a, pp. 12–13.

translation. The rest of the book is, probably also, a translation of an original work by Māshā'allāh.¹³³

- *De cogitatione* (or *De interpretatione cogitationis* or *De intentione*), translated by Johannes Hispalensis.¹³⁴
- *Liber interpretationum* (or *De inventione occultorum*), translated by Johannes Hispalensis.¹³⁵
- *De revolutionibus annorum mundi* translated by Johannes Hispalensis.¹³⁶

3.1.2.1.1.1.5 *Sahl b. Bishr* (fl. 821–850)

- The most important and popular work of Sahl is *Liber iudiciorum*, in five parts –hence *Liber sextus* given to *Fatidica* below –, namely *Introductorium*, 50 *precepta*, *De interrogationibus*, *De electionibus* and *Liber temporum*, most probably translated by Johannes Hispalensis.¹³⁷ Another translation of the section *De interrogationibus* (*Liber Zael de iudiciis .lvi capitulis distinctus = Kitāb al-aḥkām 'alā l-nisba al-falakiyya*) is possibly due to Hermann of Carinthia and it was included in the collection called *Liber trium iudicum*.
- *Fatidica* or *Pronostica*, also known as *Liber sextus astronomie* was translated in 1138 by Hermann of Carinthia (*Kitāb taḥāwīl sinī al-'ālam*).

3.1.2.1.1.1.6 *Abū 'Alī al-Khayyāt* (fl. ca. 854)

- *De nativitatibus* (*Kitāb al-mawālīd*), on nativities,¹³⁸ translated by Plato of Tivoli in 1136 and, again by Iohannes Toletanus in 1152–1153 (Johannes Hispalensis?).

3.1.2.1.1.1.7 *al-Kindī* (d. ca. 873)

- *Liber Alkindi De iudiciis .xlvi capitulis discretus* – a translation of al-Kindī's *Kitāb fī madkhal ilā 'ilm al-nujūm/ Risāla fī madkhal al-aḥkām 'alā l-masā'il/ al-Madkhal ilā 'ilm aḥkām al-nujūm* also known as *Arba'ūna bāban* – by Hugo Sanctelliensis and included in the *Liber trium iudicum*. The same book – under the title of *Iudicia Alkindi astrologi* – was also translated reluctantly by Robert of Kettene, because his real interest was in geometry and astronomy.¹³⁹

133 The attribution to Māshā'allāh has been discarded by Dykes, 2009 (information given by David Juste).

134 Carmody, 1956, pp. 28–29 no. 5; Thorndike, 1956, pp. 53–54.

135 Carmody, 1956, pp. 33–35 nos. 9–10; Thorndike, 1956, pp. 54–56.

136 Hasse, 2016.

137 Carmody, 1956, pp. 40–41; Juste, 2011, p. 31 n. 20.

138 Carmody, 1956, pp. 49–51.

139 Al-Kindī's *Iudicia* have been analysed by Burnett, 1993, where (pp. 95–97) he justifies his attribution of the collection of the *Liber trium iudicum* to Hugo Sanctallensis; see also Burnett, Yamamoto and Yano, 1997 (edition of chapter 35 of Robert's translation). An

3.1.2.1.1.1.8 *Abū Bakr al-Ḥasan Ibn al-Khaṣīb al-Kūfī (Alive in 845)*

- Alheacib Alcufr on nativities,¹⁴⁰ translated by Hugo Sanctelliensis. This work does not survive but we do have a translation (1218) of that text by Salio of Padua.

3.1.2.1.1.1.9 *al-Qabīṣī (fl. ca. 950)*

- *Introductorius*¹⁴¹ translated by Johannes Hispalensis before 1135.

3.1.2.1.1.1.10 *Alī b. Aḥmad al-ʿImrānī (d. 955)*

- *Liber de electionibus horarum Haly filii Achamet Ebreny* translated in 1133 or 1134 by Abraham bar Ḥiyya¹⁴² = *Kitāb ikhtiyārāt* of ʿAlī b. Aḥmad al-ʿImrānī (d. 955). This treatise on elections says that astrologers should not predict the future of a newborn from his nativity horoscope, but from the horoscope of the conception, determined by the use of the animodar (see 3.2.4) and that the moment of conception should be chosen carefully using astrological criteria.

3.1.2.1.1.1.11 *Unidentified Sources*

- *Iudicia Almansoris/ Centiloquium/ Capitula stellarum/ Centum (or Centum-quinquaginta) propositiones*,¹⁴³ translated by Plato of Tivoli in 1136. This text contains a collection of 150 (or 164) aphorisms. There have been several unsuccessful attempts to identify its Arabic source¹⁴⁴ but the identification proposed by Fuat Sezgin (al-Isrāʾīlī, the Jewish astrologer of the Fatimid Caliph al-Ḥākim al-Manṣūr, r. 996–1021)¹⁴⁵ has recently been confirmed by François Charette, who has a working edition of the Arabic and Latin texts.
- In an unknown date Plato translated a *De nativitate revolutionibus (taḥāwīl sinī al-mawālīd)* by Alkasem filius Alchasith, identified by Carmody¹⁴⁶ with Abū Bakr al-Ḥasan ibn al-Khaṣīb. This identification is problematic because one of the sources mentioned in the text is al-Qabīṣī, who lived in the second half of the 10th c.

edition of the two Latin translations (by Hugo and Robert of Ketene) in Burnett, 1993a. See also Burnett, 1987a, pp. 11–12.

140 Haskins, 1924, pp. 76–77. Not mentioned by Burnett.

141 Burnett, Yamamoto & Yano, 2004.

142 The first chapters were edited by Millás, 1942, pp. 328–339; Carmody, 1956, pp. 137–139.

143 See the edition and French translation by Vadet, 1963.

144 See for example Carmody, 1956, pp. 132–134; Sarton, 1931, pp. 177–179; Millás, 1942, pp. 155, 175; Vadet, 1963, pp. 44–46.

145 Sezgin, *GAS* VII, pp. 175–176.

146 Carmody, 1956, p. 137.

3.1.2.1.1.2 The Translators as Active Astrologers: Horoscopes of the Translations
 The interest on astrology shown by this group of translators may have led some of them to become active astrologers and cast horoscopes in order to find the propitious moment for beginning or finishing a translation.¹⁴⁷ Plato of Tivoli is a good example, although the planetary positions given in the text could only be the result of his attempt to fix the chronology of his translations and not part of an horoscope. In the *Iudicia Almansoris*, he gives the following details of the moment in which he completes his work:

in civitate Barchinona anno arabum 530 18 die mensis dulkigida [var. dialkiada] Sole in Virgine 1° 5' Luna in Ariete 15° 16'.

The date is 18th of Dhū l-Qa'da 530/ 18th August 1136. The longitudes of the Sun and the Moon seem to be what is left of a complete horoscope in which the planetary positions had been calculated with al-Battānī's *zīj*, whose canons were translated by Plato. The positions calculated with al-Battānī's parameters for the 17th August 1136 at 2^h PM, are:

Sun 151;2,26°

Moon 15;29,48°.

Plato's explicit of the *Liber embadorum*,¹⁴⁸ is more elaborate:

Finit liber embadorum a Savasorda Iudaeo in hebraico compositus et a Platone Tiburtino in latinum sermonem translatus anno Arabum DX mense saphar, die XV eiusdem mensis, hora tertia
 Sole in XX gradu et XV minuto Leonis
 Luna in XII gradu et XX minuto Piscium
 Saturno in VIII gradu et LVII minuto Tauri
 Iove in Arietis XXVI gradu et LII minuto
 Marte in Libra XXVII.XV
 Venere in Libra II.XXVIII
 Mercurio in Leone XIII.XLV
 Capite in Cancro 0.1
 Cauda in Capricorno 0.1

¹⁴⁷ David Juste calls my attention on two other examples: two manuscripts contain the horoscope of the translation of al-Imrānī's *Kitāb al-ikhtiyārāt* by Abraham bar Ḥiyya (25 September 1133); the second one appears in the translation of Ptolemy's *Quadripartitum* made in Toledo in 1234; see <https://ptolemaeus.badw.de/work/33>.

¹⁴⁸ Curtze, 1902, p. 182.

A long time ago Haskins¹⁴⁹ established that the Hijra year cannot be correct and that it should be 540/1145 instead of 510/1116, because the planetary positions correspond to the 15th Šafar 540/ 7th August 1145. Recomputation proves that Haskins was right and confirms Nallino's hypothesis who stated that Plato of Tivoli used al-Battānī's *zīj* to calculate the aforementioned planetary longitudes.¹⁵⁰

My recomputation is for the 6th August 1145 (astronomical date). As for the hour, the *hora tertia* corresponds to three seasonal hours after sunrise or before midday. The solar longitude being 140;15°, three seasonal hours correspond to 3;33^h before midday, approximately 8;30^h in the morning. Al-Battānī's mean motion tables are calculated for the meridian of Raqqa, in Northern Syria, whose longitude, according to al-Battānī, is 73;15° from the Western meridian. This source does not give the longitude of Barcelona, but it contains a longitude of 9;20° for Hispania Betica, which is the longitude of Córdoba in many Islamic sources. As we can assume a longitude difference of some 7° between Córdoba and Barcelona, the difference in longitude between Raqqa and Barcelona will be 73;15° – 9;20° + 7° = 70;55°, equivalent to a time difference of 4;45^h. The *hora tertia* of our text will begin, therefore 1;15^h after midday at Raqqa. Table 3.5 gives the planetary longitudes calculated with al-Battānī's parameters for the midday of Raqqa and for one and two hours after midday. The agreement obtained is good enough and shows that our translator was competent in the use of al-Battānī's *zīj*:

TABLE 3.5 Planetary positions for the 6.8.1145

Planet	Text	Rec. 6.8.1145 midday (Raqqa) ^a	Rec. 6.8.1145 1h PM	Rec. 6.8.1145 2h PM
Sun	140;15°	140;9°	140;11°	140;14°
Moon	342;20°	342;27°	342;59°	343;32°
Saturn	38;57°	38;59°	38;59°	38;59°
Jupiter	26;52°	27;38°	27;38°	27;38°
Mars	207;15°	206;12°	206;14°	206;16°
Venus	182;29°	182;29°	182;32°	182;35°
Mercury	134;45°	134;32°	134;36°	134;41°
Asc. node	90;1°	90;1	90;1°	90;1
Desc. node	270;1°	270;1°	270;1°	270;1

a See an almost identical recomputation in Mercier, 1987, p. 112.

149 Haskins, 1924, p. 11.

150 Nallino, 1899–1907, I, p. 55.

3.1.2.1.2 *Transmission through Hebrew Authors: Abraham bar Ḥiyya and Abraham ibn Ezra*

The first half of the twelfth century is also a period of time in which a new cultural phenomenon takes place: the use of Hebrew as a language of science. Until then, in al-Andalus, Arabic was the language used in science, while Hebrew was only used in religious studies, liturgy and poetry. We can find a good example in the figure of Maimonides (1135–1204) who wrote all his medical works in Arabic and used Hebrew in his great biblical commentary (*Mishneh Torah*). This, in spite of the fact (see 2.3) that book 3 of the work deals mainly with astronomy.

As the Christian conquest advanced (mainly after the fall of Toledo in 1085), Muslim scholars migrated towards Southern Muslim lands in the Peninsula or the Maghrib, while Jewish scholars did not see any advantage in following them and stayed in Christian lands where they could profit of their knowledge of the Arabic language. Some Jews played an important role in the process of transmission of Arabic science to the Latin world. Such is the case of Mosheh Sefardi/ Petrus Alfonsus (fl. 1062), of Avendauth/ Abraham ibn Dāwūd (fl. ca. 1148–1180),¹⁵¹ probably of Abraham bar Ḥiyya (ca. 1070–ca. 1145) and, in the 13th c., of the Jewish collaborators of king Alfonso x.

Abraham bar Ḥiyya also launched the process of using Hebrew as a scientific language by writing a series of mathematical and astronomical works the purpose of which was to introduce the new Arabic science in the Jewish European community who had no access to the original Arabic works. This appears clearly in his own words in the presentation of his book *Yesodeh ha-tebunah u-migdal ha-emunah*: “I have not dealt with this topic as a result of my own will or in order to attain glory but as a result of the fact that many illustrious scholars of my own generation, whose advice I always follow, have insisted in my doing so because in all the land of *Sarfat* [Central and Northern France] there is not a single Hebrew book dealing with these sciences and this is why I have translated them from Arabic books in the best possible way, according to my own capacities”.¹⁵² A similar remark appears in his *Sefer ha-ibbur*: “If I had found in France a Hebrew book containing the rules of the calendar, I would not have bothered to write this book”.¹⁵³ Finally, in the prologue of Bar Ḥiyya’s *Šurat ha-‘areš*, he states “Having spent time in looking for a book *in our language*, which explained methodically the aforementioned science, without any success, I felt obliged in my own spirit to write it according to my possibilities”.¹⁵⁴

151 D’Alverny, 1954.

152 Millás, 1952, p. 37.

153 Romano, 1977. See p. 228 of the reprint, no. 7.

154 Millás, 1956^a, p. 28.

3.1.2.1.2.1 Abraham bar Ḥiyya's Astrological Works

Bar Ḥiyya's scientific production is mainly centered in mathematics and astronomy and only secondarily in astrology,¹⁵⁵ although it has the merit of including the only example I know in the Iberian Peninsula and the Maghrib of a complete astrological history: that of the nation of Israel in chapter 5 of his *Megillat ha-megalleh*¹⁵⁶ which I will analyse in 3.2.5.3. He is also the author of a famous letter to the Barcelona rabbi Yehudah ben Barzillai in which he defends the licit character of the practice of astrology.¹⁵⁷ The starting point of that letter is the advice given by Bar Ḥiyya to another Jewish scholar concerning the adequate day and hour in which he should get married, something which suggests the possibility of our author's actual practice of astrology. Supporting this hypothesis is the fact that Bar Ḥiyya compiled a set of astronomical tables, the main purpose of which is casting horoscopes, and the last three chapters of the canons deal with mathematical astrology: division of the houses of the horoscope (chapter 18), anniversaries or revolutions of the years (chapter 19) and astrological aspects (chapter 20).¹⁵⁸ Besides, his *Yesodeh ha-tebunah*, an encyclopedic book which, apparently, was conceived as a resumed exposition of the whole *quadrivium*, as well as logic – although only the parts concerning arithmetic, geometry, optics and music are extant – was supposed to contain a part on the “science of stars” (*ilm al-nujūm* in Arabic)¹⁵⁹ and it has been suggested that astrology might have been included in it.

Whatever the case, a *Sefer tiqqun ha-tequfot min ha-luḥot ha-‘eleh* (Method for determining the revolution from these tables), attributed to Bar Ḥiyya has recently been discovered.¹⁶⁰ This text contains canons explaining the use of two series of tables preserved in Bar Ḥiyya's astronomical tables and related to the revolutions of nativities (*taḥāwīl sinī al-mawālīd* in Arabic) for years and months. Concerning the revolutions of the months he mentions the “astrological months”¹⁶¹ resulting from the division of the solar year into 13 months of 28^d 2^h 18^m, which imply the use of a tropical year of 365^d 5^h 54^m. Another division of the year into 15 “astrological months” of 24^d 8^h 24^m – 20^s implies a tropical year of 365^d 5^h 55^m and this value agrees with the *revolutio anni* of 88;48° which the tables ascribe correctly to Ptolemy as it implies a tropical year of

155 On Bar Ḥiyya's astrology see Rodríguez Arribas, 2011, pp. 55–120.

156 On this work see Töyrylä, 2014.

157 Spanish translation in Rodríguez Arribas, 2011, pp. 299–310.

158 Millás, 1959, pp. 96–106.

159 Millás, 1952, p. 36.

160 Sela, 2013a.

161 Goldstein & Chabás, 2013.

365^d 5^h 55^m 12^s.¹⁶² Sela suggests that the second part of the *Sefer tiqqun* seems to be based on book VI of Ibn Abī l-Rijāl's *Kitāb al-Bārī' fī aḥkām al-nujūm* (see 3.2.2.2.2.3) which deals with the revolutions of the months and the days. This is, to the best of my knowledge, the earliest reference to Ibn Abī l-Rijāl's work in the Iberian Peninsula.

3.1.2.1.2.2 Abraham ibn 'Ezra's Great Collection of Astrological Handbooks¹⁶³
Between 1148 and 1160, Abraham ibn 'Ezra (ca. 1089–1161) compiled two or three redactions of a set of independent astrological treatises,¹⁶⁴ considered by some modern scholars to form a kind of encyclopedia whose importance can only be compared, in the geographical area we are studying, to Ibn Abī l-Rijāl's *Kitāb al-bārī'*. These books were written in Hebrew, although we also have Latin versions of some of them.¹⁶⁵ These Latin texts have different origins: there are treatises whose Hebrew original is lost but survive in Latin translations, while in other cases the Hebrew treatises were translated into Latin more than one hundred years after Ibn 'Ezra's death. The different recensions of the same chapter were probably written as a result of the request made by patrons Ibn 'Ezra met in different cities of Europe (in Italy, Provence, France and probably England) which he visited throughout his life, after he left the lands of the Iberian Peninsula ca. 1140. As explained by Sela, Ibn 'Ezra “would write a new version of an old work for a new patron when he arrived in a new town”.¹⁶⁶ His travels took him to Rome, Lucca, Pisa, Mantua, Verona, Béziers, Narbonne, Rouen, Angers, and probably London and Winchester.¹⁶⁷

The first version of his collection of astrological books was written in five months of 1148 in Béziers. A second redaction has been tentatively considered as written by Ibn 'Ezra between 1148 and 1158 in Southern France (*'Olam 11*) and, more towards the north, probably in Rouen (the rest). This is a list of the extant books:¹⁶⁸

162 Millás, 1959, pp. 99 and 115.

163 Shlomo Sela has revised this section and I am very grateful for his advice and corrections, although the responsibility for the errors that may have remained is entirely mine.

164 These sources have been published in critical editions of the Hebrew texts of the extant versions, English translations and commentaries by Shlomo Sela, in Sela 2007, 2010, 2011, 2013 and 2017.

165 Epstein, 2015.

166 Sela, 2013, p. 4.

167 On Ibn 'Ezra's biography see Roth, 2012. On the chronology of his works see Sela & Freudenthal, 2006. A general introduction to Ibn 'Ezra's scientific production in Sela, 2003. See also Rodríguez Arribas, 2011, pp. 121–270.

168 Sela, 2017, pp. 2–5.

- *Re'shit Hokhmah* I and II (“The beginning of wisdom”), a general introduction to astrology dealing with the fixed stars, the zodiacal constellations and their astrological characteristics. Only a fragment of *Re'shit* II has been preserved.¹⁶⁹
- *Sefer ha-Te'amim* I and II, the “Book of Reasons” in which he explains the “reasons” of the concepts used in *Reshit Hokhmah*.¹⁷⁰
- *Sefer ha-Moladot* I and II, on nativities.¹⁷¹ *Moladot* II is extant in two Latin manuscripts (*Liber nativitatum*) and there is also a *Liber de nativitatibus* with different contents. Smithuis¹⁷² has suggested that this latter source seems to have been written ca. 1154 by Ibn 'Ezra himself or by somebody else in his name and under his direction.
- *Sefer ha-Tequfah*, on anniversaries of nativities.¹⁷³
- *Sefer ha-Mivharim* I and II, on elections. There is a fragment of *Mivharim* III.¹⁷⁴
- *Sefer ha-She'elot* I and II on interrogations. A fragment of *She'elot* III is also extant.¹⁷⁵
- *Sefer ha-'Olam* I and II on world astrology, including meteorological astrology.¹⁷⁶
- *Sefer ha-Me'orot*, on medical astrology.¹⁷⁷

To this one should add the *Mishpetei ha-Mazzalot*, hypothetically written ca. 1154 in Rouen.

The existence of four Latin translations,¹⁷⁸ made from intermediary French versions, entitled *Liber nativitatum*,¹⁷⁹ *Liber revolutionum*,¹⁸⁰ *Liber electionum* and *Liber interrogationum* has led Smithuis¹⁸¹ to hypothesize the existence of a third collection – composed in an unknown date in Northern France or

169 Sela, 2010a; Sela, 2017.

170 Sela, 2007 and 2015.

171 Edition and translation of *Moladot* I in Sela, 2013.

172 Smithuis, 2006. Erhardus Ratdolt's printed edition of the *Liber de nativitatibus* (Venice, 1484) is accessible in warburg.sas.ac.uk/pdf/fah875ab.pdf.

173 Edition and translation in Sela, 2013.

174 Edition and translation of the three versions in Sela, 2011.

175 Editions and translations in Sela, 2011.

176 Edition and translation in Sela, 2010. Quite recently (Sela *et al.*, 2019), fragments of a third version of *'Olam*, in a Latin translation by Henry Bate, have been discovered and analysed.

177 Edition and translation in Sela, 2011.

178 Thorndike, 1944.

179 Sela, 2019c.

180 Shlomo Sela tells me of the existence of a Hebrew text corresponding to the *Liber revolutionum*.

181 Smithuis, 2006.

England –¹⁸² in which the aforementioned Latin titles would correspond to *Moladot* III, *Tequfot* (or *Tequfah* II?), *Mivḥarim* III and *She'elot* III. Sela has accepted this hypothesis and discovered fragments of the Hebrew texts of *Mivḥarim* III and *She'elot* III.¹⁸³ He also suggests the existence of a lost *Me'orot* II which corresponds to the same period and location.¹⁸⁴

These books of Abraham ibn 'Ezra do not show great originality except for their introductions in which our author conveys creative and original ideas. In Sela's own words they "are mainly textbooks or reference books"¹⁸⁵ and are "an excellent means of learning about the scientific data and sources available in al-Andalus in the twelfth century or earlier".¹⁸⁶ After all, Ibn 'Ezra's scientific knowledge derives from Arabic sources¹⁸⁷ and it has been suggested that he may have studied in Toledo or in some city of al-Andalus,¹⁸⁸ although there is no evidence about this. I could also conjecture that, being born in Tudela, he might have had access to the scientific library of king al-Mu'taman of Zaragoza (r. 1081–1085) which, between 1110 and 1130, was in Rueda del Jalón (see 1.5.1.2.2.2). Whatever the case, it is obvious that he had read many Greek (in Arabic translations) and Arabic scientific sources.¹⁸⁹ He knew Ptolemy's *Almagest* and *Tetrabiblos* (through Arabic translations)¹⁹⁰ as well as the pseudo-Ptolemaic *Karpós/ Kitāb al-Thamara/ Fructus*. This latter source mentions the 120 conjunctions of the seven planets (the sum of the combinations of two, three, four, five, six and seven planets), a topic well developed in the

182 Sela, 2011, pp. 32–36.

183 Sela, 2009.

184 Sela, 2011, pp. 32–36.

185 Sela, 2003, p. 86.

186 Sela, 2003, p. 87.

187 S. Sela ("Abraham ibn Ezra as a translator of astrological and astronomical texts from the Arabic into Hebrew: sources and methods", forthcoming) states that, in the *Re'shit Ḥokhmah*, about one fourth of the whole text is a collection of translations or close paraphrases of Arabic astronomical and astrological sources. These are mainly passages of Abū Ma'shar's *al-Madkhal al-kabūr*, al-Qabīṣī's *Madkhal*, Sahl b. Bishr's *Nawādir al-qaḍā'* and al-Farghānī's *Elements*. I have been able to read this paper thanks to the generosity of its author. See also Sela, 2017, pp. 10–13, 640–666.

188 According to Sela (2017, p. 1) "Abraham ibn Ezra (ca. 1089–ca. 1161) passed the first five decades of his life in Muslim Spain". See also the anonymous "Abraham ibn Ezra a universal European scientist" in *Iberia Judaica* 4 (2012), 19–21. and Roth, 2012, pp. 25–26.

189 Sela, 2019b, shows that many pages of *Reshit ḥokhmah* are Hebrew translations of passages of Abū Ma'shar's Great Introduction, al-Qabīṣī's *Madkhal*, Sahl b. Bishr's *Nawādir al-qaḍā'* and al-Farghānī's *Elements*.

190 Sela, 2003, pp. 238 ff.

two extant versions of the *Book of the World*,¹⁹¹ as well as in many other treatises written by him.

In spite of this, Ibn ‘Ezra does not seem to have a high opinion of Ptolemy as an astrologer. In his *Book of Reasons* he says: “Anything that Ptolemy says about the orbs is correct and no one surpasses him, but his astrological decrees and judgements do not befit his wisdom”.¹⁹² He considers more reliable the works of Doronius (Dorotheos of Sidon) and Māshā’allāh (d. ca. 815). In this work he quotes Vettius Valens, Enoch (usually identified with the mythic Hermes)¹⁹³ but also Hermes’ *Kitāb al-aṭwāl*, pseudo-Ptolemy, Andarzaghār b. Zādān Farrūkh,¹⁹⁴ considered by him as a Jew, an unidentified Ibn Abī Damīna and Abū Ma’shar. More or less the same sources are repeated in his books on interrogations and elections where he expresses his approval of Māshā’allāh’s and Abū Ma’shar’s books on interrogations, while he disapproves the latter’s astrological history, in spite of the fact that an alternative title of the *Sefer ha-‘Olam* (Book of the World) is *Sefer ha-Maḥbarot* (Book of Conjunctions),¹⁹⁵ which reminds us of Abū Ma’shar’s *Kitāb al-Qirānāt*.

In the books on interrogations and elections he adds to the aforementioned sources Sahl b. Bishr (fl. 821–850), Abū ‘Alī al-Khayyāṭ (fl. ca. 854), al-Kindī (d. ca. 873) on buried treasures¹⁹⁶ and other topics, Ibn Zakariyā’ al-Rāzī (d. 925) and an unidentified Sa’īd. Sela has suggested that some of Ibn ‘Ezra’s quotations may be indirect and that he is using Ibn Abī l-Rijāl’s *Bārī’*.¹⁹⁷

On the whole this list of sources is more or less similar to what we have seen in 3.1.2.1.1.1 and it respects the limits proposed in 1.5.1.2: works written in the Mashriq after 950 rarely reached al-Andalus. This seems to be true for Ibn ‘Ezra’s astrological sources, while a different conclusion will be reached if one analyses the sources used in his astronomical works, like the *Liber de rationibus*

191 Sela, 2010, pp. 15–16; see also Sela, 2003, pp. 313–323.

192 Sela, 2007, p. 59 and also 33, 35; Sela, 2010, p. 26: here Ibn ‘Ezra criticises Abū Ma’shar’s astrological history of the *Kitāb al-qirānāt* because the latter relies on mean planetary conjunctions instead of true ones.

193 In Sela, 2011, pp. 20–21 he speaks about “the ancient Enoch”, “Enoch the First” and “Enoch the Egyptian”; see also Sela, 2010, pp. 12–13. The source seems to be Abū Ma’shar’s *Kitāb al-Ulūf*.

194 Sezgin, *GAS* VII, 80–81; Burnett & Hamdi, 1991–92. It is clear that this source circulated, directly or indirectly, in al-Andalus and in Christian Iberian lands because it was translated into Latin by Hugo of Sanctalliensis in book IV of his *Liber Aristotilis*.

195 Sela, 2010, p. 3.

196 See Burnett, Yamamoto & Yano, 1997.

197 Sela, 2011, pp. 20–23.

tabularum.¹⁹⁸ here the sources mentioned include books that do not seem to have reached al-Andalus and I tend to believe that Ibn 'Ezra knew them as a result of his travels in Italy.¹⁹⁹

One of the most interesting aspects of Ibn 'Ezra's astrological theory is his analysis, in the *Book on Nativities*, of the eight factors that affect human beings collectively and to which the individual destiny signified by the nativity horoscope is subordinated.²⁰⁰ This means that, in spite of the importance of the nativity horoscope, modulated each year by the successive anniversary horoscopes, the future of a human being depends also on other factors which can alter significantly the results of the natal chart. This gives a coherent answer to the standard anti-astrological argument: how can one explain that two children born exactly at the same time and in the same locality have completely different destinies. Besides, this treatment explains the relation between nativities and world horoscopes. In Sela's words "Universal judgements override particular judgements". The factors in question are the following ones:

1. Nationality and religious affiliation: the examples given by Ibn 'Ezra are related to the nation of Israel which has a collective astrological fate.
2. Geographical location: the seven climates and the associated planets have an obvious influence in the individual.
- 3–4. Catastrophes: here Ibn 'Ezra explains how a Saturn-Jupiter conjunction may have consequences for the future of a person.
- 5–6. The socio-political status: two children sharing the same nativity horoscope will have different futures depending on their social origin.
7. The weather can also be the cause of catastrophes that will alter what can be predicted from the nativity horoscope.
8. The delivery from the decrees of the stars: if one knows, from an anniversary horoscope, that some evil may affect the subject during the following year, he may protect himself using rational methods or imploring God's help.

198 As Sela tells me: "It is doubtful whether Ibn Ezra himself wrote this Latin work; in all likelihood it is a later elaboration based on Hebrew treatises by Ibn Ezra". Whatever the case, the problem of the new sources mentioned in *De rationibus tabularum* remains and it can be explained by Ibn 'Ezra's stay in Pisa and the existence of a link between Antioch and Pisa: see Burnett, 2000; Burnett, 2003a; Burnett, 1999.

199 Samsó, 2012, pp. 193–195.

200 Sela, 2013, pp. 31–40; Sela, 2003, pp. 145–191.

A further question I would like to raise here is whether Ibn ‘Ezra’s is really the author of the extant horoscopes attributed to him. An analysis of such horoscopes would be extremely interesting in order to establish the kind of astrology (sidereal or tropical) practised by him, as I have suggested elsewhere²⁰¹ that Jewish scholars introduced tropical astronomy and astrology in the lands of the Iberian Peninsula.²⁰² In the list of Ibn ‘Ezra’s astrological works established by Renate Smithuis²⁰³ we find the reference to the following horoscopes, attributed to him, extant in Hebrew. They correspond to:

1. A nativity horoscope, probably cast in Verona the 14th October 1143.
2. Another nativity horoscope in Narbonne the 15th October 1160
3. Two undated horoscopes
4. A prognostication for year 1166 which contains incomplete references to three horoscopes: a) that of the Saturn-Jupiter conjunction which took place that year; b) the vernal equinox of 1166 and c) a solar eclipse in May 1166.

Both the nativity of 1160 and the prognostication for 1166 have been considered spurious and only the prognostication has been seriously analysed by Goldstein²⁰⁴ who ascribes it to an anonymous Jewish astrologer who had migrated to the Crown of Aragón after the Almohad invasion. Interestingly, Goldstein has established that the positions mentioned in the text have been calculated with al-Khwārizmī’s *zīj* and that, consequently, they are sidereal positions. A similar study of the other three horoscopes would be extremely useful to determine whether they were cast by Ibn ‘Ezra himself or not. The same could be said about the numerous horoscopes appearing in Ratdolt’s printing of the *Liber de nativitatibus* (Venice, 1484), although, according to Sela, they are later additions and they are not part of the original book. Quite recently, I have become aware of the existence of several Latin horoscopes, apparently due to Ibn ‘Ezra, which, definitely, compute tropical positions:

- A nativity horoscope calculated for 23 August 1135.²⁰⁵
- A second nativity horoscope for 29 October 1135.²⁰⁶
- Spring equinox 1067.

²⁰¹ Samsó, 2012, pp. 195–196.

²⁰² In 3.2.5.3 I have tried to answer the same question in relation to the horoscopes of Abraham bar Ḥiyya in the *Megillat ha-Megalleh*, although I have been unable to reach clear conclusions.

²⁰³ Smithuis, 2006, pp. 334–336.

²⁰⁴ Goldstein, 2004a.

²⁰⁵ Steel *et al.* 2018, p. 80.

²⁰⁶ North, 1986, pp. 108–9.

- Luni-solar opposition which preceded the Saturn-Jupiter conjunction of 1067.
- Saturn-Jupiter conjunction of 1067.²⁰⁷

Apart from the existence of Latin translations of Abraham ibn 'Ezra's books, the authorship of which is being discussed, I would like to end this passage with a reference to the translations of his books into French, Latin, Castilian and Catalan which had, therefore, a wide diffusion in Latin Europe between the 13th and the 15th centuries. In 1273 the French Jew Hagin made a translation into French of four of Ibn 'Ezra's astrological works in Malines. One of these translations was the object of, at least, a retranslation into Latin by Henry Bate in 1281 and 1292, to which he added other Latin translations of Ibn 'Ezra's collection, although we do not know the source he used.²⁰⁸ Other Latin versions were due to Pietro d'Abano (ca. 1250–1316) who used the four French translations made by Hagin,²⁰⁹ and to Arnoul de Quinquempoix (before 1326).²¹⁰ Besides, we know of the existence of two Castilian collections of texts which contain direct translations from Hebrew. In Ms. B-332 of the Archivo Diocesano of Segovia (copied in 1432) we find the Castilian text of *Reshit Hokhmah I*, *Ṭe'amim I*, *Moladot I* and *'Olam II*. Ms 2138 of the University Library of Salamanca (copied in 1520–21) contains a more complete collection of Ibn 'Ezra's works in translations which are independent from those of the Segovia manuscript: *Reshit Hokhmah I*, *Ṭe'amim II*, *Moladot I*, *She'elot I*, *Me'orot*, *Mibharim I*, *'Olam I*, and part of *Mishpetei ha-Mazzalot*.²¹¹ There was also, at least, one Catalan translation of a good part of the collection in a volume, extant in El Escorial, which bears the title of *Libre dels juhins de les estelles*. We do not know from which language the works were translated. The author (or the copyist) was a certain Martinus d'Oscá and it seems that the translation was made before the end of the 15th century. The aforementioned volume contains the *Libre de introductori* (*Reshit Hokhmah*), *Libre de les nativitats* (*Moladot*), *Libre de les demandes* (*She'elot*), *Libre de les eleccions* (*Mivharim*), *Libre del ajustament dels planetes e del entrament del Sol en Aries, de les revolucions dels anys* (*Tequfah*) and *Libre de les malalties* (*Me'orot*). We also know that a Catalan version was retranslated into Latin by Luís de Angulo in Lyon in 1448.²¹²

²⁰⁷ North, 2002 and 2003; Sela *et al.*, 2019.

²⁰⁸ Sela, 2017a; Sela *et al.*, 2019.

²⁰⁹ Sela, 2019a.

²¹⁰ Levy, 1927; Levy & Cantera, 1939; Levy, 1947.

²¹¹ Alba, Sainz & Sela, 2010.

²¹² Romano, 1990.

3.1.2.2 Castile and Aragon between the Thirteenth and the Fifteenth Centuries

3.1.2.2.1 *The Court of Alfonso X of Castile*

Enough has been said in 1.5.2 about the interest king Alfonso X of Castile (1252–1284) had in astrology. He patronized the Castilian translations of two important Arabic astrological handbooks: the *Libro de las cruces* (3.2.1.1) and the *Libro conplido en los iudizios de las estrellas* (3.2.2.2). This second work is a literal translation of Ibn Abi l-Rijāl's *Kitāb al-bārī' fī aḥkām al-nujūm*. Until recently, I believed that the Alfonsine work implied the earliest attestation of the knowledge of Ibn Abi l-Rijāl's book in Latin Europe, although Shlomo Sela has suggested that, at least concerning interrogations, Abraham ibn 'Ezra knew this work (see 3.1.2.1.2.1 and 3.1.2.1.2.2).²¹³ The *Bārī'* became the most popular treatise on astrology in Europe, from the end of the 13th c. onwards, due to two retranslations into Latin of the Alfonsine Castilian version, made by Alfonso's collaborators Alvarus [Ovetensis] and by the team formed by Egidius of Tebaldis and Petrus de Reggio.²¹⁴

King Alfonso also patronised a new *zīj*, the *Alfonsine Tables*, also used for casting horoscopes (see 7.6.2 and 6.2.7.2). As the mean motion tables of the Latin version of this *zīj* are tropical, it is obvious that the horoscopes computed with it would also be tropical. Although the prologue of the Castilian canons does not contain any astrological references, the text deals with the treatment of astrological techniques, such as transits in thickness,²¹⁵ division of the houses²¹⁶ and casting the rays.²¹⁷ Besides, we have, at least, one clear evidence of the fact that Yehudah b. Mosheh, the author of the Castilian translation of Ibn Abi l-Rijāl's *Kitāb al-Bārī'*, was also a practising astrologer because he cast the horoscope of the beginning of his translation in order to choose the most propitious moment for it.²¹⁸ This horoscope has been preserved in the Latin

213 Cifuentes, 2001, p. 212, raises the question of the possible translation of the *Bārī'* from Hebrew into Latin made by Johannes Hispalensis. This source would have been available in Majorca ca. 1396 and would have been retranslated into Catalan. David Juste (private communication 26.4.2015) informs me of the existence of another translation from Hebrew attributed to Johannes Hispalensis: an astromagical text whose author would be Hermes and Toz Grecus and entitled *De stationibus ad cultum Veneris*.

214 Hilty, 2011.

215 Chabás and Goldstein, 2003, chapter 26, pp. 47 and 170.

216 Chabás and Goldstein, 2003, chapter 28, pp. 52–53, 183–185.

217 Chabás and Goldstein, 2003, chapter 52, pp. 91–92, 222.

218 An American scholar, Lynne Fisher (Fisher, 1979) has analysed this horoscope according to the rules established in a modern astrological handbook (Ivy Goldstein-Jacobson, 1960). The purpose of these rules is the choice of a propitious moment in which an author

retranslation made by Egidius de Tebaldis and Petrus de Regio,²¹⁹ where the date is given in the following way:

Ec est figura incii translacionis huius libri die Jovis mensis Marcii .21. die Moharan. Era Arabum .652. Xpi .1253. Cesaris .1291. Alexandri .1565.

The 21st Muḥarram 652H was actually a Friday and it corresponds to the 13th March 1254 CE, 1292 of the era of Cesar (Spanish era), and the 13th Adhār 1565 of the Seleucid era. The disagreement in the year of the Christian and the Spanish eras is explained by Hilty²²⁰ as a result of the use of the Incarnation era, according to which the year begins the 25th March and, consequently, the 13th March 1254 AD / 1292 SE belongs to year 1253/1291 *ab Incarnatione*. As for the day of the month, the recomputation (see below table 3.6) shows that the horoscope was computed for Thursday 12th March (civil date) at about 6^h in the morning, which corresponds to the 11th March (astronomical date, for which the day begins at midday of the previous day).

The division of the houses is calculated according to the standard system and I add in parenthesis the differences between the values of the text and the recomputed values:

I	8°	II	38;51° (+6')	III	67;21° (-10')
IV	94;18° (+18')	V	123;8° (+5')	VI	154;35° (+7')

Using John North's programme HOROSC we can establish that the latitude for which the horoscope was cast is comprised between 42;6° and 44;33°. The horoscope may have been cast for the latitude of Toledo (about 40°), although the latitude is a little too high. The hour, calculated from the longitude of the houses and the position of the Sun, is 6;25^h AM.

The planets are in the following positions for the 11th March (astronomical) 1254

should begin the final version of a manuscript or present the work, for the first time, to a publisher. It is really surprising to discover that most of these rules agree very well with the data we find in the Alfonsine horoscope.

219 Hilty, 1954, pp. LXII–LXV and Hilty, 2005, pp. IX–XI. Thorndike, 1950, established clearly that this horoscope cannot correspond to the beginning of Egidius de Tebaldis' Latin retranslation. Gerold Hilty has repeatedly shown his agreement with Thorndike's opinion: see recently Hilty, 2011.

220 Hilty, 1954, p. LXIV.

TABLE 3.6 Planetary positions the 12.3.1254 at about 6h a.m.

Planets	Text	Toledan tables	Al-Battānī's <i>zīj</i>	Alfonsine tables
Sun	358°	358;11°	359;22°	359;13
Moon	249°	248;58°	248;37°	250;8°
Saturn	286°	286;39°	287;2°	288;24°
Jupiter	65;37°	65;20°	66;56°	65;44°
Mars	132°	136;15° *	131;52°	131;9°
Venus	32°	32;2°	33;1°	32;57°
Mercury	13;30°	13;11°	14;35°	14;13°
Asc. node	148;22°	148;27°	149;44°	141;48°

I have recomputed the planetary positions using the parameters of the three *zīj*es which were available to the Alfonsine collaborators, although the *Alfonsine Tables* were probably not ready in 1254. It is obvious that the longitudes computed are tropical and not sidereal, something which disagrees with the standard practice of Andalusī and Maghribī astrologers who usually cast sidereal horoscopes. Besides, it seems that the best recomputation corresponds to the *Toledan Tables*, in spite of the obvious mistakes in the position of Mars. I should add to this that, in spite of the fact that the mean motions of the *Toledan Tables* are sidereal, the true positions have been corrected for precession using the trepidation model.

3.1.2.2.2 *The Royal Patronage of Astrology in the Crown of Aragon in the Fourteenth Century*

Although there is a certain astrological activity in the Crown of Aragon in the 13th century, represented mainly by Raymundus Lullius/ Ramon Llull's *De nova astronomia* (cf. 3.2.1.3) but also by Arnaldus de Vilanova (1240–1311), who wrote a treatise on medical astrology (*De judiciis astronomiae* or *Capitula astrologiae*),²²¹ this cannot be compared to the importance acquired by astrology during the reigns of Pedro IV of Aragon (1336–1387) and his son King Juan I (1387–1396).²²² It has already been explained (in 1.5.3) that the cultural model

221 On the practice of medical astrology in Majorca during the 14th and the 15th centuries see Contreras, 2000, especially pp. 96–100.

222 In spite of its title, the recent book of Ryan, 2011 does not contain much information on astrology proper. See my review in *The Catholic Historical Review* 99 (2013), 134–135. On King Juan I and his interest in astrology see Roca, 1929.

created in the 13th century by Alfonso x of Castile was clearly imitated in the following century by Pedro iv of Aragon. Both he and his son were surrounded by a court of astrologers. One of them was Ya'qov ben Abī Abraham Yizḥaq al-Corsuno, from Seville, where he had written a treatise on the astrolabe in Arabic in 1375–76, which he later translated into Hebrew in Barcelona in 1378. In 1380 he appears in a document as “juheu astrolench e torcimany de casa del senyor Rey” (Jewish astrologer and translator of the royal house).²²³ He was the author of the final version of the *Tables of Barcelona* (ca. 1378, see 7.6.3). Another reference to this person has been found in a manuscript of the Arabic *Almagest*, copied in 1381 in Zaragoza by Aḥmad b. Aḥmad b. Salāma al-Ṣ.ṇ[...].ānī for his master and teacher Abū Ishāq Ya'qūb ibn Ishāq ibn Ya'qub, known as Ibn al-Qursunūh, astrologer (*munajjim*) of king Pedro iv.²²⁴

Another important character of the royal court is Bertomeu de Tresbens, who was also King Pedro's physician between 1361 and 1374 and appears in a document, dated 1359, as “magister in artibus et medicina”.²²⁵ According to Vernet & Romano, Tresbens was often consulted on astrological matters by the heir, Prince Juan, and we know that he studied carefully the horoscopes of the king and of his heir.²²⁶

Prince Juan also sent his enquiries to Dalmau Ses Planes, Guillaume Lunell – who lived in Paris in 1384 and was invited to join the Barcelona court in December 1385 and in 1388 –, Thomas of Bologna – who served Charles v and Charles vi of France and was asked by prince Juan to write astrological predictions for years 1386 and 1387 – and Cresques Vivier of Avignon – who was his main astrologer when he became king in 1387. When Cresques died in 1391, king Juan i obtained the collaboration of a German astrologer, master Ramar/Renardi/ Remaur/ Raymar Ohem, who is the only astrologer mentioned in the documents of the royal house after 1392.²²⁷

All the previous information seems mainly related to the personality of Pedro iv's heir of the throne. In spite of this it is clear that king Pedro was also deeply interested in astrology. We know that when he had to make major works in the Royal Palace, he insisted in the election of the propitious day and hour for the beginning of the task, which should be established by his astronomers and astrologers Pere Gilbert (Petrus Gileberth) and Dalmau Ses Planes (Dalmacius Planes).²²⁸ A better picture of Pedro iv's interest in astrology can

223 Millás, 1962, p. 71; Chabàs, 1992, p. 20. Giralt, 2017b discards the attribution to Arnaldus de Vilanova of *De iudiciis astronomie*.

224 Kunitzsch, 2001.

225 Torre, 1971, doc. n° 22 (p. 38).

226 Vernet & Romano, 1957–58, vol. I, p. 11.

227 Chabàs, 1992, pp. 23–27; Chabàs, 2004, pp. 503–506.

228 Millás, 1962, p. 60.

be obtained through the analysis of the written works undertaken under his patronage. One of them is the *Almanach* compiled by Pere Gilbert in 1361–62 and by his disciple Dalmau ses Planes (1362–1366) of which only the prologue and a few canons are extant.²²⁹ Although the tables are not extant, it seems clear that the work was a set of ephemerides giving solar, lunar and planetary positions, including the position of the ascending node and the true conjunctions and oppositions of the Sun and the Moon, for each day between years 1361 and 1433²³⁰ for the coordinates of Barcelona whose longitude is 33° and latitude 41°. These positions are apparently tropical, and, strangely enough, the text ends with a reference to a correction of 0;0,28,14° per month to obtain the position from the Head of Aries. The text does not say anything about entering with this value in a table of the equation of trepidation, which would give us the amount of precession for a given date (see 6.2.7.3).

It is obvious that the main application of this set of ephemerides was the direct obtention of the positions of the celestial bodies which were necessary to cast an horoscope. Besides, the prologue contains quotations from pseudo-Ptolemy's *Centiloquium*, Ptolemy's *Quadripartitum*, and Māshā'allāh (*Epistola Messallach de planetarum effiacia*), as well as reference to transits in thickness.

A similar case is that of the astronomical tables for the coordinates of Barcelona, compiled by Jacob Corsuno.²³¹ The canons of these tables are extant in Hebrew, Latin and Catalan and the prologue insists in the importance of astrology for predicting future events with references to world astrology, nativities and revolutions of nativities, interrogations and elections. For that reason, king Pedro IV ordered Pere Gilbert and Dalmau Ses Planes to compile the aforementioned *Almanach*, which gave tropical positions used by most Christian astrologers to cast their horoscopes. The king, however, is conscious of the fact that many Arab and Jewish astrologers, as well as members of other sects, compute sidereal horoscopes and wants to have a set of tables which yield sidereal positions, probably in order to have horoscopes calculated according to the two systems. This was the task undertaken by Jacob Corsuno, although the analysis of the tables made by J. Chabás shows that the situation is not clear at all: the *radix* positions are sidereal but the mean motion parameters seem to be tropical, something which is clearly incoherent. The astrological interest of the tables is also shown by the chapter on the determination of the ascendant and the division of the houses according to the

229 Editions by Thorndike, 1937; Millás, 1962, pp. 87–93.

230 This is a period of 72 years. The text says, however, “patet hoc opus solum ad 60 annos extenditur de presenti”, although this period can be extended with some corrections.

231 Millás, 1962, pp. 100–238; Chabás, 1996a; Chabás, 2004.

standard method (3.2.3.1.1.1) and by the chapter on the revolution of the years of the world and of nativities.²³²

As said in 1.5.3 a good part of the astronomical and astrological activities took place in Southern France, in areas belonging to the Crown of Aragon. In Perpignan, Jacob b. David Yomtov (= Bonjorn) (fl. 1361) computed his important astronomical tables, but a member of his family, David ben Yomtov, who seems to be Bonjorn's father or son, wrote the most important extant Hebrew treatise on medical astrology, entitled *Kelal Qaṭan*, which used, as its main source, Abraham ibn 'Ezra's *Sefer ha-Me'orot*. This work appears in a century and a geographical area that seems to have felt a keen interest in medical astrology and it deals with the choice of propitious moments for therapeutical interventions as well as with critical days which use the Moon as the chief indicator.²³³

Another important astrological work undertaken under the patronage of Pedro IV is the treatise on nativities written by Bartomeu de Tresbens²³⁴ and edited under the title *Tractat d'astrologia*. This is one of the many examples of the use of the vernacular language, Catalan, in the 13th and 14th centuries in astronomical and astrological sources as well as in other scientific disciplines.²³⁵ One of the two manuscripts containing the text of the *Llibre de les nativitats* (ms. XXI of the Lambert Mates library in Ripoll) contains also a book on interrogations, another on elections (dedicated to king Pedro IV) and a short text on the search for treasures whose author is Bartomeu de Tresbens and it is also dedicated to the king.²³⁶ This set of four texts has been interpreted by Cifuentes as being four independent treatises, authored by Tresbens himself and written in different dates: the treatise on nativities was written ca. 1369 while the other three are later, the last one being the book on the search of treasures which was written in 1372.²³⁷ On the other side, Gloria Sabaté and Lourdes Soriano²³⁸ consider that the four aforementioned texts are the four parts of an astrological treatise which follows mainly the Castilian or the Latin version of Ibn Abī l-Rijāl's *Bārī*,²³⁹ although S. Giralt has shown that he also uses other sources.²⁴⁰ It is well known that Ibn Abī l-Rijāl's book was available

232 Millás, 1962, pp. 134–135 and 138.

233 Bos, Burnett & Langermann, 2005.

234 See Ferragud, 2018; Giralt, 2017, 2017 a and 2018; Ferragud & Giralt, 2019.

235 On this topic see Cifuentes, 2001 and Cifuentes, 2004.

236 Vernet & Romano, 1957–58, I, pp. 26–28. See also Chabás, 2004, Giralt, 2017; Giralt, 2018.

237 Cifuentes, 2004, p. 209.

238 Sabaté & Soriano, 2012, p. 401.

239 This would correspond to Ibn Abī l-Rijāl's books I–III (interrogations), IV–VI (nativities and anniversaries of nativities) and VII (elections). Trebén was, apparently, not interested in world astrology (book VIII). See 3.2.2.2.

240 Giralt, 2017a.

to scholars of king Pedro IV's court: the 1410 inventory of the books of the library of king Martín I mentions a Romance (probably Catalan) translation of the *Bārī* and several copies of the Latin translation.²⁴¹ Besides, in a document of 1386, we find Prince Juan asking the governor of Majorca to send him a copy of the book entitled *Ali Abenragel* extant in the convent of "El Carme" of the city of Majorca and orders a Catalan translation of the book if the copy is in Latin. Also, in 1359, Dalmau Ses Planes is allowed by king Pedro IV to borrow all the astronomical instruments and books he needs from the Royal Palace with the only exception of two volumes containing astrological works by Ibn Abī l-Rijāl:²⁴² were these two volumes reserved for Tresbens who was preparing his astrological treatise?

In spite of the obvious influence of Ibn Abī l-Rijāl, Tresbens' book has original aspects: he gives his own opinion on the question of which is the planet having more dignities and deserves to become the *haylāj* (*yleig*) and the *kadkhudāh* (*alcocoden*).²⁴³ In a passage which seems to have some relation with Ibn 'Ezra's idea, in his *Book on nativities* (3.1.8.1.2.2), according to which the social status of the newborn has an obvious influence on his future, he gives examples of the contrary: in spite of his humble origins, Charles de Blois was elected count of Flanders (?)²⁴⁴ and a haberdasher of Toulouse became so rich that his sons attained the category of knights.²⁴⁵ He also gives original examples in one of which he obtains the *tasyūr* between two indicators by using the difference in oblique ascensions of both celestial bodies (see 3.2.3.2.2.2), calculated for the latitude of Barcelona: in an horoscope the *haylāj* is the Sun (longitude 71°) and we want to progress it to Mars (longitude 193°).²⁴⁶ He proceeds as follows:

$\alpha_{\phi}(71^{\circ}) = 48;14^{\circ}$ (in the *Tables of Barcelona*²⁴⁷ 48;57°; 48;24° in the *Toledan Tables*)²⁴⁸

241 Chabàs, 1992, pp. 31–32 (item 79), 33 (item 119), 34 (item 130), 36 (item 221) and 37 (item 253). See also Chabàs, 2004, pp. 508–512 and Cifuentes, 2001, pp. 211–212.

242 Cifuentes, 2001, p. 194. There is evidence of the existence of copies of the *Libro conplido* in the royal archives in the collection of documents gathered by Rubió, 1908–1921: documents dated in 1359 (Rubió I, 190, doc. 191), 1373 (Rubió I, 246, doc. 258), probably 1381 (Rubió I, 293–294, doc. 258), and 1384 (Rubió I, 320, doc. 353).

243 Vernet & Romano, 1957–58, pp. 61–63.

244 This cannot refer to Charles de Blois, duke of Brittany (1319–1364).

245 Vernet & Romano, 1957–58, pp. 111–112.

246 Vernet & Romano, 1957–58, pp. 126–127.

247 Millás, 1962, pp. 222–223.

248 Pedersen, 2002. Vol. III, pp. 1057–1060. Recomputed value using Benno van Dalen's *Table Analysis* for an obliquity of the ecliptic 23;33° and a local latitude of 41°: 48;38°.

α_{φ} (193°): 196;30° (*Tables of Barcelona* 196;20°; 196;28° in the *Toledan Tables*)²⁴⁹

196;30° – 48;14° = 148;16° (the arc of *tasyīr*)

In another passage,²⁵⁰ Tresbens is clearly commenting on Ibn Abī l-Rijāl's chapter 14 of Book IV of the *Bārī*.²⁵¹ Following Ibn Abī l-Rijāl, he is trying to calculate the length of his own life from the horoscope of the nativity of his son. Tresbens says that he was forty-seven years old at the moment of his son's birth and this could allow us to compute his date of birth if the data of the horoscope were clearer and more complete.²⁵² This is not the case and I can only assert that Tresbens makes several attempts to calculate the length of his own life, from the date of his son's birth, obtaining 25 years 6 months and 18 days, 30 years 2 months and 18 days, 12 years, 45 years and 79 years. He rejects this last period because it implies a life which is clearly too long.

Another Catalan astrological and astromagical compilation that has been the object of an edition and detailed study is extant in the miscellaneous ms. 1 of the Arxiu de les Set Claus of the Historical National Archive of Andorra.²⁵³ The manuscript has been dated, with paleographical criteria, between 1430 and 1440 but the author of the original collection seems to be a Catalan Jewish author who was possibly a physician and was related to the court of king Juan I (1387–1396).²⁵⁴ The work contains four different parts:

1. An astromagical text which derives from the *Ghāyat al-ḥakīm* / *Picatrix* (chapter 12 of book II) and deals with the relation of zodiacal signs with the different parts of the human body and with the making of talismans. This passage has led the editor of the text to hypothesise the existence of a Catalan translation of the *Picatrix* from the (lost) Alfonsine Castilian translation or the extant Latin translation with the same origin.²⁵⁵ This hypothetical Catalan translation is called *Tencar*, a mysterious word whose origin has not been explained.

249 Recomputed value with van Dalen's *TA*: 196;27°.

250 Vernet & Romano, 1957–58, pp. 128–132.

251 Hilty, 1954, pp. 206–207. I have checked the Castilian translation with Ms. Rabat, Hassaniyya Library 3503, fol. 124v.

252 In an unpublished paper ("Astrología matemática en el *Llibre de les nativitats* de Bertomeu de Tresbens") I have tried to establish the date of this horoscope. It seems to correspond to an unknown day of March 1364, which implies that Tresbens was probably born in 1317.

253 Vela, 1997.

254 Cifuentes, 2001, p. 216.

255 Pingree, 1986.

2. The second brief text deals on medical astrology and it begins with the passage of the Moon through the zodiacal signs and its relation to human diseases and insists in the correspondence between the four elements, the seasons of the year, the four humours and the zodiacal signs. The main source used by the author of the compilation is the pseudo-Hippocratic *De pronosticationibus egritudinum secundum motum lune* of which we have here an almost complete Catalan translation complemented by two lists of “critical” days.
3. The third text is concerned with interrogations and elections and seems to be narrowly related to passages of Ibn Abī l-Rijāl’s *Kitāb al-Bārī’*. This is followed by a treatment of world astrology, with references to planetary conjunctions, eclipses and comets. He gives the names of nine comets: “Veren”, “Tentaculum”, “Pertita”, “Miles”, “Senyor de Scon”, “Matucha”, “Argent”, “Rosa” and “Negre”.²⁵⁶ He finally recommends the use of the *Alfonsine Tables* for all the computations necessary to cast an horoscope.
4. The fourth part of the edited text deals with the problems of the Jewish calendar, the length of the lunar synodic month and the computation of the date of the new Moon.²⁵⁷

3.1.2.2.3 *Astrology in Aragon, Castile and Portugal in the Fifteenth Century*

The first half of the 15th c. is a period of decline both in Aragon and Castille and I believe that this is the result of the lack of royal patronage.²⁵⁸ Not a single Aragonese or Castilian king could be compared to Alfonso X or to Pedro IV. In Aragon, Martin I (1396–1410), the brother and successor of Juan I, rejected the practice of astrology, probably due to his religious prejudices. The analysis of sources written in Catalan made by Cifuentes²⁵⁹ shows that most of the extant information derives from small and unimportant written texts or mere notes added to an astronomical work,²⁶⁰ although we find a Catalan translation of the astrological treatise written in 1334 in Seville by Alfonsus Dyonisii (Alfons Dinis), who was the physician of king Alfonso IV of Portugal (1326–1367) and of his daughter Maria, the wife of Alfonso XI of Castile (1312–1350). The situation would not change in the second half of the century although we find a treatise on astro-numerological prognostication printed towards the end of

256 See lists of names of comets in Arabic sources in Kennedy, 1957; Martínez & Marco, 2016.

257 Nothaft, 2015, considers that this part of the compilation was written ca. 1300.

258 There is, however, some information about astrologers who served the kings of Navarra during this period: see Serrano, 2009.

259 Cifuentes, 2001, pp. 216–220.

260 Johnston, 1984.

the 15th c,²⁶¹ and a treatise on astrology, copied between 1490 and 1510, extant in the University Library of Valencia.²⁶² To this one should add the existence of an obvious interest of the local government of Catalonia in recording, in the official *Dietaris*, the appearance of comets such as Halley's comet, observed in 1456 and considered to be the cause of the death of King Alfonso V (1416–1458), three other comets seen in 1468, 1471 and 1472, as well as a lunar eclipse which took place the 21st November 1471.²⁶³

Another kind of astronomical source which had an obvious use for astrologers were the *lunaria*, lunar calendars recording the phases of the Moon (mainly new and full Moons) which could be used by physicians to determine the moments in which they could practise a phlebotomy. These sources also calculated eclipses. We have already seen how the fourth part of the *Tencar* compilation deals with this kind of problem, and we will refer in another chapter (see 7.6.4) to the lunar tables of David ben Yom Tov of Perpignan. We also have, in manuscript form, a lunar *Kalendarium* compiled by Raymond Bancal (Franciscan minister of the province of Aragon in 1326/27) and calculated for 1311–1386: one of the two manuscripts which preserve this work contains many astrological notes.²⁶⁴ Two other *Lunaris*, with astrological implications, were calculated for Valencia (ca. 1367)²⁶⁵ and Barcelona²⁶⁶ but the most popular calendar of this kind was the *Lunari* of Bernat de Granollachs (fl. 1418–1487),²⁶⁷ printed, for the first time, in 1485 and reprinted 89 times until 1582. The contents of this work were purely astronomical but, from ca. 1492 onwards, Andrés de Li, of Zaragoza, added to Granollachs' *Lunarium a Reportori dels temps* which contained the astrological materials²⁶⁸ that gave the *Lunarium* its astrological reputation.²⁶⁹

As for the kingdom of Castile, we know that, in the 14th c., there was some activity related to astrological translations from the Arabic: in Seville (1333) Pero Ferrandes translated into Catilian al-Qabīṣī's *Kitāb al-madkhal ilā šinā'at aḥkām al-nujūm* ("Introduction to the craft of astrology"), which had already been translated, some two hundred years earlier, by Johannes Hispalensis.²⁷⁰

261 Lucas, 2003.

262 Lucas, 2003, p. 29.

263 Martínez & Marco, 2014 and 2016.

264 Nothaft, 2015.

265 Chabás & Roca, 1985, pp. 57–59.

266 Cifuentes, 2001, p. 199.

267 Chabás & Roca, 1985, pp. 31–38.

268 Delbrugge, 1999.

269 Chabás & Roca, 1985, pp. 17–22.

270 Burnett, Yamamoto & Yano, 2004.

One year later, the aforementioned Alfonsus Dyonisii was working, also in Seville, in other astronomical and astrological translations and wrote an astrological treatise.²⁷¹ Juan Gil de Castiello or de Burgos (alive in 1350–1352) copied, at least, one astrological text for king Pedro IV of Aragon, added his own remarks to the Castilian translation of Ptolemy's *Tetrabiblos* and of the commentary by 'Alī ibn Riḍwān and wrote an important treatise on astronomy.²⁷² We have also information about the astrological activities of the Banū Waqār, a family of physicians active in the 14th c. A medical manuscript which belonged to this family, preserved in ms. Escorial 873, contains four folios with astronomical and astrological tables and notes, including partial data of three nativity horoscopes two of which are dated: 25.11.1406 and 22.10.1330 (nativity of the "Lord of Navarra"). The third horoscope does not contain sufficient number of legible data allowing the computation of a date, but it concerns the nativity of "The Lord of Castile, son of Leonor Núñez": the person involved is probably Enrique II de Trastámara, who was the illegitimate son of king Alfonso XI and of Leonor de Guzmán, the daughter of Pero Núñez de Guzmán.²⁷³

Alfonsus Dyonisii is probably one of the channels through which Arabic astrology was introduced into Portugal. Ms 3349 of the Biblioteca Nacional (Madrid), probably compiled in 1339, contains texts and tables, most of them in Portuguese, with both astronomical and astrological contents and includes a Portuguese version of the *Almanach* of 1307 (7.7.3.2).²⁷⁴ We know that, in 1411, the Jew Josef Franco, son of Gedalia, translated into Portuguese the Alfonsine Castilian translation of 'Alī ibn Abī l-Rijāl's *Bārī* (*Libro conprydo en os juyzos das estrelas*).²⁷⁵ We also know the existence of a Castilian text entitled *Secreto de los secretos de astrología compuesto por el infante don Enrique de Portugal* which was bought in Salamanca in 1525 by Fernando Colón (the son of Christopher Columbus).²⁷⁶

A good example of the decline of astrology during the first half of the 15th c. can be found in the *Tratado de astrología*²⁷⁷ attributed to the well known author Enrique de Villena (d. 1434), although its real author is probably

271 Beaujouan, 1967, p. 22; Beaujouan, 1969, pp. 7–8.

272 Beaujouan, 1969, p. 11.

273 Castells, 1991.

274 Chabás & Goldstein, 2015, pp. 389–406.

275 The end of book 6 as well as book 7 of this Judeo-Portuguese translation have been edited by Hilty, 2005, pp. 78–190.

276 Beaujouan, 1961.

277 Cátedra & Samsó, 1983.

Andrés Rodríguez who compiled this work in 1438–39 and dedicated it to Alonso de Cartagena, bishop of Burgos. One should not be deceived by the title of this book, as the Castilian word *astrología* does not necessarily mean astrology in its modern sense. As a matter of fact, in the book we can find astrology, but also astronomy, ecclesiastical computus, cosmography, geography etc. and the work is a handbook containing the information that readers of the 15th century expected to find in a book with such title. Its author is, obviously, an educated man, who was interested in the topic, but who was not a professional astronomer or astrologer. His knowledge of such disciplines is based on second-hand sources although, among the astrological ones, we find Sahl b. Bishr, Abū Ma'shar and Māshā'allāh. He seems, at least, to have read Māshā'allāh's *Book on eclipses*, translated into Hebrew by Abraham ibn 'Ezra and into Latin by Johannes Hispalensis. The astrological materials contained in this book are extremely limited and we only find the nature and properties of planets and zodiacal signs, classifications of the signs, triplicities and predictions based on eclipses. To this one should add many references to talismanic magic.

The situation changed entirely in the second half of the 15th century in which a chair of Astrology was created in the University of Salamanca and we know the names of the scholars who occupied this chair between ca. 1460 and 1504 (see 1.5.4).²⁷⁸ This University was, at that time, seriously interested in Astronomy and Astrology as it is shown by the vault of the University Library, painted by Fernando Gallego (1440–1507) with a representation of the sky at the moment of the foundation of the building. This painting, of which only a third is extant, represents the zodiacal signs and the positions of the planets in them, forming a kind of rudimentary horoscope which has been the object of different attempts to date it. All these essays agree that the illustration corresponds to the month of August 1475 but the specific day varies between the 6th and the 29th of that month.²⁷⁹

I should remark here, as in the previous paragraph, that the chair of Astrology was not only concerned with astronomy proper. We can extrapolate some information about the subjects which were taught by considering the statutes of 1561 of Salamanca University in which we find the sketch of the programme of “astrological” studies:

- First year: The *Sphere*, *Theorica planetarum*, astronomical tables and the astrolabe.

²⁷⁸ Cantera, 1931, pp. 371–389.

²⁷⁹ García Avilés, 1994.

- Second year: six books of Euclid and Arithmetic until square and cubic roots; Ptolemy's *Almagest* or Regiomontanus' *Epitome* or Jābir ibn Aflāḥ or Copernicus; the *Sphere*.
- Third year: Cosmography or Geography, an introduction to judicial astrology, perspective or an instrument.²⁸⁰

Astrology had, therefore, little importance in the teachings of the chair. In spite of this, there is evidence that some of the scholars who occupied the chair of Astrology were also active astrologers: Diego Ortiz de Calzadilla (chairholder 1469–1476) is supposed to have predicted the victory of Juana la Beltraneja in the war of succession to the throne of Castile which took place between 1475 and 1479, after the death of Enrique IV (1454–1474). As the supporters of Juana were defeated, Diego Ortiz had to exile himself in Portugal.²⁸¹ Rodrigo de Vasurto (chairholder after 1487–1504) was more lucky because he predicted, in his *Praxis prenosticandi sub exemplo anni 1497*, that Juan, son of the Catholic Kings, would not leave the city of Salamanca when he entered the city in 1497.²⁸² The prince did actually die in Salamanca in that year.

We have more information about the astrological work of another chairholder: Diego de Torres (ca. 1480–after 1487) who wrote an astrological treatise,²⁸³ which he finished the 25th May 1487, the first part of which deals with mathematical astrology and which seems to use Abraham Zacut's *Hibbur* (7.7.3.4.1). He is also the author of a prediction related to the solar eclipse of the 16th March 1485.²⁸⁴ It is a short text which predicts a plague epidemic as a result of the eclipse and suggests a series of prophylactic measures in order to protect the population. The details given in the text²⁸⁵ show that Diego de Torres was competent in astronomy. Chabás and Goldstein have established that the data for this solar eclipse seem to derive from the table of eclipses in Zacut's *Hibbur*

280 López Piñero, 1979, p. 185. A similar syllabus can be found in a 15th century source: J. Paul de Fundis, in his commentary to the *Sphaera* of John of Holywood (Sacrobosco) says: "Ordo in tali scientia temptando talis est; nam primo legendum est de spera rotunda, 2º theoricam planetarum, 3º de algorismo, 4º de tabulis, 5º de compositione astrolabii et eius opere, 6º de iudiciis astronomie, 7º liber iudicum Ptolomei sive Halihenberagel, 8º A[l]pharganus et novo Almagesti; licet secundo ponebatur theorica planetarum sed illud erat pro majori introductione tabularum, unde et simul levi deberent." Quoted by Poulle, 1971 p. 34 fn. 1. See also Bustos, 1973. A similar case is that of the University of Pisa, analysed by Rutkin, 2015.

281 On Diego Ortiz see Chabás & Goldstein, 2000, pp. 13–15.

282 Cantera, 1931, pp. 380–381; Chabás, 2002, p. 91.

283 A detailed index of its contents can be found in Cantera, 1931, pp. 137–140; Chabás & Goldstein, 2000, pp. 165–166.

284 Amasuno, 1972.

285 Amasuno, 1972, pp. 67–69.

(see 7.7.3.4.1).²⁸⁶ As Chabás and Goldstein have established that Zacut's tables are based on the *Alfonsine Tables*, I will use the latter for my recomputations:

- 1) He begins by mentioning a conjunction of Saturn and Jupiter in Scorpio 23° the 25th November 1484. Using the *Alfonsine Tables* we can calculate for midday of the aforementioned date:

Saturn: 233;41,16°

Jupiter: 233;40,26°

- 2) There was a solar eclipse the 16th March 1485 which was almost total in Salamanca,²⁸⁷ in agreement with Zacut and Torres who state that 10 parts, out of 12, were eclipsed. According to both sources the true conjunction took place 2 hours and 20 minutes after midday while the middle of the eclipse was "slightly" after 3 h. in Torres (3;46h PM according to Zacut's table). The duration of the eclipse was one hour and forty minutes (1;48h in Zacut's table). The *Alfonsine Tables* give, for the aforementioned date, at 2h PM:

Sun: 5;25,8°

Moon: 5;31,57°

Node: 353;39,59°

The ascendant was Virgo in Salamanca and this agrees approximately with 3h PM in which we find the ascendant in 148;11,35°. ²⁸⁸ The eclipse was in the "house of death" (house VIII) which, at 3h. PM begins in 356;29,28°. According to the text the eclipse was at 5° and some minutes, which agrees with my recomputation of the conjunction using the *Alfonsine Tables*.

- 3) "The lord of the eclipse is Mars because it is stronger in the place of the eclipse and in the preceding cusp [house x]. In spite of its unfortunate

²⁸⁶ Chabás & Goldstein, 1998; Chabás & Goldstein, 2000, pp. 67 and 165.

²⁸⁷ Oppolzer, *Canon*, eclipse no. 6398, chart no. 128.

²⁸⁸ I am calculating the beginning of the ascendant and that of the VIII house assuming that they correspond to the latitude of Salamanca (41;19°), a solar longitude of 5° and 3h p.m., using John North's programme HOROSC. Chabás and Goldstein (1998, p. 8) do not reproduce the longitude of the ascendant but Chabás has sent to me a photograph of fol. 7v of ms. 110 of the Library of the Segovia Cathedral, which contains Zacut's table of solar eclipses. According to this table the position of the ascendant is Leo (Virgo in Torres). This implies that Zacut is giving the ascendant for about 2h p.m. (136;56°) while Torres calculates it for slightly after 3h p.m. (for 3h the ascendant will be 148;28,58°).

character it is moderated by Jupiter, which aspects it in opposition. [Mars] is in a sign of air, at 13° of Gemini”

This can be checked with the *Alfonsine Tables*. The 16th March at 2h PM the positions of Mars and Jupiter were:

Mars: 72;39,57°

Jupiter: 248;43,28°.

Diff. Jupiter–Mars = 176;3,31°

Mars is clearly in midheaven which, at 3h PM begins in 52;1,35°.

- 4) Mars is in sextile aspect to the place of the eclipse” and “it will return to the position of the eclipse the 16th May 1486”

Obviously $73^\circ - 5^\circ = 68^\circ$ which corresponds to a sextile aspect. The 16th May 1486, at midday, Mars is, according to the *Alfonsine Tables*, at 5;24,27°.

Also in Salamanca, we find the leading figure of Abraham Zacut (1452–1515) who did not teach in Salamanca University but had contacts with the astronomers living in the city, particularly with Juan de Salaya or Selaya, who occupied the chair of Astrology between 1464 and 1469 and who translated into Castilian, in 1481, Zacut’s great astronomical almanach *ha-Hibbur ha-gadol* (“The great composition”).²⁸⁹ It is also possible that he had contacts with Diego Ortiz de Calzadilla. After the decree of 1492 ordering the expulsion of the Jews from the kingdoms of Spain, Zacut had to move from Gata (where he stayed after leaving Salamanca) to Portugal and he seems to have been at the service of King João II (d. 1495) and Manuel I and he remained there until 1496 or 1497. As already explained, Diego Ortiz had left Castile and migrated to Portugal in 1476 where he occupied an important position in the court, and it has been suggested that he may have played an important role in Zacut’s entering the service of the king.²⁹⁰

The good reception of Zacut at the Portuguese court can easily be explained by the interest the kings of Portugal, as well as other members of the royal house, felt in astrological studies as well as by the fact that the most court astrologers had a Jewish- Castilian origin. Such is the case of Gedaliah b. Shlomo b. Yahyā (master Guedelha), who belonged to the Negro family,²⁹¹ and was, since 1428, the physician and astrologer of João I (r. 1385–1433), king Duarte

289 Edition of the Castilian text in Cantera, 1931, pp. 151–236.

290 Beaujouan, 1966, p. 77.

291 Nine generations of this family, of Castilian origin, had served Portuguese kings since Afonso Henriques (ca. 1109–1185) until Afonso V (1432–1481).

(r. 1433–1438), prince Pedro (1392–1449), Alfonso v (r. 1438–1481) and prince Henrique the Navigator (1394–1460). Astrology was common practice in the court: there is evidence of nativity horoscopes being cast on important occasions, such as the birth of the members of the royal family and events like the conquest of Ceuta (1415), and we know that the technique of elections was applied for choosing the propitious moment for Afonso v's accession to the throne in 1438, as well as for the ceremony of allegiance.²⁹²

Zacut's *Hibbur* is basically an astronomical work but it is also obvious that it was extremely useful for the computation of horoscopes, as it allowed obtaining the planetary positions with a minimum calculation. Besides this, the canons contain many astrological references of which I will give here a few examples: when he describes the table giving the entrance of the Sun in the zodiacal signs for a period of 136 years, he explains how to use this table to calculate the hour of the yearly revolution of nativities;²⁹³ in the chapter dedicated to the luni-solar conjunctions and oppositions, he begins by talking about astrological aspects (conjunction, opposition, trine, quartile and sextile);²⁹⁴ chapter 8 deals with the table giving the ascendant and the longitudes of the houses of the horoscope and explains the animodar attributed to Enoch/ Hermes (see 3.2.4) although he explains that he has checked this doctrine and found, by experience, that it is often wrong;²⁹⁵ finally, in the end of chapter 9 he discusses the famous topic of the 120 conjunctions which derives from the pseudo-ptolemaic *Karpós* and which interested also Abraham ibn 'Ezra (see 3.1.2.1.2.2).²⁹⁶

The same could be said about the *Almanach Perpetuum* (7.7.3.4.2), the abridged recension of the *Hibbur* prepared by Zacut's disciple José Vizinho, as it also contains tables for the computation of the tropical longitude of the twelve houses of the horoscope²⁹⁷ and for the calculation of the animodar (3.2.4).²⁹⁸

Apart from his astronomical works, Zacut also wrote on astrology. One of his astrological books receives the title of *Tratado breve en las ynfluencias del cielo* ("Brief treatise on the influences of the sky") written in Castilian in 1486 at

292 Avelar, 2011, 2014, 2015; Avelar & Campos, 2015. All these papers are accessible in academia.edu.

293 Cantera, 1931, pp. 159–160, 259–262.

294 Cantera, 1931, pp. 170, 269.

295 Cantera, 1931, pp. 183–184, 280–282.

296 Cantera, 1931, pp. 197 and 309–310.

297 See the facsimile of the Spanish edition of the *Almanach Perpetuum* printed in Leiria, 1496 published in Lisbon, 1986, pp. 71–73, 109–114.

298 *Almanach* pp. 100, 402–404.

the request of Juan de Zúñiga y Pimentel (d. 1504), the last master of the Order of Alcántara, in Gata (in the present province of Cáceres, Extremadura), whom he served in the quality of astrologer. The purpose of this text was to help the physicians of his patron and, consequently, a good part of it is concerned with medical astrology, although it also deals with meteorological astrology, the 28 mansions of the Moon, nativities and their yearly revolutions.²⁹⁹ This work is followed by an appendix, also dedicated to Juan de Zúñiga, on solar and lunar eclipses. In both texts Zacut mentions another, non extant, work with the title of *Juyzio del eclipse*, concerning the solar eclipse of the 16th March 1485, analysed by Diego de Torres. Finally Zacut is also the author of a short astrological Hebrew work entitled *Mishpetei ha-’išṭagnin* (“Judgements of the astrologer”) in which he analyses the consequences of a solar eclipse of 8 June 1518,³⁰⁰ a lunar eclipse of 6/7 November 1519³⁰¹ and a planetary conjunction in 1524.³⁰² These dates are in apparent conflict with the presumed date of Zacut’s death (1515) but the aforementioned eclipses and conjunction could easily have been computed in advance.

3.2 Thematic Surveys

3.2.1 Simple Systems of Astrological Prediction

3.2.1.1 The “System of the Crosses” (*ṭarīqat aḥkām al-ṣulub*)

Al-Baqqār’s *Kitāb al-amṭār wa l-as’ār*, already mentioned and used in 3.1.1.5.2, contains, among many other materials, 39 verses of an *urjūza* (poem written using the *rajaz* metre) by ‘Abd al-Wāḥid ibn Ishāq al-Ḍabbī (fl. Algeciras and Córdoba ca. 788–ca. 852, see 3.1.1.1),³⁰³ dedicated to emir al-Ḥakam I (r. 796–822) and dealing with the prediction of atmospheric events and the vicissitudes of kings. This poem is the oldest known Andalusī astrological text and a versification of chapter 57 of the Alfonsine *Libro de las Cruces*.³⁰⁴ Its antiquity,

299 Carvalho, 1927; Carvalho, 1947. The medical part has been reproduced by Cobos, 2000. A summary of its contents in Cantera, 1931, pp. 77–78, 88–93.

300 See Oppolzer, *Canon*, no. 6474 chart 130; Mucke & Meeus, 1983, pp. 465 and 840. It appears in Zacut’s list of solar eclipses: Chabás & Goldstein, 1998, p. 8 and 2000, p. 67.

301 See Oppolzer, *Canon*, no. 4212; Mucke & Meeus, 1983a, p. 189. It appears in Zacut’s list of lunar eclipses: Chabás & Goldstein, 1998, p. 7 and 2000, p. 66.

302 There was a conjunction of Saturn, Jupiter and Mars at about 340° the 5th February 1524, according to the *Alfonsine Tables*. See Chabás and Goldstein, 2000, pp. 15 and 173.

303 Edited in Samsó, 1983a; on the biography of al-Ḍabbī see Samsó, 2001a. A new edition in Guesmi & Samsó, 2018, pp. 202–204 and 54–55.

304 Ed. Kasten & Kiddle, 1961, pp. 159–160. Quite recently Charles Burnett (Burnett, 2011a) has identified a new manuscript of the Alfonsine work (Vienna, Österreichische

the fact that it was composed in a period of history in which Eastern astronomical texts had not yet reached al-Andalus, the references made both by the Castilian translation and al-Baqqār's book explaining that the system was the one used by the ancient Romans of al-Andalus, Ifrīqiya and the Maghrib and the primitive techniques used for astrological predictions led both my master, Juan Vernet, and myself to assume a late Latin origin to this system.

The poem makes very simple predictions, related to weather and crops, based on the position of the unidentified "indicators" in zodiacal signs corresponding to the four triplicities (water, earth, air, fire) or in combinations of them (earth-water, fire-air). The method used corresponds to the "system of the crosses", previously known only through the Alfonsine translation of a later revision of an Arabic source written in the 11th c. (after the Saturn-Jupiter conjunction of year 459/1066–67, quoted in the text) by a certain 'Abd Allāh b. Aḥmad al-Ṭulayṭulī.³⁰⁵ Since 1971 we know of the existence of two Arabic works which deal with the system extant in manuscripts Escorial 916³⁰⁶ and 918.³⁰⁷ The former contains the full text of al-Baqqār's "Book on rains and prices" (3.2.6), already mentioned, the first part of which is related to the astrological techniques of the "system of the crosses". Its contents correspond rather closely to chapters 1, 4, 5, 6, 57, 60, 61, 62, 63, 64 and 65 of the Alfonsine book.³⁰⁸ Ms. Escorial 918 only contains chapters 60–62. The equivalence shows that al-Baqqār was using a source related to the aforementioned revision made by 'Abd Allāh al-Ṭulayṭulī, the *Oveidalla* of the Alfonsine translation.

The predictions of the "system of the crosses" are based on few variables, that can be easily listed using combinatorial techniques. With this method the astrologer will have to establish the variables that concern a particular case and read the corresponding prediction already given to him in one of the lists, without having to bother about the computation of planetary longitudes, the division of houses and other technicalities of standard astrology. In spite of the fact that the Castilian manuscript used for the edition of the *Libro de las Cruces* by Kasten and Kiddle is obviously a manuscript copied in the royal *scriptorium* for the king himself, the system of prediction used makes me think of a kind of popular astrology used by cheap astrologers who work in the *sūq* of a city.

Nationalbibliothek, Hebr. 199, second part, fols 1v–47r.), written in Castilian but in Hebrew script.

305 Castells, 1992.

306 Vernet, 1971; Samsó, 1979; Samsó, 1985–86.

307 Muñoz, 1981. Incidentally, ms. Escorial 939 contains one page (fol. 34v) with two short chapters (on the division of al-Andalus between the zodiacal signs and between the planets) related to the "system of the crosses".

308 Guesmi & Samsó, 2008, pp. 40–55, 192–204.

The aforementioned system deals mainly with weather prediction,³⁰⁹ periods of draught and rains and their influence on the evolution of prices of agricultural products using mainly the positions of the superior planets. The horoscopes are presented in a format different from the standard ones: three straight lines intersecting in the centre of the figure and determining twelve positions. Six of them correspond to the ends of the lines (*aṭrāf al-khuṭūṭ*, “estacas” in the Castilian text), while the other six are the angles (*zawāya*, “ángulos”) between the straight lines. The signs of fire (Aries, Leo, Sagittarius) and air (Gemini, Libra, Aquarius) are placed at the end of the straight lines, while the signs of earth (Taurus, Virgo, Capricorn) and water (Cancer, Scorpio and Pisces) correspond to the angles. The same diagram can be used to represent the horoscope proper: the upper end of the vertical diameter is the place of the ascendant (1st house) and the following houses will appear, anticlockwise, at the end of the lines (houses 3, 5, 7, 9, 11) and in the angles (houses 2, 4, 6, 8, 10, 12), (see Figs. 3.1, 3.2 and 3.3).

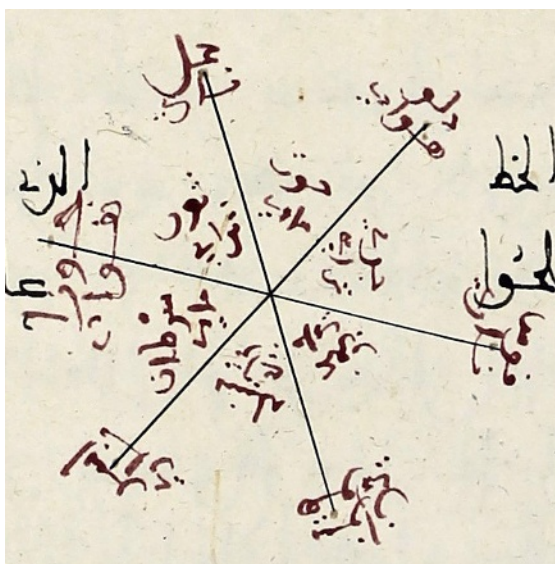


FIGURE 3.1
The horoscope of the crosses in
ms. Escorial 918

309 On weather prediction see G. Bos y C. Burnett, *Scientific Weather Forecasting in the Middle Ages: The Writings of al-Kindī*. London, 2000, although its contents bear no relation to those of the *Kitāb al-amṭār*. See also C. Burnett, “Weather Forecasting in the Arab World” in E. Savage-Smith, *Magic and Divination in Early Islam*. Ashgate-Varioum. Aldershot, 2003, pp. 201–210.

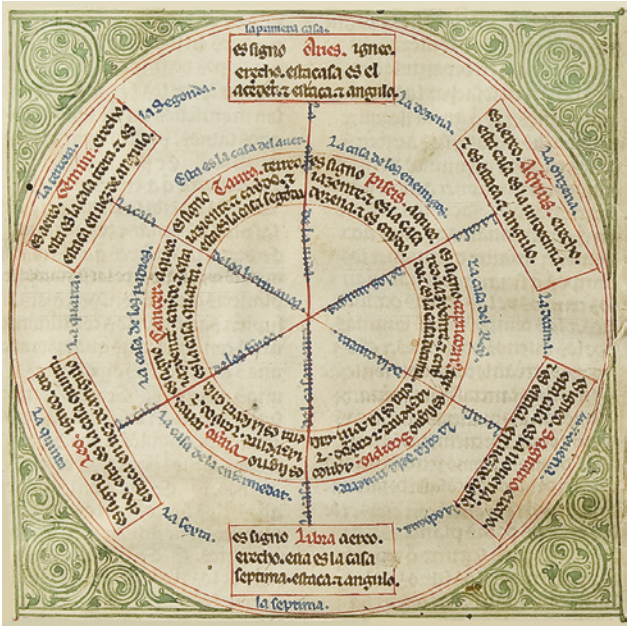


FIGURE 3.2
The horoscope of the crosses in ms. Madrid BN 9294

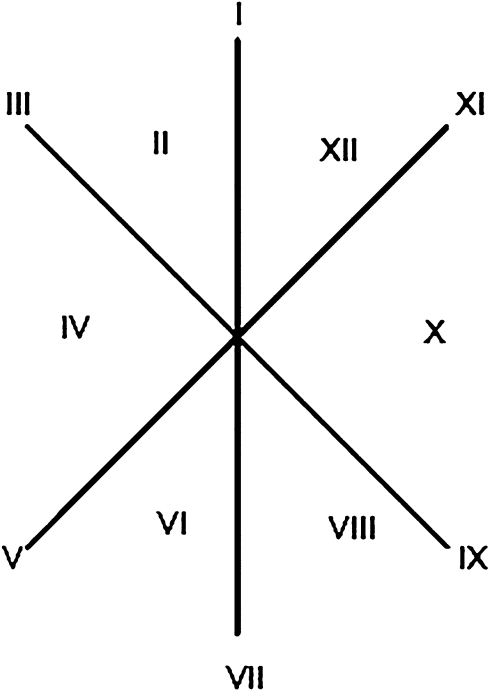


FIGURE 3.3
Diagram of the horoscope of the crosses

The system of the crosses tends to simplify astrological technicalities and identifies houses with zodiacal signs. It uses standard aspects such as quartile, together with others which are different such as the *iẖtirāq* (“quemazón”) which should not be identified with the “combustion” of classical astrology: there is *iẖtirāq* when the planets are together in the same sign or scattered either in even or in odd houses,³¹⁰ or the *ighlāq* (“lock”), in which the planets are *mughlaqa* (locked) when they retrograde. The planets can also be *mutasatṯara* (on line) when you can find them in four consecutive signs. When no sign (of these four) is empty, the disposition is called *mughlaqa* (locked); if there is an empty sign, it receives the name of *mughlaqa maftūha* (locked and open). The text also speaks about aspects, called *ittiṣāl* (application) although the Alfonsine translation uses the term “catamientos” which seems a translation of *naẓar*. These aspects are used when the author tries to establish which of the two superior planets (Saturn and Jupiter) dominate when they are in trine of quartile. A new aspect is also introduced – besides quartile, trine, conjunction and opposition – because the text speaks, strangely, about a “septile” (*tasabbuʿ*).

There are, sometimes, evidences of the practice of a more developed astrology, such as passages which mention dignities (*huzūẓ*), dejections (*suqūṭ*), aspects between propitious and malefic planets, opposition and conjunction.³¹¹ This may imply that such passages could correspond to additions made by ‘Abd Allāh al-Ṭulayṭulī who quotes other sources independent of the system of the crosses. Thus, in chapter 63 of the *Libro de las Cruces* (in a passage also preserved by al-Baqqār³¹²), the text quotes an Eastern source (*baʿḍ al-mashriqīyyīn*) or (in al-Baqqār’s book only) a quotation of the *Kitāb al-ʿarḍ fī l-asʿār* of Hermes al-Ḥakīm.³¹³ This kind of more sophisticated astrology appears in what seems to be the remnant of a possible horoscope, indicating a heavy rainfall:³¹⁴

Saturn in the House of Illness (VI) which is Scorpio (210°–240°)
 Sun in the House of Brethren (III) which is Leo (120°–150°)
 Jupiter in the House of Happiness (XI) which is Aries (0°–30°)
 Ascending node in the House of Life (I) which is Gemini (60°–90°)

310 Poch, 1980.

311 Cf. Kasten & Kiddle, 1961, chap. 5, pp. 11 a y b, 12 a.

312 Guesmi and Samsó, 2018 pp. 197–198 (section [19]).

313 Guesmi and Samsó, 2018 pp. 198 (section [21]), 224 (sections [99]–[100]).

314 Analogous information, although less complete, in Kasten & Kiddle, 1961, chap. 65, p. 167a, 2nd paragraph.

Bearing in mind the possible date in which ‘Abd Allāh al-Ṭulayṭulī lived, the aforementioned data could correspond to the 1st August 1015 in which the planetary positions, according to al-Khwārizmī’s *Zīj*, were:

	<i>True long.</i>	<i>Mean long.</i>
Saturn	234;28°	238;36°
Jupiter	33;30	19;53
Sun	125;58	127;39
Asc. node	77;14	

3.2.1.2 The *Alchandreana*

Millàs Vallicrosa’s *Assaig* (1931) contained, among many other things, a critical edition of the so-called “old corpus” (Kunitzsch) or “old collection” (Burnett) of Latin texts, based on Arabic sources, on the construction and use of the astrolabe, the astronomical quadrant and a few other astronomical instruments. Surprisingly there are very few astrological materials in them. The answer to this puzzle has been given by David Juste, with his edition of the *Alchandreana* collection,³¹⁵ which consists of a series of eight astrological texts which share common techniques of prediction: the data of the horoscope are calculated using numerological procedures (numerical values of the letters forming the name of the subject of the horoscope) and the prediction is based on isolated elements (the onomatomantic ascendant, the planetary hours, the position of the planets in the triplicities or in the lunar mansions etc.). Thus, the *Alchandreana* is another instance of the existence of a kind of popular, simplified astrology, like the “system of the crosses”, although both sources have very little in common.

The books contained are either anonymous or attributed to a mysterious writer called Alchandreus. Apart from him, the other authorities quoted are Alexander Macedo, Ascalu Hismaelita, Argafalau/ Arfarfau Caldeus and Aluaten Sarracenus. Quite convincingly Juste establishes a relation between Ascalu and the 18th lunar mansion (*al-Qalb* spelt *Alcalu* in several sources of the collection), Argafalau with mansion 26th (*al-Fargh al-Awwal*, *Algarfalaui*) and Aluaten with mansion 28th (*Baṭn al-Hūt*, *Aluaten*). As for Alchandreus, Juste seems to tend towards the identification of this name with a corruption of al-Kindī but, given the degree of creativity shown by the author of these texts who invents names of scholars very similar to those of lunar mansions, I will repeat a previous hypothesis I made a few years ago when reviewing one of the papers published by Juste on the *Alchandreana*. Bearing in mind that one

³¹⁵ Juste, 2007.

of the derivations of the corpus is the so-called *Liber Arcandam*, very popular from the 16th c. onwards, I believe that Alchandreus could be a corruption of the *Arkand*, an Indian astronomical book known with this name in Arabic: the name, at least, circulated in al-Andalus towards the beginning of the 9th c.: it is one of the sources brought from the East by ‘Abbās ibn Nāṣiḥ for al-Ḥakam I (r. 796–822).³¹⁶

The eight texts contained in the collection are:

- 1) *Liber Alchandreī*;
- 2) *Epistola Argafalau ad Alexandrum*;
- 3) *Breviarium Alhandrei summi astrologi*;
- 4) *Quicumque nosse desiderat legem astrorum*;
- 5) *Proportiones competentes in astrorum industria*;
- 6) *In principio fecit Deus caelum et terram*;
- 7) *Benedictum nomen Domini*
- 8) a collection of *Fragmenta Alchandreana* (22 chapters extant in several mss which do not appear in the other works).

Of these, Juste considers that 6) and 7) are “vulgar” texts consisting in translations or adaptations of Arabic originals, while 1), 2), 3), 4) and 5) are more elaborate versions based on previous translations from the Arabic but which also use other sources. It is very difficult to establish the Arabic sources used by the authors of the collection and it is only clear that they derive from a milieu in which we can find *Sarraceni*, *Iudaei*, *Barbari*, *Romani/Christiani* and *Melli* (*mawālī*?) (*Benedictum*). Place names of the Iberian Peninsula (*Corduba*, *Serragoza*, *Ciscitra*?) appear in the *Proportiones*. Similarities with an Andalusī source on the making of talismans related to the lunar mansions and dated towards the middle of the 10th c. (Ibn al-Ḥātim’s *De imaginibus*) have been underlined by Juste in the *Benedictum*. To this one should add the use of a Maghribī alphanumerical system (*abjad*) and the Western names of the planets: *al-Muqātil* (Saturn), *al-Aḥmar* (Mars) and *al-Kātib* (Mercury), as well as the names of zodiacal signs, also common in al-Andalus: *al-Kabsh* (Aries) and *al-Taw’amān* (Gemini). All this points to an Andalusī source or group of sources which circulated in Córdoba towards the middle of the 10th c. in a moment in which, under the conservative Caliphate of ‘Abd al-Raḥmān III (912–961), astrology only survived in a secret way and books like the *Ghāyat al-ḥakīm/Picatrix* (with a strong interest in lunar mansions) and the *De imaginibus* were written.³¹⁷ The atmosphere of the period fits very well the non-mathematical

³¹⁶ Makkī, 2003, p. 278; Sp. Translation Makkī & Corriente, 2001 pp. 169–170; Forcada, 2004–5, pp. 20–22.

³¹⁷ Oliveras, 2009.

character of the astrology represented in the *Alchandreana* collection. This is not contradicted by the fact that Juste has found a few references to Eastern astrological sources: Māshā'allāh's (fl. 762–809) *Liber de interrogationibus*, for example, is identified with the *Breviarium* in one of the manuscripts, while two manuscripts of the *Proportiones* consider that this book is Māshā'allāh's *Liber iudiciorum*. As for actual sources and, apart from the Latin ones (Martianus Capella, *Excerpta Plinii*, *In quo signo versetur Mars*, computus treatises), there are parallels with the *Secretum secretorum*, a Syriac medical compilation, the Mandeian Book of the Zodiac and the Book of Nativities attributed to Abū Ma'shar (d. 886). All this forms a bulk of information which was available in Córdoba at that time, or in the Christian lands where the translation/ adaptation took place.

Juste has argued that the collection originated in Catalonia towards the end of the 10th c., before the "old collection" of texts on the astrolabe and other instruments was compiled. He refers to the analysis made by Bernard Bischoff of the language of *In principio* (a bad Latin contaminated by the influence of a Romance dialect which he places in Catalonia). The examples mentioned by Bischoff do not seem very conclusive to me and I find a much stronger argument in Juste's analysis of the *Proportiones*. This text is particularly interesting because it contains the only reference, in the whole corpus, to a translation from the Arabic:³¹⁸ the author has not been able to find what he is going to explain (astrological predictions based on the vision of comets) in Christian or Hebrew sources, but in Arabic texts translated into Latin by the wisest master of the Arabs as he was instructing him.³¹⁹ On the other hand this book contains a date: an example is given on the computation of planetary longitudes for year 4715 (although in the process of the calculation he uses 4795 systematically) *ab initio mundi*. If the Jewish chronology (year of Creation 3761 BC) is used, this gives $4715 - 3761 = 954$, which fits nicely what one expects, given the fact that the oldest MS (Paris BNF 17868), containing five treatises of the *Alchandreana*, is dated towards the end of the 10th c. Besides, the *Proportiones*, a revision of the *Benedictum*, and the so-called "Compilation of El Burgo de Osma" (one of the mss. containing the *Fragmenta*) seem to have been written by the same author. They all use an uncommon vocabulary which, apparently, derives from the Glossaries compiled at the Monastery of Ripoll and the style of this text coincides fully with the extant works of Miró Bonfill (d. 984), bishop of Gerona since 971 and the correspondent of Gerbert of Aurillac who, in 984, asked from

318 Juste, 2007, pp. 69, 554.

319 Actually the text refers to a Chaldean source and to "a sapientissimo Caldeorum magistro" which should be interpreted as Arabic. See Juste, 2007, p. 554.

him a copy of the *De multiplicatione et divisione numerorum* written by a mysterious Joseph Sapiens. Miró Bonfill would, therefore, be the author of the *Proportiones* and of the two other related texts. Besides, Juste considers that the traditional identification of the *Liber de astrologia* which Gerbert asked, also in 984, from its “translator” (*translatum a te*), Lupitus Barchinonensis (fl. 975–995), with one of the Latin texts on the astrolabe is unjustified and that a better hypothesis would be to identify it with the three first treatises of the Alchandreana collection (*Liber Alchandreï*, *Epistola Argafalau* and *Breviarium Alchandreï*), given the fact that the first, at least, of these books bears the title of *Liber de astrologia*. In case this very reasonable hypothesis proved to be true, Lupitus would be the name of another of the authors of the collection. There are, therefore, at least two Christian scholars (Miró Bonfill and, perhaps, Lupitus), an Arab (who helped Miró with the translation) and, without any doubt, a Jewish collaborator who, in Juste’s opinion, made a serious effort to hebraize the Arabic sources used by the compilers of the collection. This explains the appearance in it of the Hebrew alphabet used as an alphanumerical system and the Hebrew names of the planets, and zodiacal signs which we find, especially, in the *Liber Alchandreï*, but also in the *Breviarium*, *Quicumque* and *Benedictum*.

It is interesting to follow Juste in his analysis of the technical contents of the collection:³²⁰ periods of revolution of the planets, domiciles, exaltations and falls, planetary apogees, *Thema mundi*, astrological characteristics of the planets, zodiacal signs and triplicities, lunar mansions, houses of the horoscope identified with zodiacal signs as in the “system of the crosses”, “Egyptian” terms, planetary and zodiacal hours, sign and planet of birth, numerical alphabets and prognostications based on single elements, such as the sign or planet at birth, the planetary hour (for interrogations), the ascending sign at the moment of interrogation, the ascending triplicity, the position of the planets in the houses, the four quadrants or the triplicities, the Moon in the lunar mansions etc. The computation of the planetary positions has an astronomical logic, as it uses the planetary periods of revolution and the number of years since the moment of the creation of the world, for which an horoscope (with the position of the planets in the signs) is supposed to be known: as a matter of fact, two different horoscopes are attested in our sources. The procedure appears in two variants which Juste calls “System A” and “System B” (no relation whatsoever to the systems with the same denomination in Babylonian astronomy).³²¹ The method explained by Juste gives a clear solution of a problem that worried me many years ago: how could astrologers, who worked in

320 Juste, 2007, pp. 99–217.

321 Juste, 2004.

al-Andalus towards the beginning of the 9th c, cast an horoscope in a moment in which there were no astronomical tables available.³²² Juste seems to have solved the problem quite definitely.

In many instances, however, the procedures explained in these texts to determine the ascendant (and therefore the signs corresponding to the houses), the sign and planet of birth are independent of any astronomical consideration and base themselves on numerological speculations. This in spite of the fact that the system of zodiacal hours contains the embryo of a vaguely approximate method to determine the ascendant and that the *In principio* seems to be aware of a more sophisticated kind of astronomy when it explains the stations and retrogradations of the superior planets as a function of their elongation from the Sun. It appears that we have here an instance of the use of a different kind of source from those standard in the *Alchandreana*. There are also some instances in which astrological predictions take into consideration several predictive elements, something which bears witness to a more developed kind of astrology.

3.2.1.3 Ramon Llull and His *Liber de nova astronomia*

In October 1297 Ramon Llull finished in Paris his *Liber de nova astronomia* which, in spite of its title, deals with astrology and not with astronomy. The text is extant both in Latin³²³ and in Catalan.³²⁴ It appeared thirty eight years after the Alfonsine *Libro de las Cruces* (completed the 26th February 1259) and, in spite of the great differences between the two books, they both share a common interest in simplifying the complexities of astrological prediction. In spite of this, it seems that they have different origins: while the *Libro de las Cruces* is clearly a translation from the Arabic, the *Liber de nova astronomia* is an entirely original work which should be analysed in the context of the whole Lullian project to engage in a formulation of a new science based on a few essential principles. In fact Llull criticises the “astronomers” of the past because their predictions are not based in solid principles and this is why they often disagree with actual facts.³²⁵ In spite of this it is possible that Llull’s astrological principles were based on some kind of popular Arabic astrology of which he might have become acquainted during his stay in Tunis in 1292.³²⁶

322 Samsó, 1985.

323 Edited by Pereira, 1989.

324 The text was edited by Jordi Gayà (Gayà, 1981) with the collaboration of Lola Badia. A more recent edition in Badia, 2002 (with a long introductory study and including a transliteration of the text in modern Catalan spelling). French translation in Llinarès, 1988.

325 Badia, 2002, pp. 254–256.

326 Later, in 1307 he stayed in Bijāya and, in 1314–15, in Bijāya and Tunis.

Llull considers that astrology is “not necessary”, meaning that astrological predictions do not necessarily become true, among other reasons because God can easily alter the planetary influences.³²⁷ In spite of this he tries to define an extremely simple astrological system:³²⁸ he does not take into consideration the ascendant or the zodiacal houses of the horoscope, which are replaced by the signs.³²⁹ His starting point considers the four classical triplicities, to which he relates the four letters A, B, C, D. Each triplicity is characterised by two qualities and Llull considers that one of them is the element’s own quality (marked in bold and italics in Table 3.7) and, thus, is stronger than the other, appropriated one. Each sign is divided in three decans, like in standard astrology, to which the letters E, F and G are assigned. The nature of the planets³³⁰ also has the same characteristics as the triplicities, according to a scheme which is also standard.³³¹ The only exception is Mercury which has a mixed nature (“Mercuri es planeta migensera”)³³² and this has led Frances Yates to consider that the four letters (ABCD) correspond to this planet.³³³ Other concepts also derived from traditional astrology, such as domicilia of the planets and aspects (“esguardaments en biaux”) are also used by Llull, although the theory of aspects is not clearly developed in this work and only conjunctions and oppositions seem to be clearly mentioned.³³⁴

TABLE 3.7 Elements, signs and planets

Letters	Elements	Zodiacal signs	Planets
B	Fire (dry and <i>hot</i>)	Aries, Leo, Sagittarius	Sun, Mars
C	Earth (<i>dry</i> and cold)	Taurus, Virgo, Capricorn	Saturn
A	Air (<i>moist</i> and hot)	Gemini, Libra, Aquarius	Jupiter
D	Water (moist and <i>cold</i>)	Cancer, Scorpio, Pisces	Moon, Venus

327 Badia, 2002, pp. 256–258.
328 Samsó, 1981–83; Bonner, 1981–83; Llinarès, 1987; Pereira, in « Introduzione » to Pereira, 1989; Badia, « Introducció » to Badia, 2002. Other studies on Lullian astronomy and astrology in Millás, 1962; Vernet, 1951–52.
329 Badia, 2002, pp. 163–168.
330 Badia, 2002, pp. 168–178.
331 Bīrūnī’s *Taḥḥīm*, pp. 231–232.
332 Badia, 2002, p. 175.
333 Yates, 1954, p. 119. Yates’ scheme is also reproduced in Badia, 2002, p. 220.
334 Badia, 2002, pp. 251–253.

Basing himself in the aforementioned principles, Llull's purpose in analysing a particular configuration of the sky is to establish which quality will be predominant and, consequently, which kind of prediction can be formulated. This is the basis of the *devictio* or *vensiment*.³³⁵ An example, extracted from Llull's casuistry, will clarify his method:³³⁶

A conjunction of Jupiter, Mars, Sun and Venus takes place in the sign of Aries. This implies:

- 3 dry qualities (Aries, Mars, Sun)
- 4 hot qualities (Aries, Mars, Sun, Jupiter)
- 2 moist qualities (Venus, Jupiter)
- 1 cold quality (Venus)

With such data the prediction should determine whether the dominant planet is Venus and Jupiter (both beneficial) or Mars (maleficent), as the solar influence is ambiguous.³³⁷ At first sight the situation seems to favour Mars, which shares a nature B with the Sun and the sign of Aries. In spite of this, one should bear in mind that Jupiter has a certain influence, as a result of the presence of four hot qualities, although heat is the appropriated quality of the planet, while humidity is its own quality and the two moist qualities are counterbalanced by three dry qualities. The influence of Venus is much smaller as its own quality is cold (only 1 cold quality in front of 4 hot qualities) while humidity is its appropriated quality (2 moist qualities in front of 3 dry qualities). As a result, Llull concludes that "in the constellation there is more evil than good".

It seems clear from the example that Llull is trying to give a set of simple rules that allow to reduce the analysis of an horoscope to a pure mechanical procedure. This can be done easily by using the instrument described by Llull³³⁸ and wrongly interpreted in the illustration appearing in fol. iv of manuscript Add. 16434 of the British Library (see Fig. 3.4). The original disposition of the instrument, according to Llull's text, has been proposed by Frances Yates³³⁹ (Fig. 3.5). The figure consists of eight concentric circles the outer one divided into the twelve signs of the Zodiac, with an indication of its nature (A,B,C,D) appearing in each one. The seven inner circles can be rotated independently and they correspond to the five planets, the Sun and the Moon, each one

335 Badia, 2002, pp. 179–182.

336 Badia, 2002, p. 227.

337 Badia, 2002, p. 173.

338 Badia, 2002, pp. 221–222, 265.

339 Yates, 1954, p. 119.



FIGURE 3.4
Lull's instrument
in fol. 1v of
manuscript
Add. 16434 of the
British Library

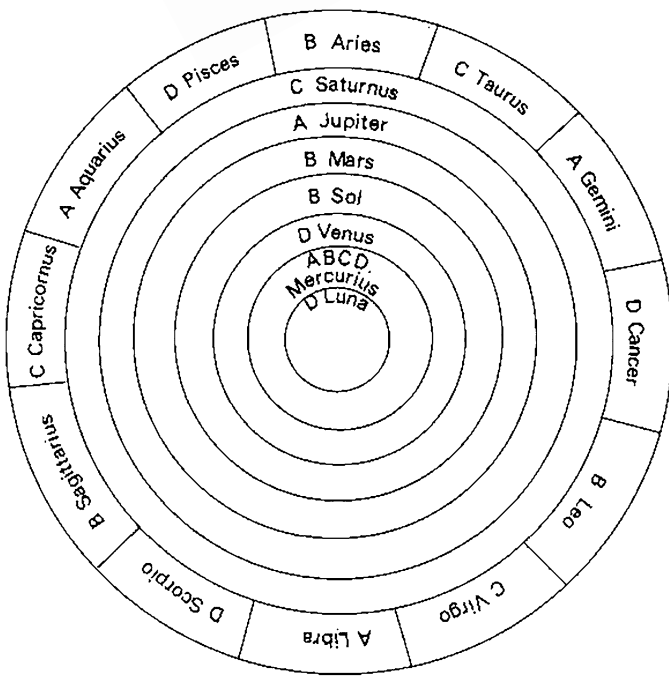


FIGURE 3.5
Lull's instrument
in Frances Yeats'
reconstruction

bearing the indication of the corresponding nature. Obviously the procedure consists in rotating the circle corresponding to each planet until the name of the planet is placed in front of the sign of its location. In this way, we will be able to obtain a graphical configuration of the horoscope and we will see immediately the natures and predominant qualities. To this one should add that such graphical configuration will also show the planetary aspects.

It is obvious that both Llull's method and instrument can also be applied to medicine and to medical astrology. The four natures symbolised by the letters ABCD are to be found not only in the four elements of nature, the zodiacal signs and the planets, but also in the four humours of the human body (A. Blood; B. Yellow bile; C. Black bile; D. Phlegm), as well as in simple and compound drugs. When facing an illness, one can easily calculate the nature and the graduation of the drug taking in account the analysis of the horoscope of the sick person.

3.2.2 *Standard Astrology Based on Classical and Eastern Arabic Sources:* *ʿAlī ibn Abī l-Rijāl*

3.2.2.1 A Biographical Sketch

In a period of the history of the Maghrib in which we have practically no information about the development of astronomy and astrology, it becomes a real surprise to discover the existence of an author of the importance of Abū l-Ḥasan ʿAlī b. Abī l-Rijāl al-Kātib al-Shaybānī, born in Tāhart but educated in Qayrawān. He was in charge of the secretarial office of Bādīs b. al-Manṣūr (r. 996–1016) who charged him of the education of his son al-Muʿizz b. Bādīs (r. 1016–1062). Ibn ʿIdhārī states that he taught the young prince the principles of mālikism and he says that Ibn Abī l-Rijāl was *wazīr* of either Bādīs or al-Muʿizz (Ibn ʿIdhārī's sentence does not seem clear to me) and he seems to charge Ibn Abī l-Rijāl with the intellectual responsibility of the persecution and death of the Fāṭimids which took place during al-Muʿizz' reign.³⁴⁰ The date of death commonly ascribed to Ibn Abī l-Rijāl (426/1034–35)³⁴¹ is doubtful for the following reason:

Ibn Abī l-Rijāl was also the astrologer of al-Muʿizz who ordered him to cast the horoscope of the Kalbī emir of Sicily Aḥmad b. Abī l-Ḥusayn's and to predict the length of his reign (d. 427/1035–36). Al-Muʿizz had told him of the date and hour of his arrival to power (Muḥarram 410/ 9th May–7th June 1019)

340 Colin & Levi-Provençal, 1980, I, 273–274.

341 Idris, 1962, see II, 810–811 where the author remarks that he has been told by Ḥasan Ḥusnī ʿAbd al-Wahhāb that Ibn Abī l-Rijāl gravestone is extant and that it bears an inscription with the date 426H. On Ibn Abī l-Rijāl see also Idris, 1962, I, 273 and II, 525–526, 784–785.

which implies that Ibn Abī l-Rijāl used this date in his horoscope. He calculated that the ascendant was Cancer 20° and predicted that he would reign during 17.5 solar years (RHL, 89r; BL, 138b; *Conplido* I, 145).³⁴² Ibn Abī l-Ḥusayn was killed one week after the predicted date (*qutila wa-kāna min amri-hi mā kāna*): al-Mu‘izz had sent a great expedition, commanded by his son ‘Abd Allāh, to Sicily in 427/1035–36. ‘Abd Allāh conquered Palerm and besieged Aḥmad ibn Abī l-Ḥusayn in his palace. Some Sicilians, who were in favour of the Zīrid emir, murdered Ibn Abī l-Ḥusayn and sent his head to ‘Abd Allāh who sent it back to al-Mu‘izz.³⁴³ This horoscope and the fact that Ibn Abī l-Rijāl knew the date in which the Kalbī emir died in 427/1036 makes me doubt about the evidence furnished by his gravestone.³⁴⁴ Besides, the last chapter of Book VIII (RHL 265r; BL, 380b; *Conplido* II, 326–327) states that Ibn Abī l-Rijāl only added this chapter (*Fī inqīḍāḍ al-kawākib*, “fall of stars” which the Alfonsine translation identifies with “comets”) after checking its validity through experience (*‘indamā ṣaḥḥa lī bi l-tajriba*) of the death of Ibn Abī l-Ḥusayn in 1036 and of Sa‘īd b. Khazrūn (429/ 1037–38).³⁴⁵ This gives us an approximate date for the completion of the book and establishes that he could not have died in 426/1034–35.

3.2.2.2 The *al-Kitāb al-Bārī’ fī aḥkām al-nujūm*

3.2.2.2.1 The Sources of the *Bārī’*

My master, Juan Vernet, emphasized, in 1978, the importance of the large library Ibn Abī l-Rijāl’s had at his disposal³⁴⁶ when he wrote the *Bārī’*, which is

342 In this chapter I will quote Ibn Abī l-Rijāl’s *Kitāb al-Bārī’* using ms. Rabat Hassaniyya Library 3503 (RHL hereafter) as well as the Castilian Alfonsine translation edited by Hilty, 1954 and 2005. My reading of these two sources has convinced me of the fact that the Alfonsine version is a very faithful and reliable translation. Besides, in some instances I have also checked ms. British Library Add 23399 (BL) which is not foliated and I will quote the number of the photograph followed by *a* (right page of the photograph) or *b* (left page). On the Alfonsine translation see Nykl, 1954 and Hilty, 1955.

343 Idris, 1962, I, 157–170 and II, 811.

344 Pingree in *El2*, vol. III, p. 688.

345 Idris, 1962, pp. 106, 164, 165, 811. According to this source, al-Mu‘izz invited Ibn Abī l-Rijāl to calculate the length of life not only of Ibn Abī l-Ḥusayn, but also of Sa‘īd b. Khazrūn b. Sa‘īd who reigned in Tripoli (Libia) until his death. This reference appears in the *Bārī’* in the last chapter of book VIII: in RHL 265r the name appears as Sa‘īd Khazrūnah; in BL (380b) I read Ḥardūn; finally in ms Teheran 6240 (305a) the name is correctly transcribed: Sa‘īd b. Khazrūn. In the Alfonsine translation (Hilty, 2005, 326–327) the name is clearly corrupt (*Aged, fijo de Abenligeron*) but Gerold Hilty (private communication) tells me that, in agreement with the Judeo-Portuguese and Latin translations it should be amended to *Zaed fijo de Aben Hazron*.

346 Vernet, 1978b, pp. 200–203. Reprinted with a different title in Vernet, 2006, pp. 308–313. This important book has been the object of German, French, Arabic and Polish translations.

a book full of quotations of old Greek and Iranian sources as well as of authors of the Islamic period who wrote their works between the end of the 8th and the first half of the 10th c. On the whole, and in spite of the fact that this book marks the origin of a Maghribī astrological tradition, one must realise that nothing seems to be Western in the *Bārī*³⁴⁷ which is clearly deeply rooted in the Mashriqī tradition of astrology. This leads us to wonder about the veracity of the famous identification of Ibn Abī l-Rijāl with the Abū l-Ḥasan al-Maghribī who, according to Ibn Qiftī,³⁴⁷ was one of the astronomers who observed the summer solstice and the autumn equinox in Baghdad in 378/988, under the direction of al-Kūhī and the patronage of Sharaf al-Dawla. Whatever the case, it is reasonable to imagine that Ibn Abī l-Rijāl had made the *riḥla* to some important cities of the Mashriq, where he was able to read or buy many of the books he quotes. His use of sources is clearly documented in several prologues to the books of the *Bārī* in sentences like: “I have collected in this book the [main] concepts (*maʿānī*) of Astrology as well as its peculiar secrets, selected from many books written by scientists, to which I have added the results of my own thoughts as well as those of my experience (*tajribatī*)” (RHL 3503 fol. 1v, BL Add. 23399, 4a). A similar expression can be found at the beginning of book IV (RHL 3503 97r, BL Add. 23399, 150a) to which he adds: “Chapter on bringing up children (*tarbiyya*): in this chapter as well as in others, I will mention things which are not well known and which, in the books of ancient scholars, were only hidden symbols, to which I add other things that can be known through the rest of the books which deal with this art.”³⁴⁸ This does not mean that the *Bārī* is a mere compilation of quotations from other sources: quite often the doctrines explained are expressions of Ibn Abī l-Rijāl’s own thoughts or, when mentioning somebody else’s opinions, we find that they are followed by our author’s critical remarks.

In the following pages I will try to make a list of the authorities mentioned by Ibn Abī l-Rijāl. This should be followed by a systematic attempt to compare his quotations with the original sources and some pioneering work has been recently done, in this respect, by Montse Díaz-Fajardo³⁴⁹ and Aina Puig-Pey,³⁵⁰ apart from many others. This kind of research has shown that our author also uses sources he does not mention such as, for example, Ibn Hibintā’s *Mughnī* only quoted once (RHL 262r) although used in many other passages.

Besides general reference to the Indians (*al-Hind*), Iranians (*Fāris*, *al-Furs*), Egyptians, Babylonians (*ahl Bābil*) and Greeks, I have tried to collect a list of

347 Lippert, 1903, pp. 351–353.

348 See also the prologue of book VIII (RHL 3503, 230r).

349 See Díaz-Fajardo, 2008 and Díaz-Fajardo, 2011.

350 Puig-Pey, 2015.

the sources explicitly mentioned in the *Bārī*. To this the reader should add Ibn Abī l-Rijāl's bibliography on *ikhtiyārāt* which I reproduce below in 3.2.2.2.4. The most remarkable feature of this list is the importance of Iranian sources and of astrologers of the Islamic period who transmit them.

Indian sources:

- Junnah: GAS VII, 94.
- Arsā al-Hindī: GAS VII, 93.

Iranian sources:

- *Kitāb al-Bīzīdhāj*, Iranian translation of Vettius Valens commented by Buzurjmihr. Cf. Ibn al-Nadīm;³⁵¹ GAS VII, 38, 80.
- *Kitāb al-amthāl* / *al-faylasūf ṣāhib kitāb al-amthāl* / *al-faylasūf* / *K. al-amthāl alladhī ijtama'at 'alayh 'ulamā' Fāris wa l-Hind wa l-Rūm*: GAS VII, 17–18. It is an anonymous compilation of Greek, Indian, Persian and Arabic sources extant in ms. Teheran, Majlis 6452. Also called *K. al-Nawādir*³⁵²
- *Kitāb al-akwān*. Cf. GAS VII p. 18: *K. al-akwān fī wilāyat al-kawākib* mentioned in the *K. al-amthāl*.
- Ibn Z.j.m.h.r (RHL 213v); Ibn Rajmuhr: RHL 104r. Probably Buzurjmihr (GAS VII, 80)
- Zarādusht: GAS VII, 81–86. The quotations appear in book v, on nativities (RHL 98v, 100r, 143r, 144r)³⁵³ and it is interesting to remark that a Maghribī copy of Zarādusht's *Kitāb al-mawālīd* is preserved in ms. Escorial 939 (fols. 18v–34r) dated in 813/1411. The copyist of this text, as well as of four other astrological works of the same collection, seems to have been Abū 'Abd Allāh Muḥammad b. 'Alī al-Baqqār, the Moroccan author of the *Kitāb al-amṭār wa l-as'ār* (see 3.2.1.1 and 3.2.6).³⁵⁴
- Abū Q.māsh (RHL 262r) identified, conjecturally, by Aina Puig-Pey, with Gāmāsb, a Persian scholar, contemporary of Zaradusht, cited by Abū Ma'shar. GAS VII, 86–87.
- Ibn Marzubān: Māhānkard b. Marzubān (GAS VII, 83–100). According to ms. Escorial 939 this author translated Zarādusht's *K. al-mawālīd* from the

351 Flügel, 1871–72, p. 376.

352 This source is probably the *Kitāb al-amthāl li l-Furs* which seems to have reached Toledo in the 11th c. and is quoted by al-Istijī in his *Risāla fī l-tasyīrāt wa-maṭāriḥ al-shu'ā'āt*. See Samsó & Berrani, 2005, p. 193). The same source is quoted by al-Baqqār in his *Kitāb al-amṭār wa l-as'ār* and in the *Kitāb al-adwār fī tasyīr al-anwār*. See Guesmi & Samsó, 2018 pp. 7, 152, 264.

353 Kunitzsch, 1993.

354 Samsó & Berrani, 2005, pp. 171–175; Derenbourg & Renaud, 1941, pp. 54–57.

prestigious language (*al-lisān al-fāḍil*) used by the author [into another kind of Persian]. The aforementioned ms. contains quotations of passages attributed to Māhānkard.

Greek sources:

- Anṭ.x.hūs,³⁵⁵ Anṭīqūs or Anṭīqūn (RHL 11r. 16v, 17r, 34v): Antiochos of Athens (GAS VII, 31).
- Dorotheos of Sidon (1st c.AD): GAS VII, 32–38.
- Vettius Valens: GAS VII, 38–41.
- Ptolemy: Ibn Abī l-Rijāl mentions the *Tetrabiblos*, *Karpós*, *Geography* (*fī kitābi-hi alladhī waṣafa fīhi al-mudun wa l-jazā'ir wa l-biḥār*; *Kitāb al-aqālīm*) and *Handy Tables* (*Zīj Baṭlamīyūs*).
- Awṭ.fiyūs or Awṭūqiyūs: Eutokios, GAS VII, 48.
- Stephanos (GAS VII, 48–49): quoted indirectly through Yahyā b. Abī Manṣūr.
- Theophilos (Thawfil b. Thūmā): GAS VII, 49–50.
- Hermes (GAS VII, 50–58): the *Kitāb al-aṭwāl* is explicitly mentioned in RHL 201r

Islamic sources:

- Māshā'allāh (d. ca. 815): GAS VII, 102–108. In RHL 167v (*Conplido* I, 272) Ibn Abī l-Rijāl criticises Māshā'allāh who, in his *Kitāb al-ma'ānī*, explained the retrogradation of planets by saying that they are tied to the Sun by a rope. When the planet reaches its maximum distance from the sun, the latter pulls on the rope and the planet moves backwards. According to our author “Māshā'allāh ignores the cause of retrogradation”. In RHL 258r Ibn Abī l-Rijāl quotes Māshā'allāh on the evolution of prices, which seems to refer to his *Kitāb al-as'ār*.
- Ibn Farrukhān al-Ṭabarī (end of the 8th and beginning of the 9th c. AD): GAS VII, 111–113.
- al-Faḍl b. Abī Sahl b. Nawbakht: astrologer of Hārūn al-Rashīd and supervisor of the library of the *Bayt al-Ḥikma* (d. ca. 815). Cf. Labarta, *Kāmil* pp. 17–18.
- Banū Nawbakht without an identification of the individual to whom the quotations belong (RHL 103v, 210r, 217r). On the Banū Nawbakht family see Labarta, *Kāmil* pp. 15–21.
- Abū 'Awn *'an abī-hi bn K.sūn* (RHL 262r): Aina Puig-Pey identifies him with Abū 'Awn b. Kaysūn, probably the author quoted by al-Khwārizmī as a

355 The x corresponds to an undotted letter in the manuscript which could be *bā'*, *tā'*, *thā'*, *nūn* or *yā'*.

- source of his *Ṣūrat al-arḍ*.³⁵⁶ Yāqūt, in his *Muʿjam al-buldān*, gives a list of the geographical coordinates of 30 localities borrowed from the *zīj* of Abū ʿAwn Ishāq b. ʿAlī.
- al-Faḍl b. Sahl [al-Sarakhsī?]: in RHL 117v he is called *Dhū l-rīʾasatayn*. Astrologer of Hārūn al-Rashīd (786–809), who died in 818: see GAS VII, 115–116. It seems that he can be identified with al-Faḍl b. Sahl b. Zadhānfarrūkh, minister of al-Maʾmūn (813–833), who was actually called *Dhū l-rīʾasatayn* and died also in 818.³⁵⁷
 - Yaḥyā b. Abī Manṣūr *ʿan Iṣṭifan wa Iṣṭ.rātūs*: RHL 103r; 103v. *Mumtaḥan Zīj* (RHL 264v; BL 379b; Burnet & Yamamoto, *Historical astrology* pp. 558–559).
 - Abū ʿAlī al-Khayyāt, student of Māshāʿallāh, who died in the first quarter of the 9th c.: GAS VII, 120–121.
 - al-Ḥasan b. Sahl: he is either al-Ḥasan b. Sahl b. Zādānfarrūkh (782–851), the brother of al-Faḍl b. Sahl [al-Sarakhsī] and minister of al-Maʾmūn (813–833) (GAS VII, 122), or al-Ḥasan b. Sahl b. Nawbakht, astrologer of al-Wathīq (842–847 and al-Mutawakkil (847–861) (Labarta, *Kāmil*, p. 18).
 - Abū l-Ḥusayn ibn al-Khaṣīb: in RHL 127v Ibn Abī l-Rijāl mentions an anecdote related to a nativity horoscope in which one of the characters involved is Abū l-ʿAnbas [al-Ṣaymarī] in 230/844–45. He is Abū Bakr al-Ḥasan b. (Abī) l-Khaṣīb: GAS VII, 122–124.
 - Muḥammad b. al-Jahm (GAS VII, 124). He lived in the time of al-Maʾmūn (813–833).
 - Sahl b. Bishr, fl. 1st half of the 9th c. GAS VII, 125–128.
 - Khurrazād (1st half of the 9th c.): Khurrazād in RHL 81v y 87v, 206v; Agh. rzād: RHL 70v. Kharrād: BL 109a. He is Khurrazād b. Dārshād: GAS VII, 129.
 - Al-Kindī (d. ca. 870): GAS VII, 130–134. Ibn Abī l-Rijāl quotes him on buried treasures (RHL 27r, *al-Kindī fī l-dafīn*),³⁵⁸ and mentions his great book (*Kitābu-hu al-aʿẓam*) and his “Forty chapters” (*Arbaʿūna faṣḥan*: RHL 38v, 39r).³⁵⁹ His opinions are taken into consideration concerning pregnancy and childbirth (RHL 31r), marriage (RHL 51r) and elections (RHL 200r) (his *Ikhtiyārāt al-ayyām?*). Ibn Abī l-Rijāl has a high opinion of al-Kindī’s astrological knowledge: “Al-Kindī occupies, among Islamic astrologers, a position similar to that of Ptolemy” ... “among his writings we find two chapters, which occupy eight folios and encompass the whole of astrology (*ʿilm al-nujūm*): a commentary on these pages would fill more than eight

356 Nallino, 1894, p. 23.

357 D. Sourdel in *El2*, vol. 11, pp. 731–732.

358 Burnett, Yamamoto & Yano, 1997.

359 Burnett, 1993, pp. 87–93.

- volumes" (RHL 38v; *Conplido* I, 63); "al-Kindī would have been the most skillful of men (*aḥdhaq al-qawm*) in astrology (*fī l-aḥkām*) had it not been for his lack of commentaries and the difficulty of his writings. He is completely opposed to Abū Ma'shar, as the latter very rarely hits his target in his predictions (*qalīl al-ṣawāb*), is too wordy in his justifications (*kathīr al-asbāb*), very rarely corrects his writings (*yasīr al-tanqīh*) and has a confuse mind (*ḥāṭib layl*)" (39r).
- Abū Ma'shar (787–886): GAS VII, 139–151. Ibn Abī l-Rijāl quotes the *Mudhakarāt* (RHL 38r, 38v); his *Kitāb al-haylāj wa l-kadkhudāh*; a *Kitāb ṣaghīr fī l-mawālīd* and his *Kitāb fī 'uyūn al-nukat* (RHL 102v). He also mentions the *K. al-tabā'i* (RHL 223r). In RHL 200r he criticises Abū Ma'shar due to his disagreement with al-Kindī and Māshā'allāh on elections.
 - Abū l-'Anbas al-Ṣaymarī (828–888) (GAS VII, 152–153). Mentioned above under Abū l-Ḥusayn ibn al-Khaṣīb. In RHL 31r a *Kitāb B.r.dār'an* (*Berdy Rafferen* in *Conplido* I, 47) is attributed to him. In BL 49a his name becomes Abū l-Qiyās al-Ḍamīrī, author of a *K. Būdāzfārān*.
 - Ḥabash al-Ḥāsib (fl. second half of the 9th c.): GAS VI, 173–175. Ibn Abī l-Rijāl mentions his *zīj* (RHL 263v) which seems to be the source of his list of geographical coordinates of cities. This list has been edited and analysed by Ayna Puig-Pey, 2015.
 - al-Battānī (d. 929): GAS VII, 158–160. As we will see in 3.2.5.1, both Ptolemy and al-Battānī considered important the predictions based on eclipses which take place in the years of Saturn-Jupiter conjunctions (RHL 230v).
 - Ibn Hibintā (1st half of the 10th c. GAS VII, 162–164). The *Mughnī* is mentioned, without the name of the author in RHL 262r. This seems to be the only explicit quotation of Ibn Hibintā, although, as Montse Díaz-Fajardo has proved,³⁶⁰ Ibn Abī l-Rijāl explains a method to calculate the *tasyīr* which derives directly from the *Mughnī* (see below 3.2.2.2.2.1). In fact, about a 40% of the contents of the chapter on *tasyīr* (RHL 3503, fols. 104r–108v, *Conplido* I, 173–180) is borrowed from Ibn Hibintā. The method in question was influential in the development of Maghribī astrology and we find it again in the Hyderabad recension of Ibn Ishāq's *zīj*, Ibn 'Azzūz and al-Baqqār. Besides this, Aina Puig-Pey has also shown that chapter 37 of book VIII (RHL 262v–264r; *Conplido* II, 313–322) derives from *al-Mughnī* (pp. 18–27, 35–37).
 - al-Hamdānī (d. 945): GAS VII, 164–165.
 - 'Alī b. Aḥmad al-'Imrānī (d. 955): GAS VII, 166.

360 Díaz-Fajardo, 2011, pp. 354–357, 366.

- ‘Abd Allāh b. Masrūr, who was one of Abū Ma’shar’s disciples: GAS VII, 166–167; GAS VI, 205.
- Al-‘Āṣimī (fl. 961?): GAS VII, 167.

Unidentified sources

- Aṣṭ.wāṭ.r: RHL 10v
- B.hā Y.kūd (Beheco in *Conplido* I, 222): RHL 133v
- Mahlah: RHL 124v
- Aqrīṭūn: RHL 124v
- Aqṭ.x.‘ūs: RHL 124v
- F.nd.rūs: RHL 204v
- Bārīṭūs: RHL 206r
- F.n.ṭ.miyūs RHL 210r: I wonder whether F.n.ṭ.miyūs and Bārīṭūs refer to the same author who could be Vindanius Anatolius of Berytos, the author of a treatise on agronomy (GAS VII, 318).
- ‘Abd Allāh b. Muḥammad al-Hāshimī: RHL 264r, BL 379a.
- al-Sh.rw.nī/ al-Sh.rwīnī (Azaroni, in *Conplido* I, 172–173): RHL 103v, BL 159b (Sh.rwīnī): mentioned as one of the great astrologers of Iraq together with Yaḥyā b. Abī Maṣṣūr and Muḥammad b. al-Jahm. Ibn Abī l-Rijāl reproduces six nativity horoscopes of this author (103v–104r, see 3.2.2.2.6).
- Qustā b. Q/Fūm.s al-Ḥarrānī “who taught astronomy (*al-nujūm*) to al-Ma’mūn”: RHL 104r, BL 160b.
- al-Mughīra: RHL 264r (BL 379a)
- Ibrāhīm al-Yahūdī (RHL 258r) also mentioned by al-Baqqār in his *Kitāb al-amṭār*.

3.2.2.2.2 The Contents of the Book

Ibn Abī l-Rijāl’s masterwork is a complete treatise on general astrology divided into eight books dealing with:

3.2.2.2.2.1 Introduction and Interrogations (Books I–III)

Introduction: Zodiacal signs, *ḥudūd* (terms), *wujūh* (decans), planets, interrogations, elections, aspects. It is somehow surprising not to find in this introduction any standard treatment of the *ḥālāt* (conditions of the planets).³⁶¹ This introduction also deals with the *anwār al-kawākib* (planetary lights) (RHL 14r; *Conplido* I, 23–25) giving the following values:

³⁶¹ See Burnett, Yamamoto & Yano, 1994, pp. 32–51.

Sun 15°
 Moon 12°
 Saturn, Jupiter 9°
 Mars 8°
 Venus, Mercury 7°

These values appear also in Abū Ma'shar's *Abbreviation*³⁶² and *Great Introduction*³⁶³ and in Ibn Qunfudh's commentary of Ibn Abī l-Rijāl's astrological *urjūza* with the only change of 9° for Mars.³⁶⁴ Ibn Qunfudh considers that these values correspond to the apparent diameters of the planets (*maqādir ajrāmi-hā*), something which is difficult to accept. Abraham ibn 'Ezra explains them as limits of visibility: a planet will be visible if it has 15° of angular distance from the Sun, 12° from the Moon, 9° from Saturn or Jupiter etc.³⁶⁵ This explanation seems also difficult to accept in the case of Saturn, Jupiter, Mars, Venus and Mercury. Although I cannot find any justification for the 15° of the Sun, the rest of the *anwār* seem to correspond to angular distances of the Moon and the planets from the Sun which make visibility possible.³⁶⁶

The rest of **Book I**, as well as **Books II and III** deal with interrogations (*masā'il*) a topic which begins with the determination of the *mubtazz* of the interrogation.³⁶⁷ The treatment of interrogations – as well as that of nativities (books IV and V) and elections (book VII) – follows the order of the twelve houses of the horoscope in which house I means life, house II fortune, house III brothers, house IV parents (also properties and hidden treasures), house V children (as well as pregnancy, child birth, messages), house VI illness (also servants, sorcerers and bewitched persons), house VII marriage (also fugitive slaves and animals, thieves, buying and selling, partnership, wars and battles), house VIII death, house IX travels (as well as dreams, and how to know the character of a man), house X power and authority (how long will a man

362 Burnett, Yamamoto & Yano, 1994, pp. 34–35.

363 Lemay, 1995, vol. III, p. 526.

364 Oliveras, 2012, p. 24 (Arabic), item [16] and p. 82 of the Spanish translation; see also Samsó, 2009, pp. 29–31. On these values see also Neugebauer, 1975, II, 831.

365 Sela, 2007, p. 209.

366 The *anwār* are also dealt with in the treatise on interrogations by Sahl b. Bishr (see ms. British Library Or. 12802, fol. 4v) who gives the same values and explains, in the case of the Sun, that if between the Sun and a planet there is a distance comprised between 1° and 15°, the Sun throws its light and both celestial bodies are in application (*muttaṣila*). The same idea is applied to the other planets.

367 The *mubtazz* is the planet having more power in the horoscope. See Bīrūnī, *Taḥḥīm* pp. 307–308 and Oliveras, 2012, pp. 90–93 (Sp. Translation) and pp. 32–37 (Arabic edition).

stay in power, list of trades and professions), house XI friends and enemies (love and friendship) and house XII animals.

Ibn Abī l-Rijāl's treatment of illness has a certain interest. In Book II-2 he establishes the moments of crisis of illness (RHL, 41v–42r; *Conplido* I, 66–67) depending on the Moon's distance from the position it occupied at the beginning of the disease: 10°, 20°, 45°, 90° (7th day), 120° (9th day), 180° (180°). The same chapter also deals with the determination of the moment of death. For that purpose he gives (RHL 42r and BL 65b) two different sets of values, the first of which is the *muntahā firāri-hi min al-shams* (*La fin del fuymiento del sol* in *Conplido* I, 67). I cannot explain the meaning of these values which are:

Venus	47;52°
Mercury	24;3°
Saturn	6;37°
Mars	49;13°
Moon	7;40°
Sun	2°

(Jupiter is missing in both Arabic manuscripts and in the Alfonsine translation)

The second set is clear as it corresponds to the maximum equations of anomaly (*akthar ikhtilāfāt fī falak tadwīri-hā*) at mid-distance of the planet from the Earth, with the exception of the value for the Moon, which correspond to its maximum distance. The planetary values agree with those attributed by al-Bīrūnī to Theon of Alexandria³⁶⁸ which are the same as those of the *Almagest*:

Sun	2°
Moon	5;1°
Saturn	6;13°
Jupiter	11;3°
Mars	41;10°
Venus	45;59°
Mercury	22;2°

368 Saffouri, Ifran & Kennedy, 1959, pp. 58, 160; Pingree, 1968, pp. 48–49. Ibn Abī l-Rijāl refers to the *Handy Tables* (*Zīj Baṭlamīyūs*) in RHL fol. 264v. See the edition of the corresponding passage (chapter 39 of book VIII) in Burnett & Yamamoto, 2000, pp. 558–559.

3.2.2.2.2.2 Nativities (*mawālīd*) (Books IV and V)

As said previously, the treatment of this topic follows the same structure as in the previous books. House I deals with whether the newborn will survive his nursing period (*tarbiya*), whether he is the son of his legal father or of somebody else, the identification of the *kadkhudāh* (the donor of years),³⁶⁹ and the interpretation of combustion (*iḥtirāq*),³⁷⁰ *hubūt* (fall)³⁷¹ and retrogradation. House II is concerned on whether the newborn will be rich, ... house V on the number of children he will have, house VI with diseases of the body and the spirit, as well as cattle, other animals and slaves the newborn will own, ... house VIII with his death, ... house X with his profession, courage and cowardice etc.

3.2.2.2.2.2.1 *The Computation of the tasyīr*

One of the interesting chapters of Book IV deals with Ibn Abī l-Rijāl's treatment of *tasyīr* (prorogation) (RHL 104r–108r; *Conplido* I, 173–178),³⁷² in which he uses, without mentioning them, two sources: Ibn Hibintā's *Mughnī* and the book on nativities ascribed to Zaradusht. The *tasyīr* (progression or prorogation) is an astrological technique usually applied to determine the amount of time required until a certain event (quite often the death of the subject of the horoscope) will take place. This is usually calculated by giving a time equivalent to the degrees of rotation on the equator of an indicator (*al-musayyir*) until its path meets a malefic element (*al-musayyar ilay-hi*) which cuts it (*al-qāṭiʿ*, the cutter) and interrupts the process. The computation of the *tasyīr* on the equator makes sense, as time is measured by the rotation of the Earth on the plane of the equator, but there is an Andalusī-Maghribī tradition which calculates the *tasyīr* by establishing the difference in ecliptical longitude between the *musayyir* and the *musayyar ilay-hi*.³⁷³ This technique is used by Ibn Abī l-Rijāl for the prorogation of the *tasyīr* of the anniversary (*tasyīr al-taḥwīl*).³⁷⁴

According to Ibn Abī l-Rijāl, the indicator which marks the beginning of the prorogation can be:

369 See Birūnī, *Taḥfīm* p. 324: the *kadkhudā* is the planet having more dignities (*muzāʿama*) in the nativity horoscope. A maximum, middle and minimum number of years is assigned to each planet and the length of life of the newborn will be determined according to the position of the *kadkhudā* in the horoscope. On the list of years corresponding to each planet see Burnett, Yamamoto & Yano, 1994, p. 89.

370 A planet is combust when its position is very near the Sun.

371 A planet is in *hubūt* when it is placed in the sign opposite to its exaltation (*sharaf*): see Burnett, Yamamoto & Yano, 1994, pp. 24–25.

372 This chapter has been studied by Díaz-Fajardo, 2011. I am following this article here.

373 Samsó & Berrani, 1999; Samsó & Berrani, 2005.

374 Díaz-Fajardo, 2011, pp. 352–354.

- 1) One of the five *hayālīj* (sing. *haylāj*): Sun, Moon, ascendant, the *sahm al-sa'āda* (*pars fortunae*) and the degree of the previous conjunction or opposition of the Sun and the Moon.
- 2) The *dalīl* (indicator) of properties, brothers, parents or sons and the degree of midheaven.
- 3) One of the five planets
- 4) The *sahm al-ghayb* (the part or lot of what is hidden).

The *Bārī'* specifies which kind of indicator should be chosen for every particular need. Besides this, it gives also a list of the *qawāṭī'* (plural of *qāṭī'*, cutter) which can be a malefic body, the opposition and quartile aspect of a malefic body, the *hudūd* (terms) of the malefic planets, the final degrees of each sign, the four fixed stars called *Qalb*,³⁷⁵ the opposition and quartile aspect of the Sun and, finally, the Sun and the Moon. Other *qawāṭī'* are mentioned by Ibn Abī l-Rijāl in quotations from Zaradusht's *Mawālīd*: the degree of the eclipse, Saturn and Mars, the occidental (*mugharrab*) and oriental (*musharraḡ*) planets in relation to the Sun, and the comets.

When the indicator is placed on the meridian (houses IV and X of the horoscope) the arc of the equator between the indicator and the *qāṭī'*, which will be transformed into units of time (years, months, days), is calculated by the difference in right ascension (α_o) between the two bodies or points of the ecliptic. If it is placed on the horizon (houses I and VII) the difference in oblique ascension (α_ϕ) will be used. The problem rises when the indicator is placed in an intermediate position between the meridian and the horizon. For such cases "mixed" ascensions (*maṭālī' mumtazaja*) (α_m) are used. The procedure described by Ibn Abī l-Rijāl to calculate this kind of ascensions uses "rising times", corresponding to the difference in right (τ_o) or oblique ascensions (τ_ϕ) between the beginning and the end of each house. It is easier, however, to explain the procedure using ascensions,³⁷⁶ in which λ_1 is the ecliptic longitude of the ascendant and $\Delta\alpha$ the ascensional difference ($\Delta\alpha = \alpha_\phi(\lambda_1) - \alpha_o(\lambda_1)$):

$$\begin{aligned}\alpha_\phi(\lambda_1) - \alpha_\phi(\lambda_1 + 180^\circ) &= \text{day arc of } \lambda_1 = 180^\circ \pm 2 \Delta\alpha \\ (180^\circ \pm 2 \Delta\alpha) / 6 &= 30^\circ \pm \Delta\alpha/3 \\ \alpha_\phi(\lambda_1) \pm (30^\circ \pm \Delta\alpha/3) &= \alpha_m(\lambda_2) \\ \alpha_m(\lambda_2) \pm (30^\circ \pm \Delta\alpha/3) &= \alpha_m(\lambda_3)\end{aligned}$$

where λ_2 and λ_3 are the longitude of the beginning of the second and third house of the horoscope. More generally, if i is the number corresponding to a house of the horoscope:

375 *Qalb al-Thawr* (α Tauri, Aldebaran), *Qalb al-Asad* (α Leonis, Regulus), *Qalb al-'Aqrab* (α Scorpii, Antares) and *Qalb al-Hūt* (β Andromedae, Andromeda).

376 Samsó, 2009, pp. 31–35.

$$\alpha_m(\lambda_i) \pm (30^\circ \pm \Delta\alpha/3) = \alpha_m(\lambda_{i+1})$$

If $\Delta\alpha/3 > 0$, the sign of the aforementioned operation will be positive between houses I and X, VII and IV, and negative in the rest of the cases. If $\Delta\alpha/3 < 0$, the sign will be negative between I and X, VII and IV and positive in the other cases. Besides this, in the expression $30^\circ \pm \Delta\alpha/3$, the sign will be positive if the ascendant has a positive declination (northern signs) and negative when the declination is also negative (southern signs).

It is clear that with this procedure we will obtain weighted values which combine a component in right ascension (used for the meridian houses IV and X) and another in oblique ascension (used in the horizon houses I and VII). When using it, the component in right ascension increases by thirds while the component in oblique ascensions decreases also by thirds between houses I and X, VII and IV. The opposite takes place between X and VII, IV and I.

A variant of the same procedure is also explained by Ibn Abī l-Rijāl who calculates:

$$|(\alpha_\varphi(\lambda) - \alpha_o(\lambda))| / 3 = |90^\circ \pm \Delta\alpha/3|$$

in which the 90° are due to the fact that while oblique ascensions are calculated with origin in Aries 0° , right ascensions are calculated, in medieval tables, with origin in Capricorn 0° . Besides this, the procedure is identical to the other one and it also causes an increase and decrease by thirds of the components in right and oblique ascension. It is a simplification of the so-called “Position semicircle method” (3.2.3.2.2.4) and Ptolemy’s *Tetrabiblos* considers it as an approximate method. It appears in many Arabic sources among which we find Ibn Hibintā’s *Mughnī* which is clearly the source used by Ibn Abī l-Rijāl.³⁷⁷

3.2.2.2.2.3 Anniversaries of Nativities (*taḥāwīl sinī al-mawālīd*) (Book VI)

We have already seen in 3.1.1.6 an example of an anniversary horoscope and, here, Ibn Abī l-Rijāl explains the techniques to be used to adjust the predictions made on the basis of the nativity horoscope of an individual to the new conditions which appear in the anniversary horoscope. One of the factors to bear in mind is to consider which is the planet governing every stage of a man’s life. For this purpose Ibn Abī l-Rijāl considers that the seven ages of man are governed by the seven planets in the following way (RHL 168r–168v; BL247 a and b; *Conplido* II, 3–5):

³⁷⁷ Ibn Hibintā, *Mughnī*, I, pp. 144–148.

Moon	4 yrs (according to the text 4 years correspond to the tenth part of the Moon's smallest years. This does not seem to be true, for the smallest years of the Moon are 25) ³⁷⁸
Mercury	10 yrs (half of its smallest years which are 20)
Venus	8 yrs (its smallest years)
Sun	19yrs in RHL and <i>Conplido</i> ; 17 yrs in BL (19 correspond to the Sun's smallest years)
Mars	15 yrs (its smallest years)
Jupiter	12 yrs (Its smallest years)
Saturn	30 yrs (<i>Conplido</i>); until the moment of death (RHL and BL)

Besides this, Ibn Abī l-Rijāl considers several kinds of cycles whose positions should be established for the moment of the anniversary. The cycles used are:

Qisma (not defined, RHL 168v–170v; *Conplido* II, 6–10): it corresponds to a period of 360 years in which the indicator advances with a velocity of 1° per year.

Intihā' (not defined, RHL 170v–177r; *Conplido* II, 7–29): it corresponds to a period of 12 years, called, in other Maghribī sources, *al-dawr al-aṣghar*, *burj al-muntahā* or *burj al-intihā'*. In it the indicator progresses one sign per year.³⁷⁹

Fardāriyyāt (RHL 177r–184r, *Conplido* II, 29–47) which are the same as in Abū Ma'shar's *Madkhal ṣaḡūr*:³⁸⁰

Sun	10yrs
Venus	8 yrs
Mercury	13yrs ³⁸¹
Moon	9 yrs
Saturn	11 yrs
Jupiter	12 yrs
Mars	7 yrs
Asc. node	3 yrs
Desc. node	2 yrs ³⁸²

378 Burnett, Yamamoto & Yano, 1994, p. 89.

379 Díaz-Fajardo, 2011 a, p. 38. See also the useful glossary in Labarta, 1982, pp. 237–238.

380 Burnett, Yamamoto & Yano, 1994, p. 81.

381 10 years according to Ibn Nawbakht: cf. Labarta & Mestres, 2005, p. 0.23.

382 3 years in Ibn Nawbakht's *Kitāb al-azmina*: Labarta & Mestres, 2005, p. 0.23.

These periods correspond to the smallest *fardāriyya* (*al-fardāriyya al-sughrā*) and, in Ibn Abī l-Rijāl's version, the sum of the years is 75. They are also called *al-dawr al-awsaṭ* or *al-tasyīr al-firdārī* in other Maghribī astrological sources.³⁸³ Both the *fardariyyāt* and the aforementioned seven periods of life give us an idea about the expected length of human life. These values correspond to the periods of influence of each planet in human life. If childbirth takes place during the day, the series begins with the Sun and, otherwise, with the Moon. Every period is divided into 7 equal subperiods during which there are sub-*fardāriyyas* governed by Sun, Moon and the 5 planets.

Chapter 15 of this book deals with the revolutions of the months (*Fī qismat al-shuhūr wa l-ayyām*) (RHL 196r–197v; *Conplido* II, pp. 82–86) and it contains a table of the *shuhūr taḥwiliyya* (astrological months), in which the length of an “astrological month” results from the division of the length of one tropical or sidereal year into 13 “months” (RHL 196v and *Conplido* II, p. 84). This table, edited by Goldstein and Chabás,³⁸⁴ is extremely interesting because it is the only evidence we have about whether Ibn Abī l-Rijāl is practising a tropical or a sidereal astrology, the latter being standard of Andalusī and, later, Maghribī astrology. We do not have, in fact, any detailed horoscope cast by Ibn Abī l-Rijāl himself reproduced in the *Bārī*. In the aforementioned table the length of one “month” is 28d 2;17,30h and thirteen astrological months correspond to a tropical year of 365d 5;47,30h.

3.2.2.2.2.4 Elections (*ikhtiyārāt*) (Book VII)

Ibn Abī l-Rijāl gives a list of the sources he is using in RHL 198v–199r; *Conplido* II, 90–91. Most of them correspond to books written from the end of the 8th to the end of the 9th c. The best books on this topic, according to him, are those written by:

Dorotheos of Sidon (1st c. AD): he probably refers to book v of his *Kitāb al-khamsa* which, according to Ibn al-Nadīm, deals with the beginning of activities (*Fī ibtidā' al-a'māl*).³⁸⁵

Vettius Valens (2nd c. AD): maybe his *Ikhtiyār al-aḥkām*.³⁸⁶

Māshā'allāh (end of the 8th and beginning of the 9th c. AD): Ibn al-Nadīm ascribes to him a *al-Kitāb al-ma'rūf bi l-sābi' wa l-'ishrīn*, whose first book deals with *ibtidā' al-a'māl*.³⁸⁷

383 Montse Díaz-Fajardo, 2011 a, p. 38.

384 Bernard R. Goldstein and José Chabás, 2013. Other tables of the same kind, though unrelated to Ibn Abī l-Rijāl, in 'Abd al-Raḥmān, 1996 a.

385 Flügel, 1871–72, p. 268. GAS VII, pp. 32–38.

386 GAS VII, 38–41.

387 Flügel, 1871–72, pp. 273–274; GAS VII, pp. 102–108.

Banū Nawbakht: Ibn Abī l-Rijāl does not specify to which member of this well known family he is referring.³⁸⁸ Abū Sahl b. Nawbakht (d. ca. 815AD) did write a *Kitāb al-mas'ala wa ibtidā' al-a'māl*.³⁸⁹

Theophilos (Thawfil b. Thūmā, fl. second half of the 8th c.).³⁹⁰

al-Khayyāt, Abū 'Alī (fl. end of the 8th c. and beginning of the 9th c.).³⁹¹

al-Kindī (d. ca. 870): among his astrological works we find *Ikhtiyārāt al-ayyām* (Election of days).³⁹²

The rest of his bibliographical references is considered by Ibn Abī l-Rijāl to be less good:

Ibn Marzubān: Māhānkard b. Marzubān.³⁹³

Sahl b. Bishr (first half of the 9th c.): author of a *Kitāb al-ikhtiyārāt 'alā l-buyūt al-ithnay 'ashar* (Elections according to the twelve houses of the horoscope).³⁹⁴

Abū Ma'shar (787–886) who wrote, at least, four works related to elections: the *Kitāb al-ikhtiyārāt*, *Kitāb ikhtiyārāt al-sā'āt* (election of hours), *Kitāb al-ikhtiyārāt fī l-a'māl wa l-ḥawā'ij min umūr al-salāṭīn* (elections of the propitious moments to begin activities as well as those things corresponding to monarchs) and *Abwāb al-masā'il wa mā ba'da-hā min al-ikhtiyārāt* (on interrogations and the elections which follow them).³⁹⁵

'Alī b. Aḥmad al-'Imrānī (d. 955) whose only extant work is precisely a *Kitāb al-ikhtiyārāt*.³⁹⁶

al-'Abbās b. Sa'īd: probably 'Abbās b. Sa'īd al-Jawharī, an astronomer and mathematician of the time of Caliph al-Ma'mūn.³⁹⁷

In the same way as with interrogations and nativities, elections are treated in the order of the houses of the horoscope. It is illustrative to give examples of the human activities for which Ibn Abī l-Rijāl considers suitable to apply the technique of elections:

House I: Entering the bath; cutting one's hair; bloodletting; cutting one's nails

House II: Buying and selling; changing one's house.

388 On the Banū Nawbakht see Labarta, 1982, pp. 15–21.

389 GAS VII, 114.

390 GAS VII, 49–50.

391 GAS VII, 120–121.

392 GAS VII, 130–134.

393 GAS VII, 83, 100.

394 GAS VII, 125–128.

395 GAS VII, 139–151.

396 GAS VII, 166.

397 GAS VI, 138–139.

House III: Recovering friendship with brothers and relatives; beginning to learn *fiqh*.

House IV: Building a city and a house; drawing water; building channels; renting lands and houses; buying land; building a mill; sowing and planting; expelling the devil from the house.

House V: Having sexual intercourse with a woman and having a baby; nursing and weaning of a child; circumcising a child;³⁹⁸ making and wearing new clothes; sending a present; sending messengers; writing letters; eating and drinking; preparing a perfume; sending pigeons; extracting a dead foetus from its mother's womb.

House VI: Giving a medicine to an ill person; curing the eyes; taking a laxative and an astringent medicine; buying a slave and an animal.

House VII: getting married; fights and battles; buying weapons for the war; making war and peace; destroying enemy's castles; making weapons and engines for the war; looking for a fugitive; forcing a robber and a spy to confess; hunting and fishing; playing chess and other games; having sex with women.

House VIII: Writing one's will.

House IX: Choosing a route; promenades; getting out of a place without being seen; traveling by boat; buying a ship, entering in it and making it move; putting the ship in the water; learning science and arts; learning to sing and to play musical instruments; entering a city.

House X: Tying a flag at the beginning of a military campaign; obtaining a position; choosing between equal men; dealing with a king; learning a profession, to use weapons and to swim

House XI: Obtaining a good reputation; fulfilling promises and asking for favours; asking for love and friendship.

House XII: Horse racing; choosing the moment for the king's capture of an enemy.

3.2.2.2.2.5 Revolutions of the Years of the World (*taḥāwīl sinī al-ʿālam*)
(Book VIII)

3.2.2.2.2.5.1 *Saturn-Jupiter Conjunctions and Other Cycles*

In the introduction (RHL 230v; *Conplido* II, p. 194) Ibn Abī l-Rijāl says that there are many different systems to make predictions using the revolutions of the world years and that they employ several cycles such as conjunctions,

398 *Khitān* in RHL 208v. In a curious attempt to christianise, the text the Latin translation of the title of the chapter has "in circumcidendo et baptizando" (Hilty, 2005, p. 118).

the ascendant of the beginning of the year,³⁹⁹ the position of the *intihā'* of the ascendant, the *intihā'* of the sign of the *dawr*, the *intihā'* of the degree of the *qisma*, the position of the ascendant of the year from the ascendant of the new (*al-jadīd*) conjunction and the sign of the conjunction. These are the bases (*uṣūl*). There are astrologers who make their predictions based on the ascendant of the beginning of the year and on the position of the last lunisolar conjunction /opposition before the spring equinox and its ascendant. The events that can be predicted using these techniques are:

1. Calamities which affect the whole mankind: epidemics, fertility and draught, atmospheric changes, rain etc.
2. Earthquakes, eclipses and floods
3. Events affecting a determined community of people such as wars
4. Atmospheric events: *nīrān*, shooting stars and comets.

There are scholars who enhance the importance of eclipses in the year of a planetary conjunction such as Ptolemy and al-Battānī. This remark is absolutely correct and seems to allude to al-Battānī's *Dalā'il al-qirānāt wa l-kusūfāt* in which this author insists in the importance of "eclipses accompanying conjunctions in their places in the ecliptic" and quotes Ptolemy's *Tetrabiblos* (II, 4 and 6) who emphasizes the interest of solar and lunar eclipses for general prognostications related to the future of countries and cities.⁴⁰⁰

Ibn Abī l-Rijāl insists in the importance in world astrology of [Saturn-Jupiter] conjunctions.⁴⁰¹ Planetary conjunctions and their effects are studied in chapter 5 of book VIII (RHL 236v–238r, *Conplido* II, pp. 215–220) according to Hermes, Abū Ma'shar and 'Abd Allāh b. Masrūr.⁴⁰² In chapter 13 (RHL 246v; *Conplido* II, 252–255) we find a description of all kinds of events affecting common people (wars and other misfortunes) according to the position of Mars in the [Saturn-Jupiter] conjunction, as well as the situation of the king, how long will he reign, and when will he die. In chapter 16 (RHL 248r–249r; *Conplido* II,

399 A variant reading, followed in the Alfonsine translation, has *min ṭālī' al-Milla* which should be interpreted as "the ascendant of [the conjunction] of the Religion", which refers to the Saturn-Jupiter conjunction, with shift in the triplicity, which took place in 571 CE and announced the birth of Prophet Muḥammad and the advent of Islam. I am using a draft of the PhD thesis of Aina Puig-Pey which edits critically Book VIII of the *Bārī'*.

400 Al-Battānī's book has been edited by Kennedy *et al.*, 2009–10, pp. 18–19, 80.

401 On Saturn-Jupiter conjunctions in Islamic astrology and its influence in Medieval and Renaissance Europe see Bezza, 2015.

402 Astronomer and astrologer of the first half of the 10th c. who wrote a *Kitāb taḥāwīl sinī al-'ālam wa l-ḥukm 'alayhā*. See GAS VI, 205–206; GAS VII, 166–167.

pp. 258–262) the conjunction of 571 CE in the sign of Scorpio (*qirān al-Milla*)⁴⁰³ is mentioned and Scorpio becomes the sign of the Arabs, while the relevance of Venus and Mars in the conjunction horoscope makes them to become indicators of the Arab community. This is confirmed by a passage of chapter 38 (RHL 264r; BL 379a; *Conplido* II, 322–323) where the text states that the small years of the lord of the Religion (*sinī ṣāhib al-Milla al-ṣuḡhrā*) are 8, which is the number of small years of planet Venus. The same chapter specifies that the degree of the conjunction of the Religion was Scorpio 4;12⁴⁰⁴ and in chapter 39 (RHL, 264v; *Conplido* II, 323–325)⁴⁰⁵ we find that the ascendant of the year of the conjunction (i.e. of the spring equinox of 571) was Gemini, according to the school of Ptolemy's *zīj* (no geographical locality specified),⁴⁰⁶ while the ascendant for the centre of the world (Arin) was Aries 6°, according to the *Sindhind*, and Cancer according to the *Mumtaḥan*.⁴⁰⁷ Finally, in chapter 40 (RHL 264v–265r; *Conplido* II, 325–26), Ibn Abī l-Rijāl adds that the ascendant of the conjunction itself was Libra 18°. ⁴⁰⁸

Some more information can be found in chapter 39 on the chronology of the conjunction of Religion because the text states that 61 Persian years, 2 months, 12 days and 16 hours correspond to the period of time between the aforementioned conjunction and the beginning of the Yazdijird era.⁴⁰⁹ This

403 See for example, Kennedy *et al.*, 2009–10, pp. 34–41, 88–92.

404 Ibn Qunfudh, in his commentary on Ibn Abī l-Rijāl's astrological *urjūza* says that the conjunction took place in the first decan of Scorpio. See Samsó, 2009, p. 324, Oliveras, 2012, pp. 201 (Ar. text), 251 (Sp. translation); According to Māshā'allāh the conjunction was in Scorpio 4;2°: see Kennedy & Pingree, 1971, p. 48.

405 On both passages see the edition and English translation by Burnett and Yamamoto, 2000, pp. 558–559.

406 This seems to be part of a quotation of al-Kindī and the same information, derived from this source, appears in Burnett, Yamamoto & Yano 2004, pp. 118–121. See also Guesmi & J. Samsó, 2018 pp. 124–130, 249–250, no [137].

407 The sources do not agree on this matter and, often, they do not specify for which latitude the ascendant is calculated: in Ibn Nawbakht horoscope the ascendant is Sagittarius 27° for the centre of the Earth and Aries 11;20° for the centre of the 4th climate (see Labarta, 1982, p. 78); Sagittarius is also found in al-Battānī's astrological history: see Kennedy *et al.*, 2009–10, pp. 35–37, 89; according to Māshā'allāh, Abū Ma'shar and al-Sizjī the ascendant was Libra (Kennedy & Pingree, 1971, pp. 13, 49 (on this conjunction see also pp. 98–102); Burnett & Yamamoto, 2000, I, 494–495, 568 and II, 334–335; Pingree, 1968, p. 95).

408 See Burnett and Yamamoto, 2000, II, 562–563.

409 Ibn Hibintā (*al-Mughnī*, ms. Munich 852, fol. 230v) gives a different value: 61 Persian years, 2 months, 17 days and 16 hours, which place the conjunction on the 15th April 571. For that date the positions of Saturn (213;15°) and Jupiter (213;18°), calculated with al-Battānī's parameters, are nearer to an exact conjunction. Ibn Hibintā, therefore, might have been Ibn Abī l-Rijāl's source whose 12 days could be a copying mistake.

amount of time is equivalent to 22337 days (assuming months of 30 days each) and 16 hours. As the Julian day number which corresponds to the beginning of the Yazdijird era is 1952063, we can calculate the JDN of the conjunction:

$$1952063 - 22338 = 1929725$$

which is the JDN of the 20th April 571. With this date, and using al-Battānī's parameters, we can calculate the positions of Saturn and Jupiter for 8 hours after midnight of the 20th April:

Saturn: 212;51

Jupiter: 212;40

which is a good approximation to the exact conjunction and proves two things: 1) that the source Ibn Abī l-Rijāl is using calculates the tropical true conjunction (not the mean one) and 2) that the position obtained does not agree with Scorpio 4;12° mentioned in chapter 38.

Another Saturn-Jupiter conjunction during the caliphate of al-Mutawakkil (r. 847–861) is also mentioned and the reference corresponds to the true conjunction which took place in May 848 in the sign of Pisces⁴¹⁰ (RHL 249r; *Conplido* 11, p. 262). Finally in chapter 17 (RHL 249r; *Conplido* 11, 263) we find the rounded lengths of the Saturn-Jupiter periods which, surprisingly, are called *qismas*:

[*al-qisma al-ṣughrā*], a mean conjunction which takes place every
20 years

al-qisma al-wuṣṭā, a mean conjunction with shift of the triplicity: every
240 years

al-qisma al-ʿuẓmā, a mean conjunction which takes place in the same
sign as the conjunction used as a radix: every 920 years

We should now try to understand Ibn Abī l-Rijāl's use of the cycles of the *intihāʾ*, *dawr* and *qisma*, not defined in the text but standard in Eastern astrology. The *intihāʾāt* (also called *adwār*, pl. of *dawr*) are periods of 12,000, 1,200,

⁴¹⁰ The mean conjunction took place at the beginning of January 849 in Aries, according to al-Khwārizmī's *zīj*. A horoscope of the spring equinox of year 848 was cast by Māshā'allāh: see Kennedy & Pingree, 1971, pp. 60–61, 116–117.

120, and 12 years,⁴¹¹ the shortest one being used in anniversaries of nativities, as we have already seen. The *dawr* may also correspond to cycles of 120, 72 or 75 and 12 years, of which the second one is equivalent to the *fardāriyya*.⁴¹² The *qismas* are cycles of 36000, 3600 and 360 years.⁴¹³ In chapter 39 of book VIII, the author explains the techniques for establishing the position occupied by the three cycles in a given date. The radix positions, as explained here (although other positions could also be used), are related to the conjunction of year 571:

- *Intihā'*: the 12-year cycle is used calculating from the ascendant of the conjunction (*min ṭālī' intiḡāl al-mamarr al-dāll 'alā Millat al-'arab*). The number of years between the year of the prediction and that of the conjunction corresponds to the number of signs to be subtracted from the sign of the ascendant of the conjunction which is assumed to be Libra. The same technique is applied to calculate the *intihā'* of the conjunction itself and, in this case, the radix position is obviously Scorpio.
- *Dawr*: again the 12-year cycle seems to be used, although it is not specified in the text. It is used to calculate, using the same procedure as in the *intihā'*, the position of the *dawr* with radix in Gemini, considered as the ascendant of the spring equinox of year 571.
- *Qisma*: the cycle of 360 years is used and the radix position is Pisces 20° which, according to Ibn Hibintā,⁴¹⁴ corresponds to the position of the Sun in the moment in which the cycle of the Arabs reached the sign of Gemini, related to planet Venus. As a matter of fact, the arrival of the cycle to the couple Gemini-Venus took place in year 582 AD, near enough to 571.

3.2.2.2.2.5.2 *Astrological Geography*

Chapters 34–37 of book VIII⁴¹⁵ deal with astrological geography, a topic which has already been the object of some interest in chapter 8 (RHL 243v; *Conplido* II, 240–243) which deals with the part of the inhabited world – extending from the Canaries to the end of China – that will be affected by the predicted events. It is interesting to remark that Ibn Abī l-Rijāl identifies the centre of the inhabited world (*Qubbat al-arḡ*) with *Tārah*, something which we also find in early authors like Ya'qub b. Ṭāriq (fl. in the second half of the

⁴¹¹ Labarta, 1982, p. 232.

⁴¹² Díaz-Fajardo, 2011 a, p. 38.

⁴¹³ Labarta, 1982, p. 237.

⁴¹⁴ E.S. Kennedy, 1983 a, p. 357. Kennedy's source is Ibn Hibintā's *Al-Mughnī* (ms. Munich 852, fol. 230v).

⁴¹⁵ Chapters edited, translated and analysed in detail by Puig-Pey, 2015.

8th c. CE).⁴¹⁶ This chapter also contains information about the hour difference between Baghdad and the *Qubba*: 1h + 1/3h, which implies a difference of longitude of 20°. This agrees with the longitude of Baghdad measured from the East: 110° (= 90° + 20°), corresponding to a longitude of 70° from the extreme West, which can be found in many sources among which we find a Khwārizmian source reported by al-Battānī.⁴¹⁷ 70° is also the value quoted by Ibn Abī l-Rijāl in chapter 37 (RHL 262v–264r; *Conplido* II, 313–322).⁴¹⁸

Ibn Abī l-Rijāl's geographical chapters contain a curious mixture of Ptolemaic, Persian and early Islamic materials. Chapter 34 (RHL 260r–260v; *Conplido* II, 304–306) explains the relation between climates, planets and zodiacal signs, although the climates mentioned (Hind, Bābil, Turk, Rūm, Ḥijāz, Miṣr, Ṣīn) do not correspond to the Ptolemaic ones and they remind, somehow, of the Persian *kishwar* organisation where the centre of the world corresponds to Irānshahr (here replaced by Bābil), surrounded by the other six *kishwars*: Maghrib (here Miṣr), Syria (replaced by Ḥijāz), Rūm, Turk, al-Ṣīn and al-Hind.⁴¹⁹ Chapter 35 (RHL 260v–261v; *Conplido* II, 307–310) gives much more detailed lists of countries and cities and their relations to the twelve zodiacal signs. The Ptolemaic climates are introduced in chapters 36 (RHL 261v–262r; *Conplido* II, 310–312) and 37 (RHL 262v–264r; *Conplido* II, 313–322) and the latter contains a description of the seven climates including, for each one, its maximum latitude and length of daylight in the summer solstice, followed by a list of some 234 localities with their geographical coordinates, rounded or truncated to the nearest degree. The identification of the place names is obvious in many cases and extremely difficult in others due to the corruptions in the manuscript transmission. Obviously the Castilian and Latin translations do not help the identification, but Aina Puig-Pey has achieved quite reasonable results in this kind of task.

The source explicitly quoted for the list of cities is the *zīj* of Ḥabash al-Ḥāsib (d. 869 AD) and, although I do not know of any clear evidence of the existence of such list in the extant *zīj* of Ḥabash,⁴²⁰ Aina Puig-Pey has established the existence of important links between this list, Ptolemy's *Geography* and *Handy Tables*, al-Khwārizmī's *Ṣūrat al-'arḍ* and the lists preserved by al-Farghānī and

416 Pingree, 1968 a, pp. 98–99. The correct name of the locality is Bāra (although al-Bīrūnī also uses Tāra) and it should be placed 90° East of both the Eastern and Western limits of the inhabited world.

417 Kennedy & Kennedy, 1987, pp. 55–56.

418 Puig-Pey, 2015.

419 See a diagram of the division of the world in *kishwars* in al-Bīrūnī's *Tafhīm*, p. 240.

420 It does not appear in the Istanbul manuscript of his *zīj*, which is the only one that has been seriously studied: see Debarnot, 1987.

Ibn Hibintā. To this she adds two important details: 1) the latitude of Baghdad (33;27°) is the only geographical coordinate of the list reaching the precision of minutes and this value coincides with that obtained by Ma'mūnī astronomers in their attempt to establish the value of 1° of the earth's meridian, according to the account of Ḥabash himself;⁴²¹ and 2) the maximum latitudes for each climate together with the longest length of daylight allow the computation of the obliquity of the ecliptic involved and Aina Puig-Pey's results oscillate between 23;31° and 23;35°, the latter value used by Ḥabash in his *zīj*.⁴²² As a result of all this, Puig-Pey concludes that Ibn Abī l-Rijāl's geographical table derives, indirectly, from the revision of Ptolemy's *Geography* undertaken by the collaborators of caliph al-Ma'mūn (r. 811–833 AD).

3.2.2.2.5.3 *Astrometeorology*

Ibn Abī l-Rijāl dedicates chapters 26–31 of book VIII to the prediction of meteorological phenomena and chapter 33 of the same book to prices. This part of the book is, obviously, one of the main sources used, at the beginning of the 15th c. by Abū 'Abd Allāh al-Baqqār in his already mentioned *Kitāb al-amṭār wa l-as'ār* ("On rains and prices"). Thus, it is significant that both sources consider that the entrance of the Sun in the 20th degree of Scorpio corresponds to a moment in which rains are to be expected (*Fī l-waqt alladhī yurjā fī-hi l-maṭar*) (RHL 256r–256v; *Conplido* II, 292–293).⁴²³ This idea can be found in al-Kindī⁴²⁴ and it can be related to Ibn Hibintā's remark that the period of rains in Iraq, Bābil and northern regions begins when the Sun reaches Scorpio 19° and ends with its passage through Taurus 19°. ⁴²⁵

Two other techniques for weather prediction have also been borrowed by al-Baqqār from Ibn Abī l-Rijāl:

1) The "opening of the door" (*fath al-bāb*) to rain, winds, heat and cold: this concept appears in many Eastern sources like 'Umar ibn Farrukhān, al-Kindī,⁴²⁶ al-Qabīṣī,⁴²⁷ Kūshyār b. Labbān⁴²⁸ or al-Bīrūnī.⁴²⁹ This phenomenon takes place when a lower planet applies to a superior planet whose domicile is

⁴²¹ King, 2000, p. 219.

⁴²² Debarnot, 1987, pp. 43 and 47.

⁴²³ Guesmi & Samsó, 2018, pp. 103–4, 237–8 ([122]), 108–9, 240 ([126]), 110–113, 242–3 ([129]).

⁴²⁴ Bos & Burnett, 2000, pp. 47, 257.

⁴²⁵ Ibn Hibintā, *Mughnī*, I, 203. *Jafar Indus* says that, according to the Indians, the 20th degree of Scorpio is meaningful for the prediction of rains: Burnett, 2004, p. 69.

⁴²⁶ Bos & Burnett, 2000, pp. 19, 385–394, 446–447.

⁴²⁷ Burnett, Yamamoto & Yano, 2004, pp. 136–137.

⁴²⁸ Yano, 1997, pp. 86–89, 90, 95.

⁴²⁹ Bīrūnī, *Tafhīm* pp. 314–315.

opposed, in the ecliptic, to that of the lower planet. Such is the case of Venus (Libra and Taurus) and Mars (Scorpio and Aries), or of Mercury (Virgo and Gemini) and Jupiter (Sagittarius and Pisces). This is al-Qabīṣī's definition but both Kushyār and al-Bīrūnī consider also the reciprocal case (a higher planet applies to a lower one) and al-Bīrūnī also accepts the Sun and the Moon with Saturn because the Sun's domicile (Leo) is opposed to the domicile of Saturn during the day (Aquarius) and the domicile of the Moon (Cancer) to that of Saturn during the night (Capricorn). Ibn Abī l-Rijāl (RHL 256r; *Conplido* 11, 291), followed by al-Baqqār,⁴³⁰ considers that the Moon plays an active role in the process and that the opening of the door takes place when the Moon leaves one of the planets of a couple and applies to the other one.⁴³¹

2) *Ta'sīs* or *Fashīsh* (according to al-Bīrūnī's *Tafhīm*⁴³²): the *ta'sīsāt* are a series of lunar positions, on its deferent circle, beginning with its conjunction with the Sun. Al-Kindī, al-Bīrūnī and Ibn Abī l-Rijāl (RHL 256r; *Conplido* 11, 291–292) consider that these positions are twelve and they are 0°, 12°, 45°, 90°, 135°, 168°, 180°, 192°, 225°, 270°, 315°, 348°. ⁴³³ Al-Baqqār's list is longer:⁴³⁴ it reaches 24 positions, 12 on each side of the axis formed by the conjunction-opposition, at unequal intervals which keep a symmetry (12°, 22;30°, 45°, 60°, 67;30°, 90°, 112;30°, 120°, 135°, 157;30°, 169;30°, 180°, 192°, 202;30°, 225°, 240°, 247;30°, 270°, 292;30°, 300°, 315°, 337;30°, 349;30°, 360°). It can be checked that the intervals between successive values are 12°, 10;30°, 22;30°, 15°, 7;30°, 22;30°, a series repeated in the inverse order.

430 Guesmi & Samsó, 2018, pp. 105–6, 238 ([123]).

431 The definition of “opening of the door” is not very clear in the first version of Abraham ibn ‘Ezra's *Sefer ha-Olam* although it seems to agree with al-Bīrūnī's opinion: “the lord of the seventh place (counting) from the (house of an) upper planet is always its key. Thus the luminaries are Saturn's keys, because their houses [Cancer, Leo] are opposite to its [Saturn's] houses [Capricorn, Aquarius]”. In the second version of the same book, Ibn ‘Ezra gives a different definition: one has to determine the lord of the sign of the ascendant at the moment of a luni-solar conjunction or opposition. If the ruling planet aspects the lord of the seventh place (after it), this is called “opening of the door”. See Sela, 2010, pp. 22–23, 83, 181.

432 Bīrūnī, *Tafhīm* p. 153, although in his *Athār*, p. 340, line 22, he speaks of *kawn al-qamar fī mawḍi‘ ta’sīsāti-hi*. Ibn Hibintā (*Mughnī* 1, 215) refers to the *mawḍi‘ al-ta’sīs*.

433 Abraham ibn ‘Ezra speaks of the “twelve keys of the Moon” and gives the same list of elongations of the Moon from the degree of conjunction. See Sela, 2010, pp. 23–24, 85, 137–139, 169, 210–211.

434 Guesmi & Samsó, 2018, pp. 103–4, 237–238 ([122]).

3.2.2.2.2.6 Horoscopes

The *Bārī* contains references to a certain number of horoscopes, scattered in books III (interrogations) and IV (nativities), which usually appear as examples of a particular point of the astrological theory explained. In some cases we have evidence of Ibn Abī l-Rijāl's practice of astrology and the corresponding horoscopes have clearly been cast by him. In book III, chapter 22 (RHL, 89r; *Conplido* I, 145) he refers to four cases in which he had predicted the removal of a governor of his position of power and remarks: "I have experimented (*jar-rabtu*) and predicted (*ḥakamtu*) about countless cases of governors (*ʿummāl*), rulers of frontiers (*wulāt al-thughūr*), cities, administrations (*dawāwīn*) and the treasure (*bayt al-māl*)":

1) One of them is Ibn Abī l-Ḥusayn's horoscope mentioned in 3.2.2.1 that allows us to delay the date of Ibn Abī l-Rijāl's death until after 1036.

2) Another case is that of a certain governor (*wālī*) Muḥammad b. al-Ḥasan about whom he predicted the moment in which he lost his position. My guess is that this reference concerns the Muḥammad b. al-Ḥasan who was the governor of Tripoli between ca. 1011 and 1016. In 19th Šafar 407/28th July 1016 he was named vice-emir, in charge of the control of the army and the general administration of the country. He kept this position until the 7th Rabīʿ II 413/11th July 1022 in which he was executed.⁴³⁵

3) A more difficult case is that of Ḥabūs b. Ḥumayd who was *wālī* of Ifrīqiya during 79 days: Pingree⁴³⁶ identifies him with the governor of Nefta with the same name who lost his *wilāya* in 439/1047–48. This would delay Ibn Abī l-Rijāl's date of death until ca. 1048.

4) The fourth case is that of ʿAbd Allāh b. Muḥammad who was in charge of the treasury in al-Manšūra. Here it is al-Muʿizz who, apparently, ordered Ibn Abī l-Rijāl to calculate how long he would keep such position: the astrologer's prediction was fifty-five months and ʿAbd Allāh lost his job nine days later. This identification is really difficult and I can only suggest, with no certainty at all, that this passage refers to ʿAbd Allāh b. Muḥammad al-Tamīmī al-Kātib, who governed Ifrīqiya since 974, imposed very high taxes and built a house made of iron and another of timber in Manšūriyya to store the treasure in 373/983–984. When al-Manšūr b. Buluggīn (984–996) reached the throne the 19th Rajab 374/16th December 984 he named him tax-collector of Qayrawān, Mahdiyya and the whole of Ifrīqiya, a position which he occupied until the 11th Rajab 377/

435 Idris, 1962, I, pp. 105–106, 133, 159–160 and II, p. 516.

436 Pingree in *EL* 111, 688.

6th November 987 in which he was executed.⁴³⁷ This identification is suggestive but it has two important drawbacks: on the one side the emir who asked Ibn Abī l-Rijāl to cast the horoscope could not be al-Muʿizz but his grandfather al-Manṣūr; on the other, the number of months between the 19th Rajab 374 and the 11th Rajab 377 is 35 (not 55, although a confusion between 35 and 55 is easy if the *abjad* system is used) and the number of days is 22, not 9.

Another set of horoscopes are concerned with nativities and Ibn Abī l-Rijāl insists in the fact that he has cast many nativity horoscopes and confirmed his predictions by experience (*jarrabtu dhālika fī ʿiddati mawālīd fa ṣaḥḥa lī*, RHL 124v; *Conplido* 1, 207). We know that a nativity horoscope was sent to him from Sicily because the Sicilian astrologers could not identify the *kadkhudāh* and Ibn Abī l-Rijāl was able to do it (RHL 100r; *Conplido* 1, 167). In another passage he seems to have cast the horoscope of Jaʿfar b. Yaḥyā b. Khālid, who lived 36 years (RHL 103r; *Conplido* 1, 172) and we also know that he cast his own horoscope and determined the length of life of his father (RHL 124v; *Conplido* 1, 207). Unfortunately in all these cases, no details are given and we cannot, therefore analyse the techniques used by him to cast his horoscopes.

In the rest of the cases Ibn Abī l-Rijāl is clearly borrowing his examples from other sources. Such is the case of an anecdote, probably derived from a Persian source, of the birth of the son of a king who died after twenty-four hours (RHL 97r; *Conplido* 1, 163); six nativity horoscopes by al-Sh.rūnī/Sh.rwīnī (perhaps Shirwānī) (?) (RHL 103v–104r; *Conplido* 1, 172–173) the purpose of which is to show how the *haylāj* and the *kadkhudāh* should be determined: the details of these horoscopes are not sufficient to try to date them; an anecdote related to a nativity horoscope dated in 230H/ 844–45 and attributed to al-Ḥusayn b. al-Khaṣīb (RHL 127v; *Conplido* 1, 211); there is also a collection of eleven nativity horoscopes, with little details, derived from Zaradusht's *Kitāb al-mawālīd*, with changes in the numerical values of the longitudes of the celestial elements⁴³⁸ (RHL 106v–107v; *Conplido* 1, 177). Another two nativities are also copied from Zaradusht (RHL 144r, *Conplido* 1, 239) and one of them is about a newborn who died in Ḥarrān and, in it, the planetary positions are given although I have not been able to date it. The details are the following ones:

The ascendant is Virgo 13°

All the planets are assembled between houses VII and IX. The beginning of House VIII is Aries 21°.

⁴³⁷ Idris, 1962, 1, pp. 48–50, 52, 54, 64, 68–69.

⁴³⁸ Díaz-Fajardo, 2011, pp. 357–360.

Mars and Jupiter are conjunct in Aries 22°
Saturn in Aries 23°
Venus in Aries 24°
Sun in the beginning of Aries
In house VIII we find the Moon in the degree of its exaltation in
Taurus (3°)

Finally we find three horoscopes with good details which were, apparently, copied from an unknown astrological treatise probably compiled during the Caliphate of al-Mahdī (775–785): the events to which these horoscopes refer could correspond to the rebellion of al-Muḡannaʿ (ca. 775–783) or to expeditions against the Byzantines (780, 782).⁴³⁹ We obviously do not know which kind of astronomical tables were used to calculate the planetary positions of this horoscope but it seems that these positions are sidereal. Therefore I have dated them and recomputed them using al-Khwārizmī’s *zīj*, which gives reasonable good results except for the position of the Sun in horoscope 1 and that of Venus in horoscopes 2 and 3. There is no way to recompute the ascendant because the local latitude or the position of midheaven are not given:

Horoscope 1: a man is looking for his brother who is engaged in a war defending his king (RHL 67v; *Conplido* I, 106–107):

	Text	Recomp. (17.3.785, 12 PM)
Ascendant	166°	
Sun	336;40°	357;44°*
Moon	28°	27;48°
Saturn	299°	301;53°
Jupiter	229° r	232;26°
Mars	21°	20;50°
Venus	339°	340;3°
Mercury	349°	348;15°

* My guess is that Pisces 6;40° given in the text is erroneous and should be replaced by Pisces 26;40°

439 H. Kennedy, “al-Mahdī” in *El2*, vol. v, pp. 1238–1239; Ed., “al-Muḡannaʿ” in *El2*, vol. vii, p. 500.

Horoscope 2: when will a long war finish? (RHL 67v–68r; *Conplido* I, 107)

	Text	Recomp. (9.1.779, 1 PM)
Ascendant	54°	
Sun	289°	289;43°
Moon	136;40°	136;23°
Saturn	233;42°	235;51°
Jupiter	34;30°	32;52°
Mars	282°	281;56°
Venus	301;2°	292;5°*
Mercury	291° r	292;48°

* I have no explanation for this error

Horoscope 3: if the enemy will get hold of a city (RHL 72v; *Conplido* I, 113)

	Text	Recomp. (10.8.782, 6 PM)
Ascendant	305°	
Sun	136;9°	137;1°
Moon	99°	98;50°
Saturn	262°	264;40°
Jupiter	149°	149;0°
Mars	201°	201;58°
Venus	165°	174;56°*
Mercury	123;20°	122;20°

* Again I can only guess that the position of Venus is not Virgo 15° but Virgo 25°

3.2.2.3 Other Works of Ibn Abī l-Rijāl

Ibn Abī l-Rijāl mentions, in the *Bārī*, three other works of his which do not seem to be extant. One of them is called “Our great book on the secrets of the stars” (*Kitābu-nā aʿẓam fī asrār al-nujūm*: RHL 39r, *Conplido* I, p. 61) which could refer to the *Bārī* itself. The second is the “Book on signs” (*Kitāb al-rumūz* RHL 89r, 93v, 99r; *Conplido* I, 154, 165–166) and the only information we have about it is that it contained five chapters on the *hayālīj* and the *kadkhudah*

and that part of the information contained in them had been borrowed from Zaradusht.

The third book was, apparently, the first *zīj* ever compiled in the Maghrib. Its title was *Ḥall al-ʿaqad wa bayān al-raṣad* ("Untying of knots and explanation of observations"). The insistence on observations appears in a reference which we can find in RHL 104v, 115v/BL 174r, *Conplido* I, pp. 174, 191–192 where he speaks about *zījū-nā alladhī raṣadnā-hu* ("our *zīj* which we observed") although we have no way to establish whether this is true or not and whether Ibn Abī l-Rijāl did actually participate in some kind of astronomical observations unless we identify our author with the Abū l-Ḥasan al-Maghribī, one of the astronomers who observed the summer solstice and the autumn equinox in Baghdad in 378/988 (see 3.2.2.1). As I have already said, we have no information about the school of thought to which this *zīj* belonged: did it calculate sidereal positions, following the Indian tradition of al-Khwārizmī's *zīj*, or tropical ones in agreement with the *Mumtaḥan* and al-Battānī's *zīj*es. As we have seen in 3.2.2.2.2.3, our only information can be found in the table of the revolution of the months (*al-shuhūr al-taḥwīliyya*) which clearly points to a use of the tropical solar year. Apart from this we only know that the *zīj* dealt with problems of mathematical astrology. In it, Ibn Abī l-Rijāl explained how to calculate the mixed ascensions for the *tasyīr* (see 3.2.2.2.2.2.1; RHL 104v, 115v/BL 174r, *Conplido* I, pp. 174). In RHL 197r/ *Conplido* II, 85 he says "Prorogate (*sayyir*) the five *hayālīj* in agreement with what I have described in the beginning of this book or in our *zīj* whose title is ...". Finally in RHL 38r and 106v (*Conplido* I, 61 and 176–177) he rejects the opinion of Dorotheus, Abū Ma'shar and al-Hamdānī, according to whom the opposition of two planets is not a real opposition if one of them has a positive latitude and the other a negative one. This is a problem he had solved in chapter 36 of his *zīj*.⁴⁴⁰

Besides these three (or two) non extant works, Ibn Abī l-Rijāl is the author of two *urjūzas* which have been preserved. One of them is the *Urjūza fī dalāl al-ra'd* ("On the indications of thunders"), which has never been studied although it has been considered the source of predictions based on thunders appearing in Andalusī and Maghribī calendars as well as in magical texts.⁴⁴¹

The other *urjūza* deals with the *aḥkām al-nujūm* and is, therefore, purely astrological. It has been edited by Marc Oliveras together with the commentary

440 See Díaz-Fajardo, 2011, pp. 363–365 and 2012–14 pp. 39–41. Abraham ibn 'Ezra says that two planets are not in opposition if both have the same northern or southern latitude. In order to have a real opposition it is necessary that one of the planets has northern and the other southern latitude. Sela, 2010, p. 136.

441 Two manuscripts, apparently corresponding to two different recensions, have been preserved in the National Library of Rabat: see Díaz Fajardo, 2008, p. 36; Albarracín, 1977.

of the Maghribī historian, astronomer and mathematician Ibn Qunfudh al-Qusanṭīnī (1339–1407).⁴⁴² This work, formed by 467 verses, has the same structure as the *Bārīʿ* and it can be summarised in the following way:

1. *Introduction* (vv. 1–21): zodiacal signs, Sun, Moon, the planets as well as certain astrological concepts like the *mubtazz* of the planet and the sign and the lord of the hour.
2. *Interrogations* (vv. 22–342): as in the *Bārīʿ*, the treatment of *masāʾil* follows the order of the houses of the horoscope related to each topic. It is interesting to remark that, out of the 320 verses of this part, 75 (vv. 243–318), almost a fourth part of the collection, deal with the analysis of the situation of the sovereign, whether his reign will be lucky or unlucky, how long it will last or what can one obtain from him.
3. *Elections* (vv. 343–423) dealing mainly with aspects of the life of common people like those we find in Book VII of the *Bārīʿ*. Only in a few cases the activity is directly related to the exercise of political power: how to choose the moment of the sultan's *bayʿa* (vv. 343–347) or of the ceremony of tying the knots (*ʿaqd al-liwāʾ*) or untying them (*ḥall al-liwāʾ*) in the flag (vv. 348–351) with which a military campaign begins or ends.
4. *Anniversaries of world years* (vv. 424–467) for which *tasyīr* techniques are used, as well as the theory of the great conjunctions of Saturn and Jupiter.

It has been said that the *urjūza* is the result of an abridgement of the *Bārīʿ*⁴⁴³ but the detailed analysis made both by Oliveras⁴⁴⁴ and by myself⁴⁴⁵ points in a different direction. First of all we should emphasize the fact that the *urjūza* omits the treatment of nativities and anniversaries of nativities which are dealt with in books IV–VI of the *Bārīʿ*. Concerning interrogations, both sources follow the same order of exposition (that of houses of the horoscope), but only in a very limited number of cases we can consider that the *urjūza* is summarising the contents of the *Bārīʿ*. In most instances coincidences are merely occasional and even in some of them there are clear disagreements. The situation is clearer in the part of the *urjūza* related to elections: here the verses do not follow the order of the houses used in the *Bārīʿ* and I have not been able to discover a logic in the order of the contents of the *urjūza*. As for the fact of whether the verses contain a summary of the *Bārīʿ*, I have divided vv. 343–423 into 29 thematic parts and in only 7 of them I have concluded that we have authentic abridgements. The situation is even clearer in the part related to world

⁴⁴² Oliveras, 2012; see also Samsó, 2009.

⁴⁴³ Hermosilla, 1978.

⁴⁴⁴ Oliveras, 2012, pp. 42–45.

⁴⁴⁵ Samsó, 2009, pp. 14–17, 356–360.

astrology in which I have not been able to find any correspondence between the chapters of the *Bārīʿ* and the verses of the *urjūza*.

We know nothing about the relative chronology of the two works although my impression is that the *urjūza* was composed at an earlier stage than the *Bārīʿ*. We do not know, either, which kind of readers had Ibn Abī l-Rijāl in mind when he wrote the *urjūza*. At first sight one could think that it was intended for popular astrologers who could learn the verses by heart and use them in the practice of their profession. The fact that the *urjūza* omits the treatment of nativities makes me doubt about the accuracy of this hypothesis. On the other side its insistence in the problems related to political power, the prediction of the government of a sovereign and the length of his reign makes me think that the *urjūza* was written having in mind important persons of the Zīrī court of Tunis.

3.2.3 *Mathematical Astrology in al-Andalus and the Maghrib*

3.2.3.0 Introduction

The problems of mathematics applied to Western Islamic astrology have been the object of very important studies during the last thirty years due to the research made by John D. North,⁴⁴⁶ E.S. Kennedy,⁴⁴⁷ J.P. Hogendijk,⁴⁴⁸ Josep Casulleras,⁴⁴⁹ Mercè Viladrich & Ramon Martí,⁴⁵⁰ Emilia Calvo⁴⁵¹ and Montse Díaz-Fajardo.⁴⁵² The interrelated problems that are going to be analysed in this part of the chapter are the following ones:

- 1) The division of the houses of the horoscope (*taswīyat al-buyūt al-ithnay ʿashar*): most astrologers agree in how to calculate the four cusps (*al-awtād al-arbaʿa*), namely the ascendant (*al-ṭālīʿ*, house I), intersection of the ecliptic with the eastern horizon; descendant (*al-ghārib*, house VII), intersection of the ecliptic with the western horizon; midheaven (*wasat*

446 North, 1986; North, 1996.

447 Kennedy & Krikorian-Preisler, 1983; Kennedy, 1996; Kennedy, 1995.

448 Hogendijk, 1989; Hogendijk, "Progressions, Rays and Houses in Medieval Islamic Astrology: a Mathematical Classification". Paper presented at the Dibner Institute Conference, *New Perspectives on Science in Medieval Islam* held in Cambridge, Mass, 6–8 November 1998. This paper was not published in the Proceedings of the conference but an updated version has been published by Casulleras & Hogendijk, 2012; Hogendijk, 2005.

449 Casulleras, 2004; Casulleras, 2007; Casulleras, 2008–9; Casulleras, 2009; Casulleras, 2010; Casulleras, 2013; Casulleras & Hogendijk, 2012.

450 Viladrich & Martí, 1983.

451 Calvo, 1998.

452 Díaz-Fajardo, 2008; Díaz-Fajardo, 2010; Díaz-Fajardo, 2011; Díaz-Fajardo, 2011 a; Díaz Fajardo, 2012–14; Díaz Fajardo, 2013.

al-samā', house x), intersection of the ecliptic with the local meridian in its upper culmination; cusp of the earth (*watad al-arḍ*, house iv), intersection of the ecliptic with the meridian in its lower culmination. In order to calculate the longitude of the beginning of the aforementioned four cusps one needs to know the solar longitude at midday and the hour. The computation will be made using a table of right ascensions and another one of oblique ascensions, calculated for the local latitude. As a matter of fact it is important to remember that houses i and vii are diametrically opposed and the same thing happens with houses x and iv. One can obviously avoid computation by using an astronomical instrument, usually an astrolabe.⁴⁵³ There is no agreement, however, on the system used to calculate the rest of the houses, for which several different systems have been designed. I will deal here with those used in al-Andalus and the Maghrib.

- 2) The *tasyīr*, prorogation or progression: this astrological technique, used to calculate the time that will elapse until a certain event takes place, has already been explained in 3.2.2.2.2.2.1. A simple way to calculate the *tasyīr* is to take into consideration the difference in ecliptic longitude between the *musayyir* and the *musayyar ilay-hi*, but the situation becomes more complicated if one considers that the *tasyīr* calculates time and time is measured on the equator. This implies that the positions of the two points have to be transferred, somehow, from the ecliptic to the equator, something which can be done in different ways.
- 3) The projection or casting of rays (*maṭraḥ al-shu'ā'āt*): a planet, considered to be on the ecliptic or near to it, casts its rays, right and left, at angles of 60° (sextile), 90° (quartile), 120° (trine) and 180° (opposition). The rays will reach another point of the ecliptic which will be affected by the positive or negative influence of the casting planet. The problem involved has similar alternatives as those of the *tasyīr*.

3.2.3.1 The Division of the Houses

3.2.3.1.1 *Standard and Prime Vertical Methods: the School of Maslama al-Majrīṭī*

3.2.3.1.1.1 The Standard Method

The study of this kind of problems in al-Andalus began, apparently, towards the end of the 10th century and it was undertaken by the school of Maslama al-Majrīṭī (d. 1007). It is common knowledge that the extant version of al-Khwārizmī's *zīj*, translated into Latin by Adelard of Bath, is the result of a

⁴⁵³ See Casulleras, 2013.

revision made by Maslama. In this *zīj* we find a rule for the computation of the houses of the horoscope using the so-called “standard method”,⁴⁵⁴ according to which we should obtain the right ascensions of the beginning of the four cusps: $\alpha_1, \alpha_4, \alpha_7, \alpha_{10}$ and obtain:

$$\begin{aligned}\Delta\alpha_n &= (\alpha_4 - \alpha_1) / 3 \\ \Delta\alpha_d &= (\alpha_1 - \alpha_{10}) / 3\end{aligned}$$

As $\alpha_1 - \alpha_{10} = D/2$ (half the length of daylight of the ascendant) and $\alpha_4 - \alpha_1 = N/2$ (half the length of the night arc of the ascendant), obviously

$$\begin{aligned}\Delta\alpha_n &= \text{two unequal night hours} \\ \Delta\alpha_d &= \text{two unequal day hours}\end{aligned}$$

This equivalence is the basis of the method explained in the canons of the *zīj*, which replace $\Delta\alpha_n$ and $\Delta\alpha_d$ by its equivalence in night and day hours. This seems to imply the use of a table of the length of daylight which has not been preserved in the extant version.

Then,

$$\begin{aligned}\alpha_2 &= \alpha_1 + \Delta\alpha_n \\ \alpha_3 &= \alpha_2 + \Delta\alpha_n\end{aligned}$$

and

$$\begin{aligned}\alpha_{11} &= \alpha_{10} + \Delta\alpha_d \\ \alpha_{12} &= \alpha_{11} + \Delta\alpha_d\end{aligned}$$

Which gives us the right ascensions of houses II, III, XI and XII. Using inversely the table of right ascensions we will obtain the corresponding longitudes of the houses:

$$\lambda_n = \alpha^{-1}(\alpha_n)$$

And, as houses VIII, IX, V and VI are opposite to II, III, XI and XII, the longitudes of the twelve houses have been adequately determined.⁴⁵⁵

454 Suter, 1914, pp. 25, 29–30; Neugebauer, 1962, pp. 76–78. The term “standard method” was coined by North, 1986, pp. 3–5.

455 Kennedy, 1996, pp. 538–540.

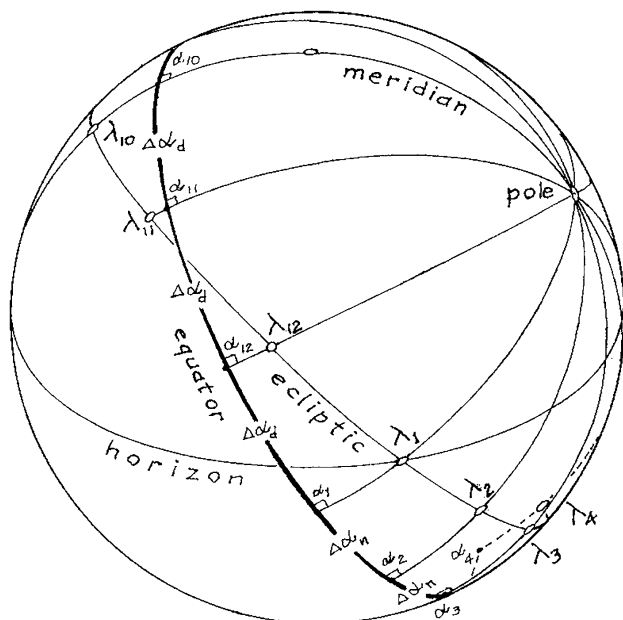


FIGURE 3.6
Standard method for
the division of the
houses borrowed from
E.S. Kennedy

Al-Khwārizmī's *zīj* contains also a table for the division of the houses according to the standard method calculated for each degree of longitude of the ascendant.⁴⁵⁶ The same table reappears in the *Toledan Tables*.⁴⁵⁷ Toomer⁴⁵⁸ studied this table and established that it was calculated for a latitude $38;30^\circ$ (Córdoba) and an obliquity of the ecliptic of $23;35^\circ$ (instead of the $23;51^\circ$ used by al-Khwārizmī). All this points to the fact that this table is probably due to Maslama al-Majrīṭī.

The standard method is probably the most frequently used one in the history of Islamic astrology and it often appears in Andalusian treatises on the use of the astrolabe or similar instruments.⁴⁵⁹ I will concentrate here on *zīj*es. In his revision of 28 sources (mostly *zīj*es), Kennedy has found it in twenty-seven ones. In our context, apart from al-Khwārizmī-Maslama's *zīj*, the method is described in the *Toledan Tables*,⁴⁶⁰ which also contain a set of tables calculated with this method for the latitude of Toledo ($39;54^\circ$), and an obliquity

456 Suter, 1914, pp. 194–205; Neugebauer, 1962, pp. 128–129.

457 Pedersen, 2002, vol. III, pp. 1075–1092.

458 Toomer, 1968, pp. 140–143. See also North, 1986, pp. 14–17.

459 North, 1986, p. 59; Calvo, 1998; Casulleras, 2013, pp. 519–520.

460 Pedersen, 2002, I, pp. 320–321 (canons Ca); II, pp. 424–427 (canons Cb), 628–631 (canons Cc).

of the ecliptic of about $23;35^{\circ}$.⁴⁶¹ The method also appears in chapter 25 of the *Tabulae Jahan*, a set of canons of a *zīj* composed, for the coordinates of Jaén (Jayyān) by *qāḍī* Abū ‘Abd Allāh Muḥammad b. Ibrāhīm b. Muḥammad b. Mu‘ādh al-Sha‘bānī (d. 1093),⁴⁶² only extant⁴⁶³ in Gerard of Cremona’s Latin translation printed in Nüremberg, 1549 under the title *Scriptum antiquum Saraceni cuiusdam de diversarum gentium eris, annis ac mensibus et de reliquis astronomiae principiis*. Ibn Mu‘ādh states that he does not approve it (“Illum quidem quod Alchorismus in libro suo dixit et quod narravit ex Ptolomeo non probo”).

We can find more references to the method in Ibn al-Kammād’s *Muqtabas zīj*,⁴⁶⁴ the Hyderabad recension of Ibn Ishāq’s *zīj*,⁴⁶⁵ Ibn al-Bannā’s *Minhāj*,⁴⁶⁶ Ibn al-Raqqām’s *Shāmīl*,⁴⁶⁷ *Qawīm*⁴⁶⁸ and *Mustawfī*⁴⁶⁹ *zīj*es, as well as in Ibn ‘Azzūz al-Qusanṭīnī’s *Muwāfiq zīj*.⁴⁷⁰ The *Muwāfiq* also contains a set of tables for the division of the houses, according to the standard method, for each degree of the ascendant and for the latitude of Fez.⁴⁷¹ Another table for the division of the houses according to the same method can be found in the *Tables of Barcelona*⁴⁷² and in Abraham Zacut (1452–1515) and José Vizinho’s *Almanach Perpetuum*,⁴⁷³ a source which was translated to Arabic in Morocco ca. 1624 and had a great diffusion in this country, as well as in the rest of the Islamic world.⁴⁷⁴ The table in question gives the houses for midday of every day of the radix year (1473).

461 Pedersen, 2002, III, pp. 1093, 1095–1101. See also Toomer, 1968, p. 146 and North, 1986, p. 17.

462 On Ibn Mu‘ādh see Samsó, 2011, pp. 137–144, 152–166, 240–244; E. Calvo & J. Casulleras, in *EA*, vol. 4, pp. 197–201; E. Calvo, in *BEA*, vol. 1, pp. 562–563.

463 Although some passages of this work are preserved in Arabic in the Hyderabad recension of the *zīj* of Ibn Ishāq. See 7.2.2.

464 On this *zīj* see Chabás & Goldstein, 1994. The description of the method appears in chapter 24: see ms. Biblioteca Nacional de Madrid Lat. 10023 fols. 12v–13r.

465 Mestres, 1996, pp. 400–401.

466 Vernet, 1952, pp. 55 (Arabic ed.) and 120–121 (Sp. trans.); Kennedy, 1996, p. 568.

467 Kennedy, 1996, pp. 558–559.

468 Ms. Rabat National Library 260 pp. 41–42; Kennedy, 1996, pp. 557–558.

469 Samsó, 2014, p. 307.

470 Ms. Rabat National Library 2461 pp. 368–369. On this *zīj* see Samsó, 1997.

471 Ms. RNL 2461 pp. 366–377. Actually, the standard method is used by Ibn ‘Azzūz in the retrospective horoscopes he cast in relation to the battle of El Salado (Faḥṣ Ṭarīf, 1340). See Samsó, 1999, pp. 116–119.

472 Millás, 1962, pp. 134–135.

473 Albuquerque, 1986.

474 Samsó, 2002–3 and 2004.

3.2.3.1.1.2 The Prime Vertical Method

Another method for the division of the houses is related to one of Maslama's disciples, Abū l-Qāsim ibn al-Samḥ (d. 1035).⁴⁷⁵ It is called "Prime vertical (fixed boundaries) method" in North's terminology.⁴⁷⁶ In it, we see the first appearance in the Islamic West, of "incident horizons" (*al-ufq al-ḥādith*),⁴⁷⁷ or position circles, which are arcs of a great circle passing through the North and South points of the horizon and through a third significant point on the sphere. They correspond to horizons for latitude ξ , different from the local latitude φ and comprised between 0° and φ . The prime vertical is divided, from the East and West points, into equal arcs of 30° and the incident horizons pass through the ends of such arcs. The intersections of these horizons with the ecliptic determine the beginning of the houses. The method seems to be the invention of al-Bīrūnī who, in the *al-Qānūn al-Masʿūdī*,⁴⁷⁸ qualifies it as "the method which I prefer" (*al-ṭarīq alladhī āthartu-hu*). In two other works of his, the *Maqālīd ʿilm al-hayʾa*⁴⁷⁹ and the *Kitāb fī istīʿāb al-wujūh al-mumkina fī ṣanʿat al-aṣṭurlāb*⁴⁸⁰ he says that it is his own invention.⁴⁸¹ It has been known for quite a long time that this method was used in al-Andalus because it is mentioned in three parts of the Alfonsine *Libro del Saber de Astrología: Libro del astrolabio llano* (where it is assigned to Hermes and Ibn al-Zarqālluh),⁴⁸² *Libro de la lámina universal* (attributed to Hermes)⁴⁸³ and *Libro de la aṣaḥḥa*.⁴⁸⁴ According to this latter source, Ibn al-Samḥ knew the method and computed a table for the division of the houses according to it.⁴⁸⁵ To the aforementioned Alfonsine sources, one should add the *Libro de la Alcora*,⁴⁸⁶ in a passage which is an Alfonsine addition and cannot be found in Qusṭā b. Lūqā's original text.⁴⁸⁷ The use of this method by Ibn al-Samḥ is also confirmed in the treatise on "the plate for all

475 See Comes, "Ibn al-Samḥ" in *EA*, vol. 5, pp. 227–230; Rius, "Ibn al-Samḥ" in *BEA* vol. 1, p. 568.

476 North, 1986, pp. 33–40.

477 The term *al-ufq al-ḥādith* is not used in Western Islamic texts. It seems to have been coined by Ibn Abī Shukr al-Maghribī (d. 1283). The translation "incident horizon" is due to E.S. Kennedy.

478 Bīrūnī, *Qānūn*, III, 1359–1369. North, 1986, pp. 32–33.

479 Bīrūnī, *Maqālīd*, pp. 284–291.

480 Ms. Istanbul Carullah 1451, fols. 24r and v; Ms. Tunis National Library 5540, fols. 35r and v.

481 Samsó, 1996, pp. 588–594. See also Kennedy, 1996, pp. 550–552.

482 Rico, 1863–1867, vol. II, 193.

483 Rico, 1863–67, III, 72.

484 Rico, 1863–67, III, 209–211. See also a modern translation of Ibn al-Zarqālluh's Arabic original text in Puig, 1987, pp. 84–85.

485 All this information was gathered by North, 1986, pp. 34–35.

486 Rico, 1863–67, I, 206–209.

487 Samsó, 2005 a, pp. 73–74.

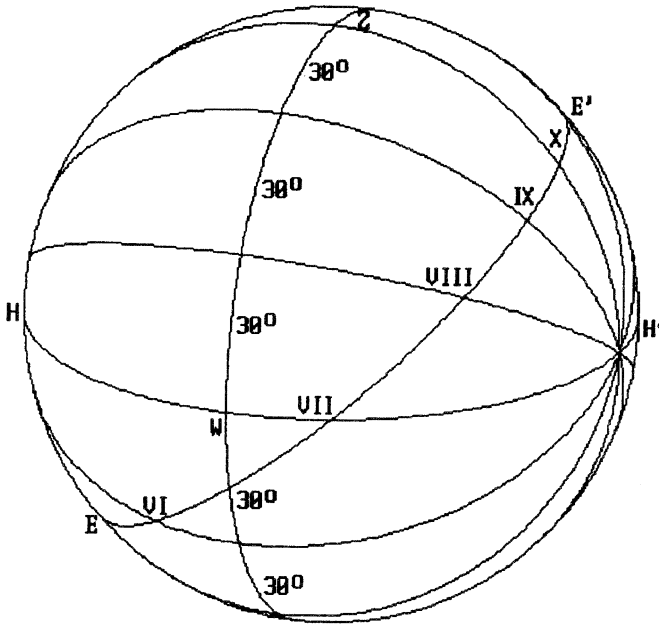


FIGURE 3.7
Prime vertical
(fixed boundaries)
method

latitudes" (*al-ṣaḥīḥa al-jāmi'a li-jamī' al-urūd*) by Ḥusayn b. Bāṣo (d. 1316) who refers to the lost *zīj* compiled by Maslama's disciple.⁴⁸⁸ Finally the first vertical method is also described in canons *Ca* of the *Toledan Tables*.⁴⁸⁹

The first information about Ibn al-Samh's algorithm for calculating the houses according to the prime vertical method was found in the *Tabulae Jahan*, The passage is to be found in chapter 25 (*De aequatione domorum duodecim*) which was difficult to understand. The correct solution was found with the study of another work of Ibn Mu'ādh, the *Risāla fī maṭraḥ al-shu'ā'āt* which offered a better text that was understood and explained by Jan Hogendijk⁴⁹⁰ and E.S. Kennedy.⁴⁹¹ Now we have an edition, translation and commentary of the complete text published by J. Casulleras.⁴⁹²

The *Maṭraḥ* contains a wealth of information about the techniques of mathematical astrology used at the time and Ibn Mu'ādh wrote this *risāla* as a result of his astonishment and scandal when he realised the mathematical ignorance of practising astrologers who applied complicated mathematical methods

488 Calvo, 1993, pp. 89 and 199 of the Spanish text, 175 of the edition.

489 Pedersen, 2002, I, p. 320.

490 Hogendijk, 2005.

491 Kennedy, 1995; Kennedy, 1996, pp. 541–543, 552–553.

492 Casulleras, 2010 pp. 239–266 (translation), 267–302 (edition).

when casting their horoscopes and made gross mistakes. It is not clear whether Ibn Mu'ādh was or not a believer in astrology, but his treatise seems to imply that he is saying to astrologers: "If you want to use these methods, at least use them properly".

Ibn Mu'ādh, in the *Maṭraḥ*, is very critical about the work of Ibn al-Samḥ whom he considers as being too verbose and careless in his demonstrations and computations.⁴⁹³ All his criticisms are centred in Ibn al-Samḥ's "great *zīj*" (*zījū-hu al-kabīr*) and confirms that it contained an algorithm for the computation of the houses and a table with the houses calculated according to the prime vertical method. The algorithm was erroneous as a result of Ibn al-Samḥ's assuming that

$$\sin a \pm \sin b = \sin (a \pm b)$$

Ibn Mu'ādh quotes literally the formulation of Ibn al-Samḥ's algorithm,⁴⁹⁴ and finds the cause of his error which, according to Ibn Mu'ādh, was repeated in other occasions in which Maslama's disciple faced similar problems. He, finally, gives the correct solution.⁴⁹⁵

It seems that Ibn al-Samḥ ascribed the prime vertical method to Hermes and, obviously, there is no reference to al-Bīrūnī. Besides, as remarked by Kennedy, "Ibn al-Samḥ's solution was quite different from any of Bīrūnī's three".⁴⁹⁶ This seems to discard the possibility that the method could have reached al-Andalus through one of al-Bīrūnī's works and during the lifetime of the Eastern astronomer (d. after 1048).

Whatever the case, it is interesting to remember al-Bīrūnī's remark that the method used to compute the houses according to the prime vertical method is long and tedious, but it becomes extremely simple with an astrolabe. For that purpose he designs, in the *Istī'āb*, a special astrolabe plate.⁴⁹⁷ Such design can be followed in Fig. 3.8 (not to scale):

Let ABGD be the projection of the tropic of Capricorn in a standard astrolabe plate and KEHMF the projection of the equator. He first determines which, among the vertical circles is the prime vertical (KZM) and divides it into 30° arcs starting from the East point. The next step is to determine the centre of the projection of the horizon which he can do using the same

493 Casulleras, 2010, pp. 240–241 (translation) and 269 (edition).

494 Casulleras, 2010, pp. 252 (translation) and 283–284.

495 Casulleras, 2010, pp. 253–257 (translation) and 284–290.

496 Kennedy, 1996, p. 541.

497 Samsó, 1996, pp. 589–591.

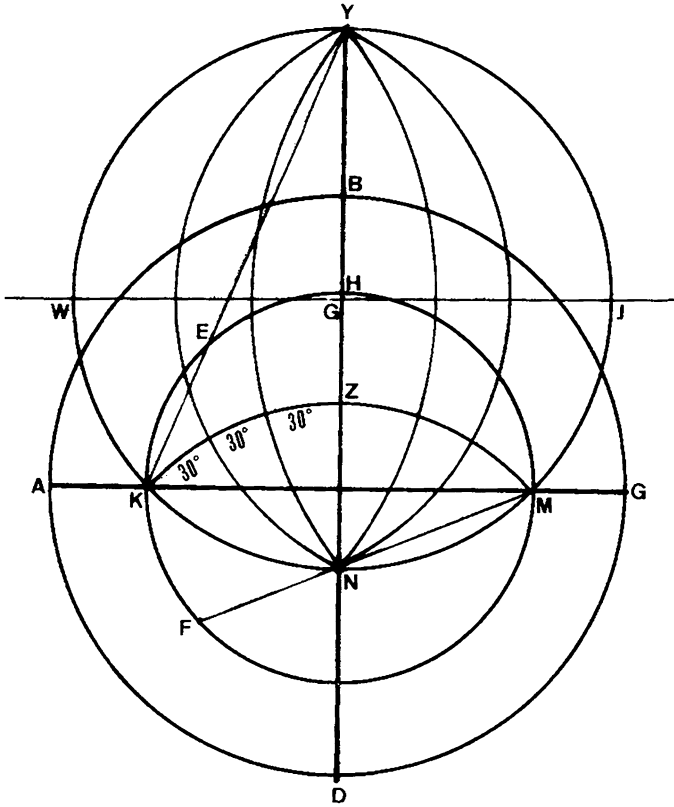


FIGURE 3.8
Al-Bīrūnī's
design of an
astrolabe plate
for casting the
houses according
to the prime
vertical method

procedure he has employed for the equatorial method (see below): from point K he takes arcs KE and KF, on the projection of the equator, both equal to the local latitude. Straight lines KE and MF will determine, on the meridian line, points Y (south point of the horizon) and N (North point of the horizon). G will be the midpoint of segment YN and, thus, the centre of the projection of the horizon. WGJ, perpendicular to the meridian line, will be the line on which we will find the centres of the circles which will divide the ecliptic. These circles will necessarily go through points Y and N and through the 30° divisions of the prime vertical.

We actually find, in the Andalusian tradition, a certain number of astrolabe plates traced for the use of the prime vertical method in the three aforementioned astrological techniques (division of houses, *tasyīr* and projection of rays).⁴⁹⁸ Obviously these plates contain more position circles than the six

498 On these instruments see Casulleras, 2010, pp. 126–128; Casulleras, 2013, pp. 525–526; Casulleras & Hogendijk, 2012, pp. 76–78.

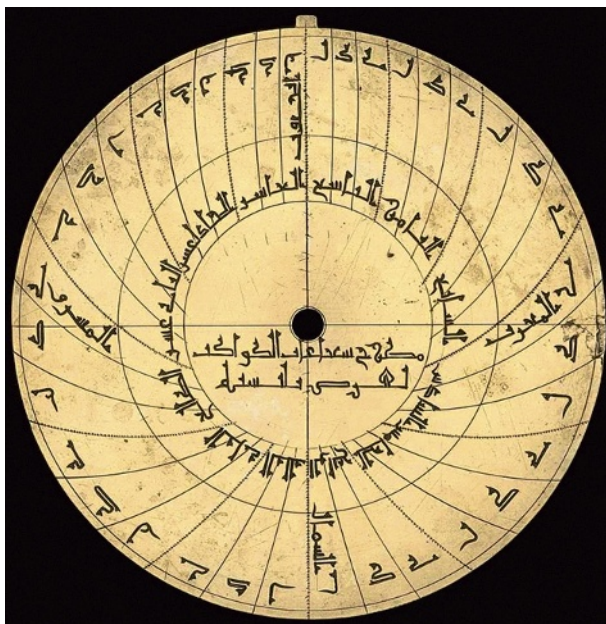


FIGURE 3.9
Plate for the projection
of rays, according to the
prime vertical method,
for the latitude of
Valencia in the astrolabe
of Muḥammad al-Ṣabbān
INV. 52473 © HISTORY
OF SCIENCE MUSEUM,
OXFORD UNIVERSITY

needed for the division of the houses but they constitute a clear proof of the fact that the three techniques used similar instruments:

- 1) Hogendijk drew attention to a Toledan astrolabe made in 1029–1030 (with two plates for latitudes $38;30^{\circ}$ and 42°) traced for the projection of rays, but with evidence showing that it was also used for the division of the houses. The plate has position circles which intersect the prime vertical at intervals of 6° .⁴⁹⁹
- 2) A second instrument, dated 1081–82 bearing two plates for the projection of rays has been analysed by David King.⁵⁰⁰ The maker was Muḥammad al-Ṣabbān and the two aforementioned plates were designed for Valencia (latitude $39;28^{\circ}$, see Figure 3.9) and Zaragoza. The position circles have an interval of 10°
- 3) An astrolabe signed by Aḥmad ibn Ḥusayn ibn Bāšo, the son of Ḥusayn ibn Bāšo, already mentioned as the author of a treatise on the plate for all latitudes, dated in 1304–1305, contains a plate with markings for the projection of rays at ten-degree intervals, as well as the inscription “Method of Hermes for the latitude $37;30^{\circ}$ ” (Granada).⁵⁰¹

499 Hogendijk, 2005, p. 99. Description of this astrolabe in Hernández, 2018 a, pp. 51–58.

500 King, 2005, pp. 936–937, 940. Description in Hernández, 2018 a, pp. 111–120.

501 Description in Hernández, 2018, pp. 219–226.



FIGURE 3.10
A plate for the division
of the houses for latitude
37;30° extant in the Vienna
Technical Museum

- 4) Finally, another astrolabe extant in the Technical Museum of Viena,⁵⁰² which seems also to be the product of one of the Banū Bāṣo and contains the plate for all latitudes, has a plate (see Fig. 3.10) for the division of the houses with position circles traced with an interval of 30°. It has been calculated for a latitude 37;30° (Granada) and is the only known extant plate designed specifically for this purpose.

3.2.3.1.2 *Ibn Mu'adh and the Equatorial Method*

The equatorial method for the division of the houses is analogous to the prime vertical system. Once more, six position circles are used which pass through the South and North points of the horizon and through the thirty-degree divisions of the equator. The intersection of these position circles with the ecliptic determine the beginning of the houses (see Fig. 3.11).

In the *Maṭraḥ*, Ibn Mu'adh disapproves both the standard and the prime vertical methods for the division of the houses and favours the equatorial method which is the only one he considers absolutely correct.⁵⁰³ Besides, he believes that Ibn al-Samḥ made a mistake, in the circle used as a base, when he ascribed the prime vertical method to Hermes, and suggests the possibility that the equatorial method was the system invented by Hermes himself for the

⁵⁰² Firneis, 1987. Description in Hernández, 2018 a, pp. 235–242.

⁵⁰³ Casulleras, 2010, pp. 200, 249, 258–259, 281, 292.

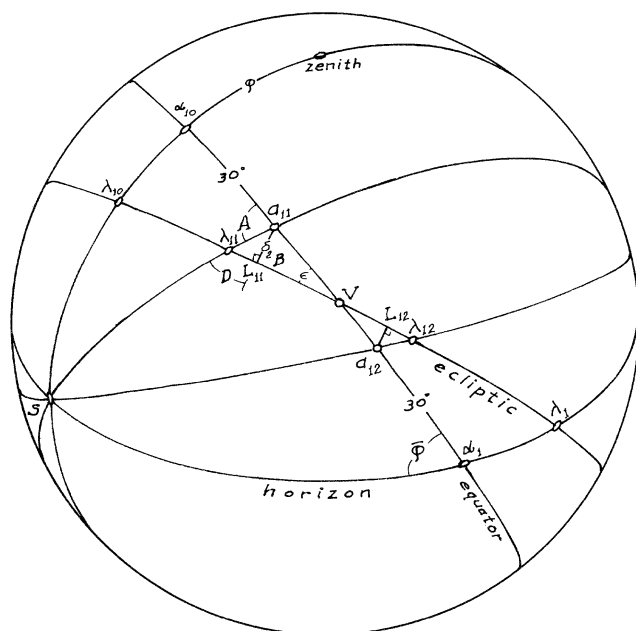


FIGURE 3.11
Equatorial method
for the division of the
houses

division of the houses.⁵⁰⁴ This implies, logically, that he does not pretend to have invented the method. In the *Tabulae Jahen* he describes an algorithm to calculate the beginning of the houses according to this method, but the Latin text remained incomprehensible⁵⁰⁵ until it was analysed, together with the description in the *Maṭrah*, not of the procedure for the division of the houses but of the system for casting the rays according to the method of the four position circles, which he considers analogous to the equatorial method for the division of houses.⁵⁰⁶

The equatorial method has only been found by Kennedy in Western sources.⁵⁰⁷ Apart from Ibn Mu‘adh, Ibn al-Raqqām gives two solutions for it in his *Shāmīl Zīj*.⁵⁰⁸ Universal instruments such as Ibn al-Zarqāllūh’s *ṣafiha*⁵⁰⁹ and Ibn Baṣo’s plate for all latitudes⁵¹⁰ use their universal plates to divide the houses according to this method. The Alfonsine corpus contains references which can

⁵⁰⁴ Casulleras, 2010, pp. 257, 290.

⁵⁰⁵ North, 1986 pp. 36–38.

⁵⁰⁶ Hogendijk, 2005; Casulleras, 2004.

⁵⁰⁷ Kennedy, 1996, p. 543.

⁵⁰⁸ In chapters 102–103. See Kennedy, 1996, pp. 559–568. See the edition of the canons in ‘Abd al-Raḥmān, 1996, pp. 139–146.

⁵⁰⁹ Puig, 1987, pp. 82, 85–86.

⁵¹⁰ Calvo, 1993, pp. 179, 181, 202, 179–181.

be found in the *Libro de las armellas*,⁵¹¹ and in the *Libro del ataçir*.⁵¹² There is no agreement concerning the method (prime vertical or equatorial) used in the *Libro del astrolabio redondo*.⁵¹³ In spite of this, I do not believe that this method was purely an Andalusian or Maghribī invention as there is evidence of the use of the method in the East for *tasyīr* and projection of rays: in al-Bīrūnī's *Istī'āb*⁵¹⁴ we find a description of how to project an astrolabe plate which the author calls *ṣafīḥat al-tasyīr* (*tasyīr* plate) although it is better known as *ṣafīḥat maṭraḥ al-shi'ā'* (plate for the projection of rays). In it we find the projection of great circles which pass through each degree of the equator as well as through the two intersections of the meridian and the horizon. The system described by al-Bīrūnī is analogous to the one described in 3.2.3.1.1.2 for the prime vertical method and it is illustrated in Fig. 3.12 (not to scale). ABGD is, again, the tropic of Capricorn, KSML the equator, KOM the horizon for a given latitude, Y and O are the south and the north points of the horizon (determined as before) and S is the midpoint of YO and the centre of the projection of the horizon. The line passing through S, parallel to the East-West line AG, will be the locus of all the centres of the circles of position which will pass through points Y and O as well as through equal divisions of the equator (30° intervals in the figure). A plate of this kind has been found in an Eastern astrolabe signed by al-Khujandī and dated in 984–985 for a latitude of 33°: the position circles intersect the equator at five-degree intervals.⁵¹⁵

With minimum differences the same procedure is applied by Rabiḥaḡ (Rabbi Ishāq b. Sīd) to draw a similar plate which, combined with a standard astrolabe, has astrological uses such as *tasyīr*, projection of rays and division of the houses.⁵¹⁶ The only difference is that Rabiḥaḡ plate contains the whole projection of the horizon, while al-Bīrūnī's outer limit is, as usual in an astrolabe plate, the projection of the tropic of Capricorn.⁵¹⁷

Another plate of the same kind has been described by North⁵¹⁸ in an already mentioned astrolabe (3.2.3.1.1.2) made by Aḥmad b. Ḥusayn b. Bāṣo, dated in 1304–1305, for the latitude of Granada (37;30°) and with the circles of position traced with an interval of 10°. Curiously enough the plate bears the inscription

⁵¹¹ Rico, 1863–1867, vol. II, p. 67.

⁵¹² Rico, 1863–1867, II, 295–309. North, 1986, pp. 35–36.

⁵¹³ North, 1986, p. 34; Calvo, 1998, p. 36; Casulleras & Hogendijk, 2012, p. 82.

⁵¹⁴ Ms. Carullah fols. 15v–16v.

⁵¹⁵ This astrolabe has been described by D.A. King in several publications mentioned by Casulleras & Hogendijk, 2012, p. 51.

⁵¹⁶ Rico, 1863–67, II, 302–304. See Viladrich & Martí, 1983.

⁵¹⁷ Samsó, 1996, pp. 591–593.

⁵¹⁸ North, 1986, pp. 60–65.

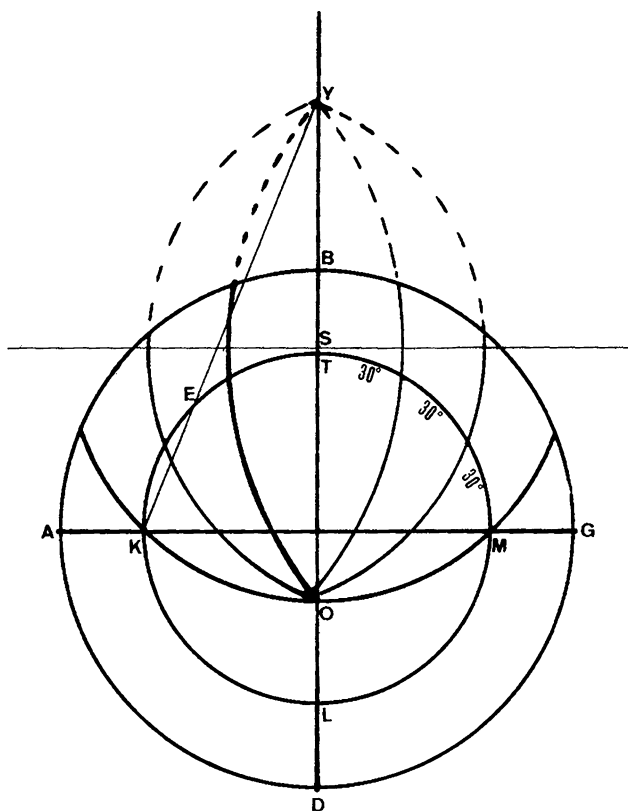


FIGURE 3.12
Design of an astrolabe
plate for the division of
the houses (equatorial
method)

madhhab al-Ghāfiqī (al-Ghāfiqī's method). We cannot be sure of the identity of this al-Ghāfiqī although North suggests that he might be Ibn al-Ṣaffār (d. 1035), another disciple of Maslama al-Majrīṭī, whose complete name was Abū l-Qāsim Aḥmad b. ʿAbd Allāh b. ʿUmar al-Ghāfiqī.⁵¹⁹

Besides Ibn Bāṣo's plate, the same kind of instrument has been found in an 11th c. Andalusian plate (latitude 35°) preserved in an Ottoman astrolabe (!),⁵²⁰ and in a Moroccan astrolabe dated in 1208 containing four plates for the casting of rays and for latitudes 31°, 33;40°, 37;30° and 38;30°.⁵²¹

519 Samsó, "Ibn al-Ṣaffār" in *EA*, vol. 5, pp. 69–73; Rius, "Ibn al-Ṣaffār" in *BEA*, vol. 1, pp. 566–567.

520 King, 2014, pp. 333–334.

521 Casulleras & Hogendijk, 2012, pp. 51–52.

3.2.3.1.3 *Other Methods for the Division of the Houses*

There is evidence of the use of three other methods for the division of the houses:

- 1) **Hour lines method:** it uses the diagram of unequal hours which we usually find in a standard astrolabe plate projected for a particular latitude. These hour lines are usually considered as approximations to the position circles. If one places the rete in position, in such a way that the ecliptic degree of the ascendant (I) will be on the Eastern horizon and, consequently, the three other cusps will be on their corresponding positions, the intersections of the ecliptic with the curves for the even hour lines will determine the beginning of houses II (10th hour), III (8th hour), IV (6th hour), V (4th hour) and VI (2nd hour). Houses VII–XII will be diametrically opposed to houses I–VI.

This system is extremely simple for anyone using an astrolabe and, on first sight, it seems an approximation to the standard method. It is, however, extremely difficult to calculate using an algorithm because it implies solving a cubic equation.⁵²² This is probably why E.S. Kennedy did not find it described in any of the 28 *zīj*es he used as sources for his analysis of methods of house division in Islamic times. Nevertheless, the hour lines method appears frequently in treatises on the use of the astrolabe like those written Ibn al-Samḥ – where the method is attributed to Ptolemy in a quotation derived from Ḥabash al-Ḥāsib,⁵²³ Abraham b. ‘Ezra,⁵²⁴ the Alfonsine treatise on the spherical astrolabe (method attributed to Ptolemy and Vettius Valens)⁵²⁵ as well as that on the plane astrolabe.⁵²⁶

- 2) **Dual longitude method:** this is another simple method described in Greek and Eastern Islamic sources. Al-Bīrūnī mentions it as the method of the ancients (*al-awā’il*), while later authors of the Mashriq ascribe it to Western Islamic practice.⁵²⁷ Kennedy has found it in seven sources (out of twenty eight), three of which are Maghribī.

⁵²² North, 1986, pp. 21–23.

⁵²³ Viladrich, 1986, pp. 66–67, 124.

⁵²⁴ North, 1986, pp. 21–25. This method is described by Ibn ‘Ezra in his Latin treatise on the use of the astrolabe, the *De rationibus*, the *Book of nativities* and the *Book of reasons*: see Millás, 1940, p. 22; Millás, 1947 a, pp. 159–161; Sela, 2007, pp. 97, 99–103.

⁵²⁵ Rico, 1863–67, II, 193.

⁵²⁶ Rico, 1863–67, II, 268–269; Martí & Viladrich, 1983, pp. 47–49. On this list of sources see Calvo, 1998 p. 36; Casulleras, 2013, pp. 530–531.

⁵²⁷ North, 1986, p. 33; Kennedy, 1996, pp. 540, 553; Casulleras, 2009, p. 55.

The method is based on the trisection of the arc of the ecliptic between two cusps. Thus, if λ_I is the longitude of the beginning of the ascendant and λ_{IV} the longitude of the beginning of the IV house:

$$(\lambda_I - \lambda_{IV}) / 3 = d_I$$

$$\lambda_I + d_I = \lambda_{II}$$

$$\lambda_{II} + d_I = \lambda_{III}$$

The same will be done with the longitudes of the beginnings of houses IV and VII and we will be able to obtain houses V and VI. For the rest of the houses we will apply the symmetry which establishes that

$$\lambda_{n+6} = \lambda_n + 180^\circ$$

This method is described in the Hyderabad recension of Ibn Ishāq's *zīj*, where it is called *al-ʿamal al-thulāthī*, followed by a description of the standard method which should only be applied to important horoscopes such as those of conjunctions, anniversaries and nativities which require a better precision.⁵²⁸ Both methods are also described in Ibn al-Kammād's *Muqtabas zīj*⁵²⁹ and in Ibn al-Raqqām's *Qawīm*⁵³⁰ and *Mustawfī*⁵³¹ *zīj*es. Besides this, the dual longitude method is used systematically by Ibn Qunfudh al-Qusanṭīnī in the twelve horoscopes he includes, as examples of the theory, in his commentary on Ibn Abī l-Rijāl's astrological *urjūza* (see 3.2.2.3).⁵³² Finally, a late use of the method has been found in ʿAbd Allāh Aṣṇāk al-Marrākushī (fl. ca. 1655) in a *risāla* on the use of Abraham Zacut and José Vizinho's *Almanach Perpetuum*. This author, after explaining the method, gives a worked example calculated for a latitude of 33° .⁵³³

- 3) **Split differences method:** this method seems to have been described in only one source: Ibn al-Raqqām's *Shāmil Zīj*⁵³⁴ where the division of the houses is said to have been calculated "from the difference of the rising and setting degrees" (*min qabl tafāḍul darajatay al-ṭulūʿ wa l-ghurūb*).

528 Mestres, 1996, pp. 400–401.

529 See ms. Biblioteca Nacional de Madrid Lat. 10023 fol. 13r.

530 Ms. Rabat National Library 260 pp. 41–42; Kennedy, 1996, pp. 557–558.

531 Samsó, 2014, p. 307.

532 Samsó, 2009, pp. 35–37; Samsó, 2004 a, pp. 500–502.

533 Samsó, 2004, pp. 87–89.

534 Kennedy, 1996, pp. 544–545, 568; edition of the corresponding canon in ʿAbd al-Raḥmān, 1996, pp. 147–148.

The procedure is relatively simple: α_{10} being the right ascension of the beginning of midheaven (house x), we add 30° to it and determine a point on the equator (a_{11}):

$$\alpha_{10} + 30^\circ = a_{11}$$

Then, using inversely tables of right (α_o) and oblique (α_φ) ascensions we obtain $\alpha_o^{-1}(a_{11})$ and $\alpha_\varphi^{-1}(a_{11})$ and obtain the difference between them:

$$|\alpha_o^{-1}(a_{11}) - \alpha_\varphi^{-1}(a_{11})| = d_{11}$$

This arc will be trisected ($d_{11}/3$) and the result added to $\alpha_o^{-1}(a_{11})$ obtaining the longitude of house XI:

$$\lambda_{11} = \alpha_o^{-1}(a_{11}) + d_{11}/3$$

For the computation of the longitude of house 12, the operation will be similar beginning with

$$\alpha_{10} + 60^\circ = a_{12}$$

We will obtain $\alpha_o^{-1}(a_{12})$ and $\alpha_\varphi^{-1}(a_{12})$ as well as d_{12} , and $d_{12}/3$. Finally,

$$\lambda_{12} = \alpha_o^{-1}(a_{12}) + 2d_{12}/3 = \alpha_\varphi^{-1}(a_{12}) - d/3$$

The procedure for the computation of the longitudes of houses II and III will be the same, beginning by the trisection of the arc between the east point of the equator and lower midheaven: λ_3 will have a higher component of right ascension and λ_2 of oblique ascension.

Hogendijk has considered this procedure as an approximation to the hour line method⁵³⁵ which, as we have seen, is difficult to reduce to an algorithm. I would like to add that it shares a similar geometrical approach with the method used by Ibn Abī l-Rijāl and Ibn Qunfudh (mixed ascensions) to calculate the *tasyīr*, which derives from Ibn Hibintā (see 3.2.2.2.2.1). It has analogies with the approximate method used by Ibn Mu‘ādh for casting the rays (see below 3.2.3.3.6).

535 Casulleras, 2009, p. 62; Casulleras & Hogendijk, 2012, pp. 85–87.

3.2.3.2 The *tasyīr*, Progression or Prorrogation

3.2.3.2.1 *Tasyīr and Astrological Cycles*

In 3.2.2.2.2.2.1 I have already analysed Ibn Abī l-Rijāl's method for the computation of *tasyīr* and explained that the main purpose of this technique is to establish the amount of time needed until a certain event takes place by calculating the difference in position between two heavenly bodies or significant astrological points placed on the ecliptic. It is frequently applied to nativity horoscopes in order to predict the moment of death of the newborn but it can also be used to establish the length of the reign of a monarch or for longer periods in world astrology. The difference between the two positions expressed in degrees (of longitude on the ecliptic, or of right or oblique ascension on the equator) have a system of equivalences in time. These equivalences follow different rules, one of them being the kind of cycles used. I have already mentioned in 3.2.2.2.2.3 and 3.2.2.2.2.5.1 the cycles used by Ibn Abī l-Rijāl. Another 11th c. source is the *Risāla fī l-tasyīrāt wa-maṭāriḥ al-shu'ā'āt* written by the Toledan astronomer and astrologer Abū Marwān 'Abd (or 'Ubayd) Allāh al-Istijī (fl. 2nd half of the 11th c.).⁵³⁶ In his case these cycles correspond to the so-called *intihā'āt* of Eastern sources, although he does not use the term *intihā'*.⁵³⁷

- 1) The mighty world-*intihā'* which moves through a zodiacal sign in 1000 years and corresponds to a cycle of 12000 years. This cycle regulates events that affect the whole world: floods, drought, periods of anarchy, wars, deposition of kings, everything concerning moral rules and laws as well as those (prophets and kings) who dictate them.
- 2) The big world-*intihā'* which moves through a zodiacal sign in 100 years and completes a revolution in 1200 years. It affects nations and parts of the world, events related to religions and their great personages, the transfer of power from one nation to another or from one dynasty to another.
- 3) The middle world-*intihā'* which moves through a zodiacal sign in 10 years and completes a revolution in 120 years. It affects individuals because 120 years is the natural (*ṭabī'i*) duration of human life. The portion (*ḥiṣṣa*) corresponding to 1 year is 3° (= 360° / 120) and the [standard] duration of human life is between 60 and 70 years. Al-Istijī's explanation is clear: if one uses as indicator (*haylāj* or *mutaqaddim*) the ascendant of the horoscope cast at the nativity of the subject, a *tasyīr* of 3° per year will result in an arc of 180° in sixty years or 210° in seventy years, which implies that

⁵³⁶ See Samsó, "Ibn Jalaf al-Istijī", *BA*, vol. 6, pp. 565–568.

⁵³⁷ Samsó & Berrani, 2005, pp. 183–185, 228–229.

the *tasyīr* has reached the descendant (house VII) or the house of death (VIII), assuming a mean length of 30° for each house. Both are most unhappy places which act as *thānī* or *qāṭi* and produce death. Another reference to the “natural length of human life”⁵³⁸ is 75 years, a period identified with the *fardārāt* of the seven planets and of the ascending and descending lunar nodes.⁵³⁹

- 4) The small world-*intihā* which moves through a zodiacal sign in 1 year and corresponds to a cycle of 12 years. Individual men are submitted each year to the influence of one zodiacal sign, which is called *burj al-dawr*, *burj al-muntahā* and *burj al-intihā*. This is the kind of period that affects the nativity horoscopes, interrogations and elections.

In the case of nativity horoscopes, the four main cycles used in the Maghrib have been analysed by Díaz-Fajardo⁵⁴⁰ on the basis of the three following sources:

- Ibn al-Bannā (fl. Marrakesh, 1256–1321) in a short tract entitled *al-Kalām ‘alā l-tasyīrāt wa-maṭāriḥ al-shu‘ā’āt*.⁵⁴¹
- Ibn ‘Azzūz (d. Constantina 1354) in his *Kitāb al-fuṣūl fī jam‘ al-uṣūl* (extant in ms. Rabat Ḥasaniyya 1110).⁵⁴²
- al-Baqqār (fl. Fez 1411–1418) in his *al-Adwār fī tasyīr al-anwār*.⁵⁴³

These cycles are the minor ones in Abū Ma’shar’s system explained in his *Kitāb al-ulūf*⁵⁴⁴ and they seem to have been very popular in the Maghrib, as they are ascribed to the *madhhab jamā‘at al-munajjimīn* (the school of the collectivity of astrologers) and appear in a marginal note in manuscript Rabat Ḥasaniyya 3503 (fols. 104r–105r) containing Ibn Abī l-Rijāl’s *Kitāb al-Bārī*. The four cycles in question are:

- *Qisma*: a cycle of 360 years which advances 1° per year on the equator.
- *Al-dawr al-akbar* (the great *dawr*) also called *al-tasyīr al-ṭabī‘ī*: a cycle of 120° which advances 3° per year on the ecliptic. It is equivalent to al-Istijī’s middle world *intihā*.

538 Samsó & Berrani, 2005, pp. 196–197, 232.

539 Kennedy, 1983 a, pp. 356–358. See also 3.3.1.2.2.3 *supra*.

540 Díaz-Fajardo, 2011 a.

541 Jabbār & Aballāgh, 2001, pp. 160–165.

542 Samsó, 1997.

543 The astronomical part of this work has been edited and analysed in Díaz-Fajardo, 2001. Many passages of the astrological part are also edited in Díaz-Fajardo, 2008. Finally, the part concerning the four cycles is published in Díaz-Fajardo, 2011 a, pp. 54–56. Díaz-Fajardo is now preparing a complete critical edition of this work.

544 Pingree, 1968.

- *Al-dawr al-awsaṭ* (the middle *dawr*): it is a cycle of 75 years ($4;48^{\circ}$ per year) according to al-Baqqār, or 72 years (Ibn al-Bannā' and Ibn 'Azzūz, 5° per year). It is also named *al-tasyīr al-firdārī* because the cycle of 75 years corresponds to the *firdār/fardār/fardāriyya* of the seven planets and both nodes (see 3.2.2.2.2.3). The progressions are made on the ecliptic.
- *Al-dawr al-aṣghar* (the small *dawr*) also called *al-tasyīr al-dawrī* or *burj al-intihā'*. It is a cycle of 12 years which advances 30° per year on the ecliptic. It is also al-Istijī's small world-*intihā'*.

3.2.3.2.2 *Methods for the Computation of the tasyīr*

3.2.3.2.2.1 The Direct Method on the Ecliptic

We have just seen that the direct method for prorogation on the ecliptic, which means simply to establish the difference of longitudes between the two indicators – the *haylaj*, also called *al-mutaqaddam* or *al-musayyir*, and *al-qāṭi'* – has been used in the Maghrib when the cycles of the three *dawrs* (great, middle and small) are employed. The corresponding number of degrees will have an equivalence in units of time, in spite of the fact that time should be measured by the earth's rotation on the plane of the equator.

This simple system was known in the East but, apparently, it was not commonly applied.⁵⁴⁵ In al-Andalus and the Maghrib, on the contrary, it was a usual method, although it is not mentioned in the Andalusian and Maghribī *zīj*es studied so far. Ibn Qunfudh, who uses the system in his horoscopes (see 3.1.1.8) only says “Some scholars (*wa min al-ulamā'*) take their degrees on the ecliptic”.⁵⁴⁶ In the 11th century al-Istijī is very critical of the work of other astrologers of his time. One of the reasons for this criticism is that our author favours the direct computation of both *tasyīr* and the projection of rays on the ecliptic, while other astrologers make projections on the equator, using right or oblique ascensions.

While, as we will see, al-Istijī praises al-Battānī for his use of the direct method for casting the rays, he blames him for his treatment of the *tasyīr*, for which he uses the hour line method (see below 3.2.3.2.2.3)⁵⁴⁷ and calculates an arc of the equator instead of an arc of the ecliptic. As a result, our author considers that “the cause of his [error] was his lack of knowledge of the meaning of *tasyīr* and the fact that he had done little research into its causes and objectives”.⁵⁴⁸

⁵⁴⁵ Casulleras & Hogendijk, 2012, p. 45.

⁵⁴⁶ Oliveras, 2012, pp. 97 (translation) and 41 (Arabic edition).

⁵⁴⁷ Casulleras & Hogendijk, 2012, pp. 53–59.

⁵⁴⁸ Samsó & Berrani, 2005, pp. 204, 235.

Al-Istijī gives some explanations of his preference for the ecliptical method in both *tasyīr* and casting the rays on the basis that everyone agrees in looking to the eighth sphere (the “sphere of the signs”, and consequently the ecliptic) for indications (*dalāʾil*) which allow astrological predictions, while the ninth sphere (the sphere that transmits the daily motion to the inner spheres) is only used to measure time, a variable needed for the computation of the ascendant:

As for those who obtain the arc of *tasyīr* from an arc of the equator they are people who drop from this science the indications furnished by the ecliptic and do not ascribe to it any power or influence. All philosophers ascribe such indications to it [i.e. the sphere of the signs]. This is the most clear and obvious idea we can obtain ... We will not find in any of the outstanding philosophers any attribution of indications – about changes which take place in persons (*ashkhāṣ*) and other kinds (*anwāʾ*) [of beings] – to the sphere of the equator. The only purpose of the calculation of ascensions is to know the times of night and day in any place and the amount of rotation of the sphere since a given moment in order to obtain the ascending degree of the ecliptic.⁵⁴⁹

I have been able to find that the ecliptical method is used in several Maghribī horoscopes I have studied such as the two horoscopes related to the battle of El Salado (Faḥṣ Ṭarīf, 1340) cast by the aforementioned Ibn ʿAzzūz al-Qusanṭīnī who calculated the *tasyīr* in five different ways, using the four aforementioned cycles, for which he used a “mixed” system:⁵⁵⁰ he added to the oblique ascension of the *haylāj* the number of ecliptical degrees which each cycle has progressed in the corresponding number of years. Ibn ʿAzzūz’ fifth application is the *tasyīr* of the Saturn-Jupiter conjunctions (on which see 3.2.2.2.2.5.1), for which he used a purely ecliptical method.⁵⁵¹ Some years later, another astrologer also from Constantina, Ibn Qunfudh (1339–1407) used this method⁵⁵² in the horoscopes of the *bayʿa* of the Merinid sultan Tāshufīn (20.9.1361),⁵⁵³

549 Samsó & Berrani, 2005, pp. 208, 237.

550 Díaz Fajardo, 2008, p. 27.

551 Samsó, 1999.

552 Although in the theoretical part of his commentary he says that the *tasyīr* can be calculated directly on the ecliptic or using right, oblique or mixed ascensions (see 3.3.1.2.2.2.1). See Oliveras, 2012, pp. 96–101 (translation), 39–46 (Arabic ed.).

553 Samsó, 2009, pp. 335–336.

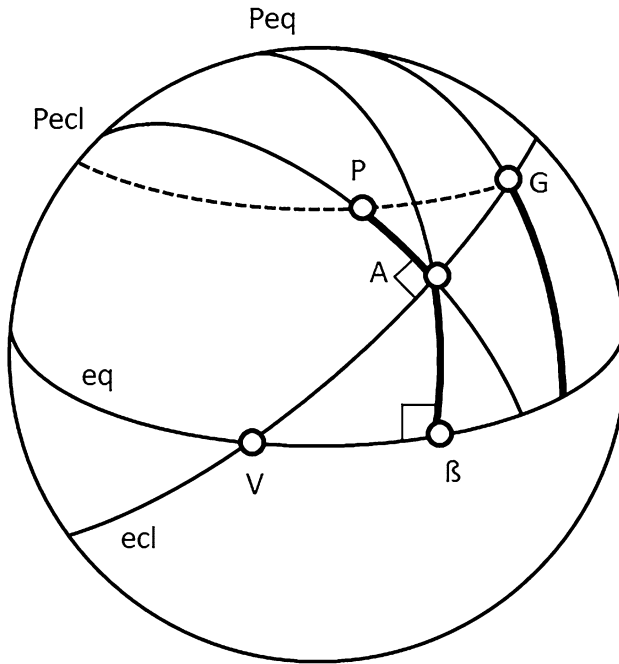


FIGURE 3.13
Parallel of declination
(borrowed from M.
Díaz-Fajardo)

of a nativity dated the 30.1.1363,⁵⁵⁴ and of the *bay'a* of the sultan Abū Fāris (30.8. 1366).⁵⁵⁵

Ibn Abī l-Rijāl (who derived his information from Ibn Hibintā), the anonymous Tunisian compiler of the Hyderabad recension of Ibn Ishāq's *zīj*, and Ibn 'Azzūz in his *Fuṣūl*⁵⁵⁶ consider the case in which one indicator or both of them (*al-musayyir* and *al-musayyar ilay-hi*) has latitude. The procedure they describe can be applied both for the *tasyīr* and the projection of rays. According to them, the *tasyīr* will only be precise if the two indicators are located on the same path (*majrā*). For this purpose one must know the approximate parallel of declination (PG in fig. 3.13) of the first indicator (P), its latitude (PA) and the declination of its degree of longitude (AB). With these elements the astrologer will calculate the approximate value of the declination of P which will be PA + AB (when both coordinates are in the same hemisphere, as in fig. 3.13) or PA – AB (if they are in different hemispheres). Using a declination table we will obtain the degree of the ecliptic (G) which has a declination equivalent to PA ± AB. The same operation will be repeated with the second indicator, when

554 Samsó, 2009, p. 346.

555 Samsó, 2009, p. 355.

556 Díaz Fajardo, 2008, pp. 95–106; Díaz Fajardo, 2011, pp. 354–355; Mestres, 1999, pp. 112–114.

it is not placed on the ecliptic. When both indicators move on the same path, the influence will be complete and the *tasyīr* precise. When they are displaced on different paths, the influence and precision of the *tasyīr* will diminish as a function of their divergence in declination. Instructions are also given in case $PA \pm AB$ exceeds the maximum solar declination.

3.2.3.2.2.2 *Tasyīr* on the Equator: Right, Oblique and Mixed Ascensions

According to al-Istijī, some Persians (*ṭāʾifa min al-Furs*) use right ascensions in both *tasyīr* and the projection of rays.⁵⁵⁷ This means that the arc of the *tasyīr* is merely the difference in right ascension between the positions of the two indicators. The same method is described in the Latin treatise on the use of the astrolabe by Abraham ibn ʿEzra.⁵⁵⁸ Another possibility mentioned in the sources is considering that the arc of the *tasyīr* is the difference in oblique ascension between the two indicators. Both systems are mentioned by Ptolemy in the *Tetrabiblos*⁵⁵⁹ although he considers that right ascensions can only be used if the indicator is on the meridian, while oblique ascensions correspond to an indicator placed on the Eastern horizon. This line of thought is followed by Ibn Abī l-Rijāl who considers that right ascensions should be used if the indicator is in houses IV and X, oblique ascensions correspond to houses I and VII, while for the rest of the cases “mixed” ascensions have to be used⁵⁶⁰ (see 3.2.2.2.2.1). The same attitude is also adopted by Ibn Qunfudh in his commentary to Ibn Abī l-Rijāl’s astrological *urjūza*.⁵⁶¹

3.2.3.2.2.3 *The Hour Line Method*

The hour line method appears in Ptolemy’s *Tetrabiblos* III,10.⁵⁶² As described by Casulleras and Hogendijk,⁵⁶³ if we consider that our two indicators are placed on points P and F on the celestial sphere and the arc of the *tasyīr* is q , if P is rotated around the celestial axis through q degrees, its image will be placed on the same hour line as F. The system is obviously easy to apply when using an instrument like the astrolabe and a special plate for the computation of the *tasyīr* (as well as the projection of rays) was described by the Granada instrument maker Ḥusayn ibn Bāšo (d. 1316): this plate had hour lines

557 Samsó & Berrani, 2005, pp. 201, 234; Casulleras & Hogendijk, 2012, pp. 45–46, 65–66.

558 Viladrich & Martí, 1983, p. 90.

559 Robbins, 1940, p. 290.

560 Ms. Rabat Hassaniyya 3503 fol. 104v; Hilty, 1954, I, 173–174.

561 Oliveras, 2012, pp. 96–101 (translation), 39–46 (Arabic ed.).

562 Robbins, 1940, pp. 286–307.

563 Casulleras & Hogendijk, 2012, pp. 53–59.

distributed along 180 divisions on each side of the horizon.⁵⁶⁴ An astrolabe made by Ḥusayn ibn Bāṣo's son, Aḥmad ibn Ḥusayn, contains plates designed for the hour line method for latitudes 33° and 37;30°. ⁵⁶⁵

An algorithm for the computation of the *tasyīr*⁵⁶⁶ according to this method appears in sources like Ibn al-Kammād's *Muqtabas*,⁵⁶⁷ Abraham ibn 'Ezra's treatise on the astrolabe,⁵⁶⁸ the Hyderabad recension of Ibn Ishāq's *zīj*,⁵⁶⁹ Ibn al-Bannā's *Minhāj*⁵⁷⁰ or Ibn al-Raqqām's *Mustawfi*,⁵⁷¹ *Shāmil*⁵⁷² and *Qawīm*⁵⁷³ *zīj*es. These sources follow the standard rules when the first indicator is on the meridian (houses IV and X, right ascensions) and on the horizon (houses I and VII, oblique ascensions). The algorithm is used for intermediate positions between the meridian and the horizon. The procedure, as explained in Ibn al-Raqqām's *Mustawfi zīj*, is the following one:

If the first indicator (P) is above the horizon, one must find the hour angle of P from midheaven (X) or from lower midheaven (IV) if it is beneath the horizon. This is equivalent to the difference in right ascension between P and the corresponding cusp ($\Delta\alpha_o(P)$). This hour angle must be expressed in unequal hours of P (H_P), which is:

$$H_P = 15^\circ \pm [\alpha_o(P) - \alpha_\varphi(P)] / 6$$

The same operation will be repeated with the second indicator (F) and we will obtain $\Delta\alpha_o(F)$ and H_F . The arc of *tasyīr* (q) will be:

$$q = ([\Delta\alpha_o(F) / H_F] \pm [\Delta\alpha_o(P) / H_P]) \times H_P$$

The sign being positive if P and F are in different (Eastern or Western) hemispheres and negative if they are in the same hemisphere.

We can find a different algorithm in the rest of the aforementioned sources. Here, as in the first procedure, we obtain $\Delta\alpha_o(P) / H_P$. Then we calculate the differences between the right and oblique ascensions of F and P:

⁵⁶⁴ Calvo, 1993, pp. 193–197 (Sp. trans.) and 162–170 (Arabic ed.).

⁵⁶⁵ North, 1986, pp. 61–63.

⁵⁶⁶ See Díaz Fajardo, 2010.

⁵⁶⁷ ms. Biblioteca Nacional de Madrid Lat. 10023 fols. 17v–18r.

⁵⁶⁸ Viladrich & Martí, 1983, pp. 90–92.

⁵⁶⁹ Mestres, 1999, pp. 116–117.

⁵⁷⁰ Vernet, 1952, pp. 71–72 (Arabic text) and 141–142 (Spanish translation).

⁵⁷¹ Ms. Rabat National Library 2461, pp. 221–222.

⁵⁷² 'Abd al-Raḥmān, 1996, pp. 150–152.

⁵⁷³ Ms. Rabat National Library 260 pp. 60–63.

$\Delta'\alpha_o = \alpha_o (F) - \alpha_o (P)$ [*dalīl al-falak al-mustaqīm*, indicator in right ascension]

$\Delta'\alpha_\varphi = \alpha_\varphi (F) - \alpha_\varphi (P)$ [*dalīl al-balad*, indicator in oblique (ascension)]

$(|\Delta'\alpha_o - \Delta'\alpha_\varphi| / 6) \times \Delta\alpha_o (P) / H_p = e$

e is the “equation” (*ta’dīl*) and

$$q = \Delta'\alpha_o \pm e$$

where the sign will be positive if $\Delta'\alpha_o < \Delta'\alpha_\varphi$ and negative if $\Delta'\alpha_o > \Delta'\alpha_\varphi$.

3.2.3.2.2.4 Position Semicircle Method

We have already analysed in 3.2.3.1.1.2 and 3.2.3.1.2 the use of position circles to divide the houses according to the prime vertical and equatorial methods. A position semicircle is also applied to the computation of the *tasyīr*. In fig. 3.14 F and P are the first and the second indicators and NPS is a position semicircle which passes through P and through the north and the south points of the horizon and can be considered as an horizon for latitude ξ . The *tasyīr* implies that F has to be rotated over an angle n so that its image reaches the position circle NPS. If P is on the meridian the position circle will be NZS, the meridian itself, and angle n will be the difference in right ascension between F and P. If P is on the horizon, the position circle will be the horizon itself and angle n will correspond to the difference in oblique ascension between the two indicators. For the rest of the cases we will have:

$$n = \alpha_\xi (F) - \alpha_\xi (P)$$

where α_ξ is an oblique ascension for latitude ξ which can be obtained from a table of oblique ascensions for such latitude.

It seems that an approximation to this method is implied in the use of “mixed ascensions” by Ibn Hibintā, Ibn Abī l-Rijāl and Ibn Qunfudh (see 3.2.2.2.2.2.1). This procedure is described in Ibn al-Raqqām’s *Mustawfi zīj*,⁵⁷⁴ and several treatises on astronomical instruments apply this method: the Alfonsine *Libro de la alcara*, *Libro segundo de las armellas* and the treatises on universal astro-labes designed by Ibn al-Zarqālluh and Ibn Bāṣo.⁵⁷⁵ Besides, we have already seen in 3.2.3.1.2 that the equatorial plate described by Rabiṣag (Rabbi Ishāq b. Sīd) can be used for the computation of the *tasyīr*, projection of rays and

574 Ms. Rabat National Library 2461 p. 221. See Samsó, 2014, p. 311.

575 Casulleras & Hogendijk, 2012, pp. 52–53.

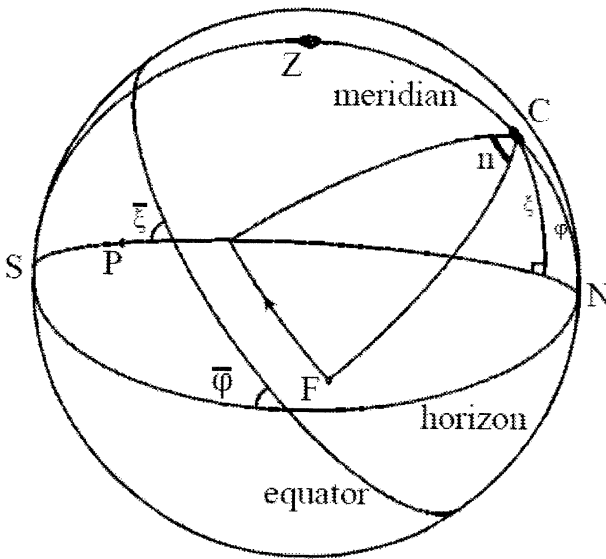


FIGURE 3.14
Position semicircle
method borrowed from
Casulleras & Hogendijk

division of houses⁵⁷⁶ and that this kind of plates appear in an astrolabe made by Aḥmad b. Ḥusayn b. Bāṣo, dated in 1304–1305, for the latitude of Granada (37;30°),⁵⁷⁷ in an Andalusian plate (latitude 35°) preserved in an Ottoman astrolabe⁵⁷⁸ and in a Moroccan astrolabe dated in 1208 (plates for latitudes 31°, 33;40°, 37;30° and 38;30°).⁵⁷⁹

3.2.3.2.2.5 Distance Method

In this method the arc of *tasyīr* is the distance between the two indicators on a great circle which passes by the two indicators. This method has been found in two sources: Ibn Bāṣo's treatise on the use of his universal astrolabe⁵⁸⁰ and Ibn al-Raqqām's *Mustawfī zīj*.⁵⁸¹ This second source considers three subcases: 1) the two points are on the ecliptic and, consequently, the direct method is applied; 2) one of them has latitude and the other one is on the ecliptic: in this case Ibn al-Raqqām calculates the distance with the same procedure as in chapter 52⁵⁸² which derives from al-Battānī's chapter 26⁵⁸³ and deals with the

⁵⁷⁶ Rico, 1863–67, II, 302–304. See Viladrich & Martí, 1983.

⁵⁷⁷ North, 1986 pp. 60–65.

⁵⁷⁸ King, 2014, pp. 333–334.

⁵⁷⁹ Casulleras & Hogendijk, 2012, pp. 51–52.

⁵⁸⁰ Calvo, 1993, pp. 174 of the Arabic edition and 199 of the translation.

⁵⁸¹ Ms. Rabat National Library 2461 pp. 220–221. See Samsó, 2014, p. 311.

⁵⁸² Ms. Rabat National Library 2461 pp. 196–197.

⁵⁸³ Nallino, 1899–1907, I, pp. 37–40, 199–204; III, pp. 57–61.

determination of the distance between two celestial bodies or two points on the sphere on the great circle which passes by the two bodies or points; 3) both points have latitude: here he follows the different subcases also explained by al-Battānī in his chapter 26.

3.2.3.3 The Projection of Rays (*maṭraḥ al-shu'ā'āt*)

Planets exert their influence not only as the result of the “aspects” (conjunction, opposition, trine, quartile and sextile) their bodies form with other planets or significant points on the ecliptic, but also due to the fact that they emit rays in seven directions which correspond to opposition, trine, quartile and sextile both in the right and in the left direction. The theory of projecting or casting the rays has developed a series of methods of which I will summarise only those present in the Andalusian and Maghribī tradition, following the classification and the terminology coined by Casulleras and Hogendijk,⁵⁸⁴ who have clearly established the relationship between these methods and those used for progressions and for the division of the houses. Among the sources dealing with this topic, I would like to emphasize the importance of Ibn Mu'ādh's *Maṭraḥ al-shu'ā'āt* and Ibn al-Raqqām's *Mustawfī zīj*. Both contain a lot of information on the different methods used for casting the rays.

3.2.3.3.1 *The Direct Method on the Ecliptic*

This procedure has the same geometrical approach as the ecliptic method in progressions (see 3.2.3.2.2.1). Imagining a planet P placed on the ecliptic and having longitude λ_P , the seven points towards which it casts its rays will be the points in the ecliptic having longitudes $\lambda_P + 180^\circ$, $\lambda_P + 120^\circ$, $\lambda_P - 120^\circ$, $\lambda_P + 90^\circ$, $\lambda_P - 90^\circ$, $\lambda_P + 60^\circ$ and $\lambda_P - 60^\circ$. This is the method approved by al-Istijī⁵⁸⁵ and by Ibn 'Azzūz⁵⁸⁶ and, according to al-Baqqār, it was the method used by Zarādusht.⁵⁸⁷ It was accepted by al-Battānī only in the case of planet P having 0° latitude. If the planet has latitude, this does not affect the quartile aspect or the opposition, but produces a deviation in the case of the trine or the sextile, which is solved by al-Battānī with a method of computation which is the object of al-Istijī's praise.⁵⁸⁸ Al-Battānī's procedure,⁵⁸⁹ which we have already seen as the “distance method” for progressions (see 3.2.3.2.2.5), is a refinement

⁵⁸⁴ Casulleras & Hogendijk, 2012, pp. 40–41, 62–79. See also Casulleras, 2007.

⁵⁸⁵ Samsó & Berrani, 2005, pp. 207 and 236–237.

⁵⁸⁶ Casulleras, 2007, pp. 52, 72, 75.

⁵⁸⁷ Díaz-Fajardo, 2008, pp. 282–283.

⁵⁸⁸ Samsó & Berrani, 2005, pp. 203–204, 234–235.

⁵⁸⁹ Nallino, 1899–1907, I, 129–131, 305–313; III, p. 197.

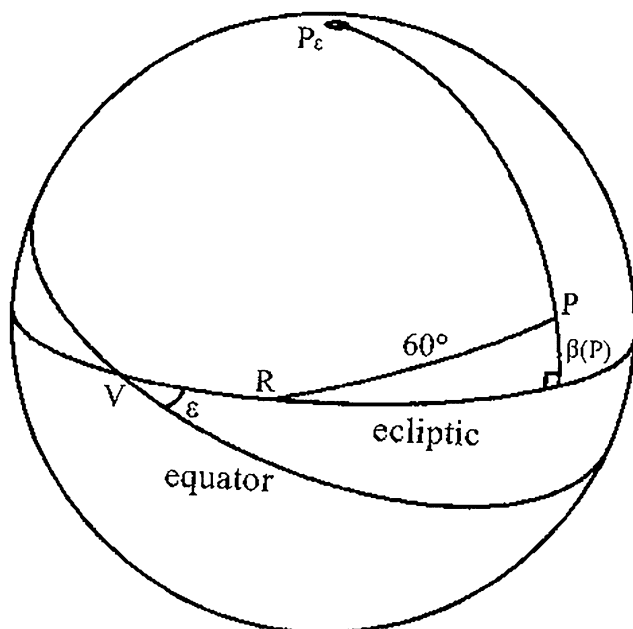


FIGURE 3.15
Borrowed from
Casulleras &
Hogendijk

of the simple ecliptical one and it appears in Ibn al-Raqqām's *Mustawfi zīj*:⁵⁹⁰ the planet is in one of the vertices of a spherical right-angled triangle in which the hypotenuse is an arc of a great circle the length of which equals the aspect (60° for the sextile, 120° for the trine) and which links the planet with the ecliptic, while the two other sides are the latitude of the planet and the length (on the ecliptic) of the projection which we want to know (see fig. 3.15). Two sides being known, the third one can be determined using more or less precise techniques.

It is interesting to remark that both al-Battānī and Ibn al-Raqqām's texts seem to be corrupt and this points to the possibility that the manuscript used by the Tunisian astronomer belonged to the same family as Escorial 908, the only extant manuscript of al-Battānī's *zīj*.⁵⁹¹ An attempt to reconstruct the procedure was made by Schiaparelli in his commentary to al-Battānī's

⁵⁹⁰ Samsó, 2014, p. 310.

⁵⁹¹ According to al-Baqqār, in his *Kitāb al-adwār*, the master Abū Ishāq al-Naqqāsh [Ibn al-Zarqālluh] had discovered this mistake and wrote, with his own hand, a marginal note in his manuscript of al-Battānī's *zīj* explaining it with mathematical proofs, and adding that the same error could be found in all the copies of the book. See Díaz Fajardo, 2008, pp. 280–282.

chapter 54.⁵⁹² This reconstruction is based on the hypothesis that the method used by al-Battānī is the one described in his chapter 26 (and in Ibn al-Raqqām's chapter 52): how to find the distance between two celestial bodies one of which has latitude while the other one is on the ecliptic. In our case the distance is known (60° or 120°) and we need to obtain the difference in ecliptic longitudes between R and P ($\Delta\lambda$).

Besides this, Ibn al-Raqqām gives, in both the *Mustawfī* and in the *Shāmi*,⁵⁹³ *zīj*es another solution, which is a simple application of the cosine law. The same solution appears in the Castilian canons of the Alfonsine Tables,⁵⁹⁴ in Ibn al-Bannā's *Minhāj*,⁵⁹⁵ and in al-Baqqār's *Kitāb al-adwār*.⁵⁹⁶ This latter source proves, however, that the deviation produced by planetary latitude is very small: al-Baqqār computes a table in which the argument is the planetary latitude from 1° to 9° and the results are given to the precision of minutes. The maximum error, however, reaches only 25'. A similar argument appears in Ibn al-Bannā's *al-Kalām 'alā l-tasyīrāt wa maṭāriḥ al-shu'ā'āt*.⁵⁹⁷

In application of the cosine law, Ibn al-Raqqām, Ibn al-Bannā' and al-Baqqār state that, in the case of the sextile, $\Delta\lambda$ will be:

$$\Delta\lambda = \cos^{-1}(1800 / \cos \beta)$$

where $1800 = \cos 60^\circ$.

Once obtained the value of $\Delta\lambda$ for the sextile, the left trine will be in opposition to the right sextile and the right trine to the left sextile.

To the aforementioned sources, Casulleras and Hogendijk add that the direct method is also described in several treatises on astronomical instruments such as the books on the universal astrolabe by Ibn al-Zarqālluh,⁵⁹⁸ 'Alī b. Khalaf⁵⁹⁹ and Ḥusayn b. Bāṣo⁶⁰⁰ as well as in the Alfonsine *Libro segundo de las armellas*.⁶⁰¹

592 Nallino, 1899–1907, I, 307–308.

593 'Abd al-Rahmān, 1996, pp. 158–159.

594 Chabás & Goldstein, 2003, pp. 91–92, 222.

595 Vernet, 1952, pp. 70–71 (Arabic text) and 141 (Spanish translation).

596 Díaz Fajardo, 2008, pp. 273–275; Díaz Fajardo, 2012–14.

597 Jabbār and Aballāgh, 2001, pp. 160–165 (see p. 163).

598 Puig, 1987, p. 82; Rico, 1863–67, vol. 3, p. 205.

599 Rico, 1863–67, vol. 3, p. 123.

600 Calvo, 1993, pp. 171–174 of the Arabic edition and 197–199 of the Spanish translation.

601 Rico, 1863–67, vol. 2, p. 65.

3.2.3.3.2 *Right Ascension Method*

According to al-Istijī, a group of the [ancient] Persians (*ta'ifa min al-Furs*) used right ascensions for the projection of rays.⁶⁰² The same attribution can be found in Ibn al-Zarqālluh's,⁶⁰³ and Ibn Bāṣo's⁶⁰⁴ treatises on the universal astrolabe, as well as in Ibn al-Raqqām's *Mustawfī Zīj*⁶⁰⁵ which adds to his descriptions of the right and oblique ascension methods, a third, more elaborate, procedure ascribed to "the Persians" which uses only right ascensions, although it does not give the same results as the right ascension method.⁶⁰⁶ Interestingly, in spite of al-Istijī's clear preference for the direct ecliptical method, he recommends to those who favour the projection on the equator the use of right ascensions and not oblique ones for casting of the rays. It is clear that he considers that the projection of rays is independent of the local latitude:

As for me, I feel somewhat surprised by those who ascribe to the rays of stars and planets twists and deviations on the sphere due to the inclinations of the horizons and consider that such inclinations cause an effect in the configurations (*ashkāl*) of celestial bodies on the sphere, that is on their mutual positions.⁶⁰⁷

The procedure used in this method is quite simple and considers that the planet has no latitude. To determine the casting of the rays of a planet with longitude λ in a right of left quartile, the astrologer operates:

$$\alpha_o^{-1}(\alpha_o(\lambda) \pm R) = \lambda' \text{ of the projection}$$

where R will be 60° , 90° , 120° or 180° .

This right ascension method is described in Ibn al-Raqqām's *Mustawfī zīj*.⁶⁰⁸

3.2.3.3.3 *Oblique Ascension Method*

This method is analogous to the previous one and it also considers that the planet has no latitude. As before:

$$\alpha_\phi^{-1}(\alpha_\phi(\lambda) \pm R) = \lambda' \text{ of the projection}$$

602 Samsó & Berrani, 2005, pp. 201, 234.

603 Puig, 1987, pp. 32, 80; Rico, 1863–67, vol. 3, p. 206.

604 Calvo, 1993, p. 174 of the Arabic edition and 199 of the Spanish translation.

605 Samsó, 2014, p. 310.

606 Casulleras, 2010, p. 164.

607 Samsó & Berrani, 2005, pp. 203–204, 235. See also pp. 206–207, 236.

608 Ms. Rabat National Library 2461 p. 217.

This method is mentioned, although rejected, by al-Istijī who ascribes it to Ptolemy.⁶⁰⁹ It also appears in Ibn al-Raqqām's *Mustawfī*.⁶¹⁰

3.2.3.3.4 *Single Position Semicircle Method*

This method is described in Ibn al-Raqqām's *Shāmil zīj*⁶¹¹ and it is analogous to the position semicircle method for the *tasyīr* (3.2.3.2.2.4). It uses an incident horizon of latitude ξ , passing through the planet and through the North and South points of the local horizon, and obtains the oblique ascension (α_ξ) of the rising or the setting point of the planet in the aforementioned incident horizon. As in the case of *tasyīr*, if the planet is on the meridian, the position circle will be the meridian itself and the ascensions will be right ascensions; if it is on the horizon, the position circle will be the horizon and the ascensions will be oblique ascensions for the local latitude. In the other cases, we will have:

$$\alpha_\xi^{-1}(\alpha_\xi(\lambda) \pm R) = \lambda' \text{ of the projection}$$

As established by Casulleras and Hogendijk⁶¹² this method receives graphical solutions in Ibn al-Zarqālluh's⁶¹³ and Ibn Bāṣo's treatises on the universal astrolabe⁶¹⁴ and in the Alfonsine treatise on the armillary sphere.⁶¹⁵ These three sources attribute the method to Hermes.

3.2.3.3.5 *Single Hour Line Method*

This method has a similar geometrical approach as the hour line method for the computation of the *tasyīr* (3.2.3.2.2.3). As explained by Casulleras and Hogendijk,⁶¹⁶ P is the planet which casts its rays on points of the ecliptic L (left aspect) and R (right aspect). If L or R is rotated around the celestial axis an angle of 60° (or 90° or 120°) in the direction of the daily motion (or in the opposite direction), its image after rotation is on the hour line through P.

This method is attributed to Ptolemy and, although it is not to be found in Ptolemy's works, it is obviously related to the corresponding method for the computation of the *tasyīr* which we find in the *Tetrabiblos* III, 10. It appears in Western Arabic sources on the use of astronomical instruments such as the

609 Samsó & Berrani, 2005, pp. 199–200, 234; 206, 236.

610 Ms. Rabat National Library 2461 p. 217.

611 'Abd al-Raḥmān, 1996, p. 159.

612 Casulleras & Hogendijk, 2012, p. 68.

613 Puig, 1987, p. 85; Rico, 1863–67, vol. 3, p. 211.

614 Calvo, 1993, pp. 178–179 of the Arabic edition and 201–202 of the Spanish translation.

615 Rico, 1863–67, vol. 2, p. 65.

616 Casulleras & Hogendijk, 2012, pp. 68–71.

treatise on the astrolabe attributed to Māshā'allāh (although it seems to be related to Maslama al-Majrīṭī, see 4.2.4.2.2)⁶¹⁷ and in Ibn al-Samḥ's treatise on the use of the astrolabe.⁶¹⁸ Ḥusayn ibn Bāṣo also mentions it in his treatise on the universal astrolabe and describes a special astrolabe plate with hour lines distributed along 180 divisions on each side of the horizon.⁶¹⁹ We have also seen in 3.2.3.2.2.3 that Aḥmad ibn Ḥusayn, the son of Ḥusayn ibn Bāṣo is the author of two of such plates traced for latitudes 33° and 37;30°.

Several *zīj*es describe an approximate algorithm which is one of the several attempts to combine right and oblique ascensions in such a way that the component in right ascension increases when the planet approaches the meridian, while the oblique ascension component also increases when it approaches the horizon. This algorithm was studied by Kennedy and Krikorian-Preisler⁶²⁰ on the basis of a text extant in al-Bīrūnī's *al-Qānūn al-Mas'ūdī*. It was known in al-Andalus where it reached Ibn al-Samḥ through Abū Ma'shar or another Eastern source. Ibn Mu'ādh criticised it due to the inconsistencies of the method.⁶²¹ It reappears in Ibn al-Kammād's *Muqtabas*,⁶²² as well as in Ibn al-Raqqām's *Mustawfi*⁶²³ and in the *Muwāfiq zīj* by Ibn 'Azzūz, in spite of the fact that he seems to favour the direct ecliptical method (see 3.2.3.3.1). This latter source includes a set of tables for the computation of the casting of the rays calculated for the latitude of Fez.⁶²⁴

The algorithm, as presented – with small variants – in the aforementioned sources, is the following one:

$$\begin{aligned}\alpha_o^{-1}(\alpha_o(\lambda) \pm R) &= \lambda' \text{ of the projection} \\ \alpha_\varphi^{-1}(\alpha_\varphi(\lambda) \pm R) &= \lambda'' \text{ of the projection} \\ \Delta\lambda &= |\lambda' - \lambda''|\end{aligned}$$

We also find the difference in right ascension ($\Delta\alpha_o[h]$) between the planet and the nearest meridian cusp (IV o X) expressed in seasonal hours. With these data we can calculate the “equation” (*e*, *ta'dīl*):

$$e = \Delta\alpha_o[h] / 6 \cdot \Delta\lambda$$

617 Viladrich and Martí, 1983, p. 94.

618 Viladrich, 1986, pp. 68–70, 147–149.

619 Calvo, 1993, pp. 163–167 of the Arabic edition and 193–195 of the Spanish translation.

620 Kennedy & Krikorian, 1983, pp. 374–376.

621 Casulleras, 2010, pp. 268–273 (Arabic) and 175, 203–210, 240–243 (Spanish).

622 Ms. Biblioteca Nacional de Madrid Lat. 10023 fols. 16v–17v.

623 Ms. Rabat National Library 2461 pp. 217–218, 220; Samsó, 2014, pp. 310–311.

624 Casulleras, 2007. This paper edits the canons and the tables and contains an analysis and a recomputation of the tables. See also Díaz Fajardo, 2008, pp. 287–292.

Then,

$$\lambda' + e \text{ (if } \lambda' < \lambda'') \text{ or}$$

$$\lambda'' + e \text{ (if } \lambda'' < \lambda')$$

The starting point, in this algorithm, is the planet's degree of longitude and this is criticised by Ibn al-Raqqām who considers that this degree should be replaced by the planet's degree of mediation, as stated by Wālis al-Miṣrī (Vettius Valens) and in agreement with the community of astrologers (*jamā'a min aṣḥāb al-aḥkām*). This confirms Casulleras and Hogendijk opinion who consider that this method was the standard one among Islamic astrologers.⁶²⁵

3.2.3.3.6 Four Position Circles Method

Following Hogendijk's description,⁶²⁶ in Fig. 3.16:

- NESW is the local horizon where N,S,E,W are the cardinal points,
- EMW is the equator,
- P is the planet who projects the rays, placed on the ecliptic,

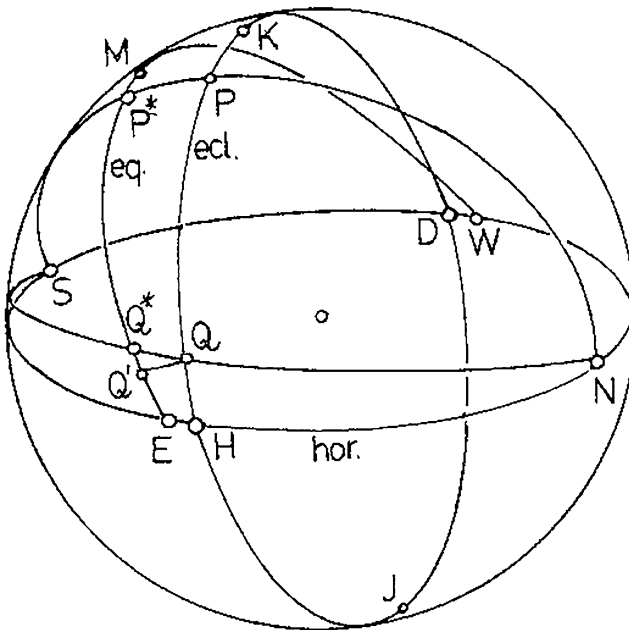


FIGURE 3.16
Borrowed from
J.P. Hogendijk

⁶²⁵ Casulleras & Hogendijk, 2012, p. 68.

⁶²⁶ Hogendijk, 1989, p. 172.

- Semicircle NPS is the semicircle of position, or incident horizon, through P which intersects the equator at point P* and has latitude ξ . P* is the end-point of an arc of oblique ascension ($\alpha_{\xi}(P)$) and it is the starting point from which the projection of rays is calculated. R being equal to $\pm 60^{\circ}/90^{\circ}/120^{\circ}$, we have $P^*Q^* = R$. A second semicircle of position, NQ*S, with latitude ξ' , is traced through point Q*, and intersects the ecliptic at point Q, which will be the point receiving the projection of rays. Obviously $\lambda_Q = \alpha_{\xi'}^{-1}(Q^*)$.

This method seems to be related to the one used by al-Khwārizmī in his approximate tables for the projection of rays calculated for a latitude of 33° (Baghdad). These tables are preserved in Ibn Hibintā's *Mughnī*,⁶²⁷ as well as in the *Toledan Tables*,⁶²⁸ and, in them, al-Khwārizmī does not compute oblique ascensions for a latitude ξ but approximates position circles by using hour lines.⁶²⁹ The tables extant in al-Khwārizmī's *zīj*,⁶³⁰ which are also approximate, are calculated for a latitude $38;30^{\circ}$ (Córdoba) and are the result of a new computation by Maslama al-Majrīṭī who did use position circles and improved considerably al-Khwārizmī both in precision and facility of usage.⁶³¹ A graphical solution for this method can be found in the Alfonsine *Libro del Ataqyr*⁶³² where we find the description of a special astrolabe plate, in which the circles of position intersect equal divisions of the equator, that can be also used for the division of houses and the *tasyīr*. We have already seen other instances of astrolabe plates which could be used for the same purposes (3.2.3.1.2).

The method attracted the attention of Ibn Mu'adh al-Jayyānī in his *Risāla fī maṭraḥ al-shu'ā'āt* – edited by Casulleras⁶³³ and studied by Hogendijk⁶³⁴ and Casulleras⁶³⁵ – as well as in his *Tabulae Jahan*.⁶³⁶ This author considers it as the only correct method for casting the rays and states that it is analogous to the equatorial method for the division of the houses. Both the *Risāla* and the *Tabulae Jahan* explain an exact algorithm for the computation of the projection of rays, studied and analysed by Hogendijk, while the *Risāla* gives an approximate method that has been clarified by Casulleras. One should add that a

627 Kennedy & Krikorian, pp. 373, 376–383; Hogendijk, 1989, pp. 180–187.

628 Toomer, 1968, pp. 147–151; Pedersen, 2002, IV, pp. 1520–1528 (see also the canons in II, p. 475, 677).

629 Casulleras & Hogendijk, 2012, p. 73.

630 Suter, 1914, pp. 206–229.

631 Maslama's tables also reappear in the *Toledan Tables*: Pedersen, 2002, IV, p. 1529.

632 Rico, 1863–67, II, pp. 295–309.

633 Casulleras, 2010, pp. 267–302 (Arabic edition), 259–265 (Spanish translation) and 210–228 (commentary).

634 Hogendijk, 2005, pp. 90–98.

635 Casulleras, 2004.

636 Ibn Mu'adh's, Jaén *zīj*, chapter 26.

passage of al-Istijī's *Risāla fī l-tasyīrāt wa-maṭāriḥ al-shu'ā'āt* mentions a school (*tā'ifa*) of mathematicians (*muhandisūn*) who transform the rays projected by the stars or the arc of the *tasyīr* into an arc of the equator. Here, al-Istijī's words are too vague, because no precision is given about what is meant by this transformation. The attribution of this procedure to Hermes leads, however, to the hypothesis of the use of position circles or semicircles,⁶³⁷ and to the possibility that al-Istijī is referring to the four position circles method.

3.2.3.3.7 *Seven Hour Lines Method*

This method considers that a planet P casts its rays to the intersections of the ecliptic with the seasonal hour lines of the astrolabe which indicate the seasonal hour of the planet plus or minus 4 (sextile), 6 (quartile), 8 (trine) or 12 (opposition). The method does not seem to appear in any of the known treatises on the astrolabe of the Andalusian and Maghribī tradition, although it could be easily used with the help of the special astrolabe plates mentioned above in 3.2.3.2.2.3. It was obviously known, however, given the existence of such plates and because Ibn Mu'ādh considers this method as an approximate alternative to the Four Position Circles Method. Besides, a passage of Ibn al-Raqqām's *Mustawfī zīj*⁶³⁸ describes an algorithm to determine the point of incidence in the ecliptic of the rays cast by a planet, which has been identified by Casulleras and Hogendijk⁶³⁹ with this method. According to this interpretation the hour lines are called by Ibn al-Raqqām *dawā'ir al-azmān* (time circles)⁶⁴⁰ and the method is considered demonstrative (*wajh burhānī*) and is ascribed to "modern astrologers" (*al-muta'akhirūn*) and to those who practice astrology basing themselves on experience (*aṣḥāb al-tajārib bi l-ṣinā'at al-aḥkāmīyya*).

3.2.4 *The Animodar*

At least five of our sources are concerned with the computation of the animodar (*namūdār*, *nīmudār* or *ḡaymūdār*):

- 1) Al-Istijī's *Risāla fī l-tasyīrāt* dedicated to his patron Ṣā'id al-Andalusī.⁶⁴¹
- 2) Ibn al-Kammād, a disciple of Ibn al-Zarqālluh, who lived in Córdoba ca. 1116, in one of his *zīj*es (probably *al-Kawr 'alā l-dawr*)⁶⁴² as well as

⁶³⁷ Casulleras & Hogendijk, 2012, pp. 48–53, 67–68.

⁶³⁸ Ms. Rabat National Library 2461 pp. 320–322.

⁶³⁹ Casulleras & Hogendijk, 2012, p. 76. In Samsó, 2014, p. 311. I interpreted this algorithm as corresponding to the Four Position Circles Method.

⁶⁴⁰ The Alfonsine *Libro del Ataṣir* calls the position circles "cercos de los tiempos".

⁶⁴¹ Samsó & Berrani, 2005, pp. 210–214, 238–240.

⁶⁴² Johannes de Dumpno's Latin canons of Ibn al-Kammād's *Muqtabis*, preserved in ms. Biblioteca Nacional (Madrid) 10023, seem to end in fol. 18v, but they are followed by an

in his *Kalām fī l-nīmūdār li-taṣḥīḥ ṭawālīʿ al-mawālīd* preserved in ms. Escorial 939, fols. 1v–8v.⁶⁴³ This manuscript is an astrological collection, whose first five texts were copied, around 1411, probably by the Moroccan astronomer and astrologer Abū ʿAbd Allāh al-Baqqār (fl. 1418).⁶⁴⁴ This is a fragment of Ibn al-Kammād's book entitled *Kitāb mafātīḥ al-asrār* which was divided into 15 *fuṣūl* of which only *faṣls* 10–15 are extant.⁶⁴⁵ Between *faṣls* 14 and 15 we find a page (fol. 5r) with a quotation by Ibn Hilāl⁶⁴⁶ on animodar. Díaz Fajardo has remarked that ms. Escorial 939 seems to derive from Ibn Hilāl's own copy which included this author's notes. This is followed by a set of tables (fols. 5v–7r) for the computation of the duration of pregnancy for 9-month and 7-month gestations. Fol. 8v ends with an astrological note on hunting and fishing.

- 3) Our third source is the anonymous Tunisian recension (ca. 1266–1281) of Ibn Ishāq's *zīj*, which deals with the problem of animodar in chapter 37 of the canons.⁶⁴⁷ The sources he mentions are Ptolemy (*Tetrabiblos*), Vettius Valens, Hermes, two unidentified authors (al-Ghārib and Sulaymān b. Yaḥyā al-Munajjim), Abū Ma'shar and, especially, Ibn al-Kammād, who is quoted extensively.
- 4) Abraham ibn ʿEzra studies this technique in his *Book on Nativities*⁶⁴⁸

appendix, containing a new series of canons, translated by Johannes de Dumpno himself, the first of which is "secundum cuiusdam libri tabulas dictas caur hale eddaur". Animodar is mentioned in fol. 20r, in an unnumbered chapter with the following heading: "Porta in scientia operationis per tabulas quantitatis durationis creature in ventre matris per tabulas ad hoc constitutas". This chapter was edited by Millás, 1942, pp. 240–241. The corresponding tables, for 9-month gestations only, are preserved in fols. 62v–64r. These tables are different from those appearing in the *Kalām fī l-nīmūdār* and they are incomplete: only the number of days and minutes appear, the column of the hours is empty. Giovanni Bianchini (fl. Ferrara 1440–1464) reproduces the tables of the Latin manuscript in his astronomical tables: see Chabás & Goldstein, 2009, pp. 129–130.

643 This text was studied and translated into Catalan by Vernet, 1949. I will refer to this paper with the double page number (first edition and reprint). An edition, English translation and commentary has been published by Díaz-Fajardo, 2016–17.

644 See Samsó & Berrani, 2005, pp. 171–172.

645 Díaz Fajardo has remarked that the 15 *fuṣūl* correspond, probably, to only a part of the book which may have contained several parts.

646 This is probably Muḥammad ibn Hilāl al-Sabtī, who lived in Ceuta in the first half of the 14th c.: see King, 2007, pp. 181, 184; Lamrabet, 2012, M142 (p. 182).

647 Edition in Mestres, 1999, pp. 230–244; commentary pp. 141–144. The tables (182–188) have not been edited by Mestres because they are the same as those appearing in Ibn al-Kammād's *Mafātīḥ al-asrār*.

648 Sela, 2013, pp. 41–45; Sela, 2019.

- 5) Finally, in the *Almanach Perpetuum* authored by Abraham Zacut and José Vizinho, we find a set of tables for the computation of the duration of pregnancy as well as a canon describing the use of such tables.⁶⁴⁹

The main purpose of the animodar is to calculate, indirectly, the longitude of the ascendant of the moment of childbirth. Al-Istijī, for example, is skeptical about the precision that can be attained with the direct computation of the ascendant of a nativity, because of the difficulty of establishing the hour using solar instruments (sundials) or water-clocks⁶⁵⁰ and he prefers the technique of the animodar which he uses in its two variants:

3.2.4.1 The Animodar of the Ruling Planet (*namūdār al-mustawlī*)

This kind of animodar derives from Ptolemy's *Tetrabiblos* 3,2⁶⁵¹ and it is described by al-Istijī with the following words:

When we cast the horoscope (*naṣba*) of a human nativity, we find that the distance, expressed in parts (*ajzā'*) of one of the cusps from the beginning of its sign is the same as the distance, also from the beginning of the corresponding sign, of the planet ruling (*al-mustawlī*) the sign of the conjunction or opposition [of the Sun and the Moon] which took place before the nativity, the corresponding "parts" being the result of the system of division adopted⁶⁵²

The source used is obviously the aforementioned chapter of the *Tetrabiblos* although, according to Ptolemy, with this technique we obtain the ascendant and not one of the cusps. This technique is mentioned, although not described, in the extant fragment of Ibn al-Kammād's *Kitāb mafātīḥ al-asrār*, but it is the object of a detailed description in the Tunisian recension of the *zīj* of Ibn Ishāq, which quotes frequently Ibn al-Kammād.⁶⁵³ The *mustawlī* is determined by analysing the planets' positions, houses, exaltations, triplicities, terms (*ḥadd*) and decans and the Tunisian recension contains Ibn al-Kammād's table allowing to calculate the strength and weight of each planet in order to establish which one of them deserves to become the *mustawlī* (*Jadwal aqdār al-muwāzana*

649 Albuquerque, 1986, chapter 23 of the canons (p. 100) and tables (pp. 402–404); Chabás & Goldstein, 2000, pp. 150–153. See also in Chabás & Goldstein, 2000, pp. 86–87 a description of a table, included in Zacut's *Ha-ḥibbur ha-gadol*, which gives the lunar mean motion in longitude and anomaly according to the duration of pregnancy from 259d to 287d.

650 On the different techniques for the determination of the ascendant see Kennedy, 1995–96.

651 Robbins, 1940, pp. 229–235.

652 Samsó & Berrani, 2005.

653 Mestres, 1999, pp. 230–232 (Ar. ed.) and 142–145 (comm.)

bayn al-kawākib fī quwwat al-ḥayyiz al-ḥazzawī wa l-muzāʿimī al-namūdḥārī wa l-ijtimāʿī wa l-istiqbālī.⁶⁵⁴

According to the Tunisian recension, Ibn al-Kammād considers that the Ptolemaic rule is only valid if the ascendant of the nativity has been “observed” (*marṣūd*), the *mustawli* has no latitude and the local latitude is northern and smaller than the obliquity of the ecliptic. If the ruling planet has latitude, the astrologer will operate with the ecliptical degree with which the planet rises (if it is in the area of the ascendant), sets (near the descendant) or mediates (when it is placed in houses iv and x). Abraham ibn ʿEzra criticises Ptolemy’s method and says that he has checked it in many experiments and has been able to establish that it does not give accurate results.

3.2.4.2 The Animodar of the Conception (*namūdār masqaṭ al-nuṭfa*):

Ibn al-Kammād

A second kind of animodar is ascribed by Islamic sources to Hermes (Enoch, according to Abraham ibn ʿEzra) and it is also described in the pseudo-ptolemaic *Kitāb al-Thamara* (*Karpós* or *Centiloquium*).⁶⁵⁵ It establishes that the lunar longitude at the instant of birth is the ascendant at the instant of conception; conversely, the lunar longitude at conception is the ascendant of the nativity. Thus, if the moment of birth is known approximately and the duration of pregnancy can be established, the astrologer can easily calculate the lunar longitude at the moment of conception and, consequently, he will also know the ascendant of the nativity. This kind of animodar is named by our Arabic sources *namūdār masqaṭ al-nuṭfa* (the animodar of the fall of the drop [of sperm]).⁶⁵⁶

The main problem of this kind of animodar is the determination of the moment of conception, obviously related to the length of gestation. Both Ibn al-Kammād⁶⁵⁷ and the Tunisian recension deal with nine-month and seven-month periods and do not take into consideration eight-month pregnancies. The non-viable character of this period was explained by the Andalusian physician and historian ʿArīb b. Saʿīd (ca. 912–ca. 980) using two different arguments: one of them is that, when the child reaches the seventh month of gestation, he tears the membranes that join him to his mother and reverses his position in the womb. After this, he is absolutely exhausted and, if he is not

654 Tables 209–210 of the Hyderabad manuscript. On the determination of the ruling planet (*al-mustawli*) in other Maghribī sources see Díaz-Fajardo, 2013.

655 Aphorism 51. See the edition by Ae Boer in *Claudii Ptolemaei Opera quae exstant omnia*. Vol. III, 2. Teubner, Leipzig, 1961, pp. 48–49.

656 Samsó & Berrani, 2005, pp. 210–212, 238.

657 Ibn al-Kammād’s text on the animodar has been edited and analysed in Díaz Fajardo, 2016–17.

born at this moment, he has to wait until the ninth month in order to have a safe birth. ‘Arīb’s second argument is purely astrological: each month of pregnancy takes place under the influence of a planet, beginning with Saturn (first month), followed by Jupiter (second month) and so on.⁶⁵⁸ The seventh month corresponds to the Moon, which is beneficial, but the cycle starts again and the eighth month suffers the evil influence of Saturn, while the ninth month is, once more, under the beneficial power of Jupiter.⁶⁵⁹

The length of pregnancy depends, in most cases, on the position of the Moon at birth.⁶⁶⁰ The astrologer should determine the moment of birth with a maximum error of half an hour, cast the horoscope for the approximate hour and calculate the lunar longitude. If the Moon is above the Western horizon in the descendant (house VII) or below it with a maximum negative altitude of 5°, pregnancy will reach its minimum duration (9.5 lunar months for 9-month pregnancies, 7.5 months for 7-month pregnancies). If it is on the Eastern horizon, in the ascendant, pregnancy will have its mid value (10 and 8 lunar months). If it is below the Western horizon with a negative altitude exceeding 5°, pregnancy will reach its maximum (10.5 and 8.5 lunar months). The precise values of these basic periods, as mentioned in our sources are:

TABLE 3.8 Basic periods for 9-month pregnancies

9-month pregnancy	Vettius Valens and al-Ghārib	Sulaymān b. Yaḥyā al-Munajjim	Ibn al-Kammād	Ibn ‘Ezra	Abraham Zacut
Minimum (9.5 lunar months)	258d	259d 1;20h	259d 1;20h ^a	259d	[259d13;15h]

a 259d 1;20h appears both in the text and the tables (Vernet, 1949, pp. 85/289 and 91/295). In another passage of the text (Vernet, 1949, p. 84/288) we find 259d 3;45h (259d 2;45^h in Díaz Fajardo’s edition), a value which is clearly corrupt. In ms. BN Madrid Lat 10023 fol. 20r the periods are given to the precision of days and only the initial 2 ... is visible, although Millás (1942, p. 240) edited 287 days. In the table, however, we find 259 days.

658 This distribution of the months of gestation among the planets can be found in several Eastern astrological sources as well as in Abraham ibn ‘Ezra’s *Book on nativities*, *Mishpeṭei ha-Mazzalot* and *Liber de nativitatibus* (see Sela, *Nativities* p. 44).

659 Jahier & Abdelkader, 1956.

660 Abraham ibn ‘Ezra (Sela, 2013, pp. 44–45) introduces other factors determining the length of gestation which will not reach nine months if, in the dates near to childbirth, Venus and Mercury, on the one side, and Mars, on the other, enter the position of the Moon at the time of birth.

TABLE 3.8 Basic periods for 9-month pregnancies (*cont.*)

9-month pregnancy	Vettius Valens and al-Ghārib	Sulaymān b. Yaḥyā al-Munajjim	Ibn al-Kammād	Ibn ‘Ezra	Abraham Zacut
Mid (10 lunar months)	273d	272 17;12h	272d17;12h ^b	273d	273d 5;15h
Maximum (10.5 lunar months)	288d ^c	286 9;13h	286d 9;3 ^d	287d	286d 21;15h

- b 272d 17;30h is the value mentioned in Vernet, 1949, p. 84/288 for the mid period of 10 lunar months. It also appears in Díaz Fajardo's edition. In the tables I find 272d 17;52h for an argument of 180° in the table for 9-month pregnancy with the Moon above the horizon. It should be equal to the argument 0° with the Moon below the horizon but, in the latter case, we have, again, 272d 17;30h. See Vernet, 1949, pp. 91–92/295–296. The analysis of tabular differences shows clearly that the correct value is 272d 17;12h instead of 17;52, a value that also appears in the text of Díaz Fajardo's edition. In any case 52 and 12 are easily confused in Arabic *abjad* notation. In ms. BN Madrid Lat 10023 fol. 20r this period appears as 313d (!). Millás (*Traducciones orientales* p. 241) edits 273 days, in agreement with the tables.
- c The three basic periods, as they appear in two notes (*fā'ida*) in the Egyptian mss. Escorial 961, Cairo Ṭal'at *miqāt* 102 and Cairo Muṣṭafā Fāḍil *miqāt* 58, which contain texts related to Vettius Valens, are 260, 273 and 288 days. See King, 2004 a, p. 29.
- d In the text the quoted value is 286d 8;15h (Vernet, 1949, p. 84/288 and in Díaz Fajardo's edition), while in the tables we find 286d 9;3h (in Díaz-Fajardo's edition) while Vernet (1949, p. 92/296) has 286d 9;0h. The correct value should be 286d 9;4h. In ms. BN Madrid Lat. 10023 fol. 20r I read 288 days, a value which agrees with the tabular maximum (fol. 64r). Millás (1942, p. 241) edited 284 days.

TABLE 3.9 Basic periods for 7-month pregnancies

7-month pregnancy	Sulaymān b. Yaḥyā al-Munajjim	Ibn al-Kammād
Minimum (7.5 lunar months)	204d 11;35h	204d 11;15h
Mid (8 lunar months)		218d 2h
Maximum (8.5 lunar months)		231d 16;45h

The difference between each two periods in the case of Vettius Valens (?) is 15d and this points to the use of a solar month of 30 days and 8 hours, and that the corresponding periods are 8.5, 9 and 9.5 solar months. The other sources use lunar sidereal months which are not always coherent, for the difference between the periods should always be equivalent to half a sidereal lunar month and, in the case of Sulaymān b. Yaḥyā al-Munajjim the differences are

13d 15;52h and 13d 16;1h which point to a mean sidereal lunar month comprised between 27d 7;44h and 27d 8;2h. Ibn 'Ezra's values are conditioned by the fact that he expresses his values in days and in weeks and, consequently, the number of days must be a multiple of 7. Zacut's values show differences of 13d 16h, which imply a sidereal lunar month of 27d 8h, a rounded value of the 27d 7;43,5 ... h that we obtain with the Alfonsine lunar mean motion in longitude of 13;10,35,1,15,11,4,35° per day.⁶⁶¹

In the case of Ibn al-Kammād the values involved in 9-month gestations are:

$$\begin{aligned} 272d\ 17;12h - 259d\ 1;20h &= 13d\ 15;52h \\ 286d\ 9;3h - 272d\ 17;12h &= 13d\ 15;51h \end{aligned}$$

These values correspond to a sidereal lunar month comprised between 27d 7;42h and 27d 7;44h. We can confirm this parameter by an analysis of the tabular differences between consecutive values for each degree of the argument between 1° and 180° for 9-month gestations (the Moon being over the horizon). These differences are 1;49h or 1;50h unequally distributed yielding also a mean value of the lunar sidereal month of 27d 7;44h.

27d 7;44h is very near to the 27d 7;42h quoted by Ibn al-Kammād in the extant fragment of the *Mafātiḥ al-asrār*,⁶⁶² which agrees approximately with the mean sidereal lunar month of 27d 7;43,12h, mentioned by the Tunisian recension.⁶⁶³ The latter value is precisely what we obtain with the daily mean motion in longitude of the Moon of Ibn al-Kammād's *Muqtabas zīj*: 13;10,34,52,46°. ⁶⁶⁴

In spite of the aforementioned considerations, it can be shown that Ibn al-Kammād is using an accurate mean lunar sidereal month of 27d 7;43,12h. He explains his method of computation in the text by saying that he multiplies 27d 7;42h by 9.5 months and obtains the shortest period of 259d 1;20h "after subtracting 12 hours because this is the distance between the ascendant and the descendent".⁶⁶⁵ We do not obtain this result, however with 27d 7;42h but with 27d 7;43,12h and the same happens with the two other periods:

$$\begin{aligned} 27d\ 7;43,12h \cdot 9.5 - 12h &= 259d\ 1;20h \\ 27d\ 7;43,12h \cdot 10 - 12h &= 272d\ 17;12h \\ 27d\ 7;43,12h \cdot 10.5 - 12h &= 286d\ 9;3,36h \end{aligned}$$

661 See Poulle, 1984, p. 135.

662 Vernet, 1949, pp. 83/287.

663 Mestres, 1999, p. 237.

664 Chabás & Goldstein, 1994, p. 30.

665 Vernet, 1949, pp. 85/289. I have modified his translation slightly using Díaz-Fajardo's edition.

This explains Chabás and Goldstein's remark who state that "[Ibn al-Kammād's] entries in the first table differ by about 12h from those of the *Almanach Perpetuum*, but exhibit the same sequence of differences between consecutive values".⁶⁶⁶

It seems obvious that Ibn al-Kammād is using here the same parameters that we find in his extant *Muqtabas zīj*. We find two other examples of such usage in a passage in which he recommends calculating the lunar position in longitude and anomaly one year after and one year before the nativity. For that purpose he says that the increase or decrease in longitude in one solar calendrical year will be 4–12, which should be interpreted as 4° 12', because lunar motion in longitude of 13;10,35,1,15,11,4,35°/ day yields 4° 12;41'. Also with a mean motion in anomaly of 13;3,53,56,19°/ day we find an increase of anomaly in one solar year of 3° 1;59' which agrees with the 3-1-59 found in the text.⁶⁶⁷

With the three basic periods for 7-month gestations we do not obtain, however, the same parameter:

$$\begin{aligned} 218\text{d } 2\text{h} - 204\text{d } 11;15\text{h} &= 13\text{d } 14;45\text{h} \\ 231\text{d } 16;45\text{h} - 218\text{d } 2\text{h} &= 13\text{d } 14;45\text{h} \end{aligned}$$

These values correspond to a mean sidereal lunar month of 27d 5;30h. A near value can be obtained by an analysis of the tabular differences between arguments 1° and 180°, the Moon being above the horizon. Such differences are 1;49h in 156 cases, 1;48h in 19 cases and 1;50 in 5 cases. Half a month is, therefore, 13d 14;46h and one sidereal month 27d 5;32h. Besides, the values of the three periods have been computed by adding a correction of 6 hours, instead of subtracting 12h as in the periods for 9-month gestations:

$$\begin{aligned} 27\text{d } 5;30\text{h} \cdot 7.5 + 6\text{h} &= 204\text{d } 11;15\text{h} \\ 27\text{d } 5;30\text{h} \cdot 8 + 6\text{h} &= 218\text{d } 2\text{h} \\ 27\text{d } 5;30\text{h} \cdot 8.5 + 6\text{h} &= 231\text{d } 16;45\text{h} \end{aligned}$$

My impression is, therefore, that the table for 7-month gestations was copied by Ibn al-Kammād from another unidentified source.

Ibn al-Kammād's *Mafātiḥ al-asrār*⁶⁶⁸ and the Tunisian recension of Ibn Iṣḥāq's *zīj* contain an approximate rule to compute the length of pregnancy

⁶⁶⁶ Chabás & Goldstein, 2000, p. 153.

⁶⁶⁷ Vernet, 1949, p. 88/292. I use the mean motion parameters squeezed by Chabás and Goldstein, 1994, p. 29.

⁶⁶⁸ Vernet, 1949, pp. 84/288.

if the Moon is not in one of the three basic positions.⁶⁶⁹ For this purpose our source calculates:

$$\text{Half a month} / 6 = 2.5 \text{ days}$$

which implies a length of half a “solar” month of 15 days and a “solar” month of about 30 days. The source is probably Vettius Valens⁶⁷⁰ or al-Ghārib and, starting from the position of the Moon in the VII house (258 days), 2.5 days will be added if the Moon is in the VIII house, 5 days if it is in the IX house and consecutively. This rule respects the basic periods of 273 days for the ascendant and 288 days if the Moon is in the VII house with a negative altitude greater than 5° below the Western horizon.

Ibn al-Kammād adds a more elaborate rule:⁶⁷¹ one must find the difference ($\Delta\lambda$) between the sidereal longitude of the Moon and the sidereal longitude of the descendant (*al-ghārib*, house VII), if the Moon is above the horizon, or the ascendant if the Moon is below the horizon. Then:

$$\Delta\lambda / 12 = N \text{ (number of days)}$$

The remainder of the division (R) multiplied by 2 will give the number of hours and

$$N + 2R = T$$

This value, T, has to be corrected:

$$T' = T - T/12 \text{ (if the Moon is above the horizon)}$$

$$T' = T + T/12 \text{ (if the Moon is below the horizon)}$$

T' is added to the shortest duration of 9-month or 7-month pregnancies, if the Moon is above the horizon, or to the mean duration of the corresponding pregnancy, if the Moon is below the horizon. The result is the amount of time the child will stay in his mother's womb.

669 Mestres, 1999, pp. 239–240.

670 As shown by King (2004 a, p. 29) this rule is implied in the set of values for the duration of pregnancy mentioned in a note added in three Egyptian manuscripts to a text which explains Vettius Valens' table for the duration of human life. Valens' procedure for the computation of the animodar is more complex: see King 2004 a, pp. 24, 30–31.

671 Vernet, 1949, pp. 89–90 / 293–294. On other cycles mentioned by Ibn Abī l-Rijāl for the anniversaries of nativities see 3.2.2.2.2.3.

This procedure seems to be based on the fact that the Moon travels at a mean speed of about 13° /day in which $\Delta\lambda / 12$ is used instead of $\Delta\lambda / 13$.⁶⁷² Besides, the approximate mean speed of the Moon per hour is $13^{\circ} / 24 = 0;32,30^{\circ}$, replaced here by $0;30^{\circ}$, as

$$R / 0;30^{\circ} = 2R$$

Obviously the value obtained is greater than the correct one and this forces Ibn al-Kammād to introduce the negative correction of $T/12$. It is easy to check that this approximate rule yields values which are reasonably near those of the tables:

TABLE 3.10

Argument	$T' = T - T/12$	Computed value	Table value	Differences
30°	2d 7h	261d 8;20h	261d 7;59h	21m
60°	4d 14h	263d 15;20h	263d 14;38h	42m
90°	7d 12h	265d 22;20h	265d 21;16h	1h 4m
120°	9d 4h	268d 5;20h	268d 3;55h	1h 25m
150°	11d 11h	270d 12;20h	270d 10;33h	1h 47m
180°	13d 18h	272d 14;20h	272d 17;[1]2h	2h 20m

3.2.5 World Astrology: Planetary Conjunctions

3.2.5.0 Introduction

We have already considered Ibn Abī l-Rijāl’s interest in world astrology to which he dedicates book VII of his *Kitāb al-Bārīʿ* (cf. 3.2.2.2.2.5.1 on *Tahāwīl sinī al-ʿālam*). We also know that world astrology is generally based on Indo-Iranian cycles.⁶⁷³ The knowledge of such cycles is documented in al-Andalus since the second half of the tenth century. The evidence is furnished in the Latin translation, by Johannes Hispalensis (fl. 1120–1135) of ʿUmar ibn Farrukhān’s *Liber Universus*, a short tract dealing with the astrological period called the “mighty *fardār*”, which lasted 360 years. The current mighty *fardār*, in ʿUmar’s lifetime, began in 580 CE and was associated to the birth of the Prophet. The

672 Apparently Ibn al-Kammād is not using the mean motion of the Moon per day, but the difference between the mean daily motions of the Moon and the Sun.

673 See also Díaz Fajardo, 2008, pp. 338–342.

interesting thing for us, in this text, is that John of Seville's translation includes a reference to the first mighty *fardār* after 580 (580 + 360 = 940) for which an horoscope had been cast for the latitude of Córdoba, using al-Khwārizmī's *zīj* in Maslama's recension.⁶⁷⁴ This agrees with the anecdote, already analysed in 3.1.1.4, according to which, towards the beginning of the eleventh century, Cordoban astrologers (Maslama was one of them) made predictions based on Saturn-Jupiter conjunctions, which form the basis of shorter astrological periods frequently used in astrological history.

The interest in world astrology reappears in al-Istijī's *Risāla fī l-tasyīrāt* which describes the importance of cycles, equivalent to the Eastern *intihā'ūt*, of thousands (cycle of 12,000 years), hundreds (1,200 years), tens (120 years) and units (12 years) on which see 3.2.5.0 and 3.2.5.3.⁶⁷⁵ In this work, al-Istijī states that he had studied historical horoscopes and checked the corresponding predictions against the well-known historical reality. This seems to be implied in his remark:⁶⁷⁶

We have studied them for the different kinds of astrological predictions – I mean the *tasyīrs* for states, religions, horoscopes (*ṭawālī'*) of the accession to power of kings and anniversaries – obtaining, out of all this, a set of very clear results which are wonderful and an extraordinary science.

3.2.5.1 Saturn-Jupiter Conjunctions

One of the topics dealt with in medieval astrology is related to planetary conjunctions, from the conjunction of Sun and Moon that takes place at the beginning of each lunar month⁶⁷⁷ to the conjunction of the seven planets in the first minute of Aries that occurred at the moment of Creation. Following aphorism 50 of the pseudo-Ptolemaic *Karpós* or *Centiloquium*, our sources mention the 120 types of conjunctions, a number which corresponds to the sum of the combinations of two, three, four, five, six and seven planets.⁶⁷⁸

⁶⁷⁴ Pingree, 1977.

⁶⁷⁵ On other cycles used in world astrology by Ibn Abī l-Rijāl see 3.2.2.2.5.1. See also the cycles mentioned in the Hyderabad recension of the *zīj* of Ibn Ishāq in Mestres, 1999, pp. 118–121. The cycles of 12000, 1200 and 120 years are also used by Abraham b. 'Ezra in his *Reshit Hokhmah* and in his commentary on *Exodus* 33:21: see Sela, 2010, pp. 30, 273.

⁶⁷⁶ Samsó & Berrani, 2005, pp. 210, 238.

⁶⁷⁷ Díaz Fajardo, 2008, p. 313.

⁶⁷⁸ The 120 types of conjunctions is a topic developed by Ibn 'Ezra and it also appears in John of Seville's *Liber primus de gentibus*: see Sela, 2010, pp. 15–16, 27–28, 32–33 53–55, 102–103, 157, 273–275; for Maghribi sources see Díaz Fajardo, 2008, pp. 301–307.

Historical astrology is often based on Saturn-Jupiter conjunctions, to which I should add the Saturn-Mars conjunction in the sign of Cancer which takes place every thirty years (*al-qirān al-thalāthīnī, qirān al-naḥsayn*).

To understand the theory underlying the conjunctions of Saturn and Jupiter, I will repeat here the standard classification of the zodiacal signs into four triplicities which are, in their turn, related to the four elements: fire (Aries, Leo, Sagittarius), earth (Taurus, Virgo, Capricorn), air (Gemini, Libra, Aquarius) and water (Cancer, Scorpio, Pisces). A Saturn-Jupiter conjunction takes place, approximately, every twenty years and each conjunction advances two thirds of a revolution (240°): Saturn's period of revolution is about 30 years, while Jupiter's amounts to some 12 years. In 20 years, Saturn will have moved approximately 240° , while Jupiter will have made a complete revolution (12 years) and $2/3$ of a second one (8 years). This is why conjunctions tend to stay in the same triplicity, although the second conjunction will take place in a position placed slightly forward, in its zodiacal sign, in respect to the previous one. The displacement progresses slowly until the conjunction takes place in a different triplicity.

The information concerning the theory of Saturn-Jupiter conjunctions seems to have reached al-Andalus and the Maghrib through Abū Ma'shar's *Kitāb al-milal wa l-duwal* (also called *Kitāb al-qirānāt*), which is mentioned in al-Istijī's *Risāla* (3.1.1.5.1), in the Hyderabad recension of Ibn Ishāq's *zīj*⁶⁷⁹ and in al-Baqqār's *Kitāb al-adwār* who also quotes Abū Ma'shar's *Kitāb al-ulūf*.⁶⁸⁰

Al-Istijī seems to prefer the *intihā'āt* to Saturn-Jupiter conjunctions,⁶⁸¹ which are studied in the third part of the *Risāla*⁶⁸² where he establishes a logical correspondence between them and the cycles of the *intihā'āt*:

- 1) The mighty conjunction which returns to the same triplicity where the cycle began, after having passed through the four triplicities, and takes place after 970 years: it corresponds to the 1000-year *intihā'*.
- 2) The middle conjunction, with change in the triplicity (242.5 years), belongs to the category of the hundreds.
- 3) The small conjunction (the period of time between two Saturn-Jupiter successive conjunctions, every 19.5 years) corresponds to the category of the tens.

679 Mestres, 1999, p. 127. This *zīj* adds another Eastern source: Ibn Nawbakht's *Kitāb al-Kāmil*.

680 Díaz Fajardo, 2008, pp. 333, 338–342.

681 See Samsó & Berrani, 2005, pp. 195–196 and 232. On *intihā'āt* and Saturn-Jupiter conjunctions see Kennedy, 1983 a.

682 Samsó & Berrani, 2005, pp. 185–199, 229–234.

Obviously in this system of correspondences, the small world-*intihā'* (twelve-year period) has no equivalent in the cycles of the conjunctions and al-Istijī puts the anniversaries in the fourth place, and in the category of the units.⁶⁸³

The periods used by al-Istijī for the three kinds of Saturn-Jupiter conjunctions seem to be new and I have not been able to find them in any known Islamic source dealing with this kind of conjunctions or compute them using al-Khwārizmī's *zīj* or the *Toledan Tables*. Our author states that the portions (*ḥiṣṣa*) which correspond to one year in the *tasyīrs* of the three types of conjunctions are the following ones:⁶⁸⁴

small conjunction: 18;29°

middle conjunction: 1;29,4°

great conjunction: 0;22,16,5°

These values allow us to compute the length of the three periods which will be:

$$360^\circ / 18;29^\circ = 19.477 \text{ yrs}$$

$$360^\circ / 1;29,4^\circ = 242.515 \text{ yrs}$$

$$360^\circ / 0;22,16,5^\circ = 969.999 \text{ yrs}$$

The period of the middle conjunction is confirmed explicitly in the *risāla* which mentions a rounded value of 242.5 years.⁶⁸⁵

The aforementioned numerical values are obviously based on Saturn and Jupiter's mean motion parameters. Al-Baqqār and Ibn 'Azzūz, on their side, consider that this kind of predictions should derive from true conjunctions and not mean ones and this is why al-Baqqār considers that the period of a middle conjunction varies between a minimum of 240 years and a maximum of 259 years.⁶⁸⁶ Besides, the computation of true planetary conjunctions is one of the reasons that justifies the existence of tables of planetary velocities which have only been found in a set of tables appearing in Ibn 'Azzūz's *Muwāfiq zīj*⁶⁸⁷ and in many Latin and Hebrew manuscripts, in the tradition of the *Alfonsine Tables*⁶⁸⁸ (see 7.4.1.6).

683 The cycles of thousands, hundreds, tens and units, as well as the *fardār* of 75 years are also mentioned by Abraham ibn 'Ezra in his *Reshit Ḥokmah*. See Sela, 2010, pp. 30, 265–267.

684 Samsó & Berrani, 2005, pp. 190–192, 231.

685 Samsó & Berrani, 2005, pp. 197–199, 233–234.

686 Díaz Fajardo, 2008, pp. 307–308, 319.

687 Samsó, 1997, pp. 88–91, 104–105.

688 Goldstein, Chabás & Mancha, 1994; Chabás & Goldstein, 2003, pp. 170–182.

Al-Baqqār's *Kitāb al-adwār* gives also a useful precision which clarifies the conditions under which we can consider that a true conjunction has taken place. According to him there are three kinds of planetary conjunctions:

- *mujāma'a*: if the two planets coincide in the same minute,
- *mumāzaja*: if the distance between both planets is less than the sum of their apparent radii,
- *mujāsada*: if the distance equals the sum of their radii.⁶⁸⁹

3.2.5.2 Astrological History: the Conjunction of 571 CE (*qirān al-Milla*)

Ibn 'Azzūz remarks that the starting point of the astrological history of the world is the conjunction of the seven planets in the first minute of Aries and that 360,000 years later another mean conjunction of the seven planets will take place.⁶⁹⁰ His source seems to be Abū Ma'shar's *Kitāb al-Ulūf* who considers that the conjunction of the seven planets in Aries 0;1° occurred in – 183,101 and that this corresponded to the moment of Creation. According also to the *Kitāb al-Ulūf*, another planetary conjunction in the same position took place 180,000 solar years later, in – 3,101, and produced the Flood. A third total conjunction will happen 180,000 years later, in 176,899.⁶⁹¹

This period, defined by a conjunction of all planets, can be divided into an integer number of Saturn-Jupiter conjunctions, of which the conjunctions with change of triplicity produce great changes in world history: a Jewish author like Abraham bar Ḥiyya remarks that the births of Jesus Christ and the Prophet Muḥammad were announced by two Saturn and Jupiter conjunctions in the signs of Leo and Scorpio.⁶⁹² Concerning the conjunction of 571 in Scorpio (*qirān al-Milla al-Islāmiyya*) that foreshadowed the rise of Islam and the birth of Muḥammad, we have already seen Ibn Abī l-Rijāl's references (see 3.2.2.2.2.5.1). Two other Maghribī sources preserve the horoscope of the spring equinox of the year of this conjunction: the Hyderabad recension of Ibn Ishāq's *zīj*⁶⁹³ and Ibn Qunfudh's commentary on Ibn Abī l-Rijāl's astrological *urjūza*.⁶⁹⁴ They have many points in common and they probably have the same origin.

Both horoscopes divide the houses according to the "dual longitude method" (see 3.2.3.1.3), something which is surprising, especially in the Hyderabad recension which establishes that this method (called *al-ʿamal al-thulāthī*)

689 Díaz Fajardo, 2008, pp. 327–328.

690 Díaz Fajardo, 2008, pp. 304–307.

691 Pingree, 1968, p. 29; Kennedy & van der Waerden, 1963, pp. 320–321.

692 Sela, 2010, pp. 20, 165.

693 Mestres, 1999, pp. 122–123.

694 Samsó, 2009, pp. 322–326; Oliveras, 2012, pp. 250–253.

should be used in ordinary life, while the “standard method” (3.2.3.1.1.1) should be reserved for important matters such as conjunctions, nativities and anniversaries which require more precise values. The longitudes of the first six houses in both horoscopes are the following ones:

TABLE 3.11 Houses in the horoscope of the spring equinox of 571 AD

Houses	Hyderabad recension	Ibn Qunfudh
I	332°	332°
II	3° (−0;40°)	3° (−0;40°)
III	35° (−0;20°)	36° (+0;40°)
IV	67°	67°
V	95° (−0;20°)	96° (+0;40°)
VI	123° (−0;40°)	123° (−0;40°)

Both horoscopes have been calculated with the parameters of Ibn Ishāq’s side-real *zīj* which I am using for the recomputation.⁶⁹⁵ According to the Hyderabad recension the date is 19th March 571 AD at 9;30 hours after dawn, although the best agreement with the recomputed values correspond to three hours after midnight for the meridian of Tunis. Ibn Qunfudh’s commentary says that the horoscope has been calculated for the longitude of Mecca, but the division of the houses points to a geographical latitude of about 6° (with a possible error of ± 6°).⁶⁹⁶ This implies that these horoscopes were probably calculated for the “cupola of the Earth” (*qubbat al-ard*), a place whose latitude is 0°. This is what should be expected in a horoscope affecting the whole mankind. As the geographical longitude of Tunis is 41;45° from a Western prime meridian, according to Ibn al-Raqqām,⁶⁹⁷ a follower of Ibn Ishāq, the difference in longitude between Tunis and the cupola is 48;15° and the hour difference will be 3;13h. This means that the hour for the centre of the world should be about 6;13 hours after midnight of the 19th March which corresponds, approximately, to the moment of dawn. In these horoscopes, the planetary positions are:

695 I use a computer programme prepared by my friend and colleague Josep Casulleras.

696 I am using John North’s programme HOROSC.

697 Samsó, 2014, pp. 298–299.

TABLE 3.12 Planetary positions in the spring equinox of 571 AD

Planets	Hyderabad recension	Ibn Qunfudh	Recomputed values (19.3.571 at 3AM in Tunis)
Sun	0;1°	0;0,1°	359;31,19°
Moon	82;7°	84	84;33,28°
Saturn	215;43°	215	215;17,6°
Jupiter	214;36°	214	214;28;45°
Mars	74;30°	67 (?)	74;2,14°
Venus	8;20° (?)	2°	2;0,19°
Mercury	9;30°	10°	9;27,56°
Ascending node	41°	41°	39;3,49°

The Hyderabad recension also contains an horoscope of the birth of the Prophet, dated on the 20th April 571⁶⁹⁸ at 9 hours after dawn which, at the cupola, corresponds to 12;13h, also after dawn or, approximately, 6;13h PM. As shown below, the best results are obtained for the 19th April at 10h PM (Tunis), which corresponds approximately to the 20th April 1;13h after midnight for the centre of the Earth. In it, the longitudes of the first six houses are exactly the same as in the horoscope of the spring equinox of the same year. The local latitude is also 6° (± 6°).The planetary positions are:

TABLE 3.13 Planetary positions at the birth of the Prophet

Planets	Hyderabad recension	Recomputed values (19.4.571 at 10h PM in Tunis)
Sun	31;10°	30;24°
Moon	139;18°	139;17°
Saturn	213;19°	213;4°
Jupiter	226° (?)	210;55°
Mars	91;46°	91;19°
Venus	48;17° (?)	41;8°
Mercury	–	7;2°
Ascending node	39;22°	37;23°

698 The date is the same in al-Battānī’s horoscope of the birth of the Prophet. See Kennedy *et al.*, 2009–10, p. 93.

3.2.5.3 The Astrological History of Abraham bar Ḥiyya⁶⁹⁹

I do not know of any attempt to use Saturn-Jupiter conjunctions in an astrological history of Western Islam, like the works compiled, in the East, by Ibn Nawbakht,⁷⁰⁰ Māshā-'allāh,⁷⁰¹ Abū Ma'shar⁷⁰² or al-Battānī,⁷⁰³ which contain collections of horoscopes related to important events in the history of the Mashriq. In the Maghrib, I am only aware of the horoscopes cast by Ibn 'Azzūz in relation to the battle of El Salado (Faḥṣ Ṭarīf, 1340)⁷⁰⁴ – which are based on the middle conjunction of Saturn and Jupiter which took place in 1305 – or by Ibn Qunfudh's collection of eleven horoscopes – which do not use the cycles of Saturn-Jupiter conjunctions – illustrating the history of Merinid Morocco between 1348 and 1372, which give an astrological explanation to the dynastic crisis that took place in the country after the murder of sultan Abū 'Inān (1348–1352).⁷⁰⁵ See 3.1.1.8.

An entirely different, and extremely interesting, case is what we find in the *Megillat ha-Megalleh* by Abraham bar Ḥiyya (ca. 1065–ca. 1145):⁷⁰⁶ an attempt to interpret the history of mankind and, more specifically, that of the Jewish nation, and of predicting the date of the advent of the Messiah, using both rabbinical texts and astrology. In spite of the fact that his starting point is that the years of the world are 6000, which should be divided into the six days of Creation, he uses a series of cycles having different origins (Indian, Persian and Jewish):⁷⁰⁷

- 4,320,000,000 years which derive from Brahmagupta's *Brahmasphuta-siddhānta*:⁷⁰⁸ at the moment of Creation all planets, as well as their apogees and their nodes were in Aries 0°. From that moment they all started to rotate with different velocities. At the end of the period (*kalpa*) they will meet again in the same position.

699 This section has been published in Samsó, 2018.

700 See Labarta, 1982; Labarta & Mestres, 2005. On this collection of horoscopes see also Brummelen, 1999.

701 See Kennedy & Pingree, 1971.

702 See Pingree, 1968.

703 See Kennedy *et al.*, 2009–10.

704 Samsó, 1999.

705 Samsó, 2004 a and 2009.

706 I am giving here a revised version of Samsó, 2004 d, pp. 307–314. On this work of Bar Ḥiyya see Sela, 2005; Rodríguez Arribas, 2011 (pp. 55–120) and 2011 a; North, 1986, pp. 94–96. This section has been revised by Bernard R. Goldstein and Shlomo Sela who have corrected my errors and given to me very interesting information.

707 Poznanski & Guttman, 1924. The account of the cycles is on pp. 10–13. Due to my incapacity to read Hebrew I am using Millás-Vallcrosa's Catalan translation: Millás, 1929. On cycles see Millás, 1929, pp. 19–23.

708 Kennedy & van der Waerden, 1963.

- 4,320,000 years (*mahāyuga*, used in the astronomical system of Āryabhaṭa, ca. 499 AD). This number is the result of multiplying $360 \times 12,000$.
- 36,000 years (33,000 years in Millàs' Catalan translation p. 20).
- 360,000 years, which is the world-year of the Persians, used by Abū Ma'shar. It is equivalent to the twelfth part of the *mahāyuga* and it is frequently used in Islamic astrology where it is named "mighty world *tasyīr*".⁷⁰⁹
- 12,000 years, corresponding to the "mighty world *intihā*":⁷¹⁰ it is clear that Bar Ḥiyya is mixing two different systems (*tasyīr* and *intihā*).
- 49,000 years
- 7,000 years.

These two last cycles seem to have a Jewish origin and to be related to the ideas of the sabbatical and jubilar year. Interestingly, these cycles were used by the Jewish astronomers of Alfonso X, as the Alfonsine model for precession is based on the combination of constant precession, with a cycle of 49,000 years, and a trepidation model which uses a cycle of 7,000 years.

Chapter 5 of the *Megillat ha-megalleh*⁷¹¹ contains a astrological history based on Saturn-Jupiter conjunctions⁷¹² and gives some rounded values of the parameters used. The interval between two small conjunctions is 20 years – $1/8$ of a year = 19;52,30 years, while, between two middle conjunctions the period is 238 years + $1/3$ of a year (238;20^y). Both parameters do not agree very well, as $19;52,30 \times 12 = 238;30$ (not 238;20) and we will obtain better approximations in this section.

Bar Ḥiyya's cycles are more or less standard:

- small conjunctions
- middle conjunctions (characterised by a change of the triplicity),
- great conjunctions (which return to the original sign, after passing through the four triplicities)
- mighty conjunctions (2,859 years, with a return of the conjunction to the same degree as in the first one, $238;20 \times 12 = 2,860$).⁷¹³ This kind of conjunction was already used in Eastern Islamic astrology.⁷¹⁴

⁷⁰⁹ Kennedy, 1983 a, p. 355.

⁷¹⁰ Kennedy, 1983 a, p. 356.

⁷¹¹ Poznanski & Guttman, 1924, pp. 111–155; Millàs, 1929, pp. 183–252.

⁷¹² Linking Saturn-Jupiter conjunctions with the advent of the Messiah was also favoured by Solomon ibn Gabirol (ca. 1021–1058) and severely criticised by Abraham ibn 'Ezra (ca. 1089–ca. 1161). See Sela, 2010, pp. 9–11.

⁷¹³ Millàs, 1929, p. 193; Sela, 2005, p. 134 gives a period of 2,869 years (probably a misprint). See Rodríguez Arribas, 2011, pp. 102, 313–314.

⁷¹⁴ Kennedy, 1958, p. 259/332.

Bar Ḥiyya also considers a period of 60 years (three small conjunctions), in which the conjunction returns to the same sign (within the same triplicity) as in the first one. The same period appears also in al-Baqqār's *al-Adwār fī tasyīr al-anwār*.⁷¹⁵

An analysis of the horoscopes cast by Bar Ḥiyya may give us some more information on the parameters used by the author, who divides the history of Israel into periods of 238 years, marked by conjunctions with shift in the triplicity and, within each of these cycles, he gives information about small conjunctions or about eclipses having a special relevance. His horoscopes do not correspond to the moment of the conjunction but, in most cases, to the spring equinox of the year of the conjunction, which allows us to obtain the kind of solar year he is using. His starting point (which I will call conjunction 0) is the "maximum conjunction of the kingdom of Israel", with a shift to the triplicity of water (Aquarius), which took place the 25th Adar of year 2365 of Creation/ 23rd March 1396 BCE). This was the year of the birth of Aaron and, three years later, of Moses. The list of spring equinoxes of the years of mean middle conjunctions given in the text is the following one:⁷¹⁶

- No 12, fire (Aries): 16th Adar II 2603/ 23.3.1158 BCE
- No 24, earth (Taurus): 8th Nisan 2841/ 22.3.920 BCE (period of the kings of Israel)
- No 36, air (Gemini): the text translated by Millàs⁷¹⁷ does not give any date, but it should correspond to year 3079/682 BCE.
- No 48, water (Cancer): 27th Adar 3318/ 20.3.443 BCE (instead of 3317/442). Reign of Nebuchadnezzar.
- No 60, fire (Leo): 19th Adar 3556/ 19.3.205 BCE.
- No 72, earth (Virgo): 11th Nisan 3794/ 19.3.34 CE. Jesus died in the previous year. 34 years after this conjunction the "exile of the second temple took place, decreed by Titus".
- conjunction, reign of Constantine, who converted to Christianity, and Manes began to preach mazdeism.
- No 96, water (Scorpio): 1st Nisan 4271 / 17.3.511 CE (Millàs' translation has 4471/711). Within this cycle Bar Ḥiyya mentions a small conjunction of year 4331/571, which was the year of the birth of Prophet Muḥammad. The power of Islam lasted for 584 years.
- No 108, fire (Sagittarius): 21st Adar II 4509/ 16.3.749. Fall of the Umayyads and beginning of the Abbasid Caliphate.

⁷¹⁵ I am using Díaz Fajardo unpublished edition of this work.

⁷¹⁶ See a complete list of conjunctions in Rodríguez Arribas, 2011, pp. 97–101.

⁷¹⁷ Millàs, 1929, pp. 213–215.

- No 120, earth (Capricorn): 14th Nisan 4747/ 16.3.987. During this cycle we find the fall of the Umayyads of al-Andalus, the period of the petty-kingdoms (*ṭawāʾif*) (1031) and the invasion of the Almoravids (1086). The sixth small conjunction of this triplicity (No 125) took place in the year which began the 9th Nisan 4847/17.3.1087 and, after twelve years (1099), the Crusades began. [The first crusade was preached in 1095 and it lasted from 1096 until 1099].⁷¹⁸
- No 132, air (Aquarius): 14th Adar 11 4986/ 15.3.1226. This conjunction announced the end of Islamic power: in conjunction no 96 Bar Ḥiyya said that this power would last 584 years, which corresponds to 1155 (from 571) or to 1206 (from the beginning of the Hijra in 622). Within this period he mentions the conjunction no 138 (the year begins the 10th Nisan 5105/ 14.3.1345) for which he prognosticates a period of great wars which is a basic nucleus of his messianic speculation.⁷¹⁹
- No 144, water (Pisces): 5th Nisan 5224 (5214 in Millàs' translation)/ 14.3.1464. This conjunction completes the great cycle of the "maximum" conjunction. As $1,396 \text{ BC} = -1,395$, $1,395 + 1,464 = 2,859$. The beginning of the solar year (spring equinox) coincides with the Saturn-Jupiter conjunction in Aquarius 25° , while the true conjunction of both planets in no. 0 was in Aquarius 24° .⁷²⁰ This cycle corresponds, approximately, to the year in which Bar Ḥiyya expects the arrival of the Messiah (1448 or 1468).⁷²¹

This list contains precise chronological data which allows us to obtain some parameters used by Bar Ḥiyya. It is easy to calculate that the period between two middle conjunctions of Saturn and Jupiter is 86,929 days minus about 7 hours (the information about hours is, generally, not very reliable). Our basic period will be 86,928.292 days which correspond to 238 years. We can, therefore, calculate, approximately, the length of the solar year used by Bar Ḥiyya:

⁷¹⁸ An Andalusian Jew, exiled in some locality of the Crown of Aragon, as a result of the Almohad invasion, is the author of a prognostication (dated in 4914/1153–54) based on the middle conjunction 129 (not mentioned by Bar Ḥiyya) which took place on Monday 1 Shawwāl 561H / Sunday 31st July 1166. Goldstein has shown that he uses al-Khwārizmī's *zīj* for his computation. See Goldstein, 2004 a.

⁷¹⁹ This conjunction was considered to be the cause of the Black Death of 1348: see Goldstein & Pingree, 1990, p. 52. See also Samsó, 2011, p. 435: the Granadan physicians al-Shaqūrī and Ibn Khātima considered that the Black Death was caused by the conjunction of *three* planets. Computing with the Alfonsine Tables we can see that there was a conjunction of Jupiter and Mars in Aquarius 14;32° the 1.3.1345; another one of Mars and Saturn in Aquarius 17° the 4.3.1345; a third one of Jupiter and Saturn in Aquarius 18;45° the 21.3.1345. Properly speaking, one cannot say that there was an exact conjunction of the *three* planets but it is clear that Mars, Saturn and Jupiter were very near to each other in March 1345.

⁷²⁰ Millàs, 1929, p. 197.

⁷²¹ Millàs, 1929, pp. 250–251; Sela, 2005, p. 133; Rodríguez Arribas, 2011, p. 110.

$86,928.292 : 238 = 365;14,41,43 \dots^d$ or $365;14,52,26$ if we use a period of 86,929.

Obviously Bar Ḥiyya is using a tropical year which is slightly longer than the one used by al-Battānī ($365;14,26,0^d$),⁷²² in spite of the fact that Bar Ḥiyya's astronomical tables are based on al-Battānī's *zīj* (7.6.1). Bar Ḥiyya's value is not very far from Ptolemy's tropical year ($365;14,48^d$),⁷²³ something which agrees with his remark saying that all his computations are based on Ptolemy's theories.⁷²⁴ It is also near the value used by Bar Ḥiyya himself, in his *Sefer ha-Ibbur*, and by Maimonides, in his *Sanctification of the New Moon*, who speak of a year having $365^d + 5^h + (977 / 1080)^h + [48 / (1080 \times 78)]^h$, which equals $365;14,45,47^d$.⁷²⁵ Besides, Abraham ibn 'Ezra attributes to Jewish authors a tropical year of $365^d + 1/4 - 1/320$, equivalent to $365;14,48,45^d$ and we know of other similar parameters.⁷²⁶ All this evidence does not identify with certitude the year length used by Bar Ḥiyya and the only thing we can consider secure is that he is using a tropical year, not a sidereal one.

We can continue with the comparison between Bar Ḥiyya's data and the Ptolemaic parameters. In the *Almagest*⁷²⁷ we find the following values of the mean motion per day of Saturn (M_s) and Jupiter (M_j):

$$M_s = 0;2,0,33,31,28,51^\circ$$

$$M_j = 0;4,59,14,26,46,31^\circ$$

$$\text{and } M_j - M_s = 0;2,58,40,55,17,40^\circ$$

From this value we can calculate the interval of days (n) corresponding to a small conjunction:

$$n \times M_s + 360^\circ = n \times M_j$$

$$n = 360^\circ / (M_j - M_s) = 7253;6,28,6,58 \text{ days}$$

$$7253;6,28,6,58 / 365;14,48 = 19;51,29,11,21,41 \text{ years}$$

⁷²² Nallino, 1899–1907, I, 42.

⁷²³ *Almagest* III, 1; translation Toomer, 1984, p. 140.

⁷²⁴ Millàs, 1929, p. 196.

⁷²⁵ Gandz, Obermann & Neugebauer, 1956, p. 97.

⁷²⁶ Millàs, 1947 a, pp. 75–76 and 100. Al-Khwārizmī gives, for the solar year of the Jewish calendar, $365^d + 5^h + 3791 / 4104$ ($365;14,48,33,35^d$): see Kennedy, 1964. The same parameter is mentioned by al-Bīrūnī (Gandz, Obermann & Neugebauer, 1956, p. 97). Šā'id of Toledo gives a different value: $365;14,58,45^d$ (Šā'id, *Ṭabaqāt*, tr. Blachère, 1935, p. 156).

⁷²⁷ *Almagest* IX, 3; translation Toomer, 1984, p. 126.

This result is not far from the 19;52,30 years that we have obtained from Abraham bar Hiyya himself. The period corresponding to a middle conjunction with shift in the triplicity will be:

$$19;51,29,11,21,41 \times 12 = 238;17,50,16,20 \text{ years}$$

which is near the 238;20 years given by Bar Hiyya. Finally, these parameters are coherent with the cycle of 144 small conjunctions at the end of which the Saturn-Jupiter conjunction will occur in the same degree and sign of the first one:

$$7253;6,28,6,58 \text{ days} \times 144 \times 0;2,58,40,55,17,40^{\circ} = 143 \text{ revolutions} + 59;59,59 \dots^{\circ}$$

We may try to obtain some more information from the horoscopes themselves. Concerning the mean motion of Saturn and Jupiter, we can easily see that their mean position in conjunction 0 (JDN 1,211,616) is 324° and 325° in conjunction 144 (JDN 2,255,857). The interval of time between both dates is 1,044,241 days, during which Saturn makes 97 rotations and Jupiter 241. We can, therefore, calculate the mean motion per day of both planets as well as the difference between them:

$$\begin{aligned} M_s &= (97^r + 1^{\circ}) / 1,044,241 = 0;2,0,23,22,1^{\circ} \\ M_j &= (241^r + 1^{\circ}) / 1,044,241 = 0;4,59,6,24,32^{\circ} \\ M_j - M_s &= 0;2,58,43,2,31^{\circ} \end{aligned}$$

These parameters do not seem to be related to those of Ptolemy ($M_s = 0;2,0,33,31,28,51^{\circ}$ and $M_j = 0;4,59,14,26,46,31^{\circ}$) or to al-Battānī's ($M_s = 0;2,0,35,50,27,26^{\circ}$ and $M_j = 0;4,59,16,54,58,16^{\circ}$),⁷²⁸ while they are nearer to al-Khwārizmī's ($M_s = 0;2,0,22,57^{\circ}$ and $M_j = 0;4,59,9,8^{\circ}$)⁷²⁹ or to those used in the astrological history of Māshā'allāh ($M_s = 0;2,0,22,17^{\circ}$ and $M_j = 0;4,59,7,18^{\circ}$).⁷³⁰ This seems to point to Bar Hiyya's use of sidereal mean motion parameters in spite of the fact that his solar year was tropical.

⁷²⁸ Parameters squeezed by E.S. Kennedy for his programme BATLN.

⁷²⁹ Neugebauer, 1962, p. 93.

⁷³⁰ Kennedy & Pingree, 1971, p. 79.

With the parameter obtained for the difference between mean motions per day we can calculate that the interval of time between two small conjunctions is 7251;40,25 days, equivalent to 19;51,15,23 years ($19^y 312^d 0;28,54^h$): the *Zīj al-Shāh*, used by Māshā'allāh to compute his horoscopes gives an interval of $19^y 311^d$.⁷³¹ In Bar Ḥiyya's system each small conjunction advances towards the shift in the triplicity $2;30,25^\circ$ ($2;25^\circ$ in the *Zīj al-Shāh* and Māshā'allāh; $2;25,17^\circ$ in Abū Ma'shar's horoscopes; $2;53,49^\circ$ according to the *Almagest*).⁷³² For a middle conjunction we will need:

$$19;51,15,23 \times 12 = 238;15,5 \text{ years}$$

a value which seems a little bit too low if we compare it to the 238;20 years mentioned in the *Megillat ha-Megalleh*. A good check of this parameter can, however, be obtained computing with it the interval between two maximum conjunctions:

$$238;15,5 \times 12 = 2859;1$$

A cursory study of the collection of horoscopes in the *Megillat ha-megalleh* shows, as we have seen, that they are cast, in most cases, for the spring equinox of the year in which a Saturn-Jupiter conjunction occurred.⁷³³ Obviously, the true conjunction very rarely coincides with the spring equinox and, in many horoscopes, the positions of Saturn and Jupiter are near to each other but not the same. Surprisingly in eight cases the same position is given for the two planets as if the conjunction had taken place the day of the equinox. These are:

Conjunction 0 (25 Adar 2365/ 23.3.1396 BC): $324^{\circ 734}$

Conjunction 7 (22 Adar II 2484/ 23.3.1277 BC): $337^{\circ 735}$

⁷³¹ Kennedy & van der Waerden, 1963, p. 324.

⁷³² Kennedy, 1983 a, p. 359.

⁷³³ This is an astrological commonplace. See, for example, the following quotation from Abraham ibn 'Ezra's *Book of the World*: "Dorotheus the king said that he found in the *Book of Secrets* by Enoch that he enjoined that at the revolution of the year in which a Saturn-Jupiter conjunction takes place, whether a great, a middle or a small conjunction, one should always observe the location of the planets when the Sun first enters Aries ..." (Sela, 2010, p. 73).

⁷³⁴ Millàs, 1929, p. 197.

⁷³⁵ Millàs, 1929, p. 203.

Conjunction 99 (6 Nisan 4331/ 17.3.571): 228^{o736}

Conjunction 125 (9 Nisan 4847/ 17.3.1087): 38^{o737}

Conjunction 132 (14 Adar 11 3986/ 15.3.1226): 303^{o738}

Conjunction 138 (10 Nisan 5105/ 14.3.1345): 315^{o739}

Conjunction 141 (2 Nisan 5164/ 14.3.1404): Aquarius⁷⁴⁰

Conjunction 144 (5 Nisan 5224/ 14.3.1464): 325^{o741}

It is, obviously, difficult to accept that, in the aforementioned eight conjunctions, the astronomical event takes place precisely in the day of the spring equinox, but it is difficult for me to find an explanation for this coincidence, as I do not know which kind of tables were used by Abraham bar Ḥiyya. One of these conjunctions, no. 132 is particularly interesting because Bar Ḥiyya gives details (longitudes of the twelve houses⁷⁴² and diagram of the horoscope) which we only find in another horoscope of the collection: that of conjunction 0. Besides, this is the only horoscope for which the positions, calculated with al-Battānī's *zīj*,⁷⁴³ give results which can be considered approximate to those of the text.

Bar Ḥiyya says that the division of the houses has been calculated for the latitude of Israel, which is 32°, ⁷⁴⁴ in agreement with al-Battānī's geographical table which gives 31;50° for the latitude of Jerusalem⁷⁴⁵ and 31° for Palestine.⁷⁴⁶ Surprisingly, Bar Ḥiyya adds that the horoscope has also been calculated for the geographical longitude of the centre of the world: the "cupola" (*qubba*) of Islamic tradition, placed at 90° of distance from the extreme Western and Eastern limits of the inhabited world (*al-ma'mūra*). I wonder why Bar Ḥiyya does not use the corresponding latitude of the *qubba*, 0°, as it is supposed to lie on the equator.

The hour of the horoscope is "a quarter after the beginning of the tenth hour of the night": as the horoscope is cast for a date very near the spring equinox, the sunset takes place at about 6^h PM and the hour will be a quarter of an

⁷³⁶ Millàs, 1929, p. 229.

⁷³⁷ Millàs, 1929, p. 234.

⁷³⁸ Millàs, 1929, pp. 241–242.

⁷³⁹ Millàs, 1929, p. 246.

⁷⁴⁰ Millàs, 1929, p. 247.

⁷⁴¹ Millàs, 1929, p. 248.

⁷⁴² Usually Bar Ḥiyya only mentions the longitude of the ascendant and that of midheaven.

⁷⁴³ I am using the computer programme BATLN prepared by Prof. E.S. Kennedy, during one of his visits to Barcelona, and later revised by Honorino Mielgo.

⁷⁴⁴ 36° in Millàs, 1929, p. 241.

⁷⁴⁵ Nallino, 1899–1907, II, 54.

⁷⁴⁶ Nallino, 1899–1907, II, 35.

hour after 3^h AM. The same hour and near latitude are obtained by entering the longitudes of houses I–VI in John North’s HOROSC programme. I suspect that Bar Ḥiyya used al-Battānī’s *zīj* to divide the houses of this horoscope and to calculate the planetary longitudes for the astronomical date 14th March (15th March civil date) 1226 at 3^h in the morning. The data of the horoscope and the recomputation of the planetary longitudes are shown below:

Houses

Divided according to the standard method. The calculated latitude is 34;37° (± 2°) and the hour 3;3^h AM. In parenthesis I note the difference between the values given in the text and the recomputation with HOROSC.

I –	303°	VII –	[123°] ⁷⁴⁷
II –	338°	VIII –	15[8] ⁰⁷⁴⁸
III –	16°	IX –	196°
IV –	52°	X –	23[2] ⁰⁷⁴⁹
V –	74° (–2°)	XI –	254°
VI –	99°	XII –	279°

Planetary longitudes

TABLE 3.13 Planetary positions for the conjunction of 1226

Planets	Longitudes (text)	Recomputation 14.3.1226, 3h AM
Sun	4°	1;56°
Moon	175°	174;6°
Saturn	303°	302;34°
Jupiter	303°	307;18°
Mars	218° R ^a	218;21°
Venus	321°	323;6°
Mercury	8°	355;54°
Ascending node	328°	331;7°

a R means retrograde motion of the planet.

747 148° in Millàs, 1929, p. 242.
748 151° in Millàs, 1929, p. 242.
749 235° in Millàs, 1929, pp. 241–242.

The aforementioned horoscope deserves a few comments. First of all, I wonder why an horoscope which is, in principle, calculated for the spring equinox gives a solar longitude of 4° . It is obvious that the date does not correspond to al-Battānī's equinox (the 12.3.1226 at about 3^h AM), but the position of the Moon is in agreement with the date and the hour. I also wonder whether the longitude of the Sun is the result of a corruption in the manuscript transmission. It is obvious that the longitudes of Venus and the node have a difference of 2° and 3° and that Mercury's longitude is much worse. I should also remark that Mars is not in retrograde motion according to al-Battānī's *zīj*. The most interesting thing is, however, that Saturn's position agrees with the presumed source, but such is not the case of Jupiter. Interestingly, the date chosen corresponds to that of al-Battānī's mean Saturn-Jupiter conjunction at about 302° . The mean longitudes of the two planets at the hour of the horoscope are:

Saturn	302;29°
Jupiter	302;12°

The true conjunction had taken place more than a month before, not in the sign of Aquarius but in Capricorn. The true longitudes of both planets were the 30th Shebat 4986/ 30.1.1326 at midday:

Saturn	298;4°
Jupiter	298;3°

This is clearly an explanation for the coincidence between the conjunction and the equinox in year 1326, as Bar Ḥiyya seems to acknowledge when he says⁷⁵⁰ that the two planets in conjunction will be in the sign of Aquarius, if computed with their regular motion, while, according to their irregular motion, they will be in that sign at the beginning of that year.⁷⁵¹ It does not work, however, for the rest of the cases. The only thing I can suggest is that, in the aforementioned eight cases, Bar Ḥiyya is including in each one of these equinox-horoscopes the mean or true position of Saturn and Jupiter at the moment of the conjunction.

⁷⁵⁰ Millàs, 1929, p. 241.

⁷⁵¹ I do not understand the meaning of "the beginning of the year". It cannot be the astronomical beginning (spring equinox) or the beginning of the Jewish year, as the 1st Tishri 4986/ 4.9.1225 both planets were in Capricorn, not in Aquarius.

The analysis of the horoscope of 1326 shows that, at least in this case, Abraham bar Ḥiyya has cast a tropical, not sidereal, horoscope, something which disagrees with the common practice in al-Andalus and the Maghrib as well as with the collections of Eastern Islamic historical horoscopes compiled by Māshā'allāh, Abū Ma'shar or Mūsā b. Nawbakht. This hypothesis is confirmed by what we have found about the length of the solar year. In spite of this, it is not at all clear that Bar Ḥiyya uses al-Battānī's *zīj* to compute his horoscopes. I have tried to recompute the horoscopes of the middle conjunctions after the Christian era (numbers 72–144) with negative results: the differences increase as we move back in time.

We must, therefore, consider the possibility that Bar Ḥiyya could be following a tradition entirely different from that of al-Battānī and could have copied some of the horoscopes, at least, from another source.⁷⁵² He might also have used a different set of astronomical tables which I have been unable to identify in spite of having done many attempts to recompute Bar Ḥiyya's horoscopes using al-Khwārizmī's *zīj*, Yaḥyā ibn Abī Maṣṣūr's *Mumtaḥan* tables, the *Toledan Tables* and, of course, those of al-Battānī. The comparison with the planetary longitudes obtained with al-Khwārizmī's sidereal tables makes me think that, in spite of the fact that the calculated parameters for the mean motion of Saturn and Jupiter seem to be sidereal, the positions in the horoscopes for the rest of the planets have probably been corrected to obtain tropical longitudes.

Finally, five conjunctions of dates after the beginning of the Christian era, computed using al-Battānī's *zīj* (1226 and 1464 are the exceptions) do not agree with Bar Ḥiyya's shifts in the triplicity, which are the base of his argumentation.

This table shows that, in five cases, the zodiacal signs of both the mean and true conjunctions, calculated with al-Battānī's *zīj*, do not coincide with those of Bar Ḥiyya. This is important if we have in mind our author's historical analysis and shows a clear difference with the evidence we find in Islamic astrological histories, all of which consider that the conjunction of 571 (no. 99 in Bar Ḥiyya's series), the *qirān al-Milla al-Islāmiyya* which announced the advent of Islam, coincided with the shift to the triplicity of water in the sign of Scorpio. Interestingly, Bar Ḥiyya considers that the change in triplicity took place in conjunction no. 96 in 511 and, therefore, the conjunction of 571 is only a small

⁷⁵² According to Sela (2005, p. 136, fn. 34) the positions of planets and nodes are similar, though not identical, to those of horoscopes calculated for the same date by Māshā'allāh, Abū Ma'shar and al-Sijzī, although I have not been able to find such similarity. See Pingree, 1962, pp. 488 and 494.

conjunction, thus diminishing the importance of the event. One has to bear in mind that Bar Ḥiyya is well informed about the chronology of the Prophet's life and dates his birth in the month of Yiar of year 4331, which began the 11th April 571,⁷⁵³ his death in year 4392/632, and knows that in the middle of the month of Yiar 4331 there was a lunar eclipse in the sign of Scorpio.⁷⁵⁴

It has a certain interest to compare Bar Ḥiyya's horoscope, calculated for the 6th of Nisan 4331/ 17.3.571 not only with al-Battānī's recomputation but also with other horoscopes for the same event in the astrological histories of Māshā'allāh, Abū Ma'shar and Mūsā ibn Nawbakht (see also 3.2.5.2). When comparing these four horoscopes we should not consider the lunar positions because of the different dates involved and the fast motion of the Moon. As for the rest, we can easily see that the differences are much greater for Saturn and Jupiter than for the other planets. I believe, therefore, that Bar Ḥiyya's horoscopes are based on Saturn-Jupiter conjunctions cleverly handled in order to obtain adequate results.

TABLE 3.14 Bar Ḥiyya's conjunctions calculated with al-Battānī's *zīj*

No of the conjunction	Mean middle conjunction (Battānī)	True middle conjunction (Battānī)	Sign of the conjunction (Bar Ḥiyya)
72	4.10.34 (124°, Leo)	12.10.34 (136°, Leo)	Virgo (true conj. in April, 163°)
84	20.1.273 (160°, Virgo)	10.8.273 (169°, Virgo)	Libra
96	8.5.511 (196°, Libra)	12.10.511 (204°, Libra)	Scorpio
108	20.8.749 (231°, Scorpio)	25.11.749 (236°, Scorpio)	Sagittarius
120	1.12.987 (267°, Sagittarius)	31.12.987 (267°, Sagittarius)	Capricorn
132	14.3.1226 (302°, Aquarius)	30.1.1226 (298°, Capricorn)	Aquarius
144	4.7.1464 (338°, Pisces)	4.3.1464 (330°, Pisces)	Pisces

753 We have seen in 3.2.5.2 that the Hyderabad recension dates the birth of the Prophet in 19.4.571.

754 Millàs, 1929, pp. 229–231. In al-Battānī's astrological history both the birth of the Prophet and the lunar eclipse occurred the 25.4.571. See Kennedy *et al.*, 2009–10, p. 89.

TABLE 3.15 Horoscope of the advent of Islam according to Bar Ḥiyya and early Islamic astrological sources

Planets	Longitudes (Bar Ḥiyya)	Recomputation 17.3.571, 1h AM	Māshā'allāh 19.3.571 ^a	Abū Ma'shar 19.3.571 ^b	Ibn Nawbakht 15.3.571 ^c
Sun	[0°]	357;57°	0;1°	0°	0°
Moon	67°	63;26°	91;10°	91;45°	81;9°
Saturn	228° R	215;15°	216;13° R	217;26°	219°
Jupiter	228° R	216;23°	214;26° R	213;32°	216;26°
Mars	79°	73;40°	72;15°	73;14°	72;26°
Venus	5°	0;7°	2;16°	2;18°	3;40°
Mercury	11°	9;58°	18;26° R	6;17° R	8;3°
Ascending node	41°	38;55°	37;53°	–	38;45°

a Kennedy & Pingree, 1971, pp. 49, 98.

b Pingree, 1968, p. 95.

c Labarta, 1982, p. 78.

3.2.6 Meteorological Astrology: al-Baqqār's Kitāb al-amṭār wa l-as'ār

I have already mentioned this book of al-Baqqār (fl. Fez 1411–1418) in 3.1.1.5.2 (the verses of Ibn al-Khayyāt's *Lāmiyya* extant in the *Kitāb al-amṭār*) and in 3.2.1.1 (where I have tried to analyse the first part of this book, dealing with the ancient system of the crosses for astrological prediction). I have also used the information of this book when dealing with the treatment of astrometeorology by Ibn Abī l-Rijāl (3.2.2.2.5.3) and I have referred to his second work, the *Kitāb al-adwār fī tasyīr al-anwār* in many passages of 3.2.3. Here, I intend to analyse the part of the *Kitāb al-amṭār* which follows his explanation of the system of the crosses and the *Lāmiyya* of Ibn al-Khayyāt and which proves al-Baqqār's use of more sophisticated astrological techniques.⁷⁵⁵

Ibn al-Khayyāt deals with the analysis of the passage of Saturn through the zodiacal signs and al-Baqqār continues with the same subject using other sources and methods. Thus he refers to *tasyīr* when he uses terms such as *ṭālī'* (ascendant), *qaṭ'* (cut) or *haylāj* (indicator). Among his sources he quotes "Some Roman/ or Greek wise men" (*ba'd ḥukamā' al-Rūm*), which contain references to al-Andalus. This makes me think in the possibility of a relation of this source

755 Guesmi & Samsó, 2018.

with the aforementioned “system of the crosses”. Other authorities are Hermes and ‘Umar ibn Farrukhān al-Ṭabarī (second half of the 8th c.–beginning of the 9th c.) whom he seems to know through an Andalusian recension of a book of his written by somebody called ‘Abd al-‘Azīz ibn al-Bayṭār, possibly the author of a set of tables of planetary latitudes (perhaps derived from those of Ibn al-Kammād) mentioned by the anonymous author of the Tunisian recension of the *Zīj* of Ibn Ishāq extant in the Hyderabad manuscript.⁷⁵⁶ Al-Baqqār also mentions another anonymous source considered reliable by him: he refers to a conjunction of Saturn and Mars, in the sign of Pisces in 626/1228–29, which brought prosperity to al-Andalus and poverty in the Eastern part of the Maghrib, Egypt and Syria. It is easy to check that, using modern tables, such a conjunction took place in 324° (near Pisces) the 21.11.1228.⁷⁵⁷ Similar results could be obtained with the medieval tables used at that time.

This is followed by another passage in which al-Baqqār adds to the transit of Saturn through triplicities and tropical, fixed and bicorporal signs, other variables such as conjunctions of Saturn-Mars, Jupiter-Venus and Saturn-Jupiter. He insists in the importance of Venus and the Moon, whose relation to rainfall is a well known astrological commonplace: he makes predictions based on the lunar mansions in which Venus rises and on the direct or retrograde motion of the superior planets.⁷⁵⁸ He applies medical ideas (such as the moments of crisis of an illness) to the study of astrometeorology: the position of the Moon allows him to predict moments of crisis (*marākiz al-buhrān*) which imply a change of weather in a particular month. He also introduces refinements such as the projection of rays, the idea of *ḥayyīz* (domain) of every planet,⁷⁵⁹ decans, *ḥudūd* (terms),⁷⁶⁰ as well as the *ziyāda/nuqṣān* (increase/decrease) in *ḥisāb* (calculation) or *masīr* (velocity).

The last part of the book is divided into three chapters which deal with 1) meteorology and rains, 2) prices and 3) solar and lunar eclipses. In the first chapter a new source appears: Ibn Abī l-Rijāl’s *Bārī* which offers new techniques for weather prediction (see 3.2.2.2.2.5.3). Such predictions can be made for yearly periods with the standard method of casting the horoscope of the conjunction or opposition of the Sun and the Moon before the vernal equinox, although there are other significant moments, such as the entrance of the Sun in Scorpio 20°. The importance of Scorpio 20° may have a purely

756 Mestres, 1996, p. 428.

757 Stahlman & Gingerich, 1963, p. 456; Tuckerman, 1964, p. 632.

758 On predictions based on the Moon see Burnett, 2004.

759 Burnett, Yamamoto & Yano, 1994, pp. 40–41.

760 Bīrūnī, *Tafhīm*, p. 265.

meteorological explanation because Ibn Hibintā⁷⁶¹ states that the period of rains in Iraq, the climate of Babel and northern parts begins with the Sun's passage through Scorpio 19° and ends with its passage through Taurus 19°. This is reinforced by a quotation of the *Filāḥa Nabatīyya* which recommends casting the horoscope of the conjunction of the Sun and the Moon in the sign of Scorpio which, according to al-Baqqār, is the horoscope of the yearly birth of the plants.

There are also predictions for seasons (horoscope of the conjunction or opposition before equinoxes and solstices), for half months (application of the Moon to a planet when the former leaves the place of the conjunction or opposition), for each week (horoscopes of syzygies and quadratures), and for periods of two and a half days (passage of the Moon through the twelve zodiacal signs). For this kind of predictions al-Baqqār introduces cycles of world-astrology (*qisma* and *ifrādīyya*) and defines new theoretical instruments:

- a) Three lots (*sihām*) which are the *sahm al-maṭar*, *sahm al-riyāḥ* and *sahm al-ayyām* which correspond, respectively, to the lots of rain, winds and "days". Only the *sahm al-riyāḥ* is calculated with the same method as in Eastern sources, while this is not the case of the *sahm al-maṭar*.⁷⁶² Al-Baqqār ascribes the *sahm al-ayyām* to Abū Ma'shar and the same is done by Ibn Abī l-Rijāl, although I have not been able to find such *sahm* in the *Madkhal Kabīr*.
- b) *Ta'sīsāt* (see also 3.2.2.2.5.3) which seem equivalent to the *fashīshāt* mentioned by al-Bīrūnī.⁷⁶³ a series (12 according to Bīrūnī, 24 according to al-Baqqār) of elongations of the Moon from the Sun, at unequal intervals, which are considered sensitive points in the monthly displacement of the Moon.⁷⁶⁴
- c) "Opening of the doors" (3.2.2.2.5.3) (*abwāb al-futūḥ*, *abwāb al-futuḥāt*). A concept which appears, with minor differences, in many Eastern sources such as 'Umar ibn al-Farrukhān, al-Kindī,⁷⁶⁵ al-Qabīsī,⁷⁶⁶ Kūshyār ibn

761 Ibn Hibintā, 1987, I, 203. *Jafar Indus* says that, according to the Indians Scorpio 20° is significant for the prediction of rains (Burnett, 2004, p. 69) and similar references can be found in al-Kindī (Bos & Burnett, 2000, p. 257), in 'Alī ibn Abī l-Rijāl (3.2.2.2.5.3) and in Abraham ibn 'Ezra (Sela, 2010, p. 187).

762 Al-Baqqār's procedure to calculate the *sahm al-maṭar* is the same we can find in Abraham ibn 'Ezra's *Reshit Ḥokmah* (Sela, 2010, pp. 223–224 and Bos & Burnett, 2000, pp. 51–52).

763 al-Bīrūnī, *Taḥfīm* p. 153.

764 Abraham ibn 'Ezra mentions the *ta'sīsāt* as the "twelve keys of the Moon": Sela, 2010, pp. 23–24, 83–85, 135–139, 210–211, 220–221.

765 Bos & Burnett, 2000, pp. 19, 385–394, 447–448.

766 Burnett, Yamamoto & Yano, 2004, pp. 136–137.

Labbān⁷⁶⁷ and al-Bīrūnī.⁷⁶⁸ Al-Baqqār's definition follows, as we have seen, that of Ibn Abī l-Rijāl which is not the same as that of his Eastern predecessors: there is "opening of the doors" when the Moon separates from one planet of the two couples which have their domiciles in opposite signs⁷⁶⁹ and enters in application with the other one.

- d) Increase and decrease in number (*ʿadad*), equation (*taʿdīl*) and light (*nūr*) as well as *ṣāʿid/hābiṭ fī falaki-hi* (ascending/ descending in its sphere) and *baṭīʿ/sarīʿ al-sayr* (slow/ fast motion).

The second chapter of this last part deals with the oscillation of prices of agricultural products which, according to al-Baqqār, depends on periods of rainfall or draught. His ideas on this topic are interesting because he states that an increase of the offer of products in the market does not necessarily imply a diminution of their prices. In his opinion, the Moon is the cause of an opposition between merchandises and *ʿayn*: he never explains the meaning of *al-ʿayn* although it is clear that he refers to gold and silver coins.⁷⁷⁰ It is not always clear which is the attitude of the author in relation to high and low prices although, in most cases, he associates good (*khayr*) with the increase of prices and evil (*al-sharr wa l-fasād*) with their decrease. Thus, when propitious indicators (*suʿūd*) are in a favourable situation, prices go up, and the contrary happens when their position is unfavourable, but similar things are also said about the malefics (*nuḥūs*), which is a clear contradiction. In most of the cases, however, the second attitude corresponds to quotations of other authors such as ʿAlī ibn Riḍwān's commentary on the *Tetrabiblos* or an unidentified Ibrāhīm al-Yahūdī, also mentioned in Ibn Abī l-Rijāl's *Kitāb al-Bārīʿ*. Al-Baqqār's attitude, however, seems to change in his analysis of the oscillation of prices using Saturn-Jupiter conjunctions: when the malefic Saturn is in the sign of the last conjunction with Jupiter, or in the *Burj al-Milla* (sign of the Religion) or in the *Burj al-qirān al-dāll ʿalā l-Milla* (sign of the conjunction which indicated the rise of Islam) – which should be considered privileged positions – the price of food will go up and there will be a generalised famine. It is clear that the *al-qirān al-dāll ʿalā l-Milla* is the conjunction of Saturn and Jupiter in 571 in the sign of Scorpio (see 3.2.5.2 and 3.2.5.3). As for the *Burj al-Milla*, the interpretation is doubtful although al-Baqqār possibly refers to the sign of Cancer, which was the sign

767 Yano, 1997, pp. 86–89, 90, 95.

768 al-Bīrūnī, *Taḥfīm* pp. 314–315. Abraham ibn ʿEzra's description of "opening the doors" is not very clear but seems to agree with Bīrūnī's: Sela, 2010, pp. 22–23, 83, 181.

769 Venus (Libra and Taurus) and Mars (Scorpio and Aries); Mercury (Virgo and Gemini) and Jupiter (Sagittarius and Pisces).

770 Dozy, 1967, II, p. 197.

of Saturn at the moment of the spring equinox of the year of the beginning of the Hijra (19.3.622). Two days later (21.3.622) there was a conjunction of Saturn and Mars, also in Cancer, according to Abū Ma'shar.⁷⁷¹ Cancer corresponded, again, to the mean longitude of Saturn in the day of the beginning of the Hijra (14.7.622), while its true longitude was in the beginning of Leo. If we realise that the sign of the last mean conjunction of Saturn and Jupiter before the beginning of the era (25.9.610) was Pisces, we have three signs (Cancer, Scorpio and Pisces) which, curiously enough, form the triplicity of water.

The predictions on the oscillation of prices use the same methodology as those related to meteorology: the same periods of time are used (years, seasons, months, half months and weeks) and the Moon, whose importance for the prediction of rainfall has already been mentioned, – followed by the Sun, Venus and Mercury – is the most important indicator.

The chapter gives frequent lists of the products subject to the oscillation of prices and they correspond to what can be expected in the economy of the Maghrib towards the beginning of the 15th century: agricultural products used for human nourishment or for weaving, as well as dyeing, all kinds of cattle but not a single reference to fishing. Allusions to metals and mining are scarce and only one luxury product is mentioned: perfumes. The aforementioned products have astrological associations which may be the triplicities or specific planets.

It is interesting to remark that al-Baqqār makes three different attempts to quantify the increase or decrease of prices. The clearest one establishes that the difference in degrees of longitude between the planet which indicates the change in the price and the planet with which it enters in application will correspond to the number of dinars or dirhams of the raise or diminution.

The third chapter deals with the indications of solar and lunar eclipses and it is the only part of the whole book unrelated to rains or prices. On the other hand, it is most interesting to remark that it contains quotations from Ibn Hibintā and passages where the *al-Kitāb al-Mughnī* has clearly been used although its author is not mentioned. Thus, using information gathered in the *Tetrabiblos*, in the *Karpós* and in Ibn Hibintā's *Mughnī*, al-Baqqār studies the period of time during which the influence of an eclipse will last as well as the moment in which such influence will be more powerful. He also deals with the persons that will be affected by the eclipse according, among other things, to the zodiacal sign or the lunar mansion in which the eclipse will take place.

⁷⁷¹ Burnett & Yamamoto, 2000, I, 124–125 and Pingree, 1968, p. 80.

Astronomical Instruments

4.0 Introduction

Ibn al-Zarqālluh (d. 1100) who, according to Isaac Israeli's *Yesod 'Olam* (written in the fourteenth century), was an instrument maker, gives us a classification of astronomical instruments in his treatise on the *ṣafiḥa zarqāliyya*.¹ This classification could, perhaps, give us an idea of the kind of instruments available to him and to his colleagues in eleventh-century Toledo. The main purpose of the instruments he mentions is “to determine the hour, the variations in the length of days and nights in any location, as well as other applications related to the aforementioned purposes”. He classifies them in two main categories:

1. Shadow instruments (*ẓilliyya*) in which we observe the shadow projected by a gnomon. According to Ibn al-Zarqālluh himself, their obvious drawback is that they can only be used in daytime. Within this group he mentions horizontal sundials (which, as we have seen in 2.4.2.1.3.2, are the only ones for which we have archaeological evidence in al-Andalus), and also vertical, cylindrical (*uṣṭuwāniyya*) and conical (*makhruṭiyya*) models. The two latter kinds were mainly used as portable sundials (*horologia viatorum*) and they were known in both the Islamic and the Latin traditions. One of the conical sundials is the *mukḥula* mentioned, in the twelfth century, in the *Kitāb al-anwā'* by al-Umawī al-Qurṭubī (d. 1206).²
2. Instruments known as *shu'ā'iyya*, in which the rays of the Sun or of another celestial body are observed directly. In this group he mentions astronomical quadrants (*arbā' al-dawā'ir*), the [celestial] sphere, the astrolabe, the spherical astrolabe, the armillary ring (*ḥalqa*), the armillary sphere (*al-ḥalaq*) and the alidades (*al-ʿaḍā'id*). Our author does not give us precise indications about the kind of quadrants to which he refers. As for the armillary rings, they are, obviously, more or less sophisticated graduated rings of different sizes to which an alidade may be added. In

1 I am using ms. 4824 of the Bibliothèque Nationale de France, fols 8r–10r, as well as the Castilian Alfonsine translation: Ms. Villa Amil 156, fols. 113v–114r; Rico, 1863–67, III, pp. 149–150. An excellent facsimile edition of codex Villa Amil 156 has been published in Barcelona, 1999 (two volumes). A digital version of this manuscript can be found in alfama.sim.ucm.es/dioscorides/consulta_libro.asp?ref=B1850071&idioma=0.

2 On the *mukḥula* see Livingston, 1972. See also Charette, 2003, pp. 150–153; Charette & Schmidl, 2004, pp. 133–134, 159–160, 183–185. On this instrument in al-Andalus see Forcada, 1990.

Ptolemy's *Almagest* we find the description of a meridian or solstitial ring (I, 12), located on the plane of the meridian that is used to determine the meridian solar altitude and, from this, the obliquity of the ecliptic and the local latitude. *Almagest* III, I also mentions the equinoctial ring, located on the plane of the equator and used to determine with precision the moment of the passage of the Sun through the equinoxes. To these two types we should also add the azimuthal ring. References to instruments of this kind are often found in Islamic observatories, due to the importance of solar observations at these institutions. As Ibn al-Zarqālluh also made observations of this kind, it is natural to assume that he had meridian and equinoctial rings both in Toledo and in Córdoba. As for the alidades (called *reglas*, rulers in the Alfonsine translation), I believe that they can be identified with the *triquetrum* or parallactic instrument, called in Arabic *al-īdāda al-ṭawīla* (the long alidade), a Ptolemaic instrument consisting in three rulers linked together: one of them is perpendicular to the horizon, while the other two can be oriented in the meridian or in any other plane. It can be used to measure altitudes of any kind. This instrument was probably known in al-Andalus through the *Almagest* (V, 12) or through al-Battānī's *zīj*. The instruments mentioned by Ibn al-Zarqālluh are those often used in "observations" (*qiyāsāt*) and, in relation to the rings, alidades and quadrants, their basic application was to determine altitudes [by direct observation] and shadows [using a table or a graphical scale of cotangents or tangents]. As regards the armillary sphere he adds that it is rarely used because it is difficult to handle and it is only applied in order to determine star longitudes and latitudes. I have already dealt with the available information on sundials (2.4.2.1.3) and two types of horary quadrant (2.4.2.1.2). I know practically nothing about cylindrical or conical sundials or about the use of rings or the parallactic instrument in al-Andalus and the Maghrib. In this chapter I will try to gather together information about spherical instruments (the celestial sphere, spherical astrolabe and armillary sphere), standard and universal astrolabes, other types of quadrants, equatoria and Jābir ibn Aflah's observational instrument. In many instances I will use the wealth of data given by the Alfonsine *Libro del saber de astrología*.³

3 Laura Fernández (Fernández, 2013, p. 214) has drawn attention to the fact that *Libro del saber de astrología* is the title of the Alfonsine collection appearing in the royal codex (ms. Villa Amil 156), although we are used to seeing the title *Libros del saber de astronomía*, adopted by Rico y Sinobas in his well-known edition (Rico, 1863–67).

4.1 Spherical Instruments

4.1.1 *Celestial Globe*

4.1.1.1 The Celestial Globe in Tenth-Century Catalonia and al-Battānī's *bayḍa*

The earliest reference we have to some kind of celestial globe is contained in the tenth-century Latin corpus of texts, based on Arabic sources and compiled somewhere in Catalonia. We find it in a brief text that bears the title of “De horologio secundum alkoram id est speram rotundam” and was interpreted by Millàs as a description of a spherical astrolabe.⁴ Later, Richard Lorch argued that the text describes a celestial globe,⁵ a kind of instrument which was also known to Gerbert of Aurillac (945–1003), who described it in a short text in which there does not seem to be any evidence of an Arabic influence.⁶

My own reading of “De horologio secundum alkoram” leads me to confirm Lorch's interpretation: the description corresponds to a celestial globe bearing the circle of the equator (*circulus equalitatis*), the ecliptic (*circulus casarum*) and the position of some stars. Besides, the instrument also has a horizon (*circulus alofac id est medie terre*) and an alidade (*alhidada*) with sights (*foramina*), through which the light of the Sun can penetrate; however, the light is insufficient to make an observation of a star, and the observer has to use an astrolabe for this purpose. The alidade can, probably, move on a ring (*circulus horarum*) which pivots on the axis zenith-nadir, and a needle (*acus*) is used to fix the alidade to the solar degree or the position of a star. The instrument also has a meridian ring (*circulus meridiei*).

This description presents an instrument which has points in common with the celestial globe named *bayḍa* in al-Battānī's *zīj* (Fig. 4.1).⁷ The latter has a horizon ring, a double meridian ring (one of them is stationary, while the other can be moved), a ring corresponding to the vertical circle passing through the east and west points of the horizon and another outside ring, representing also a vertical circle through which an alidade with two sights and a needle is inserted. The author of our Latin text probably had at his disposal a written description of al-Battānī's instrument which he imitated with some simplifications (there is no mention of a *double* meridian ring or of the vertical circle passing through the E and W points of the horizon). The use of these sources can be attributed to the fact that the astrolabe texts in our Latin collection seem to be influenced by the works on this instrument undertaken by

4 Millàs, 1931, pp. 288–290.

5 Lorch, 1980.

6 Bubnov, 1899, pp. 24–28, 378–380.

7 Nallino, 1899–1907. Vol. I (1903), pp. 138–142, 319–321; Savage-Smith, 1985, pp. 18–21, 88–89.

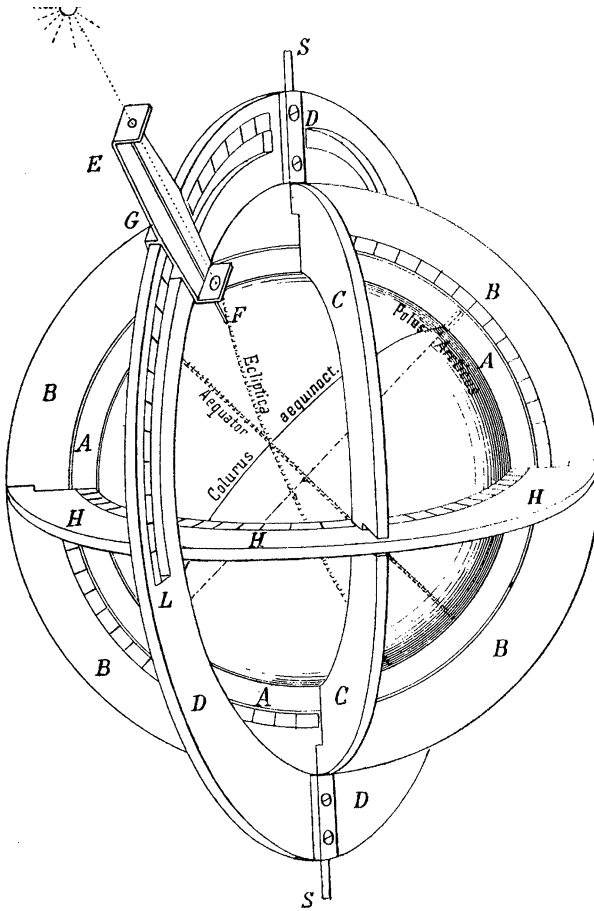


FIGURE 4.1
al-Battānī's *bayda* in
Nallino's reconstruction

Maslama al-Majrīṭī (d. 1007) and his disciples (4.2.2.3.1) and that this group of scholars knew al-Battānī's *zīj* very well.

On the other hand, it seems that the author of our Latin text had some difficulty in understanding the Arabic text he was reading. His main interest is the use of the instrument for timekeeping and he describes, more or less clearly, the procedure to calculate the rotation of the sphere between sunrise and the moment in which the Sun reaches the observed altitude. In a similar way, he calculates the day arc of the Sun for a given date. The other chapters related to timekeeping (determination of the night hour from a star altitude observed with an astrolabe; conversion of equal to seasonal hours) are more difficult to understand, either because the author has not really understood what he was explaining or because the text we have is corrupt. The same can be said of the chapters on the determination of the solar meridian altitude, from which one obtains the local latitude, and the determination of the altitude of a star.

4.1.1.2 Standard Celestial Globes in the Iberian Peninsula

4.1.1.2.1 Introduction

As for the Iberian Peninsula, the celestial globe seems to have been known in al-Andalus and, as we have seen, in the Christian kingdoms in the tenth century, even though evidence of it is scarce. Both the *Almagest* and al-Battānī's *al-Zīj al-Ṣābi'*, which contained the description of a precession globe⁸ and of the *bayḍa* respectively, were available to Maslama al-Majrīṭī (d. 1007).⁹

In the eleventh century, the great Toledan astronomer Ibn al-Zarqālluh criticised the defects in spheres made in his time by other artisans and mentioned that he had built a celestial globe which could be adapted to any latitude with the precision of one degree.¹⁰ His contemporary, Ibrāhīm ibn Sa'īd al-Sahli (fl. Toledo and Valencia 1071–1090), was a well-known instrument maker and five astrolabes made by him are still extant.¹¹ He is also known to be the author of one of the two Andalusī celestial globes known today, and has been proposed as the author of the second one.¹² The former, signed by Ibrāhīm ibn Sa'īd al-Sahli al-Wazzān with the collaboration of his son Muḥammad, is now in the Museum of the History of Science in Florence. The instrument was made in Valencia on 1 Ṣafar 473/ 22 July 1080 or 478/ 29 May 1085 for the supreme commander (*al-qā'id al-a'lā*) and holder of the double vizirate (*dhū l-wizāratayn*) Abū 'Īsā ibn Lubbūn.¹³ The globe has a graduated ecliptic and equator and six great circles passing through the poles of the ecliptic and the beginnings of the zodiacal signs. It also has 1015 stars with 47 labelled constellations. The magnitude of these stars is indicated by circles surrounding the stars whose diameter diminishes from the first to the sixth magnitude. Millàs measured the longitude and latitude of 23 stars in this globe and obtained a mean increase

8 *Almagest* VIII, 3. See Toomer, 1984, pp. 404–407; Neugebauer, 1975, II, 890–892; Savage-Smith, 1985, pp. 8–11.

9 Nallino, 1899–1907, vol. I, pp. 139–142 and 320–321; Savage-Smith, 1985, pp. 18–20.

10 Samsó, 2011, pp. 173–175.

11 King, 2005, p. 1006; Hernández, 2018 a, pp. 71–93, 121–136.

12 I have no information about celestial globes in the Medieval Maghrib. Only Ibn al-Bannā' (1256–1321) seems to have written a non-extant *Risāla 'alā l-kura* in Marrakesh although we do not know whether the text dealt with a celestial globe or with a different topic. See Jabbār & Aballāgh, 2001, pp. 60, 125, 199; Lamrabet, 2012, M134, p. 167.

13 On this important politician and poet see Aragón, "Ibn Lubbūn", in *BA* vol. 4, pp. 32–39. The doubt regarding the date of the instrument is due to the difficulty in ascertaining whether the last digit of the Islamic date (473 or 478) is a *jīm* (3) or a *hā'* (8). Ibn Lubbūn was actually vizier of al-Ma'mūn (1043–44–1075) and al-Qādir (1075–1085) of Toledo but, after the conquest of the city by Alfonso VI of Castile in 1085, al-Qādir reigned in Valencia between 1086 and 1092 and Ibn Lubbūn was the governor of this kingdom in the name of al-Qādir. This makes me think that 1085 is the most likely date for the instrument.

of their longitudes from Ptolemy's star catalogue of $14;10^{\circ}$,¹⁴ while Destombes, using 44 stars, obtained $14;5^{015}$ and, more recently, Dekker and Kunitzsch have confirmed Destombes's mean value of $14;5^{\circ}$.¹⁶ These differences agree with the results one obtains with the two variants of Ibn al-Zarqālluh second model of trepidation (6.2.2.4): $14;10^{\circ}$ with model 2A and $14;8^{\circ}$ with model 2B for 1075.¹⁷ The longitude of the star Qalb al-Asad (Cor Leonis, Regulus), which is placed on the ecliptic, is $136;40^{\circ}$, while the longitude established by Ibn al-Zarqālluh ca. 1075 was $136;35^{\circ}$.¹⁸ All this seems to indicate that Ibrāhīm ibn Sa'īd was using Ibn al-Zarqālluh's star longitudes. The second celestial globe, preserved in the Paris National Library, has also been attributed to Ibrāhīm ibn Sa'īd al-Sahlī due to its similarity to the Florence globe,¹⁹ although Dekker and Kunitzsch reject this attribution and consider that neither globe is a copy of the other, although both are Andalusī, are based on the same source material and are more or less contemporary: the mean value of precession calculated on the Paris globe is $14;3^{\circ}$. This does not mean that they can be attributed to the same artisan or to the same workshop.²⁰

In the thirteenth century, in addition to the Alfonsine treatise on the celestial globe (4.1.1.2.2), an instrument of this kind, extant in the library of the hospital founded by Nicholas Cusanus in Bernkastel-Kues, has also been considered to have an Alfonsine origin,²¹ although this attribution does not seem to be well established.²² About a century later, in December 1361, Pere Gilbert and Dalmau ses Planes, astronomers of Pedro IV of Aragon (1336–1387), finished the construction of a great sphere in Barcelona with a diameter of seven spans, which they used to display the positions of fixed stars according to the observations they made between 1360 and 1366.²³

The Alfonsine book on the solid sphere is, as we will see presently, a good introduction to elementary spherical astronomy. This is why I would like to complement the previous list of references to celestial globes in the medieval Iberian Peninsula with others concerning astronomical or geometrical treatises which deal with the sphere. It is well known that Classical books on the

14 Millàs, 1931, pp. 55–65.

15 Destombes, 1966, p. 319.

16 Dekker & Kunitzsch, 2008–9, p. 190.

17 Dekker & Kunitzsch, 2008–9, pp. 161–162, use a constant precession of 1° in 66 years and obtain year 1066–7.

18 Samsó, 2011, pp. 236–237.

19 Savage-Smith, 1985, pp. 24, 217, 236, 285.

20 Dekker & Kunitzsch, 2008–9, pp. 158, 161–162, 177–8, 190.

21 Domínguez, 1984.

22 Comes, 1987.

23 Millàs, 1962, 66–68 and 124.

sphere written by Autolycus of Pitane (fl. ca. 300 BC)²⁴ Menelaus (fl. 98 CE), and Theodosius of Tripoli (first century CE),²⁵ circulated in Latin in translations made by or ascribed to Gerard of Cremona, and that there was also a Latin translation of the short elementary treatise entitled *De recta imaginatione spere et circulorum eius*, ascribed to Thābit b. Qurra.²⁶ Finally, the inventory, made in 1410, of the library of the Royal Palace in Barcelona mentions a Latin book with the title of *Spera solida*.²⁷ I can add to this two quotations from Theodosius and Autolycus extant in Spanish vernacular texts between the thirteenth and the beginning of the fifteenth century. Proposition 1, 16 of Theodosius' book is quoted by the Alfonsine astronomers both in the treatises on the sphere²⁸ and on the spherical astrolabe,²⁹ and the first proposition of Autolycus' *De sphaera mota* appears quoted (as *D'espera mobili* or *Tractado de la esfera movable* by Talocus/ Thalocus) by the Spanish writer Enrique de Villena (1384–1434) in his *Glosas a la Eneida*.³⁰

4.1.1.2.2 The Alfonsine Treatise “*Libro de la fayçon dell espera*”

4.1.1.2.2.1 Introduction

This book is the second of the *Libro del saber de astrología*, in the order in which they appear in the royal codex of the collection (Villa Amil 156, Universidad Complutense de Madrid, fols. 27r–42r).³¹ A short prologue, totally independent of any Arabic source, states that it deals with the instrument called the *alcora* (Ar. *al-kura*, i.e., sphere) or *vet* [*sic* instead of *det*] *alcorcy* (Ar. *dhāt al-kursī*, i.e., the globe, with a chair or stand) and it establishes a dependence with the book written by the Oriental scholar *Cozta*: the source mentioned is obviously the *Risāla fī l-kura al-falakiyya*, *Kitāb al-‘amal bi l-kura al-falakiyya/ al-nujūmiyya*, *Risāla fī l-‘amal bi l-kura [dhāt al-kursī]* – as it appears in the different manuscripts – written by the well-known scientist and translator Qusṭā b. Lūqā (fl. second half of the ninth century).³² However, the prologue explains clearly that the book is divided into three parts:

24 Mogenet, 1948.

25 Ver Eecke, 1959; Millás, 1942, p. 208.

26 Carmody, 1960, pp. 118–119 and 140–144.

27 Chabás, Roca & Rodríguez, 1992, p. 29.

28 Ms. Villa Amil 156, fol. 41r; Rico, 1863–67, I, 206.

29 Ms. Villa Amil 156, fol. 46r; Rico, 1863–67, II, 127.

30 I know this reference through the unpublished doctoral dissertation by Pedro M. Cátedra (*Sobre la vida y la obra de Enrique de Villena*. Universidad Autónoma de Barcelona, September 1981).

31 Rico, 1863–67, I, 153–209.

32 Sezgin, *GAS* VI, pp. 180–181. On this treatise, see Worrell, 1944. I have not been able to see H. Schnell, *Die Kugel mit dem Schemel*, Inaug. Diss., Erlangen, 1924, which contains a

1. Four initial chapters dealing with the construction of the instrument, a topic which does not appear in Qusṭā b. Lūqā's book. No author is mentioned for this part of the Alfonsine work, but it has been reasonably ascribed to Iṣḥāq b. Sid: this is the Alfonsine collaborator who often appears as the author asked by the king to write an original treatise on the construction of astronomical instruments when an adequate Arabic source cannot be found.³³
2. The 65 chapters of the Castilian translation of Qusṭā b. Lūqā's book on the *use* of the instrument. The translators were Yehudah b. Mosheh ha-Kohen³⁴ and the cleric Johan Daspa. They finished their work on 6 February 1297 of the Spanish Era (1259 CE). The translation was revised in 1315 SE/1277 CE, and this is the translation that appears in the royal codex.
3. An additional chapter appears after Qusṭā b. Lūqā's treatise. Following the king's orders, a certain Don Mosheh³⁵ describes an auxiliary quadrant and an armillary semicircle which he uses, in combination with the globe, to divide the houses of the horoscope and to calculate the *ataçir* (i.e., *al-tasyīr*), an astrological technique that aims to determine the time in which a future event will take place.

4.1.1.2.2.2 Description of the Instrument

The description of the instrument appears both in the first four chapters of the Alfonsine text, on the construction of the celestial globe, and in the fifth chapter (the first chapter of Qusṭā b. Lūqā's treatise)³⁶ which is the standard opening of a work on the use of an astronomical instrument. Obviously, the Alfonsine treatise offers many details on the construction of the instrument

complete German translation of Qusṭā's book. See also Samsó, 2005 a. For the comparison between the Arabic and the Castilian texts I have used here mss. Teheran, A. Mahdawī 503 and Istanbul, Ahmet III 3505; photocopies of these two mss. were sent to me by Richard Lorch to whom I would like to express my gratitude.

33 See Romano, 1971, pp. 689–690, 693, 705. On the scientific collaborators of Alfonso X, see also Procter, 1945; Roth, 1985; Roth, 1990.

34 On this author see Hilty, 1955.

35 The spelling *Don Xosse* which appears in Rico's edition (I, 206) is an obvious misprint. The spelling in Ms. Villa Amil 156, fol. 31r is "Mosse".

36 The Alfonsine treatise (69 chapters) follows the chapter division of the Arabic text (65 chapters), but as it adds four extra chapters at the beginning, there is a constant difference of 4 between the chapter numbers of the two versions. My references will be to the corresponding chapter number, so that *chapter 1/5* means first chapter of the Arabic original/ fifth chapter of the Alfonsine translation.

which are not mentioned in the Arabic original,³⁷ and even the general design of the celestial globe presents substantial differences between the two treatises which I will try to outline in the following pages (Fig. 4.2). This offers me the first opportunity to state that, even if the *Alcora* can be considered, in general terms, a faithful translation of the Arabic original,³⁸ Yehudah b. Mosheh and Johan Daspa are quick to expand upon or modify it when they deem it necessary.

Both texts (chapters 1/5) describe a celestial globe on which the circles of the ecliptic and the equator appear, as well as their poles and the great circles of longitude which correspond to the beginnings of the zodiacal signs.³⁹ Both the ecliptic and the equator are divided into 360° , although the ecliptic appears divided into twelve sections of 30° each, which correspond to the zodiacal signs, while the division into degrees of the equator is consecutive from 1° to 360° . While the Alfonsine chapter 3 uses an obliquity of the ecliptic near 24° ,⁴⁰ chapter 1/5 does not mention a value for this parameter, but $23;33^\circ$ appears in chapter 30/34 on the use of the instrument,⁴¹ and 24° is implicit in several others (47/51, 49/53, 50/54, 51/55, 52/56). An important difference between the two texts (chapters 1/5) appears in relation to the stars projected on the surface of the globe itself: the Arabic treatise mentions the 28 lunar mansions (*manāzil al-qamar*), represented by circles of different size, to which Qusṭā b. Lūqā adds a small number of brilliant stars (small circles with the same diameter) which he identifies with those usually projected on the astrolabe. The Alfonsine translators seem to have considered the lunar mansions to be inadequate for a cultural milieu which did not use a lunar calendar, and eliminate all references to them in their translation of this chapter: in fact the Castilian text replaces them with the 48 constellations of Ptolemy's star catalogue (not mentioned in Qusṭā's treatise) and by an undetermined number of stars within each constellation (represented by circles of different size according to the magnitude of the star). Thus, the two texts apply an entirely different approach to the stars that appear on the globe. Although the Castilian text does not state this explicitly, the implication is that the aim of the Alfonsine globe, like Ibrāhīm al-Sahlī's Florence instrument, was probably to depict the full set of stars listed

37 On the materials used and the construction techniques, see Savage-Smith, 1, 1985 pp. 81–82.

38 A classification of the Alfonsine astronomical translations and original works can be found in Samsó, 1994, pp. 558–566.

39 Both texts mention twelve circles: they are, in fact, six.

40 Ms. Villa Amil 156 fol. 29v; Rico, 1863–67, 1, 166.

41 Rico, 1863–67, 1, p. 187; this chapter is missing in ms. Villa Amil 156. It is the value established in 213 H./828–29 by Yahyā ibn Abī Manṣūr in Baghdad. See Ali, 1967, p. 60.

in star catalogues (class A in Savage-Smith's classification of Islamicate celestial globes); the globe described by Qusṭā b. Lūqā corresponds to class B (in the same classification) in which there are no constellation outlines, the number of stars is limited, and astrolabe stars are nearly always included.⁴²

Qusṭā b. Lūqā mentions two other circles: the horizon and the meridian. The former is a ring (*ḥalqa*) placed on the upper surface of the stand (*kursī, siella* or "chair"), divided into 360°. There is no description of the stand in chapter 1/5, but we find it in chapter 4 of the Alfonsine text, which is entirely independent of Qusṭā's treatise.⁴³ It has the shape of a square box with a graduated circular ring in its upper surface (horizon) and rectangular openings in its four sides: the user can introduce his hand through these openings in order to move the sphere. The meridian is a ring, also divided into 360°, which is fixed on the two equatorial poles of the sphere by means of two nails (*mismār, priego*) which allow the sphere to rotate inside the meridian ring (chapter 1/5). The meridian ring, which holds the sphere, stands on a pivot placed in the centre of the lower surface of the *kursī/ siella* and is fixed onto it by two guides in the northern and southern points of the horizon: in this way the meridian ring can slide so that the pole forms with the horizon an angle equal to the local latitude.⁴⁴

Qusṭā b. Lūqā's instrument corresponds, therefore, to the simplest kind, also described by Ḥabash al-Ḥāsib (fl. 831–860),⁴⁵ equipped with only two rings (meridian and horizon). The same can, obviously, be said of the Alfonsine text between chapters 5 and 69. As for the first four Alfonsine chapters on the construction of the instrument, they introduce a third graduated ring, called the smaller armilla (*armilla menor*) to distinguish it from the meridian ring (*armilla mayor*). The smaller ring is fixed to the two poles of the ecliptic and can rotate freely around them and underneath the meridian ring, being able to represent any great circle of celestial longitude. The purpose of the ring is obvious: it can be used to determine the position of any celestial body in longitude and latitude, as is explicitly stated in chapter 3 of the Castilian text. A ring of this kind appears in Ptolemy's description of his precessional globe, but I do not know of any Islamic text or instrument which mentions or includes one. It is possible that the Alfonsine collaborator who wrote the four chapters on the construction of the instrument used the *Almagest* as his source of inspiration for the *armilla menor*: the *Almagest* is quoted in relation to the construction of

42 For this classification see Savage-Smith, 1985, p. 61.

43 Ms. Villa Amil 156, fols. 29v–30r; Rico, 1863–67, I, 168–169.

44 See chapter 1/5. Further details on the mounting of the instrument in chapter 4 of the Alfonsine treatise (Ms. Villa Amil 156, fols. 29v–30r; Rico, 1863–67, I, 168–169).

45 Lorch & Kunitzsch, 1985.

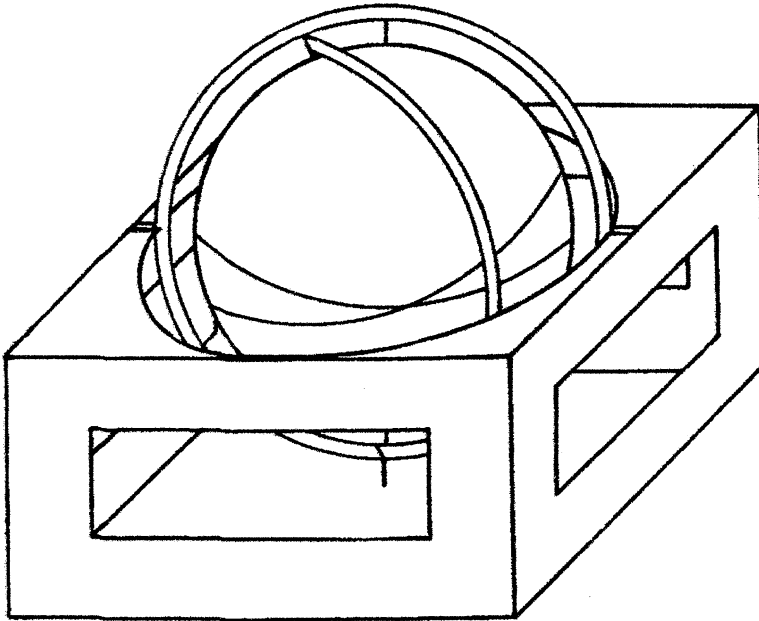


FIGURE 4.2 A reconstruction of the Alfonsine celestial globe

the celestial globe in the Alfonsine treatise on the spherical astrolabe⁴⁶ and MS Escorial 915, which contains Ishāq b. Ḥunayn's translation of Ptolemy's work, seems to have been copied at the Alfonsine court.⁴⁷

4.1.1.2.2.3 The Auxiliary Quadrant

It is interesting to examine the procedure described by Qusṭā b. Lūqā, and followed by the Alfonsine translation, to determine the position of a celestial body in longitude and latitude with a globe that does not have this smaller ring.⁴⁸ In 53/57 Qusṭā b. Lūqā describes an auxiliary graduated quadrant to determine the solar altitude by fixing one of its ends on the zenith and the other on the horizon. The same quadrant is also used in 58/62 to determine the longitude of the Moon or of any planet: now, one of the ends of the quadrant is fixed on the pole of the ecliptic and the other one on the ecliptic itself. This auxiliary quadrant appears described in other globe texts such as those written by

46 Ms. Villa Amil 156, fol. 44v; Rico, 1863–67, II, 121.

47 Samsó, 1987, p. 27.

48 Abū l-Ḥasan 'Alī al-Marrākushī easily solves the problem because his “meridian” armilla can be pivoted on the poles of the equator or on the poles of the ecliptic. See Marrākushī, 1984, II, pp. 7–8.

al-Šūfī (903–986),⁴⁹ al-Khāzinī (fl. 1115–1130),⁵⁰ Abū l-Ḥasan ‘Alī al-Marrākushī (thirteenth century)⁵¹ and in a Latin treatise on the solid sphere ascribed to Johannes of Halebeke or Harlebeke (fl. Paris 1303).⁵² It is rather surprising that Qusṭā does not mention this useful auxiliary quadrant until chapter 53 of the Arabic text, and uses approximate methods instead. Thus, in chapter 20/24 Qusṭā describes how to determine the longitude of a star and is forced to use the meridian ring: the degree of the ecliptic that crosses the meridian together with the star (i.e., the mediation of the star) is considered to be equal to the star’s longitude.⁵³ This equivalence is only approximate and the Castilian text adds a brief description of the auxiliary quadrant which is used exactly to establish the longitude of the star. The following chapter (21/25) describes how to determine the latitude of the star and, again, the Arabic text of Qusṭā uses only the meridian ring, while the Alfonsine translation adds the exact method based on the use of the quadrant, showing clearly that the translator was well aware of the relative inaccuracy of the method used by the Arabic original.⁵⁴ In chapters 62/66 (on the determination of the longitude of a star which is not projected on the sphere if we know its meridian altitude and the meridian altitude of one of the stars which appear on the sphere), and 63/67 (on the determination of the latitude of a star not projected on the sphere if we know its meridian altitude and its longitude), we no longer have Alfonsine additions but rather entirely different texts in the Spanish translation: once more, Qusṭā b. Lūqā uses only the meridian ring while, here, the Alfonsine text (no longer

49 Kennedy, 1989.

50 Lorch, 1980 a.

51 See for example Sédillot, 1844, p. 115; Marrākushī, 1984, 11, p. 5.

52 Lorch, 1980.

53 In chapter 29/33 both the Arabic text and the Castilian translation explicitly state this equivalence:

وانظر الجزء الذي توسط معه السماء من دائرة فلك البروج فذلك الجزء جزء توسط السماء وهو جزءه الحقيقي من فلك البروج.

“Look at the degree of the ecliptic which crosses the meridian together with it [= star]. That degree is the degree of its mediation and it is [also] its true degree of the ecliptic”. The Castilian text says: “Et aquel grado con que se para en el medio cielo en esta oriella desta armella, aquel es su logar dela estrella” (Ms. Villa Amil 156, fol. 34v; Rico, 1863–67, 1, p. 186).

54 “Et esta obra del quarto del cerco, quier por saber el logar de la estrella, quier por saber su ladeza, es mas certera, et mas derecha que la otra obra que dixiemos que se faze por la armiella de mediodia” (Ms. Villa Amil 156, fol. 33v; Rico, 1863–67, 1, p. 181) (“And this use of the quadrant, in order to determine the place [= longitude] of the star or its latitude, is more adequate and precise than the other aforementioned method which uses the meridian ring”).

a translation) describes methods based on the use of the auxiliary quadrant. These additions and corrections do not appear, however, in all the pertinent cases in which the auxiliary quadrant should have been used: in chapter 40/44, for instance, both texts use the meridian ring to determine the longitude and latitude of the Moon or of a planet, and the same happens in chapter 59/63, which aims to establish the latitude of the Moon or of a planet if we know its longitude and meridian altitude.

4.1.1.2.2.4 The Alfonsine Appendix

When Don Mosheh wrote the appendix on the procedure for the use of the celestial globe⁵⁵ in order to calculate the *tasyīr* (see 3.2.3.2) and to divide the houses according to the method of Hermes, he described, once more, how to make an auxiliary quadrant, divided into 90 equal parts and provided with a nail (*clauo*) in order to fix the quadrant to either of the holes made in the sphere in the northern and southern poles of the ecliptic. The text also carefully explains how to use the quadrant in order to establish, on the globe, the position in longitude and latitude of a celestial body. Interestingly, Don Mosheh seems to have written his appendix without reading either the Alfonsine translation of Qusṭā b. Lūqā's treatise (where, as we have seen, the auxiliary quadrant is described and used) or the first four chapters of the book on the construction of the instrument: the auxiliary quadrant is not needed for the determination of ecliptic coordinates if we have the *armilla menor*, fixed to the poles of the ecliptic. This is not the only instance of Alfonsine collaborators working independently of each other and being unaware of the work of their colleagues.⁵⁶

Don Mosheh also adds to the instrument half an armilla (*media armilla dell ataḡyr*), provided with a movable index (*demostrador*), the ends of which are fixed to the northern and southern points of the horizon. This semiarmilla is used to calculate the *tasyīr*⁵⁷ and can also be used to divide the houses according to the method of Hermes, the "prime vertical method" (3.2.3.1.1.2), in North's terminology.⁵⁸ This is the second method for the division of the houses of the horoscope which appears in the Alfonsine text, for chapter 55/59 explains how to use the globe in order to divide the houses according to the so-called *standard* method (3.2.3.1.1.1). For the prime vertical method, Don Mosheh

55 Ms. Villa Amil 156, fols. 41r–42r; Rico 1863–67, I, pp. 206–209.

56 See Samsó & Castelló, 1988.

57 Viladrich & Martí, 1983, pp. 88–90.

58 North, 1986, pp. 36–38. This method is also described in three other Alfonsine books: *Libro del Astrolabio Redondo*, *Libros de la Lámina Universal* and *Libro de la açafeha*. On the prime vertical method and its introduction in al-Andalus see also Kennedy, 1994; Kennedy, 1996; Samsó, 1996.

needs a second semiarmilla which is attached to the east and west points of the horizon and which passes by the zenith, thus representing the prime vertical. This semiarmilla will be divided into six equal parts (i.e., six arcs of 30° each)⁵⁹ and, obviously, if the *tasyīr* semiarmilla is pivoted on the north and south points of the horizon and is free to move in the north-south direction, its intersection with the 30° divisions of the “prime vertical” semiarmilla will determine, on the ecliptic, the beginnings of the houses according to this method.⁶⁰ However, the Alfonsine text is somewhat confusing; when it describes the prime vertical semiarmilla for the first time, it states that its use is to fix the *tasyīr* semiarmilla so that it will not move when the sphere rotates.

4.1.1.2.2.5 The Alfonsine Translation of Qusṭā b. Lūqā's Treatise

I have classified elsewhere⁶¹ the Alfonsine scientific works into faithful translations (*Aḡaḡeḡa*,⁶² *Canones de Albateni*), interpolated translations (*Picatrix*, *Lapidario*, *Libro de las Cruces*, Ibn al-Haytham's *Cosmology*), translations with original additions (*Lamina universal*, *Alcora*), adaptations (*Astrolabio redondo*) and false translations (*Ochaua Espera*). The *Alcora* can be considered a faithful translation of the Arabic original, but as I noted above Yehudah b. Mosheh and Johan Daspa do not hesitate to expand upon it or modify it when necessary. I have already mentioned changes introduced by the Alfonsine translators in relation to the stars projected on the sphere or on the use of the auxiliary quadrant. Other examples can easily be quoted. They are, sometimes, unimportant amplifications, or changes in the order of presentation,⁶³ of the Arabic text, attempting to explain it in a more complete way. This is the case of a passage (chapter 4/8) in which Qusṭā deals with the people who live south of the equator, of whom he states:

59 Viladrich & Martí, 1983, p. 98 n. 31, note that there is a mistake in Rico's edition (1, 208). In the edition of Kasten and Nitti, the text states clearly that the semiarmilla is divided into six parts, hypothetically equal.

60 “ponemos ell armilla del ataḡr sobre cada parte destas partes, et mouremos el demostrador fasta que se encuentre con el zodiacho, et mostrarnos ha las casas segund la opinion de Hermes” (Ms. Villa Amil 156, fol. 42r; Rico 1863–67, 1, 208).

61 Samsó, 1994, pp. 558–566.

62 Millás noted the literalism of Alfonsine translations but his analysis was based only on Ibn al-Zarqālluh's *Aḡaḡeḡa*: see Millás, 1933. Puig, 1987 has proved that this treatise is, actually, a very literal translation.

63 See the amplifications in chapters 6/10, 7/11, 18/22, 28/32; changes of order in 13/17, 28/32. In chapters 32/36 and 33/37 which deal, respectively, with the use of the sphere to obtain the rising times in right and oblique ascensions of the zodiacal signs, the Alfonsine translation tends to give examples for specific cases (signs of Aries, Taurus, Gemini), while Qusṭā often limits himself to a general reference to a zodiacal sign.

وَأَنَّهُمْ شَيْئُهُ بِالْبَهَائِمِ لَا بِنَاءَ لَهُمْ وَلَا صِنَاعَةَ وَلَا عِلْمَ

They are like beasts and have no buildings, techniques or science.

The Castilian translation is modified slightly, and insists on the idea of law as an important element in human civilisation – a not unexpected emphasis in the court of Alfonso X, who was a legislator of note:

Et son yentes semblantes de bestias, que non an ley, nin reglas, nin derechos, nyn saber nyn maestria⁶⁴ (“They are like beasts and have no laws, rules, rights, knowledge and crafts”)

In other instances the Castilian translation explains concepts which are not common knowledge outside the Islamic community. This is the case of chapter 57/61, which explains an approximate, and obviously incomplete,⁶⁵ method to determine the *qibla*: the Spanish version translates the Arabic text faithfully but adds “la linna dell alquibla, que quier dezir la parte haza qual los moros fazen oracion”⁶⁶ (“The *qibla* line, which means the direction towards which the Moors say their prayers”). This reference seems to be quite exceptional for, in the rest of the Alfonsine astronomical works, there is no mention of the *qibla*, and the problem is dealt with in a general way: how to find the direction of a given place from your location.⁶⁷ Only the book on the use of the plane astrolabe states, at the end of the corresponding chapter, that the direction of Mecca from Córdoba is at 45° between east and south.⁶⁸

64 Ms. Villa Amil 156, fol. 31r; Rico, 1863–67, I, 172.

65 Both the Arabic and the Castilian texts determine the *qibla* as a function of the difference of longitudes between the place in question and Mecca and forget about the latitudes of the two cities.

66 Ms. Villa Amil 156, fol. 40r; Rico, 1863–67, I, 202.

67 See *Armellas* chapters 37–38 (Ms. Villa Amil 156, fols. 148v–149r; Rico, 1863–67, II, 53–54); *Astrolabio redondo* chapters 81–83 (Ms. Villa Amil 156, fols. 64r–64v; Rico, 1863–67, II, 202–203); *Lámina Universal* chapters 53–55 (Ms. Villa Amil 156, fols. 105r–105v; Rico, 1863–67, III, 116–117); *Açafeha* chapter 39 (Ms. Villa Amil 156, fol. 123v; Rico, 1863–67, III, 192). This latter example (*Açafeha*) is less significant than the others, because Ibn al-Zarqālluh's original Arabic text also deals with the problem in a general way and does not mention the *qibla*: cf. Puig, 1987, pp. 72–73. See also Viladrich, 1987, p. 113: the author presents evidence of other instances of “censorship” in the *Astrolabio redondo* in matters related to *mīqāt* (astronomy specialised in Islamic worship).

68 *Astrolabio llano* chapter 56 (Ms. Villa Amil 156, fols. 82r–82v) = 57 (Rico, 1863–67, II, 288–289).

The *qibla* is not the only instance in which the Alfonsine translation gives an explanation of an Arabic technical term. Another interesting case is chapter 52/56, in which Qusṭā explains that in places with latitudes less than 24° , the Sun transits the local zenith twice during the year. This happens when the Sun is in a given degree of longitude [such that its declination equals the local latitude] and “in its *naẓīr* in declination” (*fī naẓīri-hi fī l-mayl*). This Arabic expression may easily create a misunderstanding, for the term *naẓīr* of the degree (of longitude) of the Sun usually means a point on the ecliptic placed at a distance of 180° . The Castilian translation interpolates a clear explanation:

“la una quando fuer en algun grado sennalado, et la otra quando fuer en el grado que es su opposito, *et quier dezir, el que es arredrado de la cabeça de Cancro tanto quanto ell otro*”⁶⁹ (“the first time when [the Sun] is in a determined degree, and the second when it is in the degree which is its opposite, *which means, the degree whose distance from Cancer equals the distance* [also from Cancer] *of the first degree*”).

Two degrees equidistant from a solstitial point must have the same declination. Clarification of an Arabic passage which seems excessively sparse may be useful for readers who are not always expert astronomers, and makes sense in a treatise on the celestial globe which, as we have seen, may be considered an introduction to spherical astronomy. Thus, chapter 51/55 deals with the northern or southern direction of the shadow in places with latitudes greater or smaller than 24° . The Alfonsine translation clarifies the question by stating that Qusṭā refers to the noon shadow:

“Et esto que dezimos, siempre es en mediodia. Ca las sombras que se fazen escuantra oriente et escuantra occidente, que se fazen por el mouimiento del sol cada dia, non hy fablamos, ca manifesta cosa es por si”⁷⁰ (“We mean at noon, for it is obvious in itself that the shadows move towards the East or towards the West as a result of the daily motion of the Sun”).

In another instance, the Castilian translation omits a passage instead of giving an explanation of a technical term – in chapter 41/45, which deals with the determination of the setting time of the Moon or of one of the planets. For this purpose, the longitude and latitude of the Moon or planet has to be established

69 Ms. Villa Amil 156, fol. 39r; Rico, 1863–67, I, 199.

70 Ms. Villa Amil 156, fols. 38v–39r; Rico, 1863–67, I, 198.

and Qusṭā b. Lūqā recommends the use of a *taqwīm* or a *zīj*, while the Alfonsine text just skips the problem and refers to the previous chapter (40/44, on the determination of the rising time of the Moon or a planet) which merely states that one should establish the ecliptical coordinates of the celestial body involved. The omission of the *taqwīm* (ephemerides calculated for one year) is reasonable, for this kind of astronomical document was not very common in the Western Islamic tradition, which cultivated perpetual almanacs instead; but one may wonder why the Alfonsine translators did not recommend the use of a set of astronomical tables (= *zīj*). Finally, in the translation of an Arabic text which (like other similar ones)⁷¹ is characterised by a systematic repetition of operations which are common to many chapters,⁷² it is interesting that, in a few cases such as the aforementioned chapter 40/44, the translators avoid repetition by referring to the other chapter that deals with a similar problem. A typical example can be found in chapters 13/17 and 14/18 which deal with the determination of the time of the day, expressed in equal (13/17) or unequal (14/18) hours, if one knows the ascendant: in both cases the process is the same and involves obtaining the hour angle. Qusṭā b. Lūqā repeats the whole set of operations in both chapters, while the Alfonsine text refers in 14/18 to the earlier explanation in 13/17.

4.1.1.2.2.6 Conclusions

The Alfonsine book on the *Alcora* follows the basic structure of many other texts extant in the *Libro del saber de astrología*. The king wanted a collection of treatises dealing with the construction and use of astronomical instruments and, for that purpose, he ordered his collaborators to translate the best available Arabic literature on the topic. Books on the use of the instruments were easy to find and this is why most of the treatises of this kind are actual translations of Arabic originals, like that of Qusṭā b. Lūqā. Texts on the construction of instruments were not so common and Alfonso x usually ordered his collaborator Ishāq ben Sīd (Rabiḡag) to write an original work on a specific instrument. The king was also extremely interested in astrology and, especially, in *tasyīr* techniques. This interest accounts for the compilation of the *Libro del Ataḡir* (which describes a specific astrolabe plate designed for the calculation of the *tasyīr*, projection of rays and division of the houses according to the equatorial method) and the addition of an extra chapter to the *Alcora*, by the

⁷¹ Kennedy, 1989, p. 52.

⁷² Such as, for example, moving the sphere with the meridian ring so that the North Pole forms with the horizon an angle equal to the local latitude.

Don Mosheh mentioned above, which deals with two of the three questions studied in the *Libro del Ataçir*.

The three parts into which the *Alcora* book has been divided are, thus, clear and here I will only summarise a couple of points which have already been explained. First of all, the book seems to be the result of the compilation of three independent works and no attempt has been made to harmonise the whole or to avoid repetitions. We will recall the presence of an *armilla menor* (a great circle of longitude) in the construction treatise, probably following the model of the *Almagest*; this is not mentioned at all in the treatise on the use of the instrument, in which the translators do not hesitate to modify Qusṭā b. Lūqā's original in problems related to the determination of the ecliptical coordinates of a celestial body by using the auxiliary quadrant, but not the *armilla menor*. In addition, the auxiliary quadrant, which appears in the treatise on the use of the globe, is described again in Don Mosheh's additional chapter.

Furthermore: even if Qusṭā's book on the use of the celestial globe is the main source used by the Alfonsine authors, and even if the translation is usually faithful to the Arabic original, the translators and authors of this collective work do not consider fidelity to the original as their main purpose: it is very clear that they seek to improve on their sources and to offer the king the best possible instrument they are able to describe. There is, no doubt, a certain spirit of originality in Alfonsine scientific works (for which there are clear precedents in twelfth-century Latin translations) which will culminate in the *Alfonsine Tables*.

4.1.2 *Spherical Astrolabe*

The spherical astrolabe is not a very common instrument and it has been debated whether it derives from the celestial globe or from the plane astrolabe.⁷³ Whatever the case, it is obvious that its two parts (the sphere and the rete) have their corresponding counterparts in the plate and rete of the plane astrolabe. The sphere contains the horizon, circles of altitude and vertical circles, as well as the hourlines, while the rete contains the ecliptic and the projection of fixed stars. The rotation of the rete simulates the daily motion of the celestial

73 King, 2018a, states that while the plane astrolabe was used by Muslim scientists in the eighth century, the spherical astrolabe appeared in the ninth century. A treatise by Ḥabash al-Ḥāsib (ninth century) has been preserved in ms. Istanbul Topkapı AIII 3475,3 (fols. 79^a–89^a). In the late ninth century we also have two other treatises on the spherical astrolabe written by Qusṭā b. Lūqā and al-Nayrizī. In the tenth century Hāmid b. 'Alī al-Wāsiṭī (fl. 950) wrote another text on the instrument (ms. Istanbul Topkapı Ahmet III 3509,2, fols. 261^a–281^a). A general survey of the information about this instrument in Islamic lands in Sezgin & Neubauer, 2011, pp. 120–133.

sphere around the observer. The spherical astrolabe is, in principle, universal, as it can be adapted to any latitude, although the seasonal hourlines are, obviously, latitude-dependent; the same could be said about the circles of position, which appear in the Alfonsine treatise (see below), for the division of the houses and the computation of the *tasyīr* using the equatorial method.

We know of the existence of only two instruments of this kind.⁷⁴ One of them, clearly Eastern and complete, is preserved in the Oxford Museum of the History of Science (Fig. 4.3): it is signed by Mūsā and dated in 885/1480. David King has suggested that this Mūsā could be identified with the Jewish scholar Moses Judah Galeano, also known as Mūsā Jalīnūs, who spent most of his life in the lands of the Ottoman Empire and worked at the court of the Sultan Bayazid II (1481–1512) (see 7.7.3.4.2.1.1). The instrument contains the two standard parts. The first is the sphere, divided into two hemispheres by the horizon. In its lower half we have the hour lines corresponding to seasonal hours for a latitude of 41° (Istanbul), while in the upper half we can see a network of almucantars or altitude circles, traced for every 2°, as well as vertical circles. One of these vertical circles has one of its quadrants with 45 holes. These are used to adapt the instrument to the local latitude: a peg will hold the point of the rete corresponding to the pole of the equator to the corresponding hole, so that the pole forms with the horizon an angle equivalent to the latitude. The second is the hemispherical rete superimposed on the sphere. It bears indexes for 19 stars and also has the ecliptic, a declination parallel used to measure equatorial coordinates and a vertical graduated quadrant which measures celestial latitudes, as well as polar and zenith distances. In this quadrant we also find a movable gnomon.⁷⁵

The second extant Islamic spherical astrolabe is part of a private collection belonging to the Cannobio family, and is incomplete, since the hemispherical rete is missing.⁷⁶ In King's opinion it can be dated to the fourteenth or early fifteenth century. The markings for the seasonal hours have an inscription which states "Hours for the latitude of Tunis and any locality whose latitude is 36;40°".⁷⁷ It is not clear whether the instrument had a latitude scale

74 A third spherical astrolabe, closely related to Mūsā's instrument (see below) has recently attracted scholarly attention. It is signed by a certain *Za'im*; King has shown that it is a twentieth-century fake which used Mūsā's astrolabe as a model.

75 F.R. Maddison, "A fifteenth century Islamic spherical astrolabe". *Physis* 4 (1962), 101–109; J. Vernet and J. Samsó (eds.), *El legado científico andalusí*. Madrid, 1992, p. 220.

76 Ernesto Cannobio, "An important fragment of a West Islamic spherical astrolabe". *Annali dell'Istituto e Museo di Storia della Scienza di Firenze* 1 (1976), 37–41.

77 The only source that gives this value for the latitude of Tunis is al-Khwārizmī's *Kitāb ṣūrat al-arḍ*. See Kennedy & Kennedy, 1987, p. 363.



FIGURE 4.3
Mūsā's spherical astrolabe.
Inv. 49687
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allowing the user to adapt the spherical astrolabe for other latitudes. It also contains latitude-dependent curves for determining the times of the prayers of afternoon twilight (*shafaq*) and dawn (*fajr*) as well as midday (*ẓuhr*) and mid-afternoon (*ʿaṣr*). It has an obvious interest for us because it is Western Islamic, and is proof that this kind of instrument was known in al-Andalus and the Maghrib. We have already seen that Ibn al-Zarqālluh mentions the spherical astrolabe in the introduction to his treatise on the saphea. To this we can add the Alfonsine treatise on this instrument, which is our most important source, and the description found in Abū ʿAlī al-Ḥasan al-Marrākushī's *Jāmiʿ al-mabādī wa l-ghāyāt*.⁷⁸

Another spherical instrument, incorrectly identified with a spherical astrolabe (though apparently related to it) is described in a seventeenth-century book entitled *al-Nāqīʿa ʿalā l-ālat al-jāmiʿa* by Muḥammad b. Muḥammad b. Sulaymān al-Rūdānī (1627–1683),⁷⁹ and edited and translated by Charles

⁷⁸ Marrākushī, 1984, II, pp. 8–14. French translation in Sédillot, 1844, pp. 142–148.

⁷⁹ Al-Rūdānī was born in Taroudant and studied in Tamegrout, Tādla, Fez, Marrakesh and Cairo. He travelled to the East and visited Istanbul, Medina and Mecca before finally settling in Damascus. See Lamrabet, 2013, M560 (pp. 270–271).

Pellat.⁸⁰ The instrument is named *al-jāmi'a* by its author, who states that he finished the treatise on 18 Šafar 1072/13 October 1661, in the courtyard of the Prophet's Mosque in Medina (*bi-finā' al-masjid al-nabawī min al-Madīna al-Munawwara*).⁸¹ One year later, in 1073/1662–63, he built the instrument itself in the courthouse (*dār al-qudā'*) of the same city.⁸² I will deal with *al-jāmi'a*, comparing it to the Alfonsine treatise, although the former, as well as Abū 'Alī al-Marrākushī's *Jāmi'*, seems to be strongly influenced by the Eastern astronomical tradition. Another late unpublished Maghribī treatise on the spherical astrolabe (*al-Kawkab al-durrī fī ma'rīfat al-ašturlāb al-kurawī*) was written by Ibrāhīm b. 'Alī al-Andalusī al-Marrākushī (fl. 1140/1727).⁸³

The Alfonsine treatise on the construction of the spherical astrolabe (*Libros del astrolabio redondo*)⁸⁴ is not a translation but an original work written by R. Ishāq b. Sīd. As acknowledged in the prologue, King Alfonso could not find a source describing how the instrument should be made ("Et porque non fallamos libro en que fable de como se deue fazer de nueuo") and he asked Rabiçag to write the book on its construction.⁸⁵ There is no written evidence concerning the second part of the book, on the use of the instrument, but it was probably authored by Rabiçag himself or by another of the king's scientific collaborators. This part is extremely long, containing 135 chapters, of which 110 are literal translations, summaries or adaptations of the treatise on the use of the plane astrolabe written by Abū l-Qāsim ibn al-Samḥ (d. 1035), one of Maslama al-Majrīṭī's disciples.⁸⁶ The Alfonsine astronomers were also aware of another treatise on the use of the astrolabe authored by another disciple of Maslama: Ibn al-Šaffār (d. 1035), who is criticised in chapter 135.⁸⁷ Besides, the Alfonsine collaborators made some attempts to update Ibn al-Samḥ's work: a good example of this can be found in chapter 116 where they mention a *revolutio anni* of 92;24°, ⁸⁸ borrowed from Ibn al-Zarqālluh,⁸⁹ while in

80 Pellat, 1973–1975; Baklī, 'Aysānī& Ilhām, 2014, pp. 19–21.

81 Pellat, 1973–75, p. 155 (French translation) and 19/64 (Arabic edition).

82 This instrument was recently in the news, as it was auctioned by Christie's on 8 October 2015.

83 Lamrabet, 2013, M370, p. 241.

84 Ms. Villa Amil 156, fols. 43r–68v; Rico, 1863–67, II, 113–222.

85 Ms. Villa Amil 156, fol. 43r; Rico, 1863–67 II, 113. On the Alfonsine treatise on the construction of the instrument see Seeman, 1925; see also Savage-Smith, 1985, pp. 82–83.

86 Viladrich, 1987 and 1986.

87 Ms. Villa Amil 156, fol. 68v; Rico, 1863–67, II, 221–222.

88 Ms. Villa Amil 156, fol. 66v; Rico, 1863–67, II, 214.

89 Samsó, 2011, p. 213.

the corresponding chapter Ibn al-Samḥ uses $93;2^{\circ}$,⁹⁰ rounded from the $93;2,15^{\circ}$ which is al-Khwārizmī's value.⁹¹

The instrument described has the standard two parts, the sphere and the rete.⁹² In the sphere we find three great circles: horizon (divided into 360°), meridian and first vertical circle which passes by the east and west points of the horizon.⁹³ The quadrant of the first vertical circle comprised between the zenith and the eastern point of the horizon is divided into 90° which will be used to trace the almucantars.⁹⁴ A number of holes are made on the meridian circle in its intersections with the almucantars: in the drawing in which these holes appear, there is one for each 10° between 0° and 80° .⁹⁵ These holes allow the rete to be fixed to the sphere by means of a "nail" and a "horse", as in the plane astrolabe. The aim is that the northern pole of the equator should coincide with the hole corresponding to the local latitude.

We also have:

1. Almucantars (called *empontizos* in the text),⁹⁶ circles of equal altitude around the local zenith which, depending on the size of the instrument, will be traced for each degree of altitude, or every 2° , 3° , 4° or 5° . In the drawings of the treatise the almucantars are traced for every 10° .
2. The vertical circles (*zontes*).⁹⁷
3. Another circle, parallel to the equator, mentioned in the text as the "çerco del yguador del día".⁹⁸ This is used to measure arcs related to the equator such as right ascensions, day and night arcs of heavenly bodies or hour angles, using an instrument which, unlike the plane astrolabe, does not have an index (*muri*).⁹⁹
4. Four semicircles join (according to the description of the instrument at the beginning of the book on the use of the instrument), the North and

90 Viladrich, 1986, p. 47.

91 Samsó, 2011, p. 154.

92 Ms. Villa Amil 156, fols 57v–58r; Rico, 1863–67, II, 171–173.

93 Ms. Villa Amil 156, fols. 46r–46v; Rico. 1863–67, II, 129.

94 Ms. Villa Amil 156, fol. 46v; Rico. 1863–67, II, 130–131.

95 Ms. Villa Amil 156 fol. 55r; Rico, 1863–67, II, 151–152.

96 The Arabic word *almuqaṭṭar* is considered to be the result of the article *al* + a participle of a non existent verb *qaṭṭara*. As the noun *qaṭṭara* means "bridge" (*pont* in thirteenth-century Castilian, modern *punte*), the Alfonsine collaborators created the adjective *empontizo* in an attempt to coin a new technical term derived from the Arabic.

97 Ms. Villa Amil 156 fols. 47r–47v; Rico, 1863–67, II, 132. In Arabic the vertical circles are called *dawā'ir al-sumūt* and the Castilian word *zonte* derives from the Arabic *samt*, pl. *sumūt*.

98 Ms. Villa Amil 156 fol. 52v; Rico, 1863–67, II, 145–146.

99 Viladrich, 1986, p. 108.

the South poles (“comiençan del polo septentrional et uan fastal polo miridional”)¹⁰⁰ and are used to divide the houses of the horoscope and to calculate the *tasyīr*. I believe that, in the text, the word “poles” is misused and that these semicircles join the north and south points of the horizon (not the poles). The figure (Fig. 4.4) which appears in the first book on the construction contains an important detail: these semicircles are position circles which pass through 30° divisions of the equator.¹⁰¹ The text is obviously using them to divide the houses and calculate the *tasyīr* using the equatorial method (3.2.3.1.2).¹⁰²

5. Twelve arcs of a circle, located in the lower hemisphere, correspond to the seasonal hour lines. These arcs cross the equatorial parallels of Cancer, Aries-Libra and Capricorn. This diagram for the seasonal hours is obviously dependent on the local latitude and does not have a universal value.¹⁰³

In al-Rūdānī’s treatise¹⁰⁴ we find, on the sphere, the meridian, first vertical, vertical circles, almucantars and horary arcs as in the Alfonsine text. There are no position circles: al-Rūdānī deals with the problem of dividing the houses of the horoscope in chapter 35 of this treatise but he describes the hour lines method, the dual longitude method (3.2.3.1.3) and the standard method (3.2.3.1.1.1).¹⁰⁵ As in the Alfonsine treatise, in the lower half of the sphere, we have the arcs of the seasonal hours, and the arc of altitudes (*qaws al-irtifāʾ*). Al-Rūdānī’s sphere also has the shadow scale which, in the Alfonsine text, appears on the rete. To these, al-Rūdānī adds a scale of sines and cosines (*qaws al-jayb*), the hour arcs corresponding to the prayers of dawn (*fajr*) and twilight (*shafaq*),¹⁰⁶ and the parallels corresponding to the latitudes of the climates.

100 Ms. Villa Amil 156 fols. 57v–58r; Rico. 1863–67, II, 172.

101 Ms. Villa Amil 156 fol. 48v; Rico. 1863–67, II, 135.

102 Viladrich, 1986, pp. 114–115.

103 Ms. Villa Amil 156 fols. 47v–48r; Rico. 1863–67, II, 133–134.

104 Pellat, 1973–75, pp. 85–90 (French translation) and 4–6/77–79 (Arabic edition).

105 Pellat, 1973–75, pp. 136–137 (French translation) and 45–46/37–38 (Arabic edition). Al-Rūdānī deals also with the projection of rays in p. 138 (French tr.) and 47/36 (Arabic).

106 In chapter 16 (Pellat, 1973–75, pp. 113–114 of the Fr. tr., 57/26 of the Ar. text) al-Rūdānī assumes, in principle, a value of -19° for the *fajr* and -17° for the *shafaq*, but adds a note in which he mentions other values: -18° for both periods, -19° (*fajr*) and -16° (*shafaq*), or -20° (*fajr*) and -18° (*shafaq*). He states that the well-known Cairo *muwaqqit* Jamāl al-Dīn al-Māridīnī (d. 1406–7) did not accept the last couple of values and adds that the duration of twilight depends on atmospheric conditions. See also 2.4.1.2.4.

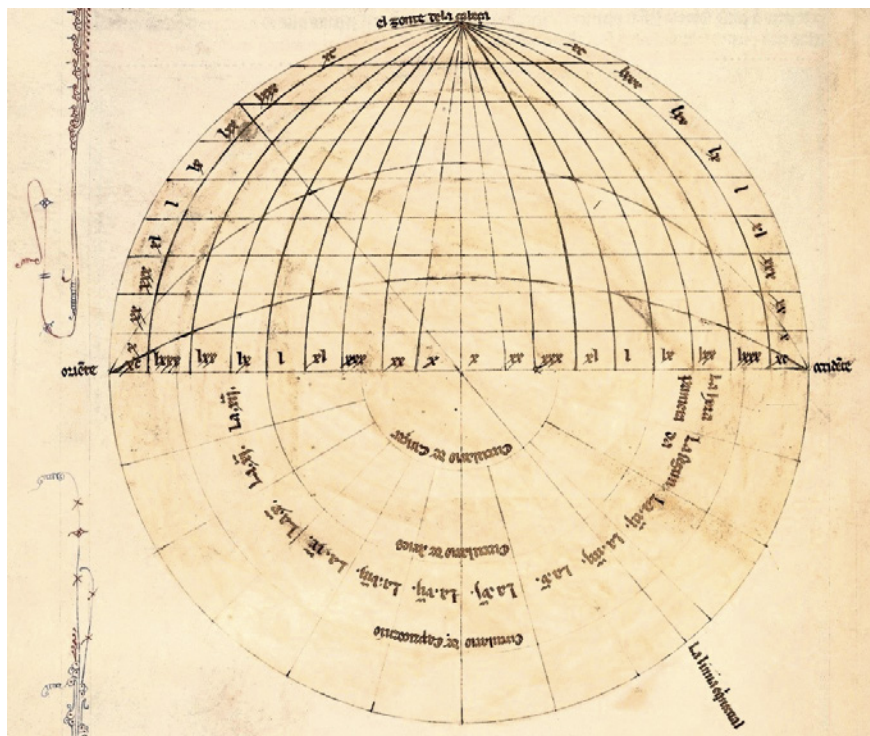


FIGURE 4.4 Vertical circles, almucantars, hour diagram and position circles in the Alfonsine treatise on the spherical astrolabe
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BH MSS 156, FOL. 48V

The Alfonsine hemispherical rete contains:

The eccentric “equator” (a declination circle) divided into 360° .¹⁰⁷ On this circle we find the peg (*piértega*) which fastens the rete to the sphere.¹⁰⁸

The ecliptic, divided into signs and degrees.¹⁰⁹ Its pole is the pole of the rete.¹¹⁰

Another circle, parallel to the ecliptic, is named *el centro saliente* (the eccentric, *khārīj al-markaz*) which is the *cerco de la yguacion del sol* (circle of the solar equation). This circle is divided into 365 parts, the number of days of the solar

107 Ms. Villa Amil 156 fol. 52v; Rico, 1863–67, II, 145–146.

108 Ms. Villa Amil 156 fol. 58r; Rico, 1863–67, II, 172.

109 Ms. Villa Amil 156 fol. 49v; Rico, 1863–67, II, 137–138.

110 Ms. Villa Amil 156 fol. 54r; Rico, 1863–67, II, 149.

year. The names of the months are also written in it.¹¹¹ The illustration on fol. 50v of ms. Villa Amil 156 shows that the solar apogee is placed in Cancer 0°. ¹¹² Consequently, the ecliptic and the eccentre form a zodiacal scale.

The construction treatise gives the solar positions for the beginning of each month of the solar year.¹¹³ Similar positions, with very few differences, are to be found in the book of the plane astrolabe (4.2.4.2.3.2),¹¹⁴ where they are given for each day of the solar year, the *Quadrante para rectificar*,¹¹⁵ and the *Relogio dell'agua*.¹¹⁶ All of them seem to have been calculated using the *Alfonsine Tables* or al-Battānī's *zīj* for dates ca. 1277:

TABLE 4.1 Solar longitude at the beginning of each month

Date	Astrolabio redondo
1 March 1250	Psc 19°
1 April	Ar 19°
1 May	Tau 18°
1 June	Gem 18°
1 July	Cnc 17°
1 August	Leo 16°
1 September	Vrg 16°
1 October	Lbr 16°
1 November	Scr 17°
1 December	Sag 18°
1 January 1251	Cpr 19°
1 February	Aqr 21°

A quadrant for measuring altitudes (graduated from 0° to 90°) is placed in the space left between the ecliptic and the *centro saliente*. This quadrant goes between Libra 0° and Cancer 0°.¹¹⁷

A shadow scale is placed in another quadrant, also in the space left between the ecliptic and the *centro saliente*. It gives the values of the tangent of the

111

Ms. Villa Amil 156 fol. 50r; Rico, 1863–67, II, 138–139.

112

Ms. Villa Amil 156 fol. 50v; Rico, 1863–67, II, 141.

113

Ms. Villa Amil 156 fol. 50r–50v; Rico, 1863–67, II, 140.

114

Chabás, 2003.

115

Ms. Villa Amil 156 fol. 172v; Rico, 1863–67, III, 298–299.

116

Chapter 19 of the *Relogio dell'agua* is missing in ms. Villa Amil 156. It is preserved in ms. Escorial h-1-1 fol. 293v; Rico, 1863–67, IV, 50–51.

117

Ms. Villa Amil 156 fol. 51r; Rico, 1863–67, II, 141–142.

altitude for a gnomon of 12 digits. A table of arc tangents in which the argument is given in intervals of 0;15^d appears in the construction treatise.¹¹⁸

The indexes on the rete mark the position of a number of northern stars.¹¹⁹ The construction treatise contains a table of 14 northern stars “observed” (*rectificadas*) in Toledo, as a result of a royal order.¹²⁰ The longitudes obtained are tropical:

TABLE 4.2 Stars ecliptical coordinates

Castilian names	Arabic name and identification	Longitude	Latitude	Longitude in the Alfonsine Tables
La cabeça de la serpiente	----- (γ Draconis)	256;18°	25;30 ^{oa}	256;48°
El ramec	al-Simāk al-rāmiḥ (α Bootis)	193;38°	31;30°	194;8°
Alfaca	al-Munīr min al-Fakka (α Coronae Borealis)	211;18°	44;30°	211;48°
Buytre cayente	al-Nasr al-wāqī' (α Lyrae)	273;58°	62;0°	274;28°
Arridf	al-Ridf (α Cygni)	325;48°	60;0°	326;18°
Cabeça dalgol	Ra's al-ghūl (β Persei)	46;18°	23;0°	46;48°
Alaioc	al-'Ayyūq (α Aurigae)	71;38°	22;30°	72;8°
Buytre uolant	al-Nasr al-ṭā'ir (α Aquilae)	290;28°	29;10°	290;58°
Cabeça dela mugier	Ra's al-musalsala (α Andromedae = δ Pegasi)	4;18°	26;0°	354;58 ^{ob}

- a There is a mistake in the latitude of this star which should be 75;30° instead of 25;30°. The copyist omitted an L and wrote xxv xxx instead of Lxxv xxx.
- b This longitude found in the *Alfonsine Tables* is clearly wrong as confirmed by the value we find in al-Šūfī's *Uranography* (0;32°): see Schjellerup, 1874, p. 115. The correct value is Aries 4;58° mentioned in the *Alfonsine IIII Libros de la Ochaua Espera* (Rico, 1863–67, 1, 52). The longitude difference between 4;58° and 4;18° is 0;40° which makes me think that 4;18° of the *Astrolabio redondo* should be emended to 4;28°.

118 Ms. Villa Amil 156 fols. 51r–52r; Rico, 1863–67, 11, 143–145.

119 Rico, 1863–67, 11, 147.

120 This table has been edited in Rico, 1863–67, 11, 147 and it appears in ms. Villa Amil 156 fol. 53v. This leads me to reject Hilty's opinion (Hilty, 1955, 43–44) who doubts about the authenticity of the table. Hilty's opinion is probably due to the fact that the same table is reproduced, again, by Rico, 1863–67, 1, 144, within the *IIII Libros de la Ochaua Espera* but, this time, it has no correspondence with the Villa Amil ms.

TABLE 4.2 Stars ecliptical coordinates (*cont.*)

Castilian names	Arabic name and identification	Longitude	Latitude	Longitude in the Alfonsine Tables
Ala del caualllo	Janāḥ al-faras (γ Pegasi)	358;48°	12;30°	359;18°
Aldebaran	al-Dabarān (α Tauri)	59;18°	5;10°	59;48°
La cabeça dela entrada de Gemini	Muqaddam al-dhirā'ayn or Ra's al-Tawwām (α Geminorum)	99;58°	9;40°	100;28
Coraçon del leon	Qalb al-asad (α Leonis)	139;8°	0;10°	139;38°
Açarfa	al-Şarfa (β Leonis)	161;8°	11;50°	161;38°

The difference between the longitudes in the *Alfonsine Tables*¹²¹ – which are the same as those of the *IIII Libros de la Ochaua Espera* – and those of the *Astrolabio redondo* is 0;30°, with the sole exception of the star *Cabeça de la mugier*, for which the values found in both sources are clearly wrong. This implies an increase in the star longitudes of 16;38° since the epoch of Ptolemy’s star catalogue (20 July 137). If we assume al-Battānī’s rate of constant precession (1° per 66 years):

$$16;38^{\circ} \times 66 = 1097;48 \text{ years}$$

which implies that this star table probably derives from a previous source composed about 1235 CE.

An alidade with two sights, which pivots on the pole of the ecliptic, can be used to measure altitudes on the altitude quadrant, concentric with the ecliptic itself.¹²² As explained in the treatise on construction¹²³ and in the figure below (Fig. 4.5), the alidade is composed of three different pieces: one of them is semicircular, with the same radius as the radius of the sphere, and should be placed on the surface of the sphere. The other two pieces, provided with sights, are erected vertically on both sides of the semicircular piece.¹²⁴

121 I am using the *princeps* edition printed by Erhardus Ratdolt in 1483 in Venice. On this star table see Kunitzsch, 1986.
122 Ms. Villa Amil 156 fol. 58r; Rico, 1863–67, II, 173.
123 Ms. Villa Amil 156 fol. 49r; Rico, 1863–67, II, 136–137.
124 This solution seems to have a certain similarity to the one adopted by Abū ‘Alī al-Marrākushī (see Marrākushī, 1984, p. 14 and Sédillot, 1844, p. 148): he describes a

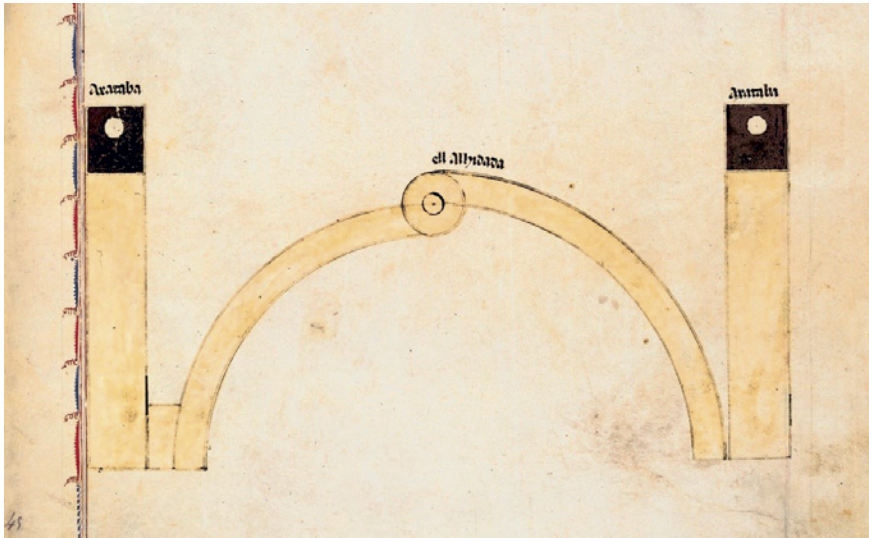


FIGURE 4.5 Alidade and sights in the Alfonsine spherical astrolabe
BIBLIOTECA HISTÓRICA DE LA UNIVERSIDAD COMPLUTENSE DE MADRID,
BH MSS 156, FOL. 49R

A ring and a suspension system (*colgadero*) used for hanging the instrument. See the entire Alfonsine instrument in fig. 4.6.

In al-Rūdānī's spherical astrolabe we also find ecliptic and equator, as well as the corresponding indexes of stars. To this, the latter author adds the equinoctial and solstitial colures, and the alidade is replaced by two sights fixed to the colure of the equinoxes. There is no zodiacal scale and the text explains two approximate procedures to calculate the solar longitude.¹²⁵ As we have already seen, the shadow scale has been moved to the sphere of the instrument. Finally I should stress that al-Rūdānī is particularly interested in the uses of his spherical astrolabe to solve the problems of religious timekeeping, a topic which, obviously, is almost totally absent in the Alfonsine treatise: we have already seen that the instrument has arcs used to calculate the beginning of dawn and the end of twilight; besides this, our author deals with the

triangular "plate" (*ṣaḥīḥa*), in which the line traced from the vertex to its base is a quadrant of a great circle of the sphere. The base has a hole of the same size as the hole of the pole of the equator. When the base is fixed to the pole by an axis (*miḥwār*), the vertex, when rotated, will move along the equator. Then, a gnomon in the direction of a radius of the sphere will be used to determine the altitude of the pole or of a star if we consider the ecliptic as the altitude circle.

¹²⁵ Pellat, "Astrolabe sphérique" p. 99 (French translation) and 70/13 (Arabic edition). Abū 'Alī al-Marrākushī's *jāmi'* does not contain any reference to a zodiacal scale.



FIGURE 4.6
The spherical astrolabe in
Biblioteca Histórica de la
Universidad Complutense de
Madrid, BH MSS 156, fol. 55v

problem of determining the time for the *ẓuhr* and the *ʿaṣr* prayers according to the Mālikī and the Ḥanafī schools,¹²⁶ the latter being the consequence of his long period of residence in cities of the Mashriq. To this he adds an extensive treatment of the problem of determining the *qibla*,¹²⁷ and deals with the problem of crescent visibility.¹²⁸

4.1.3 *The Armillary Sphere*

4.1.3.1 Introduction

The armillary sphere¹²⁹ was known in al-Andalus in the first half of the ninth century, when the poet and polymath ʿAbbās ibn Firnās (d. 887)¹³⁰ built an instrument of this kind for the emir ʿAbd al-Raḥmān II (r. 821–852). One century later, in 344/955–6, the Tunisian Jew Dūnāsh ibn Tamīm (900–960)¹³¹ wrote the first extant Western-Islamic treatise on the armillary sphere, the purpose of which was to accompany the instrument he had built for the *shaykh* Abū

¹²⁶ Pellat, 1973–75, pp. 112–113 (French translation) and 57–58/25–26 (Arabic edition).

¹²⁷ Pellat, 1973–75, pp. 117–123 (French translation) and 49–54/29–34 (Arabic edition).

¹²⁸ Pellat, 1973–75, pp. 139–140 (French translation) and 34–35/48–49 (Arabic edition).

¹²⁹ On the history of this instrument see Comes, 2012.

¹³⁰ Terés, 1960.

¹³¹ Lamrabet, 2013, M18, p. 128; King, 2007.

l-Ḥasan Muḥammad b. al-Ḥusayn who was probably the governor of Mahdiyya,¹³² a city mentioned, as we will see, in the examples given in the treatise. In the eleventh century, the Andalusian astronomer Ibn al-Zarqālluh (d. 1100) wrote a treatise on the construction of the armillary sphere which has been preserved in an Alfonsine Castilian translation together with an Alfonsine treatise on the use of the instrument.¹³³ After this text, I only know of a single reference to the armillary sphere in the Iberian Peninsula and the Maghrib: the Maghribī astronomer Ibn ‘Azzūz al-Qusanṭīnī (d. 1354) used an instrument of this kind in the observations he made in Fez in 745/1344–5, the results of which were used in his *al-Zīj al-Muwāfiq*.¹³⁴

In the following pages, I will concentrate on the treatises by Dūnāsh ibn Tamīm, Ibn al-Zarqālluh and the Alfonsine book on the use of the instrument, which are the only sources that provide accurate information on the armillary sphere in our geographical area.

4.1.3.2 Dūnash ibn Tamīm and the Armillary Sphere

4.1.3.2.1 *The Author*

Dūnash (or Dunas) b. Tamīm b. Ya‘qūb al-Isrā‘īlī al-Qarawī was a Tunisian scholar, born in Qayrawān and disciple of the well-known physician and philosopher Ishāq ibn Sulaymān al-Isrā‘īlī (ca. 858–955). He worked as a physician of the Fatimid caliphs and wrote a certain number of books dealing with astronomy, as well as an Arabic commentary on the *Sefer Yezirah*, an ancient Hebrew esoteric book on cosmogony, extant in Judeo-Arabic documents and in several Hebrew versions.¹³⁵ This book was written in 955–6 and it contains many passages on astronomical topics. In it, Dūnash mentions that he dedicated a book on astronomy to Ḥasdāy b. Shaprūt (ca. 905–975), the physician of the Umayyad Caliph ‘Abd al-Raḥmān III (912–961), composed of three parts dealing with *hay’a*, mathematical astronomy and astrology respectively. The aforementioned commentary also refers to another astronomical work, on the “structure of the spheres” (*hay’at al-aflāk?*) which he wrote for the Fatimid Caliph al-Manṣūr Ismā‘īl b. al-Qā‘im (946–953). Dūnash’s interest in *hay’a*¹³⁶ has been confirmed by Taro Mimura¹³⁷ who has argued, very convincingly, that the Latin translation of *De orbe*, attributed to Māshā’allāh, is actually a work

132 Stern, 1956; Mimura, 2015 pp. 27–29; Mimura, 2015 a.

133 Comes, 2013 a; Samsó, 2011, pp. 175–180.

134 Samsó, 1997.

135 Georges Vajda, *Le commentaire sur le Livre de la Création de Dūnāsh ben Tamīm de Kairouan (X^e siècle)*. Ed. Paul B. Fenton. Leuven, 2002.

136 This interest was shared by his Cordoban contemporary Qāsim ibn Muṭarrif al-Qaṭṭān: see Casulleras, 1998.

137 Mimura, 2015.

on *hay'a* by Dūnash ibn Tamīm.¹³⁸ In this paper Mimura compares this Latin translation to the original text extant in two Arabic manuscripts, one of them the fourteenth-century manuscript Pennsylvania University Library LJS 439, written by an Andalusian copyist.¹³⁹ This, together with the aforementioned reference to the contacts between Dūnash and Ḥasdāy b. Shaprūt, seems to be a confirmation of the arrival of Dūnash's works in al-Andalus and of the influence of Qayrawān in the early development of Andalusian astronomy in the tenth century.

4.1.3.2.2 *The Treatise on the Armillary Sphere*

4.1.3.2.2.1 The Contents of the Book

I will deal here with Dūnash's *Risālat al-ʿamal bi-dhāt al-ḥalaq*, dedicated to the *shaykh* Abū l-Ḥasan Muḥammad b. al-Ḥusayn (considered by Stern¹⁴⁰ to be the governor of Maḥdiyya) as a result of Dūnash's visit to his residence, during which they discussed astronomical instruments like the astrolabe (which Dūnash often called *dhāt al-ṣafā'iḥ*). During this conversation, Dūnash showed his knowledge of Eastern astronomical instruments such as the *munbataḥ* (probably *mubattakh*, "melon-shaped")¹⁴¹ and *ḥūtī* ("fish-shaped") astrolabes,¹⁴² to which he added the *bayḍa* (which might be a simple celestial globe or the more sophisticated instrument described by al-Battānī, see 4.1.1.1), and the armillary sphere. Our author considers that the armillary sphere has clear advantages over the astrolabe because it rotates around two poles (those of the equator and of the ecliptic) while the astrolabe only rotates around the pole of the equator. The armillary sphere is, therefore, an exact counterpart of the celestial sphere.

The governor then asked Dūnash to build for him a *bayḍa* or an armillary sphere. Finally the latter instrument was chosen, because Dūnash mentioned that he had heard Abū 'Alī Aḥmad b. 'Uthmān al-Aṣṭurlābī, probably an astrolabe maker, praising the multiple uses of the armillary sphere. The instrument was built and the governor helped by placing at his disposal a number of artisans who took care of the preparation of the various rings and Dūnash wrote

138 I wonder whether the *De orbe* could be identified with the book on *hay'a* dedicated to al-Manṣūr.

139 This manuscript can be downloaded from <http://hdl.library.upenn.edu/1017/d/medren/4830256>. The second manuscript is Berlin Stabi ms. or. oct. 273 also accessible in <https://ismi.mpiwg-berlin.mpg.de/om4-ismi/public/publicCodex.jsp?eid=181096>

140 Stern, 1956, p. 375.

141 Kennedy, Kunitzsch & Lorch, 1999.

142 This seems to be the astrolabe called *samakī* described by al-Sijzī (second half of the tenth century): see Lorch, 1994, pp. 231–236; Charette, 2003, pp. 71–73.

the present epistle on the description and use of the instrument in which, practically, all the examples are given for the latitude of Mahdiyya.

The *risāla* has been preserved in only one manuscript (ms. Istanbul Ayasofya 4861).¹⁴³ This copy was finished on 9 Dhū l-Ḥijja 613 H./19 March 1217 CE by ‘Abd al-Qawī b. ‘Abd al-Mu‘ṭī b. Hilāl b. Aṭīyya b. Tamīm b. al-Mu‘izz b. Bādīs for his own use. The writing is clearly Eastern but, as noted by Stern,¹⁴⁴ the aforementioned genealogy shows that the copyist was the great-great-grandson of Tamīm b. Mu‘izz b. Bādīs, a member of the Zīrid dynasty, who reigned in Ifrīqiya between 1062 and 1108.

The manuscript seems to be incomplete: on fol. 11v, Dūnash states that the *risāla* is divided into three parts (*funūn*) but the text of only two of them seems to be extant. The first part (fols. 1v–11v) is an introduction, while the second one is a collection of 43 applications of the armillary sphere (fols. 11v–60v). Only chapter 1 of the third section, dealing with mathematical geography, appears in our manuscript between chapters 28 and 29 of the second section (fols. 35r–38v).

The introduction contains a description of the basic great circles (*manāṭiq*) of the celestial sphere (equator, meridian, horizon and ecliptic), of three *awtār* (diameters of the celestial sphere)¹⁴⁵ and of seven points (*nuqaṭ*) which correspond to the four cardinal points, midheaven (*wasat al-samā’*), the pivot of the Earth (*watad al-arḍ*) and the centre of the Earth (*markaz al-arḍ*).

When Dūnash describes the ecliptic, he notes that the Sun moves in an orbit which is eccentric in relation to the equator, with which it forms an angle of 24°. ¹⁴⁶ The author adds (fol. 4v) that he has written a *Risāla fī l-burhān ‘alā khurūj markaz al-shams ‘an markaz al-kull bi-darajatayn wa-niṣf* (“Proof of the sun’s eccentricity of 2;30 in respect to the centre of the world”). It is clear that he is referring to a Ptolemaic solar eccentricity of 2;30°. This is not the only title of Dūnash’ works mentioned in the *risāla*. On fols 8r and v he refers to a *Kitāb al-ḥass wa l-maḥsūs* although it is not very clear whether this book was written by Dūnash himself or by somebody else.¹⁴⁷ In fol. 34v, within his description of

143 A photocopy of this manuscript was obtained by Mercè Comes during one of her visits to the Institut für Geschichte der Arabisch-Islamischen Wissenschaften in Frankfurt. After Mercè’s death, this photocopy was given to me by her sister Rosa Comes, to whom I would like to express my gratitude.

144 Stern, 1956, p. 377.

145 They correspond to the diameters that join 1) midheaven with the opposite point on the ecliptic; 2) the north and south poles of the equator; 3) the east and west points of the horizon and the equator. See Mimura, 2015, p. 28.

146 In fol. 32v he says that the obliquity of the ecliptic is 24° minus a few minutes.

147 He may be referring to Aristotle’s “On sense and the sensible”.

the inhabited world (*al-ma'mūra*), he speaks of a disc or a round table (*sufra*), in which he had engraved or painted a map of the Earth, which he presented to "our lord and sovereign" (*ilā mawlānā wa-sayyidinā*);¹⁴⁸ he adds that he is the author of a book in which he explains how this disc should be made, and notes that it does not need to be a square or a polygon (*dhāt al-zawāyā*).

The introduction ends with a long and confusing description of the rings in the armillary sphere (fols. 5v–10r) which I will try to clarify in 4.1.3.2.2.3.

The second part of the book (*al-fann al-thānī*) deals with the use of the armillary sphere, described in 43 chapters, most of which correspond to the standard applications of analogical computers such as the astrolabe or the celestial globe for solving problems of spherical astronomy. To this, Dūnash adds lengthy digressions concerning topics which are not related to the use of the armillary sphere, or in which the use of the instrument is only a visual aid to understand the theoretical principles explained.¹⁴⁹ In some cases these digressions are cosmological,¹⁵⁰ and the author's interest in *hay'a* can be seen in the terminology used throughout the book: in chapter 23 (fols. 27r–28r) he mentions the ninth sphere which is starless (*al-ghayr mukawkab*) and in chapter 24 (fols. 28r–29r) he states that *al-falak al-mudīr min al-mashriq ilā l-maghrib* and *al-falak al-mudīr al-ghayr mukawkab* is the origin of daily motion. This can also be seen in the first section of the book (fols. 10r–11v) in which he insists on the spherical shape of the celestial sphere, the Earth and the four elements, or in God being the mover of all celestial motions. Interestingly, Dūnash poses the problem of the Earth's motion, although finally he rejects this possibility. The main topic of these digressions (fols. 32r–50v, which include chapter 1 of the third section) is the division into climates of the inhabited part of the Earth, according to the maximum duration of daylight which depends on the geographical latitude, as well as the variation of the length of day and night throughout the year, and the determination of the hour which depends on the geographical longitude, etc. For this geographical part, Dūnash mentions two sources: Ptolemy's *Geography*, and a book by al-Kindī (fol. 46v). On fols. 32r

148 The text of the manuscript apparently has a couple of errors: *wa-anta tarā dhālika fī l-sufra al-ma'mūla al-[probably li-] ma'mūrat al-arḍ allatī raḥa'tu-ka* [sic, I understand *raḥa'tu-hā*] *ilā mawlānā wa-sayyidi-nā ...*. The aforementioned "lord" could be either al-Qā'im [ibn] al-Mahdī (934–946) (mentioned in fol. 38v) or his son al-Manṣūr Ismā'īl b. al-Qā'im (946–953).

149 Such is the case of chapter 19 (fols. 24v–25r) in which the instrument is used to show the differences, in the earth, between the sky as seen by people living on the equator and by those living in a location with latitude; and also of chapter 23 (fol. 27r–28r) used to show the differences in the length of day and night throughout the year and in the different climates.

150 See also Mimura, 2015, pp. 28–29.

and 38r and v, he gives the latitude values of the middle of each one of the seven climates, which are not the same as Ptolemy's (table 4.3):

TABLE 4.3 The seven climates according to Dūnash ibn Tamīm

Climates	Max. daylight	Latitude (Dūnash)	Latitude (Ptolemy) ^a
I	13 h	16;12 ^{ob}	16;27°
II	13;30	23;45 ^{oc}	23;51°
III	14	30;22°	30;22°
IV	14;30	36;12 ^{od}	36;0°
V	15	41;15°	40;56°
VI	15;30	45;32°	45;34°
VII	16	49°	48;38°

- a Neugebauer, 1975, vol. 1, pp. 38 and 44.
- b 16° in fol. 32r.
- c 24° in fol. 41r.
- d 36;6° in fol. 32r.

Dūnash's conception of the inhabited world is standard: the *ma'mūra* is surrounded by the *al-Baḥr al-Muḥiṭ* (the Atlantic Ocean) to the west and the *al-Baḥr al-Muẓlim* (Pacific Ocean) to the east, on the coast of China. The text seems to contain a reference to the Great Wall of China, because he mentions the *al-sudd* (the obstacle) which separates this country from the land of Gog and Magog (fol. 46r). The Ancients used the prime meridian of the Fortunate Isles (*al-Jazā'ir al-Khālīdāt*) but the text continues, surprisingly (?), saying that in his *Geography* Ptolemy computed the longitudes of the inhabited world represented on the disc (*al-sufra*) from the cities located on the sea of Tangier, which begins in the Bronze Idols (*al-aṣnām al-nuḥās*),¹⁵¹ whose longitude is 7;12°, ¹⁵² something which was criticised by al-Kindī (fol. 47r). The text also mentions the Strait of Gibraltar, between al-Andalus and Tangier which has a width of seven miles.

151

On fol. 33v he mentions *al-ṣanamān al-nuḥās* (*sic*) attributed to Hercules (Hirqil). This is a reference to the Isle of Cádiz in which there was a very high column with a human statue on its top. The "idol of Cádiz" is described in many Arabic sources. See A. Huici, "Qādis" in *El2* IV, pp. 383–384.

152

The Isle of Cadiz has a longitude of 5;30° in the *Handy Tables*. Cf. Kennedy & Kennedy, 1987, p. 88. No locality with a longitude of 7;12° is mentioned in the Kennedys' collection.

In chapter 35 (fols. 48v–52v), Dūnash deals with the problem of the determination of geographical longitudes as well as of the distance, expressed in miles, between two localities. For that purpose he uses a value of the degree of the meridian of 66;40 miles which he ascribes to the Ma'mūnī determination made on the plane between Tadmur and Raqqa (fol. 34r),¹⁵³ although it derives from the 180,000 stadia used by Ptolemy for the circumference of the Earth, using an erroneous conversion factor of 7.5 between miles and stadia. Interestingly, this value is used by Dūnash as equivalent to the distance a traveller can cover in two days.¹⁵⁴ This allows him to establish that in one day, he will be able to travel along a distance equivalent to 0;30° of the Earth's meridian.¹⁵⁵

In order to obtain the longitude of a locality of known latitude, if one knows the longitude and latitude of another locality, Dūnash describes the standard method based on the observation of a lunar eclipse in both places. To this he adds another, extremely crude, method based on the reduction of itinerary measures, which is a simple application of the Pythagorean relation. The extant text refers to a figure which does not appear on fol. 50v, although the copyist has left an empty space for it.

The aforementioned figure must have been more or less similar to Fig. 4.7 in which K is a locality whose longitude and latitude are known, while N is a second locality with unknown longitude but whose latitude (= HE) is known. AB is the Earth's equator and DEG is the geographical meridian of K.

We need to obtain NH (Δ_L , the difference of longitudes), expressed in degrees of the Earth's meridian. We know KH (Δ_φ , the difference of latitudes between K and N) as well as KN, the distance, in days of travel, (d_m), between K and N, equivalent to $KN/2$ degrees of the Earth's meridian. By applying the Pythagorean relation, we find:

$$\Delta_L = \sqrt{([d_m / 2]^2 - \Delta_\varphi^2)}$$

Our text gives an example in which locality K has a latitude of 36° and N has 30°. 36° is the latitude of Mahdiyya in several Arabic sources,¹⁵⁶ although in the majority of examples of the *risāla* (fols. 13r, 16r, 19v) Dūnash uses a latitude of 34° for that city.¹⁵⁷ The distance, in days of travel, between N and K is 20:

¹⁵³ King, 2000.

¹⁵⁴ See "marḥala" in *EL2*, vol. VI, pp. 558–559.

¹⁵⁵ This equivalence is confirmed in chapter 30 (fol. 44v–45r), where he says that the distance between the equator and the middle of the first climate is 16° of the meridian, equivalent to 1066 + $\frac{2}{3}$ miles, which correspond to 32 days of travel, as one needs two days for each degree.

¹⁵⁶ Kennedy & Kennedy, 1987, p. 205.

¹⁵⁷ A latitude of 36° appears also in fol. 53v–54r.

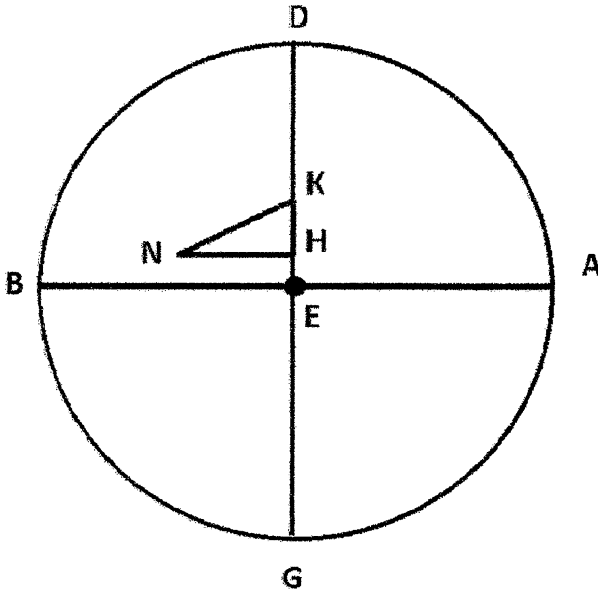


FIGURE 4.7
Dūnash ibn Tamīm
method to calculate the
difference in geographical
longitude between two
localities

$$\begin{aligned}\Delta\varphi^2 &= 6^2 = 36 \\ [d_m / 2]^2 &= [20 / 2]^2 = 100 \\ \Delta_L &= \sqrt{100 - 36} = 8^\circ\end{aligned}$$

The result obtained is obviously inaccurate. Dūnash himself remarks, at the end of the chapter (fol. 52v) that on the surface of the sphere, one operates not with straight lines but with sines, chords and radii, although he does not offer a better procedure.¹⁵⁸ This is not the only case in which Dūnash uses an approximate procedure. In chapter 9 (fols. 17v–18v) he deals with the rising and setting of the fixed stars depending on the local latitude. An explanation of this problem appears at the beginning of the chapter, based on the following conditions for stars visible in the northern hemisphere:

If $90^\circ - \delta \leq \varphi$, the star will always be visible and it will neither rise nor set.
If $90^\circ - \delta > \varphi$, the star will rise and set,

(δ being the star's declination and φ the local latitude).

¹⁵⁸ The same procedure can be found in Ḥabash al-Ḥāsib's *Kitāb al-ajrām wa l-ab'ād* although, in this case, the author intends to obtain the distance NK expressed in degrees of a great circle in order to compare it with the actual distance along the roads; besides, he reduces the difference of longitudes between Baghdad and Mecca (3°) to $2;48^\circ$, taking into account the latitude of Mecca. Cf. Langermann, 1985 pp. 109–110, and King, 2000, pp. 221–222.

However, Dūnash does not use δ , probably because he works with a star table giving longitudes and latitudes but not declinations; he replaces it with the well-known approximation $\delta' = \delta_\lambda \pm \beta$, where δ_λ is the declination of the degree of longitude of the star (λ) and β the star's latitude. He describes how to solve the problem using the armillary sphere and gives an example for the latitude of Mahdiyya (34°) and for the star *al-Kaff al-Khaḍīb* (α And = δ Peg)¹⁵⁹ whose coordinates are $\lambda = 344^\circ$ and $\beta = +29^\circ$.¹⁶⁰

The text makes an unnecessary use of the armillary sphere for the solution of the problem: after adapting the instrument to the local latitude he rotates the ecliptic until its degree 344° coincides with the meridian and makes a mark on the meridian ring (*dhāt al-ʿilāqa*). Then he determines the distance in degrees between the North Pole and the mark and obtains 92° . This should correspond to $90^\circ - \delta_\lambda$ and it implies that he assumes that $\delta_\lambda(344^\circ) = -2^\circ$, an obviously wrong value because the declination of Pisces 14° , for an obliquity of $23;51^\circ$, is $-6;24^\circ$. After this, he subtracts:

$$92^\circ - 29^\circ = 63^\circ$$

a value which corresponds to $90^\circ - (\delta_\lambda + \beta)$. He takes 63° degrees on the meridian ring northwards and says that this is the position of the star (when it crosses the meridian). As $63^\circ > 34^\circ$, the star will rise and set in Mahdiyya.¹⁶¹ If the user wishes to visualise this with the instrument, the position of the star on the *dhāt al-ʿilāqa* is transferred to ring no. 7 (an equatorial meridian?) and, by rotating the ring, he will be able to see that the star rises and sets. The same procedure is repeated in chapter 10 (fols. 19r–19v) which intends to determine whether a star is always visible because it is circumpolar: in it, Dūnash gives an example for the latitude of Mahdiyya (34°) and an unidentified star with $\lambda = 10^\circ$ and $\beta = +70^\circ$. He obtains:

$$90^\circ - \delta_\lambda = 86^\circ$$

159 *al-Kaff al-Khaḍīb* should be identified with β Cas, but there is a Western Islamic tradition, represented by this text as well as by Qāsim ibn Muṭarrif, Maslama al-Majrīṭī, Ibn al-Zarqālluh, Ibn al-Kammād, Abū ʿAlī al-Marrākushī and Ibn al-Bannā' which identifies this star with α And = δ Peg. See Comes, 1994, pp. 100–101/ 128–9 of the reprint.

160 These coordinates seem to derive ultimately from Ptolemy's star table (Toomer, 1984, p. 358) which has $\lambda = 347;50^\circ$ and $\beta = +26^\circ$. The confusion in the longitude between 344 and 347 is easy to explain, in *abjad* notation, due to the similarity between a *dāl* (4) and a *zayn* (7). The latitude of 29° (instead of 26°) is also used by Qāsim, Maslama, Ibn al-Kammād and Ibn al-Bannā': see Comes, 1994, pp. 101/129.

161 The correct value for the declination of *al-Kaff al-Khaḍīb*, for an obliquity of $23;51^\circ$, is $+20;14,19^\circ$. Therefore $90^\circ - 20;14,19^\circ = 69;45,41^\circ$. As for the approximation, if we use the correct value for the declination of Pisces 14° ($-6;24^\circ$), we will obtain $96;24^\circ - 29^\circ = 67;24^\circ$.

(which implies $\delta_\lambda = 4^\circ$, a correct value as the computed declination of the tenth degree of Aries is $4;2^\circ$). Then:

$$90^\circ - \delta_\lambda - \beta = 86^\circ - 70^\circ = 16^\circ$$

and $16^\circ < 34^\circ$. Therefore, the star will always be visible.

One of the problems raised by Dūnash's treatise on the armillary sphere is to determine the author's competence as an astronomer. This is difficult given the fact that only one manuscript is extant and we do not know whether its errors should be attributed to the author or to the copyist. Whatever the case, it seems that Dūnash uses a terminology which is not standard and which I have tried to interpret:

miḥwar (axis) is often used instead of pole, although the term *quṭb* (pole) is repeatedly used in the introduction;

ilāqa (hanger) has an obvious correspondence with zenith, and *qubba* (cupola) has apparently the same meaning. I have not found the standard term *samt al-ra's* in the text;

majrā is the parallel of declination of an ecliptical degree, instead of the standard *madār*.

Another example of Dūnash's peculiar terminology is his use of *maṭālī' fī l-falak al-mustaqīm* for both right and oblique ascensions, as we can see in chapter 7 (fols. 15v–16r) which deals with the variations in the rising and setting times of the zodiacal signs: this chapter is pure theory, and there is little use of the armillary sphere. He gives an example for the latitude of Mahdiyya (34°) but, surprisingly, he speaks of rising times in right ascension (*fī l-falak al-mustaqīm*) instead of in oblique ascension. According to the text, the rising time of Aries and setting time of Libra is 19° , which corresponds to the rising/setting time for 34° of latitude (the calculated value is $19;50^\circ$ for an obliquity of the ecliptic of $23;51^\circ$). The value given in the manuscript for the rising time of Cancer (20°) is clearly wrong (the correct value should be 35°).

The same kind of error appears in chapters 35 (fols. 53v–54r) and 36 (fols. 54r–54v). Both deal with the rising times of the signs, called *maṭālī' al-burūj fī l-falak al-mustaqīm*, although a distinction is made as, in chapter 35, he specifies “for any locality of the Earth where we are, belonging to any climate or latitude” (*ḥaythu kunnā min al-arḍ min al-aqālīm wa l-urūd*): this specification makes one think that the latitude is not involved and, therefore, that we are dealing with rising times in right ascension. In spite of this, an example is given for a latitude of 34° (Mahdiyya) and the text states that the rising time, in right (sic) ascension, of the sign of Aries is 20° and that the same

result can be obtained with the table of right ascensions of a *zīj*. The result is correct for the rising time in oblique ascension which, for latitude 34° , is $19;50^\circ$. The specification in chapter 36 is “right ascension of the zodiacal signs for the people living on the equator” (*maṭālī‘ al-burūj fī l-falak al-mustaqīm li-ahl al-istiwā*) and here, obviously, the ascensions are right ascensions, although the examples given, confirmed with those read in tables (*jadāwil*), are the result of a confusion between the rising times of Aries (32° according to the text, while the correct result for an obliquity of $23;51^\circ$ is $27;50^\circ \approx 28^\circ$) and those of Gemini 28° (instead of $32;16^\circ \approx 32^\circ$). The rising time of Taurus is correctly given as 30° ($29;54^\circ$). Another expression used in chapter 14 (fol. 21r) for rising times in right ascension is “ascension of every sign in the meridian” (*maṭla‘ kull burj fī wasaṭ al-samā’*).

4.1.3.2.2.2 Dūnash’s Sources

As for the sources available to Dūnash, we have already seen that he mentions Ptolemy’s *Geography*. The *Almagest* is not mentioned explicitly but we do find references to the star catalogue (chapter 10, fols. 19r–19v),¹⁶² and to the *Kitāb fī iqtisāṣ* (“Planetary Hypotheses”, chapter 34, fols. 52v–53v) which Dūnash seems to use to determine the mean motion of the Sun and the length of the tropical year. He says that the Sun, in 150 Egyptian years and 37 days, returns to the apogee of its eccentric circle 150 times, according to Ptolemy’s observations and those of his predecessors.¹⁶³ This period is equivalent to $54750 + 37$ days = 54787 days. He divides this amount by the number of revolutions and obtains 150. The corresponding solar year is $365 + 37/150^d$, equivalent to $365;14,48^d$.¹⁶⁴ In chapter 39 (fols. 56v–57v) on the precession of the equinoxes, he seems to be referring to the *Handy Tables*, as he says that according to Ptolemy the star *Qalb al-asad* (Regulus, α Leonis) had a longitude of $27;54^\circ$ of Cancer, in the beginning of the year after the death of Alexander, on the first day of the [first] Egyptian month, at midday, in Alexandria. This position appears in the *Handy Tables* for 1 Thoth of year 1 of the era of Philip, not Alexander (12 November 324 BC),¹⁶⁵ a value calculated backwards, using a precession of $1^\circ/100$ years,

162 A star table (*jadwal al-kawākib al-thawābit*) with no specific author is mentioned in chapter 43 (fol. 59v).

163 See Neugebauer, 1975, II, 901: Ptolemy states the equivalent relation: $300^y + 74^d = 300$ tropical years.

164 This value for the tropical year appears both in the *Planetary Hypotheses* (Neugebauer, 1975, II, 901) and in the *Almagest* III.1 (Neugebauer, 1975, I, 54–55; Toomer, 1984, p. 140).

165 Neugebauer, 1975, II, 986.

from Leo 2;30°, the position observed by Ptolemy on 23 February 139. This is the longitude that appears in the star catalogue of the *Almagest*.¹⁶⁶

Our book often refers to *zīj*es, which are used to determine the position (in longitude, or longitude and latitude) of the Sun (chapter 4, fols. 13v–14r; chapter 21, fols. 25v–26r; chapter 38, fols. 56r–56v), the Moon or a planet (chapter 40, fols. 57v–58r). Dūnash mentions tables (*jadāwil*) of right ascension (chapter 36, fols. 54r–54v) and tables that give the ascendant and the descendant for each hour (chapter 42, fols. 58v–59v). There is a specific reference to the *al-Zīj al-Mumtaḥan* for determining the solar longitude (chapter 15, fols. 21r–22r) and we have already seen that he mentions the Ma'mūnī determination of the length of one degree of the Earth's meridian (fol. 34r). Interestingly, he mentions tables that give the solar altitude for each hour throughout the solar year: *al-jadāwīlāt* [sic] *al-mawḍū'a li-irtifā' al-sā'āt*. Finally, we have already seen that he refers to al-Kindī (fol. 46v) in his geographical descriptions of the world, and we find Thābit ibn Qurra's definition of *al-samt* (azimuth) in chapter 24 (fols. 28r–29r).

4.1.3.2.2.3 The Rings

The nine rings of his armillary sphere are described twice on fols. 5v–9v in a confusing way, and they are identified by an ordinal number (from first to ninth). My interpretation of the function of the nine rings is the following:¹⁶⁷

1. The first ring, usually called *dhāt al-'ilāqa* (bearer of the hanger). It is a fixed ring which corresponds clearly to the meridian.¹⁶⁸ It is divided into degrees which allow the instrument to be adapted to the local latitude. As this ring has a hanger (*'ilāqa*), the instrument described by Dūnash is a small-sized armillary sphere: the chapters describing how to use the instrument to determine the solar altitude by day (chapter 1 of the second *fann*, fol. 11v–12r) or the altitude of the Moon or a star by night (ch. 2, fol. 12r) begin by saying that the user of the instrument should hang it with his right hand.
2. Equator (*ḥalqat al-falak al-mustaḳīm*): *al-mu'tariḍa li-dhāt al-'ilāqa* (“opposed to the bearer of the hanger”). It corresponds to the celestial equator, with which one knows the [terrestrial] longitudes. The description adds that it can be used to determine the altitude of the sphere (i.e., the altitude of the beginning of Aries equivalent to the local colatitude).

¹⁶⁶ Toomer, 1984, pp. 328 and 367.

¹⁶⁷ My interpretation may be wrong. I hope this will be clarified with the edition of the text which Mimura is planning to publish (see Mimura, 2015, p. 347, fn. 60).

¹⁶⁸ “[*ḥalqa*] *dhāt al-'ilāqa allatī hiya ḥalqat niṣf al-nahār*” (fol. 7r; see also fol. 8v).

3. *Ḥalqat al-ufq* (horizon ring), used to determine the ascendant and the descendant, and the altitude of celestial bodies above it. It is the limit (*ḥadd*) between the visible and the invisible parts of the sky.
4. Movable ring, whose axis links the north and south poles of the equator. It can be rotated so that the pole forms, with the horizon, an angle equivalent to the local latitude, using the graduation of the meridian ring.
5. *Ḥāmilat al-burūj* (bearer of the zodiacal signs): an ecliptic meridian ring. It is used to measure celestial latitudes.
6. *Ḥalqat al-burūj* (ring of the zodiacal signs, the ecliptic).
7. An ecliptic meridian ring, as can be seen in chapters 20 (fols. 25r–25v), 37 (fol. 54v–56r), 38 (fols. 56r–56v), 40 (fols. 57v–58r), 42 (fols. 58v–59v), 43 (fols. 59v–60r). It is used to measure the latitudes of both stars and planets. In chapter 9 (fols. 17v–18v) it seems to correspond to an equatorial meridian.

I have a serious problem understanding rings 8 and 9. On the one hand, according to fol. 7v, they are linked together and it seems clear that ring n°. 8 is a movable equinoctial ring called *ḥalqat al-falak al-mustaqīm*, like ring n°. 2 (fol. 9r). Besides, on fol. 9r, I read *ju'ilat qibālat al-ḥalqat al-thālitha* (it is placed facing the third ring), which does not make much sense if it refers to the aforementioned third ring (horizon). The third ring of fol. 9r is the second ring of fol. 6r (equator). This makes me think that ring n°. 8 is an equatorial meridian.

As for ring n°. 9, the situation is still more confusing. On fol. 9r it is called *al-ḥalqa allatī tadūru 'alā āfāq al-ard* (“the ring which rotates over the horizons of the Earth”) and it is used to measure azimuths. The problem is: if rings 8 and 9 are linked, one would expect two rings corresponding to the same system of coordinates and, therefore, if ring n°. 8 is an equatorial meridian, ring n°. 9 would correspond to another ring referred to the plane of the equator.¹⁶⁹ A ring of this kind could not be used, however, to determine azimuths or altitudes of the Sun and the Moon. On the other hand, if we consider that ring n° 9 is a vertical circle whose axis goes from the zenith to the nadir, it is difficult to understand how it could be linked to ring n°. 8.

The chapters on the use of the instrument add to the confusion: in chapters 4 and 5 the altitudes of celestial bodies are always marked in ring n°. 9. In chapter 24 (fols. 28r–29r) the pole of the ninth ring is placed under the *ilāqa*, which corresponds to the local zenith: the altitude of the Sun is marked on the ninth ring and the purpose of the operation is to obtain the solar azimuth. In chapters 37 (fol. 54v–56r) and 40 (fols. 57v–58r) the ninth ring is also a vertical

¹⁶⁹ In chapter 30 (fol. 44v) he says that the eighth ring is the carrier (*ḥāmila*) of the ninth.

circle. In chapter 39 (56v–57v) the altitude of *Qalb al-asad* (α Leo) is marked on the ninth ring.

On the other hand, in chapters 14 (fol. 21r), 23 (fol. 27r) and 30 (fol. 44r) we find that “the pole of the equator is the pole of the ninth ring” (*miḥwar al-falak al-mustaqīm ... huwa miḥwar al-ḥalqa al-tāsi’a*). In chapters 23 (fol. 27r), 30 (fol. 44v) and 42 (fols. 58v–59v) the author adds (or implies) that the ninth ring is the equator (*al-ḥalqa al-tāsi’a hiya al-falak al-mustaqīm*). In chapter 35 (fols. 56r–56v), the ninth ring is placed on the equator (fol. 53v–54r). In chapter 38 (fols. 56r–56v) the ninth ring is an equatorial meridian.

Should we understand that the ninth ring can be used either as a vertical circle or as an equatorial meridian, depending on the circumstances?

4.1.3.2.2.4 The Use of the Armillary Sphere

Dūnash b. Tamīm’s armillary sphere is not an observational instrument. We have already seen that it is small and it seems more closely related to Ptolemy’s meteoroscope¹⁷⁰ than to the armillary sphere described in the *Almagest* v.1.¹⁷¹ Only very rarely is the instrument used to make an observation: for example, to determine the altitude of the Sun (chapter 1, fol. 11v–12r), the Moon or a star (chapter 2, fol. 12r). This is confirmed by the fact that none of the rings of the armillary sphere has a pair of sights which would make the observation easier. In the case of the two aforementioned observations one has to direct one ring towards the Sun or the star until, in the case of the Sun, the shadow projected by one side of the ring coincides with the other side or, in the case of the Moon or another celestial body, both sides of the ring prevent the star from being seen by the observer. Besides, if an observation is needed for one of the uses of the instrument, we often find that the author recommends the use of an astrolabe or another astronomical instrument like a graduated gnomon (chapter 16, fols. 22r–23r; chapter 17, fols. 23r–23v; chapter 27, fol. 30r–31v). He adds that, for the determination of the latitude of a city, precise observations require the use of great instruments having large diameters (*al-ālāt al-‘izām al-wāsi’a al-aqtār*).

Like the astrolabe, Dūnash’s armillary sphere is a computing device that can be used to solve problems of spherical astronomy. Dūnash himself remarks (fol. 7v) that, with the astrolabe, a single operation can give several results: as

170 Mercè Comes, 2012, pp. 81–83: Ptolemy’s meteoroscope also has nine rings, but they do not seem to correspond to those of Dūnash b. Tamīm.

171 Toomer, 1984, pp. 217–219. Ptolemy’s armillary sphere is described by al-Marrākushī, 1984, vol. II, pp. 113–116 (including a passage on the use of the instrument). Al-Marrākushī’s source is somebody called al-Lawkarī or al-Lūkārī, whom I have been unable to identify.

an example, he notes that, after determining the solar altitude, if we place the degree of the Sun in the position (the almucantar) which corresponds to that altitude on a plate designed for the local latitude, the ascendant will be on the horizon, the degree opposed to the solar degree will be on the corresponding hour in the horary diagram placed below the horizon, and the cusps will be in their appropriate place. This is not so with the armillary sphere, which requires an operation for the solar altitude, another one for establishing the ascendant and a third one for obtaining the hour.

I transcribe here the list of the chapters on the use of the armillary sphere in Dūnash's book:

Fols. 11v–12r: determination of the solar altitude.

Fol. 12r: id. of the Moon or a star.

Fols. 12v–13r: ascendant and the other three cusps with an example for the latitude of Mahdiyya (34°).

Fols. 13v–14r: how many hours have elapsed since sunrise.

Fols. 14v–15r: ascendant during the night using the moon, a star or a planet of known longitude and latitude.

Fols. 15r–15v: obtaining the hour during the night.

Fols. 15v–16r: variations in the rising and setting times of the zodiacal signs.

Fols. 16v–17r: rising and setting amplitude.

Fols. 17v–18v: rising and setting of the fixed stars depending on the local latitude.

Fols. 19r–19v: how to establish if a star is always visible and does not rise nor set.

Fol. 20r: how many hours a star will be visible.

Fol. 20v: how many hours a star will be invisible, as it will be below the horizon.

Fols. 20v–21r: theoretical chapter in which the author explains the different positions occupied by the zodiacal signs as the ecliptic rotates with its daily motion.

Fol. 21r: how to determine the rising times, in right ascension, of the zodiacal signs.

Fols. 21r–22r: determining the meridian line at any place.

Fols. 22r–23r: meridian altitude of a star.

Fols. 23r–23v: longitudes and latitudes of celestial bodies (*kawāḳib*) although the contents of the chapter are only related to the latitude.

Fols. 23v–24v: another procedure to obtain the longitude and latitude of a star.

Fols. 24v–25r: differences, in the terrestrial sphere, between the sky as seen by people living on the equator and by those living in a location with latitude.

Fols. 25r–25v: how to know the degree of the ecliptic with which a star rises, crosses the upper and lower meridian, and sets.

Fols. 25v–26r: arc of the day for any day and any latitude.

Fols. 26r–27r: equal and unequal hours.

Fols. 27r–28r: differences in the length of day and night throughout the year and in the different climates.

Fols. 28r–29r: on the azimuth.

Fols. 29r–29v: determining the meridian line from a solar azimuth.

Fols. 29v–30r: obtaining the azimuths of the hours.

Fols. 30r–31v: obtaining the latitudes of cities.

Fols. 32r–34v: on climates and the division of the inhabited part of the Earth (*al-ma'mūra*).

35r–38v: chapter 1 of the third *fann*: again on climates.

Fols. 38v–44r: again on climates and length of the day.

Fols. 44r–45r: how to know the latitude of the middle of the climate (*wasaf al-iqlīm*).

Fols. 45r–45v: determining the beginning and the end of each climate.

Fol. 45v: establishing which climate corresponds to a particular city.

Fols. 45v–52v: longitudes of the cities.

Fols. 52v–53v: periods of revolution of the celestial bodies (*maqādīr 'awdāt adwār al-kawākib*) and mean motion of planets.

Fols. 53v–54r: obtaining the right (*sic*) ascension of the zodiacal signs for any locality.

Fols. 54r–54v: right ascension of the zodiacal signs for the people living on the equator.

Fols. 54v–56r: conjunction of the Sun and the Moon, position of the ascending and descending nodes, and distance of the Moon from them. In spite of the title, the chapter deals only with the distance of the Moon from the node.

Fols. 56r–56v: obtaining the ascendant if the hour is known.

Fols. 56v–57v: on the displacement of fixed stars from west to east (precession of the equinoxes).

Fols. 57v–58r: crescent visibility. The text seems obviously incomplete for, after obtaining the difference in longitude between the Sun and the Moon, it does not seem to use this value and the only criterion used is the lunar altitude, which should be 12° or more. This would make some sense if we understand that the moment of the observation corresponds to sunset, in which the solar altitude is 0°.

Fols. 58r–58v: the five methods on the sphere (*al-ṭarā'iq al-khams fī l-falak*).

Fols. 58v–59v: on the number of hours the Moon or a planet stays above the Earth and is visible, or stays below the Earth and cannot be seen, assuming

the longitude and latitude of the Moon or the planet at the moment it rises is known.

Fols. 59v–60r: on the number of hours a fixed star stays above the Earth and is visible, or stays below the Earth.

4.1.3.3 Ibn al-Zarqālluh and Alfonso x on the Armillary Sphere

Ibn al-Zarqālluh wrote a treatise on the construction of the armillary sphere which we know through the Castilian translation (*Libro de las armellas*) preserved in the Alfonsine *Libro del saber de astrología*.¹⁷² As usual in this collection, the treatise on the armillary sphere contains two sections related to the construction (11 chapters) and use of the instrument (70 chapters). The first part is clearly attributed to Ibn al-Zarqālluh¹⁷³ while the second is due to Rabiçag (Rabbi Ishāq b. Sīd),¹⁷⁴ since the Alfonsine collaborators did not have access to any Arabic works on the use of the instrument that they could translate.

There is no reason to doubt the authenticity of the attribution to Ibn al-Zarqālluh of the first part of the book. I can give a few arguments in favour of this hypothesis:

The text insists on many details related to the techniques used to build the instrument¹⁷⁵ and these details are exactly what one would expect from someone like Ibn al-Zarqālluh who began his professional career as an instrument maker.

When the text deals with the obliquity of the ecliptic, three values are mentioned: 23;51° (attributed to Hipparchus), 23;33° (the value obtained in the Ma'mūnī observations and finally adopted in the text) and 23;35° (al-Battānī). The preference shown for 23;33° is significant because it is a parameter confirmed by Ibn al-Zarqālluh himself by observation.¹⁷⁶ Besides, the three afore-

¹⁷² Ms. Villa Amil 156 fols. 135v–155v; Rico, 1863–67, II, 1–79. On this work see Nolte, 1922, pp. 26–47; Comes, 2012, pp. 188–203; Comes, 2013 a; Samsó, 2011, pp. 175–180.

¹⁷³ “Dixo el sabio abuçaḥ azarquiel” (Villa Amil 156, fol. 135v; Rico, 1863–67, II, 3).

¹⁷⁴ “Tenemos por razon de mostrar del otro estrumento que fizo Ptolomeo a que dizen en arabigo det alhalac [*dhāt al-halaq*] et en latin armillas. Et mostraremos otrossi en qual guisa deuen obrar con ellas, maguer este libro de como obran con ellas non era fallado en esta nuestra fazon. Et por ende mandamos a nuestro sabio Rabiçac el de Toledo que lo fiziesse bien complido et bien llano de entender en guisa que pueda obrar con el qual omne quier que cate en este libro” (Villa Amil 156, fol. 135v).

¹⁷⁵ Size and shape of the rings, how to join them together, how to cut the rings, how to polish them, how to prepare the junctures and how to divide the rings: Comes, 2013 a, p. 367.

¹⁷⁶ In canons Cb and Cc of the *Toledan Tables* we find the following remark: “quae [i.e. al-Ma'mūn's value] apud nos ducitur verior, quia primam novimus rumore, et hanc didicimus per consideracionem” (“and among us this is considered truer, since we know the

mentioned parameters seem to be those used by Ibn al-Zarqālluh in his design of a geometrical model to justify the variations of the obliquity of the ecliptic (cf. 6.2.2.5).

The text mentions three localities whose geographical coordinates have been well established with precise observations: Toledo (where Ibn al-Zarqālluh himself observed), Alexandria (Ptolemy) and Raqqa (al-Battānī).

To this, Mercè Comes adds two more arguments:¹⁷⁷

- 1) In other instances that we can check because the Arabic original text is extant, we see that whenever the Alfonsine translation attributes a text to Ibn al-Zarqālluh, this attribution is correct.
- 2) The text gives the Arabic names of the circles and rings, a terminology which is not to be found in the treatises on the construction and use of instruments written by Alfonso's collaborators.

In the prologue to his treatise on the construction of the armillary sphere, Ibn al-Zarqālluh remarks that the main uses of the instrument are the observation of the longitudes and latitudes of the Sun, Moon, planets and fixed stars. To this he adds that the sphere is also used to study the variations in the length of day and night in different latitudes, to determine the hour, and to solve problems of mathematical astrology. This double set of applications shows, as noted by Mercè Comes,¹⁷⁸ that the instrument designed by Ibn al-Zarqālluh presents features of the Ptolemaic *astrolabon*, whose purpose is precisely observation, and of his meteoroscope (an analogical computer). This is confirmed by the topics studied in the second part of the treatise, on the use of the instrument, which deals with seven different types of problem:

Time reckoning (13 chapters)

Astrological problems (13 chapters)

Horizontal, equatorial and ecliptic coordinates and how to transform one set of coordinates into another one (33 chapters).

Geographical coordinates (five chapters).

Trigonometry (shadows) (one chapter).

Terminology (two chapters)

Solar and lunar observations (three chapters).¹⁷⁹

former from hearsay but we have learnt the latter from observation"). See Pedersen, 2002, vol. II, 410–411 and 612–613.

177 Comes, 2013 a, pp. 366–367; Comes, 2012, pp. 194–195.

178 Comes, 2013 a, p. 366.

179 Comes, 2013 a, p. 367.

Concerning the possible applications of this instrument to observations, although we know that Ibn al-Zarqālluh made observations of the Sun, Moon and stars, we have not a single description of any of them and we do not know which instruments he used. However, George Saliba¹⁸⁰ has studied a *Risālat al-hay'a* written by an anonymous Toledan astronomer, a contemporary of Ibn al-Zarqālluh, who reports that Ibn al-Zarqālluh used an instrument that had some problems of stability, something that makes me think that it was a large-sized instrument. Was this instrument the armillary sphere described in the Alfonsine translation?

Mercè Comes has identified the rings and pieces of Ibn al-Zarqālluh's armillary sphere and I will copy here, almost literally, her description:¹⁸¹

1. Supporting piece or foot of the instrument: it is the T-shaped piece in fig. 4.8.
2. *Armella mayor* (prime vertical circle): a fixed ring supported by the foot of the instrument. It is graduated from 0° to 360° , beginning at one of the horizontal ends. It is the outer ring in fig. 4.8.
3. Rotating set placed inside the *armella mayor* and composed of:
 - a) *Medio cerco de altura* (semicircle of altitude), supported by 3b). It is made up by two quadrants (graduated from 0° to 90° each) fixed perpendicularly to 3b) and 4b). It rotates horizontally.
 - b) *Armella del leuador del medio cerco de altura* (carrier of the semicircle of altitude): it is joined to 4b).
4. A set fixed to the foot, composed of two fixed rings intersecting at right angles:
 - a) *Armella del mediodía* (meridian ring): a graduated ring placed vertically and parallel to the *armella mayor*. It rotates around the poles of the equator. In fig. 4.8 it is the second concentric ring counting from ring n° 2 towards the centre of the figure.
 - b) *Armella del horizonte* (horizon ring): fixed at a right angle to the meridian ring. Graduated in four series going from 0° to 90° , beginning at the north and south points of the horizon and ending at the east and west points. In fig. 4.8 it is represented by the horizontal straight ruler.
5. *Armella mayor de rectificar* (great ring for observation): a ring with two sights, placed inside the meridian ring, which rotates around the poles of the ecliptic. It is used to observe longitudes. It is the third concentric ring counting from ring n° 2 towards the centre.

¹⁸⁰ Saliba, 1999.

¹⁸¹ Comes, 2013a, pp. 368–369.

6. A rotating set composed of:

- a) *Armella de los polos* (polar ring): solstitial colure. It is the fourth concentric ring counting as before.
- b) *Armella del yguador del día* (equatorial ring): it intersects the polar ring at 90° from the poles. Graduated from 0° to 360° .
- c) *Armella zodiacal* (ecliptic ring) which intersects the equator at an angle of $23;33^\circ$. It is divided into the twelve zodiacal signs, each one of them graduated from 0° to 30° . Both the *armella zodiacal* and the *armella del yguador del día* appear in fig. 4.8 as the two graduated straight rulers which intersect the horizon ruler (4b).
- d) *Armella menor de rectificar* (small ring for observation of latitudes). It is a ring placed inside 6a. It rotates around the poles of the ecliptic and it is the smallest ring of the sphere. Graduated in four series from 0° to 90° , it has a complementary ring with two sights. The *armella menor de rectificar* can be replaced by an alidade.

Ibn al-Zarqālluh's armillary sphere contains, therefore, ten rings and is substantially different from Ptolemy's instrument (*Almagest* v,1) which has only seven: two for the local meridian, ecliptic, solstitial colure and three great circles of longitude, the smallest of them bearing two sights which allow determinations of latitude. It is clear that the purpose of Ptolemy's armillary sphere is only to observe longitudes and latitudes of celestial bodies, while Ibn al-Zarqālluh's instrument can be used to determine ecliptic, equatorial and horizontal coordinates although only the rings used to obtain longitudes and latitudes by observation (5 and 6d) have a system of sights and are the only ones described, in the Alfonsine translation, as "de rectificar" (for observation).¹⁸² Equatorial and horizontal coordinates are supposed to be obtained by means of a transformation of ecliptic coordinates, using the instrument as an analog computer.

Mercè Comes¹⁸³ has drawn attention to the interest of Ibn al-Zarqālluh's (?) suggestion that the *armella menor de rectificar* can be replaced by an alidade. This is an improvement on the instrument traditionally attributed to Mu'ayyad al-Dīn al-'Urḍī (d. 1266), active at the Marāgha observatory in northern Iran. Al-'Urḍī justifies the advantage obtained with the use of the alidades with two arguments: 1) its construction is easier, and 2) an observation made through the two sights of a ring may be difficult, in a large-sized armillary sphere, because

¹⁸² However, chapter 1 of the treatise on the use of the instrument (Ms. Villa Amil 156 fols. 143v–144r; Rico, 1863–67, II, 33–34) mentions an *armella del rectificar de la altura* (ring for the observation of altitudes) which has two sights and is used in chapter VI of the same treatise (Ms. Villa Amil 156 fol. 145r; Rico, 1863–67, II, 38) to observe the altitude of the Sun or of any other celestial body.

¹⁸³ Comes, 2013 a, pp. 370–372.

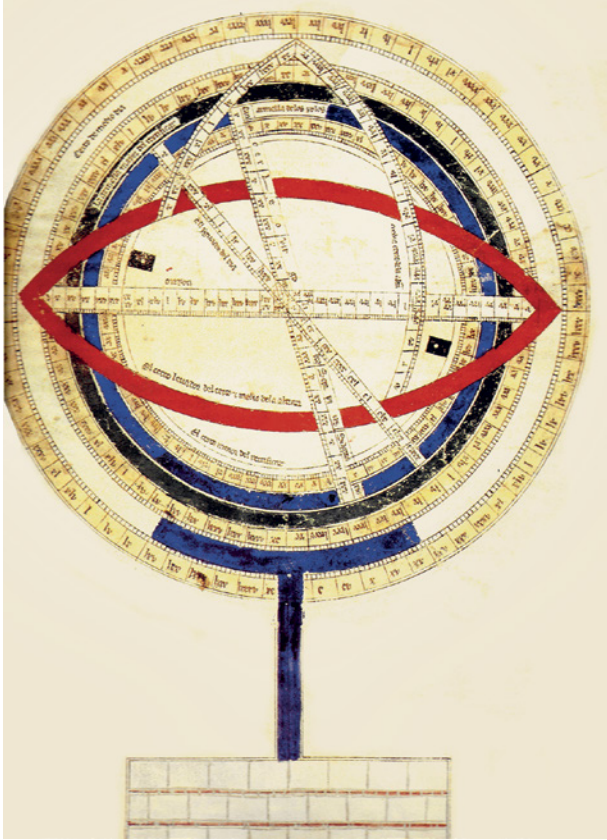


FIGURE 4.8
The Alfonsine
representation of the
armillary sphere
BIBLIOTECA
HISTÓRICA DE
LA UNIVERSIDAD
COMPLUTENSE DE
MADRID, BH MSS 156,
FOL. 142R

of the distance between the sights; while in an alidade, the sights can be placed at any distance, regardless of its longitude. The problem is: was Ibn al-Zarqālluh the first to introduce this improvement, or does the reference to the alidade correspond to a later Alfonsine interpolation? Finally, is this interpolation the result of the Alfonsine astronomers' knowledge of al-'Urḍī's work?¹⁸⁴ If so, we would have another indication of the possible existence of a communication between the Alfonsine astronomers and those of the Marāgha observatory.¹⁸⁵

¹⁸⁴ Roser Puig reminds me that al-'Urḍī knew Ibn al-Zarqālluh's *ṣaḥīḥa* (4.3.2): cf. Saliba, 1990, p. 135.

¹⁸⁵ Comes, 2004.

4.2 The Astrolabe

4.2.1 *The Introduction of the Standard Astrolabe in al-Andalus*

It is clear that the astrolabe, the most popular astronomical instrument in the Middle Ages, was known in both al-Andalus¹⁸⁶ and the Maghrib in the second half of the tenth century. We have already seen that Dūnash ibn Tamīm mentions it repeatedly in his treatise on the armillary sphere (4.1.3.2) where he refers not only to the standard plane astrolabe, in its two variants (northern and southern), but also to the astrolabes called *al-mubattaḥ* and *al-ḥūtī*. Besides, he also gives the name of an instrument maker called Abū 'Alī Aḥmad b. 'Uthmān al-Aṣṭurlābī. Apart from this, I do not know of any other Maghribī reference to the instrument until the thirteenth century.

The situation is different in al-Andalus, where the information is more reliable and complete from the second half of the tenth century onwards. On the one hand, manuscript Paris B.N. Lat. 7412 contains a detailed series of drawings of an Andalusī astrolabe (with inscriptions in Arabic and their corresponding Latin translations) made by Khalaf ibn al-Mu'ādh, towards the end of the tenth century,¹⁸⁷ in which we can observe certain characteristics (see figs. 4.9 and 4.10):

The astrolabe is archaic because the vertical circles (*dawā'ir al-sumūt*) do not appear in its plates,¹⁸⁸ as in later instruments. Besides, the instrument is universal, with plates for the seven Ptolemaic climates, while no plate corresponds to a specific locality.

The instrument belongs clearly to the Western Islamic tradition because its back has a zodiacal scale in which two circles represent 1) the ecliptic, divided into the twelve zodiacal signs and into 360°, and 2) the solar year, divided into the twelve Julian months and the corresponding number of days. With this diagram it is easy to obtain the solar degree of longitude for any day of the solar year by using the alidade which we will place on the corresponding day: it will mark, on the ecliptic, the solar longitude.

The zodiacal scale has always been considered to be characteristic of "Hispano-Moresque" astrolabes, in Michel's terminology,¹⁸⁹ and I believed for many years that this was a Western Islamic invention, born in a part of the world in which the Julian calendar was in use. I was forced to change my mind

186 A general survey of the astrolabe treatises in al-Andalus and the Christian states of the Iberian Peninsula in Viladrich & Martí, 1981; Martí & Viladrich, 1983.

187 Kunitzsch, 1998; Hernández, 2018 a, pp. 27–33.

188 These circles were first mentioned by al-Khwārizmī: see King, 2005, p. 367.

189 Michel, 1947, pp. 151–152.

as a result of the publication, by David A. King,¹⁹⁰ of an important paper on the instrument designed by Naṣṭūlus (fl. 890–930) – a Muslim scholar and instrument-maker, probably with Nestorian origins – the purpose of which was to determine the hour as a function of the solar instantaneous and meridional altitude. The instrument has a shape similar to that of the astrolabe and bears a zodiacal-calendar scale like the one we find in Andalusī and Maghribī astrolabes. In spite of this evidence, it is clear that this scale appears in Eastern astrolabes at a later date.

Khalaf ibn al-Mu'ādh's instrument will often be recalled in this section, where we will analyse its relation to the Latin Catalan texts on the astrolabe and to its contemporary instrument, the so-called Carolingian astrolabe, also made towards the end of the tenth century. We should, however, begin by analysing the interest of the works on this instrument produced by Maslama al-Majrīṭī (d. 1007)¹⁹¹ and his disciples Ibn al-Ṣaffār (d. 1035)¹⁹² and Ibn al-Samḥ (d. 1035).¹⁹³

Maslama, who was a brilliant mathematician, was mainly interested in the theory of stereographic projection which underlies the astrolabe. This projection was the object of a detailed analysis in Ptolemy's *Planisphaerium*, a book translated into Arabic.¹⁹⁴ Maslama is known to have revised the translation of the Ptolemaic work, and it has been conjectured that he may have been the translator. During Maslama's life time, the Byzantine monk Nicholas was in Córdoba where, together with a group of Cordoban physicians, he prepared a revision of the Arabic translation of Dioscorides' *Materia medica*; it has been suggested that Maslama might have met him, learned Greek with his help and translated the *Planisphaerium*.¹⁹⁵ Nonetheless, this hypothesis has been soundly rejected by Paul Kunitzsch.¹⁹⁶ Maslama was probably the author of a recension of the Eastern Arabic translation of the *Planisphaerium* which included his own comments. Maslama's notes have been edited by Vernet & Catalá¹⁹⁷ and Kunitzsch & Lorch¹⁹⁸ and, in them, the Andalusī astronomer tries to complement Ptolemy's work, adding to it a series of new procedures,

190 King, 2008.

191 See HATA no. 23; M. Rius in *BA* VI, pp. 535–39; J. Casulleras en *BEA*, pp. 727–8.

192 HATA no. 39; J. Samsó in *BA* V, pp. 69–73; M. Rius in *BEA*, pp. 566–7.

193 HATA. no. 40; M. Comes in *BA* V, pp. 227–230; M. Rius in *BEA*, p. 568.

194 An edition of this translation was prepared by Anagnostakis, 1984 and by Sidoli & Berggren, 2007; see also Kunitzsch, 1994.

195 Vernet & Catalá, 1965.

196 Kunitzsch, 1995–96.

197 Vernet & Catalá, 1965.

198 Kunitzsch & Lorch, 1994.

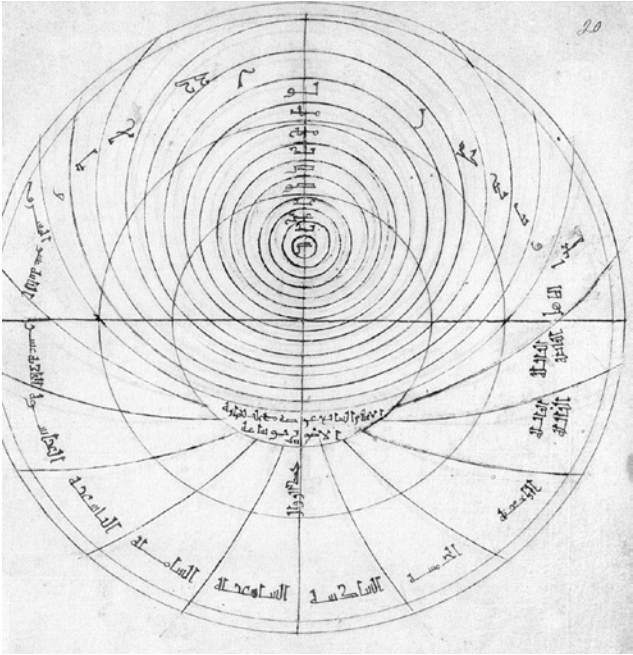


FIGURE 4.9
Plate for the seventh climate in Khalaf ibn al-Mu'adh's astrolabe. The image preserves the projection of the zenith and those of the almucantars (circles of altitude) but not the vertical circles. In the lower part we can see the horary diagram. Ms. BNF Lat. 7412, fol. 20r

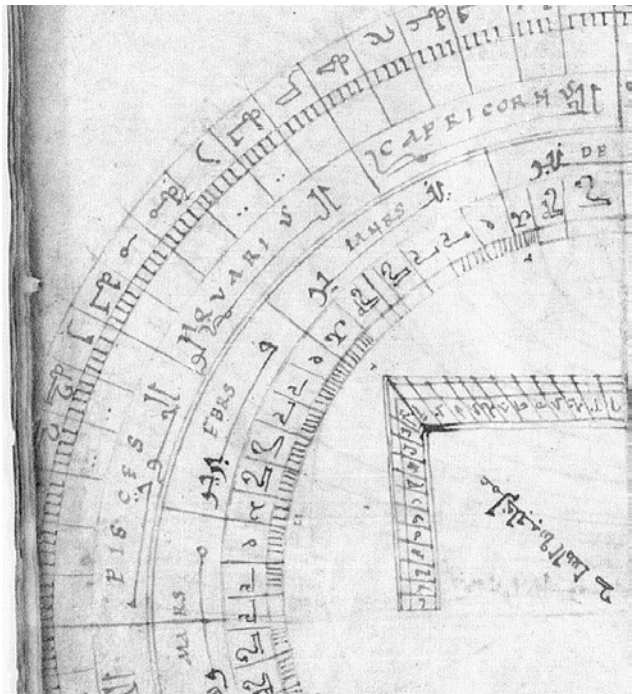


FIGURE 4.10
Part of the back of Khalaf ibn al-Mu'adh's astrolabe. His name is clearly visible along the diagonal of the shadow square. The drawing also shows the zodiacal scale corresponding to the signs of Capricorn, Aquarius and Pisces and to the months of January, February, March (ms. BNF Lat. 7412 fol. 23v)

of his own invention, for projecting the different circles of the celestial sphere on the astrolabe. He describes three new procedures to divide the ecliptic on the astrolabe (Ptolemy gives two more); another three procedures to divide the projection of the horizon, analogous to those used for the ecliptic, which fill a gap in Ptolemy's *Planisphaerium*, and three procedures to project the fixed stars of the rete on the astrolabe, using ecliptic, equatorial and horizontal coordinates. In the second part of his work he uses the only trigonometric tool available to him at that time in order to solve right-angled spherical triangles: Menelaus' theorem, on which he had written a set of notes preserved in a Latin translation.¹⁹⁹ He also deals with the determination of the right ascension of the beginning of each zodiacal sign, using a procedure analogous to the method described above for the division of the horizon using right ascensions, and with the determination of the declination of a celestial body, of the mediation of a star or planet (using al-Battānī's formulas), and of the degree of the ecliptic which rises or sets at the same time as a celestial body. Finally he calculates a table which allows the computation of the arc of a vertical circle comprised between equator and horizon for a latitude of $38;30^\circ$ (Córdoba), while in the first part of his work he uses a latitude of 39° .

Maslama's recension of the *Planisphaerium* was read widely in Medieval Europe. A partial (?) Latin translation of the Ptolemaic work circulated in Catalonia towards the end of the tenth century.²⁰⁰ Hermann of Carinthia translated the Ptolemaic-Maslamian text into Latin in 1143,²⁰¹ and a Hebrew translation appeared later. Even though Maslama's work was not a treatise on the construction of the astrolabe, it had a profound influence on later Islamic and European treatises of this kind. An example of this influence appears in the treatises on the construction and use of the instrument which, as we will see, cannot be attributed to Māshā'allāh (fl. Bagdad 762–ca. 815). These treatises, extant in Latin, were probably the most popular works of their kind in Medieval Europe, and became the main source of Geoffrey Chaucer's (1340?–1400) treatise on the use of the astrolabe for the education of his son Lewis (see 4.2.4.2.2 and 4.2.4.2.3).

Maslama's interest in the astrolabe was reflected in the works on this same instrument written by his disciples Ibn al-Šaffār and Ibn al-Samḥ. The former wrote a brief treatise on the use of the astrolabe²⁰² of a mainly prac-

199 Björnbo & Suter, 1924, pp. 23–24, 39, 79, 83; Lorch, 2001 a, pp. 30, 33, 35, 57, 61, 116–121, 145, 152–153, 360, 410, 421.

200 Kunitzsch, 1993 a.

201 Drecker, 1927.

202 An edition was published by Millás, 1955. Millás had also published a Catalan translation of the text in Millás, 1931, pp. 29–48. Millás' edition was based on only one manuscript

tical character, which enjoyed great popularity in both the Islamic and the Latin worlds. His work was translated twice into Latin, by Plato of Tivoli²⁰³ and Johannes Hispalensis.²⁰⁴ Both texts, as well as ms. 2738 of the Tunis National Library,²⁰⁵ which I have been able to examine, contain a last chapter (absent in the Arabic text edited by Millás in 1955) which describe a calendar diagram placed on the back of the astrolabe that allows the computation of the weekday corresponding to 1 January of a Julian year and the weekdays of the beginnings of each month of the same year. This diagram is also found in at least two Andalusī astrolabes made by Ibrāhīm b. Saʿīd al-Sahli and dated in Toledo in 1067 and 1068.²⁰⁶ This last chapter also contains a series of examples of correspondences between Islamic and Christian dates. The years in question are 413/1023–24, 416/1026, 419/1029, 420/1030, 421/1031, 423/1032 and 424/1033. To this list Johannes Hispalensis' translation adds 427/1036. As Ibn al-Ṣaffār died towards the end of 426 (ca. November 1035), we can conclude that he wrote this treatise in the final stages of his life, when he was staying in Denia.²⁰⁷

His colleague Ibn al-Samḥ wrote a book on the construction of the astrolabe and another one on its use. Only two chapters of the former are extant,²⁰⁸ while the latter contains 129 chapters.²⁰⁹ Due to its length, this book was not as popular as Ibn al-Ṣaffār's treatise but it has an obvious interest, as it contains the most complete treatment of the subject written in al-Andalus and in the Christian kingdoms of the Iberian Peninsula of the Middle Ages. It was well known (cf. 4.1.2) among the astronomers of the court of Alfonso X who used it as their main source for the descriptions of the use of the spherical astrolabe. Besides this, it contains chapters on topics which are not at all common in astrolabe treatises. I am referring here to the use of the astrolabe for questions related to the Moon (determination of its longitude and latitude, the degree of the ecliptic which rises or sets together with the Moon, crescent visibility, etc.) as well as to the planets. These unusual chapters are attributed by Ibn al-Samḥ to a scientist named Ḥanash, probably the well-known Ḥabash al-Ḥāsib

(Escorial 964): as this work has been preserved in many other manuscripts (see an updated list in H.A.T.A no. 39.2) which seem to be more complete, it is obvious that a critical edition is needed in order to be able to evaluate Ibn al-Ṣaffār's work.

203 Edition by Lorch, Brey, Kirschner & Schöner, 1994.

204 Edition by Millás, 1942, pp. 261–284.

205 Ms. BN Tunis 2738 fol. 15v.

206 Hernández, 2018, pp. 71–88.

207 See Martí & Viladrich, 1983, pp. 65–66.

208 Viladrich, 1986 a.

209 An edition of this treatise was prepared by M. Viladrich in her Ph.D. thesis (University of Barcelona, 1984) and remains unpublished. A Catalan translation and commentary appeared in Viladrich, 1986.

(fl. 831–870), and derive from his non-extant book on the use of the instrument. These quotations by Ibn al-Samḥ are the earliest references to Ḥabash in an Andalusī source and give information on a lost work by this author. Finally, Ibn al-Samḥ's treatise shows that Maslama and his disciples were particularly interested in al-Battānī's *zīj*, since, in its instructions for the construction of the zodiacal scale in the back of the astrolabe, he gives the solar longitude (calculated to the nearest half degree) for days 1 and 15 of each Julian month, positions which were probably calculated with the aforementioned astronomical tables.

4.2.2 *The Introduction of the Astrolabe in Catalonia (Tenth Century)*²¹⁰

4.2.2.1 The Texts

The period in which the first Andalusī Arabic texts on the astrolabe appear coincides approximately with the first set of Latin texts on the instrument. These belong to the so-called “old collection” (Burnett)²¹¹ or “old corpus” (Kunitzsch)²¹² to which I have already referred in 2.4, 2.4.2.1.1, 3.2.1.2 and 4.1.1.1. Burnett²¹³ has established the existence of three successive stages in the compilation of the collection. The first one contains the following texts which correspond to crude Latin versions of Arabic materials and to literary *remaniements* of the crude texts:

- The prologue to a treatise on the construction and use of the astrolabe, attributed by Poulle²¹⁴ to Gerbert of Aurillac: inc. *Ad intimas summe philosophie disciplinas ...*²¹⁵
- On the construction of the instrument: *De mensura astrolapsus* (inc.: *Philosophi qui sua sapientia motus siderum ...*) (h' in Millàs);²¹⁶ *De mensura astrolabii* (inc.: *Philosophi quorum sagaci studio visibilium ...*) (h' in Millàs).²¹⁷ To these texts we could add the already mentioned fragment of a Latin translation of Ptolemy's *Planisphaerium*.
- On the use of the astrolabe: *Astrolabii sententiae* (inc.: *Quicumque vult scire certas horas noctium ...*) (J' in Millàs);²¹⁸ *De utilitatibus astrolabii*

210 This part is a revised abridgement of Samsó, 2004 b. It also contains materials from Samsó, 2000.

211 Burnett, 1998.

212 Kunitzsch, 2000.

213 Burnett, 1998, pp. 338–340.

214 Poulle, 1985.

215 Ed. in Millàs, 1931, pp. 271–275.

216 Millàs, 1931, pp. 293–295.

217 Millàs, 1931, pp. 296–299.

218 Millàs, 1931, pp. 275–288.

(J in Bubnov);²¹⁹ *De divisione igitur climatum que fit per almucantarath ...*²²⁰ (Millàs, 1931, pp. 320–322).

– On the celestial sphere: *De orologio secundum alchoram* (J'a)²²¹ (see 4.1.1.1). The second stage in the compilation combined materials of the first phase with others related to the pseudo-Gerbertian *Geometria incerti auctoris*. Burnett does not list the corresponding titles but one may wonder whether this second collection included the following texts:

- Construction: *De astrolabii compositione* (inc.: *Cum hominum habitationes equales sibi fore non patiaturs ...*) (h''' in Millàs);²²² *Incipit compositio astrolapsus secundum Ptolomeum*.²²³
- Use: *Si fuerit nobis propositum invenire quando vel quelibet ...*²²⁴

A third and final stage was the result of Hermann Contractus' (1013–1054) compilation and contains the *De mensura astrolabii* (Hermann's own composition),²²⁵ the *De utilitatibus* (J) and *De horologio viatorum* (see 2.4.2.1.3.3), also Hermann's own composition.

4.2.2.2 Chronology and Geographical Origin of the Collection

These texts seem to have been written in Catalonia towards the end of the tenth century. The chronology was established by Millàs in 1931 as a result of an early dating of manuscript Ripoll 225, which was the basis of his edition. This dating has been disputed by more recent research²²⁶ which tends to believe that the manuscript was copied in the time of Oliva, abbot of Ripoll between 1008 and 1046. In spite of this, and even though other manuscripts have been known and studied²²⁷ (thus diminishing the importance of the Ripoll manuscript) Borst has argued that the old collection already existed ca. 995 in the circle of Constantin de Fleury, dean of the monastery of St Benoit de Fleury (988–996) and later abbot of the same monastery (1011–1120/21).²²⁸ Besides, I have offered arguments showing that one of these texts (*De mensura astrolabii*), at least, was written after 978.²²⁹ This implies that a part of the collection, at least, was written between ca. 978 and ca. 995, two dates that agree surprisingly well with

219 N. Bubnov, 1899, pp. 114–147.

220 Millàs, 1931, pp. 320–322.

221 Millàs, 1931, pp. 288–290.

222 Millàs, 1931, pp. 308–315.

223 Millàs, 1931, pp. 322–324.

224 Millàs, 1931, pp. 324–327.

225 Drecker, 1931.

226 Beaujouan, 1985; Borst, 1989; Burnett, 1998.

227 See two lists in Borelli, 2008, pp. 226–237.

228 A. Borst, 1989.

229 Samsó, 2000.

the period during which Lupitus Barchinonensis/ Seniofredus (fl. 975–995)²³⁰ was archdeacon and later archpriest of the Cathedral of Barcelona. This is important because it is well known that he corresponded with Gerbert of Aurillac (945–1003), and he has been considered to be the author of the earliest translations of astronomical texts from Arabic into Latin.

The geographical origin of the “old corpus” has been discussed since the early 1930s. Thomson²³¹ and Welborn²³² believed that this early introduction of Arabic scientific materials took place in Lotharingia (Lorraine) as a result of the embassy sent to Córdoba in 953 by Oton the Great (936–973), whose leader was the monk John of the Abbey of Gorze in the Lorraine. John stayed in Córdoba during three years and met scholars like Bishop Recemundus and the Jewish counsellor of the Caliph ‘Abd al-Raḥmān III, Ḥasday ibn Shaprūt. When John returned to Gorze, he took with him a collection of scientific manuscripts which led to the scientific revival of the Lorraine towards the end of the tenth century. The influence of the region spread southwards and was the origin of the treatises on the astrolabe written by Ascelin of Augsburg²³³ and his disciple Hermann Contractus of Reichenau (1013–1054).

Millás, in his *Assaig*, and Van de Vyver²³⁴ favoured the Catalan origin of the collection and gave a simpler explanation of the process of transmission, for which they emphasised the role of Lupitus – who would have translated these texts with the help of Mozarabs – and of Gerbert of Aurillac, who had been a student in Vic between 967 and 970 and was considered as the main vehicle of the introduction of these scientific materials into the other side of the Pyrenees. Later research downplays the role of these two figures and both Borst²³⁵ and Burnett²³⁶ have doubts about this origin, due to the provenance of the earliest extant manuscripts, and emphasise the importance of the region near Orléans.

I will, however, defend a Catalan origin of the collection with the following arguments:

The obvious relation between the old corpus and the Carolingian astrolabe (see below), which is clearly Catalan.

The texts have linguistic features which correspond both to the Latin used by Mozarabs coming from the eastern part of the Iberian Peninsula, south of

230 Pratt Latin, 1932.

231 Thomson, 1929.

232 Welborn, 1931.

233 Burnett, 1998, pp. 343–368.

234 Van de Vyver, 1936.

235 Borst, 1989.

236 Burnett, 1998.

Catalonia (*Sharq al-Andalus*) and to the Arabic dialect of the same area, as shown by the transliterations of Arabic words.²³⁷

The worked examples given in the *Astrolabii sententiae*, *De utilitatibus astrolabii*, *De horologio secundum alkoram* (see 4.1.1.1), and *De mensura astrolapsus*, give or imply values of the local latitude comprised between 41° and 42;25° which fit fairly well the latitude of Barcelona (modern value 41;25°)²³⁸ but are unacceptable for localities in northern France or southern Germany. The only exception is the *Incipiunt regule de quarta parte astrolabii* (see 2.4.2.1.2.1) in which the two values of the latitude mentioned are 30° and 53°.

4.2.2.3 The Sources

Little progress has been made in the task of establishing which Arabic sources were actually translated into Latin. Leaving aside the fragments of a translation of Ptolemy's *Planisphaerium*, and the fact that the astrolabe of Khalaf ibn al-Mu'ādh seems to have been the source of the bilingual table of climates in the Ripoll manuscript,²³⁹ we know only that the *Astrolabii sententiae* contains:²⁴⁰

1. An introduction which does not seem to be a translation from the Arabic, but an original text written by the compiler.
2. A description of the astrolabe made by a person who had, in front of him, an instrument of this kind and who was helped by somebody who could identify the different pieces and understand the corresponding terminology.
3. Instructions for the use of the astrolabe which are partially based on a treatise of al-Khwārizmī (fl. ca. 830). The text is sometimes a translation and at other times a commentary on the Arabic source. There is obviously a process of adaptation of the Eastern source as, in this text, the latitude involved in the examples is 42°.

In spite of this, it seems that al-Khwārizmī's treatise was not the only source used for the compilation of the *Astrolabii sententiae*. When this latter text deals with the problem of the determination of the solar degree for a given date of the solar year, two solutions are offered: one of them is an approximate arithmetical rule to calculate the solar longitude which could derive from an Eastern source, although it does not appear in al-Khwārizmī's book, while the

²³⁷ Samsó & Fórneas, 1980.

²³⁸ Medieval Islamic values collected by Kennedy & Kennedy, 1987, p. 67 give values of 42° and 42;18° for the latitude of Barcelona. To this I should add that the "Carolingian" astrolabe (4.2.2.4) has a plate for a latitude of 41;30° which, in my opinion, corresponds to the latitude of Barcelona.

²³⁹ Kunitzsch, 2000 a.

²⁴⁰ Kunitzsch, 1987.

second one uses the zodiacal scale on the back of the instrument which, as we have already seen, is a characteristic feature of Andalusī and Maghribī astrolabes and does not appear in Eastern instruments of this kind until a later date.

4.2.2.3.1 *Maslama al-Majrīṭī and the Star Table in the De mensura astrolabii*

We can find a second Arabic source in the works on the astrolabe authored by Maslama and his disciples. Millàs proposed a direct dependence of our text on the pseudo-Māshā'allāh's treatise on the construction and use of the instrument, and we will see (4.2.4.2.2) that, according to Kunitzsch, Māshā'allāh should be identified with Maslama. Besides, Kunitzsch himself believes that there is a link between these early Latin treatises on the astrolabe and the works of Maslama and his school.²⁴¹ Kunitzsch's remark appears in a context in which he has mentioned Maslama's star table dated in 367/978 and appended to Maslama's notes on the *Planisphaerium*,²⁴² in which 20 stars also appear in the table of 27 stars of the *De mensura astrolabii* (h').²⁴³ This led me, some years ago, to analyse in detail the possible relations between the two star tables.²⁴⁴

The *De mensura astrolabii* gives a confusing explanation of the procedure to project the stars on the rete, but it can be interpreted with the explanation of the same method which we find in similar texts: the *Compositio astrolabii*, written towards the beginning of the eleventh century by Ascelin of Augsburg who knew the contents of h',²⁴⁵ Hermann Contractus' *De mensura astrolabii*²⁴⁶ and a fourth text on the construction of the instrument which Millàs attributed to Māshā'allāh, although this attribution is probably mistaken²⁴⁷ and this may in fact be another derivation of Maslama's school.²⁴⁸ The method described in this text has been analysed by John North²⁴⁹ and by Mercè Viladrich and Ramon Martí,²⁵⁰ and is clearly related to the star table in h'²⁵¹ and to all the aforementioned texts which present the same method for the star projection.²⁵² This table assigns two coordinates to each star, named *latitudo* (α) and *altitudo* (β). In most cases the *latitudo* is a rounded value of the mediation (*mediatio*

241 Kunitzsch, 1997, p. 196).

242 Kunitzsch, 1966, Typ. I (pp. 15–18); Vernet & Catalá, 1965, p. 45; Kunitzsch, 1980.

243 Millàs, 1931 p. 300.

244 Samsó, 2000.

245 Burnett, 1998, pp. 348–351, 355–358; Kunitzsch, 2000 b.

246 Drecker, 1931, pp. 208–210.

247 Millàs, 1942, p. 319.

248 Kunitzsch, 1981, p. 49, fn. 35; Viladrich & Martí, 1981, pp. 70–71.

249 North, 1976, pp. 159–161.

250 Viladrich & Martí, 1981, pp. 70–71. See also Kunitzsch, 2000.

251 Millàs, 1931, pp. 301–302; Kunitzsch, 1966, no. 111, pp. 23–30.

252 Kunitzsch, 1999–2000; Kunitzsch, 2000; Kunitzsch, 2000 b.

coeli/ tawassuṭ al-samāʿ, degree of the ecliptic which crosses the meridian together with the star) that we find in Maslama's star table which, according to the only manuscript that preserves it, is the result of Maslama's own observations made at the end of 367/978.

In Fig. 4.11, circle *BAEC* is the projection of the circle of Capricorn and circle *FG* that of the ecliptic. Radius *OE* is the projection of the hour circle passing through the beginning (*G*) of the zodiacal sign which corresponds to a given star. Our first coordinate (α) will be taken *on the ecliptic* (arc *GF*) according to Ascelin of Augsburg and Hermann Contractus: we will thus obtain point *F* – also on the ecliptic – which is the star's mediation. Although the text in treatise *h'* is not easy to understand, it seems to agree with Ascelin's procedure. A different technique appears in pseudo-Māshā'allāh's treatise which seems to imply that α is measured *on the circle of Capricorn*.²⁵³ This would indicate that we are measuring angle α not from point *G* to obtain *F*, but from *E* to obtain *A*. Having established the position of point *A* with one of the two methods, the second coordinate, *altitudo* (β), is measured *on the circle of Capricorn*, from point *A* in both the direct and retrograde sense, so that $AB = AC = \beta$. *B* is then joined to *C* by chord *BC*. The intersection of *BC* and *OA* in point *D* determines the projection of the star and *OD* will be the radius of the projection of the parallel of declination of the same star.

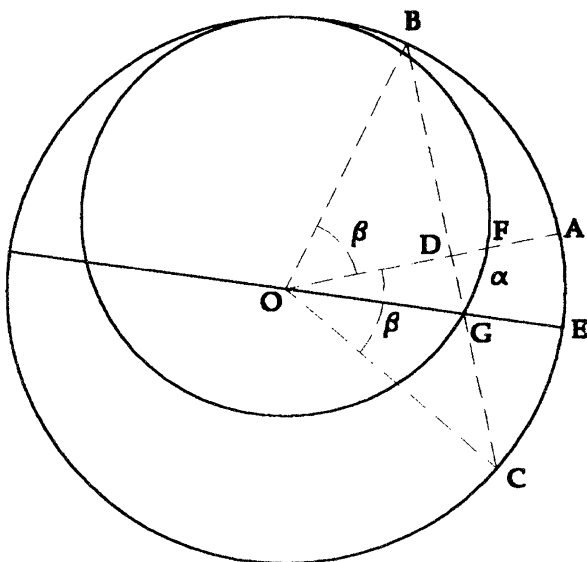


FIGURE 4.11
Projection of a fixed star on
the rete according to *h'*

²⁵³ Prof. Paul Kunitzsch (letter dated 21.10.1999) disagrees with this interpretation and believes that the pseudo-Māshā'allāh's text explains the same procedure as the other sources.

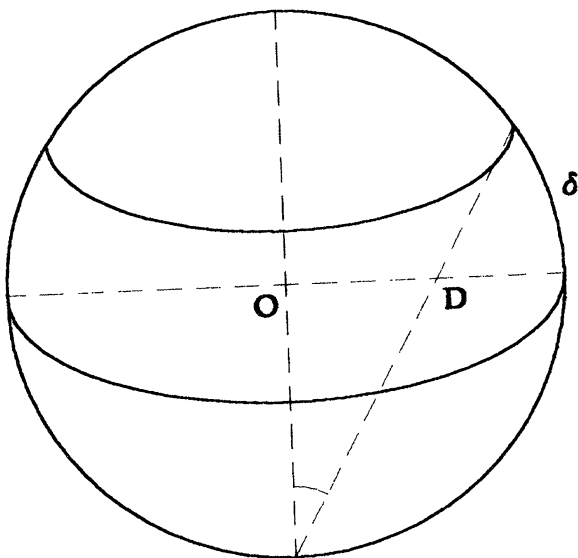


FIGURE 4.12
Procedure for the
computation of the *altitudo*

In order to determine the value of β we should consider Fig. 4.12, in which:

$$OD = r \tan \left(\frac{1}{2} (90^\circ - \delta) \right)$$

r being the radius of the equator which we take as $r = 1$.

δ is the declination of the star.

On the other hand, in Fig. 4.11:

$$OD = R \cos \beta$$

where R is the radius of the circle of Capricorn, which will measure for $r = 1$, and for an obliquity of the ecliptic $\varepsilon = 23;35^\circ$, the value used by Maslama:

$$R = \tan \left(\frac{1}{2} (90^\circ + 23;35^\circ) \right) = 1.528.$$

If we accept that:

$$OD = R \cos \beta = r \tan \left(\frac{1}{2} (90^\circ - \delta) \right)$$

then:

$$\beta = \cos^{-1} \left(\left(\frac{r}{R} \right) \tan \left(\frac{1}{2} (90^\circ - \delta) \right) \right)$$

β obviously depends on the value used for the star's declination. As coordinate α seems to agree with the degree of mediation of the star given in Maslama's table, we should now check whether we can obtain a similar agreement for coordinate β by recomputing the values of h' using Maslama's declinations.

The results for both coordinates can be seen in table 4.4, in which the star numbers are the ones we find in Kunitzsch's *Typen*.²⁵⁴ This table requires some explanations: the table of h' contains 27 stars, while Maslama's table has only 21, 20 of which are also found in h' . Of these 20 common stars, the comparison is impossible in three cases, for which Maslama only gives the longitude and latitude of the star and not its mediation and declination. My conclusions are, therefore, limited to the other 17 stars.

The results obtained are good approximations to the values of the column *altitudo*: in seven cases the table's *altitudo* is a correct rounding of the recalculated value, in three others we have truncated values and, finally, in three the error reaches a maximum of 1° . The other four values (marked with an asterisk in table 4.4) are clearly wrong. The results can, therefore, be considered satisfactory in a table which, probably, has many erroneous values and one may say that the two coordinates of the text h' have a clear relation to a star table of a Maslamian origin.

To this, one should add that the oldest version of the star table in h' is the one found in ms. BN Paris 7412 (fol. 5v) in which we find the values of Maslama's mediation and a second coordinate which is not β but the meridian altitude of the star for a latitude 39° , probably a rounding of $38;30^\circ$, the standard latitude of Córdoba, the city in which Maslama and his disciples lived and worked.

My hypothesis, therefore, is that the star table in h' derives from an Arabic source belonging to Maslama's school, although not from the actual table appended to Maslama's notes on the *Planisphaerium*. This Arabic source was a table containing 27 stars, at least, as well as all the necessary elements for projecting these stars on the rete of an astrolabe using different methods. This table did not necessarily contain the longitudes and latitudes of the stars but had, at least:

- A column for the mediation of each star (coordinate α)
- A second column with the star's declination

²⁵⁴ Kunitzsch, 1966, p. 28.

TABLE 4.4 The star table in the “De mensura astrolabii” (*h*) and Maslama’s star table

Star name	Latitudo (α)	Maslama’s mediation	Maslama’s declination	Altitudo (β)	β recalculated
1. Alramech/ al-Simāk al-Rāmiḥ (α Bootes)	[Libra] 24° [= 204°]	203;52°	+24;56°	65°	65;19°
3. Alhawi/ Ra’s al-Ḥawī (α Ophiuchi)	[Sagit.] 24° [= 264°]	264°	+8;30°	57°	55;40°
4. Altair/ al-Nasr al-Ṭā’ir (α Aquilae)	[Capr.] 14° [= 284°]	283;18°	+6;25°	54°	54;11°
6. Alferat/ Mankib al-Faras (β Pegasi)	[Aquar.] 30° [= 330°]	331;30°	+22;29°	65°	64;4°
8. Alrif/ al-Ridf (α Cygni)	[Capr.] 29° [= 299°]	299;44°	+42;4°	73;30°	73;5°
9. Wega/ al-Nasr al-Wāqī’ (α Lyrae)	[Capr.] 1° [= 271°]	270°	+38;25°	72°	71;34°
10. Benenas/ Qā’id Banāt Na’sh (η Ursae Maioris)	[Libra] 18° [= 198°]	197;35°	+55;30°	74°	*78;16°
11. Calbagrab/ Qalb al-‘Aqrab (α Scorpii)	[Scorp.] 25° [= 235°]	234;18°	−23;5°	14°	*7;53°
12. Alcimek/ al-Simāk al-‘Zal (α Virginis)	[Libra] 8° [= 188°]	187;30°	−5;10°	41°	*44;14°
14. Alhabor/ al-Shi’rā al-‘Abūr (α Canis Maioris)	[Cancer] 1° [= 91°]	90;20°	−15;35°	32°	30;27°
15. Rigel/ Rijl al-Jawzā’ (β Orionis)	[Gemin.] 8° [= 68°]	66;29°	−10;29°	39°	38;7
19. Aldeuaran/ al-Dabarān (α Tauri)	[Taur.] 27° [= 57°]	56;34°	+14;12°	41;30°	*59;22°
20. Malgevze/ Mankib al-Jawzā’ (α Orionis)	[Gem.] 18° [= 78°]	76;21°	+6;48°	55°	54;28°
21. Algoize/ al-Shi’rā al-Ghumayṣā’ (α Canis Minoris)	[Cancer] 11° [= 101°]	100;40°	+6;56°	56°	54;34°
22. Aldiraan/ Muqaddam al-Dhirā’ayn (α Hydrae)	[Leo] 6° [= 126°]	125;49°	+2;25°	52°	51;8°
23. Calbalaze/ Qalb al-Asad (α Leonis)	[Leo] 19° [= 139°]	134;20°	+16;27°	61°	60;43°
25. Alhaioc/ al-‘Ayyūq (α Aurigae)	[Gemin.] 3° [= 63°]	62;55°	+43;50°	74°	73;48°

As I believe that the method for the projection described in the pseudo-Māshā'allāh's text seems to take coordinate α on the circle of Capricorn, I suggest that the presumed Arabic source also had a column with the right ascension of the mediation of each star. This hypothesis receives some support in one of the two extant chapters of Ibn al-Samḥ's treatise on the construction of the astrolabe²⁵⁵ which says that, when applying the standard method (mediation and declination) for the projection of stars on the rete, one may use the right ascension of the mediation instead of the mediation itself: this will give a better precision because the projection of the circle of Capricorn is larger than that of the ecliptic, and dividing it is easier.

In agreement with ms. BN Paris 7412, a fourth column should give the meridian altitude of each star for the latitude of Córdoba.

4.2.2.3.2 *Translations from the Arabic or Descriptions of an Instrument?*

In a book published in 2008, Arianna Borelli²⁵⁶ proposed important changes in the way in which these early Latin treatises on the astrolabe have been considered until recent times. She notes that only two sources (al-Khwārizmī and Ptolemy's *Planisphaerium*) have been identified so far, to which I should add my own hypothesis about Maslama's star table. The number of cases in which we can be sure of the existence of an actual translation is very limited. Besides, as Borelli explains, the state of the texts in the different manuscripts makes it extremely difficult to prepare a critical edition of the corpus: they are often inexact and contain errors (something that is unlikely if they are indeed genuine translations). In her own words, the astrolabe manuscripts of the old collection "are the written traces of a process of transmission and assimilation of knowledge which took place thanks to a combination of written and non-written, verbal and non-verbal strategies of communication".²⁵⁷ According to her, one should not consider that the extant texts of the early collection are all translations of Arabic originals or revisions of earlier translations; rather they are notes taken during or after (Borelli emphasises the importance of memory) an oral explanation in which drawings were used. One of the strengths of this book is its careful study of illustrations.²⁵⁸

I fully support this theory and I will comment on two different aspects of the author's work:

²⁵⁵ Viladrich, 1986 a.

²⁵⁶ Borelli, 2008.

²⁵⁷ Borelli, 2008, p. 99.

²⁵⁸ Borelli, 2008, pp. 118–129.

1) Borelli underlines²⁵⁹ the insistence of the texts on the need to memorise the Arabic terms of the parts of the astrolabe, something which, in my opinion, is extremely important. When one reads, for example, the *Astrolabii sententiae* or *De utilitatibus astrolabii* one is somewhat puzzled by the use of Arabic terms (in some cases even a full sentence) which are totally unnecessary because they are immediately followed by a Latin translation. In the description of the instrument of the *Astrolabii sententiae*,²⁶⁰ the text states that the “HOTOT ALZACAPH [= *khuṭūṭ al-sāʿāt*] sunt linee horarum”, the hour line of the prayer of HALDOAR (*al-ḡuhr*) “est hora sexta” and that of the ALGAZAR (*al-ʿaṣr*) begins in the ninth hour, the meridian line is called VATIALZEME (*wasat al-samāʿ*) and “linea medii celi” or “linea meridiei”, the ring from which the instrument is hanged is called ALHEYLCHA (*al-ʿilāqa*) and “ansa”. The nail which fixes the mother of the instrument to the astrolabe plates and the rete is called ALMUZMAR/ ALMUZAMAR/ ALMIZAMAR/ ALMUTMAR (*al-mismār*) but also “cardinem vel clauum”. The graduated crown which surrounds the astrolabe’s mother receives the name of ALNOGIZA (*al-ḥujra*) “id est umbo” etc. This excessive use of Arabisms reaches its peak when complete sentences in Arabic are quoted, although they are immediately followed by a Latin translation. As the projection of the ecliptic is unequally divided, the semicircle from Libra to Pisces is greater than the semicircle from Aries to Libra, something explained in our text with the following words: “Et maior semicirculus incohatur AVVILALMIZENI ILIGAIR ALHAUT [= *awwal al-Mizān ilā ākhir al-Ḥūt*]”, translated immediately by “id est de capite libre usque ad postremum piscis”. The same thing happens with the titles of some chapters of this treatise: “Beberti faaschemz [*bāb irtifāʿ al-shams*, “chapter on the solar altitude”] id est titulus apprehensione altitudinis solis”²⁶¹ or “Titulus de ezmen [*azmān* = times] id est scientia temporis horarum dierum”.²⁶²

2) A second aspect is, as we have seen, Borelli’s insistence on the importance of illustrations in the process of transmission, to which I would add the use of real astrolabes. The relation between illustrations and the use of Arabic words is clarified by the explicit of the description in the *Astrolabii sententiae* where we read that the user of the astrolabe should know and understand the *names* of the different parts of the instrument, which I understand to refer to the Arabic names and not to the Latin ones: “Et qui disciplinatus vult fieri

259 Borelli, 2008, p. 110.

260 Millàs, 1931, pp. 276–279.

261 Millàs, 1931, p. 280.

262 Millàs, 1931, p. 283.

in ipsa tabula, debet scire et intelligere ea nomina sicut iam explanavimus, et dum ea scierit quanta est utilitas illorum probabit".²⁶³

An interesting passage at the end of the *Astrolabii sententiae* bears the title "Hec est figura interpretationis verborum arabicorum in latinum que sunt in astrolapsu."²⁶⁴ It describes a figure with three concentric circles which display, starting from the outer one, the following:

1. The names of the Arabic letters (*nomina litterarum arabicarum*) in transliteration.
2. The same letters in Arabic script.
3. Numerical value of the letters in circles 1 and 2 according to the Arabic alphanumerical notation (*abjad*).

This description agrees with two illustrations extant in two manuscripts containing the old corpus. The better one appears in MS Chartres 214²⁶⁵ – destroyed during the Second World War, although some photographs remain. A second, less complete, illustration is to be found in ms. British Library Old Royal 15 B, IX, fol. 71r (BM).²⁶⁶ In both drawings we can see the three concentric circular crowns I have just described. The BL manuscript has only seven unsuccessful attempts to draw the Arabic letters of the second circle, while in the Chartres manuscript, the scribe has tried to complete the series although, in many cases, instead of letters, we have a collection of unintelligible scrawls which cannot be compared to the drawings of Ibn al-Mu'ādh's astrolabe of ms. MS Paris BN lat. 7412. The author of the illustration of the BL manuscript has not filled in all the cases and more than one quadrant is empty, while the Chartres diagram is complete. Besides this, the creators of both drawings had some difficulty in understanding what they were doing: in the BL manuscript we find frequent displacements and discordances between the *abjad* crown and the crown containing the Roman numerals, while in the Chartres manuscript, the scribe has omitted the graduation 10° and, as this graduation proceeds in steps of 5° , there is a mismatch of 5° between the first and the second crown until graduation 300° .

In fig. 4.13, inside the three crowns we can see that an astrolabe plate has been drawn, containing the projections of the Capricorn circle, equator and Cancer circle, horizon, zenith and circles of altitude as well as the hour diagram, in which the hours are identified with Roman numerals and with the

²⁶³ Millàs, 1931, p. 279.

²⁶⁴ Millàs, 1931, p. 288.

²⁶⁵ Van der Vyver, 1936; Burnett, 1998, p. 356.

²⁶⁶ Millàs, 1931, plate XII.



FIGURE 4.13
Ms. Bibliothèque de
Chartres 214

transliterated Arabic names of the hours. In fig. 4.14 only the circles of Cancer, equator and Capricorn and the horizon appear.

These illustrations, and the excellent reproductions of Ibn al-Mu‘ādh’s astrolabe in ms. Paris B.N. Lat. 7412, show, in my opinion, that there were actual Arabic astrolabes circulating (the case of the “Carolingian” astrolabe translated into Latin is exceptional, see 4.2.2.4). Towards 1025 Radolphus of Liège wrote to Ragimboldus of Köln informing him that he would like to send him his astrolabe, but that he only had one instrument of this kind and that he was using it as a model to make a copy. Therefore, he invited Ragimboldus to visit Liège in order to see it there.²⁶⁷ The problem posed by this reference is to establish whether the copy Radolphus was preparing was a real instrument or a series of drawings like those of ms. Paris BN Lat. 7412. Besides, the texts of the old corpus contain information about some details of the Arabic astrolabes available: they were Andalusī (or perhaps Maghribī) astrolabes because the *abjad* system described is Western and because the zodiacal scale was traced on the back of the instrument, as we have seen in Ibn al-Mu‘ādh’s astrolabe and we will find in the Carolingian astrolabe. Besides, this calendric scale is mentioned in

²⁶⁷ Beaujouan, 1972.

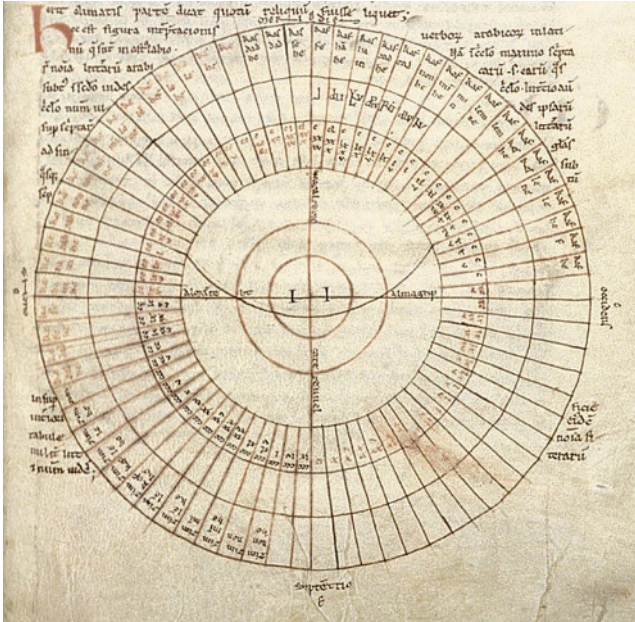


FIGURE 4.14
Ms. British Library
Royal 15 B, IX

the *Astrolabii sententiae*,²⁶⁸ the *De mensura astrolabii*,²⁶⁹ in the summarised description of the instrument (*h^m*) whose *incipit* is *Cum hominum habitaculones equales sibi fore*²⁷⁰ and in the pseudo-Gerbertian *De utilitatibus astrolabii*.²⁷¹ This reference is accompanied, in the *Sententie* and in the *De utilitatibus*, by an arithmetical rule to calculate the solar position as a function of the number of days since the spring equinox on the basis of a uniform motion of the Sun of 1° per day. In both sources the date of the spring equinox is 16 March: in the Carolingian astrolabe the spring equinox is 15²⁷² or 16 March;²⁷³ 16 March reappears in the *Calendar of Córdoba* (tenth century), according to the *Mumtaḥan* astronomical system,²⁷⁴ in the drawings of Ibn al-Mu‘ādh’s astrolabe and in Hermann Contractus’ treatise on the astrolabe.²⁷⁵ To these data we should add that the instrument described in the *Astrolabii sententiae* had the projection of

268 Millàs, 1931, pp. 279, 282.

269 Millàs, 1931, pp. 298–299.

270 Millàs, 1931, pp. 312–313.

271 Bubnov, 1899, pp. 124–127.

272 King, 1995 a, p. 377.

273 Turner, 1995, pp. 427–429.

274 Viladrich, 1996.

275 Drecker, 1931, pp. 210–211.

the hour lines corresponding to the prayers of the *zuhr* and the *ʿaṣr*.²⁷⁶ Besides, we find references to an interval of 6° between almucantars,²⁷⁷ although the pseudo-Gerberian *De utilitatibus*²⁷⁸ and the treatises of Ascelin of Augsburg,²⁷⁹ and Hermann Contractus²⁸⁰ mention intervals of 5°.

Whatever the case, the information gathered so far bears witness to the interest of the author of the *Astrolabii sententiae* in teaching his readers how to use an Arabic astrolabe, and that to do so they needed to know the Arabic technical terminology and be able to identify it in the instrument itself. In agreement with Kunitzsch's previous hints and Borelli's opinions, I suggest that a part, at least, of the texts of the old corpus are not translations from the Arabic, but descriptions of a real instrument which the author had in front of him. The clearest reference in this respect is the text placed at the end of ms. Ripoll 225 whose *incipit* is *De divisione igitur climatum que fit per almucantarath*.²⁸¹ In it, the author states that, as he did not have a master, he has written this booklet explaining what he has seen in a certain instrument named *oroscopo* (*quia non habuimus magistrum ita in hoc posuimus libello quemadmodum in quodam vidimus oroscopo*). The reference to a *magister* is important, because it implies that the authors of most of the texts of the corpus needed the help of a scholar who knew the astrolabe and had a certain knowledge of Arabic. We do not know the identity of these masters or whether they were Mozarabs, Jewish scholars or Muslim slaves, but we do know that they must have existed. This channel of transmission of scientific knowledge does not exclude the possible existence of other channels such as the diplomatic ones proposed by Zuccato,²⁸² as we will see in 4.2.2.4.

4.2.2.4 The "Carolingian" Astrolabe

In 1961, Marcel Destombes, a French merchant navy officer and collector of astronomical instruments, bought an interesting astrolabe with inscriptions in Latin (fig. 4.15) from the antiquarian Gilbert Suc in Paris. Suc had acquired it from a Spanish "collector" who was living in southern France, and, although we have no reliable information, it seems that the price he paid was extremely low. A Spaniard living in the south of France in the early 1960s was most likely a political refugee living there since the end of the Spanish Civil War in 1939,

²⁷⁶ Millàs, 1931, p. 277.

²⁷⁷ Millàs, 1931, pp. 280–281, 292, 321.

²⁷⁸ Bubnov, 1899, p. 129.

²⁷⁹ Burnett, 1998, p. 354.

²⁸⁰ Drecker, 1931, p. 206.

²⁸¹ Millàs, 1931, pp. 320–322.

²⁸² Zuccato, 2005.



FIGURE 4.15
The “Carolingian”
astrolabe preserved in
the Institut du Monde
Arabe (Paris)
© MUSÉE DE L’INSTITUT
DU MONDE ARABE,
PARIS / PHILIPPE
MAILLARD

and the story of this sale makes me suspect the possibility of the despoilment of a private collection during the civil war, at a time in which García Franco’s catalogue had not yet been published.²⁸³

Destombes immediately published an important paper²⁸⁴ describing his discovery which he called the “Carolingian astrolabe”, and gave it an early date, towards the end of the tenth century. He compared it with all the early Islamic astrolabes and early European astrolabes known to him at the time. He considered that it had been made in Catalonia and linked it to the old corpus of Latin texts on the astrolabe published by Millàs Vallicrosa. Besides, he argued that the astrolabe had plates serving five latitudes, three of which were related to Ptolemaic climates: 36° (in the mother) is the beginning of the fourth climate, while 45° and 47° (on both sides of the same plate) corresponded to the beginning and the midpoint of the sixth climate (fig. 4.16). One should add that the diagrams of these three latitudes corresponded to the old style, which represented (as in Ibn al-Mu‘ādh’s astrolabe) the horizon, the hour diagram and the circles of altitude but not the vertical or azimuthal circles (*dawā’ir*

²⁸³ García Franco, 1945.

²⁸⁴ Destombes, 1962.

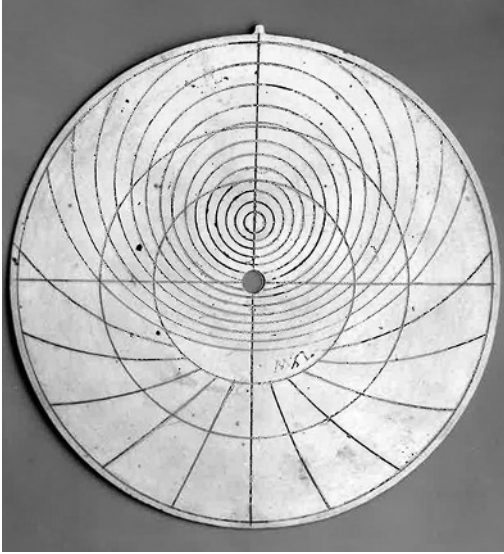


FIGURE 4.16

Plate for latitude $47;30^{\circ}$ (MZL), in the “Carolingian” astrolabe, with the projection of almucantars and hour lines but not the azimuthal curves

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al-sumūt). The third plate, on the other hand, was privileged, as it contained the projection of the vertical circles on both sides, and served the latitudes of two specific cities: $38;30^{\circ}$ (Córdoba) and $41;30^{\circ}$ (Barcelona) (fig. 4.17). The side for the latitude $41;30^{\circ}$ bore the inscription ROMA ET FRANCIA (fig. 4.18) and it is clear that this latitude agrees very well with the latitude of the city of Rome as well as that of Barcelona and Zaragoza.²⁸⁵ The problem was to understand the meaning of FRANCIA which could not correspond to any city in France.

The publication of Destombes' article in 1962 sparked a lengthy dispute on the authenticity of the instrument.²⁸⁶ On his death in November 1983, his collection of astronomical instruments was given to the Institut du Monde Arabe in Paris, and the astrolabe lay forgotten in the museum's storehouse. Jeanne Mouliérac published an excellent catalogue²⁸⁷ of the collection in 1989 which did not include the so-called “Carolingian” astrolabe, mentioned only in passing: “A ce groupe d'instruments islamiques, s'ajoute un astrolabe qui, selon M. Marcel Destombes, serait carolingien et aurait un rapport avec l'origine de nos chiffres arabes”. A series of events changed this attitude completely: on 26 January 1990 the Société Internationale de l'Astrolabe held a meeting at the Institut du Monde Arabe in Paris, at which the astrolabe was discussed, and highly qualified scholars took different sides in the debate: David A. King

²⁸⁵ See a complete description of this instrument in Hernández, 2018 a, pp. 271–278.

²⁸⁶ Turner, 1995a.

²⁸⁷ Mouliérac, 1987–89.

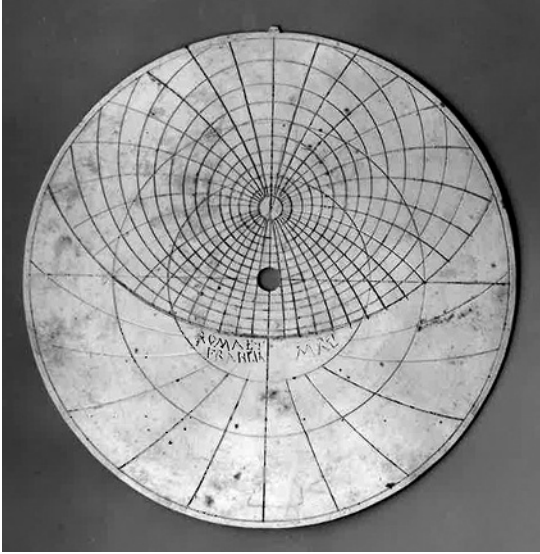


FIGURE 4.17
The plate for latitude 41:30°:
"ROMA ET FRANCIA"
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DU MONDE ARABE, PARIS /
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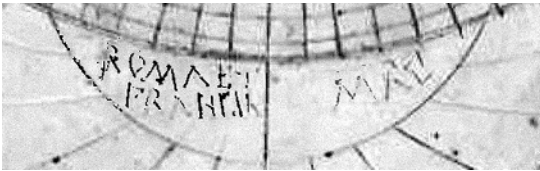


FIGURE 4.18
Detail of the inscription
"ROMA ET FRANCIA"
© MUSÉE DE L'INSTITUT
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(J.W. Goethe University in Frankfurt) and Paul Kunitzsch (Munich University) favoured the authenticity of the instrument, while Guy Beaujouan (École des Hautes Études de Paris) and Emmanuel Poulle (École des Chartres) rejected it. Two years later the astrolabe was exhibited at the National Archaeological Museum of Madrid,²⁸⁸ and, in 1993, an international symposium on the instrument was held in Zaragoza²⁸⁹ with the participation of historians of medieval astronomy (Bruce S. Eastwood, Guy Beaujouan, Emmanuel Poulle, Anne Tihon, Julio Samsó), paleographers (Wesley Stevens, Anscari M. Mundó), experts in scientific instruments (Anthony J. Turner, Raymond d'Hollander, Gerard L'E Turner) and in analysis of metals (Jean Noël Barrandon). In addition, the collector Jan de Graeve analysed Destombes's archives and, especially, his correspondence with the great Belgian expert Henri Michel.²⁹⁰

²⁸⁸ Vernet & Samsó, 1992.

²⁸⁹ The proceedings of this symposium, edited by W.M. Stevens, were published under the title of *The Oldest Latin Astrolabe* in *Physica* 22 (1995), 450 pp.

²⁹⁰ Graeve, 1995 (including eight plates with photographs of pages of Destombes' notebook).

The conclusions of this symposium mainly supported the authenticity of the Carolingian astrolabe and of its early date. The most compelling argument in its favour was provided by Mundó, who analysed each one of the letters of the instrument and proved that they had exact parallels in letters appearing in Catalan epigraphy of the tenth century.²⁹¹ Besides, the meaning of the inscription *FRANCIA* was clarified: confirming Destombes's opinion, it refers to the *Marca Hispanica*, something which does not make much sense in Latin sources, but it is easy to understand in a medieval Arabic context. *Francia* is a translation of *Ifranja*, a geographical denomination, in Arabic sources of the tenth-twelfth centuries, not only of France but of the lands of the Christian kingdoms in the north-east of the Iberian Peninsula.²⁹²

In my opinion, this astrolabe is a translated copy of an Andalusī Arabic astrolabe made especially as a ceremonial present on the occasion of an embassy to Córdoba sent by the Count of Barcelona, or as the result of the visit to Barcelona of Ḥasdāy ibn Shaprūt in 940. This theory ties in with the hypothesis formulated by Marco Zuccato related to the arrival of Arabic scientific materials in Catalonia through diplomatic channels,²⁹³ and would explain the presence of the privileged plate traced for the latitudes of Córdoba, Rome and Barcelona. A plate for the latitude of Córdoba in an Andalusī astrolabe is obviously common, but the same cannot be said about Rome and Barcelona. Indeed, the presence of these cities on both sides of the same plate could have a symbolic character.

On the other hand, it seems that the craftsman who copied the Arabic original astrolabe was new to this work, as he was forced, several times, to draw a circle twice. He was probably helped by somebody who could interpret the Arabic inscriptions and, instead of Roman numerals, he used an alphanumerical notation which corresponds exactly to the Arabic Western *abjad*: A = *alif* = 1, B = *bā'* = 2, C = *jīm* = 3, D = *dāl* = 4, etc.²⁹⁴ The latitude for 41;30° is marked as MA L, in which M corresponds to the Arabic *mīm* (40), A to *alif* (1) and L to *lām* (30). The rete contains 20 indices although it is probable that only 18 of them correspond to real stars. The stars are not identified and, in six cases, there is not enough room in the corresponding index to write the name of the star.²⁹⁵ The ecliptic is correctly divided into the twelve zodiacal signs but the subdivisions of each sign are full of errors. The names of the signs are written by a later

291 Mundó, 1995.

292 Samsó, 1995.

293 Zuccato, 2005: the author remarks that, between 941 and 974, five Catalan embassies were sent to Córdoba.

294 Kunitzsch, 1991–92.

295 See an attempt to identify them in Kunitzsch & Dekker, 1996.

hand. The back of the instrument corresponds to what one would expect in an Andalusī or Maghribī instrument with an eccentric calendric-zodiacal scale, a shadow square and an alidade with sights which, like the radial ruler of the front, are later replacements of lost parts of the instrument.²⁹⁶

4.2.2.5 The European Diffusion of the “Old Corpus”

Millàs considered that the diffusion in Europe of the old corpus and of the new astronomy was the result of the visit of Gerbert of Aurillac (945–1003, later Pope Sylvester II) to Catalonia, where he studied the sciences of the *quadrivium* between 967 and 970 in Vic and probably in Ripoll and elsewhere. After his return to France, he corresponded with the aforementioned archdeacon Lupitus, asking him for an astronomical book “translated” by him (*Itaque librum de astrologia translatum a te mihi petente dirige ...*) in 984. This book has been conjecturally identified with the *De utilitatibus astrolabii* (inc. *Quicumque astronomice peritiam discipline ...*) which, as we have seen, was edited by Bubnov as a pseudo-Gerbertian work.²⁹⁷ This document is the reason why Lupitus has been considered as a translator.

The role of Gerbert in this process of transmission of the new astronomical materials is unclear.²⁹⁸ His authentic writings show no Arabic influence, at all, and they are good representatives of the survival of a Latin astronomy of a classical origin.²⁹⁹ If Gerbert did really have access to these Catalan sources, they probably reached him after his return to France. As we have seen (4.2.2.3.1), there are reasons to believe that at least one of the sources of the corpus (*De mensura astrolabii*) was probably written after 978. I fully agree with Kunitzsch when he says that Gerbert could not have come across this work between 967 and 970.³⁰⁰

296 On this astrolabe see King, 1995; King, 2004 b; King, 2005. King disagrees with my interpretation of the instrument because the design of the astrolabe bears no relation to that of any other known early Islamic astrolabe, eastern or western, or to that of any known early European astrolabe. He has proposed that the piece might also be the sole representative of an otherwise unknown Graeco-Roman tradition, even though it was clearly made in the region of Barcelona in the late, if not the early, tenth century.

297 In the same year, 984, Gerbert also wrote to Bishop Miró Bonfill of Gerona asking him to send to Adalberon, bishop of Reims, a copy of the book *De multiplicatione et divisione numerorum a Joseph Hispano editum*. Zuccato, 2005 has suggested that the otherwise unknown “Joseph Hispanus” could be identified with Ḥasdāy b. Ishāq b. Shaprūt whose *kunya* was Abū Yūsuf.

298 Zuccato, 2014 emphasizes, once more, the role of Gerbert in the transmission but his arguments do not seem convincing to me.

299 Poulle, 1985.

300 Kunitzsch, 1997.

Besides, the problem of the arrival of the corpus in Europe has been seriously revised in recent decades by Borst³⁰¹ and Burnett³⁰² on the basis of the oldest manuscripts in the collection. Both underline the importance of the area near Orléans, where we find two monasteries (St. Benoît de Fleury and St. Mesmin de Micy) in which copies of the old corpus were made: three texts of the collection very soon reached the monastery of Reichenau, in a miscellaneous manuscript probably accessible to Hermann Contractus. The copy of St. Benoît de Fleury was made at the time when Abbon was the abbot of the monastery (988–1004). The copy of Micy and its transmission to Reichenau could be due to the initiative of Constantin of Fleury, who was Dean (988–996) and later Abbot (1011–1020/21) of Micy. His contemporary Bern de Prüm was also living in that area and later became Abbot of Reichenau and Hermann's master. Manuscript Ripoll 225 may derive from the copy of the Fleury monastery, and should be dated to the first half of the eleventh century: the contacts between Oliva of Ripoll (1008–1046) and Ganzlin, who became Abbot of Fleury after Abbon, are well documented, as the correspondence between them has been preserved.

Burnett's paper insists on the relations between Constantin of Fleury, Gerbert of Aurillac and Ascelin of Augsburg. Gerbert dedicated some mathematical works of his to Constantin, and Constantin is, probably, the *Stabilis* of Orléans, a monk of Micy, who insisted that Ascelin write his treatise on the construction of the astrolabe which (like the work on the same topic by Hermann Contractus) derives from the *De mensura astrolabii*. Besides, Burnett points out that Hermann Contractus is only known to have been in Reichenau from the age of 30, and goes back to an old hypothesis that Hermann might have been sent to Augsburg when he was seven and would have been a disciple of Ascelin. We have thus two possible channels of transmission of our materials from Fleury to Reichenau: Bern de Prüm and Ascelin of Augsburg.

Burnett, finally, notes the presence in Chartres, near Orléans, of two other copies of the old corpus: manuscript Chartres 214 – destroyed during the Second World War (see 4.2.2.3.2) – and a second manuscript related to it (Oxford, Corpus Christi College 283), in which the interesting part is dated towards the end of the eleventh century. It seems clear that the *exemplar* of both manuscripts reached Chartres from Micy and that the presence of these texts explains the verses composed by Fulbert, Bishop of Chartres (1006–1028) in which the Arabic names of some stars appear. Both these verses and the

301 Borst, 1989.

302 Burnett, 1998.

introductory notes, written by Fulbert himself, derive from the chapter *De stellis horarum*, related to the *Astrolabii sententie*.³⁰³

So, to summarise: Borst and Burnett have clarified the diffusion of the old corpus in the region of Orléans, in central France, and the connections of the monasteries there with the monastery of Reichenau in south-western Germany, on an island in the Lake of Constance. What nobody seems to have been able to explain is the arrival of these materials (probably from Barcelona, as the monastery of Ripoll seems to have lost its prominence given the late date of ms. Ripoll 225). The *Gerbert hypothesis* remains a possibility but the question has not been conclusively settled.

4.2.3 *On the Standard Astrolabe in al-Andalus and the Maghrib from the Eleventh Century Onwards*

4.2.3.1 Introduction: Some Lists of Names

The astrolabe was the most popular astronomical instrument in the Middle Ages. In our area, this can be demonstrated by the number of Andalusi and Maghribi astrolabes which have been preserved. The list compiled by D.A. King contains 21 Andalusi and Maghribi astrolabes from the tenth and eleventh centuries, either complete or fragmentary,³⁰⁴ including the copy of an astrolabe by Khalaf ibn al-Mu'adh mentioned in 4.2.1.³⁰⁵ These references, from the table of contents to King's medieval instrument catalogue, should now be expanded with the more detailed descriptions in the new catalogue of instruments of Andalusi/ Iberian provenance listed as Hernandez 2018.

Here I reproduce only the list of astrolabists who signed their own instruments:

Muḥammad ibn al-Ṣaffār (the brother of Aḥmad ibn al-Ṣaffār, also mentioned in 4.2.1): two astrolabes dated 417/1026 and 420/1029.³⁰⁶

³⁰³ Millàs, 1931, p. 292.

³⁰⁴ King, 2005, pp. 359, 951–957, 1006. These references should be completed with Hernández, 2018 a. See an interesting study on the stars projected in the spider of Andalusi and Maghribi astrolabes in E. Mercier, 2018.

³⁰⁵ The so-called “astrolabe of Pope Sylvester II” or “astrolabe of Alfonso X” (extant in the Florence Museum of the History of Science) is an unfinished Eastern instrument, made towards the end of the tenth century. The markings on the back, with Latin inscriptions, were added by a late European hand after an Andalusi model, with an unfinished and incorrect calendrical scale. King (2005, pp. 489–494) believes that the back was reworked by a forger. See a description of this instrument in García Franco, 1945, pp. 131–152.

³⁰⁶ The astrolabe of 417/1026–27 was made in Córdoba, and the second extant astrolabe (420/1029) in Toledo: see Mayer, 1956, p. 75; Hernández, 2018 a, pp. 43–58.

Ibrāhīm ibn ‘Abd al-Karīm (one of his astrolabes is dated 458/1066). Nothing seems to be known about this astrolabist, but his undated astrolabe has a mater for latitude 30° and plates for latitudes between 20° and 39°. ³⁰⁷

Ibrāhīm ibn Sa‘īd al-Sahlī: astrolabes dated 459/1067, 460/1068, 463/1071. ³⁰⁸

Ibrāhīm ibn al-Sahlī (either Ibrāhīm ibn Sa‘īd himself or his son): astrolabe dated in Valencia in 478/1086. ³⁰⁹

Muḥammad ibn al-Sahlī (a son of Ibrāhīm ibn Sa‘īd): astrolabe dated 483/1090–91; ³¹⁰

Muḥammad ibn Sa‘īd al-Ṣabbān: astrolabes dated 466/1073–74, 474/1081–82. ³¹¹

Aḥmad ibn Muḥammad al-Naqqāsh: astrolabe dated 472/1079–80. ³¹²

It is easy to see that this list is mainly composed of Andalusī astrolabists, and only Ibrāhīm ibn ‘Abd al-Karīm seems doubtful. ³¹³ The situation changes in the period between the end of the twelfth and the sixteenth centuries, for which King has listed some 72 standard astrolabes, ³¹⁴ quite a number of which were made in the Maghrib. This seems to be in agreement with my comment in 1.2 about the lack of records of astronomical activity in the Maghrib before the beginning of the thirteenth century, at which point a substantial amount of information becomes available. ³¹⁵ Among the astrolabists who signed their own productions, two made more instruments than all the others and their distinctive designs were influential for centuries:

³⁰⁷ King, 2005, p. 952.

³⁰⁸ Hernández, 2018 a, pp. 71–93; Vernet, 1985, pp. 80–83; Vernet & Samsó, 1992, pp. 223–224.

³⁰⁹ Hernández, 2018 a, pp. 121–129; Vernet, 1985, pp. 84–85.

³¹⁰ Mayer considers that Ibrāhīm ibn Sa‘īd and Ibrāhīm ibn al-Sahlī are the same person and that Muḥammad ibn al-Sahlī is Ibrāhīm's son. They were active in Toledo and Valencia: see Mayer, 1956, pp. 50–52. On the instrument made by Muḥammad in Valencia see Goldstein & Saliba, 1983 and Hernández, 2018 a, pp. 131–136. This astrolabe has the unusual characteristic that the star names on the rete are written in Hebrew or Arabic with Hebrew script.

³¹¹ Astrolabist born in Zaragoza, who lived in Egypt and Guadalajara where he made the astrolabe dated 474/1073–74: see Mayer, 1956, pp. 76–77; Hernández, 2018 a, pp. 95–102, 111–120.

³¹² This astrolabe was made in Zaragoza: see Mayer, 1956, p. 37; Hernández, 2018 a, pp. 103–109.

³¹³ Hernández, 2018 does not include it in her catalogue.

³¹⁴ King, 2005, pp. 1010–1012.

³¹⁵ King, 2005, pp. 1010–1012.

Abū Bakr ibn Yūsuf, fl. Marrakesh: maker of six standard astrolabes dated between 605/1208–9 and 615/1218.³¹⁶

Muḥammad ibn Fattūḥ al-Khamā'irī, fl. Seville: maker of 12 astrolabes dated between 609/1212–13 and 634/1236–7.³¹⁷

Other less prolific makers were:

Muḥammad b. Yūsuf b. Ḥātim, fl. ?: maker of an astrolabe dated 638/1240–41.³¹⁸

ʿUthmān b. ʿAbd Allāh al-Ṣaffār, fl. Fez: maker of an astrolabe dated 699/1299–1300.³¹⁹

Aḥmad b. Ḥusayn b. Bāṣoh, fl. Granada: maker of three astrolabes dated between 694/1294–5 and 709/1309–10.³²⁰

Muḥammad b. Qāsim al-Qurṭubī, fl. Fez: maker of an astrolabe dated 719/1319–20.³²¹

Aḥmad b. Muḥammad b. Hārūn al-Hadabī, fl. Marrakesh: maker of an astrolabe dated 728/1327–8.³²²

Aḥmad b. ʿAlī al-Sharafī, fl. Alcalá la Real (?): maker of an astrolabe dated 729/1328–9.³²³

³¹⁶ Mayer, 1956, pp. 32–33.

³¹⁷ Mayer, 1956, pp. 64–66. Splendid photographs of one of these astrolabes (dated 634/1236–7) in Pingree, 2009, pp. 2–5. Hernández, 2018 a, pp. 137–184, describes seven standard astrolabes and two Zarqāllian *ṣafīḥas*, one of which (at least) corresponds to the kind usually named *ṣafīḥa zarqālīyya* (see 4.3.2).

³¹⁸ Mayer, 1956, p. 78. Excellent photographs of this instrument in Pingree, 2009, pp. 6–9. Pingree considers that Muḥammad ibn Yūsuf ibn Ḥātim is an astrolabist from Spain, i.e., al-Andalus. This is probably true, as the astrolabe contains plates for Toledo (40°) and Burgos (42°); Hernández (2018 a, pp. 191–197) agrees with Pingree in considering Ibn Ḥātim Andalusī, although she considers that the plates for Toledo and Burgos could be a later addition.

³¹⁹ Mayer, 1956, p. 84.

³²⁰ He was the son of Abū ʿAlī al-Ḥusayn ibn Bāṣoh, d. 709/1309–10 (Calvo, “Ibn Bāṣuh, Abū ʿAlī” in *BA*, vol. 2, pp. 595–598), who designed the universal plate for all latitudes. Two astrolabes made by Aḥmad ibn Bāṣoh, and dated in 704/1304–5 and 709/1309–10, contain a plate of this kind; Mayer, 1956, pp. 35–36; a description of one of his astrolabes in García Franco, 1955, pp. 197–309. For a full description of the three instruments see Hernández, 2018, pp. 211–233.

³²¹ King, 2005, p. 873.

³²² Mayer, 1956, p. 86.

³²³ Mayer, 1956, p. 35; Hernández, 2018, pp. 249–255. Made in the locality of al-Qalʿa, identified by Hernández with Alcalá la Real. The instrument seems Andalusī: the plates for the latitudes of Toledo/Talavera and Granada/Málaga bear the curves for the prayers of dawn

Ibrāhīm b. Muḥammad b. al-Raqqām, fl. Guadix: maker of an astrolabe dated 720/1320–1.³²⁴

Muḥammad ibn Faraj, fl. Granada: maker of an astrolabe dated 881/1476–7.³²⁵

Muḥammad ibn Zawāl, fl. Granada: maker of an astrolabe dated 886/1481–2.³²⁶

‘Alī b. Muḥammad al-Azdī, fl.?: maker of an astrolabe dated 950/1543–4.³²⁷

Aḥmad b. ‘Umar al-Kabbī, fl.?: maker of an astrolabe dated 993/1585.³²⁸

‘Alī b. Muḥammad b. ‘Abd Allāh b. Faraj: maker of an astrolabe dated 1010/1607–8.³²⁹

Among these 14 names, seven of them are clearly, or seem to be, Andalusī; the other seven are Maghribī (I include those who dated their astrolabes after 1492). Among the Andalusī astrolabe-makers we find family names related to well-known astronomers like Abū ‘Alī al-Ḥusayn ibn Bāṣoh, who was also an astrolabist although no instruments of his seem to have been preserved, and Muḥammad ibn al-Raqqām. The tradition of astrolabe-making also seems to have been preserved during the last period of the survival of the Naṣrid kingdom of Granada. This tradition seems to have become important in the Maghrib from the beginning of the thirteenth century onwards.

The interest in the astrolabe is also documented in a number of treatises on its use of varying importance (works on its construction are fewer in number) but not many of them have been studied. Among the authors of treatises on the use of the instrument are al-Zubayr b. Aḥmad b. Ibrāhīm b. al-Zubayr al-Thaqafī (d. after 1300), author of the *Tadhkirat al-albāb fī l-‘amal bi l-aṣṭurlāb*/*Tadhkirat dhāwī l-albāb fī istifā’ al-‘amal bi l-aṣṭurlāb*,³³⁰ Ibn al-Bannā’ al-Marrākushī (1256–1321) who wrote a *Maqāla fī ‘ilm/‘amal al-aṣṭurlāb* (not extant, but

and twilight which do not appear on the other plates. Besides, the instrument contains Ibn Bāṣoh’s plate for all latitudes and the plate of horizons attributed to Ḥabash al-Ḥāsib.

324 He is probably the son of the well-known astronomer and polymath Abū ‘Abd Allāh Muḥammad ibn al-Raqqām, d. 715/1315 (Samsó, in *BA* vol. 4, pp. 440–444); Mayer, 1956, p. 50; García Franco, 1955, pp. 309–311; Hernández, 2018, pp. 242–248. An inscription states that this is the first instrument made by Ibrāhīm ibn al-Raqqām.

325 Hernández, 2018, pp. 257–262. One of the plates is Ibn Bāṣoh’s plate for all latitudes.

326 Mendoza, 1990; Vernet & Samsó, 1992, p. 227.

327 Mayer, 1956, p. 42.

328 Mayer, 1956, p. 38.

329 Mayer, 1956, p. 42.

330 HATA no. 123.1; *BA* vol. 6, p. 327.

apparently a treatise on the construction) and a *Mukhtaṣar risālat Ibn al-Ṣaffār*;³³¹ his contemporary Ibn al-Raqqām (d.1315) is the author of a *Risāla fī l-aṣṭurlāb*, an *Istikhrāj al-maṭālīb al-nujūmiyya bi l-aṣṭurlāb* and a *Manẓūma fī l-ʿamal bi l-aṣṭurlāb*, all of them apparently extant.³³² Slightly later, in the fourteenth century, we find the *Risāla fī ʿilm al-aṣṭurlāb* written by Muḥammad b. ʿAlī al-Anṣārī, a scholar from Fez who migrated to Granada, where he died in 1361.³³³ In the fifteenth century we have a versified version of Ibn al-Ṣaffār's treatise on the use of the astrolabe in the *Bughyat al-ṭullāb fī sharḥ al-aṣṭurlāb* by Muḥammad b. Aḥmad b. Abī Yaḥyā al-Ḥabbāk al-Tilimsānī, Abū ʿAbd Allāh (d. 1463),³³⁴ which was the object of a commentary by his disciple Muḥammad b. Yūsuf b. ʿUmar b. Shuʿayb al-Ḥasanī al-Sanūsī al-Tilimsānī (1428–1490) (*ʿUmdat dhāwī l-albāb fī sharḥ Bughyat al-ṭullāb*).³³⁵ Finally, at the very end of the Naṣrid kingdom of Granada, we find Ibrāhīm b. Muḥammad b. Futūḥ/Fattūḥ al-ʿUqaylī al-Gharnāṭī (d. 1464), teacher at the Granada *madrassa* where one of his disciples was the famous mathematician al-Qalaṣādī (d. 1486), and author of a *Risāla fī l-aṣṭurlāb*.³³⁶

As it is impossible for me to analyse all the extant Andalusī and Maghribī texts on the astrolabe, here I will concentrate on three of them, written by:

Abū l-Ṣalt Umayya b. ʿAbd al-ʿAzīz b. Abī l-Ṣalt al-Dānī (ca. 1068–1134),³³⁷ whose *Risāla fī l-ʿamal bi l-aṣṭurlāb* I have been able to examine in ms. Berlin Petermann 1 672, fols. 44r–71v.³³⁸ Abū l-Ḥasan ʿAlī al-Marrākushī quotes critically extensive passages of Abū l-Ṣalt's *Risāla* in his *Kitāb al-mabādī wa l-ghāyāt*.³³⁹

Muḥammad b. ʿAlī b. Yaḥyā b. al-Naṭṭāḥ, Abū ʿĀmir is the author of another *Risāla fī l-ʿamal bi l-aṣṭurlāb* which I have read in ms. London British Library Oriental manuscripts Add 9602, fols. 1v–24v.³⁴⁰ Nothing is known about this author but, as I will try to show later, I believe that he was Andalusī and that

331 Lamrabet, 2013, M134 (pp. 164–177, see p. 168); Jabbār & Aballāgh, 2001, pp. 121, 125.

332 Lamrabet, 2013, M126 (p. 162); HATA no. 124.1 and 124.2.

333 Lamrabet, 2013, M163 (pp. 188–189); ʿInān, 1973–77, vol. 11, pp. 287–288.

334 Lamrabet, 2013, M219 (p. 207). This work has been preserved in many manuscripts, among them Berlin Landberg 517, accessible in <https://ismi.mpiwg-berlin.mpg.de> <<https://ismi.mpiwg-berlin.mpg.de/>>

335 Lamrabet, 2013, M231 (p. 212).

336 Lamrabet, 2013, A340 (p. 110). See also HATA no. 141.

337 Comes, "Ibn Abī l-Ṣalt" in *BA* vol. 1, pp. 722–728; HATA no. 77.

338 This manuscript can be accessed in <https://ismi.mpiwg-berlin.mpg.de> <<https://ismi.mpiwg-berlin.mpg.de/>> thanks to the generosity of the Staatsbibliothek zu Berlin.

339 Marrākushī, 1984, vol. 11, pp. 256–292.

340 Accessible in the Qatar Digital Library, http://www.qdl.qa/en/archive/81055/vdc_100023486667.0x000000. This is usually considered to be the only copy of this work but the

his work was written after the death of Ibn al-Šaffār in 1035 and before the conquest of Córdoba by Fernando III of Castile in 1236. Apparently he flourished during the *ṭāʾifa*, Almoravid or Almohad periods.³⁴¹

Ibn al-Šhāṭṭ is another unknown Andalusī scholar who wrote a treatise on the use of the astrolabe which was edited and translated into Spanish in 1884,³⁴² on the basis of a Moroccan manuscript found in Tetuan and brought to Granada in 1860. At least two more manuscripts are extant in the Ḥasaniyya Library in Rabat, under the title *Risāla fī uṣūl [ʿilm] al-aṣṭurlāb*.³⁴³ Khaṭṭābī mentions two scholars named Ibn al-Šhāṭṭ although the biographical sources do not state that either wrote on the astrolabe. They are Qāsim b. ʿAbd Allāh b. Muḥammad b. al-Šhāṭṭ al-Sabtī (1246–1323) and ʿĪsā b. Aḥmad al-Bijārī, also known as Ibn al-Šhāṭṭ (alife in 890/1485). The former has been conjecturally identified by Lamrabet with the author of the *risāla*,³⁴⁴ and the same suggestion was made by Antonio Almagro in his 1884 edition of the text.³⁴⁵ In my opinion, this identification is probably correct (see 4.2.3.2).

I have also been interested in checking Lamrabet's hypothesis on the identification of the author of the *Risālat al-aṣṭurlāb al-sarṭānī al-mujannah* (on the winged crab-like astrolabe),³⁴⁶ extant in ms. Escorial 961 (fols. 17r–29r)³⁴⁷ in which the name of the author appears as Muḥammad b. Naṣr b. Saʿīd. Lamrabet³⁴⁸ suggests that he might be identified with Muḥammad b. Saʿīd al-Saraquṣṭī, known as Ibn al-Mashshāṭ al-Aṣṭurlābī, a contemporary of Šāʿid al-Andalusī (1029–1070), who notes that he visited Cairo, where he saw a treatise on the use of the astrolabe written by Jābir ibn Ḥayyān.³⁴⁹ This suggestion is extremely interesting because during his stay in Egypt Ibn al-Mashshāṭ may have been introduced to this kind of non-standard astrolabe, apparently invented by

Rabat Hassaniyya Library ms. 6493, fols. 49–80 contains a second copy. See Khaṭṭābī, 1983, vol. III, pp. 238–239 (no. 281).

341 Lamrabet suggests two possible identifications, both corresponding to Maghribī scholars: Muḥammad al-Šanhājī al-Naṭṭāḥ (fl. 684/1285) and Abū ʿAbd Allāh Muḥammad b. Abī l-Ḥasan ʿAlī al-Šanhājī (alife 684/1285). See Lamrabet, 2013, M103–M104–M104bis, p. 158.

342 Almagro, 1884.

343 Khaṭṭābī, 1983, III, pp. 203–205 (no. 237–238): mss. 6665 (fols. 6r–10v) and 6843 (fols. 85r–88r).

344 Lamrabet, 2013, M135 (p. 180); Rodríguez Figueroa, “Ibn al-Šāṭṭ” in *BA*, vol. 5, pp. 267–269.

345 Almagro, 1884, p. 17.

346 On this type of astrolabe, apparently invented by Naṣṭūlus (fl. beginning of the tenth century) see Charette, 2003, pp. 69, 249–250; King, 2005, pp. 471–472.

347 I would like to express my gratitude to Father José Luis del Valle, Director of the Escorial Library, who provided me with a copy of this manuscript.

348 Lamrabet, 2013, no. A135 (p. 72).

349 Bū ʿAlwān, 1985, p. 153; Blachère, 1935, pp. 118–119.

Nas̥tūlūs in Baghdad (fl. beginning of the tenth century). Unfortunately, my examination of the manuscript makes me doubtful of this identification: on the one hand, the date of the work appears in the manuscript as the month of Ramaḍān of 511/27.12.1117–25.1.1118,³⁵⁰ which seems slightly too late for a contemporary of Šā'id al-Andalusī; on the other, four examples of the procedure for obtaining the ascendant as a function of the solar position on the ecliptic and the solar altitude give as the local latitude 33°,³⁵¹ which cannot correspond to any Andalusī city and agrees with what one would expect for the latitude of Fez, Baghdad or Damascus. On the whole, my impression is that this *risāla* is an Eastern work, not a Western one.

4.2.3.2 Three Treatises on the Use of the Astrolabe

Abū l-Šalt was an Andalusī scholar born in Denia, but it is well known that he migrated to Egypt, for unknown reasons, and lived there between 1096 and 1112. His stay in that country does not seem to have been a very happy one:³⁵² after enjoying the protection of al-Afḍal, the powerful minister of the Fatimid caliph al-Musta'li b. al-Mustanšir (1094–1101), he fell from grace in around 1108 and was imprisoned for three years and one month between 1108 and 1111.³⁵³ It was during his imprisonment that he wrote many of his scientific works, among them his *Risāla fī l-'amal bi l-ašturlāb*, written in 503/1010.³⁵⁴ This is probably the reason why this work is not related to the Andalusī school of treatises on the astrolabe, but rather shows clear Eastern influences.

Ms. Berlin Petermann 1 672 contains 68 chapters³⁵⁵ and it does not correspond to a complete version: Ms. Milan Ambrosiana CXIX:D has 90 chapters, and Ms. 80 of the Zāwiya Ḥamzawīya in Morocco (fols. 7–87) contains 98 chapters and an appendix divided into 30 sections.³⁵⁶ Besides, the revised version

350 Ms. Escorial 961, fol. 17v.

351 Ms. Escorial 961, fols. 19v–21r.

352 Prémare, 1964–66.

353 The sources give different reasons for his imprisonment: according to Ibn Abī Uṣaybi'a, 1957, vol. 11, 86–87, the cause was Abū l-Šalt's failure to raise a boat loaded with copper, which had sunk in the harbour of Alexandria. Other explanations can be found in Comes, *BA* vol. 1, p. 723.

354 In a Cairo manuscript, King, 1986 b (p. 50, no. B89) found a reference to Abū l-Šalt's writing of the treatise on the astrolabe while he was in prison. The same information is also found in Ibn Khallikān, ('Abbās, 1968 a, 1, pp. 243–44, no. 104) and in ms. Leiden 468 (fols. 179v–180r). This manuscript contains the *Kanz al-yawāqit*, written by an anonymous Cairo *muwaqqit* who gives the date of Abū l-Šalt's work.

355 See a list of chapters in this ms. in Millàs, 1931, pp. 77–80.

356 An updated list of manuscripts containing this work can be found in HATA 77.6. On the Ḥamzawīya manuscript see Kuwayfi & Rius, 1998, p. 450.

contained in Abū l-Ḥasan al-Marrākushī's *Mabādī' wa ghāyāt* is divided into 77 chapters, many of which cannot be found in the version extant in the Berlin ms.: this is the case of chapters 53–54, 56, 59 (on shadows), 60–63 (several questions related to the ecliptic pole), 64–68 (stars), 69–70 (moon and planets) and 71–77 (astrological problems on anniversaries, division of the houses of the horoscope, projection of rays and *tasyīr*). One should note that this version is not always a literal transcription of Abū l-Ṣalt's text: even though Abū l-Ḥasan considers Abū l-Ṣalt's treatise as the best treatment of the use of the astrolabe he has been able to read (in spite of a certain amount of padding, *hashw*), there is a certain carelessness (*ihmāl*) which has led to the existence of a few mistakes (*ghalaṭ yasīr*).³⁵⁷ Therefore, Abū l-Ḥasan sometimes tends to summarise the contents of Abū l-Ṣalt's *risāla* and omits some chapters because he has already dealt with the topic in other parts of the *Mabādī'*. In other instances he corrects Abū l-Ṣalt or explains a procedure in a different way (*bi-ghayr mā dhakara-hu Abū l-Ṣalt*).³⁵⁸

The selection made by the compiler of the Berlin ms. seems to be restricted to the use of the instrument in questions of timekeeping: the position of the Sun on the ecliptic, altitude and azimuth of the Sun and the stars, day and night arcs, equal and seasonal hours, rising and setting amplitudes, shadows, dawn and twilight, hours of the *zuhr* and *ʿaṣr* prayers, *qibla* etc. To these, the final chapters (64–68) add practical solutions to problems of surveying. The most interesting part of the text concerns the zodiacal scale, standard in Andalusī and Maghribī astrolabes, which seems to have already been introduced in the Mashriq in Abū l-Ṣalt's time:³⁵⁹ in chapter 2 (fol. 46v), Abū l-Ṣalt explains how to determine the solar longitude approximately and quickly, without resorting to a *zīj* (astronomical tables) or a *taqwīm* (ephemerides), by using a table in which the solar degree is given for each day of the Syriac months of the solar year.³⁶⁰ In chapter 3 (fols. 47r–47v) he explains that on the back of some astrolabes (*fī baʿḍ al-aṣṭurlābāt*) one can find the diagram of the zodiacal scale in which the names of the months are Syriac and Coptic. This zodiacal scale belongs to the eccentric type and Abū l-Ṣalt remarks that the correspondence

357 Marrākushī, 1984, p. 256.

358 See for example Marrākushī, 1984, chapters 61–63 and 68, pp. 284–286.

359 Scales of this kind appear in Eastern astrolabes of the twelfth and thirteenth century which use Syrian and Coptic names for the months. See King, 2005, pp. 870 (fn. 127) and 888. This remark does not contradict the fact that the zodiacal scale was already known in the East towards the end of the ninth century (4.2.1) where it appears in an astronomical instrument designed by Naṣṭūlūs, which is not an astrolabe. I am referring here to zodiacal scales placed on the back of astrolabes.

360 A table of this kind is missing on fol. 47r of the Berlin ms. in which there is a blank space.

between solar degrees and days of the year should agree with the values determined for the time in which the astrolabe was made, and that the distance between the centre of the astrolabe and the centre of the circle of the months must be proportional to the distance between the centre of the world and the centre of the solar eccentric. The same reference can be found in Abū l-Ḥasan al-Marrākushī's *Mabādī' wa ghāyāt*, where the zodiacal scale is called *dā'irat ta'dīl al-shams* (circle of the solar equation).³⁶¹

Another interesting feature of this treatise relies in the fact that not all astrolabes known by Abū l-Ṣālt have vertical circles traced on each plate, something which, apparently, corresponds to the earliest Islamic instruments ever built. Thus, in chapters 49 (fols. 63r and v) and 50 (fols. 63v–64r), on the determination of the azimuth of the Sun or a star as a function of their altitude, he remarks: *wa in kāna dawā'ir al-sumūt makhtūṭa 'alā dawā'ir al-muqanṭarāt* (and if there are vertical circles traced on top of the almucantars). To this he adds the possibility of vertical circles being traced on the lower part of the plate, intersecting the hour lines: *wa in kāna dawā'ir al-sumūt makhtūṭa 'alā aqsām al-sā'āt faqaṭ* (and if there are vertical circles only drawn on top of the hours). The same expressions are repeated by Abū l-Ḥasan al-Marrākushī in his chapters 39 and 40.³⁶²

Finally, it is also interesting that the back of the astrolabe described by Abū l-Ṣālt has a scale of sines and cosines, made by straight lines traced from each degree of the altitude circle and perpendicular to the vertical diameter of the astrolabe which passes by the hanger (*ilāqa*). The values of the sines are found on another scale in the alidade.³⁶³

Our second treatise on the astrolabe was written by Muḥammad b. 'Alī b. Yaḥyā b. al-Naṭṭāḥ, an author who has not been identified so far. The contents of the *risāla* do not seem to be very different from those in Abū l-Ṣālt's, although we find two chapters³⁶⁴ dealing with the determination of the longitude of the Moon and a planet, as well as establishing whether the planet has a direct or a retrograde motion. These topics are not dealt with in the Berlin ms. of Abū l-Ṣālt's *risāla*, although we do find them in Abū l-Ḥasan al-Marrākushī's

361 Marrākushī, 1984, p. 259.

362 Marrākushī, 1984, p. 275.

363 A cosine scale appears in the astrolabe made by al-Khamā'irī on the back of his astrolabe dated in 634/1236–7. The two upper quadrants of this back contain an orthographic projection like the one we find in Ibn al-Zarqālluh's *ṣafīha* (see 4.3.2.3); cf. Pingree, 2009, p. 3; Hernández, 2018, pp. 180–82.

364 See BL 9602, fols. 21v–22r.

recension,³⁶⁵ as well as in Ibn al-Ṣaffār³⁶⁶ and Ibn al-Samḥ's treatises (in the latter case at some length).³⁶⁷

The instrument described by Ibn al-Naṭṭāḥ corresponds to a standard Western astrolabe with a zodiacal scale on its back (although the author notes that this diagram is not to be found in some astrolabes: *wa rubbamā lam yakun fī ba'd al-aṣṭurlābāt*)³⁶⁸ and the projection of vertical circles in all its plates. Surprisingly, the description includes something which, in the author's own words, has never been found in an astrolabe treatise before: a list with the Maghribī values of the letters of the alphabet from 1 to 1000. As Ibn al-Naṭṭāḥ notes, the figure of 300 would be enough for an astrolabe user.³⁶⁹

The only geographical reference we find in the whole *risāla* is that of Córdoba, for which he mentions a standard value of its latitude (38;30°).³⁷⁰ This value agrees with a note on the differences between the degree of the ecliptic with which a star rises, mediates and sets. An example is given for the star al-Simāk al-A'zal (α Virginis) in which case the difference between the rising and the mediation degrees is approximately 1°, while the difference between the rising and setting degrees is only a few degrees.³⁷¹ I have recomputed these differences for a latitude of 38;30°, an obliquity of the ecliptic of 23;30° (parameter mentioned by Ibn al-Naṭṭāḥ),³⁷² a longitude of the star of 188;20° and a latitude of -2°.³⁷³

Rising degree: 188;53°

Mediation degree: 187;28°

Setting degree: 184;34°

The reference to Córdoba and the corresponding latitude are repeated in the chapter on the determination of the *qibla*,³⁷⁴ edited and analysed by D.A. King.³⁷⁵ In this chapter Ibn al-Naṭṭāḥ remarks that most mosques in Córdoba

365 Marrākushī, 1984, p. 287.

366 Millàs, 1955, p. 74; Millàs, 1931, p. 46.

367 Viladrich, 1986, pp. 141–147.

368 Ms. BL 9602 fol. 2r.

369 Ms. BL 9602 fols. 3v–4r.

370 Ms. BL 9602 fol. 14v.

371 Ms. BL 9602 fol. 12v–13r.

372 Ms. BL 9602 fol. 13v–14r.

373 These values are found in Maslama's star table. I have not introduced a correction for precession because I do not know the date of Ibn al-Naṭṭāḥ's treatise. See, on this table, 4.2.2.3.1.

374 Ms. BL 9602 fols. 18v–19r.

375 King, 1978, pp. 370–387 and 390.

are oriented towards 30° South of East, according to “the method of al-Battānī” (see 2.5.2.1) although, as shown in King’s recomputation, the result obtained with this method using al-Battānī’s geographical coordinates, is 23° S of E, the value attributed to the astronomers (*ahl ṣināʿat al-taʿdīl*) by Ibn al-Naṭṭāḥ himself. Our author also adds that the *Jāmiʿ* Mosque of Córdoba is oriented towards 60° S. of E., which is quite a precise estimation, as the value established in recent times is 152° from the North (see 2.5.1.2). These references make me think that Ibn al-Naṭṭāḥ’s treatise was written at a time in which Córdoba was still a Muslim city, before its conquest by Fernando III of Castile in 1236. Another chronological limit can be deduced from Ibn al-Naṭṭāḥ’s criticisms of Ibn al-Ṣaffār’s treatise on the astrolabe in two passages dealing with the computation of the declination of the Sun or a star.³⁷⁶ Both of them are introduced by *Qāla Muḥammad* and they are related to Ibn al-Ṣaffār’s use of the expression:

$$h_m = 90^\circ - \phi \pm \delta$$

where h_m is the meridian altitude of the Sun or a star, ϕ the local latitude and δ the declination of the celestial body. Apparently he complains that, if ϕ has been obtained with the astrolabe, one cannot also use it to determine δ and vice versa. Therefore, he recommends obtaining the latitude by observing the maximum solar altitude in one of the equinoxes (in which $\delta = 0^\circ$) and the declination of the Sun or the star in a *zīj*. Since, in both passages, the name of Ibn al-Ṣaffār is followed by the expression “May God have mercy upon him” (*rahīma-hu Allāh*), it seems clear that the treatise was written after 1035, the year of Ibn al-Ṣaffār’s death.

As for Ibn al-Shāṭṭī’s treatise on the astrolabe, it is interesting that its author is called *al-ḥaysūbī* (arithmetician) but also *al-muwaqqit* and *al-muʿaddil*.³⁷⁷ The former seems to agree with the figure of Qāsim b. al-Shāṭṭī (1246–1323) who was considered an expert in arithmetic (*ḥisāb*) and partition of inheritances (*farāʿid*).³⁷⁸ Although Qāsim was born and lived in Ceuta, he travelled to Granada as a member of his home city’s delegation when Ceuta became a part of the Naṣrid kingdom of Granada in 1307.³⁷⁹ His condition of *muwaqqit* and *muʿaddil* agrees chronologically with the time in which the first *muwaqqits* are

376 Ms. BL 9602 fols. 15r–16v. Kunitzsch (1981, pp. 49, 52) has established the existence of a relation between the treatises of Ibn al-Ṣaffār and Ibn al-Naṭṭāḥ.

377 Almagro, 1884, p. 21. In this work the Arabic edition is not paginated. I have added the numbers of the pages, from right to left: see p. 1.

378 Rodríguez Figueroa, “Ibn al-Šāṭṭī”, *BA*, vol. 5, p. 267.

379 Rodríguez Figueroa, “Ibn al-Šāṭṭī”, *BA*, vol. 5, pp. 267–268; Inān, 1973–77, vol. IV, pp. 260–261.

documented in Naşrid Granada (see 2.0) although, if our author is identified with Qāsim, he must have practised this profession in Ceuta. All this agrees with the interest shown by the treatise in questions related to timekeeping: this kind of topic is standard in treatises on the use of the astrolabe, but not with the insistence and detail we find in Ibn al-Shāṭṭī's *risāla*. Thus, he carefully describes the hour diagram in which there are curves for the prayers of the *zuhr* (in the eighth hour) and the *ʿaṣr* (tenth hour), as well as the *fajr* and the *shafaq*, both of which are placed on almucantar 18°. ³⁸⁰ It also insists on the determination of the *qibla*, for which he gives the value of 30°, according to Ibn al-Ṣaffār, for Córdoba and its surroundings, which could be used for Granada and Seville. Ibn al-Shāṭṭī states that 30° was accepted as the rule until Ibn al-Zubayr changed this value and established that the *qibla* in Granada should be 14° S. of E. ³⁸¹ This Ibn al-Zubayr is, no doubt, Abū l-Qāsim al-Zubayr b. Aḥmad b. Ibrāhīm b. al-Zubayr (d. after 1300), ³⁸² already mentioned in 4.2.3.1 as the author of a treatise on the astrolabe. His *qibla* value for Granada is surprisingly good, as the modern calculated value is 100;21° from the north point (see 2.5.2.2).

4.2.4 *The Astrolabe in the Christian Kingdoms of the Iberian Peninsula*

4.2.4.1 Five Extant Astrolabes Made in the Crown of Aragon and the Kingdom of Castile

Rather surprisingly, D.A. King has remarked that only five astrolabes made in the Christian kingdoms of the Iberian Peninsula are extant, and that four of them are Catalan in origin; another was made in al-Andalus, to which Latin and Catalan inscriptions were added. ³⁸³ These exemplars bear witness to the existence of craftsmen making astrolabes and other astronomical instruments in which King Pedro IV (1336–1387) and King Juan I (1387–1396) were extremely interested. The main centres for the production of these instruments were the island of Majorca and the city of Perpignan, in southern France, which at that time belonged to the Crown of Aragon. ³⁸⁴

The first of the aforementioned “Catalan” astrolabes is the “Carolingian” astrolabe already analysed in 4.2.2.4. After this one, we have two astrolabes, made ca. 1300, a third one signed and dated in 1375 and an eleventh-century Andalūsī astrolabe with additional inscriptions made by a Catalan artisan.

³⁸⁰ Almagro, 1884, pp. 25–26, 33–35, 43, 46; Arabic text pp. 2, 4–5, 8, 9.

³⁸¹ Almagro, 1884, pp. 36–38; Arabic text pp. 5–6.

³⁸² HATA no. 123.1; BA, vol. 6, p. 327.

³⁸³ King, 2005, p. 839. A description of these four instruments in King, 2004 b.

³⁸⁴ Millás, 1962, pp. 77–78, 81–82.

The second instrument belongs to the London Society of Antiquaries, and shows clear Islamic influences (curves for twilight, corresponding to a solar altitude of -18° , under the horizon). The names of the stars are written in a Latin contaminated by Catalan. King dates it towards the end of the thirteenth century. The plates are traced for latitudes $32;30^\circ$ (*mater*, for Jerusalem), $38;30^\circ$ (Córdoba), $39;40^\circ$ (Valencia?), 41° (Barcelona) and 42° (Gerona). The set comprised between $39;40^\circ$ and 42° clearly covers the range of latitudes of the Crown of Aragon from south to north. We have a fuller example of an endeavour of this kind in the clock sent by Pedro IV to his daughter in which the dial was replaced by an astrolabe and the plates were traced for latitudes 38° , 39° , 42° and 43° , corresponding to the southern part of the region of Valencia (38°), the city of Valencia (39°), Barcelona and Zaragoza (42°) and Montpellier (43°) (see 2.4.2.1.6).³⁸⁵

Our third, contemporary, astrolabe is preserved in the National Maritime Museum in Greenwich, and one of its distinctive features is that the names of the months are written in Catalan.³⁸⁶ It contains seven plates engraved for latitudes between 31° and 45° ; the corresponding cities are identified and the related (and in some cases wrong) climates are also added. There is a clue suggesting that the astrolabe may have been made in Valencia.

The fourth astrolabe was made in Barcelona in 1375 by a craftsman named Petrus Raimundus.³⁸⁷ It is preserved in the Boston Museum of Fine Arts, but little is known about it. King has underlined its morphological relations to the picture of the astrolabe appearing in the Alfonsine *Libro del saber de astrología* (see 4.2.4.2.3). A peculiar detail appearing on the back of the instrument is that the calendric scale also contains a diagram showing the ecliptic positions of the 28 lunar mansions. Interestingly, a similar diagram is to be found in the Catalan Atlas by the Majorcan Jew Cresques Abraham (extant in the Paris Bibliothèque Nationale de France), also traditionally dated to 1375.³⁸⁸

A fifth astrolabe is not Catalan but Andalusī and is preserved in the Jagiellonian University Museum of Krakow. It was made in Córdoba in 446/1054–55 but the author's name is not inscribed on the instrument. A Catalan artisan added (probably in the fourteenth century) a series of Latin inscriptions as well as the names of the months in Catalan. The plates designed for the latitudes of $38;30^\circ$, 40° and 42° have the names of the corresponding cities in Arabic: Córdoba and Valencia ($38;30^\circ$), Toledo and Santarem (40°) and Zaragoza (42°),

385 King & Maier, 1996; Hernández, 2018, pp. 337–344.

386 Maier, 1994; Hernández, 2018, pp. 323–329.

387 Hernández, 2018, pp. 361–369.

388 Llompart, Pujades & Samsó, 2005, p. 98.

but the Latin additions identify other localities for which the corresponding plate could be used: Cardeya (Cartagena, 38;30°), Tortosa (40°) and Perpignan (42°).³⁸⁹ This item is a good example of something we have already seen in the drawings on the astrolabe of Ibn al-Mu'ādh (4.2.1): Arabic astrolabes were used in Christian lands, and Latin or Hebrew inscriptions were added to them.³⁹⁰ We will see below that the phenomenon also took place in the other direction.

Our next astrolabe, now preserved in the Agha Khan Foundation in Toronto, is most extraordinary because it contains inscriptions in Arabic and Latin as well as scratches in Hebrew.³⁹¹ It was made in the fourteenth century, probably in Toledo, but the Jewish artisan who began its construction did not complete it (the names of the zodiacal signs and of the months in the calendric scale were later added in Arabic). He was the author of the Hebrew scratches which register, in alphanumerical notation, the latitudes of the three original plates and whose purpose was merely to help him in the identification of each plate. Either he or a Christian collaborator made the inscriptions in Latin on the rete, back, and three plates. A Muslim named Mas'ūd finished it in the Christian part of the Iberian Peninsula and added the first set of Arabic inscriptions on the boss of the shackle, the rete, the back and the fourth plate (for Algiers and the latitude of Mecca with curves for the times of the *zuhr* and *fajr* prayers). It is possible that he was planning to leave the Christian-dominated Iberia and migrate to Algiers. A second set of Arabic inscriptions (star names) were also scratched on the rete and it is possible that its author was the same Jew who inscribed the Hebrew latitudes of the plates. In a later stage, in the sixteenth century, the astrolabe travelled to northern or eastern France, as evidenced by another layer of inscriptions around the outer rim.

To the aforementioned list of six astrolabes, Azucena Hernández has added ten more instruments which she considers to have been made in the Christian kingdoms of the Iberian Peninsula, although none of them has been signed or dated, and there does not seem to be scholarly agreement on their origin. Four of these instruments seem to have archaic characteristics: their plates are not

389 Maier, 1999; Hernández, 2018, pp. 59–69.

390 King, 2005 (p. 849) refers to three Andalusī astrolabes to which inscriptions in Hebrew, Catalan and Castilian were added. The one with Catalan inscriptions is our astrolabe dated 446/1054–55.

391 King, 2002–3 and Hernández, 2018, pp. 353–360. A revised version has appeared in King, 2005, pp. 831–914. See also King, 2000, pp. 308–316). Astrolabes with inscriptions in Hebrew are rare: see Goldstein, 1976 and Abuzayed, King and Schmidl, 2010, pp. 131–132. I have already mentioned (4.2.3.1) an astrolabe made in 483/1090–91 by Muḥammad ibn al-Sahli in which the star names on the rete are written in Hebrew or in Arabic with Hebrew characters.

designed for specific latitudes but for one of the seven Ptolemaic climates and two of them lack azimuthal circles. Finally, the spider of one of them has formal similarities with the Carolingian astrolabe.³⁹² It is also interesting that one of these ten astrolabes has its inscriptions written in Hebrew or Judaeo-Arabic,³⁹³ while another one is fully inscribed in Judaeo-Arabic and bears, in the rim of the *mater*, a corrupt version of a tenth-century Arabic poem written by the astronomer and mathematician Abū Ishāq Ibrāhīm al-Šābi'. The latter astrolabe has been considered to be Andalusī or Maghribī,³⁹⁴ but Azucena Hernández suggests a possible Castilian origin.³⁹⁵

4.2.4.2 Three Texts on the Construction and Use of the Astrolabe

We have already seen (4.2.1) that Maslama's notes on Ptolemy's *Planisphaerium* reached the Latin world through Hermann of Carinthia's translation and I will later explain (4.2.4.2.2.4) that this source exerted an important direct or indirect influence on the pseudo-Māshā'allāh's treatise on the construction of the astrolabe as well as on the Alfonsine text on the same subject (4.2.4.2.3.1). Two Latin translations were made of Ibn al-Šaffār's treatise on the use of the instrument (4.2.1). The twelfth century is also the period in which original treatises begin to appear: such is the case of Adelard of Bath (ca. 1150),³⁹⁶ although his connection with the Iberian Peninsula is not clear, of Robert of Chester (ca. 1147–1150) and of Rudolph of Bruges' text on the construction of the instrument (ca. 1144).³⁹⁷ Johannes Hispalensis seems also to be the author of two original treatises on its construction, edited by Millás: one of them is added to an incomplete copy of pseudo-Māshā'allāh's *De operatione*;³⁹⁸ the other begins with "dixit Johannes" and it is difficult to establish whether the author is Johannes Hispalensis or Johannes Hispanus.³⁹⁹ This second text contains an interesting reference to the different types of astrolabe which existed at that time: some were made for the seven climates, for which only three plates are needed if the *mater* contains the projection of one of them (see 4.2.1); others were made for specific localities; some contained nine plates, others three and others only one. To this, the author adds a most peculiar remark

392 Hernández, 2018, pp. 279–285, 287–292, 293–297, 299–306.

393 Hernández, 2018, pp. 371–375.

394 Abuzayed, King & Schmidl, 2011.

395 Hernández, 2018, pp. 331–336.

396 Pouille, 1987.

397 Lorch, 1999.

398 Millás, 1942, pp. 313–321. According to Burnett, 2002, p. 60, Johannes Hispalensis' text begins in Millás, 1942, p. 316. See also Thomson, 2019, introduction.

399 Millás, 1942, pp. 322–327. See also pp. 218–219.

about astrolabes containing plates in which mean planetary motions and values of the revolutions of years and months are inscribed.⁴⁰⁰ The text also contains a reference to a trepidation model, something which is not common in astrolabe texts.⁴⁰¹

4.2.4.2.1 *Abraham ibn 'Ezra's Treatise on the Use of the Astrolabe*⁴⁰²

In spite of the fact that Ibn 'Ezra (ca. 1089–1167) did not write this work in the Iberian Peninsula, I am including it here because there is no doubt that his astronomical knowledge has a clear Andalusī origin and that his treatise, as well as the rest of his astronomical and astrological books, follows the Andalusī-Maghribī tradition.

Ibn 'Ezra's treatise on the astrolabe is preserved in three Hebrew and one Latin versions and I will deal here with the Latin text edited by Millás.⁴⁰³ The first two Hebrew versions (*Sefer Keli ha-neḥoshet* I and II) were written in Mantua and Verona,⁴⁰⁴ respectively, in 1146, while the third version (*Sefer Keli ha-neḥoshet* III) was produced in Béziers in 1148. As for the *Tractatus de astrolabio*, it corresponds to a later date (1154) when the author was staying in Rouen.⁴⁰⁵

The *Tractatus de astrolabio* has many points of contact with the three recensions of the *Keli ha-neḥoshet*.⁴⁰⁶ Millás identified Abraham ibn 'Ezra as the author of the *Tractatus* on the grounds that a) the text mentions a “magister Abraham”, b) there is a reference to England (where Ibn 'Ezra is supposed to have stayed between 1157 and 1160)⁴⁰⁷ and c) the author refers to a set of astronomical tables composed by him which he considers more reliable than the approximate computations one can make with an astrolabe.

The text edited by Millás has certain peculiarities which should be taken into consideration. On the one hand he mentions northern and southern astrolabes, something that we also find in Abū l-Ṣalt's epistle (4.2.3.2), but he refers to them as *sinistrum* and *dextrum*, which apparently corresponds to a

400 Millás, 1942, p. 325: “quidam uero uolentes astrolabium perfectum facere sculpiunt in quibusdam tabellis medios cursus planetarum adque (*sic*) equationes annorum quoque et mensium”. See also Samsó, 1981.

401 Millás, 1942, p. 324.

402 This part is a revision of my previous paper Samsó, 2012 (see pp. 189–191).

403 Millás, 1940.

404 Only *Keli ha-neḥoshet* I has been edited by H. Edelman, Königsberg, 1845.

405 Sela & Freudenthal, 2006, pp. 19–21.

406 See a comparative study of the four texts in Sela, 2001, pp. 104–113; Sela, 2003, pp. 22–36; Rodríguez Arribas, 2014.

407 The allusion to England is interpreted in a different way by Sela, 2001, pp. 111–113, who considers that the *Tractatus* was written in Rouen in 1154.

mistranslation of Arabic in which *shimāl* and *yamīn* have two different meanings: north/left, and right/south. On the other, it contains references to old-fashioned instruments, and this is the cause of Ibn ‘Ezra’s comment that an astrolabe is less precise if it is old. The reason is, obviously, that the calendrical scale depends on the position of the solar apogee as well as on the precession of equinoxes, which also affects the longitudes of the stars projected on the rete.⁴⁰⁸ Among other features concerning the old-fashioned character of the astrolabes described by Ibn ‘Ezra, we find that our author states that astrolabes contain three plates, engraved on both sides, one side per climate, while the seventh climate is represented on the *mater* of the instrument.⁴⁰⁹ There are several references to this kind of “universal” astrolabe in one of the texts of the tenth-century Latin corpus on the instrument,⁴¹⁰ but in the “Carolingian” astrolabe there is already a plate both sides of which are engraved for the specific latitudes of Córdoba and Barcelona (4.2.2.4). Another hint of the same kind is the mention of azimuthal or vertical cycles which are *usually* found in astrolabes,⁴¹¹ in spite of the fact that the presence of projections of these circles was absolutely standard in the twelfth century. We should remember, however, that we have found the same kind of remark in the treatise by Abū l-Ṣalt (4.2.3.2). Furthermore, Ibn ‘Ezra describes the procedure to determine the solar longitude with the calendrical scale *in the back of those astrolabes which have this diagram*. We have already seen similar comments in the treatises by Abū l-Ṣalt and Ibn al-Naṭṭāḥ.

As in the treatises on the astrolabe by Ibn al-Ṣaffār, Ibn al-Samḥ, and Ibn al-Naṭṭāḥ, Ibn ‘Ezra explains uses of the astrolabe for purposes for which the instrument has not been designed, related to the positions of the Moon and planets. In the case of the Moon, he explains how to determine the hours during the night using the lunar altitude. He begins by computing the lunar longitude using a set of astronomical tables (“locum lune in tabulis coequationis planetarum secundum artem quam dedimus de coequandis

408 In *Keli ha-neḥoshet* 1, Abraham ibn ‘Ezra uses a precessional constant of 1° every 70 years (Rodríguez Arribas, 2014, p. 230). This value is attributed to al-Ṣūfī by Ibn ‘Ezra in his *Fundamenta tabularum* (Millás, 1947 a, p. 78) and it seems to be a rounded value of the constant used by the Jewish astronomer in his *zīj* (only known through Ibn ‘Ezra’s *Tabule pisane*) for the motion of the solar apogee (which depends only on precession): 1;3° in 70 years. See Samsó, 2012, p. 181.

409 Millás, 1940, p. 10.

410 See, for example: “et ipsa folia que conclusa iacent in ipso ALHOGIZA [= *al-hujra*] super ipsam matrem tabulam continent infra se scripta omnia climata mundi” (Millás, 1931, p. 279).

411 Millás, 1940, p. 11. According to Rodríguez Arribas, 2014, pp. 227 and 239, Ibn ‘Ezra considered azimuthal lines to be irrelevant.

planetis").⁴¹² Obviously the result obtained will not be as precise as in the case of the Sun or of the fixed stars, because it implies considering that the Moon is on the ecliptic; this is why the text ends by saying that, if the Moon is not on the ecliptic, the error involved will be proportional to the magnitude of this latitude ("hoc autem quod diximus recte quidem est si luna est in zodiaco, si autem in zodiaco non est secundum maiorem uel minorem eius latitudinem magis minusque falleretur").⁴¹³ He also uses the astrolabe in relation to planets and confusingly explains two procedures to determine their position, considering them, once more, as if they were on the ecliptic, or to establish whether they have a direct or retrograde motion.⁴¹⁴ He notes that the maximum elongation of Mercury and Venus in relation to the Sun is 41° and 47° : the value for Venus agrees with the corresponding one in Ibn 'Ezra's *De rationibus*, but this is not the case for Mercury, as the value in the *De rationibus* is 21° .⁴¹⁵

Finally, as in most treatises on the astrolabe, Ibn 'Ezra deals with the use of the instrument to solve standard problems of mathematical astrology (3.2.3). Our author carefully describes three of these applications: the division of the houses of the horoscope, the computation of aspects and, consequently, the projection of rays, and, finally, the prorogation or *tasyir*, which he translates as *ductus*.⁴¹⁶ Ibn 'Ezra's system for the division of the houses has been considered to be one of the oldest sources describing the hour-lines method (fixed limits, see 3.2.3.1.3), in which the beginnings of the houses are determined by the intersections of the ecliptic with the lines corresponding to the even hours, which are present in any astrolabe. This method is described in his treatise on the astrolabe,⁴¹⁷ in the *De rationibus*⁴¹⁸ and in the *Sefer ha-moledot*;⁴¹⁹ it is extremely laborious and complicated to solve by computation, while it becomes quite easy if the astrologer is using an astrolabe. Ibn 'Ezra refers to this when he says "Nos vero in astrolabio docuimus facile distinguere domus".⁴²⁰ The

⁴¹² Millás, 1940, p. 19.

⁴¹³ Millás, 1940, p. 19. The same remark concerning the planets appears in *Keli ha-neḥoshet* 11: see Rodríguez Arribas, 2014, p. 234.

⁴¹⁴ These topics are also dealt with in a chapter of Ibn al-Naṭṭāḥ's treatise: see ms. BL Add 9602, fols. 21v–22r.

⁴¹⁵ Millás, 1947 a, p. 122. As a matter of fact, the sentence on the maximum elongation of Mercury in the *Tractatus* is not very clear and seems corrupt: "et mercurius in maxima remotione a sole que est 41 secunde graduum". The maximum elongation of Mercury, in Ptolemaic theory, lies between 18° and 28° : see Pedersen, 1974, p. 295.

⁴¹⁶ Millás, 1940, pp. 22–26.

⁴¹⁷ Millás, 1940, p. 22.

⁴¹⁸ Millás, 1947 a, pp. 159–161, where he also describes the procedure for calculating the *tasyir*.

⁴¹⁹ North, 1986, pp. 20–27.

⁴²⁰ Millás, 1947 a, p. 160.

method, though easy, was rarely described in *zīj*es, although it often appears in astrolabe treatises and was well known in Europe from the fifteenth century onwards.⁴²¹

4.2.4.2.2 *Pseudo-Māshā'allāh's Treatise on the Construction of the Astrolabe*

4.2.4.2.2.1 Introduction

The treatises *De compositione* and *De operatione uel utilitatibus astrolabii* attributed to the famous astrologer Māshā'allāh who flourished in Baghdad towards the end of the eighth and beginning of the ninth centuries, are, no doubt, the most popular texts on our instrument in medieval Europe. The research undertaken during many years by Ron B. Thomson shows that some 189 manuscripts are extant.⁴²²

The attribution to Māshā'allāh was conclusively dismissed by Paul Kunitzsch⁴²³ who states in his conclusions:

Part I, *De compositione*, is a western Latin compilation, perhaps from the middle or second half of the thirteenth century AD, consisting of four or five different sections of different origin, none of which bears a recognisable relation to Messahalla. Not even Maslama, who is quoted as source for the main part of *De compositione*, seems to be involved in the history of the treatise, as far as his works are known. Part II, *De operatione*, has proved to be a purely western compilation and fabrication made on the basis of Johannes Hispalensis' Latin translation of the Arabic treatise on the use of the astrolabe by the Spanish-Muslim astronomer Ibn al-Ṣaffār, one of Maslama's disciples.⁴²⁴

In this quotation, the reference to Maslama in relation to *De compositione* is due to the fact that some manuscripts include, after chapter 16, the following declaration: "Finit opus astrolabii secundum Marcelliana", while others give this name in the Maslama-form (*macellama*, *Macelama*, *Mascelama*).⁴²⁵

In the following pages I will follow Kunitzsch's lead, although I will try to find evidence of the connection between the texts of pseudo-Māshā'allāh and the works on the astrolabe undertaken by Maslama and his disciples. Although

⁴²¹ See North, 1986, as well as Kennedy, 1996 and Casulleras, 2010.

⁴²² Thomson, 2019. This section has been revised by Prof. Ron B. Thomson who also gave me access to his unpublished edition. I would like to express here my deep gratitude to him.

⁴²³ Kunitzsch, 1981.

⁴²⁴ Kunitzsch, 1981, p. 56.

⁴²⁵ Kunitzsch, 1981, p. 46 and fn. 25; Thomson, 2019, *De compositione*, ch. 16, fn. 20.

we cannot be sure where the compilation of the Latin text was made,⁴²⁶ it seems clear to me that a part of it, at least, is based on Andalusī sources. This is clear in pseudo-Māshā'allāh's *De operatione*, which Kunitzsch has established derives directly from Johannes Hispalensis' translation of the treatise on the use of the astrolabe by Ibn al-Ṣaffār (see 4.2.1).⁴²⁷

4.2.4.2.2.2 Part I: The *Proemium*

Pseudo-Māshā'allāh's *De compositione* begins with a *Proemium* for which Kunitzsch has found an Arabic source in an anonymous treatise on the construction of the astrolabe (extant in ms. Aya Sofya 2671). King⁴²⁸ has suggested that it may be the work of the eleventh-century Toledan astronomer Ibn al-Zarqālluh (d. 1100). As Kunitzsch shows, practically all this *Proemium* is a translation from the Arabic.⁴²⁹

4.2.4.2.2.3 Part II: Chapters 1–6

Chapters 1–6 are the second part of the book according to Kunitzsch. Chapters 1 and 2 contain a general description of the *mater* and the back of the astrolabe. The second chapter gives interesting information about the calendrical scale and states that the solar apogee is placed at 84;30°, although a parenthetical note adds that this value should be replaced by 87° due to precession.⁴³⁰ These positions are not very far from those established by Ibn al-Zarqālluh who, in observations made in 1074–75, determined that the longitude of the solar apogee was 85;49°,⁴³¹ while in his *Almanac* the tables for Venus yield an apogee, which usually coincides with the solar apogee, at 87°. ⁴³² The solar eccentricity

426 The only locality explicitly mentioned is Paris where, apparently, observations were made in 1223 AD (this year should probably be replaced by 1246, when John of London composed his star table in Paris), resulting in the star-table appended to the *De compositione*: see Kunitzsch, 1966, pp. 67–71; Kunitzsch, 1981, p. 47. A reference to this star-table appears in Thomson's *addendum* to chapter 10 (lines 32–36) of *De compositione*.

427 Millàs (1931, pp. 29–48) seems to have been the first to notice the obvious similarities between Ibn al-Ṣaffār's treatise on the use of the astrolabe and the *De operatione*; see on this topic Martí and Viladrich, 1983.

428 King, 1979.

429 Kunitzsch, 1981, pp. 43–44.

430 Thomson, 2019, ch. 2, lines 27–27. The solar apogee is located in 85° according to Johannes Hispalensis: see Millàs, 1942, p. 320.

431 Samsó, 2011, p. 211.

432 Boutelle, 1967.

is $1/32$,⁴³³ equivalent to $1;52,30^p$ if we assume a radius of the solar eccentric of 60^p . Interestingly $1;52,30^o$ is the maximum solar equation (not the eccentricity) in Ibn al-Zarqālluh's equatorium, which makes me think of a possible mistake by the author of the *De compositione* in the identification of the parameter.

Chapter 2 also gives the dates for the equinoxes and solstices, which are: 14 March, 16 June, 17 September and 15 December.⁴³⁴ These dates correspond, approximately, to ca. 1132. In an *addendum* to chapter 2, we also have the solar positions for the beginning of each month,⁴³⁵ and Thomson indicates that these positions coincide with what we find in a table giving the true longitude of the Sun for the beginning of each month of year 1252,⁴³⁶ although we can find similar approximations, using the computer programme TOLLN,⁴³⁷ for dates around 1132.

Chapter 3 continues with the description of the back of the astrolabe in which we find a shadow scale and hour lines like those of the *quadrans vetus* (2.4.2.1.2.2).

Chapter 4 deals with the standard alidade, while chapter 5 is more interesting because it describes an horary alidade (fig. 4.19) in which the outer sight is used as a gnomon of 12^d , and the reversed shadow (tangent) projected by this gnomon (corresponding to a solar altitude of 15^o , 30^o , 45^o , 60^o and 75^o) is marked on the outer half of the alidade. The values of the shadow given in an *addendum* to chapter 5 are correct:⁴³⁸

15^o	$3;13^d$
30^o	$6;56^d$
45^o	12^d
60^o	$20;47^d$
75^o	$44;46^d (-0;1^d)$

433 Thomson, 2019, chapter 2, lines 28–32. The same value for the eccentricity can be found in Johannes Hispalensis' treatise on the construction of the astrolabe: see Millás, 1942, p. 320.

434 Thomson, 2019, chapter 2, lines 36–45.

435 Thomson, 2019, ch. 2, lines 57–69.

436 Chabás & Goldstein, 2012, p. 85; Pedersen, 2002, vol. 3, p. 1608.

437 A computer programme giving planetary positions according to the parameters of the *Toledan Tables*. The skeleton of this programme was prepared by E.S. Kennedy and later revised by Honorino Mielgo.

438 Thomson, 2019, ch. 5, lines 50–57.

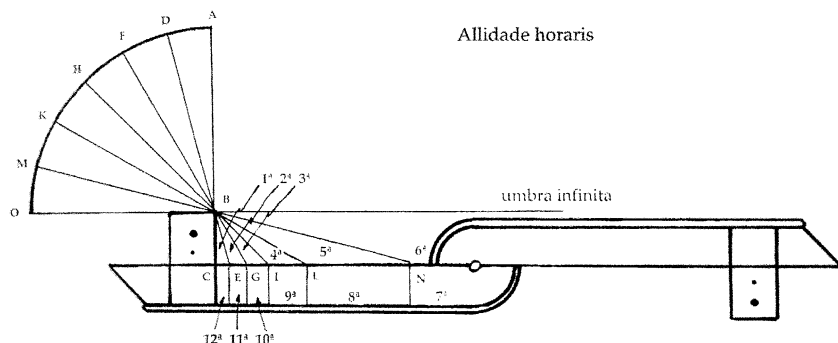


FIGURE 4.19 The horary alidade according to Ron B. Thomson, 2019

This kind of alidade, which assumes that an increase or decrease of 15° in the solar altitude corresponds to the passage of one seasonal hour, is independent of the local latitude. It was known in the Greek and Byzantine world, as well as in early Medieval Europe and in early Islamic astronomy, as described by Ḥabash al-Ḥāsib (fl. 831–860) and by Hermann Contractus (1013–1054).⁴³⁹

Chapter 5bis describes the *novella*, the radial ruler placed in the front of the astrolabe. Inscribed on it we find divisions which correspond to the intersections of the meridian line with the almucantars of the plate traced for the latitude of the observer. This implies that the divisions of the radial ruler can only be used for a particular latitude.

Chapter 6 contains a description of the pin (*al-qutb*) and the horse (*al-faras*) used to keep together the *mater*, plates and alidade. This chapter ends with an interesting sentence:

Huc usque intermissimus de diversis tractatibus, sed nunc redeamus ad librum, translated by Thomson as “up to this point we have interrupted [our text with material] from various treatises; but now let us return to our book.”⁴⁴⁰

This sentence marks the end of the second part of the book which, as we have seen, is a lengthy description of the astrolabe; it is more complete than what we usually find in the first chapter of a treatise on the use of the instrument but is basically similar.

439 King, 2005, pp. 253–255. This kind of device was not unknown in the Andalusī tradition. As remarked by Martí and Viladrich (1983, pp. 69–70) a reference to it appears in Ibn al-Ṣaffār’s treatise on the use of the astrolabe: see Millás’ Arabic edition (Millás, 1955, pp. 63–64) and his Catalan translation in Millás, 1931, pp. 39–40.

440 Thomson, 2019, chapter 6, line 8.

4.2.4.2.2.4 Part III: Chapters 7–16 and the Echoes of Maslama's School

The third part (chapters 7–16) is far more technical and, in it, we will find evidence of the presence of Maslamian materials:⁴⁴¹

Chapters 7 and 8 deal with the projection of the equator and the tropics of Cancer and Capricorn (chapter 7) and with the projection of the ecliptic (chapter 8). The procedures explained are standard and can be found in Ptolemy's *Planisphaerium*, in the old Catalan collection of Latin treatises on the astrolabe (4.2.2) and in the Alfonsine book on the construction of the instrument (4.2.4.2.3).⁴⁴² The only interesting detail is that our text projects the three circles (equator and tropics), using any one of them as the base circle.

Chapter 9, on the division of the ecliptic into signs and degrees,⁴⁴³ is extremely interesting because it derives clearly from Maslama's extra chapter to his notes on Ptolemy's *Planisphaerium*.⁴⁴⁴ In it, the pseudo-Māshā'allāh explains four methods to divide the ecliptic, the first of which is not easy to understand:⁴⁴⁵

Let ABGD (fig. 4.2o) be the equator and AZGH the ecliptic. One takes arc GT equal to half the obliquity of the ecliptic. Line AT determines point Q on the meridian line BED. Then one takes arcs DL = MB = 30° on the previously divided equator and draws arc LQM which determines, on the projection of the ecliptic, points N and P. These will correspond to the beginnings of Gemini and Sagittarius.

The first impression made when one reads the text is that arc GT should be equal to the obliquity of the ecliptic and not to half of it: point Q would then be the projection of the northern pole of the ecliptic and PLQNM would be an arc of a great circle orthogonal to the ecliptic. The construction would then be similar to the one described in chapter 15 of Ptolemy's *Planisphaerium*,⁴⁴⁶ and this seems to be the second method mentioned in the pseudo-Māshā'allāh's text.⁴⁴⁷

Nevertheless, Maslama's notes on Ptolemy's book shed some light on the question: the Andalusī author describes three different constructions for the division of the ecliptic into signs and degrees. The third of these constructions⁴⁴⁸

441 Most of the information given here can also be found in Viladrich & Martí, 1981.

442 Viladrich & Martí, 1981, pp. 81–82.

443 See on this topic Viladrich & Martí, 1981, pp. 89–94.

444 This extra chapter was edited and translated into Spanish by Vernet & Catalá, 1965. On this text see also Kunitzsch & Lorch, 1994, pp. 85–88.

445 Thomson, 2019, ch. 9, lines 6–24. This explanation is taken from Samsó, 1980 a.

446 Cf. Heiberg, 1907, pp. 251–252; Neugebauer, 1975, vol. 2, p. 866. The great circle orthogonal to the ecliptic is determined by three points: one is the projection of the northern pole of the ecliptic, whilst the other two are two diametrically opposite points on the ecliptic.

447 Thomson, 2019, *Addendum* 9.1, lines 25–26.

448 Vernet & Catalá, 1965, pp. 22–24, 30–33.

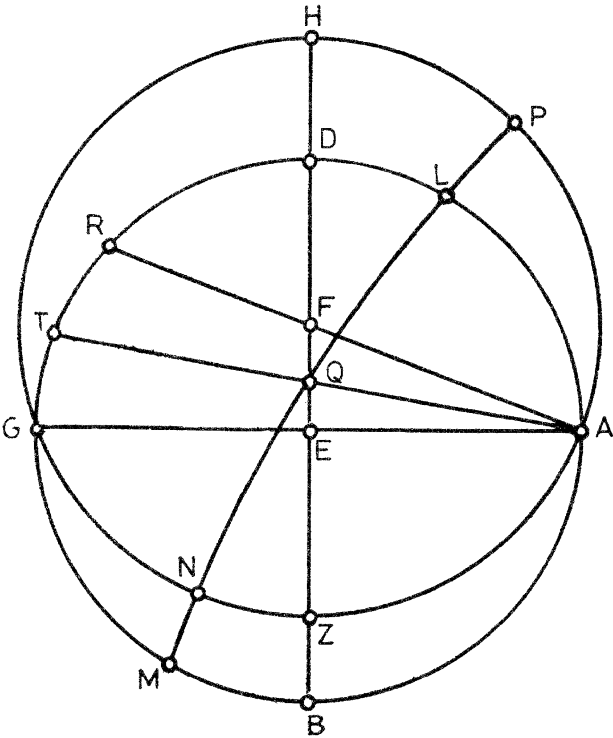


FIGURE 4.20
Maslama's method
to divide the ecliptic
traced on an astrolabe

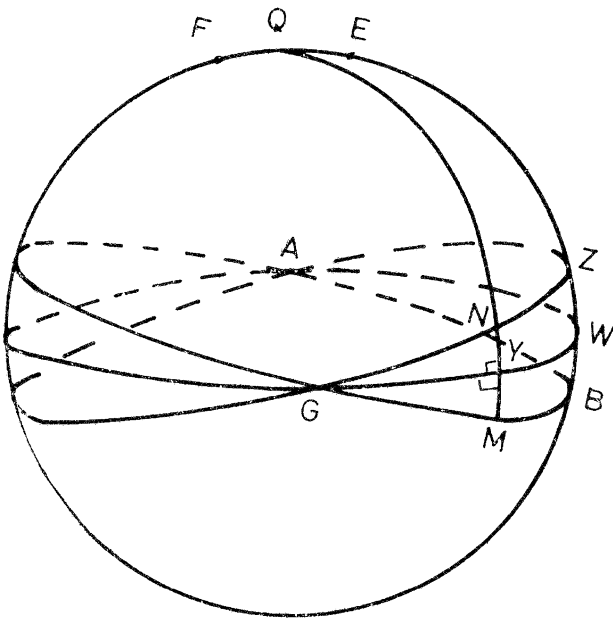


FIGURE 4.21
Maslama's auxiliary
theorem applied to the
division of the ecliptic
traced on an astrolabe

coincides precisely with pseudo-Māshā'allāh's text, with one addition: Maslama takes arc GR as the double of the obliquity (*sic* for the obliquity) and determines point F which he identifies with the pole of the ecliptic, whilst point Q is defined as being "the pole of the great circle which intersects the equator at points A and G and which divides the arc between the two solstices (*sic*: one should understand, as Vernet and Catalá do, half the distance between the two solstices) into two halves.

The role of point Q becomes clear if we take into consideration a theorem previously demonstrated by Maslama: great circles GZA (the ecliptic) and GBA (the equator) (fig. 4.21) intersect at points G and A, and their respective poles are F and E. We draw the great circle GWA which goes through points G and A and divides arc BZ into two halves at point W. If we draw the arc of a great circle QNYM it can easily be proved (as Maslama does) that GN = GM. Therefore, it becomes evident that in pseudo-Māshā'allāh's construction (fig. 4.20) point Q is not the projection of the northern pole of the ecliptic, and arc PLQNM is not an arc of a great circle orthogonal to the ecliptic, but that the purpose of the construction is attained since points N and P are effectively the beginnings of Gemini and Sagittarius on the astrolabe. Maslama's influence, here, seems fairly clear.

To this first procedure for the division of the ecliptic, in an *addendum* pseudo-Māshā'allāh adds three more:⁴⁴⁹ I have already mentioned the first one in which GT is the obliquity of the ecliptic and Q the northern pole of the ecliptic. The other two were also mentioned by Maslama, and they are based on:

1) The right ascensions of the degrees of the ecliptic: straight lines are drawn from the centre of the astrolabe, which is also the centre of the equator, to the degrees of the equator corresponding to the right ascension of the corresponding degrees of the ecliptic. This construction is extremely simple and it requires only a table of right ascensions. Pseudo-Māshā'allāh gives the following correct values of the difference in right ascension between the beginning and the end of the following signs (rising times of the signs in right ascension). They have been calculated for an obliquity of the ecliptic of $23;35^{\circ}$.⁴⁵⁰

449 Thomson, 2019, *Addendum 9-1*, lines 25-33.

450 This is actually al-Battānī's value for the obliquity of the ecliptic and not $23;36^{\circ}$ as it appears in pseudo-Māshā'allāh's text: see Thomson, 2019, ch. 7, line 16. The following rising times in right ascension are clearly derived from al-Battānī: see Chabás & Goldstein, 2012, p. 27.

Sagittarius, Capricorn, Gemini, Cancer:	32;13°
Aries, Libra, Pisces, Virgo:	27;53°
Tauro, Scorpio, Leo, Aquarius:	29;54°

2) The declinations of the degrees of the ecliptic: parallels of declination, corresponding to particular degrees of the ecliptic, are drawn on the astrolabe. Their intersections with the ecliptic will determine the corresponding degree. A table of declinations is needed here.

Chapter 10 deals with the projection of the fixed stars on the rete of the astrolabe. The method described in the text is standard and uses the declination⁴⁵¹ and mediation of the star. This procedure appears also in Maslama’s extra chapter⁴⁵² although this case is not as significant as that of chapter 9. Two examples are given in our text for stars *Vultur volans* (*al-Nasr al-Ṭā’ir*, α *Aquilae*) and *Cor Tauri* (*al-Dabarān*, α *Tauri*)⁴⁵³ for which coordinates are given which, in the case of *Cor Tauri*, are not far removed from those in Maslama’s star table (4.2.2.3.1):

TABLE 4.5

Star	Pseudo-Māshā’allāh’s mediation	Maslama’s mediation	Pseudo-Māshā’allāh’s declination	Maslama’s declination
Vultur volans	286°	283;18°	7;25°	+6;25°
Cor Tauri	56°	56;34°	14°	+14;12°

An *addendum*⁴⁵⁴ to chapter 10 refers to the possibility of projecting the stars on the rete using a star-table based on observations made in Paris with an armillary sphere. According to the *addendum* this table “continens stellas cum distancia earum ab orbe signorum, et cum longitudinibus earum secundum

451 *Longitudo ab equinoctio diei* in the text: see Thomson, 2019, ch. 10, line 4 and *passim*. This seems to be a literal translation of the Arabic expression *bu’d ‘an mu’addil al-nahār*, as the term *mayl* is only applied to the declination of a point of the ecliptic.

452 Vernet & Catalá, 1965, pp. 25, 37–29. This method is also described in the two extant chapters of Ibn al-Ṣamḥ’s treatise on the construction of the astrolabe. Another possibility is also introduced by Ibn al-Samḥ: using the right ascension of the mediation instead of the mediation. See Viladrich, 1986 a.

453 Thomson, 2019, ch. 10, lines 1–26.

454 Thomson, 2019, ch. 10, lines 32–36.

veritatem que habentur ex circulo magno eunte per polos zodiaci et per stellas ad eclipticam". This means that the two coordinates mentioned are the star's latitude ("distantia ab orbe signorum") and its tropical longitude ("cum longitudinibus earum secundum veritatem que habentur ex circulo magno eunte per polos zodiaci et per stellas ad eclipticam").⁴⁵⁵ The identification of the star table used is not easy.

Kunitzsch⁴⁵⁶ reports that six different star tables are found in the manuscripts of *De compositione*, corresponding to types v,⁴⁵⁷ vii,⁴⁵⁸ viii,⁴⁵⁹ ix,⁴⁶⁰ x,⁴⁶¹ and xi.⁴⁶² Tables of types v and vii give the stars' longitude and latitude, while types viii, ix and x contain their mediation and declination. Star tables of types v and vii do not contain any reference to the Parisian observations, though type xi does:

certificata ad ciuitatem parisius cuius latitudo est .48. gradus et .30. minuta. in anno domini nostri iesu christi. 1223.

A cursory examination of the actual table makes me doubt this interpretation. The table contains five different coordinates:

- 1) "Gradus cum quibus mediant celum"
- 2) "Gradus longitudinis ex utraque parte"
- 3) "Altitudo meridiana"
- 4) "Longitudo"
- 5) "Latitudo"

Surprisingly, the names of these coordinates do not correspond to their apparent meaning. *Longitudo* and *Latitudo* are not the stars' longitude and latitude but their mediation and declination, although the *Altitudo meridiana* does correspond to the meridian altitude for the latitude of Paris, assumed to be 48;30°. Coordinates 4 and 5 correspond to the previously mentioned standard method for projecting stars on the rete, using mediation and declination, and they are not "something new" to which the *addendum* seems to refer. The identification of coordinates 1 and 2 becomes even more surprising, as they correspond to the *Latitudo* (α) and *Altitudo* (β) of the star table in the old *De mensura astrolabii*

⁴⁵⁵ This is the procedure described in chapter 22.

⁴⁵⁶ Kunitzsch, 1981, p. 47.

⁴⁵⁷ Kunitzsch, 1966, pp. 35–37.

⁴⁵⁸ Kunitzsch, 1966, pp. 47–50.

⁴⁵⁹ Kunitzsch, 1966, pp. 51–58.

⁴⁶⁰ Kunitzsch, 1966, pp. 59–61.

⁴⁶¹ Kunitzsch, 1966, pp. 63–66.

⁴⁶² Kunitzsch, 1966, pp. 67–71.

which I have described in 4.2.2.3.1.⁴⁶³ If our *addendum* does refer to this particular table, does this mean that the procedure implied in this addendum is the same as the one we found in the old Catalan corpus, and that this is another example of Maslamian influence on pseudo-Māshā'allāh's text?⁴⁶⁴

Chapter 11 deals with the technical details for the making of the rete, chapter 12 describes an auxiliary figure in which the three basic parallels (tropic of Capricorn, equator and tropic of Cancer) are drawn and both the circle of Capricorn and the equator are divided equally into degrees or multiples of degrees. Chapter 13 deals with the projection of the horizon and the almucantars. All these chapters are more or less standard and the only unanswered question is why chapter 12 makes equal divisions on the circle of Capricorn, since these divisions are not used in the next chapter, in which the divisions of the equator are employed.⁴⁶⁵

Two manuscripts of *De compositione* contain a marginal note⁴⁶⁶ giving, apparently, the rising times in oblique ascension of the zodiacal signs, for a latitude around 48°. The values given in the text appear in table 4.6, column 1, in which Thomson has introduced two corrections which are necessary for the symmetry of the table. Column 3 gives the oblique ascensions corresponding to the end of each sign and proves that there is something wrong in column 1, as the oblique ascension for Virgo 30° should be 180° (instead of 175°), while the value for Pisces 30° should be 360° (instead of 350°). For the sake of comparison I have added columns 2 and 4 containing the rising times of the signs for a latitude of 48° and an obliquity of the ecliptic of 24° (column 2), and the oblique ascensions of the ends of the signs for the same parameters.

Chapter 14 deals with the division of the horizon. For this purpose the same three methods described for the division of the ecliptic can be used, replacing the obliquity of the ecliptic by the colatitude. Maslama also mentions the same three methods; one of them,⁴⁶⁷ which we also find in pseudo-Māshā'allāh's treatise, deserves a description in detail:

463 Kunitzsch, 1981, p. 47, notes that types VIII and XI derive from two sources: the old Catalan star table (Typ. 111) and the one composed by John of London in Paris, 1246.

464 These values also appear in one of Johannes Hispalensis' texts on the construction of the astrolabe: see Millás, 1942, p. 319.

465 Ibn al-Samḥ suggests the use of the divisions of the circle of Capricorn for the projection of the fixed stars. The reason is, obviously, that the division of the larger circle of Capricorn is easier than that of the smaller equator. See Viladrich, 1986 a.

466 Thomson, 2019, ch. 13, fn. to line 21.

467 Vernet & Catalá, 1965, pp. 24–25, 35–36; Samsó, 1980 a, pp. 7–8.

TABLE 4.6 Rising times and oblique ascensions for latitude 48°

Signs	1. Rising times in oblique ascension (text)	2. Calculated rising times for $\phi = 48^\circ$ and $\epsilon = 24^\circ$	3. Oblique ascensions of the end of the signs (text)	4. Oblique ascensions of the end of the signs for $\phi = 48^\circ$ and $= 24^\circ$
Aries	15°	14°	15°	14°
Taurus	19°	19°	34°	33°
Gemini	27°	27°	61°	60°
Cancer	36°	38°	97°	98°
Leo	39°	41°	136	139°
Virgo	[3]9° (txt 29°)	41°	175 ^{o***}	180°
Libra	39°	41°	214°	221°
Scorpio	39°	41°	253°	262°
Sagittarius	36°	38°	289°	300°
Capricorn	[2]7° (txt 37°)	27°	316°	327°
Aquarius	19°	19°	335°	346°
Pisces	15°	14°	350 ^{o***}	360°

Let ABGD be (in fig. 4.22) the projection of the horizon and EBZD that of the equator. Let us take arc DT equal to the colatitude and divide it into two halves so that DQ = QT. Then we draw BQ to intersect diameter ZE at point L (which will not be the projection of the zenith: this last point would be determined, as Maslama explicitly states, by the intersection of ZE and BT at R, DT being the double of the colatitude).⁴⁶⁸ Let us then take arc EN = ZM with an arbitrary value:⁴⁶⁹ arc XNLSM (which is not the projection of a vertical circle), going through the previously established points N, L and M, will determine points X and S on the projection of the horizon. The same system is to be used until we have divided the whole of the circle ABGD in this way.

As for the other two procedures for the division of the horizon, only one of them is mentioned in an *addendum* to chapter 14 which, in Thomson’s translation, reads: “The horizon is better divided by straight lines passing through the whole altitude [of the circle] of Aries, that is, through the zenith point and

468

This remark is not to be found in pseudo-Māshā’allāh’s text, and neither line BT nor point R appear in the diagram reproduced in Thomson, 2019, ch. 14, fig. 14.

469

10° in Thomson, 2019, ch. 14, line 13, to which the text adds *aut quantum vis* (“or however much we wish”).

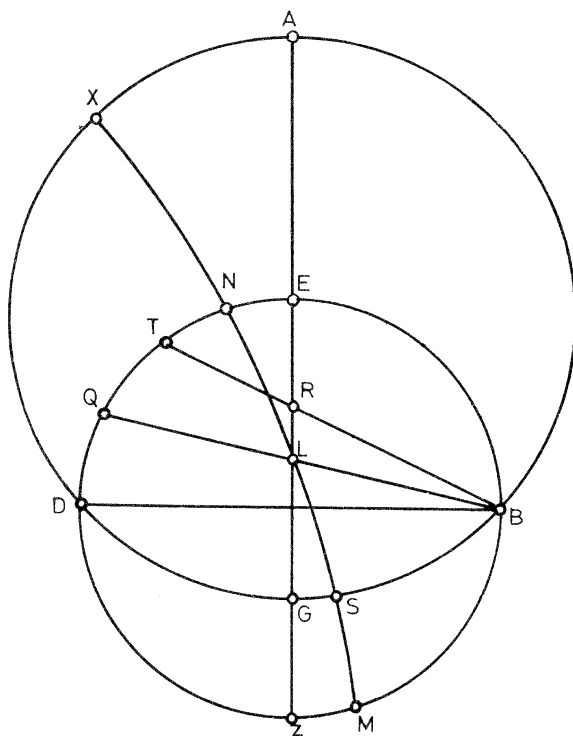


FIGURE 4.22
Maslama's method to divide
the horizon on an astrolabe

the degree of the equator; and we use this [method]". This corresponds to the second, approximate, method, also described by Maslama.⁴⁷⁰

How to project the azimuths is the subject of chapter 15. The procedure described is simple if the horizon has already been divided: it consists in determining three points through which each vertical circle will pass. One of these points is the local zenith⁴⁷¹ and the other two are two opposite divisions of the horizon.

Chapter 16, on the hour diagram below the horizon, is also standard. There are no references to curves for the *zuhr* or the *ʿaṣr*, dawn or twilight, although the latter will appear in the fourth part of the compilation (chapter 21).

4.2.4.2.2.5 Part IV: Chapters 17–22

This last section (the fourth in Kunitzsch's list, the third in Thomson's edition) repeats the treatment of some of the topics. Thus, chapter 17 explains the principle of stereographic projection and applies it to the projection of

⁴⁷⁰ Thomson, 2019, ch. 14 *addendum*, lines 20–21; Vernet & Catalá, 1965, pp. 24 and 34–35.

⁴⁷¹ The text does not seem to explain how to project the zenith, which appears later in ch. 19.

the equator, the two tropics and the ecliptic. Chapter 18 explains in detail how to project a circle parallel to the ecliptic. This is a topic treated in Ptolemy's *Planisphaerium*⁴⁷² as well as in Maslama's notes on it,⁴⁷³ and it can be applied to the projection of a star using its longitude and latitude, explained in chapter 22, which also appears in Maslama's extra chapter.⁴⁷⁴

Chapters 19 and 20 explain how to project azimuth circles using a procedure which is different from that explained in chapter 15. The method has been studied by J.L. Berggren⁴⁷⁵ who considered it anonymous⁴⁷⁶ and found it described in pseudo-Māshā'allāh's *De compositione*, al-Sijzī (ca. 950–ca. 1025), al-Bīrūnī (973–1048) and Abū 'Alī al-Ḥasan al-Marrākushī (d. 1262).⁴⁷⁷ In al-Sijzī's description we have (Fig. 4.23):

BDAG tropic of Capricorn

ZHET the equator

BA the meridian line

GD the East-West line

K the zenith

M the nadir

KHMT the first azimuth

L the centre of KHMT

RLS a diameter of KHMT perpendicular to the meridian line. All the centres of the projections of azimuth circles are contained in this line.

In order to determine the location of these centres, al-Sijzī divides the circle KHMT into the degrees or the multiples of degrees sought. As arc KX is one of these divisions, we trace the straight line KX which will intersect RLS at point O, the centre of the projection of the azimuth circle KM. In this procedure the angle MKX equals half the arc MX.

Chapter 19 of the *De compositione* begins by explaining how to project the zenith on the astrolabe for the latitude of 48°, which might correspond, approximately, to Paris, a city already mentioned in the *addendum* to chapter 10. It also explains how to find the image of the first vertical, and states that the centres of the azimuthal arcs will be placed on line RS. In contrast to al-Sijzī,

472 Heiberg, 1907, pp. 252–258.

473 Kunitzsch & Lorch, 1994, pp. 20–31.

474 Vernet & Catalá, 1965, pp. 25 and 37.

475 Berggren, 1991, pp. 330–333.

476 According to al-Bīrūnī it is “the method known among the artisans”: see Berggren, 1991, p. 332.

477 Marrākushī, 1984, II, pp. 48–50; Sédillot, 1844, pp. 160–161.

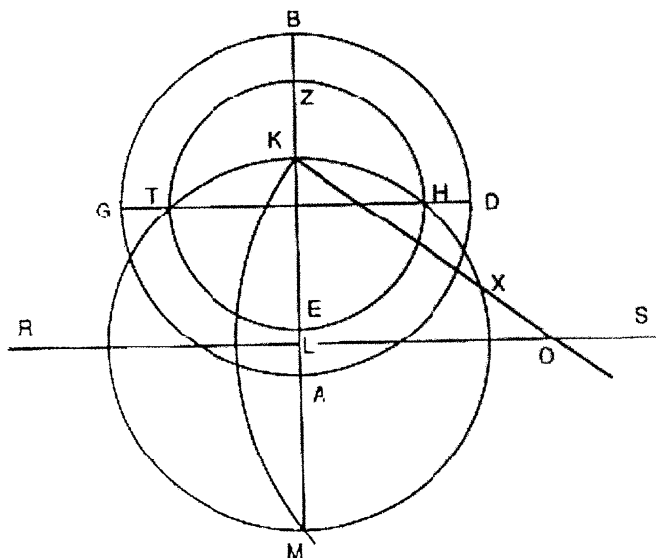


FIGURE 4.23
Borrowed from
Berggren

however, the first azimuth is not divided into equal parts but each one of the arcs KT, TM, MH and HK (the projections of the four quadrants of the first azimuth) are divided equally. This is incorrect, but it agrees with the description of the method we find in Abū 'Alī al-Ḥasan al-Marrākushī. The text seems incomplete because it does not describe how to determine the centre of each azimuth, although the figure traces a straight line from the nadir M to point X: the result of this is that the angle KMX equals half the arc KX, instead of MX. The azimuths obtained will, therefore, be different from those in al-Sijzī's procedure. Finally, in chapter 20, "another method" is described which is the same as al-Sijzī's, in the sense that the whole image of the first azimuth is divided equally, although, once more, the azimuths' centres are determined from point M and not from point K.

These two chapters pose the problem of the source used: neither al-Sijzī nor al-Bīrūnī seem to have been known in the Islamic West,⁴⁷⁸ and al-Marrākushī's *Mabādī*, a work written in Egypt which shares a common error with the *De compositione*, was never accessible in a Latin or Hebrew translation. The method probably derives from an unknown source or was actually used by astrolabe-makers. Whatever the case, it is also used in the treatise by Rudolf of Bruges.⁴⁷⁹

478 See however Samsó, 1996.

479 Lorch, 1999, p. 90.

Chapters 21 and 22 contain additional information that complements chapters 16 and 10. Chapter 21 explains how to project the curves of dawn and twilight which will appear below the horizon, crossing the hourlines. The negative solar altitude corresponding to the beginning of dawn and end of twilight is -18° , a standard parameter in Western Islamic astronomy (see 2.4.1.2.4). I wonder why the author of this part of the compilation felt the need to trace these curves, which do not seem to have practical applications beyond determining the times for the *maghrib* and *fajr* prayers. Finally, chapter 22 explains in detail how to project a fixed star using its longitude and latitude, a procedure mentioned, though not explained, in chapter 10. The same procedure, which is more or less standard, is also described in Maslama's extra-chapter.⁴⁸⁰

4.2.4.2.2.6 Conclusions

This brief revision of pseudo-Māshā'allāh's treatise on the construction of the astrolabe shows clearly the use of Andalusī sources: the *Proemium* is a translation of a treatise on the use of the astrolabe attributed to Ibn al-Zarqālluh, while chapters 7–16 show a series of instances of the influence of Maslama's extra chapter added to his notes on Ptolemy's *Planisphaerium*. To this one should add that Millàs, who believed in Māshā'allāh's authorship of the treatise, showed multiple examples of the connection between the pseudo-Māshā'allāh's treatise and the old corpus of Catalan texts on the astrolabe which, as we have seen, is clearly related to the works of Maslama and his school (see 4.2.2.3.1). Kunitzsch pointed in this direction but rejected it because Maslama does not seem to have written a treatise on the construction or the use of the astrolabe. Today, as a result of the discovery of two chapters of a book on the construction of the instrument by Ibn al-Samḥ, I believe that this could be the source used in chapters 7–16 and, perhaps, also of chapters 1–6. The other important disciple of Maslama, Ibn al-Ṣaffār, seems to have been the source of the *De operatione*, through Johannes Hispalensis' Latin translation.

The locality in which this compilation was made (in one or several stages) is unknown but it is interesting that in one instance (*addendum* to chapter 10) we have a reference to Paris, while in another (chapter 19) a latitude of 48° (which could also correspond to Paris) is mentioned. It is possible, therefore, that these parts are the result of a revision of the text made in northern France.

⁴⁸⁰ Vernet & Catalá, 1965, pp. 25 and 37.

4.2.4.2.3 *The Alfonsine Treatises on the Construction and Use of the Astrolabe*

4.2.4.2.3.1 The Construction Treatise

The Alfonsine collection of the *Libro del saber de astrología* contains a treatise on the construction of the astrolabe (*Primer libro dell astrolabio llano*)⁴⁸¹ and another one on its use (*Libro segundo dell astrolabio llano*).⁴⁸² Both texts have been studied in depth by Mercè Viladrich and Ramon Martí in the context of the Andalusī and Hispanic tradition of astrolabe treatises.⁴⁸³

The construction treatise is divided into 25 chapters and it has been clearly established that chapters 3–9 correspond exactly to chapters 7–14 of pseudo-Māshā'allāh's *De compositione*.⁴⁸⁴ The similarities between the two texts led Viladrich to formulate the bold hypothesis that “in some of the chapters of the *De Compositione Astrolabii* we might have the remains of an Alphonsine Latin version for which the Castilian translation would be the intermediate link”.⁴⁸⁵ This clarifies Viladrich and Martí's interpretation: the Alfonsine text was not a translation of pseudo-Māshā'allāh's, but both were based on the same source, which they identified with a text originated in Maslama's school.⁴⁸⁶

It is, perhaps, appropriate to revise this question here. Chapters 1 and 2 of the Alfonsine treatise are totally independent from pseudo-Māshā'allāh's *Proemium*. As a matter of fact the texts give different interpretations of the word astrolabe: pseudo-Māshā'allāh considers it to be a Greek word whose meaning is “acceptio stellarum” (“the reception of stars”),⁴⁸⁷ probably a translation of the Arabic *akhdh al-kawākib* (“taking the stars”) which is found in the anonymous treatise on the astrolabe attributed by King⁴⁸⁸ to Ibn al-Zarqālluh and considered by Kunitzsch to be the source of the *Proemium*. The Alfonsine

481 Ms. Villa Amil 156, fols. 69r–75v; Rico, 1863–67, II, pp. 225–252.

482 Ms. Villa Amil 156, fols. 76r–83r; Rico, 1863–67, II, pp. 253–292.

483 Viladrich & Martí, 1981 and Martí & Viladrich, 1983.

484 Viladrich, 1982.

485 This was based on Menéndez Pidal's analysis (1951) of the translation techniques of the Alfonsine school. At an early stage, translations were made by two or three collaborators. One of them translated the Arabic text into Castilian, while the other retranslated into Latin the previous oral Castilian translation. This retranslation was put in writing by a scribe (see d'Alverny, 1989). The originality of the Alfonsine translations would be that the oral Castilian intermediate translation was also written down by another scribe.

486 In spite of this, Pseudo-Māshā'allāh's *De compositione* could have been known in the Alfonsine court: Viladrich, 1990, edits and studies the text extant in one folio of the library of the Pedralbes Monastery (Barcelona), written in a script similar to those used by the scribes of the royal scriptorium of Alfonso X. The text contains chapters 9 and 10 of the *De compositione*.

487 Thomson, 2019: *Proemium* lines 1–2.

488 King, 1981, p. 44.

explanation is more pedestrian, as it considers that *astro* means “astra” or “stars” while *labio* has the same meaning as in Castilian: “lips”.⁴⁸⁹

Chapter 3 of the Alfonsine treatise is very similar to pseudo-Māshā’allāh’s chapter 7. Both texts deal with the projection of the two tropics and the equator in the same way, and use any one of these three basic circles as their starting point. The Alfonsine text uses an obliquity of 24°, while pseudo-Māshā’allāh mentions more precise values like 23;51° (Ptolemy), 23;36° (al-Battānī), 23;33° (al-Ma’mūn’s observers), and 24° (Indians).⁴⁹⁰ The diagrams illustrating the procedure in both sources are identical, although the letters are not always placed in the same sites.⁴⁹¹ The same equivalence can be found between:

Alfonsine treatise on construction	Pseudo-Māshā’allāh’s <i>De compositione</i>
Chapter 4	Chapter 8
Chapter 5	Chapter 9
Chapter 6	Chapter 10
Chapter 7	Chapter 11
Chapter 8	Chapters 12–13
Chapter 9	Chapter 14

Some of these equivalences deserve a brief comment: Alf 5/PsM 9 uses the procedure described by Maslama (see 4.2.4.2.2.4) for the division of the ecliptic, and both omit Maslama’s projection of the pole of the ecliptic. Each text seems to be a translation of the other,⁴⁹² although this hypothesis should be dismissed. The *addendum* to pseudo-Māshā’allāh’s chapter 9, in which three other methods for the division of the ecliptic are mentioned, does not appear in the Alfonsine work. Alf 6/PsM 10 treats the projection of star on the ecliptic. Both sources use the same stars as examples: *buytre uolant* (*vultur volans*) and *addauaran* (*al-Dabarān*, *Cor Tauri*) for which the two coordinates (declination and mediation) are not given in the Alfonsine text. Alf 9/PsM 14 deals with the

489 Ms. Villa Amil 156, fol. 69r; Rico, 1863–67, II, p. 229. The same etymology appears in one of Johannes Hispalensis’ texts on the construction of the astrolabe: “Astrolabium dicitur quasi astrorum labium per similitudinem dictum”.

490 Thomson, 2019, ch. 7, lines 16–18. The actual value used by al-Battānī is 23;35°.

491 Thomson, 2019, fig. 7; Ms. Villa Amil 156 fol. 69v; Rico, 1863–67, II, 231.

492 As shown by Viladrich, 1982, pp. 167–169, who reproduced the two texts in parallel columns.

division of the horizon in the same way. For illustrative purposes, let us compare two short texts:

“Partiras el cerco dell orizon assi cuemo partiste el cerco de los signos *en aquellas tres maneras*, saluo ende que fagas, en logar de la declinacion general, toda la altura de la cabeça de Aries en aquella uilla pora do es fecha la tabla”.⁴⁹³

“Deinde divides circulum hemisperii sicut divisisti circulum signorum *per ipsos tres modos*; sed uteris in loco totius declinationis tota altitudine Arietis et Libre in eadem regione”.⁴⁹⁴

Interestingly, the three procedures for dividing the ecliptic and the horizon were not mentioned in the Alfonsine chapter 5.

Another point of similarity can be found in the geometrical diagrams which, in most cases, are almost identical, although the letters used present a certain number of differences.⁴⁹⁵ Such is the case of figs. 4.24 and 4.25, illustrating the division of the horizon, although the Alfonsine manuscript includes the two tropics which do not appear in pseudo-Māshā'allāh's diagram. The same letters occupy the same positions, with the following exceptions:

<i>Alf</i>	<i>PsM</i>
X	O
C	G
V	B
Q	K

in which O/X probably correspond to an Arabic *shīn*, C/G to a *jīm*, V/B to a *bā'* and Q/K to a *kāf*.⁴⁹⁶

The similarities between the two sources end here, although they often deal with the same topics. Interestingly, in the description of the procedure for the construction of the calendric scale, on the back of the astrolabe, the Alfonsine text uses an eccentricity of $1/32$ ($1;52,30^p$) as in pseudo-Māshā'allāh's treatise (see 4.2.4.2.2.3).⁴⁹⁷ This eccentricity corresponds to a maximum solar equation of $1;47,27^\circ$, which is below the standard Alfonsine value ($2;10^\circ$).⁴⁹⁸ The correspondence between solar degrees and days of the year is not explained

493 Ms. Villa Amil 156 fol. 71v; Rico, 1863–67, II, 236–237.

494 Thomson, 2019, ch. 14, lines 3–5.

495 On the variations in the lettering of the text and the diagrams in pseudo-Māshā'allāh's *De compositione* see Thomson, 2019: Introduction, section F.

496 On this kind of problem see Kunitzsch, 1991–92.

497 Ms. Villa Amil 156 fol. 73v; Rico, 1863–67, II, 243.

498 Chabás & Goldstein, 2003, pp. 153–155.

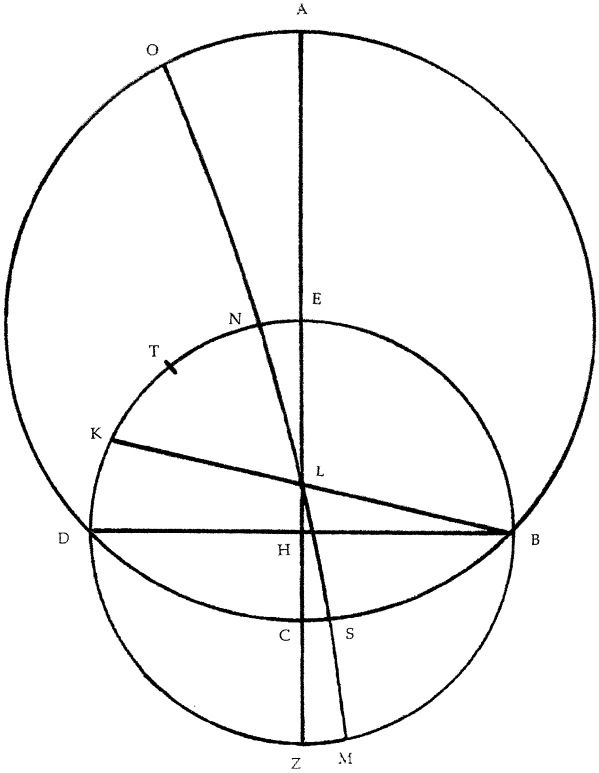


FIGURE 4.24
Pseudo Māshā'allāh's
division of the horizon
(Thomson's fig. 14)

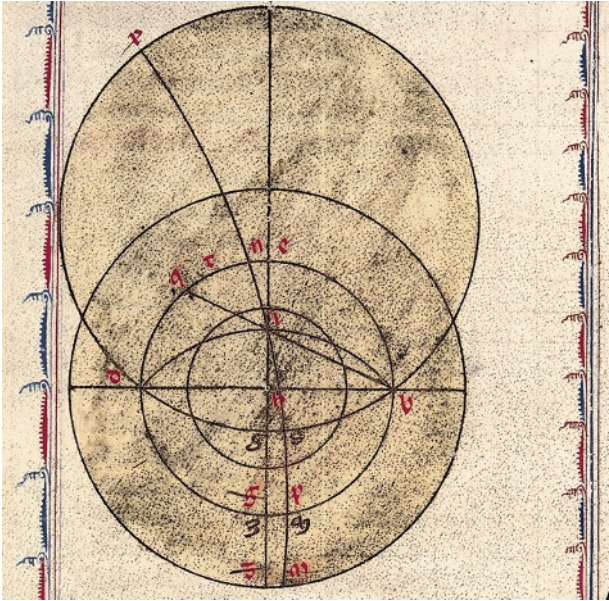


FIGURE 4.25
Alfonsine division
of the horizon
BIBLIOTECA HISTÓRICA
DE LA UNIVERSIDAD
COMPLUTENSE DE
MADRID. (BH MSS 156,
FOL. 72R)

clearly in this chapter, but we find a table which gives the position of the Sun for each day of a whole year at the end of the treatise on the use of the astrolabe.⁴⁹⁹ This table has been studied by Chabás⁵⁰⁰ who established that it was computed for ca. 1278 using a mean solar motion per day of 0;59,8,19,37° and a maximum solar equation of 2;10°, which derive clearly from the Alfonsine Tables.

4.2.4.2.3.2 The Treatise on the Use of the Instrument

The Alfonsine treatise on the use of the astrolabe contains 53 chapters, and ends with the table of solar positions to which I have already referred. I will deal here with some chapters that provide some interesting information.

Chapter 5⁵⁰¹ ends the description of the instrument and contains a reference to the five hour lines for prayers which are found in the hour diagram of Islamic astrolabes – of which it explicitly mentions the *heat toloḥ al-fagr* (*khaṭṭ ṭulūʿ al-fajr*) for the beginning of dawn and *heat maguip axafac* (*khaṭṭ maghīb al-shafaq*) for the end of twilight – and it adds “Et algunos de los xptianos ponen estas dos linnas en sus astrolabios pora saber quando comiença la claridat del sol et quando se pone” (and some Christians put these lines in their astrolabes in order to know when solar light begins and when it ends). Another reference to these hour lines appears in chapters 42 and 43,⁵⁰² where the text informs us that these two hour lines correspond to a solar altitude of –18°.

Chapters 6 and 7⁵⁰³ are concerned with a calendaric problem: that of determining the weekday for the beginning of each month of the Julian and the Muslim year if the weekday of the beginning of the year is known.⁵⁰⁴ As we have seen (4.2.1), this is a problem that interested Ibn al-Šaffār, who described a diagram for this purpose on the back of the astrolabe. The Alfonsine treatise uses a set of numbers which it calls *sennales* (signs) and are, for the Julian calendar:

January	1	July	7
February	4	August	3
March	4	September	6
April	7	October	8 (<i>sic</i> , instead of 1)
May	2	November	4
June	5	December	6

499 Ms. Villa Amil 156 fol. 83r; Rico, 1863–67, II, 291–292.
500 Chabás, 2003, 142–150.
501 Ms. Villa Amil 156 fol. 76v; Rico, 1863–67, II, 264.
502 Ms. Villa Amil 156 fols. 80r–80v; Rico, 1863–67, II, 281.
503 Ms. Villa Amil 156 fols. 76v–77r; Rico, 1863–67, II, 265–266.
504 Samsó, 1980, pp. 170–174.

The same set of values appear in Gerard of Cremona’s Latin translation of the *Calendar of Córdoba* where they are called *epactae* and in another translation of the same source, extant in ms. Vic Diocesan Museum 167, under the name of *regulae*. Interestingly, they also appear in the *Kitāb al-anwā*’ attributed to al-Kātib al-Andalusī, identified by Miquel Forcada with ‘Arīb b. Sa‘īd, one of the authors of the *Calendar of Córdoba*.⁵⁰⁵ This work, extant in ms. Istanbul Malik 2049, gives an equivalent set of values (*āya*, signs) for the Syriac Julian months in three verses authored, according to the source, by al-Fazārī, the famous astronomer of the second half of the eighth century.⁵⁰⁶

The Alfonsine description clarifies the use of these numbers: if one knows the day of the week (1 = Sunday, 2 = Monday etc.) of 1 January, and adds the “sign” of the corresponding month, the result will be the number of the weekday of that month. Thus, if 1 January is a Sunday, 1 February will be a Wednesday (by adding four days including Sunday), 1 March will also be a Wednesday, 1 April a Saturday, and so on. The same numbers can be obtained in older sources such as al-Khwārizmī’s *zīj*, the *Toledan Tables* and Ibn al-Zarqālluh’s *Almanac*.

Another set of values for the Muslim calendar can also be found in the Alfonsine treatise, as well as in al-Khwārizmī’s *zīj* and in the *Toledan Tables*. The “signs” used are, in this case:

Şafar	3	Sha‘bān	5
Rabī‘ I	4	Ramaḍān	6
Rabī‘ II	6	Shawāl	1
Jumādā I	7	Dhū l-Qa‘da	2
Jumādā II	2	Dhū l-Hijja	4
Rajab	3		

Values of this kind for both Julian and Muslim months reappear in the later Maghribī tradition, as we can see in the *Rawḍat al-azhār* of Abū Zayd ‘Abd al-Raḥmān al-Jādirī (1375–1416), as well as in the commentary on this *urjūza*, authored by Abū ‘Abd Allāh Muḥammad al-Ḥabbāk (d. after 1562).⁵⁰⁷

Chapter 27 (28 in Rico’s edition)⁵⁰⁸ describes the division of the houses of the horoscope according to the hour line method, which is more or less standard in many astrolabe treatises (3.2.3.1.3).

505 Forcada, 2000.

506 This attribution, if correct, implies an addition to the information on al-Fazārī collected by Pingree, 1970.

507 Saidi, 2013, pp. 24–39.

508 Ms. Villa Amil 156 fols. 78v–79r; Rico, 1863–67, II, 274–275.

Chapters 28 (29 in Rico's edition)⁵⁰⁹ and 35 (36 in Rico)⁵¹⁰ explain procedures to calculate the declination of a degree of the ecliptic and the latitude of a star. In both cases the almucantars are used as an auxiliary scale to measure declinations and latitudes. In the first case the degree of the ecliptic is placed on the meridian line and the user determines the degree of altitude (h') corresponding to that position; then, the operation is repeated with the point of the ecliptic corresponding to Aries 0° (h''). The difference between the two altitudes will be the declination of the degree. This use is correct if we consider that the degree in question is northern:

$$h' = 90^\circ - \varphi + \delta$$

$$h'' = 90^\circ - \varphi$$

$$h' - h'' = \delta$$

This procedure is more or less standard, but the author of the text applies it to the determination of the latitude of a star in chapter 36, something which cannot be considered acceptable. As in the previous case, the degree of longitude of the star (G) is placed on the meridian line and its meridian altitude is obtained ($h' = 90^\circ - \varphi + \delta(G)$, if the degree is northern). Then, the operation is repeated with the index (S) of the star itself ($h'' = 90^\circ - \varphi + \delta(S)$). According to our source, the difference between the two altitudes will be the star's latitude, while the result obtained will be the difference in declination between the star and its degree of longitude ($\delta(S) - \delta(G)$), an equatorial coordinate, not an ecliptic one.⁵¹¹

Chapter 33 (34 in Rico)⁵¹² is concerned with the revolutions of the years of the world, of nativities and of their ascendants (*tahāwīl sinī al-‘ālam/al-mawālīd*) and it states that the *revolutio anni* is $92;22^\circ$. This is, obviously, a sidereal parameter corresponding to a sidereal year of $365;15,23,40^d$, very near Ibn al-Zarqāllūh's sidereal year ($365;15,24^d$) as well as the year used by Ibn al-Hā'im ($365;15,23,37,33^d$).⁵¹³ One should remember that Ibn al-Kammād attributes to Ibn al-Zarqāllūh a *revolutio anni* of $92;24^\circ$.⁵¹⁴

509 Ms. Villa Amil 156 fol. 79r; Rico, 1863–67, II, 275.

510 Ms. Villa Amil 156 fol. 79v; Rico, 1863–67, II, 278–279.

511 Martí & Viladrich, 1983, pp. 38–39.

512 Ms. Villa Amil 156 fol. 79v; Rico, 1863–67, II, 277–278.

513 A slightly lower value is found in the *Toledan Tables* as well as in Ibn al-Raqqām *zīj*es: both give $365;15,23,29$ d. See Chabás & Goldstein, 2012, p. 57.

514 Samsó, 2011, p. 213.

The Alfonsine treatise, like many others we have already seen, applies the astrolabe to problems involving the Moon and the five planets. In chapter 41 (42 in Rico)⁵¹⁵ the use of the instrument for determining whether the altitude of one of these celestial bodies is eastern or western involves no difficulty, but the same cannot be said of chapter 40 (41 in Rico's edition)⁵¹⁶ which explains how to determine the longitude of the Moon or the planets with the astrolabe. The solution explained in the text involves, obviously, the assumption that the planet or the Moon is placed on the ecliptic, as the author himself acknowledges: "ca non se puede saber por el astrolabio ciertamientre por razon dela ladeza" (this cannot be established with certainty using the astrolabe due to the planet's latitude).

Finally, chapter 55 (56 in Rico)⁵¹⁷ explains how to determine the four cardinal points with the astrolabe, followed by chapter 56 (57 in Rico's edition)⁵¹⁸ which deals with the *qibla*. The word *qibla* does not appear in the title of the chapter, in which there is no reference to the city of Mecca either.⁵¹⁹ The formulation is general: how to determine the direction towards any city we like from the city where we are ("De saber en qual derecho esta qual uilla tu quisieres de la uilla en que estas sabiendo las ladezas et las longuezas damas las uillas"). To calculate the *qibla*, the text uses the following approximate expression, already seen in 2.5.2.1:⁵²⁰

$$q = \arcsin \left(\frac{\sin \Delta L \cos \phi_2}{\sqrt{(\sin \Delta L \cos \phi_2 / R)^2 + \sin^2 \Delta \phi}} \right)$$

in which

$q = \text{samt al-qibla}$,

ΔL is the difference of geographical longitudes between the two cities,

ϕ_2 is the latitude of the city whose azimuth we want to measure from our locality,

and $\Delta \phi$ is the difference between the latitudes of both cities.

⁵¹⁵ Ms. Villa Amil 156 fol. 80r; Rico, 1863–67, II, 280–281.

⁵¹⁶ Ms. Villa Amil 156 fol. 80r; Rico, 1863–67, II, 280.

⁵¹⁷ Ms. Villa Amil 156 fol. 82r; Rico, 1863–67, II, 288.

⁵¹⁸ Ms. Villa Amil 156 fol. 82r–82v; Rico, 1863–67, II, 288–289.

⁵¹⁹ There is, however, a reference to Mecca at the end of the chapter where we read that the "açent de Mecha" (*samt al-Makka*) in Córdoba is 45° (see 2.5.3).

⁵²⁰ Samsó, 1980, pp. 174–175.

The Alfonsine text, however, instructs the calculation $90^\circ - q$, and obtains, therefore, the *inḥirāf al-qibla* (measured from the meridian) instead of the *samt al-qibla* (measured from the East point).

The aforementioned expression is clearly related to another, known to Muslim astronomers since the ninth century, which was used in the time of al-Ma'mūn to find the *qibla* at Baghdad and was attributed by David King to al-Khwārizmī. The only difference is that, in the earlier text, $\sin \Delta L$ is replaced by ΔL .⁵²¹

4.3 Universal Astrolabes⁵²²

4.3.0 Introduction

In the prologue to his treatise on the *ṣaḥīḥa zarqālīyya* (4.0), Ibn al-Zarqālluh considers that the astrolabe is the best of all astronomical instruments of his time. Its use is simple, although it has one major drawback: it requires a specific plate for each latitude. If the adequate plate is not available, approximate methods have to be used, which do not guarantee exact results and involve a waste of time. This is why he has designed a universal astrolabe, the *ṣaḥīḥa*, valid for all latitudes so that “if one cannot obtain a particular result using these instruments or the operation is too back-breaking, one can fulfil this purpose and obtain the required precision with this plate”.⁵²³ This sentence, a translation from Ibn al-Zarqālluh's Arabic prologue, has a certain interest because it seems to imply that our author has designed the *ṣaḥīḥa* as an instrument complementing the astrolabe and that it should only be used if the astrolabe does not contain the plate for the adequate latitude. Ibn al-Zarqālluh was possibly aware of the difficulty and the effort of imagination required in many applications of the *ṣaḥīḥa*, compared to the simplicity of the astrolabe. This is probably the reason why we often find mixed instruments: a conventional astrolabe on one side and a *ṣaḥīḥa* on the back.

The *ṣaḥīḥa* is a contemporary of another instrument of the same kind: 'Alī ibn Khalaf's universal plate, which we only know through the Alfonsine translation.⁵²⁴ We know very little about this author. Ṣā'id al-Andalusī mentions him among the young scientists working in Toledo in his time and gives his full

⁵²¹ King, 1986c, pp. 85 and 102.

⁵²² This is a revised version of Samsó, 2011, pp. 180–199, 417–420, 488–490, 565.

⁵²³ On this kind of instrument, see Puig, 1985 Puig, 1986 and Puig, 1987.

⁵²⁴ Calvo & Puig, 2006.

name: Abū l-Ḥasan ‘Alī b. Khalaf b. Aḥmar al-Ṣaydalānī (the pharmacologist).⁵²⁵ Besides, ms. Escorial 962, containing Ibn al-Zarqālluh’s 100-chapter treatise on the *ṣafiḥa* calls him Abū l-Shajjār (“the father of the botanist”) in a final note which seems to have been added by the copyist.⁵²⁶ Ms. Hyderabad Andra Pradesh State Library 298, which contains an anonymous Tunisian recension of Ibn Ishāq’s *zīj* (see 1.4.2), names him as ‘Alī al-Shajjār and attributes to him a determination of the sidereal longitude of the solar apogee ($77;13,35^\circ$) and an observation of the obliquity of the ecliptic ($23;32,12^\circ$) which he made in Toledo in association with the well-known pharmacologist, physician and agronomist Ibn Wāfid in 477/1084–85. Ibn Wāfid died in 467/1074–75, which means that the date of the observation must be, at least, ten years earlier. My impression, however, is that the table of the Hyderabad manuscript recording this presumed observation, together with many others, is wholly unreliable⁵²⁷ (see 6.2.6.4) and that the only point that we should take from it is that ‘Alī b. Khalaf was called al-Ṣaydalānī and al-Shajjār, two terms that are almost synonymous, and that he maintained some kind of relationship with his contemporary the botanist and pharmacologist Ibn Wāfid. We should also bear in mind that Ṣā’id considered him to be a good geometer, while Ibn al-Zarqālluh was, in Ṣā’id’s opinion, the most important astronomer of his time.

In the prologue just mentioned, Ibn al-Zarqālluh’s *ṣafiḥa* is called *al-ṣafiḥa al-mushtaraka li-jamī’ al-’urūd* (the common plate for all latitudes). Like the astrolabe, this instrument, as well as ‘Alī ibn Khalaf’s universal plate (also called *orizón universal* in the Alfonsine translation) use the stereographic projection, although they change both the centre and the plane of projection: the plane is no longer the equator (astrolabe) but the solstitial colure, while the centre of projection is not the South Pole (astrolabe) but the equinoctial points (Aries 0° and Libra 0°). In Roser Puig’s words: “both [instruments] are based on the stereographic meridian of each hemisphere, superimposing the projection of a half of the celestial sphere from the vernal point (and turning it) on to the projection of the other half from the autumnal point”⁵²⁸ (see fig. 4.26). This system of meridian stereographic projection has an advantage over the equatorial stereographic projection of the astrolabe: the projection of the local horizon is a straight line identified with a diameter of the instrument and, consequently, a ruler which rotates around the centre of the *ṣafiḥa* or a diameter of the rotating rete in the universal plate can represent any horizon. We have, therefore, a

525 Bū ‘Alwān 1985, p. 180; Blachère, 1935, pp. 138–139.

526 King, 1979.

527 Comes, 1991a.

528 Puig, in *BEA*, 1, pp. 34–35.

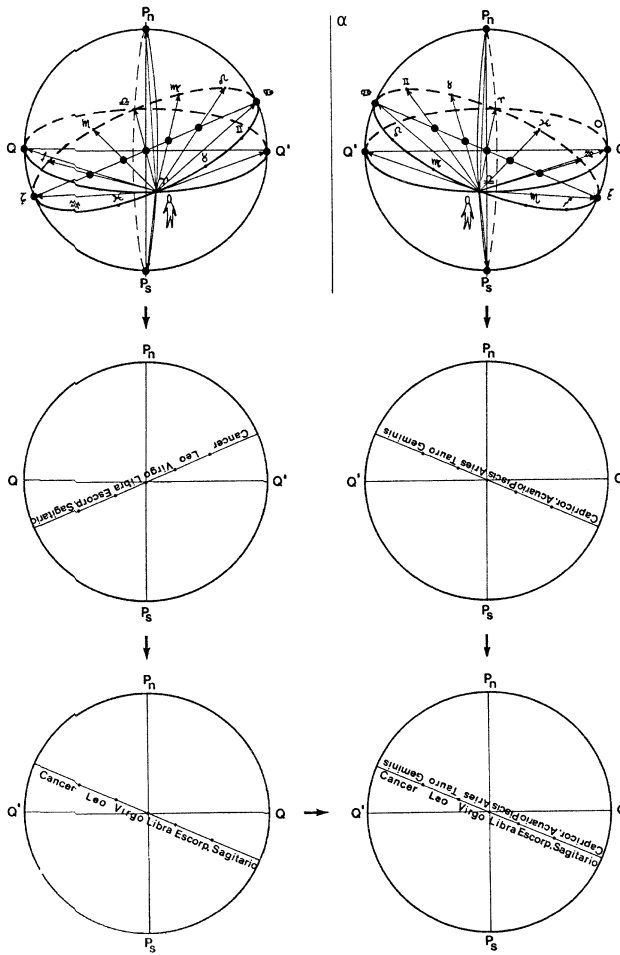


FIGURE 4.26
Meridian stereographic
projection. Borrowed
from Roser Puig

movable horizon, and the new astrolabe becomes a truly universal instrument which can be used for any latitude.

‘Alī ibn Khalaf and Ibn al-Zarqālluh’s works seem to be the first to include meridian stereographic projection. It should, however, be remembered that al-Bīrūnī’s *Taṣṭīḥ al-ṣuwar wa tabṭīḥ al-kuwar*⁵²⁹ describes two methods of non-stereographical projection (today called azimuthal equidistant projection and globular projection) which give a very similar image to the one used by the two Toledan astronomers. There is no reason, therefore, to believe in an Eastern origin of these two instruments although (as Roser Puig reminded me years ago) when al-Bīrūnī describes the cylindrical (today called orthographic)

⁵²⁹ Berggren, 1982; Kennedy & Debnort, 1984.

projection, in the *Tasīḥ*, he replaces the plane of the equator with that of the solstitial colure.⁵³⁰

There is no reason to accept the attribution to Ḥabash al-Ḥāsib (fl. 831–870) of an instrument which would be similar to the ones I am analysing here.⁵³¹ The evidence is late and it only bears witness to the fact that Ḥabash dealt with the “plate of horizons” (*al-ṣafīha al-āfāqīyya*) an expression which, in its usual meaning, designates a plate in which, using an equatorial stereographic projection, multiple lines of the horizon have been inscribed.⁵³² The problem Ḥabash was trying to solve was analogous to that posed by the Toledan astronomers, but the solution adopted is radically different and is linked far more closely to the instrument designed by Ibn Bāṣoh of Granada in the fourteenth century (4.3.5). This does not imply a rejection of a possible theoretical analysis, made in Antiquity or in Eastern Islam, which led to the meridian stereographical projection. The origins of the system could, perhaps, be related to the tradition of analemma methods that gave rise to the orthographic projection. A hint in this direction can be found in an incomplete passage of a book by ‘Alī ibn Sulaymān al-Ḥāshimī, an Eastern author of the second half of the ninth century. Al-Ḥāshimī’s problem is to determine quickly the time each zodiacal sign needs to cross the horizon, and his text gives a hint of the solution which is developed by Id⁵³³ using the analemma technique just mentioned: to determine the right ascension of a point of the ecliptic, he makes an orthogonal projection whose centre is the vernal point and whose plane is the solstitial colure.⁵³⁴

4.3.1 *‘Alī ibn Khalaf’s Universal Plate*

The treatise on the use of this instrument is dedicated to King al-Ma’mūn (r. 1037–1074) of Toledo who seems to have also sponsored the research of Ibn al-Zarqālluh.⁵³⁵ It has only been preserved in a Castilian translation included in the Alfonsine *Libro del saber de astrología*,⁵³⁶ where it appears together with a brief text on the construction of the plate authored by the well-known Rabiḥag

530 Puig, 1996.

531 King, 1979, p. 255.

532 Michel, 1947, pp. 37–38 and 91–92.

533 Id, 1969.

534 See, however, Haddad, Kennedy & Pingree, 1981, p. 277: Id’s solution is consistent with the incomplete text, but it is only one of the many possible graphical solutions.

535 Ms. Villa Amil 156 fol. 87r. Ms. Leiden University 468 calls the instrument *al-aṣṭurlāb al-ma’mūnī* and states that it was dedicated to al-Ma’mūn in 464/1071: see King, 1979, p. 252.

536 Ms. Villa Amil 156 fol. 87r–109r; Rico, 1863–67, III, 11–132.

(Rabbi Ishāq b. Sīd),⁵³⁷ one of King Alfonso X's two main scientific collaborators. In the prologue to the two chapters on the construction of the instrument we read that Alfonso told Rabiçag to write them because "el sabio que fizo esta lamina sobredicha non fizo libro de como se deue fazer de nueuo"⁵³⁸ (the scholar who made this plate did not write a book on the procedure to build the instrument). This remark is at odds with 'Alī ibn Khalaf's prologue to the long treatise on the use of the instrument, where the author says "et fiz este libro en que fabla de como se deue fazer de nueuo"⁵³⁹ (and I wrote this book explaining how to build the instrument), adding that he has read Ptolemy's book related to the projection of the sphere on a plane (the *Planisphaerium*) and that in the future he plans to write a book on the different systems to make these projections, including the required geometrical demonstrations, among which he will deal with the system used to build his universal plate.

It seems clear that the Alfonsine collaborators did not have a complete copy of Ibn Khalaf's book, and there is no evidence that our author actually wrote his book on projections. The two chapters added by Rabiçag agree with Ibn Khalaf's description of the instrument, in the treatise on its use, and they do not contain anything that could be considered original, because Rabiçag follows very closely Ibn al-Zarqālluh's treatise on the construction of the *ṣafīḥa* which, as we have seen, is also based on a meridian stereographic projection;⁵⁴⁰ he only updates the solar positions for the beginnings of each month in his description of the calendar scale in the back of the instrument. The values he gives are:⁵⁴¹

March	Pisces 19°	September	Virgo 16°
April	Aries 19°	October	Libra 16°
May	Taurus 18°	November	Scorpio 17°
June	Gemini 18°	December	Sagittarius 17°
July	Cancer 16°	January	Capricorn 19°
August	Leo 16°	February	Aquarius 22°

These solar positions also appear (with small differences due to copying mistakes in the day number expressed in Roman numerals) in the book of the

537 Ms. Villa Amil 156 fol. 83v–84v; Rico, 1863–67, III, 4–10.

538 Ms. Villa Amil 156 fol. 83v; Rico, 1863–67, III, 4.

539 Ms. Villa Amil 156 fol. 87r; Rico, 1863–67, III, 11.

540 Samsó, 1987 a.

541 Ms. Villa Amil 156 fol. 85r; Rico, 1863–67, III, 7–8.

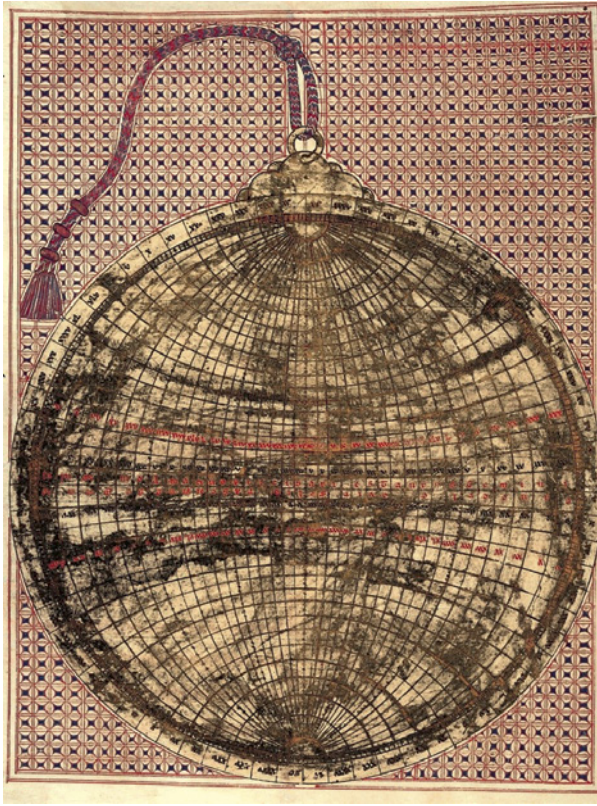


FIGURE 4.27
Mater of the universal
 plate in Biblioteca
 Histórica de la
 Universidad Complutense
 de Madrid. (BH MSS 156
 fol. 84v)

plane astrolabe (4.2.4.2.3.2),⁵⁴² as well as in the *Astrolabio redondo*,⁵⁴³ the *Quadrante para rectificar*,⁵⁴⁴ and the *Relogio dell agua*.⁵⁴⁵ All of them seem to have been calculated using the *Alfonsine Tables* or al-Battānī's *zīj* for dates ca. 1277.

As for the instrument itself, its back is very similar to that of a Western Islamic standard astrolabe. It contains an outer circle, divided into four quadrants, each one of them graduated from 0° to 90°. As we have seen, it also has a calendar scale and a double shadow square.

The *mater*, on the face of the plate, contains a set of coordinates traced according to the meridian stereographic projection (see figs. 4.27 and 4.28).

542 Chabás, 2003. There are only two exceptions: Libra 15° (instead of 16°) for 1 October and Aquarius 21° (instead of 22°) for 1 February.

543 Ms. Villa Amil 156 fol. 50r–50v; Rico, 1863–67, II, 140.

544 Ms. Villa Amil 156 fol. 172v; Rico, 1863–67, III, 298–299.

545 Chapter 19 of the *Relogio dell agua* is missing in ms. Villa Amil 156. It is preserved in ms. Escorial h-1-1 fol. 293v; Rico, 1863–67, IV, 50–51.

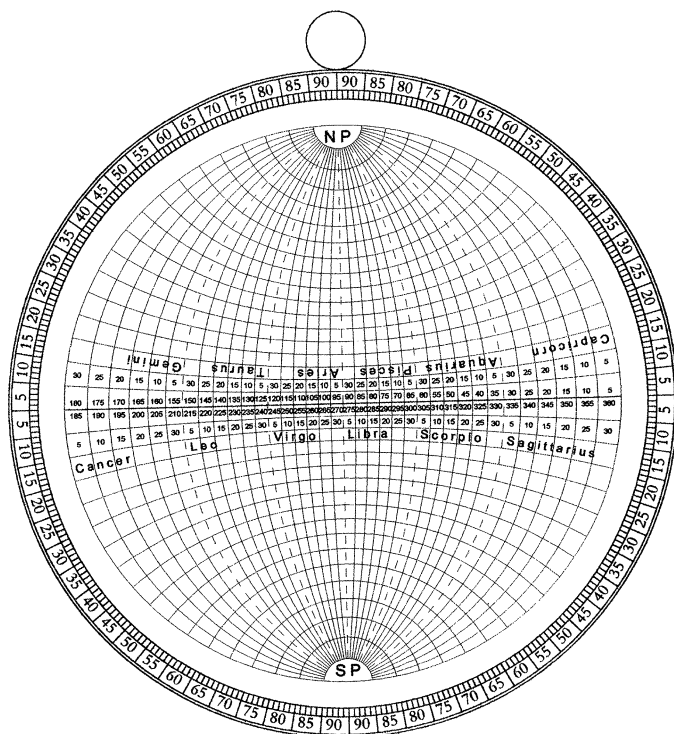


FIGURE 4.28
Mater of the
 universal plate
 reconstructed
 by E. Calvo and
 R. Puig, with the
 collaboration of
 X. Crous

The graduated horizontal diameter⁵⁴⁶ corresponds, in principle, to the ecliptic, and the poles (NP and SP in fig. 4.28) are the poles of the ecliptic. The parallels are called *cercos de la longura* (small circles of latitude), while the meridians going from NP to SP are the *cercos de la ladeza* (great circles of longitude). This set of coordinates can also be used as horizontal ones in which case the central diameter will be the horizon, NP the zenith and SP the nadir. The meridians will be the *cercos de la altura* or *cercos de los zontes* (*dawā'ir al-sumūt*), azimuthal or vertical circles, while the parallels are called *empontizos*, in an attempt to give a literal translation of *al-muqanṭarāt*.

Emilia Calvo and Roser Puig⁵⁴⁷ have carefully analysed the description of the plate in chapter 2 of Ibn Khalaf's treatise on the use of the instrument,⁵⁴⁸ and have discovered some differences with regard to the illustration of the *mater* as it appears in the Villa Amil manuscript. One of them concerns the empty circular crown that appears inside the graduated circular rim, which has the same thickness as the rete and is welded to the mater. The rim is called

⁵⁴⁶ Called *andamio*, a translation of *mamarr*.

⁵⁴⁷ Calvo & Puig, 2006. See also Calvo, 2017.

⁵⁴⁸ Ms. Villa Amil 156, fols. 87r–89r.

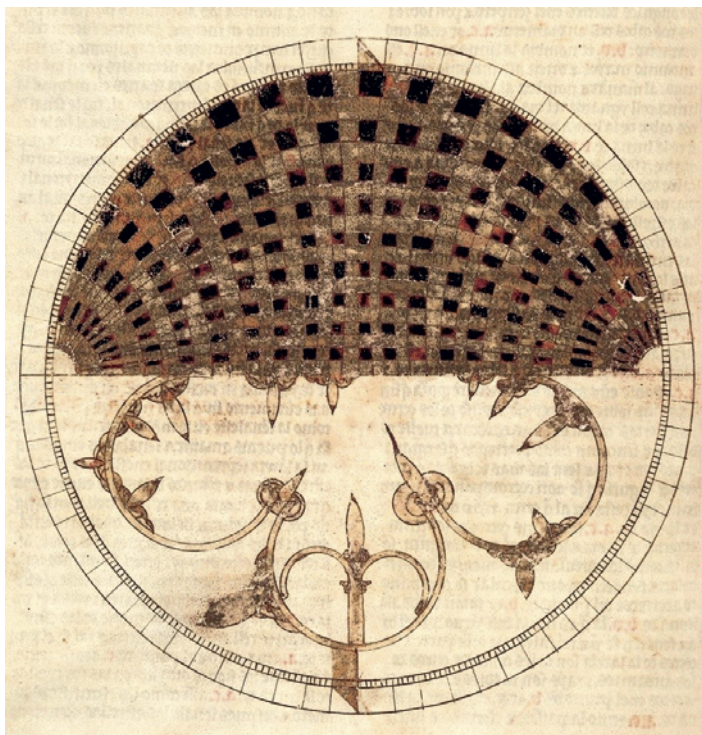


FIGURE 4.29 Rete of the universal plate Biblioteca Histórica de la Universidad Complutense de Madrid. (BH MSS 156 fol. 86v)

cercello in the Alfonsine text, while the circular crown is the *cabecón*. The purpose of this empty space becomes clear when we superimpose the rete on the mater (fig. 4.31): the graduation of the half grid of coordinates of the rete will occupy the empty circular crown. It is obvious that the author of the illustration has omitted this detail, as well as others.⁵⁴⁹

A mobile rete rotates on top of the *mater*. A diameter divides it into two halves: one of them resembles the rete of an astrolabe, as it consists in a set of indices, each one of which corresponds to the projection of a particular star. The other half of the rete is a second set of coordinates projected using the same meridian stereographic projection. This second set corresponds to equatorial coordinates: the vertical diameter is, therefore, the equator, while the horizontal one is the north-south axis. Here the parallels are called *madarat* (*madārāt*) or *circulares*, while the meridians are *mamarat* (*mamarrāt*) translated as *andamios* (figs. 4.29 and 4.30).

⁵⁴⁹ The same omission appears in fol. 200v of ms. Escorial h-1-1.

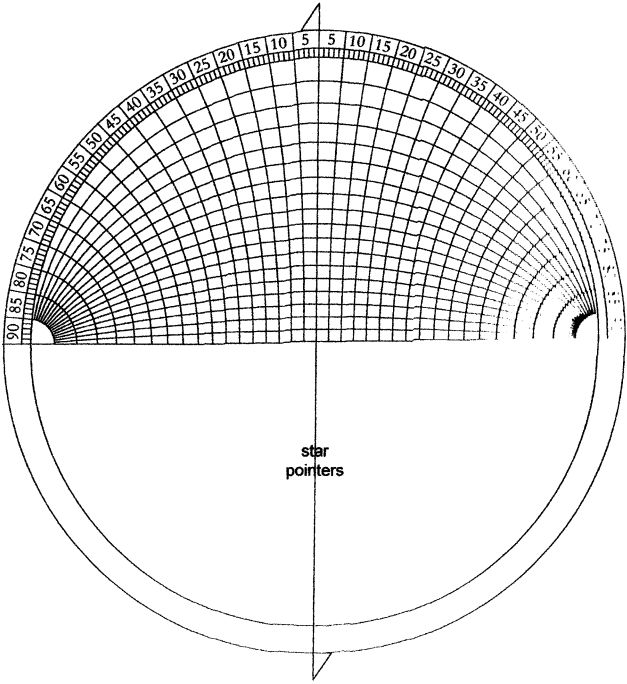


FIGURE 4.30
Rete of the universal
plate reconstructed
by E. Calvo & R. Puig,
with the collaboration
of X. Crous

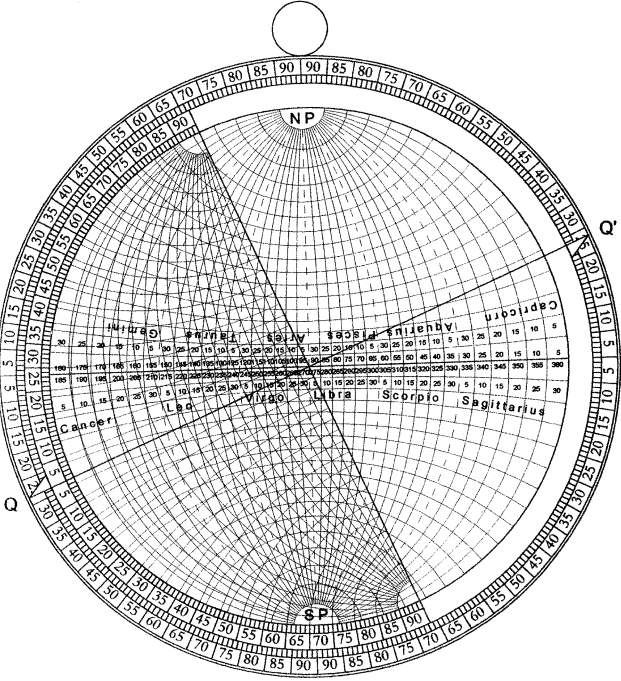


FIGURE 4.31
Rete of the universal
plate superimposed
on the mater (E. Calvo,
R. Puig & X. Crous)

‘Alī ibn Khalaf’s universal plate reached a very limited audience.⁵⁵⁰ We know, however, that the instrument was known in the Islamic East, where it was described by Najm al-Dīn al-Miṣrī as the *rūmī* astrolabe called *al-shajjāriyya*.⁵⁵¹ Besides this, the Cairo Museum of Islamic Art contains what seems to be a mater of the instrument.⁵⁵² Only one complete universal plate seems to be extant: a seventeenth-century Indian universal astrolabe, made probably in Lahore, in which the rete has a half myrtle ecliptic.⁵⁵³ To this one should add the astrolabe made by Ibn al-Sarrāj’s – (a Syrian astronomer who flourished in the early fourteenth century) extant in the Benaki Museum of Athens. This astrolabe has multiple possible configurations, in one of which a mobile rete – similar to Ibn Khalaf’s grid – is superimposed on a *shakkāziyya* plate.⁵⁵⁴ The instrument catalogued with this name (*lāmīna universal* or universal plate) in the Oxford Museum of History of Science (4.3.6) is not related to Ibn Khalaf’s plate: it has a full myrtle ecliptic and a universal plate similar to the one designed by Ibn Bāsoh (4.3.2) in the early fourteenth century.⁵⁵⁵

The plate was also known in Europe, outside the Alfonsine circle. Two Latin texts (preserved in mss. Oxford, Digby 40, and Wolfenbüttel Aug. Qu. 24) contain summarised descriptions of its construction and use. Besides, at the beginning of this century, the Spanish National Museum of Science and Technology acquired a sixteenth-century astrolabe very similar to Ibn Khalaf’s plate.⁵⁵⁶ Most interestingly, this instrument has a half myrtle ecliptic in its rete which does not appear in the Alfonsine version. A similar ecliptic appears in Ibn al-Sarrāj’s universal astrolabe, in a Latin description of the universal plate and in the treatise on astronomical instruments written ca. 1325–1340

550 As for the Maghrib I can only say that King (2005, p. 1010) has found a reference, in a Cairo manuscript, to a universal astrolabe in the tradition of ‘Alī ibn Khalaf dated 584/1188–89.

551 Charette, 2003, pp. 103–108. The name is clearly related to Ibn Khalaf who, as we have already seen (4.3.0) was also called Abū l-Shajjār. In Charette’s plate 5 (page 409), I read *shajjāziyya*, probably as a result of a contamination with the word *shakkāziyya*. See also Charette, 2003, pp. 244–246 and 22–24 of the Arabic edition.

552 Following Roser Puig’s suggestion I examined this instrument many years ago, but I have lost my notes on it.

553 King, 2005, pp. 561–562, 819–829.

554 See King, 1979, p. 254 (plate 5) and Vernet & Samsó, 1992, p. 236. On Ibn al-Sarrāj’s astrolabe see King, 1987 b.

555 The Oxford instrument was made by ‘Alī ibn Ibrāhīm al-Jazzār (or al-Ḥarrār) in Tāzā (Morocco) in 728/1327–28. See Calvo, 1990, pp. 223–225; King, 2005, pp. 559 (fig. 6) and 560.

556 Moreno, 2002. This paper proposes a new interpretation of Ibn Khalaf’s plate which does not seem convincing, because it is based on a radical change of the meaning of some well-established Alfonsine technical terms.

by Najm al-Dīn al-Miṣrī.⁵⁵⁷ Quite reasonably, Charette suggests the possibility of an ecliptic of this kind appearing in one of the versions of the instrument designed by ‘Alī ibn Khalaf or that it might be related to a derivation of Ibn al-Sarrāj’s astrolabe.

4.3.2 *Ibn al-Zarqālluh’s ṣafiḥa (“azafea”)*

4.3.2.1 Sources

Ibn al-Zarqālluh’s *ṣafiḥa* is well documented both by manuscripts and by the extant instruments. The Arabic manuscripts of the treatise on the use of the *ṣafiḥa* show the existence of at least three different recensions which correspond to the two variants of the instrument:

One of them contains 60 or 61 chapters and describes the simplified version of the instrument, the *ṣafiḥa shakkāziyya*.⁵⁵⁸ According to David King, some versions of this treatise are dedicated to al-Mu‘tamid (r. 1069–1091).⁵⁵⁹ This recension was translated into Hebrew and Latin, in 1263, by Prophatius Iudeus/Profeit Tibbon.⁵⁶⁰ The version in 61 chapters contains the same introduction as the treatise in 100 chapters (which deals with the *zarqāliyya*), except for the description of the back of the plate, which is essentially different in the two instruments.⁵⁶¹

The second one is divided into 100 chapters and it describes a more elaborate version of the instrument, usually named *ṣafiḥa zarqāliyya*.⁵⁶² The Alfonsine translation of the treatise on the use of the instrument corresponds to this second recension.⁵⁶³ A treatise on its construction is also extant in Arabic (in ms. BNF 4824): it contains a star table which, according to Charette,⁵⁶⁴ can be dated ca. 480/1087–88.⁵⁶⁵ The construction treatise was also translated into Castilian and incorporated to the Alfonsine *Libro del saber de astrología*.⁵⁶⁶ According to the introduction to the Alfonsine text, a first version of this treatise and of the instrument was dedicated to King al-Ma’mūn of Toledo

557 Charette, 2003, plate 5, p. 409.

558 Puig, 1985; Puig, 1986; Charette, 2003, pp. 96–97, 100–101.

559 King, 1979, p. 253.

560 Millàs, 1933 a.

561 Charette, 2003, pp. 96–97.

562 Puig, 1987; Charette, 2003, pp. 97–100, 101–103.

563 Ms. Villa Amil 156, fols. 113v–135v; Rico, 1863–67, III, 163–237. As stated in the introduction (Ms. Villa Amil 156, fol. 109v; Rico, 1863–67, III, 133) the treatise on the *azafea* was translated twice: by Fernando de Toledo in 1255–56 and by Bernardo el Arábigo and Abraham in 1277.

564 Charette, 2003, p. 98.

565 Kunitzsch, 1980, pp. 196–200.

566 Ms. Villa Amil 156, fols. 109v–113v; Rico, 1863–67, III, 137–146.

(r. 1037–1074) and the *ṣaḥīḥa* received the name of *al-ma'mūniyya*. After Ibn al-Zarqālluh's migration to Seville, he made an improved version of the instrument which he dedicated to al-Mu'tamid, king of that city (r. 1069–1091).⁵⁶⁷ The latter is, probably, the version translated into Castilian.

A third recension is preserved in a single Istanbul copy (Aya Sofya 2671, 1r–75r) and was discovered by David King.⁵⁶⁸ As described by Charette⁵⁶⁹ this version contains 1) a prologue, 2) a theoretical presentation, with proofs, of the principle and construction of the *zarqālīyya*, 3) a more practical description of the construction of the instrument, without proofs, and 4) a treatise on its use, divided into 80 chapters containing practically the same text as the corresponding ones in the 100 chapter recension. In the Istanbul manuscript, this recension is dated to 459/1066–67, and is dedicated to an unknown ruler who, according to Charette, should be identified with al-Ma'mūn of Toledo (1037–1074). It includes a star table whose coordinates, according to the text, are computed for the present time (459/1066–67) by adding a constant of 2;37° to the values in al-Battānī's table. This would imply an increase of 13;47° on Ptolemy's longitudes in the *Almagest*. Kunitzsch's analysis shows, however, that the actual increase on Ptolemy's longitudes is not 13;47° but 12;40° and that the table lists 29 stars, 21 of which are the same and have the same coordinates as in Maslama's table dated in 978 (see 4.2.2.3.1). This table is, consequently, an expanded version of Maslama's star table.⁵⁷⁰

4.3.2.2 Main Characteristics of the Two Types of *ṣaḥīḥa*

On its face (fig. 4.32), the *ṣaḥīḥa shakkāziyya* has a complete set of coordinates which, in principle, are defined as equatorial. Superimposed on it we find the ecliptic which, like the equator, is a straight line and the great circles of longitude corresponding to the beginning of the zodiacal signs. On the face we also find the projection of some fixed stars since the *ṣaḥīḥa* does not have a movable rete. A graduated ruler rotates on a pivot placed in the centre of the instrument. This ruler represents the horizon and can be placed on the graduated outer crown in such a way that the altitude of the pole over the horizon equals the local latitude.

The back of the instrument (fig. 4.33) has an outer crown in which the upper semicircle is divided in two quadrants graduated from 0° to 90°. This graduation, together with the alidade with sights, is used to measure altitudes. The

567 Ms. Villa Amil 156, fol. 109v; Rico, 1863–67, III, 133.

568 King, 1979, p. 253.

569 Charette, 2003, p. 99.

570 Kunitzsch, 1980, pp. 192–196.

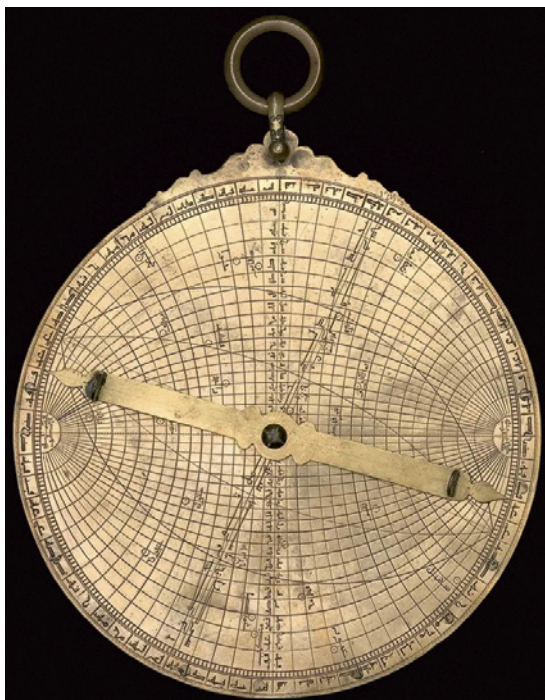


FIGURE 4.32
Face of the *shakkāziyya* (back
of an anonymous and undated
Maghribi astrolabe
INV. 41122 © HISTORY OF
SCIENCE MUSEUM, OXFORD
UNIVERSITY

lower semicircle has a shadow scale (cotangents and tangents), projected from the double shadow square placed under the horizontal diameter of the instrument. A zodiacal scale is also added to the aforementioned inscriptions.

As for the *ṣafiḥa zarqāliyya*, its face has a double set of coordinates which, in principle, should be considered equatorial and ecliptic although the treatise on the use of the instrument implies other interpretations. It also contains the projection of some stars and a ruler-horizon (fig. 4.34). The back of the instrument is far more interesting: the limb is graduated as in the *shakkāziyya*, although here the shadow scale of the lower semicircle is not superfluous to requirements, because the shadow square has disappeared. We also find a zodiacal scale, as in Western astrolabes, and an inner circle, divided into four quadrants graduated from 0° to 90° . The free space in the middle is also divided into four parts by two perpendicular diameters. Three of them (the two upper ones and the left quadrant of the lower one) contain a new set of coordinates which correspond not to a meridian stereographic projection but to a meridian orthographic one, in which the centre of the projection is located in infinity, the meridians are arcs of ellipses and the parallels straight lines.⁵⁷¹ The

⁵⁷¹ Puig, 1987 a; Puig, 1996. In this second paper, Puig suggests that Ibn al-Zarqālluh's orthographic projection could have been influenced by al-Bīrūnī's cylindrical projection.

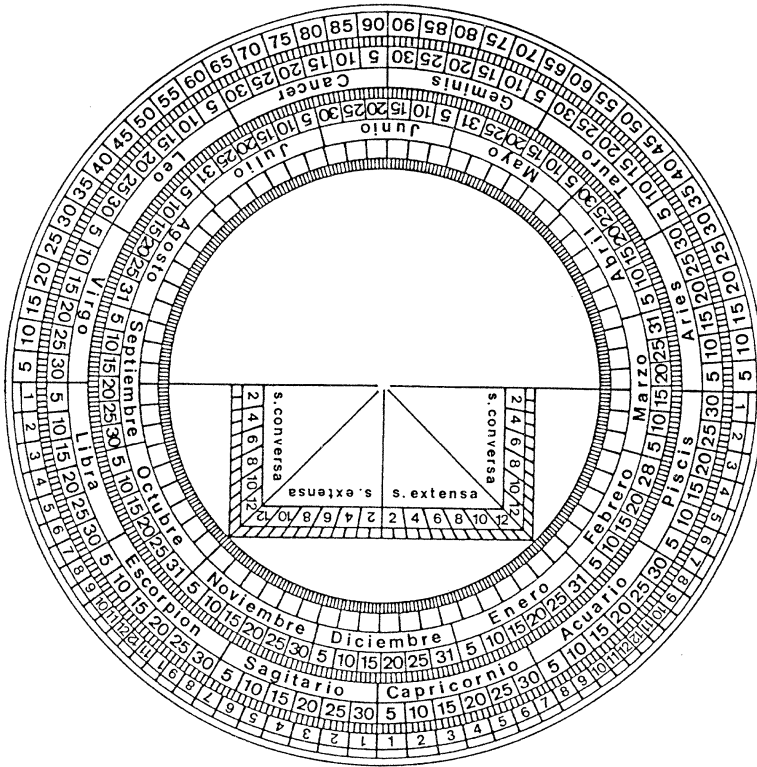


FIGURE 4.33 Back of the *shakkāziyya* in R. Puig's reconstruction

Note: Very few real instruments of the *shakkāziyya* type – in which the back is in agreement with the description found in the written treatise – seem to be extant. See an example (thirteenth-century) in Pingree, 2009, pp. 10–11. The face of the *shakkāziyya*, and sometimes that of the *zarqāliyya*, is often found as an extra plate (or engraved on the back) of a standard astrolabe. Three examples of *shakkāziyyas* on the back are mentioned by King, 2005, pp. 1011–1012.

fourth quadrant (lower right) is a sine quadrant composed of 60 perpendiculars traced from each one of the 60 divisions of the lower radius of the vertical diameter. Leaving aside the so-called *quadrans vetustissimus* described in the old collection of Latin texts on the astrolabe and other instruments (see 2.4.2.1.2.1), this seems to be the first instance of a reference to the sine quadrant in the Andalusī tradition.⁵⁷² On the lower radius of the circle (which, as we have seen, is divided into 60 parts) we find a small circle called “circle of the moon” (figs. 4.35 and 4.36). An alidade with sights turns around the centre

⁵⁷² Puig, 1991.

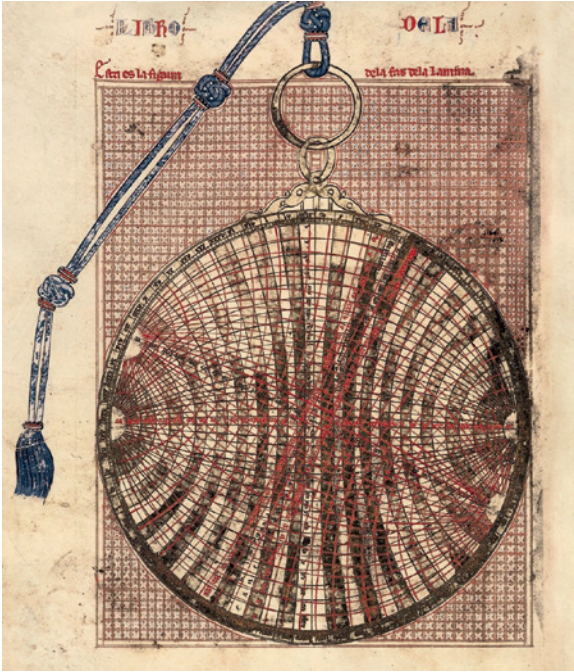


FIGURE 4.34
Face of the *şafiha zarqāliyya*
BIBLIOTECA HISTÓRICA
DE LA UNIVERSIDAD
COMPLUTENSE DE MADRID
(BH MSS 156 FOL. 112V)



FIGURE 4.35
Back of the *şafiha zarqāliyya*
BIBLIOTECA HISTÓRICA
DE LA UNIVERSIDAD
COMPLUTENSE DE
MADRID (BH MSS 156
FOL. FOL. 113R)

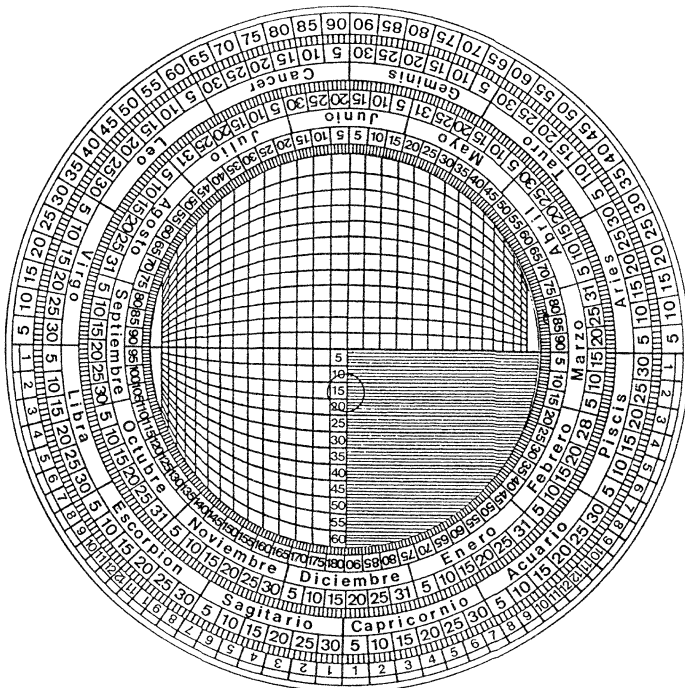


FIGURE 4.36
Back of
the *ṣafiha*
zarqāliyya in
Roser Puig's
reconstruction

of the plate, on which a second ruler (called “transversal”, *al-mu'tariḍa*) slides (fig. 4.37).⁵⁷³

It seems obvious that the *zarqāliyya* plate offers important improvements over the *shakkāziyya* and we should, therefore, make some remarks on the use of the two instruments and on the differences appearing on the back of the *zarqāliyya*:

Both instruments have one important drawback as they do not have a movable rete (like the astrolabe or the universal plate) whose rotation imitates the apparent motion of the celestial sphere around the earth. This means that they require a greater effort of imagination than that needed when a spherical instrument or an astrolabe is used. They are instruments which can be considered more “abstract” and they represent a step forward in the evolution of astronomical instruments which begins with a simple representation, drawn to scale, of the universe, and leads to the slide-rule. One should add to this that the *shakkāziyya* has only one complete set of coordinates combined with

573 The only known example of an alidade-transversal piece of this kind is preserved in an eighteenth-century Maghribī astrolabe in the London Science Museum (Inv. no. 1986–190). See Vernet-Samsó, 1992, p. 234.

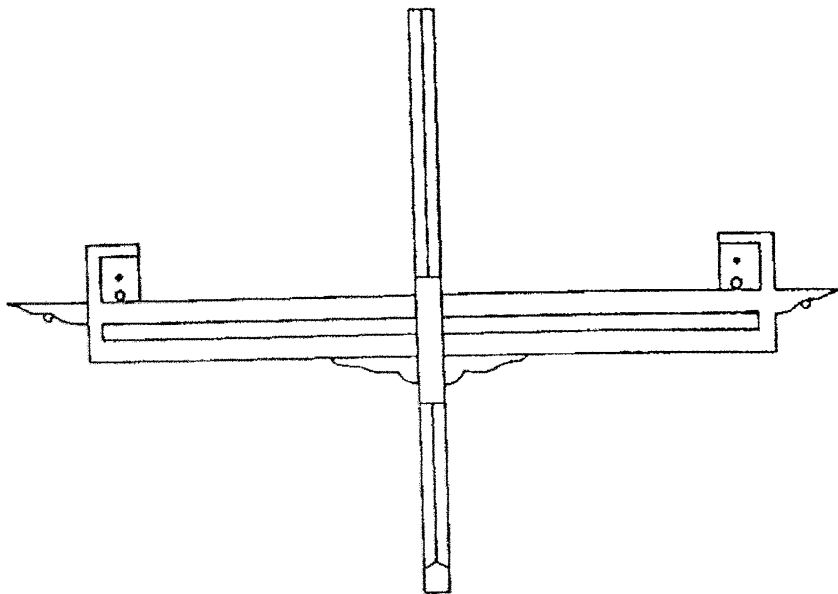


FIGURE 4.37 Alidade and transversal ruler, borrowed from Roser Puig

a ruler-horizon, something which makes the use of the instrument quite difficult. Depending on the circumstances, this unique set may correspond to equatorial, ecliptic or horizontal coordinates and in only a limited number of cases can we obtain results by reading them directly on the instrument. In most cases the procedure will be more burdensome and it will begin by determining a position using the sole existing set of coordinates. This position will then be rotated, using the ruler-horizon, through an angle equivalent to the local latitude (transformation of equatorial coordinates into horizontal ones or the other way round) or to the obliquity of the ecliptic (from equatorial to ecliptic coordinates and the reverse). This technique is used all the time both in the *shakkāziyya* and in the *zarqāliyya*, for, although the latter has a double set of coordinates (equatorial and ecliptical), it does not have horizontal coordinates.

4.3.2.3 The Orthographic Projection on the Back of the *ṣaḥīḥa zarqāliyya*
The orthographic projection is described in the treatise on construction:⁵⁷⁴ in fig. 4.38 circle ABCD (called *al-shabiha*, the “similar”, from its analogy to the

574 I am using Roser Puig’s working edition based on ms. Paris BNF 4824, fols. 5v–6v. The projection is also described in the treatise on the use of the instrument: ms. Escorial 962, fol. 10r–10v. See the Spanish translation by Puig, 1987, p. 54.

circle ABCD, and divided, as AC, into 24 equal parts. Then the points of each chord with the same graduation, on both sides of AC, are joined, resulting in approximations to an ellipse, while the parallel chords will correspond to the parallels of the projection (*al-madārāt*, of which the diameter AC is called “the greatest parallel”, *al-madār al-a‘zam*).

The construction text contains the sentence “I have explained elsewhere the procedure to draw an ellipse” although I do not know which of his own works he has in mind. He does not explain the procedure used to join the points of each ellipse, but he says, at the end of the passage, “We have already seen that some people join three consecutive points with an arc of a circle, concealing how the line was traced except to those who know the procedure.” This is what he does in his treatise on the equatorium (see 4.5), in which the curve of Mercury’s deferent, the result of the combination of two motions in Ptolemy’s model, is an ellipse. As mentioned here, he determines groups of three points of the deferent, and traces arcs of a circle passing through these three points. The process continues with several sets of three points until the whole deferent is drawn. The resulting curve is similar to an ellipse.⁵⁷⁶

The alidade and the orthographic projection are used together for several applications of the back of the instrument. I will give an example here: the determination of the hour of the day as a function of the solar altitude.⁵⁷⁷

In Fig. 4.39a the alidade (straight thick line) is on the central meridian, and the perpendicular diameter HH' represents the horizon. The edge of the transversal ruler (aa') is placed on the inner graduated circle so that the arc Ha equals the solar instantaneous altitude (h_s). In this way, aa' represents the almucantar corresponding to the solar altitude. Then (fig. 4.39b) the alidade is rotated a number of degrees equivalent to the colatitude ($90^\circ - \varphi$), while the transversal ruler remains in its previous position. Thus the system of coordinates becomes equatorial and the vertical diameter in fig. 4.39b is the equator (QQ'). The user then determines the parallel (pp') corresponding to the solar declination (δ) in that day. The intersection of pp' with the transversal ruler which, as we have seen, represents the almucantar of the sun’s instantaneous altitude (h_s), will determine the position of the Sun at this moment (S). Then, the user will see which equatorial meridian coincides with S, as well as the point of intersection of this meridian with the equator (S'). Afterwards, he will place the edge of the alidade on the equator and move the transversal ruler until it coincides with S' (fig. 4.39c). This edge will coincide with a particular

⁵⁷⁶ Samsó & Mielgo, 1994 a.

⁵⁷⁷ I am using ms. Escorial 962 fols. 24r–25r. See the Spanish translation in Puig, 1987, pp. 64–65; commentary in Puig, 1987 a, pp. 130–131.

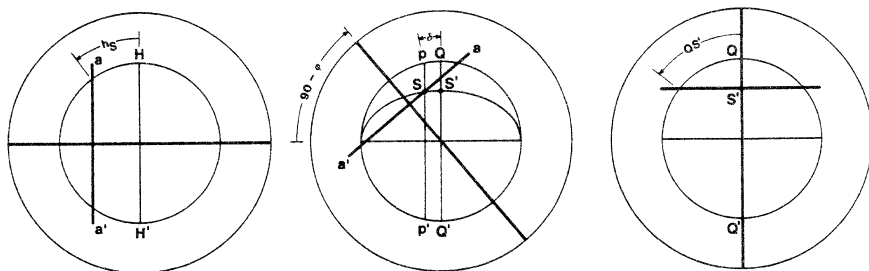


FIGURE 4.39 a), b) and c) from R. Puig

number of degrees in the graduation of the “similar” circle, and S' will be the endpoint of the arc of right ascension of the Sun. The “similar” circle will be the local meridian and the transversal ruler will mark, on the “similar” circle, the number of degrees of equatorial rotation before or after midday.

4.3.2.4 The “Circle of the Moon” and the Computation of the Lunar Distance

The alidade + transversal ruler is used by Ibn al-Zarqālluh in combination with the “circle of the moon” to obtain a graphical solution of the problem of determining the distance between the Moon and the centre of the Earth, which will be used for the computation of the lunar parallax in the altitude circle, as well as its components in longitude and latitude.⁵⁷⁸ How to draw the circle of the Moon is described in a brief passage in our author’s construction treatise, while its use is explained in chapter 68 of the treatise on the use of the instrument.⁵⁷⁹ As we have already seen, the lower radius of the “similar” circle is divided into 60 parts by the lines of the sine quadrant, and the centre of the circle of the Moon is located in the thirteenth division below the centre of the instrument, while its radius equals six of these divisions. The circle itself is also divided into 24 equal parts. This implies the use of rounded Ptolemaic parameters: 13^p is the rounded value of the Ptolemaic lunar eccentricity of $12;30^p$ (for $R = 60^p$) while $r = 6^p$ corresponds to the $6;20^p$ of the radius of the lunar epicycle. This is in clear contrast to the formulae used for the computation of the lunar parallax, corresponding to an Indian tradition which does not always derive from al-Khwārizmī’s *Sindhind*.

The procedure described by Ibn al-Zarqālluh for the determination of the lunar distance is too complicated to describe here, and I am not able to add anything to Roser Puig’s explanation. I should, however, explain that it is

⁵⁷⁸ Puig, 1989.

⁵⁷⁹ Both texts are edited (from mss. Paris BN 4824 and Escorial 962) in Puig, 1989, pp. 307–309.

divided into three steps: in the first one, the purpose is to determine the centre of the lunar epicycle and, for that purpose, the “similar” circle represents the lunar deferent, the centre of the plate will be the centre of the deferent, and the centre of the “circle of the moon” the centre of the Earth. In the second step, one needs to establish the position of the mean apogee of the lunar epicycle; this is difficult, due to the absence of a movable lunar epicycle that can be displaced on the deferent. Ibn al-Zarqālluh gives a clever solution by exchanging the roles of the sector around the centre of the instrument and the “circle of the moon”, on the one side, and the “similar” circle on the other. In this second stage, the point which, in the previous step, represented the centre of the lunar epicycle, now becomes the centre of the Earth, while the “circle of the moon” becomes the lunar epicycle. In the third step, the mean lunar anomaly, measured from the mean apogee, makes it possible to determine the lunar position on the epicycle and, using the graphical scale of the transversal ruler, we will be able to obtain the geocentric distance of the moon.

4.3.3 *On the Relative Chronology of the Three Universal Astrolabes*

It is extremely difficult to establish a relative chronology of ‘Alī ibn Khalaf’s universal plate and of the two variants of Ibn al-Zarqālluh’s *ṣafiha*. Millás believed that ‘Alī ibn Khalaf was the discoverer of the meridian stereographical projection, or of its application to a universal astrolabic instrument.⁵⁸⁰ This hypothesis was based on the fact that Ibn al-Zarqālluh was considered an artisan, dedicated to the construction of astronomical instruments, while Ibn Khalaf presented himself as a scholar who had studied Ptolemy’s *Planisphaerium* and who planned to write a book on the different ways to project the sphere on a plane. However, this argument may no longer be valid, since Charette has established that ms. Aya Sofya 2671⁵⁸¹ contains a theoretical part on stereographic projection and its application to the construction of the *ṣafiha*.

Besides, the Alfonsine text states that Ibn al-Zarqālluh designed two different versions of his instrument dedicated, respectively, to al-Ma’mūn of Toledo (r. 1043–1074) (*ṣafiha ma’mūniyya*) and, after his migration to the kingdom of Seville,⁵⁸² to al-Mu’tamid ibn ‘Abbād (r. 1069–1091) (*ṣafiha ‘abbādiyya*).⁵⁸³

580 Millás, 1943–50, pp. 425 ff.

581 Charette, 2003, p. 99.

582 According to Ibn al-Abbār (*Takmila*, II, 120, no. 358), Ibn al-Zarqālluh left Toledo to settle in Córdoba, which belonged to the *tā’ifa* of Seville, at the beginning of the reign of al-Qādir (r. 1075–85), although a note, added by the copyist himself in a fifteenth-century manuscript of the *Toledan Tables*, states that he moved south due to Alfonso VI’s conquest of Toledo in 1085 (Millás, 1943–50, p. 14).

583 Ms. Villa Amil 156, fol. 109v; Rico, 1863–67, III, 135.

Some more information can be obtained from Ibn al-Zarqālluh's prologue to his treatise on the use of the instrument, where the author says that, in the past, he had built another *ṣafiḥa* which was inferior to the one he is going to describe.⁵⁸⁴ These data may prompt us to identify *shakkāziyya* with *ma'mūniyya*, and *zarqāliyya* with *'abbādiyya*; we might also say that these identifications are confirmed by the probable Toledan origin of the *shakkāziyya*, a name that may derive from the existence in Toledo of the quarter (*ḥawmat*) of the *shakkāzīn* (belt-makers, as well as men working with leather products).⁵⁸⁵

New information related to this problem was furnished by David King in 1979,⁵⁸⁶ when he drew attention to ms. Leiden Or 468, which contains a treatise on *mīqāt* entitled *Kanz al-yawāqīt*, written by an anonymous Egyptian author of the early fourteenth century who belonged to a well-known family of Cairo *muwaqqits*.⁵⁸⁷ It is clear that the author of the *Kanz* was interested in Andalusī astronomy, since the book contained a summary of Šā'id's *Ṭabaqāt al-umam* to which he added historical materials from other sources. In relation to our problem, he says that Ibn al-Zarqālluh invented the *zarqāliyya* and wrote on it a treatise of one hundred chapters in 440/1048–49, while 'Alī ibn Khalaf designed an instrument (*al-aṣṭurlāb al-ma'mūnī*) for al-Ma'mūn (whom he calls Dhū l-Majdayn Abū l-Ḥasan Yahyā ibn Dhī l-Nūn) in 464/1071–72. Whether or not we accept these new data will depend on the value we give to the sources of the *Kanz*, and there is a detail which leads me to consider them reliable: al-Ma'mūn's full name is absolutely correct, and the author adds to it a relatively rare *laqab* (Dhū l-Majdayn).⁵⁸⁸ This makes me think that the author of the *Kanz* is using a source which is well informed and near enough, in time, to the historical events.

Another important detail can be found in the prologue to Ibn al-Zarqālluh's treatise on the construction of the *zarqāliyya*: the book is dedicated to Abū l-Qāsim Muḥammad b. 'Abbād to whom he attributes the *laqab* of al-Mu'ayyad bi-Naṣr Allāh.⁵⁸⁹ This is al-Mu'tamid's honorific title which he used from his birth in 1040 until he became crown prince in 1058, when he received the title of al-Zāfir bi-Ḥawl Allāh. So it seems clear that the *zarqāliyya* existed in 1048 and that an early version of the instrument was dedicated to the future

584 Ms. Paris BNF 4824, fol. 10r; ms. Villa Amil 156, fol. 114r; Rico, 1863–67, III, 150.

585 Corriente, 1997, p. 287.

586 King, 1979, pp. 250–255.

587 Charette, 2003, p. 13, suggests that the author is Naṣīr al-Dīn Muḥammad ibn Sam'ūn (d. 1336–37), a *muwaqqit* in the mosque of 'Amr in Fustāt.

588 This *laqab* appears in 'Abbās, 1978, vol. II, p. 578.

589 Ms. Paris BNF 4824, fol. iv.

al-Mu'tamid ibn 'Abbād who was only eight or nine years old at the time.⁵⁹⁰ Obviously Ibn al-Zarqālluh was trying to obtain the patronage of al-Mu'tamid's father, al-Mu'taḍid (1042–1069). 'Alī ibn Khalaf, for his part, designed his instrument much later, in 1071–72, and dedicated it to al-Ma'mūn a short time before the latter's death in 1074. All this interpretation agrees with the colophon of the treatise on the *zarqāliyya* contained in ms. Escorial 962, also quoted by King. The text seems corrupt; it is difficult to understand, and two different hands have participated in its making (that of the copyist of the ms. and a later hand).⁵⁹¹ It says that Ibn al-Zarqālluh “finally built [the instrument], after a discussion with Abū l-Shajjār [= Ibn Khalaf] concerning the first [version of the instrument] and, finally, [Ibn Khalaf/Ibn al-Zarqālluh?] used it”. The second hand appends that [Ibn Khalaf] added a rete (*shabaka*) to Ibn al-Zarqālluh's instrument.

It seems obvious that, after the early version of the *zarqāliyya* (1048–49), Ibn al-Zarqālluh rewrote the treatise and redesigned the instrument. Thus, in 459/1066–67 he dedicated his treatise, structured in 80 chapters and corresponding to the *zarqāliyya* type, to an unknown ruler. The final version, which was the object of the Alfonsine translation, contains a star table dated by Charette ca. 480/1066–67 (omitted in the Castilian text). This proposed date is not far away from the date of Ibn al-Zarqālluh's determination of the position of the solar apogee (ca. 1074–75), in which he obtained 85;49°, ⁵⁹² which agrees with the 86° given in the construction treatise.⁵⁹³ Nothing is known, however, of the chronology of the *shakkāziyya*; I can only say that in some manuscripts it appears dedicated to al-Mu'tamid, although this does not imply necessarily that the instrument was designed after Ibn al-Zarqālluh's migration to Córdoba.

4.3.4 *On the Transmission of Ibn al-Zarqālluh's ṣafiḥa to the Mashriq and the Maghrib*

We have already seen (4.3.1) some evidence of the limited diffusion of 'Alī ibn Khalaf's universal plate both in the Islamic world and in Europe.⁵⁹⁴ Ibn al-Zarqālluh's two instruments, on the other hand, were known in both Eastern Islam and in Europe at the beginning of the thirteenth century.⁵⁹⁵ Ibn al-Qifṭī

590 Aguiar, 2013.

591 Puig, 1987, pp. 103 and 111 (fn. 320).

592 Samsó & Millás, 1994, pp. 8–9.

593 Puig, 1987, p. 20; Ms. Villa Amil 156 fol. 110v; Samsó & Millás, 1994, pp. 8–9.

594 Poulle, 1970.

595 On *zarqāliyya* and *shakkāziyya* plates made in the Mashriq, see the list in King, 2005, p. 1009.

(1172–1248) says “when it [the *ṣafiha*] was known by the experts in this science in the lands of al-Mashriq, they became confused with it and were only able to understand it with God’s help [*ba’d al-tawfiq*]”.⁵⁹⁶ In the thirteenth century Abū l-Ḥasan ‘Alī al-Marrākushī carefully describes the *zarqālīyya* as well as the procedure for its construction.⁵⁹⁷ To this he adds a brief description of the *shakkāziyya*, which he considers to be a simplified version of the other instrument.⁵⁹⁸ Later, in the same work, he states that the *ṣafiha zarqālīyya* “is the most excellent and brilliant of all [astronomical] instruments, because it can be used for all horizons and has multiple applications, many of which cannot be obtained with other instruments” although he acknowledges that most of them are difficult to understand. In another work of his, apparently non-extant, he has described these applications exhaustively, giving the corresponding proofs. In the present book he is going to summarise the author’s [Ibn al-Zarqālluh] words and add other procedures which were not mentioned by him.⁵⁹⁹ Then, a long treatise follows on the uses of the instrument, divided into 130 chapters.⁶⁰⁰ Abū l-Ḥasan’s treatise on the use of the *zarqālīyya* has also been preserved as an independent text in manuscript 800 of Leipzig University.⁶⁰¹

Najm al-Dīn’s book on astronomical instruments contains careful descriptions of the construction of the *shakkāziyya* and *zarqālīyya* instruments.⁶⁰² In both cases there is no reference to the back of these instruments, although the author does mention the “transversal” rule (*al-mu’tariḍa*). Another treatise on the use of the *zarqālīyya*, divided into 60 chapters and extant in a Damascus manuscript, is due to an unidentified Abū l-Walid Ismā’il b. Aḥmad al-Qurṭubī.⁶⁰³ This author is considered by Lamrabet⁶⁰⁴ to be one of the Andalusī scholars who lived in the Maghrib.⁶⁰⁵ Other treatises related to the *ṣafiha* and preserved in the Damascus National Library, described by Puig, show that the diffusion of these instruments continued in the Mashriq until the nineteenth century. A scholar named Yūsuf Ḥallāj, whose son had travelled through Egypt and Syria and brought “strange astrolabes” back with him, introduced the *zarqāla*

596 Ibn al-Qifṭī, 1326 H, p. 42.

597 Puig, 2005.

598 Marrākushī, 1984, II, pp. 87–99.

599 Marrākushī, 1984, II, p. 293.

600 Marrākushī, 1984, II, pp. 293–336.

601 Millás, 1943–50, pp. 447–448; King, 1979, p. 253, fn. 21; Puig, 2004, pp. 82–83.

602 Charette, 2003, pp. 261–262, 269–271 and 41, 50–51 of the Arabic edition.

603 Puig, 2005 a, pp. 8–9.

604 Lamrabet, 2013, no. A386, p. 117.

605 Another treatise on the *zarqālīyya* by the same author, divided into 20 chapters, has been preserved in the Tunis National Library: Samsó, 1966, pp. 390–391.

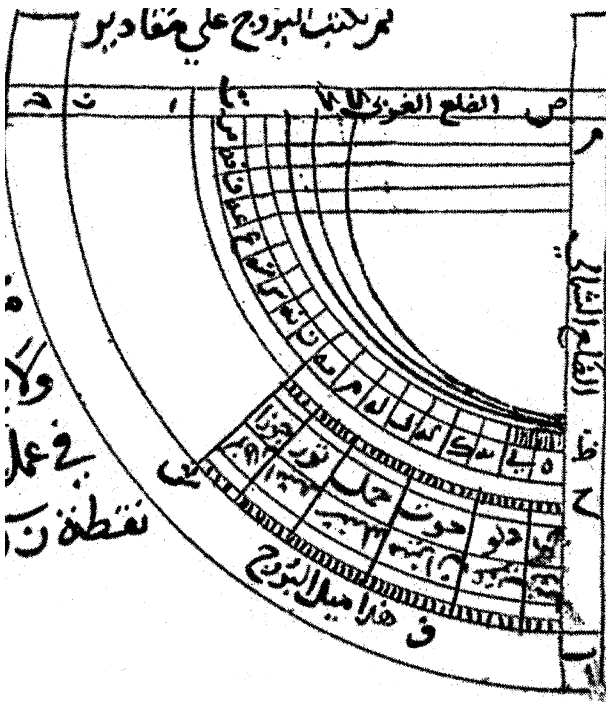


FIGURE 4.40
Ibn al-Zarqālluh's
quadrant in
al-Marrākushī's *Jāmi'*
al-mabādī (Ms. Istanbul
Ahmet III 3343)

in Samarqand, where it aroused the interest of al-Kāshī and Qāḍizāda.⁶⁰⁶ The *ṣafīḥa* finally reached India where an instrument maker from Lahore, $\frac{3}{4}$ Ḍiyā' al-Dīn Muḥammad, made a *ṣafīḥa zarqālīyya* in 1680.⁶⁰⁷

Quadrants derived from Ibn al-Zarqālluh's and Ibn Khalaf's instruments appear in the Mashriq between the thirteenth and the sixteenth centuries. Abū l-Ḥasan 'Alī al-Marrākushī's *Mabādī' wa ghāyāt* also contains a short text on the construction of a quadrant called *al-rub' al-zarqālī*, derived from the orthographic projection on the back of the *ṣafīḥa zarqālīyya* and therefore containing arcs of quarter ellipses for the meridians and straight lines for the parallels. It also has a cursor which makes it possible to obtain the solar declination throughout the year (fig. 4.40).⁶⁰⁸

The process continued in the fourteenth and fifteenth centuries. We have at least two examples of these instruments, which now receive the name of *shakkāzī* quadrants. One of them is due to Jamāl al-Dīn al-Māridīnī (d. 1406), the leading Cairo *muwaqqit* of his time. His instrument is composed by two quadrants, each one with a set of meridian stereographical coordinates.

606 Puig, 2004, pp. 84–85.

607 Sarma, 1996.

608 Marrākushī, 1984, I, pp. 375–377; Sédillot, 1844, pp. 104–106; King, 2005, p. 220.

One quadrant has a perforated grid and is superimposed on the other,⁶⁰⁹ and so this reminds us of 'Alī ibn Khalaf's universal instrument. The second instrument of this kind was invented, according to Ḥājji Khalīfa, by 'Alā' al-Dīn Ṭibughā or Ṭaybughā (d. 1394) although this attribution seems doubtful since Ibn al-Sarrāj (d. in or after 1329), Ibn al-Shāṭir (1306–1375–6) and Najm al-Dīn al-Miṣrī (fl. 1325–40) all described the instrument.⁶¹⁰ One of the extant texts seems to have been written by Ṭaybughā's son, 'Alā' al-Dīn Abū l-Ḥasan 'Alī, who was *muwaqqit* in Aleppo ca. 1400.⁶¹¹ The instrument, in this case, is a single quadrant with a meridian stereographic projection of only one set of coordinates. The back of the instrument seems to be based on a polar stereographical projection, like the astrolabe, and has the projection of a full ecliptic, a full equator, a zodiacal scale, some stars and two alidades, one of them for measuring altitudes, while the purpose of the other is to measure equatorial coordinates.⁶¹²

In the Maghrib, the diffusion of these instruments seems to have been more limited. There is no evidence of *ṣafīḥas* of either kind being made outside al-Andalus,⁶¹³ where we have the well-known *zarqālīyya* made in Murcia by Muḥammad b. Muḥammad b. Hudhayl, described by Millás Vallicrosa.⁶¹⁴ To this I can add two *ṣafīḥas* made by the well-known astrolabe maker Muḥammad ibn Fattūḥ al-Khamā'irī of Seville in 613/1216–7 and 615/1218–9, as well as three other instruments of the same kind, unsigned and undated, which also belong to al-Khamā'irī's tradition.⁶¹⁵ The latter could, of course, have been made in the Maghrib but there is no evidence supporting this hypothesis.

The well-known Maghribī astronomer Ibn al-Bannā' (1256–1321) was deeply interested in these instruments, as he wrote three epistles on the use of the *shakkāzīyya*,⁶¹⁶ the *zarqālīyya*⁶¹⁷ and, interestingly, the *shabīḥa*,⁶¹⁸ concerned with the orthographic projection on the back of the *zarqālīyya*.

609 King, 1974.

610 Charette, 2003, pp. 111–112, 219. On Najm al-Dīn's description see also pp. 271–272 and 51–53 of the Arabic edition.

611 Charette, 2003, p. 19.

612 Samsó & Catalá, 1971–75; Samsó, 1971.

613 I mean completely independent *ṣafīḥas*. As a matter of fact, the Science Museum in London has an eighteenth-century Maghribī astrolabe containing a *shakkāzīyya* and a *zarqālīyya* plate as well as the only extant specimen of the transversal ruler (*al-mu'tariḍa*). See Vernet & Samsó, 1992, p. 234.

614 Millás, 1944; Hernández, 2018, pp. 199–201.

615 King, 2005, pp. 1010–1012; Hernández, 2018, pp. 141–144, 151–153.

616 Calvo, 1989.

617 Puig, 1987 b.

618 Puig, 2004 a.

Ibn al-Bannā's treatment of the *shakkāziyya* contains a selection of the chapters of Ibn al-Zarqālluh's treatise and the addition of others, which do not appear in the latter source, dealing with the measurement of heights, distances and depths. Interestingly, the use of the instrument implies the existence of a larger number of great circles of longitude than those described in the standard *shakkāziyya*.

Ibn al-Bannā's *risāla* on the *zarqāliyya* is more interesting and more original, because it is not a mere recension of Ibn al-Zarqālluh's work or a treatise on the use of the *ṣafiha*, but a reconsideration of the multiple possibilities offered by the markings on the instrument as representations of the celestial sphere.

Ibn al-Bannā' wrote his short text on the *shabiha* in response to a request from a friend of his who was interested in the use of this projection which he had seen on the back of some astrolabes, without the complement of the alidade + transversal ruler (*al-mu'tariḍa*). This implies that the *shabiha* had somehow become independent of the *ṣafiha zarqāliyya* and was used as a universal projection supplementing the plates of a standard astrolabe. Besides, the title of the *risāla* (*Risāla fī l-'amal bi l-shabiha allatī tursam fī ba'ḍ al-aṣṭurlābāt bi-ghayr mu'tariḍa*) clearly indicates that no transversal ruler is involved, and the chapters on its use clarify that it has been replaced by a standard alidade. The description of the instrument shows that the orthographic projection occupies only the two upper quadrants and there is no mention of the sine quadrant or of the circle of the Moon, in spite of the fact that the parallel lines of the orthographic projection are also used to solve the trigonometric problems posed by Ibn al-Bannā'.

4.3.5 *Ibn Bāṣoh's Plate for All Latitudes*

Ḥasan (or Ḥusayn) b. Muḥammad b. Bāṣoh (d. 1316), as well as his son Aḥmad b. Ḥasan, were both *muwaqqits* of the *jāmi'* mosque of Granada (see 2.0), as well as instrument makers. Ḥasan b. Muḥammad, as he was named by Ibn al-Khaṭīb,⁶¹⁹ is probably al-Ḥusayn b. Aḥmad b. Bāṣoh who designed a new universal astrolabe plate (*al-ṣafiha al-jāmi'a li-jamī' al-'urūd*) in which two different traditions combine: on the one side, the Eastern "plate of horizons" (*al-ṣafiha al-āfāqiyya*), probably invented by Ḥabash al-Ḥāsib (d. ca. 864) and, on the other, that of the instruments designed by Ibn al-Zarqālluh and 'Alī ibn Khalaf. Like the standard astrolabe, Ibn Bāṣoh's plate (fig. 4.41) uses a polar stereographical projection with the equator as the plane of projection, and has multiple horizons, like the plate of horizons. The plate in question contains the following lines:

⁶¹⁹ 'Inān, 1973–77, I, 468 and 204. See Renaud, 1937.

1. Two perpendicular lines which cross each other in the centre of the plate: the meridian and the East-West line.
2. Three concentric circles: the tropic of Capricorn, the equator and the tropic of Cancer.
3. A rete of concentric semicircles placed between the equator and the tropic of Capricorn: they are parallels of southern declination comprised between the two great circles.
4. A second rete of concentric circles between the projection of the equator and that of the pole (centre of the plate). Ibn Bāṣoh calls them *madārāt* (parallels) or almucantars, because they correspond to altitude circles for a latitude of 90° .
5. A rete of horizons which are arcs of great circles that cross the parallels and pass through the two points of intersection of the equator and the East-West line.
6. A rete of arcs (*qisī*) which rotate around the E and W points and cut the horizons and the parallels.

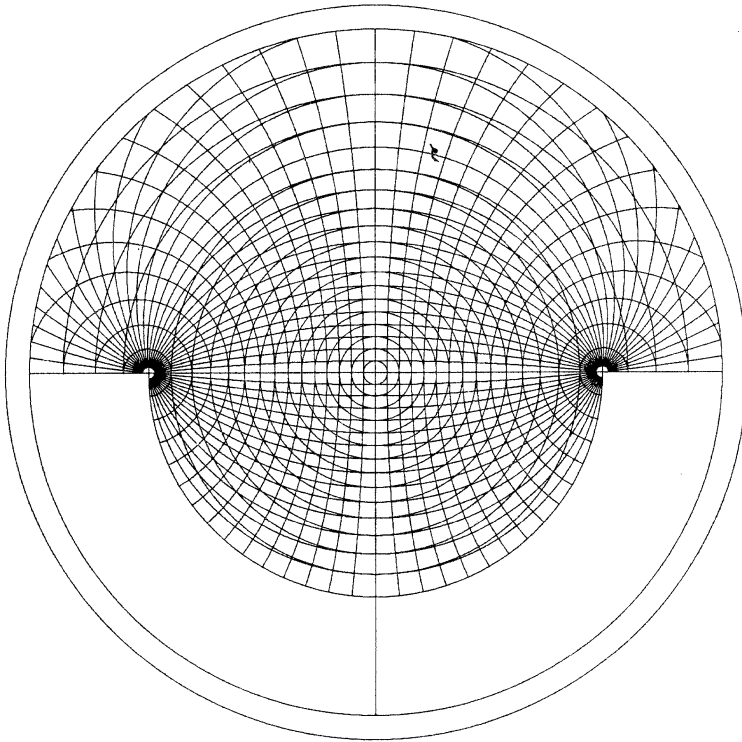


FIGURE 4.41 Ibn Bāṣoh's universal plate, borrowed from Emilia Calvo and Honorino Mielgo

Ibn Bāṣoh's long treatise (160 chapters) on the use of this plate has been edited and translated by Emilia Calvo.⁶²⁰ This treatise does not contain indications on the procedure for drawing the lines of the instrument, which are to be found in a late Maghribī epistle, authored by Abū l-Rabīʿ Sulaymān b. Aḥmad al-Fishtālī (d. Fez 1794). The title of this work is *Nubdha li-mā yataʿallaq bi l-ṣaḥīḥa al-jāmiʿa* (Note on the universal plate) and it contains an abridgement, in four chapters, of Ibn Bāṣoh's book as well as a first chapter on the construction of the instrument.⁶²¹ The analysis of this short text shows the existence of clear similarities between Ibn Bāṣoh's instrument and those of Ibn al-Zarqālluh and ʿAlī ibn Khalaf: if, in fig. 4.41, we only consider the lines between the equator and the pole (centre of the plate) and delete the rete of *madārāt*, we obtain an image that is identical to the one in Ibn Khalaf's instrument. Besides, Calvo's analysis of Ibn Bāṣoh's treatise shows that, as the plate does not contain a rete of horizontal coordinates, the user of the instrument must replace it by using the equatorial rete. For that purpose, he must perform a rotation, equivalent to the local colatitude, northwards or southwards of the equatorial position, using the rete of "arcs" in the same way as Ibn al-Zarqālluh uses the ruler-horizon.

Ibn Bāṣoh's universal plate became well known in the Maghrib, where several abridged treatises on its use were written, most of them in the eighteenth century.⁶²² However, this instrument was introduced at an earlier date, as we know of the existence of universal plates made in the Maghrib since 1224, only six years after Ibn Bāṣoh's death. Emilia Calvo has collected references to the existence of 21 astrolabes, containing plates of this kind, most of which seem to be Maghribī.⁶²³

The diffusion in Eastern Islam was less common although it had already started in the fourteenth century: Najm al-Dīn al-Miṣrī informs us that he discovered [a slight variant of] this instrument in Mecca in 723/1323–24, which he calls *al-aṣṭurlāb al-āfāqī*. When he returned to Cairo, he saw a Maghribī astrolabe containing a plate of this kind, attributed to Ibrāhīm ibn ʿAlī ibn Bāṣoh al-Andalusī, probably a member of our author's family. He obtained a treatise on its construction and use and, finally, acknowledged that Ibn Bāṣoh had preceded him in the design of this instrument.⁶²⁴ Besides this, we know that the Egyptian astronomer Muḥammad b. Aḥmad al-Mizzī (born in 1291) made

620 Calvo, 1993; see also Calvo, in *BA*, vol. 2, pp. 595–598; Calvo, 1992; Calvo, 2000.

621 Calvo, 1994.

622 Samsó, 1973 pp. 176–180; Calvo, 1991.

623 Calvo, 1996, pp. 759–763.

624 Charette, 2003, pp. 108–111, 247, 24–25 (of the Arabic edition).

a plate of this kind in Damascus in 734/1334, which was later included in an astrolabe made in Tunis by Ḥusayn ibn ‘Alī in 709/1309. The Institut du Monde Arabe in Paris also preserves an astrolabe of Ottoman style, made in 1098/1686–87 by Ibrāhīm ibn Muḥammad al-Balawī, containing a plate engraved on both sides: one side corresponds to Ḥabash al-Ḥāsib’s *ṣafiḥa āfāqīyya*, while the other has Ibn Bāṣoh’s projection.⁶²⁵

4.3.6 *al-Jazzār’s Astrolabe*

Another example of Western Islamic creativity in the field of universal astrolabes can be found in an astrolabe extant in the Oxford Museum for the History of Science made by ‘Alī b. Ibrāhīm al-Jazzār⁶²⁶ in Tāzā (now in Morocco) in 728/1327–28. The internal catalogue of the Museum describes it⁶²⁷ as an unusual form of Ibn Khalaf’s universal plate. The back of the instrument is similar to that of a standard astrolabe, with a zodiacal scale, diagram of seasonal hours, sine, cosine and shadow scales. The rete also looks like that of an astrolabe, with the projection of 58 stars, but it also bears a full myrtle (*āsī*) ecliptic, combining the northern projection of the northern signs with the southern projection of the southern signs. This ecliptic was invented in the Mashriq in the ninth or tenth century, as it is mentioned by Ibrāhīm ibn Sinān (907–946), and al-Jazzār’s astrolabe is one of the very few extant instruments with an ecliptic of this kind.⁶²⁸

The face of the astrolabe (fig. 4.42) has been interpreted in two different ways. On the one hand, the catalogue of the Oxford Museum considers that a projection of azimuthal circles corresponding to a standard astrolabe plate traced for a latitude of 90° has been superimposed on a meridian stereographic projection like that of Ibn Khalaf’s plate. This interpretation seems peculiar as it considers that two different stereographic projections (equatorial and meridian) are used in the same plate, while the rete also corresponds to that of a standard astrolabe projection. This problem was solved by Emilia Calvo⁶²⁹ who noted that the image of the face in al-Jazzār’s astrolabe is identical to what would be obtained by deleting, on Ibn Bāṣoh’s plate, the lines inscribed between the circles of the equator and the tropic of Capricorn. The instrument would

625 Vernet & Samsó, 1992, p. 237.

626 The name al-Jazzār bears no diacritic marks in the instrument; this is why it has often been read as al-Ḥarrār.

627 Or did so in the 1980s and 1990s, when I examined the instrument for the preparation of two exhibitions: see Vernet, 1985 (p. 91) and Vernet & Samsó, 1992 (p. 239); see also King, 2005, p. 559.

628 King, 2005, pp. 557–560.

629 Calvo, 1990, pp. 223–225.

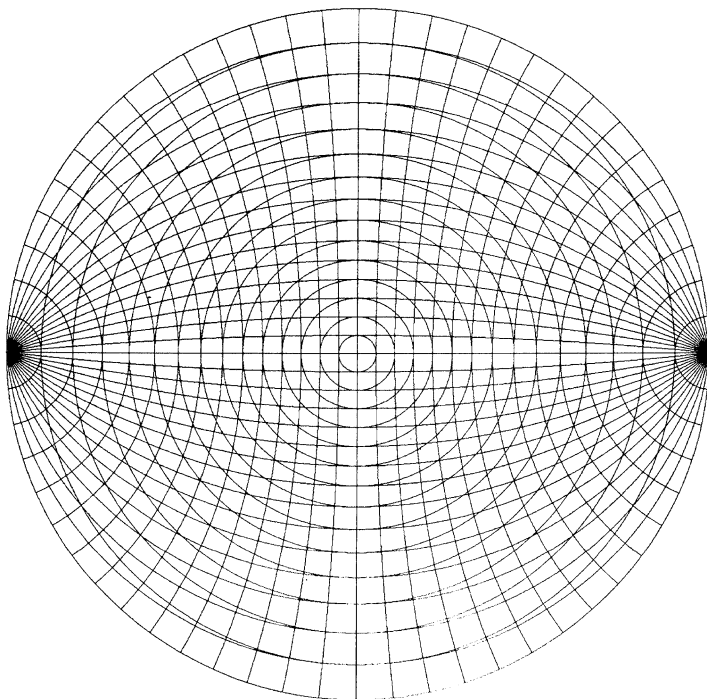


FIGURE 4.42 Face of al-Jazzār's astrolabe borrowed from E. Calvo

therefore be a development of Ibn Bāṣoh's instrument which, like al-Jazzār's astrolabe, uses an equatorial stereographic projection. The concentric circles around the centre of the instrument will, therefore, be parallel to the equator.

4.4 Quadrants

In 2.4.2.1.2 I have already dealt with the introduction of the sine quadrant in Catalonia towards the end of the tenth century. This type of instrument was called, by Millàs, *quadrans vetustissimus*. I have also analysed the *quadrans vetus* documented in the Alfonsine *Libros del Saber*. Apart from these two references, there is very little evidence of the use of these horary quadrants in the Andalusī tradition,⁶³⁰ as the sole example appears in the lower right quadrant of the back of Ibn al-Zarqālluh's *ṣafiha zarqāliyya*, where it is used for

⁶³⁰ On the sine quadrant see Schmazl, 1929, pp. 83 ff.; King, "Rub", *El2*, VIII, pp. 574–575; King, 2005, pp. 162 ff.

graphically solving trigonometrical problems,⁶³¹ in Abū l-Šalt's treatise on the use of the astrolabe (4.2.3.2) and on the back of two astrolabes by Muḥammad ibn al-Fattūḥ al-Khamā'irī of Seville (609/1212–13, 634/1236–37).⁶³² We have also seen (4.3.4) that Abū 'Alī al-Marrākushī, in his *Jāmi' al-mabādī' wa l-ghāyāt*, ascribes to Ibn al-Zarqālluh the design of a quadrant which combines elements of the tradition of the sine quadrant with the orthographic projection of the celestial sphere on the plane of the solstitial colure which also appears on the back of the *ṣafiha zarqāliyya*: in it the parallels are straight lines, perpendicular to one of the sides of the quadrant, which can be used as a graphical scale of sines and cosines, while the meridians, in Sédillot's interpretation, are arcs of ellipses.

In fourteenth-century Granada we find, again, the name of the *muwaqqit* Ibn Bāṣoh (4.3.4) who is the author of a *Risāla fī l-ṣafiha al-mujayyaba dhāt al-awtār*.⁶³³ The instrument described is not a sine quadrant but a plate of sines provided with chords and it contains, among other things, a grid of parallel and horizontal lines, unequally divided, in which the half chords of the horizontal parallels are projected perpendicularly on the vertical diameter of the plate from the graduations of each one of the four quadrants. Therefore, each one of them corresponds to the variant of the sine quadrant called *al-jayb al-tis'īnī* (nonagesimal sine quadrant).⁶³⁴ The applications of the instrument are trigonometrical calculations and the solution of problems related to *mīqāt*.

A century later, we find a new reference to the sine quadrant in the instrument called *sexagenarium*, used in Cairo and introduced in Valencia in 1450 by a *faqīh* from Paterna, who also brought with him a non-extant Arabic treatise on the use of the instrument.⁶³⁵ This is the first instance of a late Eastern connection that results in the introduction of an astronomical instrument: two centuries earlier, Ibn Arqam al-Numayrī (d. 1259) wrote an epistle with the title *Risāla fī l-aṣṭurlāb al-khaṭṭī wa l-'amal bi-hi*,⁶³⁶ on the linear astrolabe invented by the Persian astrolabist Sharaf al-Dīn al-Ṭūsī (d. 1213).⁶³⁷

The Arabic treatise on the *sexagenarium* was translated into Catalan in 1456, probably by Johannes de Bonie, a Valencian physician who seems to have

631 Puig, 1991.

632 Pingree, 2009; Hernández pp. 139, 180–2.

633 Calvo, 2001; Calvo, 2008.

634 Charette, 2003, p. 210.

635 On this instrument see Thorndike, 1951; Poulle, 1966; Poulle, 1980, pp. 417–444; Aguiar & González, 2003, pp. 98–103, 115–162; Aguiar & González, 1996.

636 Puig, 1983.

637 Michel, 1943.

been interested in astronomy.⁶³⁸ The Catalan text, divided into 20 chapters, has been preserved in ms. Paris BNF 7416A. Johannes de Bonie seems also to be the author of two other texts on the instrument extant in the same manuscript: an incomplete Latin retranslation of the Catalan text, made in 1464, and a longer treatise (46 chapters) on the use of the instrument, which includes a description (not found in the two aforementioned texts) of the back of the instrument. A second manuscript from the Paris BNF (10263) contains another treatise on the use of the face of the instrument without any allusion to its back: this version was written by Christianus de Prolianus in 1475.⁶³⁹ In 1516, a fourth text on the *sexagenarium*, written by Bartholomeus de Solliolis, was printed in Saint Pons de Thomières (department of Hérault, in France).

The instrument described corresponds to the variety of the sine quadrant characterised by a grid of 60 equally-spaced horizontal and vertical lines which was called *al-jayb al-sittīnī* (sexagesimal sine quadrant) and it seems obvious that *sexagenarium* translates *sittīnī*.⁶⁴⁰ The texts on the sine quadrant mentioned above contain collections of chapters dealing with the use of the instrument for the solution of standard problems of timekeeping, including the determination of the *qibla*. The longer text contained in ms. BNF 7416 A describes an instrument with two sides, one of them being a graphical scale giving the mean motions of all planets. The purpose of the instrument is to use the sine quadrant for the graphical computation of planetary equations, which thus qualifies it as a member of the family of equatoria. Besides this, the latter description fits the only extant instrument of this kind, preserved in the Oxford Museum of the History of Science.⁶⁴¹ We have, therefore, the unsolved problem of determining whether this planetary use of the instrument corresponds to the original Arabic treatise or is a later European addition, although M. Aguiar and J.A. González Marrero favour the latter solution.

As we have seen, this sine quadrant seems to have been little used in al-Andalus but we find frequent references to its application in the Maghrib. A large number of extant Maghribī manuscripts deal with it,⁶⁴² and it seems

638 Aguiar & González, 2005.

639 Aguiar & González, 2005 a. The sine quadrant appears, in an illustration of the manuscript, on the lower right quadrant placed in the back of an astrolabe. In the other three quadrants we find two horary quadrants and an astrolabic quadrant. The instrument is also provided with an alidade and transversal scale similar to the one found in the back of Ibn al-Zarqālluh's *ṣaḥīḥa zarqālīyya*.

640 Charette, 2003, p. 210; Aguiar & González, 1996, pp. 199–200.

641 Good photographs of the Oxford Museum's *sexagenarium* can be seen in Vernet, 1985, pp. 108–109, and in Vernet & Samsó, 1992, pp. 216–217.

642 See, for example, Lamrabet, 2013, M195 (pp. 198–199), M214 (p. 206), M219 (p. 208), M282 (p. 224) and M310 (p. 228) for authors of treatises on the sine quadrant until the sixteenth century.

to have become the favourite instrument of the *muwaqqitūn*. Unfortunately, only one of these texts has been edited: the *Risāla kāfiyat al-sayb fī l-ʿamal bi l-jayb*, written by ʿIzz al-Dīn ʿAbd al-ʿAzīz b. Masʿūd (fl. 1372–1392), known as ibn Farmīja (possibly Firmījūh = Bermejo),⁶⁴³ probably born in Tlemcen; he was *imām* and *muwaqqit* in Fez, Tunis, Jerusalem and Damascus, an instrument maker,⁶⁴⁴ and author of an astrological treatise,⁶⁴⁵ who played a role in the introduction of Ibn Abī l-Shukr’s *Tāj al-azyāj* in the Maghrib (see 7.5.1).⁶⁴⁶ The treatise on the sine quadrant was written in Cairo in 795/1392–3 and it describes a trigonometrical quadrant with a double set of perpendiculars traced from the arc to both sides of the instrument, which resemble modern graph paper. The treatise describes in detail the uses of the sine quadrant to solve standard problems of spherical astronomy, placing special emphasis on those related to the profession of a *muwaqqit*. A chapter deals with the computation of the *qibla* using the “method of the *zīj*es” (2.5.2.1). It also contains long introductory chapters on problems of chronology and calendar conversion, and ends with the use of the instrument for land surveying (*misāḥa*)⁶⁴⁷ and for trivial arithmetical operations. Interestingly, it deals with the computation of the four cusps and the division of the houses of the horoscope using the dual-longitude method (3.2.3.1.3). It shows both Western influences (quotations of Andalusian authors like Ibn al-Zarqālluh, Ibn al-Ṣaffār and Ibn Muʿādh al-Jayyānī) and Eastern influences (a precession table copied from Ibn Abī l-Shukr al-Maghribī’s *Tāj al-azyāj* which could also be the source for the longitude of Cairo, 64;40° given in the text). Ibn Masʿūd *risāla* was definitely used in the Maghrib: the only extant manuscript (Escorial 918) was copied somewhere in the Islamic West in 888/1483, and the work is quoted in the *Maḥṣalat al-maṭlūb fī l-ʿamal bi-rubʿ al-juṣūb* of the Tunisian astronomer ʿUmar b. ʿAbd al-Raḥmān al-Tawzarī (d. 1454).⁶⁴⁸

Besides texts, we should bear in mind the number of sine quadrants built in the Maghrib, on one face of which we usually find a double grid of perpendiculars, forming a double scale of sines and cosines, and, on the other, a set of astrolabic markings (*rubʿ al-muqanṭarāt, quadrans novus*). An example is a quadrant made by Aḥmad b. ʿAbd al-Raḥmān al-Dahmānī, dated in 854/1450–51, in which the astrolabic markings correspond to a latitude of 36;40° (Tunis).⁶⁴⁹ Later instruments of the same kind are the quadrant made by Muḥammad

643 Edition and Spanish translation in Aguiar, 1995.

644 See King, 2005, pp. 835–914, especially pp. 902–903.

645 Herrera, 2001.

646 Samsó, 1998. On this *zīj* see Dorce, 2002–3.

647 Aguiar, 1996.

648 King, 1986 b, p. 141 (F39); Lamrabet, 2013, M214, p. 206.

649 Millás, 1947, pp. 61–64; King, 2007, p. 192.

b. ‘Alī Ḥajjāj (1006/1597–98) for a latitude $31;30^\circ$ (Marrakesh)⁶⁵⁰ and by Muḥammad b. Aḥmad al-Baṭṭūṭī (1142/1729–30) for a latitude 34° (Meknès?).⁶⁵¹

This is not the only kind of quadrant used in the Maghrib. We have already seen that the almucantar or astrolabic quadrant appears, combined with the sine quadrant, in several extant instruments and, in the second half of the fifteenth century, Muḥammad al-Lajjām al-Lajā’ī or Luḥā’ī (fl. ca. 1485) wrote a *Risāla fī rub‘ al-muqaṭṭarāt*.⁶⁵² Interestingly, a treatise on the myrtle quadrant (*Tadhkirat al-nāsī li l-rub‘ al-āsī*), written by ‘Umar b. Muḥammad b. ‘Azm al-Tamīmī al-Tūnīsī (d. 1442), is preserved in the Hassaniyya Library of Rabat. This instrument derives from the myrtle astrolabe, which superimposes northern and southern projections;⁶⁵³ this is confirmed by the *incipit* of the extant manuscript in which we can read “the instrument called *āsī* in our locality (*fī ḥaḍrati-nā*) which is the result of a composition of northern and southern almucantars”.⁶⁵⁴

An entirely different case is that of the Alfonsine *cuadrante sennero*⁶⁵⁵ in which the adjective *sennero* seems, at first sight, to be a translation of the Arabic *mujayyab* although the text itself disproves this hypothesis. This brief treatise was authored by Rabbi Ishāq ibn Sīd (Rabiçag) following the king’s orders. Alfonso X was aware of the difficulties presented by two Ptolemaic observational instruments, the armillary sphere and the triquetrum (“las alidades”). The prologue insists on two possible uses of the quadrant: in one of them, the instrument is an altazimuthal quadrant whose centre is placed at the centre of a horizontal graduated circle. It seems obvious that this circle should be placed on top of some kind of structure allowing the observer to look from the centre of the quadrant, using an alidade with sights or a sighting tube.⁶⁵⁶ This application of the instrument is called in the text “el cuadrante movable” (the movable quadrant) because it can be rotated to determine the altitude and azimuth of any celestial body. In the second application the quadrant is fixed

650 Arribas, 1965.

651 Millás, 1947, pp. 56–61. Good photographs of another undated Maghribī sine quadrant with an, apparently, unfinished back calculated for Marrakesh in Pingree, 2009, p. 204. See a list of extant Maghribī quadrants in King, 2007, p. 205 and in <http://web.uni-frankfurt.de/fb13/ign/instrument-catalogue-TOC.html>.

652 Lamrabet, 2013, M230, p. 211.

653 Charette, 2003, pp. 85 and 275.

654 Khaṭṭābī, 1983, p. 157; Lamrabet, 2013, M208, p. 205.

655 Millás, 1956.

656 There seems to be a reference to a sighting tube in the title of chapter 12: “De saber como se deve fazer ell estrumento del cannudo con que pueden saber la quantitat del tiempo que es entre dos puntos, ell uno dellos sabudo et ell otro non” (Millás, 1956, p. 240 of the reprint). The *cannudo* is probably equivalent to modern Castilian *canuto*, a hollow tube.

on the meridian plane, which implies that it will be used to observe the meridian altitudes of celestial bodies. We have, therefore, an observational quadrant which might have been used in Alfonsine observations.

This is all the information we can gather from the prologue of the treatise, as there is no description of the instrument itself. The table of contents of the treatise gives the title of twelve chapters, of which only eight have been preserved in the two extant manuscripts. These eight chapters deal with a collection of astronomical problems with very few references to the instrument itself. The solutions are accompanied by careful proofs⁶⁵⁷ in which the “new” trigonometry is used. This trigonometry reached al-Andalus through Ibn Mu‘ādh al-Jayyānī’s *Kitāb majhūlāt qisī al-kura*⁶⁵⁸ and Jābir ibn Aflah’s *Iṣlāḥ al-Majistī*,⁶⁵⁹ although the latter seems to be the source actually used by Rabiḥag because he uses the sine and cosine laws and Geber’s theorem, but not the law of tangents which is characteristic of Ibn Mu‘ādh’s book. On the whole, the solutions and proofs bear witness to Rabiḥag’s trigonometrical competence⁶⁶⁰ in clear contrast to the elementary character of the trigonometry used in the other treatises of the *Libro del saber de astrología*.⁶⁶¹

4.5 Equatoria

4.5.1 Introduction

As we have already seen, the astrolabe is the standard tool used by astrologers to solve some of the problems related to the casting of a horoscope. It allows them an easy determination of the position of the Sun and the four cusps, as well as the division of the other houses. The astrolabe cannot be used, however, for the computation of the planetary positions; for this purpose, the astrologer has to use a *zīj*. This means that about half an hour is needed to calculate the longitude of the Moon or a planet, and to cast a horoscope the astrologer needs to obtain the longitude of the Moon and of the five planets. A horoscope cast using a *zīj* requires, therefore, about three hours for the computations, which means that it is an expensive and time-consuming task, if the work is done properly. To solve this problem two different solutions appeared

657 This is done by Rabiḥag following the king’s orders: “Et que aduga sobre cada rason su prueba de Geometria et de astrologia por toller la dubda, et por que se paresca la certedumbre de lo que avemos dicho” (Millás, 1956, p. 239 of the reprint).

658 Villuendas, 1979.

659 Lorch, 1995 a.

660 Ausejo, 1984.

661 Ausejo, 1983.

in eleventh-century al-Andalus: on the one hand, the instrument called the equatorium; on the other, the computation of ephemerides and perpetual almanacs (see 7.7). This is one of the many instances in which an astrological problem led to the development of astronomical tools.

The equatorium is an instrument that allows the graphical determination of planetary longitudes with very little computation. It is based on a series of Ptolemaic planetary models made to scale. The origin of this instrument seems to be Eastern, as it is one of the components of Abū Ja'far al-Khāzin's *Zīj al-ṣafā'ih*.⁶⁶² In spite of this, its main developments seem to have taken place in al-Andalus where three equatoria were designed by Ibn al-Samḥ, Ibn al-Zarqālluh and Abū l-Ṣalt al-Dānī.⁶⁶³

4.5.2 *Ibn al-Samḥ's Equatorium*

Ibn al-Samḥ's treatise on the construction of the equatorium is only known in the Castilian Alfonsine translation extant in the *Libros del saber de astronomía*,⁶⁶⁴ where it appears with the title "Libro primero de las laminas de las siete planetas". This instrument has a plate for each planet, and these plates are kept inside a *mater*, as in an astrolabe. Leaving aside the special characteristics of the plates for Mercury and the Moon⁶⁶⁵ (for which the Ptolemaic model is more complex) I will say that the outer circle of each plate corresponds to the ecliptic (Fig. 4.43). According to the Alfonsine text:

- AP is a diameter of the ecliptic which joins the planet's apogee (A) with its perigee (P),
- B is the centre of the universe, and of the ecliptic,
- BG is the double eccentricity,
- G is the centre of the deferent (*leuador* in the Alfonsine terminology)
- E is the midpoint of BG and the centre of the equant circle.

This is an obvious mistake because, in Ptolemy's planetary models, the centre of the deferent is always the midpoint between the centre of the equant circle and the centre of the universe. The great Jewish astronomer Levi ben Gerson (1288–1344) considered the possibility of this anomalous position of the centre of the equant in his exhaustive analysis of all the possibilities offered by

662 King, 1980. See also Samsó, 1996, pp. 594–601; Calvo, 2004 a.

663 See Comes, 1991. On the history of equatoria in Latin Europe see Poule, 1980.

664 Ms. Villamil 156, fols. 156r–165r; Rico, 1863–67, III, pp. 241–271; Comes also edited the Italian translation of the Alfonsine text made in 1341: Comes, 1991, pp. 169–188. On this source see Comes, 1991, pp. 27–78; Samsó, 1983; Mancha, 1987.

665 One of the models for the Moon described by Ibn al-Samḥ is a simple eccentric (chapter 9) and it corresponds to the lunar model with only one inequality which appears in the *Almagest* IV, 6.

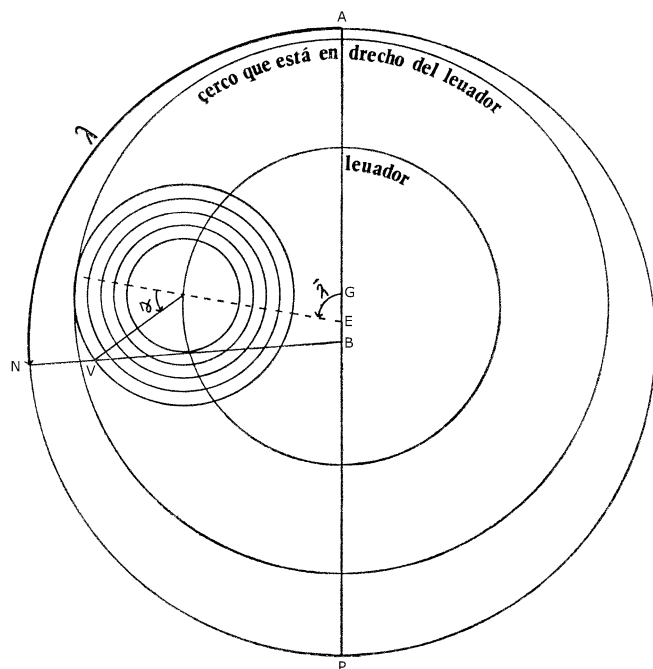


FIGURE 4.43
Determining the
true planetary
longitude with
Ibn al-Samh's
equatorium

the geometry of eccentrics. Mancha has shown however that, in this case, it is an error of the Alfonsine translator which also appears in the Alfonsine Latin translation of Ibn al-Haytham's *Fī hay'at al-‘ālam*.⁶⁶⁶ This allows me to suggest that the translator of Ibn al-Samh's treatise on the equatorium was the same Abraham Hebreo who translated Ibn al-Haytham's book.

Let us, therefore, correct the Alfonsine interpretation and consider, as usual, that

- B is the centre of the universe
- E is the centre of the deferent
- G is the centre of the equant circle

With centre in E and a radius equal to half the radius of the ecliptic, we trace the deferent circle (*leuador*). Also with centre in E we will trace the *cerco que está en drecho del leuador* (the circle in front of the deferent). The difference between the radii of the deferent and of this second circle should be equal to the radius of Venus' epicycle. Then, with an arbitrary radius, greater than all the others, and centre in G, we will trace the equant circle (named *cerco dell alaux*, circle of the apogee), an auxiliary circle which will be deleted later. The *cerco dell alaux* will be equally divided into 360° and we will project these

666 See Mancha, 1990.

divisions on both the deferent and the circle in front of the deferent, by drawing diameters which join opposite points with centre G of the equant.

This is the basic structure of each planetary plate on which the plate of the epicycles will be superimposed, in which five concentric circles appear, corresponding (in order of decreasing size) to the epicycles of Venus, Mars, Mercury, Jupiter and Saturn.

The logic of this disposition is easy to understand: the plate of the epicycles is graduated and it has an index, placed on the outer circle (epicycle of Venus), which marks the graduation 0° of the mean anomaly. Besides, the centre of this plate must necessarily be perforated. We place the plate of the epicycles in such a way that we can see, through the hole in its centre, the graduation of the *leuador* corresponding to the *mean centre* of the planet (its mean longitude calculated from its apogee) for a particular date and hour. Then we rotate the plate until the index coincides with the same graduation (that of the *mean centre*) of the *çerco que está en drecho del leuador*. Then the epicycle will be in its correct position, as the graduation of both the deferent and the circle in front of the deferent correspond, actually, to the graduation of the equant circle. The radius that joins G with the centre of the epicycle will form with BA, the apse line, an angle equal to the mean centre of the planet. The index of the plate of the epicycles, which marks the origin of the mean planetary anomalies, will also be correctly placed and in the graduation of the epicycle plate we will be able to locate, the degree corresponding to the mean anomaly (α) of the planet, which will give us its position on its epicycle (V). We will place the ruler that rotates around point B (centre of the universe) on V of the epicycle and it will intersect the ecliptic on point N and we will obtain arc AN, the true longitude of the planet measured from its apogee. One can easily see that the scheme can only work properly if the difference between the radius of the *çerco que está en drecho del leuador* and the radius of the *leuador* equals the radius of the plate of the epicycles.

Ibn al-Samḥ's treatise also gives the numerical parameters used in the construction of his instrument: the planetary apogees derive from those of al-Battānī with an increase of $2;12^\circ$, corresponding to the value of precession between 879 (the *radix* date for al-Battānī's apogees) and year 416/1025–26 (the epoch used by Ibn al-Samḥ) on the basis of 1° per 66 solar years (the precessional constant used by al-Battānī). The positions of the ascending planetary nodes derive, apparently, from al-Khwārizmī-Maslama's *zīj*, while the planetary eccentricities and radii of the epicycles are those of the *Almagest*.

So we see that the equatorium allows the measurement of the true planetary longitude with very little computational effort, as the astrologer only needs to

use a *zīj* to obtain the mean longitude and anomaly of a planet. Ibn al-Samḥ, who wishes to design a self-sufficient instrument, is conscious of the fact that his planetary plates have empty spaces between the concentric circles, and suggests that the values of the mean motions in longitude and anomaly should be inscribed in these empty spaces. Besides, he states that his planetary plates will be placed in the mater of an astrolabe, as this instrument contains an empty box in which the plates corresponding to each latitude are kept. The instrument designed by Ibn al-Samḥ is, therefore, an astrolabe-equatorium in which the outer graduated crown of the astrolabe can be used for the graduation of the ecliptic, and its radial ruler can also be used in the final operation of determining the planet's longitude on the ecliptic. These two characteristics (mean motion tables inscribed on plates, and an equatorium enclosed in the mater of an astrolabe) are exactly what we find in Hibat Allāh's astrolabe, made in 514/1120–21, which agrees with Abū Ja'far al-Khāzin's description of the *Zīj al-ṣafā'iḥ*. As a matter of fact, a reference to the first of these characteristics is also to be found in Johannes Hispanus' treatise on the astrolabe,⁶⁶⁷ in which we read:

quidam uero uolentes astrolabium perfectum facere sculpiunt in quibusdam tabellis medios cursos planetarum adque [*sic*] equationes annorum quoque et mensium ab inuicem extractiones sicut in ...⁶⁶⁸ qui dicitur liber directionum inuenitur.

Once again, this seems to be a reference to an astrolabe-equatorium, which could be called “perfect” because it can be used to solve problems related not only to the Sun and the fixed stars, but also to planets. The *equationes* mentioned in the text do not, probably, refer to the standard planetary equations, but rather to either the mean motions in anomaly or to the *revolutiones annorum et mensium*, frequently used in astrological practice.⁶⁶⁹

4.5.3 *The Equatorium of Ibn al-Zarqālluh*

Ibn al-Zarqālluh, who applied equatorium techniques to his use of the “circle of the Moon” for the computation of the geocentric distance of the Moon (4.3.2.4), is also the author of two short treatises dealing with the construction of an equatorium and its use. The former is only known through the Alfonsine

667 Millás, 1942, p. 325.

668 There is a blank space in the manuscript according to Millás.

669 Samsó, 1981.

Castilian translation, also found in the *Libro del saber de astrología*,⁶⁷⁰ while the latter is extant in the original Arabic text.⁶⁷¹ This second treatise mentions the year 474/1081–82 and was written for the library of King al-Mu'tamid of Seville, while the Alfonsine text on the construction mentions year 473/1080–81.

The instrument is called *al-ṣafīha al-zījīyya* (a plate with the same applications as a *zīj*), a denomination which bears a clear resemblance to al-Khāzin's *Zīj al-ṣafā'ih*. The treatise on the use of the instrument mentions the existence of a precedent of a similar kind, which may be an allusion to Ibn al-Samḥ's equatorium. As a matter of fact both instruments have the same basis: each planet has a system of two concentric circles called *leuador* (*al-falak al-hāmil*) and *ṣerco dell alaux* (*falak al-awj*), whose centre is the centre of the planet's deferent. Both circles are divided unequally as, with centre in the centre of the equant, a hidden circle is traced (*ṣerco dell yquador*) which is divided into 360 equal parts. As in Ibn al-Samḥ's equatorium, these divisions are projected on both the *leuador* and the *ṣerco dell alaux* by drawing diameters through each one of these divisions. The instrument will work in the same way as Ibn al-Samḥ's: we place the plate of the epicycles in such a way that its centre coincides with the graduation of the *leuador* corresponding to the mean centre of the planet, and the index of the plate of the epicycles coincides with the same degree of the *ṣerco dell alaux*. Then, the centre of the planetary epicycle will be situated correctly. The position of the planet on its epicycle will be established by taking the mean planetary anomaly in the graduation of the epicycle from the index. Finally, a ruler that rotates on the centre of the instrument (and of the universe) will pass through the position of the planet on its epicycle and will determine on the outer circle of the plate (ecliptic) the true longitude of the planet from its apogee.

We have not found, so far, any differences between Ibn al-Zarqālluh's and Ibn al-Samḥ's instruments. I should say, however, that while Ibn al-Samḥ's equatorium comprises a set of plates kept in the *mater* of an astrolabe, Ibn al-Zarqālluh's equatorium seems to be an independent, unrelated instrument. Besides, in the construction treatise, Ibn al-Zarqālluh shows that he is interested in designing an instrument that can work with a limited number of plates and, for this purpose, he concentrates all the circles of the *leuador* and the

670 Ms. Villamil 156 fols. 165v–169v; Rico, 1863–67, III, 272–284. Comes has also edited the Italian translation of the Alfonsine text made in 1341: Comes, 1991, pp. 189–202.

671 Edited and translated into Spanish in Comes, 1991, pp. 203–236. See a study of both treatises in Comes, 1991, pp. 79–138. See also Price, 1955, pp. 120–122; Poulle, 1980, I, 197–200.

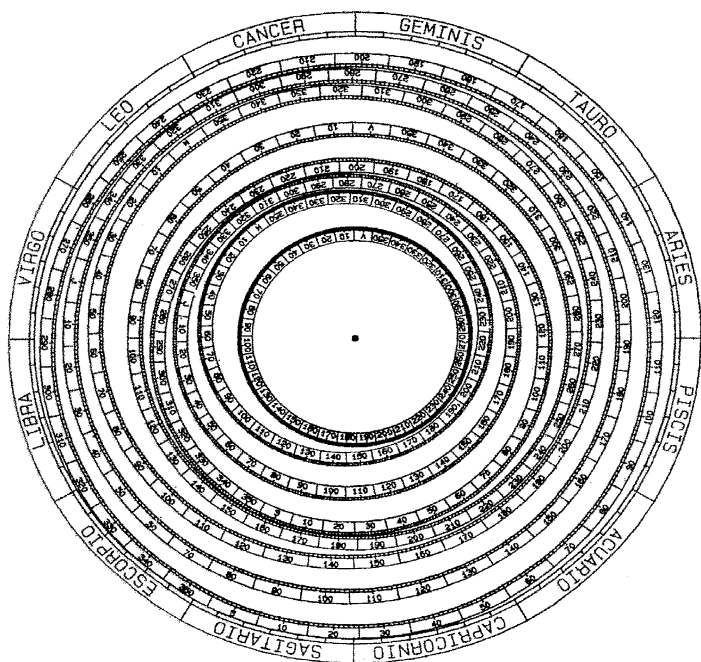


FIGURE 4.44
Circles for
Venus, Mars,
Jupiter and
Saturn,
according to
M. Comes and
H. Mielgo

cerco dell alaux on both sides of the same plate, something which forces him to use different scales for each planet. On the first side the circles appear in the following order (from the centre to the outer part of the instrument, see Fig. 4.44):

- Deferents of Venus, Mars, Jupiter and Saturn
- *Çercos dell alaux* of Venus, Mars, Jupiter and Saturn
- Ecliptic.

In this plate the difference between the radii of the deferent and of the *cerco dell alaux* for each planet equals the radius of Mars' epicycle, as the epicycle of this planet has the largest size, followed by those of Venus, Mercury, Jupiter, Saturn and the Moon.

The second face of this plate is more original; in it, we find, from its centre (Fig. 4.45):

- Deferents of Mercury and the Moon
- *Çercos dell alaux* of the Moon and Mercury
- Solar eccentric circle

One can easily see in fig. 4.45 that Mercury's deferent and *cerco dell alaux* are not circles but some kind of oval figure. This has attracted a certain amount of attention from modern scholarship. The construction of the

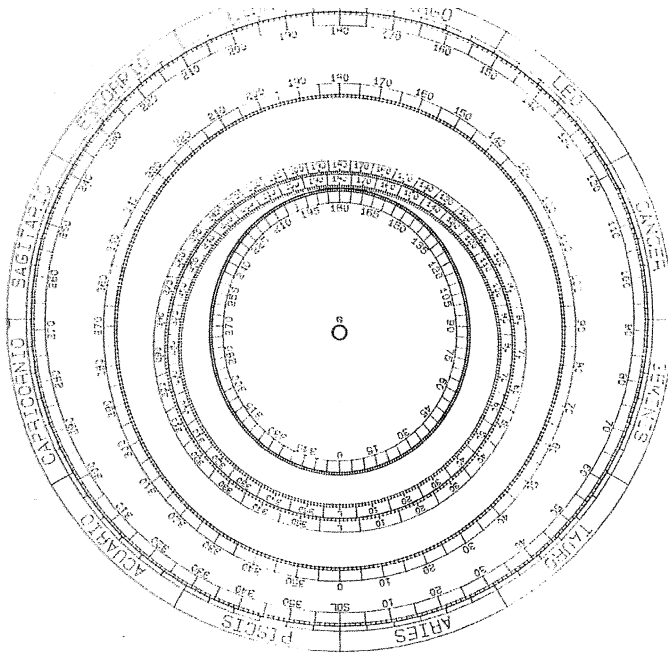


FIGURE 4.45
Circles for
Mercury, the
Moon and the
Sun according
to M. Comes and
H. Mielgo

deferent described in the Alfonsine translation⁶⁷² is as follows (Fig. 4.46, not to scale):⁶⁷³

- O is the centre of the plate (and the centre of the universe)
- E is the equant point, and
- A is the centre of the small circle of radius AE, the locus of the centres of Mercury's deferent ("çerco leuador del centro del leuador de Mercurio").
- $AE = EO = 2;21^p$.

With A as centre and radius 81^p , circle BDCG is drawn and divided into 5° arcs. Circle BDCG is an auxiliary circle used to divide the small circle of radius AE.

With E as centre and radius 80^p , circle LDHG (the equant circle) is also drawn and divided into 5° arcs.

Following this construction, points I, J, and K correspond to the first three divisions, into 5° arcs, of circle of radius AE, starting clockwise from the apse-line.

⁶⁷² Ms. Villamil 156 fols 167r–168v; Rico, 1863–67 III, pp. 278–280. The method used to draw the curve was described by Hartner, 1968, pp. 476–7; see also Comes, 1991, pp. 117–23.

⁶⁷³ The construction of Mercury's deferent and the identification of the curve was previously published in Samsó & Mielgo, 1994 a.

This passage cannot be translated without our first explaining two technical expressions that appear in it. The first is the *figura pinnonata* (*pineal figure*, shaped like a pine-cone) which attracted the attention of Willy Hartner, who considered that the oval shape of the curve resembles that of a pine-nut.⁶⁷⁶ On the other hand, the curve is described as *bayḍī* (oval) in the Arabic text of Ibn al-Zarqālluh's book on the use of the instrument,⁶⁷⁷ a much less technical work. *Figura pinnonata* is, however, a literal translation of the Arabic *shakl ṣanawbarī*, which means pineal figure but also conic figure. In this regard the Alfonsine Latin translation (based on a previous Castilian version) of Ibn al-Haytham's *Fī hay'at al-ʿālam* systematically uses *figura pinee*/*figura pineata*/*pineatum* to convey the meaning of conic figure or cone in passages in which the Arabic equivalent is *makhrūṭ* (the most common word for cone) or a term derived from this word.⁶⁷⁸

The second expression that deserves comment is *taiadura menguada*. *Taiadura* derives from *taiar* (modern Spanish “tallar”, “tajar”, cf. French “tailler”) and, therefore, means a cut or a section (Arabic *qaṭʿ*). *Menguada* is a participle of the verb *menguar* (to diminish or reduce), and thus *taiadura menguada* is a literal translation of the Arabic *qaṭʿ nāqīṣ* (deficient section), the standard Arabic expression for ellipse.⁶⁷⁹ So the translation of the aforementioned Castilian text is:

The deferent circle will have the shape of the deficient section [ellipse], of those sections which appear in the *pineal figure* [cone]

In his famous paper “The Mercury horoscope”, first published in 1955, Hartner established that the curve described by the centre of Mercury's epicycle becomes practically identical to an ellipse when the parameters of the *Almagest* are used, and that the situation does not change with Ibn al-Zarqālluh's slight modification of Ptolemy's parameters; Hartner also stated that the Alfonsine

676 Hartner, 1984, pp. 282, 287.

677 Comes, 1991, pp. 117, 207: *ʿalā shakl al-bayḍa wa-huwa al-shakl al-maʿrūf bi l-bayḍī ʿinda l-ʿarīfīn bi-ʿilm al-hayʾa* (with the shape of an egg, which is the shape called oval by those who know astronomy): Millás, 1943–50, p. 459.

678 Samsó, 1990 a, pp. 124–5; the Arabic edition of Ibn al-Haytham's treatise was published by Langermann, 1990. The Latin Alfonsine translation was edited by Mancha, 1990.

679 Roser Puig has noted that the semi-ellipses that appear in the orthographic projection of the back of the *ṣafīha zarqālliyya* are called, in the Alfonsine text, *linnas* [= lines] *de taias mingudas*. See Puig, 1987, pp. 20–22, and 1987 a, p. 127. See 4.3.2.3 where I have already noted another reference of Ibn al-Zarqālluh to the technique for drawing an ellipse by determining three points of the curve and joining them with an arc of a circle.

translation of Ibn al-Zarqālluh's lost Arabic original is the first explicit description of Mercury's true deferent. Now we also know that Ibn al-Zarqālluh identified this curve with an ellipse and this is, to the best of my knowledge, the first instance of the use of a conic section of this kind in astronomy, even if the eleventh-century Toledan astronomer had no theoretical pretensions in using an ellipse and employed it only to simplify his design of an equatorium.⁶⁸⁰

Ibn al-Zarqālluh's work on the equatorium may have been known in the Mashriq; in the fifteenth century, al-Kāshī describes an instrument similar to the Toledan astronomer's equatorium, and traces, in a less precise form than our author, an oval curve for Mercury's deferent.⁶⁸¹ Similar developments took place in Latin Europe from the fifteenth century onwards: Georg Peurbach (1423–1461) was the first European scholar to identify the oval curve of Mercury's deferent in Ptolemaic theory and was followed by Erasmus Reinhold (1511–1543) who did the same not only for Mercury but also for the Moon.⁶⁸²

Before ending this analysis of Ibn al-Zarqālluh's equatorium, something should be said about its parameters. This seems to be important because the treatise, although a minor work in Ibn al-Zarqālluh's bibliography, corresponds to the late stage of his scientific production (ca. 1080–1082), and it seems to contain new Zarqāllian planetary materials that may correspond to his observations during a good part of his life.

First of all, the construction treatise gives a double series of longitudes (tropical and sidereal) of planetary apogees. The difference, due to precession, between them should be (but is not always) $7;18^\circ$, which corresponds to Ibn al-Zarqālluh's own estimation. E. Millás has suggested that this difference may be the result of a computation made with the trepidation tables extant in both the *Toledan Tables* and in the *Liber de motu* (see 6.2.1) if we assume that the result is $7;13^\circ$ instead of $7;18^\circ$, given the similarity between the *jūm* (3) and the *ḥā'* (8) in the *abjad* notation system.⁶⁸³ Mercè Comes, however, that better results can be obtained using the second variant of the first model of trepidation, designed by Ibn al-Zarqālluh himself in his treatise on the *Fixed stars* (6.2.2.3).⁶⁸⁴ Besides, the Arabic text on the use of the instrument also records the [tropical?] positions of the apogees, although most of the values seem to be corrupt.

680 Mozaffari, 2014, has analysed a passage of al-Bīrūnī's *al-Qānūn al-Mas'ūdī*, in which the great Eastern astronomer studies the oval-shaped curve described by the centre of the lunar epicycle and proves that it is not an ellipse.

681 Kennedy, 1960 a; Kennedy, 1952.

682 See Hartner, 1968.

683 MillásVendrell, 1983.

684 Comes, 1991, pp. 88–93.

We should also consider the Zarqāllian parameters for the radii of the epicycles and eccentricities.⁶⁸⁵ The radii for Jupiter, Mars and Venus seem to contain small adjustments of Ptolemy’s radii which are difficult to justify:

Planets	<i>Equatorium</i>	<i>Almagest</i>
Jupiter	11;26,11 ^P	11;30 ^P
Mars	38;53,53 ^P	39;30 ^P
Venus	42;2,57 ^P	43;10 ^P

As for the eccentricities:

Planets	<i>Equatorium</i>	<i>Almagest</i>
Saturn	2;51,23 ^P and 2;48,48 ^P	3;25 ^P
Jupiter	2;46,4 ^P	2;45 ^P
Mars	6 ^P	6 ^P
Venus	1;3,26 ^P	1;15 ^P
Mercury	2;51,26 ^P	3 ^P
Moon	10;19 ^P	12;29 ^P

In this table we see that the eccentricities for Jupiter, Mars and the Moon are Ptolemaic. The case of the Moon is the result of a confusion, probably due to Ibn al-Zarqālluh himself: Ptolemy gives an eccentricity of 10;19^P for a radius of 49;41^P and 12;29^P for a radius of 60^P. The case of Mercury was studied by Hartner⁶⁸⁶ who suggested a possible connection between this new Zarqāllian parameter and the corrections introduced by Ptolemy in his *Planetary Hypotheses* in the values of Mercury’s eccentricity (2;30^P) and radius of the epicycle (22;15^P instead of 22;30^P, as in the *Almagest*). As we have seen, Ibn al-Zarqālluh’s eccentricity is 2;51,26^P but, as he does not change the radius of the epicycle, the results obtained are similar to those of Ptolemy’s *Hypotheses*. Finally, Venus’ eccentricity (1;3,26^P) seems to be consistent with the modifications of Ptolemaic

685 All the values of epicycle radii and eccentricities are normed for a radius of the planetary deferent of 60^P. See Comes, 1991, pp. 93–124.

686 Hartner, 1977.

parameters made by Muslim astronomers: Yaḥyā b. Abī Maṣṣūr (ca. 830) gives an eccentricity of 1;1,48^p, Ibn Yūnus 1;3,5^p and al-Bīrūnī 1;2,30^p. We also find similar parameters to those of Ibn al-Zarqālluh in later astronomers like Ibn al-Shāṭir (1;3,30^p) and al-Kāshī (1;3^p) in his treatise on the equatorium which, as we have seen, was probably influenced by Ibn al-Zarqālluh's instrument.⁶⁸⁷

4.5.4 *The Equatorium of Abū l-Ṣalt Umayya of Denia*

I have already referred to Abū l-Ṣalt's treatise on the use of the astrolabe (4.2.3.2) and I should add here that he is also the author of another text on the equatorium which might be the channel through which this instrument was reintroduced into the Mashriq. This treatise is only known through a single manuscript⁶⁸⁸ which, apparently, seems to have been lost. A microfilm of the manuscript is, however, extant and was the basis of the research on this instrument made by E.S. Kennedy⁶⁸⁹ and Mercè Comes.⁶⁹⁰

Abū l-Ṣalt calls the instrument *ṣafīḥa jāmi'a tuqawwimu bi-hā jamī' al-kawākib al-sab'a* (the comprehensive plate which allows you to compute the true longitudes of the seven planets). As suggested by Kennedy, the instrument described would never have been made as the details given in the text are often too vague. In the rare cases in which parameters are mentioned, they are Ptolemaic. Certain characteristics of the instrument show some originality: Abū l-Ṣalt is the first designer to represent, in his instrument, the equant circle and the deferent of each planet. Besides, he uses the centre of the equant and the centre of the plate: two threads leave from the centre of the plate, and one of them is linked to a peg placed in the centre of the equant and determines the position of the centre of the epicycle on the planet's deferent; the second thread replaces the rotating ruler of Ibn al-Samḥ and Ibn al-Zarqālluh's equatoria and, when it is placed on the epicycle in the graduation corresponding to the planet's mean anomaly, it allows us to obtain its true longitude in the graduated crown of the instrument (Fig. 4.48).

In other instances, Abū l-Ṣalt seems to follow Ibn al-Zarqālluh's lead: it is clear that, in his instrument, the curve drawn by the centre of Mercury's epicycle is not a circle, although the extant text does not describe how to trace it. As in Ibn al-Zarqālluh's equatorium a single plate, inscribed on both sides, bears

687 On these parameters see Samsó, 1987.

688 Ms. 223 of Saint Joseph's University in Beirut.

689 Kennedy, 1987.

690 Comes, 1991, pp. 237–251 (edition and Spanish translation of Abū l-Ṣalt's *risāla*) and 139–157 (commentary).

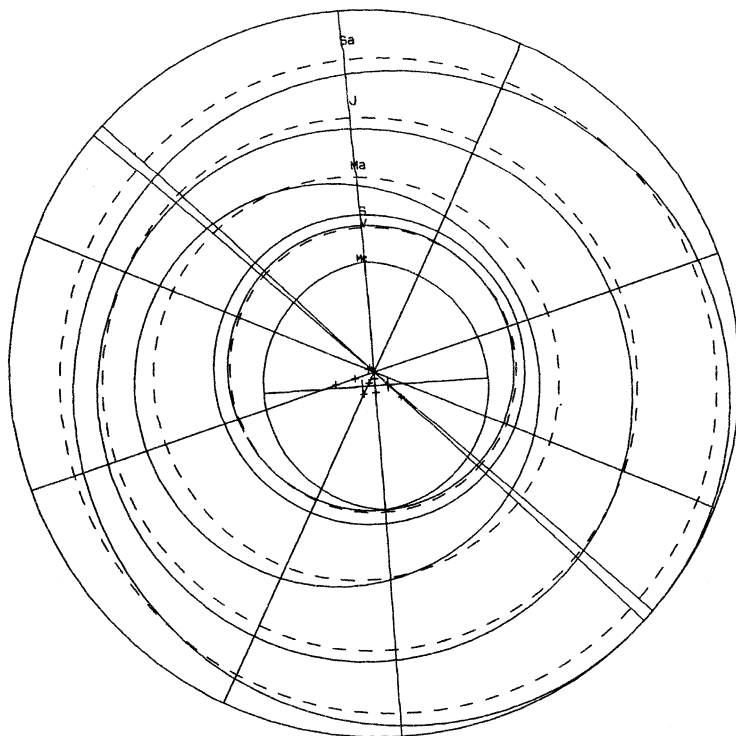


FIGURE 4.47 The Sun and the planets in Abū l-Ṣalt's equatorium: deferents in broken lines, equants in continuous lines

DRAWING BY M. COMES AND H. MIELGO

all the planetary machinery of deferents and equants and, as in both previous cases, the plate of the epicycles will be superimposed on it.

Interestingly (Fig. 4.47) all planets as well as the Sun appear on the same side of the plate (the only exception is the Moon which, due to the complexity of the model, occupies the other side) following the Ptolemaic order and, therefore, the outer circle of Saturn is tangent to the circle representing the ecliptic; then, the outer circle of Jupiter is tangent to an auxiliary circle which is also tangent to Saturn's equant, and so on. This kind of disposition could correspond to a physical structure of the cosmos, like the one adopted in Ptolemy's *Planetary Hypotheses*⁶⁹¹ which makes it possible to compute the distances and sizes of the planets, in spite of the fact that the system of tangencies is not the same. It could also be the result of the influence of Ibn al-Haytham's *Fī l-hay'a*.

691 Goldstein, 1967 a.

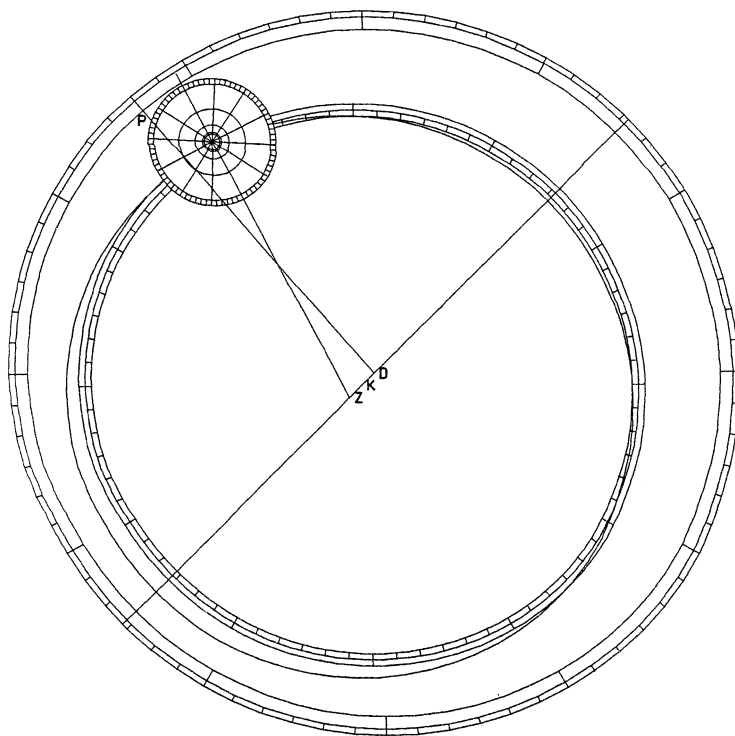


FIGURE 4.48 Use of threads to determine the true longitude of a planet: Z centre of the equant; K centre of the deferent; D centre of the Earth
DRAWING BY M. COMES AND H. MIELGO

Whatever the case, an interpretation of this kind would not be surprising in a scholar like Abū l-Ṣalt who wrote a treatise on six philosophical and cosmological questions which show evidence of these interests.

4.5.5 *Other equatoria*

4.5.5.1 The Instrument Described by al-Marrākushī

The equatorium did not become a popular instrument in the Islamic and Christian kingdoms of the Iberian Peninsula or in the Maghrib, as it did later in Latin Europe. In spite of this, we do have some information on later developments which deserve to be mentioned.

One of these instruments is the equatorium, described in vague terms in Abū l-Ḥasan al-Marrākushī's *al-Mabādī' wa l-ghāyāt*,⁶⁹² in which a plate is used

692 Marrākushī, 1984, pp. 126–131. An analysis of this source was made by Comes, 1998.

for each celestial body, as in Ibn al-Samh’s instrument, which seems to be the main source used by al-Marrākushī. The plate for the Sun is a simple zodiacal scale, calculated for an eccentricity of 2^p and a solar apogee placed at the beginning of Cancer (90°). The plate for the Moon corresponds to Ptolemy’s first lunar model (*Almagest* IV, 6) which has only one inequality. As for the planetary plates, al-Marrākushī describes only the one for Mars, considered to be the model for the other planets. In fact, his description of this plate seems to be a simple reproduction of the corresponding Ptolemaic planetary model and not of an instrument. Here, the graduated equant circle is inscribed on the plate and, therefore, placing the centre of the epicycle in its correct position becomes quite simple. Mercury’s plate, like that of the Moon, corresponds to a simplified model in which the small circle, on which the centre of Mercury’s deferent rotates, is ignored, although al-Marrākushī’s text refers to it. The only interesting detail is his remark, in the solar zodiacal scale, that the plate will only be precise for a limited amount of time, due to the motion of the solar apogee. He suggests building a rotating parecliptic (*al-falak al-mumaththal*) which can be adjusted to the circle of the months in order to locate the apogee in its correct position for a given date. The same idea could be applied to the plates of the planets, as each one of them has a graduated ecliptic.

Abū l-Ḥasan al-Marrākushī also gives some numerical parameters, like the double eccentricity for Saturn, Jupiter, Mars and Venus and the eccentricity for Mercury and the Moon (*):

Planets	al-Marrākushī	<i>Almagest</i>
Saturn	6;2 ^p	6;50 ^p
Jupiter	5;30 ^p	5;30 ^p
Mars	12 ^p	12 ^p
Venus	2;5 ^p	2;30 ^p
Mercury	3;10 ^p	3 ^p
Moon	12;30 ^p	12;29 ^p

It is easy to see that the eccentricities of Jupiter, Mars and the Moon are Ptolemaic, while the others are not, and Mercè Comes suggests that they may derive from al-Bīrūnī. Al-Marrākushī also gives the Ptolemaic values of the epicycle radii for Saturn (6;30^p) and Mercury (22;30^p).

4.5.5.2 Two equatoria Described by Jewish Scholars

Several years ago, Bernard R. Goldstein⁶⁹³ studied two equatoria designed, in Italy, by two Jewish astronomers who migrated from the Iberian Peninsula between the end of the fourteenth and the middle of the fifteenth centuries. One of them was Isaac al-Ḥadib who arrived in Syracuse, in Sicily, where he invented his instrument in 1396.⁶⁹⁴ He knew Ibn al-Zarqālluh's equatorium (as well as other instruments of the same kind made by Christian astronomers) but considered that their use was difficult and confusing due to the presence of multiple circles on a single plate.⁶⁹⁵ Interestingly, although his equatorium is unrelated to the astrolabe, he uses astrolabical terminology in his description of its different parts which are:

- The *mater*, which has a rectangular shape, and serves as a base to which the other parts of the instrument are attached. On it appears a fixed epicycle, the planetary stationary points, the centre of the deferents and a straight line (called “line of length”) which joins the centre of the deferents and the centre of the epicycle.
- The *spider*, the graduated circle of the ecliptic, drawn with an arbitrary radius. There is a hole in its centre in which a peg placed, on the *mater*, in the centre of the deferents, is inserted. The ecliptic, therefore, rotates freely on top of the *mater*. The *spider* also contains holes for the centres of the planetary deferents. Inside the circle of the ecliptic there is a second graduated circle corresponding to Mercury's equant circle, and a set of twelve holes, in a small circle, for the centres of Mercury's deferent: this allows an approximate reproduction of the motion of the centre of Mercury's deferent.
- An alidade rotates around the centre of the epicycle. On it we find a scale which marks the size of the radii of the planetary epicycles. Only one circle is used to represent all the epicycles.
- A rule also rotates around the centre of the ecliptic. When it is aligned with the position of the planet on the epicycle, it will mark on the ecliptic its true planetary longitude.

It is clear, then, that the instrument described in al-Ḥadib's work bears no relation to its Andalusī precedents, and it seems far more difficult to understand and to see its relation to the Ptolemaic models. The parameters used for eccentricities, epicycle radii and planetary stations are Ptolemaic, while the

693 Goldstein, 1987.

694 On this author see Goldstein, 1981, p. 238; Goldstein & Chabás, 2006.

695 Goldstein, 1987, p. 128.

positions of the apogees of Saturn, Jupiter and Mars seem to derive from Ibn al-Raqqām.⁶⁹⁶

Our second author is Yosef ha-Parsi, of whose life we know nothing. His equatorium is extant in two versions which describe similar instruments. The earlier description was made in Seville in 1439 and the mean motion tables mentioned in the text use 1 January 1439 as the radix date: the instrument is called *Keli Paz* (the instrument of gold). The second treatise on the instrument was written in Bologna in 1444 and it receives the name of *Keli Golah* (the instrument of exile) and the tables mentioned use 1 March 1441 or 1445 as epoch. The instrument described in both texts has a *mater* containing three plates, engraved on both sides. We have, then, a general structure similar to that of Ibn al-Samḥ's equatorium. The six sides of the three plates correspond to the Moon and the five planets, while the diagram for the Sun can be engraved on a fourth plate, the back of the instrument or on the *mater* itself. A peg in the *mater* serves for the centre of the deferents, while the centre of the Earth is marked on each planetary plate. The epicycles are represented by a semicircle (in the first version of the instrument) or by a complete circle (in the second version). An alidade rotates around the centre of the epicycles and, as in al-Ḥadib's equatorium, it contains a scale marking the radius of each epicycle. A ruler (in the first version) or a thread (in the second) is attached to the centre of the Earth and, when it is aligned with the position of the planet on its epicycle, it marks its true longitude on the ecliptic.

4.6 Jābir ibn Aflaḥ's Observational Instrument

Jābir ibn Aflaḥ⁶⁹⁷ is a well-known Andalusī mathematician who lived in Seville in the twelfth century and rewrote the *Almagest* using, in his demonstrations, the new trigonometry introduced in al-Andalus by Ibn Mu'ādh al-Jayyānī (d. 1093). His main purpose is to correct the mathematical inconsistencies in Ptolemy's work (5.3). The title of his only extant book is *Iṣlāḥ al-Majisṭī* ("Revision of the *Almagest*") and, in it, he claims that he has invented an instrument⁶⁹⁸ which can replace all the instruments in the *Almagest*, namely

696 Goldstein, "Descriptions" pp. 108–109.

697 A general introduction to Jābir and his work can be found in Lorch, 1975. See also Lorch 1995 a and Lorch, 1995 b.

698 Lorch, 1976. All the information on Jābir's instrument in this section is based on Lorch's paper.

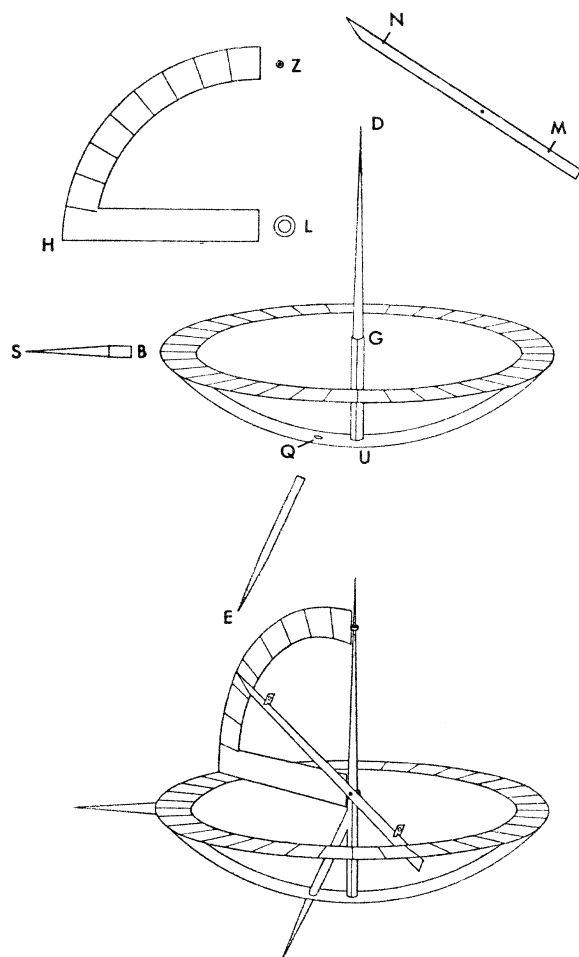


FIGURE 4.49
Jābir's instrument in Richard
Lorch's reconstruction

the *astrolabon* (armillary sphere, *Alm.*v,1),⁶⁹⁹ meridian armilla (*Alm.* I,12),⁷⁰⁰ equinoctial armilla (*Alm.* III,1),⁷⁰¹ and triquetrum or parallactic instrument (*Alm.* v,12).⁷⁰² These four instruments require eight armillas (six for the astrolabon and two for the meridian and equinoctial armillas) while Jābir's instrument is composed of an armilla, a quadrant and an alidade with sights (see fig. 4.49).

699 Toomer, 1984, pp. 217–219.

700 Toomer, 1984, pp. 61–62. Jābir does not mention the mural quadrant described in the same chapter of the *Almagest*.

701 Toomer, 1984, p. 133.

702 Toomer, 1984, pp. 244–247.

Fig. 4.49 corresponds to Richard Lorch's reconstruction of the instrument, as described in the Arabic original text of the *Iṣlāḥ*. As we can easily see it is composed by a rotating armilla, a graduated quadrant which also rotates on an axle fixed to the centre of the armilla and an alidade with sights which moves around the centre of the quadrant. The instrument is quite large, as well as heavy (being made of brass) with a thickness that does not allow the instrument to bend, and an armilla with a diameter measuring six hands-breath. A pointed rod is placed on the plane of the armilla, so that it can be fixed to a horizontal marble slab on which the meridian line has been traced: the armilla will, then, be adjusted to the plane of the meridian and it will be used in the same way as Ptolemy's meridian armilla to observe the Sun at noon throughout the year and, therefore, to determine the obliquity of the ecliptic. This disposition also indicates the Moon's zenithal distance as well as the limit of the Moon's latitude which Ptolemy obtained with the triquetrum.

After obtaining the obliquity of the ecliptic, a second rod is joined to the axle in the centre of the armilla so that it forms with it an angle equivalent to the obliquity. This makes it possible to mount the instrument so that the armilla is located in the plane of the ecliptic and we can determine the longitude and latitude of a celestial body, as Ptolemy does with the *astrolabon*. Finally, the armilla can also be mounted in the plane of the equator and be used to observe the equinoxes, for which Ptolemy employs the equinoctial armilla.

Jābir's *Iṣlāḥ* was translated into Latin by Gerard of Cremona and, later (1274), into Hebrew by Moses ibn Tibbon as well as by Jacob ben Maḥir. In 1335, the latter's Hebrew translation was revised by Samuel ben Judah of Marseilles. While the Arabic manuscripts containing Jābir's book and Ibn Tibbon's Hebrew translation describe the instrument as represented in fig. 4.49, Gerard's Latin translation and Samuel's Hebrew revision introduce a major modification that appears in fig. 4.50, in which we see that the rotating quadrant has been replaced by hinged rods on the principle of the triquetrum.⁷⁰³ It seems difficult to determine whether this second variant is an invention of Jābir's or a later addition.

Jābir's instrument is interesting because of the limited number of references to instruments designed for observation in the Andalusī and Maghribī traditions, in which I have only found Ibn al-Zarqālluh and Alfonso x's armillary sphere and Ibn 'Azzūz's use of this instrument for observations made in Fez in 1344–45 (4.1.3). To this one could probably add the Alfonsine *cuadrante sennero* (4.4), although no description of the instrument seems to be extant. In spite of this, one must have serious doubts about whether Jābir's instrument was ever

⁷⁰³ As remarked by Lorch, 1976, pp. 31–32, Samuel describes both variants of the instrument.

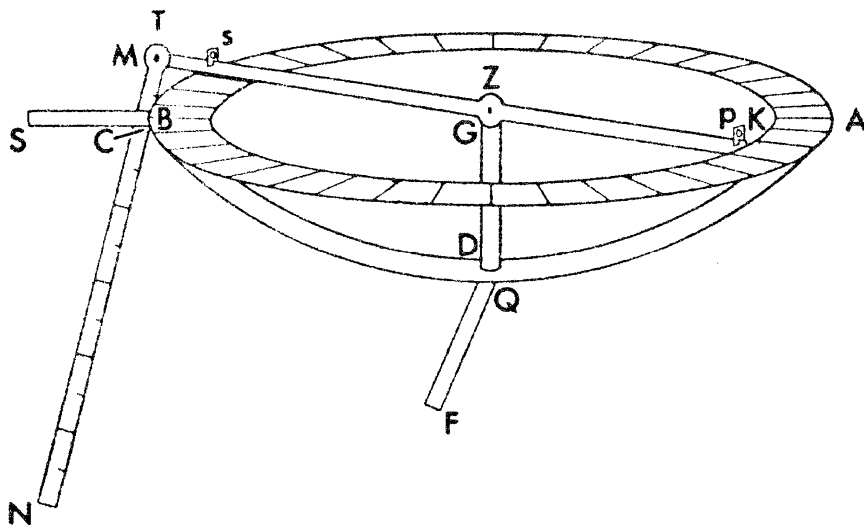


FIGURE 4.50 Jābir's instrument in the Latin translation, according to Richard Lorch's reconstruction

made and used.⁷⁰⁴ My experience with Jābir's *Iṣlāḥ*⁷⁰⁵ makes me think that he was only interested in Ptolemy's *Almagest* from a mathematical point of view, and there is no evidence that he ever made an astronomical observation.

It has also been said that there is a certain similarity between this instrument and the European *torquetum* described, for the first time, towards the end of the thirteenth century by Bernard of Verdun or by Franco of Poland.⁷⁰⁶ The latter instrument combines, in a single tool, the three basic astronomical planes – horizontal, equatorial and ecliptic – to which a vertical plane has been added, and it also has rules, alidades with sights and a plumb-line. In this regard it bears a certain similarity to Jābir's instrument. There are, however, several differences between them, of which I will emphasise just one: the Andalusī instrument has a fixed mounting, unlike the *torquetum*, and cannot be adapted easily to a change of latitude. If the *torquetum* is a successor of Jābir's instrument, its conception implies a radical change; whereas the Andalusī instrument has been designed for observation and is medium-sized, relatively heavy and fixed, the *torquetum* is only theoretically used for observation and its basic applications are related to demonstrations as well as computations, and its uses are sometimes similar to those of the astrolabe.

⁷⁰⁴ Delambre, 1819, p. 183, already remarked "Cet instrument, dont ne parle aucun auteur, pourrait fort bien n'avoir jamais été exécuté".

⁷⁰⁵ Samsó, 2001 b.

⁷⁰⁶ Poulle, 1964.

Hay'a (Cosmology)

5.1 Introduction: on the Meaning of Hay'a in Western Islam

Many years ago¹ I was surprised to realise that there were references to *hay'a* in Šā'id's *Ṭabaqāt al-umam*, when the author talks of astronomers of the second half of the tenth and the eleventh centuries: he states that Maslama al-Majrīṭī knew, much better than all his predecessors, the “science of spheres and of the motion of stars” (*‘ilm al-aflāk wa ḥarakāt al-nujūm*). In this expression we might understand the existence of a certain difference between an *‘ilm al-aflāk*, perhaps in the line of the *hay'a* tradition, and a *ḥarakāt al-nujūm*, concerned with *zīj* literature and mathematical astronomy in general. The terminology used by Šā'id becomes clearer when he deals with the members of Maslama's school: Ibn al-Samḥ (d. 1035), Ibn Barghūth (d. 1056), al-Wāsiṭī (d. 1045–46?), Ibn al-Jallāb (still alive ca. 1069?), and Abū Ishāq ibn Lubb, named al-Quwaydis (d. 1062). To all these authors, Šā'id attributes a thorough knowledge of *‘ilm hay'at al-aflāk* and, again, of *ḥarakāt al-nujūm*:² there seems to be no doubt that we are dealing with a “science of the physical structure of celestial spheres”. This kind of remark appears again when Šā'id mentions later authors: he ascribes, for example, an interest in *‘ilm hay'at al-aflāk* to the mathematician 'Abd al-Raḥmān ibn Sayyid (fl. 1063–1096), as well as to the famous astronomer Abū Ishāq Ibrāhīm ibn Yahyā al-Naqqāsh, known as Ibn al-Zarqiyāl or Ibn al-Zarqālluh (d. 1100).³ The reference to Ibn Sayyid is interesting, for he taught Mathematics to Ibn Bājja, one of the leaders of the group of Aristotelian philosophers who criticised Ptolemy in the following century. However, Šā'id's mention of Ibn al-Zarqālluh's interest in this topic makes me doubt my own interpretation, for there is absolutely nothing in the whole of Ibn al-Zarqālluh's extant works which might suggest this kind of approach.

It is, therefore, appropriate to investigate the meaning of *hay'a* in an Andalusī-Maghribī context, continuing Ragep's work concerning the Islamic East.⁴ One of the sources that Ragep quotes is the volume on *Astronomia* of

¹ Samsó, 1994 c.

² Bū 'Alwān, 1985; Blachère, 1935, pp. 129–136.

³ Bū 'Alwān, 1985, pp. 180–181; Blachère, 1935, pp. 138–139.

⁴ Ragep, 1993, vol. 1, pp. 29–41. See also Ragep, Sally, 2016, pp. 27–32; Forcada, 2017.

the Epistles of the Brethren of Purity,⁵ a source that is significant for us, for this collection of texts was first introduced in al-Andalus towards the middle of the tenth century by Maslama b. Qāsim al-Qurṭubī (d. 964), the author of the *Ghāyat al-ḥakīm* and of the *Rutbat al-ḥakīm*.⁶ In epistle 3 we read “the science of the stars is divided into three types. One type is knowledge of the arrangement of the orbs, the quantity of the stars, the divisions of the zodiacal signs, their distances, their motions, and related things from this discipline. This division is called the science of configuration (*hay'a*). Another division is knowledge of understanding *zīj*es, making almanacs (*taqwīm*), deriving calendars, and similar things. [Another] division is knowledge of the manner of indication – from the rotation of the celestial sphere, the ascendants of the zodiacal signs, and the motions of the stars – on existents by their being under the orb of the Moon. This kind is called the science of judgements”. This expression is complemented by a description of the “orbs” (*aflāk*) which are “spherical, transparent, and hollowed-out bodies: there are nine orbs that are set one inside the other like the rings of an onion”. So in these passages we have a clear reference to understanding *ʿilm al-hay'a* as a science dealing with the description of a physical cosmos, with solid spheres and spherical celestial bodies.

This is not, however, the only meaning of *hay'a*, for Ibn Sīnā (980–1037) considers *ʿilm al-hay'a* to be the general term for the discipline of astronomy, from which astrology has been excluded. This means that *hay'a* includes, for example, computing *zīj*es, designing astronomical instruments or finding the direction of Mecca.⁷ A similar attitude is adopted in the Maghrib by Ibn Khaldūn (1332–82) who, in his *Muqaddima*, divides the astronomical discipline into two: *ʿilm al-hay'a* and *ḥisāb al-azyāj* (*zīj* computation),⁸ while he rejects the validity of astrology (*ṣināʿat al-nujūm*).⁹ *Hay'a* studies the motions of stars and planets, the modalities of these motions, the [geometrical] figures (*ashkāl*) they describe, and the placement of spheres which produce the sensible movement of celestial bodies. The methods used for these descriptions are based on geometry (*bi-ṭuruq handasiyya*). He mentions that *hay'a* proves that the centre of the Earth is displaced from the centre of the solar sphere and that there is a motion of accession and recession (*al-iqbāl wa l-idbār*), and he gives indications about the existence of direct and retrograde planetary motions due to the existence of small spheres (epicycles) which carry the planets and move

5 Ragep & Mimura, 2015, pp. 25–26 of the translation and 6–9 of the edition.

6 Callataÿ, 2013.

7 Ragep, 1993, p. 34.

8 Quatremère, 1858, III, pp. 105–108.

9 Quatremère, 1858, III, pp. 220–228.

inside their great sphere (*dākhil falaki-hā a'zam*). Ibn Khaldūn also proves the existence of an eighth sphere producing the motion of fixed stars, while each planet has multiple spheres. This discipline is based on observations.

As for the science of *zīj*es, Ibn Khaldūn remarks that it is a computational discipline (*ṣinā'a ḥisābiyya*) which follows numerical rules (*qawānīn 'adadiyya*) that are specific for each planet and derive from the laws to be found in the books on *hay'a*. The purpose of this discipline is to determine planetary positions on which astrological judgements are based.

Overall, in spite of the fact that Ibn Khaldūn seems to have in mind physical spheres carrying stars and planets, he insists on the identification of *hay'a* with mathematical astronomy. This makes it difficult for us to establish accurately the meaning of the word *hay'a* when we find it in an astronomical text. In the following pages I will consider *hay'a* as cosmology, implying a qualitative description of the Universe, based on simplified Ptolemaic geometrical models, with very limited references to numerical parameters and without any demonstrations. This is sometimes combined with ideas derived from Ptolemy's *Planetary Hypotheses*, such as geocentric distances and the sizes of planets and stars. Such is the case of the books written by two tenth-century authors, the Tunisian Dūnash ibn Tamīm (see 4.1.3.2) and the Cordoban Qāsim ibn Muṭarrif, mentioned frequently in chapter 2, and the book by an anonymous Toledan astronomer of the eleventh century discovered by Saliba,¹⁰ which seems far more elaborate and complete than those of his tenth-century predecessors. This astronomer also seems to be the author of another lost work, entitled *Kitāb al-Istidrāk* [*'alā Baṭlamyūs?*], which contained criticisms of the *Almagest*. This tendency will, later, be reinforced by the circulation of Ibn al-Haytham's *Kitāb fī hay'at al-'ālam* which was translated by the Alfonsine circle. *Hay'a* understood as simple cosmology might correspond to the contents of Ibn Abī l-Ṣalt's (ca. 1067–1134) *Al-wajīz fī 'ilm al-hay'a*, a book which has been preserved but has never been studied. It seems to have been written in Egypt, according to some sources at the request of al-Afḍal, the vizier of the Fatimid caliph al-Musta'li b. al-Mustanṣir (1094–1101); according to other sources, it was dedicated to al-Afḍal while the author was imprisoned in Alexandria (4.2.3.2). Al-Afḍal had the book read by the Aleppo astronomer Abū 'Abd Allāh who considered that it contained nothing new or interesting.¹¹ Whatever the case, Abū l-Ṣalt's interest in *hay'a* is easy to understand since he was a scholar who was deeply concerned with philosophy and wrote the answers to six questions posed by al-Afḍal on problems related to philosophy and physics: for example,

¹⁰ Saliba, 1999.

¹¹ Comes, "Ibn Abī l-Ṣalt", in *BA*, vol. 1, 722–728; Lamrabet, 2012, M47, p. 134.

whether the supra-lunar universe, in which the motions are circular, is affected by generation and corruption, and the determination of the distances and sizes of the Sun and the Moon and their ratio in respect to the Earth.¹²

In other cases *hay'a* is more ambitious and it contains an attempt to describe a non-Ptolemaic physical universe as if it truly existed. This was the attitude of the Aristotelian philosophers of the twelfth century (Ibn Bājja, Ibn Rushd, etc.) and, especially, of al-Bīṭrījī, the only one to describe an (unsuccessful) non-Ptolemaic universe. Much later, Naṣīr al-Dīn al-Ṭūsī's *al-Tadhkira fī 'ilm al-hay'a* and al-Jaghminī's *Mulakhkhaṣ fī l-hay'a* seem to have circulated in the Maghrib, where they exerted a limited influence.¹³

There is, finally, a significant case in which *hay'a* is clearly identified with mathematical astronomy: that of Jābir ibn Aflah's *Iṣlāḥ al-Majisṭī*. In it, *'ilm al-hay'a* is defined as "the study of the motions of the Sun, Moon and the stars (*nujūm*) and the knowledge of their spheres (*aflāk*)", its main source being Ptolemy's *Almagest*, which is "sufficient in order to master this science since it gathers together all of its contents".¹⁴ In spite of its reference to "spheres", the book does not mention physical astronomy of any kind. Jābir's criticisms of Ptolemy's *Almagest* are purely mathematical.

5.2 The Treatises on *hay'a* by Dūnash ibn Tamīm, Qāsim ibn Muṭarrif and an Anonymous Eleventh-Century Toledan Astronomer

5.2.1 *Dūnash ibn Tamīm*

In 4.1.3.2 I already dealt with Dūnash ibn Tamīm's (fl. 955) treatise on the use of the armillary sphere. I summarised the few known details about his life, and mentioned the identification made by Taro Mimura of the Latin translation entitled *De orbe* (ca. 1130), traditionally attributed to Māshā'allāh (d. ca. 815), with an Arabic book on *hay'a* by Dūnash himself.¹⁵ This book is preserved in two manuscripts (Berlin Staatsbibliothek Ms. or. oct. 273 and Philadelphia,

12 Millàs, 1931, pp. 80–81.

13 Lamrabet, 2012: Dāwūd b. Muḥammad al-Tāmīlī (d. ca. 1494) in M240 (p. 214); 'Alī ibn Māmī Karbāša (alife in 1663) in M333 (p. 233). Other instances in which the word *hay'a* appears in a title are M274 (p. 223), M295 (p. 226), M333 (p. 233), M344 (pp. 235–6), and M560 (pp. 270–1).

14 Bellver, 2008 p. 183.

15 Mimura, 2015; Mimura, 2015 a. On the coincidences between the cosmological passages of the book on the armillary sphere and the *hay'a* treatise, see Mimura, 2015, pp. 27–29. See also Sela, 2012.

Pennsylvania University Library Ms. LJS 439).¹⁶ The Philadelphia ms. has no title, while the title appearing in the Berlin ms. (*Ikhtiṣār al-Majisī*) should, obviously, be rejected.

The book is a treatise on cosmology (*hay'a*) and we have already seen (4.1.3.2) that Dūnash wrote two treatises on *hay'a*, one of them for his friend the Cordoban Jewish physician Ḥasdāy b. Shaprūt (ca. 905–975) and another for the Fatimid Caliph al-Manṣūr Ismā'īl b. al-Qā'im (946–953). The contents of the first one are described in his commentary on the *Sefer Yezirah*:

We have already explained this [i.e., lunar phases] and have put figures about it in our book which we composed and sent to Abū Yūsuf Ḥasdāy to reply the questions which reached us from the city of Constantinople. It consists of three parts: the first part is on the science of the configuration of the orbs; the second part is on the knowledge of the orb according to calculation; and the third is on the judgement of the stars.¹⁷

There is no clear evidence allowing us to decide whether the book on *hay'a* we are analysing here is the text dedicated to Ḥasdāy or to Caliph al-Manṣūr, except for a few small details: one of them is that, in a passage in which he criticises astrology, he promises to explain astrological notions “in a more specific place” (*fī mawḍi'ī-hi al-akhaṣṣ bi-hi*).¹⁸ Mimura interprets this as “in a later section”, an astrological section which does not appear in the extant text of the *hay'a* book. If this interpretation is correct, Dūnash ibn Tamīm's *hay'a* treatise could be the first part of the book dedicated to Ḥasdāy b. Shaprūt. This hypothesis is not certain, as the treatise dedicated to al-Manṣūr is described in similar terms.¹⁹ We should bear in mind, however, that our text bears witness to Dūnash's obvious contacts with al-Andalus, as it refers to a solar eclipse which was observed in al-Andalus two hours after sunrise of a Friday in the month of Ramaḍān of year 327 and which was visible in Baghdad after midday,²⁰ thus implying an acceptable time difference of about four hours between Córdoba and Baghdad. This refers obviously to the solar eclipse of Thursday 27 Ramaḍān 327/18 July 939 in which the line of totality crossed the middle of the Iberian Peninsula and an area to the north of Iraq. This eclipse was obviously observed

16 Both manuscripts can be downloaded from <http://hdl.library.upenn.edu/1017/d/medren/4830256> and <https://ismi.mpiwg-berlin.mpg.de/om4-ismi/public/publicCodex.jsp?eid=181096>.

17 Mimura, 2015 a, p. 93.

18 Mimura, 2015, p. 341.

19 Mimura, 2015 a, p. 93.

20 Mimura, 2015, pp. 344–45.

in and outside Córdoba as we find two references to it in vol. 5 of Ibn Ḥayyān's *Muqtabis*.²¹ In both cases the time of the eclipse is defined as "early morning" or "full morning" (*daḥwa* and *duḥā*).

Hay'a in this book is understood in a very broad sense, as it deals not only with the description of the supralunar world, but also with phenomena related to the spheres below the Moon. It contains descriptions of many meteorological phenomena (oceans, mountains, water, earthquakes, volcanoes in Rome and Sicily, tides and their relation to the Moon, Nile floods, winds, clouds, rain, thunder, lightning, and so on). Celestial spheres move circularly, while terrestrial motions are rectilinear, either centrifugal or centripetal. The spherical Earth is surrounded by the spheres of the four elements and Dūnash states that celestial spheres consist of an element that differs from the four sublunar ones.

Our author describes the structure of the supralunar world and states, as in his treatise on the armillary sphere (4.1.3.2.2.1), that the components of the celestial orb are three circles (equator, meridian and horizon), three chords (*awtār*) (diameter EW, the diameter which joins the upper midheaven, the centre of the Earth and the lower midheaven, and the diameter joining the N and S points with the centre of the Earth) and seven points (*nuqaṭ*) (the centre of the Earth, points N, S, E and W, and the upper and lower midheaven).

Ancient astrologers (*aṣḥāb al-tilasmāt*)²² claimed that the cosmos was composed of nine spheres, but Ptolemy (according to Dūnash) showed that it contained ten concentric spheres: the supreme sphere (*al-falak al-a'zam*), the sphere of the signs (*falak al-burūj*) which is starless, the sphere of the fixed stars, and the seven planetary spheres. The supreme sphere and the sphere of the signs rotate from E to W, while the other eight spheres rotate from W to E. On this point, Dūnash adds that, according to Ptolemy, the motion of planets and stars from W to E is the same, with the same nature (*ṭab'*) and the same structure (*tarkīb*), although, apparently, Saturn is slower than the Moon. This idea is followed later by al-Biṭrūjī.

The reference to a motion of the fixed stars implies that the author was aware of precessional motion. In fact we find an allusion to this motion in a passage in which he quotes the *Qānūn* and says that, to obtain the position of a star, one should consider the longitude of *Qalb al-Asad* (Regulus, α Leonis):²³ the *Qānūn* is, obviously, the *Handy Tables* which use Regulus as a marker of precession, due to its position (with a very small latitude) practically

21 Chalmeta, Corriente & Şubh, 1979, pp. 434 (where the date corresponds to 20 July) and 447 (date 18 July). Spanish translation by Viguera & Corriente, 1991, pp. 324 and 336.

22 Philadelphia ms. p. 3.

23 Ms. Berlin fol. 51r: *wa-lā yutamm al-ta'dil li-kawkab fī l-Qānūn illā bi-ta'dil Qalb al-Asad*.

on the ecliptic, and add a column with the position of Regulus to the mean motion tables of each planet.²⁴ This is no doubt an indirect quotation from the Ptolemaic source. As Mimura has shown, Dūnash's quotations of Ptolemy cannot correspond to the Greek author, of whose works he explicitly mentions a *Kitāb tarkīb al-aflāk*, which can be identified with the *Planetary Hypotheses*. As a matter of fact Dūnash does not deal with the distances and sizes of the celestial bodies but states that the Sun is $166 \frac{3}{8}$ times larger than the Earth, a number which coincides with the value given by Qāsim b. Muṭarrif (5.2.2), but differs from the $166 \frac{1}{3}$ which we find in the *Hypotheses*.

As for the planetary spheres, Dūnash gives very rudimentary descriptions of the two spheres of the Sun (the supreme sphere and the eccentric sphere) and ascribes four spheres (supreme sphere, parecliptic, eccentric sphere and epicyclic sphere) to the Moon and Saturn, as well as to the rest of the planets. There is no mention of the equant point or of the specific Ptolemaic models for the Moon and Mercury. The combination of the motion of the planet on its epicyclic sphere, and that of the centre of the epicyclic sphere on the eccentric sphere – both of them from W to E – leads to the phenomena of planetary retrogradation, stations and direct motions, but this does not affect the Moon due to the velocity of the motion of the eccentric sphere. All this has been analysed by Mimura, who suggests that Dūnash “used an indirect book on the Ptolemaic astronomy containing popular non-Ptolemaic arguments”.²⁵

The book also deals with solar and lunar eclipses and with topics of astronomical geography, such as the seven climates and the length of a degree of the terrestrial meridian which he estimates to be 66 and $\frac{2}{3}$ miles, corresponding to the meridian of 24,000 miles, ascribed, in his treatise on the armillary sphere, to the Ma'mūnī determination.²⁶ He also gives a value of the terrestrial diameter equivalent to 7236.5 miles which implies that the computer is using the standard value of $\pi = 3 \frac{1}{7}$.

5.2.2 Qāsim ibn Muṭarrif al-Qaṭṭān

Qāsim ibn Muṭarrif, whose book I quoted several times in chapter 2, is an author known as a *muqri'* (Qur'ānic reader). We do not know the dates of his birth or death but Ibn al-Faraḍī²⁷ states that he was from Córdoba and gives a list of six of his masters. The presence of Abū Ḥafṣ 'Umar ibn Ḥafṣ ibn Abī

²⁴ Neugebauer, 1975, II, p. 986.

²⁵ Mimura, 2015 a, p. 112.

²⁶ In fact this value derives from the 180,000 stadia used by Ptolemy in the *Hypotheses* and in the *Geography* with a conversion factor of 1 mile = 7.5 stadia.

²⁷ Codera, 1891–2, no. 1072; ed. Cairo, 1966, no. 1074.

Tammām (d. 928), among these masters implies that Qāsim must have been born before 918. He must have had a fairly long life, for he quotes Maslama al-Majrīṭī (d. 1007). On the whole I have the impression that his *Kitāb al-hay'a* must have been written during the third quarter of the tenth century. His *hay'a* book is extant in only one manuscript, Istanbul Carullah 1279,²⁸ and it has been studied by Josep Casulleras,²⁹ who is also the author of the unpublished edition and Castilian translation of the text I am using here.

The book was written in respond to the request of an unnamed person who asked him to write a text on *hay'a* explained in a simple way that could be understood by a beginner. In spite of this, the contents of the book relating specifically to *hay'a* are scarce,³⁰ and this seems to be another case in which the precise meaning of this technical term in al-Andalus seems doubtful. An important chapter in this regard is chapter 10 (fol. 316v) in which Qāsim possibly follows ideas derived from Dūnash ibn Tamīm's book: motions in the sublunar world are either centrifugal (light bodies) or centripetal (heavy bodies), while the celestial sphere is characterised by circular motions. The world is composed of ten spheres of which the tenth is the "sphere of the intellect" (*ḥalāk al-'aql*), the ninth is the "sphere of the signs" (*ḥalāk al-burj*) and the eighth is the "sphere of the fixed stars" (*ḥalāk al-kawākib al-thābita*) which is the "sphere of the soul" (*ḥalāk al-naḥs*), followed by the seven planetary spheres – which are considered to be "an element of nature" (*unṣur al-ṭabī'a*) – and the spheres of the four elements. Like Dūnash, Qāsim also mentions "the three circles", seven points and three axes (*awṭād*) (fols. 320 r and v).

Hay'a contents are also found in chapter 30 (fols. 320v–321r), at the end of the book, in which we find a list of distances, sizes and radii of the celestial bodies, along the lines of Ptolemy's *Planetary Hypotheses*, although the source actually used by Qāsim seems to be Ibn Rustah's (early tenth-century) *Kitāb al-a'lāq al-naḥisa*. This is something that we do not find in Dūnash's treatise although, interestingly, as we have already seen, Qāsim (like Dūnash) gives a size of the Sun 166 $\frac{3}{8}$ times the size of the Earth. Both sources agree on the values given for the Earth's meridian (24,000 miles) and one degree of the meridian (66 $\frac{2}{3}$ miles) although Qāsim's book contains a mistake in the value of the diameter of the Earth: 7,636 miles instead of 7,236.5. This incorrect value is confirmed by the geocentric distances of the celestial bodies which are given in terrestrial radii.

28 A microfilm of this manuscript reached Barcelona thanks to Prof. Fuat Sezgin, who was kind enough to lend it to me.

29 Casulleras, 1998.

30 Casulleras, 1998, pp. 340–342.

The rest of the book is basically astronomical and Qāsim seems to be interested in questions related to timekeeping. We have already seen his values for the meridian shadows projected by a gnomon of 12 digits for the latitude of Córdoba on fols. 318v–319r (2.4.1.2.1), his general formula for calculating the hour as a function of the instantaneous shadow and the meridian shadow (2.4.1.2.2), his description of a sundial called *balāṭa* in fol. 319r (2.4.2.1.3.3) and of a candle clock (*thurayyā*) on fols. 319r–320r (2.4.2.1.5), which could derive from a Tunisian source, and his use of the lunar mansions to calculate the hour during the night in fol. 320r.

Stars are also among the topics dealt with in Qāsim's book. Chapter 12 (fols. 317r and v) contains a table of 16 stars calculated for year 300/912–3, and gives for each one of them its longitude, latitude, day arc and degree of the ecliptic which rises together with the star.³¹ Mercè Comes' analysis of the table shows that the text preserved in the extant manuscript is in a very poor shape, with a great many errors. Comes hints at the possibility of the use of an astrolabe, with a plate traced for the latitude of Córdoba, for the computation of the numerical values. Besides this, al-Battānī's and Maslama's star tables may have been used as sources. An example of this is the erroneous identification of the star *al-Kaff al-khaḍīb* with γ Peg/ α And, instead of with β Cas, a mistake that will reappear in Maslama, Ibn al-Zarqālluh, Ibn al-Kammād, Abū l-Ḥasan 'Alī al-Marrākushī and Ibn al-Bannā'. In chapters 3–5³² and 6³³ the book describes the lunar mansions, the corresponding zodiacal signs and the figure formed by the stars corresponding to each mansion, accompanied by illustrations of the kind we often find in *anwā'* books.

Qāsim follows Ptolemy in considering that precession advances at a rate of 1° per one hundred years. There is, however, an interesting passage showing that Maslama might have had some information about observations made in Baghdad and Damascus under the patronage of al-Ma'mūn (813–833):

Maslama ibn Aḥmad said: the Sun and the planets were observed in year 1150 of Alexander which corresponds, approximately to year 220 of the *hijra*. We discovered that they had advanced 5° in respect to Ptolemy's method and what he transmitted of his observations.³⁴

³¹ Comes, 1994.

³² Ms. Istanbul Carullah 1279 fol. 315v.

³³ Ms. Istanbul Carullah 1279 fol. 316r.

³⁴ Ms. Istanbul Carullah 1279 fol. 316v.

Qāsim does not seem to believe these results and ascribes the differences to mistakes made when copying numerical values represented by the *abjad* Arabic letters, errors that are frequent when copying a *zīj*.

There seems to be a mistake in the above text. 1150 of Alexander corresponds to 223–4 H/838–9. Assuming that the epoch of Ptolemy's star catalogue is 137 AD, the interval between the two dates is about 700 years, in which the value of precession, according to Ptolemy, is 7°. As for the Ma'mūnī value, if we use the parameter of Yahyā ibn Abī Maṣṣūr reproduced in the *zīj* of Ḥabash (Istanbul manuscript), 0;0,0,9,16,40°/ day,³⁵ we obtain 10;59°, which implies an increase of about 4° (not 5°) over the value obtained using Ptolemy's precessional value.

Qāsim's book does not describe solar, lunar or planetary models but it contains some elements that are worth mentioning. One of them is the terminology used. Saturn (Zuḥal, Muqātil, Kaywān), Jupiter (Mushtarī, Hurmuz, Birjīs) and Mars (Mirriḥ, al-Aḥmar, Bahrām) are given the standard Arabic name, the Maghribī and the Persian names, while Venus is called al-Zuhara and Nāḥid, and Mercury 'Uṭarid and al-Kātib.³⁶ Something similar happens with the zodiacal signs. Two of them receive a double nomenclature according to the Eastern and Maghribī terminology: Aries is al-Ḥamal and al-Kabsh, Gemini is al-Jawzā' and al-Taw'amān.³⁷ We also find an interesting list of corrupt *rūmī* (Greek?) names for the signs: Afrūs (Aries), Ṭaurus, Dirm.n.s (Gemini), T.rm.s (Cancer), Liyūn (Leo), B.r.bās (Virgo), Rūyb.sh (Libra), Squrbūs (Scorpio), Yaqsūtūtīs (Sagittarius), 'Ajīr.s (Capricorn), Abūj.s (Aquarius) and al-F.ḥ.s (Pisces).³⁸

The text gives the approximate periods of revolution of the planets on the ecliptic and states that the maximum elongation of Venus with respect to the Sun is 48°, while that of Mercury is 26°. ³⁹ It also describes the retrogradation, direct motion and stations of the five planets,⁴⁰ although it says that these should be looked for in another book (*dhālika ltamis-hu min ghayr hādhā l-kitāb*). As for the Sun and the Moon, it describes their motion along the

35 Debarnot, 1987, p. 57.

36 Ms. Istanbul Carullah 1279 fols. 316 r and v.

37 Ms. Istanbul Carullah 1279 fol. 315r.

38 Ms. Istanbul Carullah 1279 fol. 317r.

39 Ms. Istanbul Carullah 1279 fol. 316v. The *Almagest* XII, 10 (Toomer, 1984, p. 596) gives 46;47° (morning star) and 47;35° (evening star) for Venus and 28;37° (morning) and 27;37° (evening) for Mercury.

40 Ms. Istanbul Carullah 1279 fols. 316v–317r.

ecliptic signs and the lunar mansions and gives approximate rules to calculate the solar and lunar longitudes.⁴¹ Solar and lunar eclipses are also described.⁴²

The book also deals with chronological questions, often found in *mīqāt* treatises: the day of the week corresponding to the beginning of each lunar month using a “mark” (*‘alāma*) for each month: 1, 3, 4, 6, 7, 2, 3, 5, 6, 1, 2, 4;⁴³ the same applied to the solar months, given in their Syriac and Latin names, for which the “marks” are 1, 4, 4, 7, 2, 5, 7, 3, 6, 8, 4, 6.⁴⁴ These “marks” are more or less standard in *zīj*es, as well as in the *anwā’* and *mīqāt* literature: the same or similar lists appear in al-Khwārizmī-Maslama’s *zīj*,⁴⁵ in the *Toledan Tables*,⁴⁶ in Ibn al-Zarqālluh’s *Almanac*,⁴⁷ as well as in the Latin translation of the *Calendar of Córdoba*, where they appear at the beginning of each solar month,⁴⁸ and in the *Kitāb al-anwā’* by al-Kātib al-Andalusī, identified by Miquel Forcada as ‘Arib b. Sa’īd.⁴⁹ This latter source quotes a series of three verses, attributed to al-Fazārī, the famous astronomer of the second half of the eighth century, which determines the number of days of each Syriac month and the corresponding “characteristic” (*āya*).⁵⁰

Finally, Qāsim deals with the day of the lunar month which corresponds to 1 January, using the Metonic cycle of 19 years,⁵¹ and with the bissextile years and the determination of the day of the week which corresponds to 1 January.⁵²

5.2.3 *An Anonymous Toledan (?) Astronomer of the Eleventh Century*

Some years ago, George Saliba⁵³ described manuscript Hyderabad, Osmania University Library 520 RH, which contained an anonymous *Risālat al-hay’a*, a simple elementary introduction to astronomy written by somebody who was far more competent than Dūnash or Qāsim.⁵⁴ For the author, astronomy is a discipline based on observations, on which geometrical models should be

41 Ms. Istanbul Carullah 1279 fol. 318r.

42 Ms. Istanbul Carullah 1279 fol. 317v.

43 Ms. Istanbul Carullah 1279 fol. 318r.

44 Ms. Istanbul Carullah 1279 fols. 318r and v. 8 should be replaced by 1.

45 Suter, 1914, pp. 110, 114.

46 Toomer, 1968, pp. 25–26; Pedersen, 2002, III, pp. 931, 933.

47 Millás, 1943–50, p. 154.

48 Dozy & Pellat, 1961.

49 Ms. Istanbul Malik 2049. See Forcada, 2000.

50 Samsó, 1980, pp. 170–174.

51 Ms. Istanbul Carullah 1279 fol. 318r.

52 Ms. Istanbul Carullah 1279 fol. 318v.

53 Saliba, 1999.

54 I have been able to see photographs of a few pages of the manuscript thanks to the kindness of George Saliba.

built and which “should not deviate from that which has been accepted from the physical sciences” (fol. 16r).⁵⁵ It is divided into four sections, called *fanns*, which deal with the general principles of astronomy (*fann* 1, *Fī mabādi' hādhā l-ilm*, fols. 1–16), the position of the earth with respect to the celestial sphere (*fann* 2, fols. 17–30), the conditions of the planets (*aḥwāl al-kawākib*, *fann* 3, fols. 31–85), the parallax, the apparent diameters of the Sun and the Moon and planetary distances and sizes (*fann* 4, fols. 86–94v). Chapter 14 of the first *fann* deals with “the methods of observation and the instruments adopted for them in a summary fashion” (*fī kayfiyyat al-arṣād wa l-ālāt al-muttakhadha la-hā 'ala tariq al-ikhtiṣār wa l-ijmāl*) and, in it, the causes of inaccurate observations are discussed. One of them is the instability (*takhalkhul*) of the instruments, and the author says that this “has happened in the city of Toledo, in al-Andalus, with the instrument that was set up in it, as I was told by the one who conducted (*al-mutawallī*) observations with it, Abū Ishāq Ibrāhīm ibn Yahyā, known as al-Zarqiyāl” (fol. 15v). He also notes that Ibn al-Zarqālluh found that the obliquity of the ecliptic was 23;33,8°. This led Saliba to identify the author of the *risāla* with a contemporary of Ibn al-Zarqālluh whom he met in Toledo or, maybe later, in Córdoba. In an earlier chapter (4.1.3.3) I suggested that the observational instrument used by Ibn al-Zarqālluh could have been an armillary sphere, on which he wrote a treatise regarding its construction. Besides, Saliba explores a possible identification of the author of the *risāla*,⁵⁶ and some of his candidates are also to be found in the list of tenth- and eleventh-century astronomers to whom Ṣā'id attributes an interest in *hay'a*.

The main interest of Saliba in this paper is not the *Risāla fī l-hay'a* but the quotations made, in this work, from a *Kitāb al-istidrāk*, which our anonymous author is planning to write in the future, and which the Lebanese scholar considers may be an *Istidrāk 'alā Baṭlamyūs*, something like “Correcting Ptolemy”. Saliba mentions three allusions to the *Istidrāk* in the *hay'a* text which give us an idea of the topics the author was planning to deal with in his future book: 1) the contributions to astronomy of Islamic authors and their discoveries which are not to be found in the books of the ancients (fol. 16r); 2) the motion of the solar apogee established by the astronomers of al-Ma'mūn in the ninth century (fol. 41v); and 3), most interestingly, the author's objections to Ptolemy's lunar model (fol. 48r).

We do not know for sure whether the *Istidrāk* was ever written and, in any case, it does not seem to have been preserved. However, the evidence discovered by Saliba points to the possible existence of a certain amount of criticism

55 Saliba, 1999, p. 14.

56 Saliba, 1999, pp. 18–20.

of Ptolemaic astronomy in eleventh-century al-Andalus, due, perhaps, to the influence of Ibn al-Haytham's *Shukūk* which, as we will see in 5.4.2.1, was known to Ibn Bājja (d. 1138).

5.3 Jābir ibn Aflāḥ's Mathematical Criticism of the *Almagest*

5.3.1 *Jābir's Chronology and Scientific Works*

In 4.6 I have already dealt with Abū Muḥammad Jābir ibn Aflāḥ's description of an observational instrument, and something should be added here about his personality and extant works. First of all, we have no biographical information, but his chronology (first half of the twelfth century) is fairly clear due to a series of indirect references:⁵⁷

- Maimonides (1135–1204), in his *Guide for the perplexed*,⁵⁸ says that he was acquainted with Jābir's son.
 - Ibn Rushd (1126–98), in his *Mukhtaṣar al-Majisṭī*, states that Jābir lived in his own century, which began in 500/1106.
 - Ibn Bājja (d. 1138) read Jābir's *Iṣlāḥ*.
 - Joseph ben Yehudah ben Sham'un (d. 1226) brought the *Iṣlāḥ* from Ceuta to Fustāṭ before 1187, where he revised the book with Maimonides.
 - Gerard of Cremona (ca. 1114–1187) translated the *Iṣlāḥ* into Latin before 1175.
- Jābir was, as we will see, basically a mathematician and the following works have been attributed to him:⁵⁹
- A small commentary on Thābit b. Qurra's (ca. 824–901) treatise on Menelaos' theorem, preserved in a Hebrew translation by Qalonymus ben Qalonymus: *Ha-dibbur be temunah ha-ḥittukit* (Commentary on Menelaos' *Figura sectoris*). In it, Jābir presents the 18 modes of the theorem and introduces proofs which are, sometimes, different from Thābit's.⁶⁰
 - A commentary on Menelaos' *Spherics*, also extant in a Hebrew translation by Qalonymus ben Qalonymus.
 - A fragment of a commentary on Euclid.
 - A brief astronomical text on the spherical shape of the Universe, preserved in a Latin translation, has been attributed to Jābir.⁶¹

⁵⁷ The best introductions to the figure of Jābir are Lorch, 1975; Bellver, 2008 Bellver, 2009.

⁵⁸ Pines, 1963, pp. 268–9.

⁵⁹ Bellver, 2009, pp. 87–88.

⁶⁰ Lorch, 2001 a, pp. 387–390.

⁶¹ Millás, 1942, pp. 203–204.

Jābir's most important work, and the one which established his reputation, is his *Iṣlāḥ al-Majisṭi* ("Revision of the *Almagest*"). Richard Lorch⁶² has listed and studied three Arabic manuscripts in Arabic script (Berlin 5653, Escorial 910 and 930) and another two (British Library Or. 10725, Cambridge University Library Add. 1191) in Arabic, but in Hebrew script. To this he adds the manuscripts of Gerard of Cremona's Latin translation and of the two Hebrew translations made by Moses ibn Tibbon in 1274 and by his nephew Jacob b. Maḥir (1236–1304). Jacob's translation was later (1335) revised by Samuel ben Jehudah of Marseille. Quite recently, however, another Arabic manuscript has been found in ms. Teheran, Majlis 1440SS.⁶³

The three Arabic manuscripts studied by Lorch and Bellver (Berlin and the two Escorial manuscripts) present significant differences. One of them is the title of the work: *Iṣlāḥ al-Majisṭi* in the Berlin ms. and, surprisingly, given the contents of the book, *Kitāb al-hay'a* in the Escorial mss.⁶⁴ Besides, the Berlin ms. is mathematically more accurate and corresponds to a recension of the text which seems different from the one represented by the two Escorial mss, which give, according to Lorch, "a more complete but less prolix exposition" and correspond to the revised text.⁶⁵ Lorch suggests that the revision was done by Maimonides and Joseph b. Yehudah although he also accepts the possibility that Jābir himself was the author of the revision.⁶⁶ As we have seen in 4.6, the description of Jābir's observational instrument extant in Gerard of Cremona's Latin translation and in the Hebrew translations is totally different from the one described in the Arabic texts; this poses, once more, the problem of whether the revision was made by Jābir himself or by somebody else. Finally, as is well known, book VI of the *Iṣlāḥ* omits the description of the Milky Way which, in Ptolemy's *Almagest*, is followed by that of a solid globe.⁶⁷ This description appears, however, in Gerard of Cremona's Latin translation of the *Iṣlāḥ*,⁶⁸ and María José Parra⁶⁹ has noted that this description appears, in a kind of appendix, at the end of ms. Escorial 930 (fols. 142v–150), which is a

62 Lorch, 1975, pp. 88–94; Lorch, 1995 b.

63 Private communication from S.M. Muzaffari. This manuscript can be downloaded from http://dlib.ical.ir/faces/search/bibliographic/biblioFullView.jspx?_afPfm=-hviow3mcw.

64 Lorch, 1975, p. 88. Ms. Tehran does not seem to bear a title.

65 Bellver (2009, p. 89) considers that ms. Escorial 910 contains the revised version, while Escorial 930 mixes materials from the two recensions represented by Berlin 5653 and Escorial 910.

66 Lorch, 1995 a, p. 34, fn. 49.

67 Toomer, 1984, pp. 400–7.

68 Lorch, 1975, p. 88; Bellver, 2009, p. 91.

69 Parra, 2016–17.

literal copy of the Arabic translation of the *Almagest* made by Ishāq b. Ḥunayn and Thābit ibn Qurra.

5.3.2 *The Aim of the Iṣlāḥ*

Jābir's purpose in writing the *Iṣlāḥ* is explained in his introduction to the book, edited, translated into Castilian and analysed in detail by Bellver.⁷⁰ He had a high opinion of Ptolemy's *Almagest*, which he considered to be a great book (*kitāb 'aẓīm*) although he believed that a student who read this work could find it difficult to understand, for several reasons:

1) Ptolemy mixed theoretical and practical contents in the *Almagest*. Jābir was only interested in the theory, which is the part of the book that was still valid in his time, when the practical applications had been superseded by the *zīj* literature. He planned to write a book explaining only the astronomical theory of the *Almagest*.

2) He wanted to explain in detail the procedures and proofs which were either presented too briefly in Ptolemy's book or which were mathematically inconsistent or purely approximate.

3) Ptolemy's only trigonometric tool was Menelaos' theorem (*al-shakl al-qatṭā'*), based on compound ratios which established the relation existing between six elements (angles or arcs) of a complete spherical quadrilateral. A new trigonometry existed which used a battery of theorems (rule of four quantities, sine law, cosine law, etc.) in which only four elements of a spherical triangle were related. This is why he dedicated the first book of the *Iṣlāḥ* to the study of plane and spherical trigonometry. This book became extremely influential in Europe due to the Latin and Hebrew translations of the whole work, and it became the main source of Regiomontanus' *De triangulis* and of the first book of Copernicus' *De revolutionibus*.⁷¹ As for the proofs appearing in the rest of the *Iṣlāḥ*, they are re-elaborations of Ptolemy's proofs using the new trigonometry which therefore make them easier to understand.

4) Some of the difficulties of the *Almagest* may be due to the Arabic translations of the Greek original text. This is one of the reasons which led Jābir to rewrite Ptolemy's work in a more understandable Arabic. This purpose is, in my opinion, similar to that of the great tenth-century physician Abū l-Qāsim al-Zahrāwī when he wrote the medical encyclopaedia entitled *Kitāb al-taṣrīf li-man 'ajiza 'an al-ta'ālīf*.

⁷⁰ Bellver, 2009.

⁷¹ See Lorch, 1995 a.

5.3.3 *Jābir's Criticisms of the Almagest*

In the introduction to the *Iṣlāḥ*, Jābir makes some critical remarks on some passages of the *Almagest*. One of them refers to the four observational instruments that Ptolemy used, which could be replaced by only one instrument, as described in 4.6.⁷² Another is concerned with the bisection of the eccentricity (e) of the three superior planets: the fact that the distance between the centre of the equant (E) and the centre of the ecliptic (C) is bisected and at its midpoint (M) we find the centre of the planetary deferent, so that $EM = MC = e$. Ptolemy assumed this to be true but did not give any proof, and designed a procedure for obtaining the parameters of the planetary model (eccentricity and direction of the apse line) using the observations of three oppositions of the planet and the mean Sun because, in the oppositions, the planet will be in the perigee of its epicycle and will be located on the straight line which joins the centre of the Earth and the centre of the epicycle. From this, Ptolemy used an iterative computation in which he began by assuming that the centre of the equant and the centre of the deferent are the same point. The procedure is correct, but extremely time-consuming and it may involve many possible errors. Jābir developed a new (?) method⁷³ which required observations of two pairs of oppositions between the planet and the mean Sun of which one pair was placed on one side of the apse line and the second pair on the other. To this he added that the interval of time between the two members of each pair should be the same. Under these conditions, determining the direction of the apse line was very simple and Jābir explained the mathematical procedure for obtaining the double eccentricity and the distance between the centre of the deferent and the centre of the equant. In this way, he was not forced to assume the bisection of the eccentricity as a kind of postulate, as in Ptolemy's method. In his exposition of his new method, Jābir reveals both his mathematical ability and his lack of experience as an astronomer: obtaining the two pairs of observations in the required conditions is very difficult in a sufficiently short period of time, in which the position of the planetary apogee will not have been appreciably displaced; if the apogee has moved, the method will be invalid.⁷⁴

Besides these two critical remarks, at the end of his introduction Jābir gives a list of 15 errors (*awhām*) that Ptolemy makes in the *Almagest*, to which one should add other minor mistakes found in the text of the *Iṣlāḥ*, although not

⁷² Lorch, 1976.

⁷³ Actually the method had already been described by al-Bīrūnī in his *al-Qānūn al-Mas'ūdī*, although, as shown by Mozaffari (2013), "the mathematical procedures applied by the two authors are substantially different" (Mozaffari, 2013, p. 210). I wonder, as does Mozaffari, whether Jābir had access to the *Qānūn*: see Samsó, 1996.

⁷⁴ Swerdlow, 1987; Hugonnard, 1987.

mentioned in the introduction. Six of the errors in the list are related to the lunar model and to solar and lunar eclipses and have been carefully studied by Bellver:⁷⁵

1. In *Almagest* IV, 2 Ptolemy discusses Hipparchus' method for determining the lunar period of anomaly using two pairs of lunar eclipses which must fulfil a series of conditions. Jābir improves on it by adding two extra conditions related to lunar velocity, which make the method easier and more elegant.⁷⁶

2. In *Almagest* V, 10 Ptolemy studies the extent to which the lunar eccentricity and the equation of anomaly of his third lunar model are significant in true syzygies and, thus, have an influence in the computation of eclipses. He concludes that the use of his simplified first model does not introduce any significant error. Jābir considers that the Ptolemaic method is basically correct, and that it gives acceptable approximations in the results. In spite of this, he draws attention to a mistake appearing in Ptolemy's introduction to the problem which is at odds with the procedure explained immediately afterwards by Ptolemy himself. Jābir suspects that the error might be due to a faulty Arabic translation of the *Almagest* and checks the translations of Ishāq ibn Ḥunayn and al-Ḥajjāj, concluding that sources agree in their contents; he therefore rejects the latter hypothesis.⁷⁷

3. Jābir's third criticism concerns *Almagest* VI.5, in which Ptolemy explains his procedure for calculating eclipse limits, the maximum intervals in the argument of latitude (distance of the mean Moon from one of the nodes) within which an eclipse can take place. The purpose is to avoid the need to compute an eclipse at every luni-solar conjunction or opposition and to restrict this kind of effort to cases in which the mean Moon is placed, on its inclined orbit, within the calculated limits of its argument of latitude. The computation is simple in the case of lunar eclipses, as there is no effect due to lunar parallax, and Jābir accepts Ptolemy's method. However, he discovers a mistake in Ptolemy's method for obtaining the limits of a solar eclipse, in which the lunar parallax plays a role. Consequently, Jābir corrects the whole method and obtains limits which are slightly lower than Ptolemy's.⁷⁸

75 Bellver, 2007. Its main contents have been published in several articles which I will quote in the following footnotes: in all these papers, Bellver edits and translates the corresponding passages of the *Iṣlāḥ*. I summarise here Jābir's criticisms following Bellver, 2008, pp. 184–187.

76 Bellver, 2006.

77 Bellver, 2008–9.

78 Bellver, 2011.

4. In the *Almagest* VI, 7–9, Ptolemy deals with the method for calculating the magnitude and the phases of both lunar and solar eclipses, using plane geometry and certain approximations which are criticised by Jābir. Besides, Ptolemy computes two tables which give the magnitude and phases of a lunar eclipse when the Moon is in its apogee and perigee of its epicycle, and adds an interpolation table for any other value of the lunar anomaly. Jābir describes a slightly different method which avoids the need to use tables, and claims to have discovered an error in Ptolemy's interpolation method. This is based on a passage of the Arabic *Almagest* which he quotes, although his quotation does not coincide with the translations by Ishāq b. Ḥunayn or al-Ḥajjāj, where this mistake does not appear. It is clear, therefore, that there is a section missing in the copy of the Arabic *Almagest* used by Jābir.⁷⁹

5. *Almagest* VI, 7–9 is, again, the object of Jābir's criticisms, now related to the magnitude and phases of a solar eclipse. Jābir mentions three "errors", two of which are the result of his use of an incorrect copy of the Arabic *Almagest*; only the third criticism has some justification. Whatever the case, Jābir demonstrates his ability to restore Ptolemaic methods even though he is using a corrupted text.⁸⁰

Another criticism on the topic of eclipses (not mentioned in the list of errors in the introduction) refers to the inclination of eclipses in the horizon, that is, the angle formed between the horizon and the great circle which joins the centres of the Sun and the Moon (solar eclipse) or of the Moon and of the cone of shadow of the Earth (lunar eclipse). The computation of this angle has no astronomical significance but it seems to have been used for astrometeorological predictions. Ptolemy's treatment of this topic uses plane geometry and gives only a rough approximation in its results. Jābir designs a new method in which he uses spherical trigonometry and obtains exact results, which nonetheless require a considerable effort of computation.⁸¹

6. Once more, referring to *Almagest* VI, 7–9, Jābir mentions another error in the computation of the solar eclipse: Ptolemy adds the lunar parallax in latitude to the lunar longitude on the ecliptic, instead of adding it to its position in its inclined orbit. Jābir is not interested in this topic, which appears only in the list of errors in the introduction; he considers it to be a purely practical question, needed only for the computation of the tables of solar eclipses.

79 Bellver, 2008a.

80 The analysis of these criticisms has not been published by Bellver; they can only be found in his Ph.D. thesis (Bellver, 2007, pp. 424–572).

81 Bellver, 2007a.

The rest of the *awhām* are concerned with planetary theory:

7–8. Both these “errors” are related to the planetary order and they have attracted the attention of many readers of the *Iṣlāḥ*. Ptolemy, in 1X.1⁸² of the *Almagest*, accepts the planetary order of the ancient astronomers, according to whom Mercury and Venus are placed below the Sun, although some later scientists place them above the Sun, due to the lack of observations of transits of both inner planets on top of the solar disk. Ptolemy seems to doubt the certainty of this argument, as he considers that planets placed below the Sun are not necessarily in the plane through the Sun and our point of view. This is rejected by Jābir in error no. 8 and, in the text of the *Iṣlāḥ*, he proves the existence of points of intersection between the epicycles of Venus and Mercury and the ecliptic and, therefore, that, if Mercury and Venus are below the Sun, transits should necessarily occur.

The problem of transits attracted the attention of several Andalusī scholars of the twelfth century: Ibn Rushd (1126–1198), in his summary (*jāmiʿ*) of the *Metaphysics*, remarks that a nephew of Ibn Muʿādh (d. 1093) saw (probably in May 1068)⁸³ two spots on the solar disk, which were interpreted by Ibn Rushd as transits of Mercury and Venus after checking, with a *zīj*, that both planets were in conjunction with the Sun. Similarly, Quṭb al-Dīn al-Shīrāzī (d. 1311) reports that Ibn Bājjā (1070?–1138) also observed two spots on the solar disk at dawn of a given day, from the terrace of his house, and, after the necessary computations, he concluded that they corresponded to Mercury and Venus. In both cases, there were no simultaneous solar transits of Mercury and Venus during the corresponding period of time, and there were no transits of Venus between 1040 and 1053. One should conclude, therefore, that Ibn Muʿādh's nephew and Ibn Bājjā were using a faulty *zīj* or, more probably, that they did not take account of the planetary latitudes.⁸⁴ It therefore seems that in both cases we should conclude that we are dealing with observations of sunspots.⁸⁵

Jābir reinforces his argument on transits by adding a reference to parallax. Ptolemy states that the Sun has a perceptible parallax, while the planets do not. Jābir, therefore, concludes that, if both Venus and Mercury are nearer the

82 Toomer, 1984, pp. 419–420.

83 Using the Toledan Tables the longitudes of the three celestial bodies involved were, the 17th May 1068, Sun (62;11°), Venus (62;59°) and Mercury (62;40°). Similar results are obtained with Battānī's *zīj*: Sun (62;13°), Venus (62;58°) and Mercury (63;41°).

84 Goldstein, 1985 a. Concerning Ibn Bājjā, one should also remember that he confessed, in one of his letters, that he did not fully understand the theory of Ptolemaic latitudes: 'Alawī, 1983, p. 78.

85 Sarton, 1947; Sayili, 1960, pp. 184–185.

Earth than the Sun, according to the Ptolemaic order, they should consequently have parallax. The conclusion is, therefore, that both planets are beyond the Sun, and the planetary order becomes: Earth, Moon, Sun, Mercury, Venus, Mars, Jupiter and Saturn.

9–10. Another “error” in the *Almagest* concerns Ptolemy’s determination of the position of the apogees of the inferior planets. His criticism is rather trivial in the case of Venus, but not in the case of Mercury. It seems clear that Andalusī astronomers were aware of the fact that Mercury’s apogee had been incorrectly determined by Ptolemy, as it contained an error of about 30° for his own time. Jābir revised Ptolemy’s procedure and, after correcting some small details, he states that Ptolemy had proved that two maximum and opposite (i.e., morning and evening) elongations of the planet from the mean Sun, which take place symmetrically in relation to Mercury’s apogee, are necessarily equal. The author of the *Almagest* claims, however, that the reciprocal formulation is also true: two equal maximum and opposite elongations of the planet from the mean Sun will necessarily take place symmetrically on both sides of the apogee. Jābir states repeatedly that this is not true and that a planet may have many equal maximum morning and evening elongations from the mean Sun without its apogee being at the midpoint between the two positions of the centre of the epicycle. Jābir, then, analyses the observational data used by Ptolemy and establishes the criteria which should be applied when selecting the pairs of elongations that should be used to determine the position of the apogee. In spite of this, Jābir eventually accepts Ptolemy’s data (even though they were not selected according to his criteria) and also the wrong position of Mercury’s apogee that Ptolemy obtained. It is clear that Jābir is only interested in the mathematical problem.⁸⁶

11–14. These four errors concern Ptolemy’s determination of the points of planetary stations as studied in book XII of the *Almagest*, in which the treatment of stations and retrogradations is based on the eccentricity of the planetary deferents, without taking into account the equation of anomaly.

15. Jābir’s last correction is concerned with a minor point in the theory of planetary latitudes in *Almagest* XIII,4⁸⁷ related to the effect the inclination of the epicycle has on the position in longitude of the inner planets, specifically with regard to Venus.⁸⁸

86 Samsó, 2001 b.

87 Toomer, 1984, pp. 605–631.

88 Toomer, 1984, p. 624; Pedersen, 1974, p. 382.

5.3.4 *Conclusions*

Altogether, the *Iṣlāḥ* shows that its author had read and fully understood Ptolemy's *Almagest* and made a serious effort to make this work more accessible to the readers of his own time. He is not interested in the many corrections and improvements made by Islamic astronomers of Ptolemy's astronomical theories; his main concern is related to the theory and mathematics of the *Almagest*, and he leaves aside its practical parts. In this respect, I should recall that, in spite of his criticisms of Ptolemy's choice of the observations he uses to determine the position of Mercury's apogee, he finally accepts them, although his Andalusī contemporaries were well aware of the fact that the longitude of the apogee was already wrong in Ptolemy's time and, thus, should not have been corrected in precession to determine its position in the twelfth century. Some of his criticisms of the *Almagest* are justified, though others are the result of his use of deficient manuscripts of the Arabic translations of Ptolemy's book. Whatever the case, it is obvious that Jābir was able to reconstruct Ptolemy's thought, in spite of the shortcomings of the sources he was using.

As remarked by Bellver,⁸⁹ Jābir's criticisms are unrelated to the "Andalusī revolt against Ptolemy" which, in the works of al-Biṭrūjī and many other philosophers, adopts an entirely different attitude. In spite of this, it is clear that the members of this group had read the *Iṣlāḥ*, which was their introduction to Ptolemy's *Almagest*. They were influenced by Jābir's discussion of the order of planets and considered that, if a serious mathematician had found mistakes in the book of the Greek astronomer, this must mean that the Ptolemaic astronomical system might be challenged and that another system was also possible. The influence of the *Iṣlāḥ* also reached European astronomy, especially, as we have seen, due to its first book on trigonometry.

5.4 The Twelfth-Century Andalusī Revolt against Ptolemy

5.4.1 *Introduction*

It was A.I. Sabra who coined this expression in a paper, published in 1984, which has become a classical work of scholarship.⁹⁰ It refers to the criticisms of Ptolemaic astronomy made by a group of Andalusī philosophers from a point of view entirely different from the one adopted by Jābir ibn Aflaḥ. This group was formed by Ibn Bājja (1070?–1138), Ibn Ṭufayl (before 1110–1185), Ibn Rushd (1126–1198), Maimonides (1135–1204) and al-Biṭrūjī (fl. 1185–1192). They all lived

89 Bellver, 2009, pp. 112–113.

90 Sabra, 1984.

during the periods of Almoravid (1086–1145) and Almohad (1145–ca. 1232) domination in al-Andalus and the Maghrib, which years ago I called “the century of philosophers”,⁹¹ as it coincides with a period of decline in mathematical astronomy and a flourishing of *hay'a*, understood as a discipline which aims to describe the real existing world (“science of the physical structure of celestial spheres”), following the line of thought represented by Ibn al-Haytham’s *Shukūk ‘alā Baṭlamīyūs* and *Maqāla fī hay’at al-‘ālam*. Both works, as we shall see, were in circulation in al-Andalus during this period. As noted by Ibn Rushd himself, the same phenomena may have very different explanations from the point of view of a natural philosopher or of a mathematical astronomer. If two scientists, each one belonging to one of these schools of thought, were asked why the celestial sphere is spherical, the natural philosopher would answer that it is due to the fact that the celestial sphere is neither light nor heavy or because it is made of fire; for his part, the mathematical astronomer would attribute it to the fact that all radii going from the centre of the sphere to its surface are equal, or to the fact that it has fixed poles.⁹²

This trend was probably born in Zaragoza during the *ta’ifa* period, when both mathematics and philosophy were cultivated in this city, and where Ibn Bājja lived during the reign of al-Musta’in II (r. 1085–1110), shortly after the death of the king-mathematician Yūsuf al-Mu’taman (r. 1081–1085). Recent work by Miquel Forcada has underlined the importance of Ibn Bājja who, apparently, was the originator of a line of thought that tried to create a new kind of astronomy.⁹³ There are also certain links which suggest the existence of a family tree of the members of this school: Ibn Ṭufayl was an indirect disciple of Ibn Bājja, through the latter’s written works, and became the master of Ibn Rushd and al-Biṭrūjī. Only Maimonides seems to have been absent, although we know that he studied astronomy with a disciple of Ibn Bājja and that he had contacts with a son of Jābir ibn Aflaḥ.

5.4.2 *Were These Andalusī Philosophers Competent in Astronomy?*

5.4.2.1 Maimonides and Ibn Bājja

It is interesting to analyse the extent to which this group of philosophers were competent in mathematical astronomy, in order to establish whether their criticisms of Ptolemy were well founded.⁹⁴ We have little information about Ibn

91 Samsó, 2011, pp. 307 ff.

92 Puig, J., 1983, pp. 23–24; Puig, J., 1987, pp. 121–122.

93 Forcada, 2011.

94 Forcada, 2017, shows that Ibn Bājja, Ibn Rushd and al-Biṭrūjī were well aware of the new astronomical developments that had taken place in Toledo in the eleventh century and, particularly, of Ibn al-Zarqālluh’s innovations.

Ṭufayl on this topic; we have already seen (2.3.2) that Maimonides was able to calculate lunar crescent visibility, and that, together with Joseph b. Yehudah b. Sham'un, he had prepared a revision of Jābir ibn Aflah's *Iṣlāḥ al-Majisṭī* (5.3.1). We also know that he was interested in mathematics: a set of unpublished notes on certain propositions of Apollonius' *Conics* is extant and it shows that he had some competence in mathematics, though without much originality.⁹⁵

The case of Ibn Bājja's is clearer, and far more interesting. We have already seen (5.1.) that he learnt mathematics from Ibn Sayyid, a mathematician who was also interested in *hay'a*. An anecdote recorded by al-Maqqarī shows that Ibn Bājja was able to calculate a lunar eclipse (*kāna qad 'araḥa waqt kusūf al-badr bi-ṣinā'at al-ta'dīl*).⁹⁶ Also, in a letter addressed to his friend Abū Ja'far Yūsuf ibn Ḥasdāy, he explains that he studied astronomy (*ṣinā'at al-hay'a*) intensively, once he had finished with the study of music, although he had not mastered the theory of planetary latitudes.⁹⁷ This might be the reason why Ibn Bājja misinterpreted an observation of sunspots and considered them to be transits of Mercury and Venus on top of the solar disk (see 5.3.3). It seems clear that he made astronomical observations as, in his commentary to Aristotle's *Meteorology*, we read:⁹⁸

We have witnessed (*shahadnā*) an occultation of Jupiter by Mars. Afterwards, [Jupiter] went out behind it [= Mars] in the conjunction which followed that of year 500/1106–7. When [Mars] entered in contact with it [Jupiter] or was near it, one could see that both planets had the shape of a rectangle in which both ends had a curved shape.⁹⁹

The reference to the conjunction of year 500/1106–7 seems to refer to a conjunction of Saturn and Jupiter which took place on 10 February 1107, in which the longitude of Saturn was 286°.55 and that of Jupiter 286°.63.¹⁰⁰ This conjunction must have been rather spectacular, as both planets had practically the same latitude (Saturn 0°.13, Jupiter 0°.16) and were very near the ecliptic. One year later, the conjunction of Jupiter and Mars took place and it was probably *observed* by Ibn Bājja: the use of the word *shahadnā* and the description of the shape of both planets when their contact began and a short time before that moment makes me think of a visual observation made in Zaragoza. Towards

95 Hogendijk, 1985, pp. 123–4 and *passim*.

96 'Abbās, 1968, VII, p. 25.

97 'Alawī, 1983, p. 78.

98 Lettink, 1999, pp. 86–88; Samsó, 1993–94.

99 Quoted by 'Alawī, 1983 a, p. 10.

100 Tuckermann, 1964, p. 571.

the end of several nights between 14 and 18 March 1108, both planets were very near each other. On 16 March, at 6 a.m. G.M.T. (the Sun rose that day at 6h 5m) the horizontal coordinates of both planets were, in Zaragoza:

Jupiter: altitude 13°.91
 azimuth 125°.52
 Mars: altitude 13°.64
 azimuth 126°.06¹⁰¹

A very interesting remark, which proves Ibn Bājja's knowledge of mathematical astronomy, can be found, once again, in his letter to Ibn Ḥasdāy.¹⁰² In it, after a sharp criticism of Ibn al-Zarqālluh's astronomical work, he continues with an attack on Ibn al-Haytham's *Shukūk 'alā Baṭlamīyūs*:

The same attitude [as that of Ibn al-Zarqālluh] has been adopted by others who preceded him: I feel quite astonished that such is the case of Ibn al-Haytham, in spite of his fame. If you wish to consider in detail what I am telling you, read his book entitled *Shukūk 'alā Baṭlamīyūs*, particularly the chapter in which he explains the invalidity of the method used by Ptolemy to establish the eccentricities of Venus and Mercury, and you will get a clear idea of what I am saying. If you make a detailed study of this work, you will reach the obvious conclusion that Ibn al-Haytham only studied astronomy in a superficial way [*min ashal al-ṭuruq*] <and that he did not assimilate in due time those things that were difficult for him, either because they confirmed his idea on the lack of validity of the method or because he left them aside in a careless way.>¹⁰³ He [= Ibn al-Haytham] does not belong to the group of those who have persevered in the study of this science [= astronomy] and in this respect he is farther away from it than Ibn al-Zarqālluh himself.

Rather surprisingly, Ibn Bājja is absolutely right in his criticism of Ibn al-Haytham's treatment of the Ptolemaic method for the determination of the eccentricities of the inferior planets.¹⁰⁴ As a matter of fact, Ibn al-Haytham had made two important mistakes in his analysis of the Ptolemaic method:

101 Calculated with the programme EZCosmos of Astrosoft Inc.

102 'Alawī, 1983, pp. 77–78.

103 According to al-'Alawī the passage in angular brackets <> is an interpolation in the text and should be eliminated.

104 Šabra & Shāhābī, 1971, pp. 29–32. The corresponding chapters of the *Almagest* can be read in Toomer, 1984, pp. 456–460, 472–474. A detailed analysis of Ibn al-Haytham's text in

1. As asserted by Ibn al-Haytham, Ptolemy determines the position of the centre of the eccentre of Mercury and Venus on the basis of two observations of the maximum morning and evening elongations of each planet from **the mean Sun** whose position, according to Ibn al-Haytham, **coincides with that of the centre of the planet's epicycle (!)**:

He obtained each one of both centres through two observations of the morning and evening [elongations] of the planets, when each one of them was in its maximum distance from the location of the mean Sun which coincides with the centre of the epicycle, according to what he [Ptolemy] established.¹⁰⁵

This is an obvious error: in the *Almagest* the mean motions of the centres of the epicycles of the inferior planets are the same as that of the Sun but, although the eccentricity of Venus (1;15^p) is half that of the Sun (2;30^p), that of Mercury (3^p) bears no relation to the solar eccentricity and the position of the three apogees is independent: the solar apogee is located at 65;30° from the vernal point and it is sidereally fixed, while that of Venus is 55°, and that of Mercury 190°, both moving at the same rate as the precession of the equinoxes. In neither case, therefore, can one assert – as Ibn al-Haytham says repeatedly – that the centre of the planetary epicycle coincides with the mean Sun, the only obvious thing being that the line connecting the centre of the Earth and the mean Sun is parallel to the line connecting the equant point and the centre of the planetary epicycle. It is true, however, that Islamic astronomers from the ninth century onwards were influenced by Hindu-Iranian astronomy – represented, for example, by the *Zīj al-Shāh* – and introduced an important modification in the Ptolemaic model for Venus: in the Islamic tradition the apogee of the Sun is the same as that of Venus, and both are subjected to the motion of precession.¹⁰⁶ It is understandable, therefore, that Ibn al-Haytham identifies the position of the centre of the epicycle of Venus with the mean Sun, although this cannot be considered Ptolemaic. This identification, however, does not seem to have any precedent for the case of Mercury.

Samsó, 2001 b, pp. 202–208. This passage of the *Shukūk* had only attracted the attention of one scholar, Don L. Voss, in his unpublished doctoral dissertation *Ibn al-Haytham's doubts concerning Ptolemy. Translation and commentary*, presented at the University of Chicago, 1985, pp. 47–52, 126–131. Photocopies of these pages were sent to me by Jamil Ragep.

105 Šabra & Shāhābī, 1971pp. 29–30.

106 Goldstein & Sawyer, 1977.

2. In his criticism of the validity of the Ptolemaic method, Ibn al-Haytham makes a second mistake: he forgets about the existence of the equant point and mean motion is measured from the centre of the deferent. As a result of this, his argument is of no value at all.

It seems clear that Ibn Bājja was aware of these two mistakes of Ibn al-Haytham's, which may be the reason of his assertion that the Eastern scientist "only studied astronomy in a superficial way [*min ašhal al-ṭuruq*]". It also proves that Ibn Bājja had a sound knowledge and understanding of Ptolemy's *Almagest*.

5.4.2.2 Ibn Rushd and al-Biṭrūjī

The case of Ibn Rushd also seems clear, as he is the author of a *Mukhtaṣar al-Majisṭī* ("Summary of the *Almagest*")¹⁰⁷ extant in a Hebrew translation made by Jacob Anatoli (ca. 1194–ca. 1256), who worked at the court of Frederick II of Sicily (r. 1220–1250). This book was probably written by Ibn Rushd between 1159 and 1161 and it bears witness to the fact that he had an adequate understanding of the *Almagest* and that he had also studied Euclid's *Elements*, Theodosius' and Menelaos' *Spherics*, Ptolemy's *Planetary Hypotheses*,¹⁰⁸ Ibn al-Haytham's *Maqāla fī hay'at al-ʿālam* and *Shukūk ʿalā Baṭlamīyūs*, Ibn al-Zarqālluh's *On the motion of the fixed stars* (6.2.2) and *Comprehensive treatise on the Sun* (6.3), and Jābir ibn Aflaḥ's *Iṣlāḥ al-Majisṭī* (5.3). Unlike Jābir's, Ibn Rushd's book aims to update the *Almagest* with the innovations introduced by Islamic astronomers, although the sources available to him are limited. The prevalent influences seem to have been Ibn al-Haytham's two aforementioned works and Jābir's *Iṣlāḥ* which is, sometimes, the indirect source used by Ibn Rushd to summarise the *Almagest*. Thus, he is aware of the new trigonometry used by Jābir, although he decides to stick to the old trigonometric tools used by Ptolemy, out of respect for the source he is going to summarise. He also mentions Jābir when dealing with the problem of planetary order and, although he does not take sides in the first part of his *Mukhtaṣar*, he refers to the observation, made by Ibn Muʿādh's nephew, of two sunspots on the solar disk and says that he consulted Abū Ishāq ibn Wādīʿ, an unidentified scholar who was probably Ibn Rushd's master in astronomy,¹⁰⁹ and who ordered him to

107 Lay, 1996. In 1999, Julianne Lay sent me a copy of her unpublished French translation of the first part of the *Mukhtaṣar* which I am also using in this chapter. See also Forcada in *BA* vol. 4, pp. 560–561.

108 His knowledge of the *Planetary hypothesis* is documented in his *jawāmiʿ* of Aristotle's *Metaphysics* and of *De coelo et mundo*, both works written ca. 1159: see Ibn Rushd, 1365 H, pp. 59–61.

109 He also studied mathematics with Abū Jaʿfar ibn Hārūn al-Turuḡāllī: see *BA*, vol. 4, p. 520.

calculate the positions of Venus and Mercury; he did this in front of his master, and concluded that both planets were in conjunction with the Sun.¹¹⁰ It seems, therefore, that Ibn Rushd was able to compute planetary positions using a *zīj*.

Ibn Rushd was also interested in the trepidation models studied by Ibn al-Zarqālluh (6.2.2) and he considers the Toledan astronomer to be the first scientist to succeed in establishing the equation of this motion. His interest in this topic seems to have endured until the very end of his life, as the philosopher Ibn 'Abd Rabbihi al-Ḥafid told him, in 584/1188–89, that the motion of recession (*idbār*) of the star *Qalb al-Asad* (α Leonis, Regulus) would begin in about sixty years.¹¹¹ Another example of Ibn Rushd's interest in astronomical problems is related to his possible observation of the star *Suhayl* (α Carinae, Canopus), whose importance as a qibla indicator has been analysed in 2.5. In his *Talkhiṣ al-samā' wa l-'ālam*¹¹² Ibn Rushd writes:

Suhayl is not visible in our country, the peninsula of al-Andalus, although it has been said that it can be seen from the mountain known as the mountain of Suhayl.¹¹³ It can be observed, however, in the land of the Berbers, in the other side of the sea placed between them and us. I observed in Marrākush, in the year 584/1188–9, a star which is not visible in [our] country, from the top of the mountain named Darn: they pretend that this star is Suhayl.

Ibn Rushd's *Mukhtaṣar* is divided into two parts: the first one deals with the motions of the Sun, Moon, inner and outer planets, fixed stars, and the order of the planetary spheres and it ends with a description of Ptolemy's armillary sphere. This part demonstrates Ibn Rushd's competence as an astronomer: his summary of Ptolemy's models is accurate and he adds the adequate geometrical proofs to his descriptions. Besides, it also shows his interest in cosmological problems and contains references to his criticisms of Ptolemy's astronomy, which will be developed in later works. The second part is more mathematically oriented and it deals with the mathematics needed to understand the *Almagest* (derived from this source as well as Theodosius' and Menelaos' *Spherics*), followed by an analysis of the equations of the Sun, Moon and planets, a study of parallax, eclipses, planetary stations and retrogradations and planetary visibility.

110 As we have seen in 5.3.3, this conjunction took probably place in May 1068.

111 Ibn Sharifa, 1992, p. 196; Comes, 2002.

112 Ed. 'Alawī, 1984, p. 275.

113 Probably the mountain north of Fuengirola, on top of which today we find the village of Mijas.

In 1984 Sabra¹¹⁴ suggested the existence of two stages in Ibn Rushd's astronomical thought: the first one should be dated ca. 1174 (the date in which he wrote his *talkhīṣ* of Aristotle's *Metaphysics*) and, in it, he remained faithful to Ptolemy's cosmology, although his attitude remained extremely prudent. The second stage began, probably, between 1186 and 1190, when Ibn Rushd wrote his great commentary (*tafṣīr*) on the *Metaphysics*, in which his attitude changed entirely.

Sabra's hypothesis has been nuanced by Julianne Lay, as a result of her analysis of the *Mukhtaṣar al-Majīṣṭi* which she dates, as we have seen, between 1159 and 1161. According to her interpretation, Ibn Rushd, both in the *Mukhtaṣar* and in its contemporary book the *Jawāmi' mā ba'd al-ṭabī'a* (Epitome of *Metaphysics*),¹¹⁵ states that, concerning astronomy, one has to follow what has been commonly accepted and is well known (*mashhūr*). In both works, therefore, he is not explaining his own ideas but those of Ptolemy, accepted by the community of astronomers.¹¹⁶ Ibn Rushd's attitude is clear in his critical remarks on Ptolemy's model of Mercury which he considers absurd, although he adds, in Lay's unpublished translation:

Que cette hypothèse est merveilleuse. Sa signification est-elle bien celle-la ? Cette conjecture doit susciter notre admiration à l'égard des anciens et la honte de nous-mêmes, car aucun d'entre nous n'est en mesure d'accéder au niveau le plus insignifiant de leur capacité en science.

In fact, it seems that Ibn Rushd acknowledges the predictive capacity of Ptolemy's models although he denies their physical reality and considers that new physical models should be designed which could represent the real world of planets and stars. As a matter of fact, he seems to adopt a similar attitude to that of Eastern *hay'a* "astrophysicists". According to Saliba:¹¹⁷

None of the *hay'a* writers had any objections to that aspect of Ptolemaic astronomy, i.e., Ptolemy's ability to carry out observations, to extract parameters from them, and to set up tables based on those parameters to compute the positions of the planets by combining their mean motions and equations from such tables.

¹¹⁴ Sabra, 1984.

¹¹⁵ Dated by 'Alawī, 1986, pp. 57–59, in 1161 (with an interrogation mark).

¹¹⁶ Lay, 1996, p. 55: "Nous avons jugé bon, cependant, de suivre dans cet abrégé ce qui est communément admis (*mefursam*) et, si Dieu prolonge la durée de notre vie, nous entreprendrons des recherches sur ces choses, dans la mesure de nos possibilités".

¹¹⁷ Saliba, 2000, pp. 334–5.

As for al-Bīṭrījī, his astronomical knowledge was probably similar to that of Ibn Rushd, although this cannot be proved as clearly, because he did not write a book like the *Mukhtaṣar al-Majisṭī*. The sources he has used are, more or less, the same as Ibn Rushd's: he seems to have read the *Almagest* and the *Planetary Hypotheses*, although he does not seem to have understood some parts of the text. He might have read Theon of Alexandria's *Commentary on the Almagest*, since certain characteristic ideas of his astronomical system also appear in the book of the Alexandrian scholar.¹¹⁸ Besides, as we will see, he seems to have information about Eudoxus' system of homocentric spheres, although we do not know the channel via which this conception of the world could have reached him. As for his knowledge of the works of Islamic astronomers, he is only aware of the works of Andalusi authors which are the same as those known by Ibn Rushd: Ibn al-Zarqālluh's book on the motion of the fixed stars, titled, according to him, *Maqāla fī ḥarakat al-iqbāl wa l-idbār* ("On the motion of accession and recession") (6.2.2),¹¹⁹ and the third Zarqāllian trepidation model might have been his source of inspiration for his planetary models.¹²⁰ He also knew Jābir ibn Aflah's *Iṣlāḥ al-Majisṭī* and used (competently) two theorems of Jābir's "new trigonometry".¹²¹

Al-Bīṭrījī's opinion of Ptolemaic astronomy does not differ from Ibn Rushd's; he values Ptolemy's contribution to mathematical astronomy, in spite of the fact that his models cannot have a physical reality:

I praise (Ptolemy) for what is valuable in his work concerning this great science. I marvel at his instructions for the planetary motions – which brought order to their diversity and precision to the computations by which one finds the times of their retrograde and direct motions as truly observed and similarly, for other astronomical questions. He pursued (the analysis of) these diverse motions until he mastered their irregularities and simplified them for anyone who wished to find them, arranging his computations so that anyone can find the positions of the stars in their spheres both when they are near and when they are far away. He indicated the magnitude and sizes of their spheres with respect to the earth and to each other. Moreover, (he tabulated) the times of the conjunctions and oppositions as well as the magnitude and duration of eclipses, both past and future, and other properties of the (luminaries).¹²²

118 Carmody, 1952, pp. 24–25, 40, 54.

119 Goldstein, 1971, I, 69, 80, 91 and II, 93–95, 143, 179; Carmody, 1952, pp. 85, 94–95, 102.

120 Goldstein, 1964.

121 Goldstein, 1971, I, 98 and 122; II, 207–209 and 309; Carmody, 1952, pp. 107–108 and 126.

122 Goldstein, 1971, I, pp. 59–60; II, pp. 38–41.

5.4.3 *Andalusī Criticisms of Ptolemy: Ibn Bājja and Ibn Rushd*

In general terms, the physical problems raised by Ptolemy's models derive from the fact that they disagree with his own assumption that the motions of the Sun, Moon, planets and stars are circular and uniform and that they rotate around a centre which has to be identified with the centre of the Earth.¹²³ Obviously the use of eccentrics and epicycles contradicts this principle, and the situation is complicated by the use of the equant in Ptolemy's planetary models. The equant problem, according to Saliba,

does not arise from violating the principle of uniform motion as it is so often asserted, (...) although its violation is one of its consequences. Rather it arises from the conceptual notion of having physical spheres that could not possibly move, in place, uniformly, around an axis that did not pass through their centers, a veritable physical absurdity that would be obvious to anyone giving it more than a moment's consideration.¹²⁴

Among many other things, I should add here the problem of the existence, in Ptolemy's system, of opposite motions that affect all celestial bodies: a daily motion of 360° in 24 hours, from East to West, and slower motions in longitude in the opposite sense, from West to East. How could a single prime mover transmit both motions to the celestial spheres?

Recent research by Miquel Forcada has shown clearly that Ibn Bājja has a leading role in the beginning of the process described in these pages.¹²⁵ Ibn Bājja's *Kalām fī l-hay'ā* is a short work preserved in incomplete form in a single manuscript. Although it has very little astronomical content, it has the importance of showing clearly the philosophical background to the Andalusī revolt. As Forcada has shown, this booklet was extremely influential in the early writings of Ibn Rushd: both authors insist on the idea of science understood as a path of perfection, although Ibn Rushd departs from Ibn Bājja in considering that the premises used by Ptolemaic astronomers belong to commonly admitted knowledge (*mashhūr*). Ibn Bājja, for his part, rejects Ptolemaic astronomy because it gives a "conceptualisation" of the heavens but is unable to prove its real existence. His main criticism of Ptolemaic astronomers, also adopted by Ibn Rushd,¹²⁶ is that they do not follow Aristotle's demonstrative method.

¹²³ Pedersen, 1974, pp. 34–35.

¹²⁴ Saliba, 2000, p. 331.

¹²⁵ Forcada, 2012–14.

¹²⁶ Forcada, 2012–14, pp. 132–137, 146–150; Lay, 1996, pp. 51–61.

Probably due to the defective character of the only extant manuscript, there are few references to specific criticisms of Ptolemaic astronomy, related to the fact that Ptolemaic models cannot have real existence as they contradict the principles of Aristotle's *Physics*. We only find an obscure reference to the impossibility of harmonising two opposite motions and, possibly, in Forcada's attempt to reconstruct a corrupt passage of the text, a rejection of planetary epicycles.¹²⁷ This is related to the remark, made by Maimonides in his *Guide for the perplexed*,¹²⁸ according to which Ibn Bājja "states in his extant *Discourse on astronomy*, that the existence of epicycles is impossible". To this he adds, with some doubt:

I have heard that Abū Bakr [ibn Bājja] has stated that he had invented an astronomical system in which there were no epicycles, but only eccentric circles. However, I have not heard this from his pupils. And even if this were truly accomplished by him, he would not gain much thereby. For eccentricity also necessitates going beyond the limits posed by the principles established by Aristotle, those principles to which nothing can be added.

Besides this, according to al-Biṭrūjī,¹²⁹ Ibn Ṭufayl had conceived an astronomical system which explained the real cosmos without using eccentrics or epicycles and he had promised to write a book on this subject, although he may not have fulfilled his promise. Carmody thinks¹³⁰ that he may have transmitted his ideas to both Ibn Rushd and al-Biṭrūjī and that this could be the reason for the similarities between the works of both authors.

Ibn Rushd's *Mukhtaṣar*, which, as we have seen, is more astronomically minded than Ibn Bājja's *Kalām*, contains plenty of detailed criticisms.¹³¹ They are mainly based on the impossibility of a physical interpretation of certain elements of Ptolemy's planetary theory, although, following the lines of Ibn al-Haytham's *Fī hay'at al-ʿālam*, he makes an effort to transform Ptolemy's circles into solid spheres: interestingly, in these three-dimensional descriptions, the equant point disappears, although the eccentric circles and epicycles remain.

Thus, he does not object to the eccentric solar model and, in the case of the Moon, he rejects "the motion in inclination" (*prosneusis*) of the lunar epicycle

127 Forcada, 2012–14, pp. 137–146.

128 Pines, 1963, vol. 2, p. 322.

129 Goldstein, 1971, I, 61 and II, 49.

130 Carmody, 1952, p. 33; Carmody, 1952 a, p. 559).

131 I am following here Julianne Lay's unpublished French translation of the first part of the *Mukhtaṣar*.

in Ptolemy's third lunar model; as it implies two opposite motions belonging to the same body, which cannot be justified by a system of physical spheres. This criticism derives from Ibn al-Haytham's *Shukūk* in which similar arguments appear and which is also mentioned by Ibn Rushd in this passage. As for Mercury, he follows Jābir b. Aflaḥ (5.3.3) in his remarks on Ptolemy's procedure for determining the position of its apogee: it may be significant that he makes no mention at all of the criticism of this method made by Ibn al-Haytham in the *Shukūk* (5.4.2.1). To this he adds that Ptolemy's model "is inadequate for moving bodies and even less for the properties of these bodies ascribed to them, in an explicit way, by nature."

In the case of the higher planets, he reminds the reader of the efforts made by Jābir ibn Aflaḥ to design a method to determine their eccentricity, without assuming its bisection as a kind of postulate (5.3.3). He remarks, however, that it is not absolutely necessary to assume this disposition of the centres and that scholars should research and analyse the astronomical observations which justify it. The result of this would be the design of new planetary models, something that is impossible for him in his present situation, because al-Andalus is overrun and the food supply in Córdoba has been interrupted.¹³²

Concerning the fixed stars, he discusses the possibility of the existence of multiple spheres (one for each star) although, as this hypothesis has not been proved by natural science, he accepts that of a single sphere. This problem had also been discussed by Ibn Sīnā, Ibn Bājja and Maimonides (*Guide* 11, 11). As we have already seen, Ibn Rushd also analyses the problem of precession and seems to have a high opinion of the trepidation model designed by Ibn al-Zarqālluh (6.2.2).

Astronomical problems are also dealt within other early works by Ibn Rushd. He discusses, for example, the existence of a ninth sphere (the prime mover) which would transmit motion to the lower spheres: he rejects this hypothesis because he considers that a sphere exists due to the celestial bodies that it drags along, which are its most noble components. According to this principle, the starry sphere which contains, at least, the 1022 stars of Ptolemy's catalogue, would be the most noble of all the spheres, and it should become the see of supreme motion.¹³³ In relation to the daily motion of the celestial sphere, he also considers the possibility of the Earth's rotation of 360° every

132 This is interpreted by Lay (1996, pp. 26–27) as a reference to the revolt of Ibn Mardanish against the Almohads: in 1159 he attacked Seville and Córdoba and swept through the surrounding countryside.

133 Ibn Rushd, 1365 H, p. 136 and pp. 57–58. An analysis of Ibn Rushd's ideas in *Al-samā' wa l-'ālam* can be found in Hugonnard, 1985.

24 hours, while the stars remain motionless, but rejects this hypothesis because he considers it alien to human nature and contrary to the evidence.¹³⁴ Besides, at this early stage in his career, he deals with a problem which will interest both him, in his later works, and al-Biṭrūjī: how can one conceive of a first mover which could transmit motions in opposite directions to the sphere of the stars and to the planetary spheres? He feels unable to find an answer to this question and he merely reproduces a metaphor from Ptolemy’s *Planetary hypotheses*: the cosmos is similar to an enormous animal moved by a unique mover, although each one of its parts seems to move in a different way and does not follow the motion of the whole.¹³⁵ Interestingly, in the *Mukhtaṣar*, he rejects the idea developed by al-Biṭrūjī (5.4.4.3), that “the cause of the Western motion [i.e. from West to East] cannot be found, as some [scholars] believe, in a slowing down of the daily motion”.

Both in his *Jāmiʿ* on the *Metaphysics* and in the *Mukhtaṣar* Ibn Rushd seems to adopt an attitude similar to that of Ibn al-Haytham in his *Hayʾat al-ʿālam* and accepts the existence of a mover for each one of the characteristic motions of the planets, according to the following scheme, that corresponds to his summary of the *Metaphysics*:¹³⁶

Celestial bodies	Number of movers
Fixed stars	1
Saturn, Jupiter, Mars	$5 \times 3 = 15$
Venus	7
Mercury	8
Sun	1
Moon	5

According to these values, the total number of movers is 37, although Ibn Rushd states that the result is 38. To these he adds another eight movers corresponding to the daily motion of the eight celestial spheres, which results in 45 or 46 movers and should explain all celestial motions. The passage ends, in the summary of the *Metaphysics*, with an explicit rejection of homocentrism:

134 Ibn Rushd, 1365 H, p. 53. See on this question Hugonnard, 1977.
135 Ibn Rushd, 1365 H, pp. 59–61. Cf. Sabra, 1984, who was the first to draw attention on this passage.
136 The scheme is slightly different in the *Mukhtaṣar*.

"We do not need to imagine a multitude of spheres whose centres would be the centre of the Universe and whose poles would also be those of the Universe".¹³⁷

Between 1192 and 1194, Ibn Rushd wrote his great commentary (*tafsīr* or *sharḥ*) on the *Metaphysics*,¹³⁸ in which his attitude in respect to astronomical problems has changed entirely. He rejected eccentric circles and epicycles as well as the equant point, because they implied motions in opposite directions, around centres which are not the centre of the Earth. He also introduced an idea adopted by al-Bīṭrūjī (5.4.4): if the prime mover is located in the exterior part of the system, the energy it transmits will diminish as it moves away from the mover. As a consequence of this principle, the sphere of the fixed stars will be the fastest one, followed by those of Saturn, Jupiter, Mars, and so on. This implies that, although in the *Metaphysics* he does not explicitly mention the idea of *taqṣīr* (*incurtatio*) – later explained in 5.4.4.3 – he seems to be considering, somehow, that the daily motion from East to West is the basic motion transmitted by the prime mover, while the opposite motions in longitude are only a kind of "delay" of this fundamental motion.¹³⁹ We should recall, however, that this idea was explicitly rejected in the *Mukhtaṣar*. Besides, the conception of the starry sphere as the fastest one cannot be considered original to either Ibn Rushd or al-Bīṭrūjī, as it has a clear Classical origin: Lucretius ascribes it to Democritus¹⁴⁰ and Alexander of Aphrodisia to the Pythagoreans. Martianus Capella (*De nuptiis* VIII, 853) states that the peripatheticians sustained that the planets do not move in an opposite direction to the motion of the celestial sphere, but that they are advanced by it, due to its velocity which they are unable to equal. Similar ideas also appear in Theon of Alexandria.

A Greek origin can also be found in Ibn Rushd and al-Bīṭrūjī's conception¹⁴¹ according to which the combination of the daily motion and the motion in longitude of celestial bodies produces a spiral (*lawlab* or *ḥalazūn*): the apparent motion of any of them describes a circle, parallel to the equator, in one day; however, since in this period of time the celestial body will have moved in longitude in the opposite direction, its motion will not reach the same position as where it began and it will in fact describe a spiral. This idea already

¹³⁷ Ibn Rushd, 1365 H, pp. 136–138.

¹³⁸ On the chronology of this work see 'Alawī, 1986, pp. 109–110. On the astronomical ideas of Ibn Rushd in this stage of his life, see Sabra, 1984; Carmody, 1952 a; Arnaldez & Iskandar, in *DSB*, vol. 12, pp. 1–9; Forcada, 1999.

¹³⁹ This kind of reference, as well as *shawq* (*desiderium*), also appears in the commentary on *De caelo*: Carmody, 1982, pp. 41, 48–50.

¹⁴⁰ Dicks, 1970, p. 139.

¹⁴¹ See for example Goldstein, 1971, I, 61–62, 85–86; II, 49, 51, 53, 157–159.

appears in Plato's *Timaeus*¹⁴² as well as in Theon of Alexandria. However, strangely enough, both Ibn Rushd and al-Biṭrūjī derive this spiral motion from literal quotations of translations of Aristotle's works (*Metaphysics*, *De caelo*), although it does not appear in the original Greek texts. This might be the result of interpolations,¹⁴³ which could be very old, or of mistranslations of the Aristotelian text. The corresponding passage of *De caelo* (II, 8) mentions *kylisis* (rolling) and *dinesis* (rotation about an axis), and the second term also suggests the idea of a whirlwind and it might have been translated by *lawlab*.¹⁴⁴ This is a possible explanation of the meaning of the word *lawlab*, although others have also been suggested: Forcada believes that the spiral motion could be the result of a misinterpretation of the Aristotelian text when it refers to the countervailing spheres of Eudoxus and Callipus,¹⁴⁵ while J.L. Mancha considers that, in al-Biṭrūjī's work, *lawlab* corresponds to each one of the four parts in which the two axis of symmetry divide the hippopede (see 5.4.4.2).¹⁴⁶

To sum up: it is clear that Ibn Rushd was not able to design his own astronomical system and he only formulates some general ideas, most of which will be used by al-Biṭrūjī. He is perfectly aware of the difficulties of the task, as he himself states in his great commentary on the *Metaphysics*:

In my youth I hoped it would be possible for me to bring this research [in astronomy] to a successful conclusion. Now, in my old age, I have lost hope, for several obstacles have stood in my way. But what I say about it will perhaps attract the attention of future researches. The astronomical science of our days surely offers nothing from which one can derive an existing reality. The model that has been developed in the times in which we live accords with the computations, not with existence.¹⁴⁷

5.4.4 *Al-Biṭrūjī's Astronomical System*

5.4.4.1 The Author and His Work

All the criticisms of Ptolemaic astronomy made by this group of twelfth-century Andalusī philosophers led to only one clear result: the astronomical

¹⁴² Dicks, 1970, p. 129.

¹⁴³ Carmody, 1952 a, pp. 561, 570–572; Carmody, 1952, p. 54.

¹⁴⁴ Samsó, 1971a; Carmody, 1982, pp. 36–37, 50–52: in *De caelo* the sole example of the word *lawlab* coincides with *dynesīs*. *Lawlab* also appears in Ibn Rushd's commentary on the *Metaphysics*.

¹⁴⁵ Forcada, 1999, pp. 90–94; Forcada, 2005.

¹⁴⁶ Mancha, 2004.

¹⁴⁷ Quoted by Arnaldez & Iskandar, in *DSB*, vol. 12, p. 3.

system designed by Abū Ja'far (or Abū Ishāq) Ibrāhīm b. Yūsuf [ibn] al-Bīṭrūjī,¹⁴⁸ which appears in Escorial ms. 963. Nothing is known about the biography or the precise chronology of this scholar, except for the fact that he seems to have been the disciple of Ibn Ṭufayl (d. 581/1185–6) who had already died when he wrote his only extant book, since Ibn Ṭufayl's name is followed by the expression *rahīma-hu Allāh* ("may God have mercy upon him").¹⁴⁹ It seems, therefore, that the book was written towards the end of the twelfth century and the date of its composition may have been between 1185 and 1192, the date of ms. Escorial 1636 containing the anonymous *Kitāb al-madd wa l-jazr* ("On high and low tide"), which presents ideas that might have been borrowed from al-Bīṭrūjī's book. Far more reliable, as an *ante quem* date, is 1217, the year in which Michael Scott finished his Latin translation of this work.

The title of al-Bīṭrūjī's book is *Kitāb [murta'ish] fī l-hay'a* ("[Revolutionary] book on cosmology"). Goldstein published a facsimile edition of the Escorial manuscript (with variants from ms. Istanbul Seray, arab 3302), together with a critical edition of the Hebrew translation made in 1259 by Mosheh b. Samuel b. Yehudah b. Tibbon, as well as an English translation and a commentary.¹⁵⁰ Al-Bīṭrūjī's book is the maximum expression of Andalusī astronomical Aristotelianism and the only serious attempt to build an astronomical system which tries to describe a physical cosmos (although, as we will see, with poor results). We have already seen (5.4.2.2) the astronomical sources with which al-Bīṭrūjī was acquainted, among them Jābir ibn Aflah's *Iṣlāḥ al-Majisṭī* (5.3) which spurred our author's interest in the problem of planetary order: he rejected both Ptolemy's (Moon, Mercury, Venus, Sun ...) and Jābir's (Moon, Sun, Mercury, Venus ...) orders, and proposed another one of his own (Moon, Mercury, Sun, Venus ...). According to al-Bīṭrūjī, both Mercury and Venus have their own light, something which explains the lack of Mercury transits on top of the solar disk.¹⁵¹

Al-Bīṭrūjī's astronomical system is purely qualitative, as he himself acknowledges.¹⁵² As one might expect, it is a homocentric system in which all celestial bodies are always kept at the same distance from the centre of the Earth. In spite of this, al-Bīṭrūjī's homocentrism uses eccentrics and epicycles, located on the surface of each sphere and in the zone of the pole,

148 On the variants in the name of this scholar see Samsó, "al-Bīṭrūjī", *EA*, I, pp. 276–280. For this section, I am using materials already published in the aforementioned entry of *E.A.*, Samsó, 2011, pp. 342–356 and Samsó, 1994 c.

149 Goldstein, 1971, I, 61 and II, 48–49.

150 Goldstein, 1971.

151 Goldstein, 1971, I, 125 and II, 319.

152 Goldstein, 1971, I, 76 and 154; II, 127–9 and 425–7.

probably adopting ideas derived from Ibn al-Zarqālluh's third trepidation model (6.2.2.5)¹⁵³ This implies that the centre of circular motions is not always the centre of the Earth. In spite of this, it is quite possible that, as suggested by Kennedy, one of his sources of inspiration was Eudoxus of Knidos' homocentric system.¹⁵⁴ This idea has been confirmed in a paper by J.L. Mancha¹⁵⁵ who interprets al-Bīṭrūjī's model of the motion of the fixed stars as the result of the combined motion of two homocentric spheres with different poles. This poses the problem of establishing the path through which Eudoxus' ideas, known only through Simplicius' commentary, could have reached our author. Whatever the case, the Islamic world seems to have been aware of Eudoxian models, as Ibn al-Haytham uses a Eudoxian couple.¹⁵⁶ These theories reached Medieval Europe and were used by Levi b. Gerson (1288–1344),¹⁵⁷ as well as by other Latin authors.

5.4.4.2 Al-Bīṭrūjī's Geometrical Models

Al-Bīṭrūjī uses the same model for Saturn, Jupiter, Mars and Venus and, in a superficial description, it could also be applied to Mercury and the Moon. The model is the following one: in fig. 5.1, HKT is the parallel to the Equator described by the pole of the eighth sphere and its radius OT is 24° , the obliquity of the ecliptic which, for al-Bīṭrūjī, is a constant value. Pole K , of planet A , always located at a distance of 90° from it, rotates on the polar epicycle $KMSL$ whose centre is T . Radius KT of the polar epicycle is always equal to the maximum latitude of the planet. The motion in longitude is due to the displacement of point T , the centre of the polar epicycle, which will move on the polar deferent HKT in the opposite direction to that of the daily motion. The rotation of point K on the polar epicycle $KMSL$ corresponds to the motion in anomaly in Ptolemaic models and, as in the latter, the motion of the planet in its epicycle takes place in the same direction as the motion of the centre of the epicycle on its deferent (in the opposite sense to that of the daily motion), it seems that the same rule can be applied to al-Bīṭrūjī's models, as the only change introduced by our author is to displace the machinery and most Ptolemaic parameters to the polar zone.

As we have already seen, planet A will always remain at a distance of 90° from its pole K . Consequently, A will describe another epicycle AC , whose

153 Goldstein, 1964.

154 E.S. Kennedy, review of Carmody, 1952, in *Speculum* 29 (1954), 246–51; Kennedy, 1973.

155 Mancha, 2004.

156 Mancha, 1990 a.

157 Mancha, 2003.

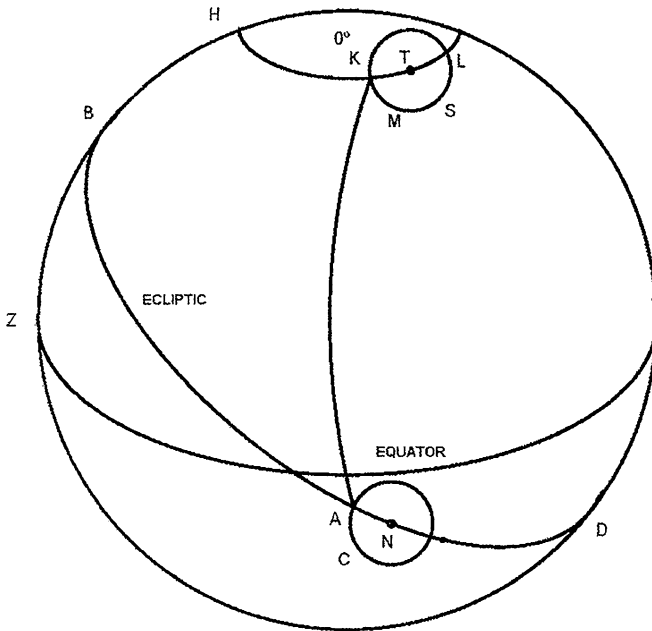


FIGURE 5.1
Al-Bitrūjī's
planetary model

centre N will move on the ecliptic with a velocity equal to the planet's mean motion. The use of a polar epicycle and of its corresponding projection on the zone of the ecliptic allows al-Bitrūjī to justify direct motions, stations and retrogradations of planets. The trouble lies in the fact that al-Bitrūjī tries to justify the planetary changes in latitude through the motion in anomaly, something which should be considered unacceptable.

Al-Bitrūjī describes his planetary models in a Ptolemaic style, although he tries to hide this by introducing changes in his terminology. The term *taqṣīr* (delay, see 5.4.3) is usually related to the motion in longitude of a planet or the Sun, while *al-taqṣīr al-akhīr* (*incurtatio postrema* or ultimate delay) means mean longitude and *al-taqṣīr al-awwal* (*incurtatio prima* or first delay) is the result of the sum of the mean longitude and the mean anomaly of the planet which, for the superior planets, will be equal to the mean solar longitude, as in Ptolemaic models.

As for the fixed stars, al-Bitrūjī traces a short history of the discovery of the precession of equinoxes by Hipparchus (ca. 127 BC), and of the various theories adopted by Greek and Islamic scientists concerning whether the velocity of precession is constant or variable. He begins with Theon's reference to the motion of trepidation which, combined with constant precession, would justify a variable speed, and continues with Islamic astronomers, al-Battānī among them, who rejected Hipparchus' and Ptolemy's precessional constant (1° /

100 years). Ibn al-Zarqālluh¹⁵⁸ managed to harmonise them (*ihtāla fī ta'alluḥi-hā*), in which *them* refers to “these unequal displacements” (*hādhihi l-intiqālāt al-mukhtalifa*) by creating a structure (*hay'a*) on which a computation can be based:¹⁵⁹ in it, the two poles of this sphere [the sphere of the fixed stars] move in parallel circles to the circle of the equator, something which necessarily implies that their motion (*ḥarakatu-hā*) [the motion of the fixed stars] follows the motion of these two poles. All this leads to an interesting sentence which seems to confirm Goldstein's hypothesis concerning the influence exerted by Ibn al-Zarqālluh's geometrical models on those of al-Bīṭrījī:

What he said on this topic attracted attention to something on which we are now working and which has not yet been thought of by anybody.¹⁶⁰

An outline of al-Bīṭrījī's model for the motion of the fixed stars can be found in fig. 5.2:

- O and S are the poles of the ninth sphere around which it makes a complete revolution in 24 hours.
- AKGM is the equator of the ninth sphere.
- ABGD is the ecliptic of the ninth sphere.
- A and G are the equinoxes.
- D and B are the solstices.
- OTS is a horizon in *sphaera recta*.
- Z is the pole of the eighth sphere at time t_0 and the distance between Z and O, the poles of the eighth and the ninth spheres, equals the obliquity of the ecliptic.

As the daily motion of the ninth sphere is transmitted to the eighth, the poles of the latter rotate around the poles of the ninth and describe the circles ZZ'F and EN, called by al-Bīṭrījī “the two circles of the path of the poles” (*dā'iratā mamarr al-quṭḥbayn*).¹⁶¹ As a result of the daily motion received from the ninth sphere, the eighth sphere rotates from E to W but it suffers a “delay” (*taqṣīr*), the result of which is that, while point A of the ninth sphere completes a whole

¹⁵⁸ In Goldstein, 1971, I, 80; II, 143 we find the title of Ibn al-Zarqālluh's book: *Maqāla fī ḥarakat al-iqbāl wa l-idbār* (“On the motion of accession and recession”).

¹⁵⁹ Goldstein, 1971, I, 91; II, 179. The Arabic sentence does not seem clear to me: *takūnu 'alayhā wa-ḥisāban*.

¹⁶⁰ *Fa-nabbaha bi-mā awrada-hu min dhalika 'alā mā waqafnā 'alay-hi al-ān mimmā lam yutaḥaṭṭan ilayhi*: Goldstein, *Bīṭrījī* II, 179. Goldstein's translation (I, 91) does not seem to include this sentence, although I find it in Mancha's translation, made from the Latin versions: “What he said induced us to discover something never considered by others”. See Mancha, 2004, p. 172.

¹⁶¹ Goldstein, 1971, I, 81; II, 147.

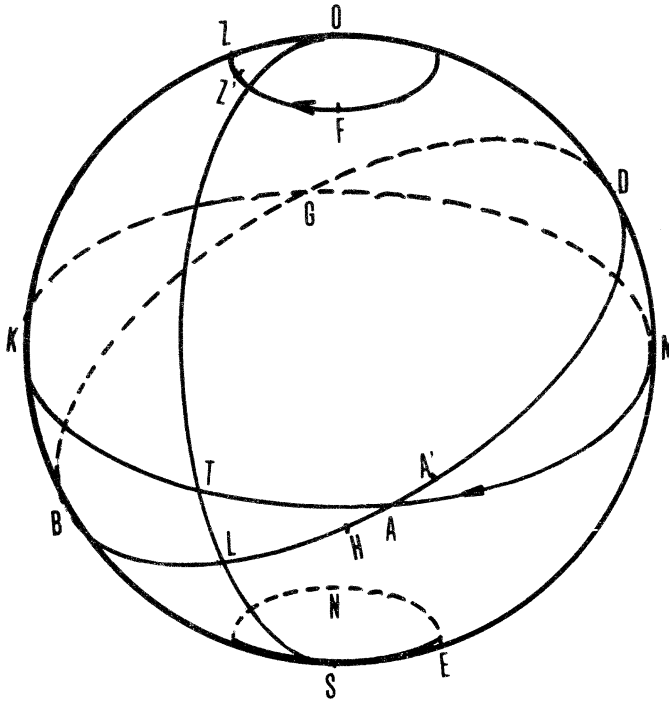


FIGURE 5.2
Al-Bīṭrūjī's model
for the motion of
the fixed stars.

revolution in 24 hours, Z, the pole of the eighth sphere, will not return to its original position (Z) but will only reach Z'. Consequently it will give the impression of a displacement in the opposite direction to the universal motion. Besides, if we consider a star placed, on the eighth sphere, at a point corresponding to point A of the ninth, at a distance of 90° from pole Z, when the latter apparently moves to Z', A will also be displaced to A' and, when Z reaches F, the star which was originally in A will now occupy the position of point D.

Until now, al-Bīṭrūjī has used the *taqṣīr* to justify the change in declination of a star, located on the equator of the eighth sphere (similar to the ecliptic) with latitude 0° . To the *taqṣīr* he adds, however, a second westward motion which he calls *istīfā'* (completion), which corresponds to the rotation of the eighth sphere around its own poles, in the same direction as the daily motion, but on the plane of its own equator which, as we have seen, is equivalent to the ecliptic.¹⁶² This motion is due to the desire the eighth sphere feels to imitate the ninth sphere (see below 5.4.4.3). The velocity of both motions (*taqṣīr* and *istīfā'*) is the same, although in the opposite direction and on different planes. Consequently, as a result of the *istīfā'*, the star which at the beginning was at

¹⁶² For the sake of simplicity, in the following lines I will call the "equator of the eighth sphere" the "ecliptic".

the point of the eighth sphere corresponding to point A, will after a certain amount of time be located at L.

Let us consider that, at a given time, OTLS is a horizon in *sphera recta*. Using two trigonometric theorems which he knows through Jābir ibn Aflaḥ, Al-Bīṭrūjī proves that $AL > AT$. Consequently, while arc AT crosses the horizon OTLS, the arc of the ecliptic AL will not be able to cross it simultaneously, and it will be delayed by an amount $AH = AL - AT$. It will seem, therefore, that point L has moved, on the ecliptic, in the opposite direction to the daily motion.

This will be the case as long as point L is located at a maximum distance of 45° , on both sides of A. However, in the quadrant of the ecliptic, the midpoint of which is the solstice B, we will have:

$$AB = AL + LB = 90^\circ$$

$$AK = AT + TK = 90^\circ$$

and, as we have seen $AL > AT$, we have $TK > LB$.

Consequently, arc LB will cross horizon OKBS faster than the arc of the equator TK. Apparently, therefore, it will seem that the motion of the fixed stars has taken place in the direction of the daily motion,¹⁶³ and we will have an accession-recession motion, as in Ibn al-Zarqālluh's trepidation models.

Both Goldstein¹⁶⁴ and Mancha¹⁶⁵ agree on the above description of the motions of the eighth sphere. The great step forward we find in Mancha's paper is to identify that

a point at A, on the equator of the eighth sphere, will generate with these combined motions Simplicius' *hippopede*; its length can therefore account for changes in declination of a star placed in A at t_0 , whereas its width can account for eastward and westward oscillations in longitude (fig. 5.3).¹⁶⁶

The problem now lies in determining how far al-Bīṭrūjī was aware of the result obtained by the rotation of two concentric spheres which move around two different axes. For this purpose, Mancha shows the evidence of the following passage, which I quote here in Goldstein's translation:

163 See a discussion of this passage in Mancha, 2004, pp. 158–159.

164 Goldstein, 1971, I, 22–26.

165 Mancha, 2004, pp. 147–151.

166 Mancha, 2004, p. 151.

(Point A) rises from some point and then reaches another by the rotation, and it always generates curves [*dawā'ir*] which are incomplete [*ghayr tām*, i.e. not closed], since all the curves are nonplanar, as we explained previously, and the figure so described is called a spiral [*shakl ḥalazūnī*]. When the pole moves from point Z to the point opposite it on the circle, point E [not in fig. 5.2], point A moves with that motion from L to B, and a figure similar to the first one is again generated; in the two remaining quarters, two figures similar to these (are generated). Point A returns to its original place; as we have shown, it has described four figures by the combination (*majmū'*) of the two motions. This is what we intended to illustrate.¹⁶⁷

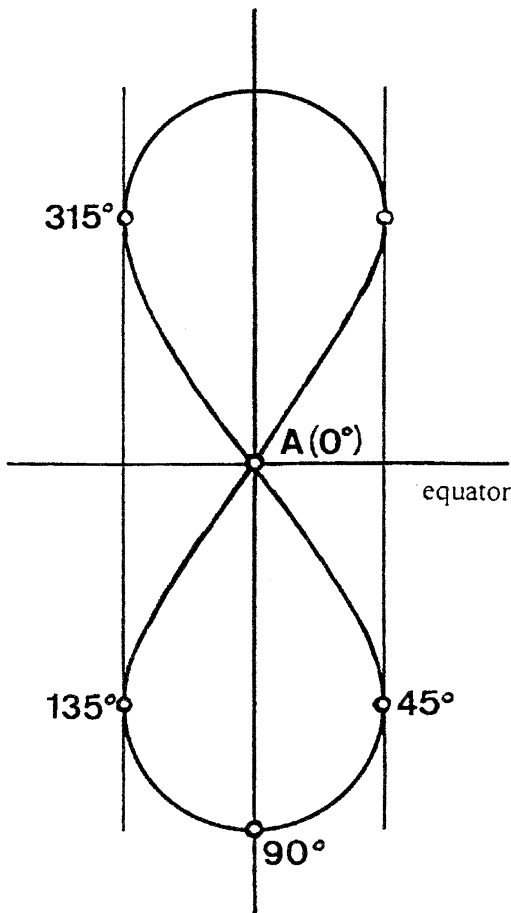


FIGURE 5.3
Borrowed from Mancha

¹⁶⁷ Goldstein, 1971, I, 87 and II, 163–165; see Mancha's translation in Mancha, 2004, p. 168.

Mancha's interpretation of this passage¹⁶⁸ is that the four figures mentioned are the four parts in which the two axes of symmetry divide the hippopede. This, of course, makes sense, although my impression is that al-Bīṭrūjī is copying this passage from another source. In its first part ("(Point A) rises ... and the figure so described is called a spiral"), however, he states that a star rises, in each rotation, at a different point of the horizon, producing incomplete (*ghayr tām*) circles in different planes, and the resulting figure, when completed, is a spiral. All this makes me think that al-Bīṭrūjī identifies *lawlab* (mentioned in other passages) and *ḥalazūn* (used here) as a true "spiral", the curve generated by a celestial body as a result of the composition of its daily motion and its motion in longitude, and does not understand the second part of the text ("When the pole moves from point Z ... by the combination of the two motions"), which justifies Mancha's interpretation and corresponds to the original source. It is difficult for me to accept that such standard technical words as *lawlab* and *ḥalazūn* might have an entirely different meaning, especially the term *ḥalazūn*, which also means snail.

5.4.4.3 Transmission of Motion in al-Bīṭrūjī's Astronomical System

One of the most interesting aspects of al-Bīṭrūjī's astronomical ideas is how he explains the physical cause of celestial motions and the transmission of energy from the first mover, located in the ninth sphere, towards the inner spheres. The ninth sphere rotates with the strongest, fastest and most simple motion, from E to W, every 24 hours.¹⁶⁹ This motion is transmitted to the inner spheres which slow down as the motion progresses towards the centre of the system. In this way, following an idea which we have already found in Ibn Rushd and other sources, the sphere of the fixed stars rotates faster than the sphere of Saturn which, in its turn, is faster than the sphere of Jupiter. This decrease in velocity continues until it reaches the sphere of the Moon, which is the slowest of all planetary spheres. The speed of rotation of the different spheres is the criterion used by al-Bīṭrūjī to establish his characteristic order of planetary spheres. The transmission of motion to the inner spheres is explained in three different ways using ideas derived from Aristotelian and Neoplatonic dynamics:

A possible allusion to an Aristotelian explanation – in which the transmission of motion would take place through contact between the different spheres – appears in the following passage:

¹⁶⁸ Mancha, 2004, pp. 156–157.

¹⁶⁹ Goldstein, 1971, I, 63 and 66; II, 59 and 77.

It is well known to everyone that the heavens comprise ten [*sic*] spheres, each separated from the other, but each touching the other perfectly. Since each moves inside the other, they all have perfectly round and equal surfaces and they touch each other, for there is no other body to come between them [...] and each one is in contact with the whole surface of the other.¹⁷⁰

We should note that neither here nor in other parallel passages do we find an explicit mention of the transmission of motion from one sphere to the other: spheres rotate one inside the other but this does not necessarily imply that one sphere moves the other.

The first Neoplatonic explanation is based on the *impetus* theory developed by John Philoponus (sixth century):

The highest body [*al-jirm al-a'lā*, the first mover] is distinct [*mufāriq*] from the power [*quwwa*] which it bestows on those spheres below it, just as one who throws a stone or shoots an arrow is distinct from the object thrown, and he is not bound to the power which he imparts to them, or like the stone propelled by staff as long as it continues to move it. This power is similar to that which moves the arrow shot by an archer which becomes weak as it moves away from its mover until finally it is exhausted when the arrow falls down. Similarly, the power which the highest body imparts to those [spheres] below it continues to diminish until it reaches the earth, which is at rest by its nature.¹⁷¹

It is interesting to note here that the *impetus* theory was formulated, in principle, to justify violent motions in the sublunary world; here we find it applied, although perhaps in a metaphorical sense, to natural motions of the celestial spheres. Al-Biṭrūjī may be using the same kind of dynamics for the whole cosmos, something which is confirmed by the fact that the power transmitted by the prime mover reaches the sphere of fire and produces comets, shooting

170 I am quoting here a slightly revised translation of Goldstein, 1971, I, 65–66, II, 73–75; Carmody, 1952, p. 82; similar passages can be found in Goldstein, 1971, I, 65 and 69–70, II, 71 and 99; Carmody, 1951, pp. 82 and 86.

171 Slightly revised version of Goldstein, 1971, I, 78; II, 137. Here Michael Scott does not quote the first part of al-Biṭrūjī's argument and mentions the example of the stone and the arrow only to justify the slowing down of motion (Carmody, 1952, p. 93 no. 11). See also Goldstein, 1971, I, 79; II, 139; Carmody, 1952, p. 94 no. 13.

stars (*ashbāh al-kawākib*) and similar phenomena, and also reaches the sphere of water, where it causes tides.¹⁷²

A second Neoplatonic explanation is based on the idea of *shawq* or *tashawwuq* (*desiderium*). Here al-Biṭrūjī seems to be borrowing a “theory” from the *Kitāb al-muṭabar* written by the Oriental philosopher Abū l-Barakāt al-Baghdādī (eleventh–twelfth centuries),¹⁷³ According to this author, each celestial sphere feels a strong desire (*yashtāq*) for the sphere that is immediately above it: in the same way, each one of the four elements feels a similar desire to occupy its own natural place. This desire is felt by the whole of the sphere, and by each one of its parts. Since, at a particular moment, a part of the inner sphere can only approach another part of the higher sphere, this desire will never be completely satisfied. Consequently, each sphere will rotate in a circular motion which will be the result of the effort made by each one of its parts to come near each one of the parts of the higher sphere. As has been stated by Pines, here we also have an attempt to apply the same kind of dynamics to the two worlds below and above the Moon.

It is not difficult to find ideas in al-Biṭrūjī which are similar to those of Abū l-Barakāt. I will only quote two examples:

All the spheres below it [i.e. below the highest sphere] move by its [= the highest sphere's] motion and seek its goal either by nature [*bi l-ṭab*] or by desire [*shawq*] in order to imitate [*tashabbuh*] its motion or to catch up with it.¹⁷⁴

Each of these lower spheres desires to imitate the highest sphere and trails along according to the amount of power which it retains from the highest sphere.¹⁷⁵

Each sphere wishes to attain perfection (*kamāl*, a word mistranslated by Michael Scott as *complementum*) and the spheres located nearest the sphere of the first mover are those which resemble it most closely and, consequently, have the fastest motion.¹⁷⁶

172 This seems to contradict al-Biṭrūjī's Aristotelian remark: “Circular motion does not persist to what is below the heavens, for motion in a straight line is natural there”. Goldstein, 1971, I, 78; II, 135; Carmody, 1952, p. 93 no. 7.

173 Pines, 1979.

174 Goldstein, 1971, I, 75; II, 125; Carmody, 1952, p. 90 no. 18.

175 Goldstein, 1971, I, 76–77; II, 129–131; Carmody, 1952, p. 92 no. 2.

176 Goldstein, 1971, I, 63, 78; II, 61, 135; Carmody, 1952, pp. 63, 93 (no. 9). See also Goldstein, 1971, I, 78–79; II, 139 (Michael Scott omits this passage: cf. Carmody, 1952, pp. 93–94).

Al-Biṭrūjī is not the first Andalusī author to use the idea of *tashawwuq* to explain motion in the planetary spheres: we have already seen that Ibn Rushd does the same in his *jāmi'* of the *Metaphysics*¹⁷⁷ and in his commentary on the *De caelo*. It seems clear that this notion should be interpreted within the Neoplatonic thoughts of Abū l-Barakāt al-Baghdādī, whose works were probably introduced in al-Andalus by Isaac, the son of Abraham ibn 'Ezra, who was his disciple in Baghdad and who wrote a Hebrew poem in his honour in 1143. We should also note that, in spite of al-Biṭrūjī's efforts to base his criticisms of Ptolemy on Aristotle's authority, the dynamics he is using is not Aristotelian but Neoplatonic.

Impetus and *tashawwuq* are used by al-Biṭrūjī to explain the apparent motions of celestial bodies without the contradictions implied in Ptolemaic motions in two different directions. The first mover transmits daily motion from East to West to the planetary spheres, and this transmission will take place in a similar way to that of the *impetus* of the world under the lunar sphere. The power received from the prime mover will diminish in each of the planetary spheres as the distance from the source of power increases, and this reduction is considered as a delay (*taqṣīr*, *incurtatio*) which is analysed by al-Biṭrūjī as a motion in the opposite direction (motion in longitude). This *taqṣīr* is, however, compensated by the desire (*tashawwuq*) that each sphere feels to attain perfection and to resemble the sphere of the prime mover. This desire is greater in the spheres which are nearest the highest sphere, and planetary velocities diminish when their spheres approach the centre of the system because their *taqṣīr* increases and their *tashawwuq* decreases.¹⁷⁸

Al-Biṭrūjī clearly identifies *taqṣīr* with motion in longitude in the case of the planets and the Sun. As for *tashawwuq*, my impression is that it corresponds to planetary motion in anomaly. Most of the passages in the *Kitāb fī l-hay'ā* that explain *tashawwuq* insist that planetary spheres rotate about their own poles when they perform this compensatory motion and, in al-Biṭrūjī's planetary models, the pole of each planet moves in an epicycle (mean motion in anomaly), the centre of which rotates on a deferent placed around the pole of the Universe. This compensatory motion should, logically, take place in the same direction as the daily motion of the celestial sphere (E-W), but al-Biṭrūjī, who is never careful with details and who uses the epicycle, in the same way as Ptolemy, to justify direct and retrograde motions as well as planetary stations,

Parallel ideas can be found in Goldstein, 1971, I, 76, 102; II, 129 and 223–225; Carmody, 1952, pp. 91 (no. 24–25) and 110 (no. 12).

177 Ibn Rushd, 1365 H, p. 136.

178 Goldstein, 1971, I, 102–103; II, 225–227; Carmody, 1952, p. 111 no. 14–17.

only states that the motion of the planet's pole on its epicycle takes place, sometimes, in the E-W direction and, others, in the opposite direction. Only in the case of the Moon, whose motions in longitude and anomaly take place in opposite directions, is he in complete agreement; he feels that his theory is confirmed and he obviously emphasizes the fact.¹⁷⁹

Another problem arises from the fact that, according to al-Biṭrūjī, *tashawwuq* diminishes in planetary spheres the farther away they are from the sphere of the first mover: mean motion in anomaly should also diminish progressively in planetary spheres as they approach the centre of the system, and this is in accordance with the three superior planets. Unfortunately, when we come to the inferior planets and the Moon, we find exactly the opposite situation and al-Biṭrūjī omits any commentary. The eighth sphere (fixed stars) is a special case in which al-Biṭrūjī uses the notion of *taqṣīr* to justify the motion of declination of the stars, while the *istifā'* ("completion", which I think is the specific *tashawwuq* for stars)¹⁸⁰ produces the precession of equinoxes.

5.4.4.4 Conclusions

Al-Biṭrūjī's *Kitāb al-hay'a* has the obvious merit of being the only serious attempt made by one of this group of twelfth-century Andalusī philosophers to apply their abstract principles and to design an astronomical system to replace Ptolemy's. Interestingly, even though its starting point was Aristotelian, the physical ideas applied by our author were Neoplatonic, something which fits neatly with the fact that Ibn Bājjā, apparently the instigator of this school of thought, is an extremely important representative of Neoplatonic dynamics.¹⁸¹

One should acknowledge al-Biṭrūjī's capacity to imagine solutions to problems which were impossible to solve if one accepted its author's premises. Twelfth-century physics was at an extremely primitive stage, if one compares it to mathematics and mathematical astronomy, and could not become the basis of an astronomical system. Al-Biṭrūjī's astronomy is a step backwards in comparison to the level attained by mathematical astronomers of the eleventh century and their successors in the twelfth. Besides, we have to accept that al-Biṭrūjī did not always respect his own principles. His system is homocentric, but his circular motions do not take place around the centre of the Earth: the centres of his eccentres and epicycles are located on the surface of the corresponding planetary sphere. This implies the abandonment of the orthodoxy

179 Goldstein, 1971, I, 144; II, 393; Carmody, 1952, p. 143 no. 8.

180 Goldstein, 1971, I, 80–81, 91–92, 97; II, 145–147, 149, 181–183, 203; Carmody, 1952, p. 95 (no. 5–7), 96 (no. 19), 102 (no. 1–2), 106–107 (no. 29).

181 Pines, 1964; Lettinck, 1994.

represented by the last stage of Ibn Rushd's astronomical thought; one cannot object to this, as he is obliged to do so if he wants to obtain results. Far more important are his inconsistencies related to the direction of the rotations of planetary poles on its epicycles and his clear attempts to cover them up.

5.4.4.5 The Diffusion of al-Biṭrūjī's System

The *Kitāb fī l-hay'a* attained a success and a diffusion it did not deserve. It was translated into Latin by Michael Scott in 1217, in a period of the history of Iberian translations marked by the interest in translating Aristotle's and Ibn Rushd's philosophical works. This Latin version was the main reason for the spread of this book in Europe between the thirteenth and sixteenth centuries. The work was well accepted in scholastic circles during the thirteenth century, which discussed al-Biṭrūjī's system and considered its value as an alternative to Ptolemy's.¹⁸² In the first stage of the European reception of al-Biṭrūjī's book (1217–1240) great scientists like William of Auvergne (ca. 1185–1249) and Robert Grosseteste (ca. 1175–1253) accepted its ideas, although the latter suggested the need for astronomical observations which could confirm the new system. This was followed by a period of confrontation between Ptolemy's and al-Biṭrūjī's system (1241–1270) during which institutions seem to become involved: the Parisian Faculty of Theology was in favour of al-Biṭrūjī, while the Faculty of Arts (where Ptolemy was taught) rejected his ideas. Albertus Magnus (ca. 1193–1280)¹⁸³ made significant criticisms of the Andalusī astronomer when he left Paris in 1248, while Roger Bacon (ca. 1214–ca. 1292) later abandoned his initial Ptolemaic position and became a supporter of al-Biṭrūjī, although he was aware of some of the deficiencies of the latter's system and, like Grosseteste, believed that new observations were needed. In the last third of the thirteenth century (1270–1300) the controversy had, once again, an institutional basis: Dominicans (Ulrich of Strasbourg [d. ca. 1278], Thomas Aquinas [ca. 1225–1274]) opposed al-Biṭrūjī, while Franciscans (Roger Bacon, John Pecham [ca. 1230–1292], Bernard of Verdun [ca. 1300], Duns Scotus [ca. 1266–1308]) first supported the Andalusī astronomer, but later turned against him using different lines of thought from those of the Dominicans. According to Avi-Yonan, the change of attitude of the Franciscans might have some relation with the condemnation of Averroism and Thomism in 1277, since the acceptance of al-Biṭrūjī was mainly due to its coincidence with Aristotelian principles. After the condemnation of some of Aristotle's theses in 1277, it no longer seemed necessary to follow a system based on respect for the physics of the Greek

182 Avi-Yonan, 1985.

183 Cortabarría, 1980; Cortabarría, 1982.

philosopher. After 1300, universities no longer seem to have been interested in al-Biṭrūjī. Nevertheless, echoes of his work are found in the fifteenth and sixteenth centuries: Regiomontanus (1436–1476) had a copy of Michael Scott's Latin translation and seems to have owned a miscellaneous manuscript containing a *Refutatio errorum Alpetragii de motibus celestis*.¹⁸⁴ Finally, Copernicus (1473–1543) himself mentioned al-Biṭrūjī's planetary order, although his source is probably Regiomontanus' *Epitome of the Almagest*.

As for the Hebrew tradition, Mosheh b. Samuel b. Yehudah b. Tibbon translated the book into Hebrew in 1259, and this version was retranslated into Latin by Calo Calonymos and printed in Venice in 1531. The book circulated in the Jewish scientific circles of the Iberian Peninsula quite early: Samuel b. Tibbon (ca. 1165–1232) summarised the contents of the *Kitāb fi l-hay'a* in his glossary of the technical terms used in Maimonides' *Guide for the perplexed* (1213), in his commentary of the Ecclesiastes (after 1213) and in the *Ma'amar yiqqawu ha-mayim* (1221 or 1231). Yehudah b. Solomon ha-Kohen wrote another summary of the work in a scientific encyclopaedia written in Arabic in 1247 and later translated into Hebrew, and al-Biṭrūjī's book reappeared, also in the thirteenth century, in the *Midrash Hokhmah* of the Toledan scholar Judah b. Mattqa. It was criticised in Isaac Israeli's *Yesod ha-'Olam* (1310),¹⁸⁵ and defended enthusiastically by Joseph ibn Naḥmias (ca. 1400) in his *Nūr al-ʿālam*, although he was aware of its deficiencies (5.6.1).¹⁸⁶ Finally, Mosheh b. Tibbon's translation was known to the great Jewish astronomer Levi b. Gerson (d. 1344) who also knew full well the limitations of al-Biṭrūjī's astronomical system.

The book was also known in Eastern Islamic countries from an early date. An Egyptian Christian named Ibn al-ʿAssāl finished his copy of the book, now preserved in the El Escorial Library, on 28 Muḥarram 680/ 18 May 1281. I suspect, however, that it was introduced into Egypt earlier than this, by Maimonides, who lived in Fustāt from 566/ 1165 until his death in 601/ 1204. This Eastern transmission may have had a certain importance: in the prologue of his *Nihāyat al-sūl fi taṣḥīḥ al-uṣūl*, Ibn al-Shāṭir (d. 777/ 1375), the great astronomer of Damascus, gives a list of astronomers who proposed alternative geometrical models to those of Ptolemy.¹⁸⁷ This list includes somebody called al-Majrīṭī. As it seems unlikely that Maslama al-Majrīṭī (d. ca. 1007) designed models of this kind, Saliba (private communication) suggested a possible confusion between the *nisbas* al-Majrīṭī and al-Biṭrūjī.

¹⁸⁴ Carmody, 1951; Shank, 1992.

¹⁸⁵ Robinson, 2003.

¹⁸⁶ Langermann, 1999 a.

¹⁸⁷ Saliba, 1994, pp. 258 and 280.

5.5 Hay'ā in Castile during the Reign of Alfonso X (1252–1284)

5.5.1 Petrus Gallecus' Summa de astronomia

5.5.1.1 Sources and Influence of Twelfth-Century Andalūsī Philosophers
Besides the Alfonsine translation of Ibn al-Haytham's *Fī hay'at al-ālam* (5.5.2), in the second half of the thirteenth century we find another cosmological work in the Iberian Peninsula which deserves mention: Petrus Gallecus' *Summa de astronomia*,¹⁸⁸ a short book written by the Franciscan Pedro González Pérez (ca. 1200–1267) who was the confessor of Prince Alfonso (the future Alfonso X of Castile) and who became bishop of Cartagena in 1250 after the conquest of the city in 1245. The book is dedicated to “Fatri Martino Egidio, abbati de Morerola”, identified with Martin Gil, abbot of the Cistercian monastery of Santa María de Moreruela since 1260 and who was still abbot when Petrus Gallecus died in 1267. It is clear, therefore, that the book was written between 1260 and 1267. The only complete copy extant (ms. Madrid BN 8918) is dated 1 May 1273, as it contains the planetary positions for noon of that day,¹⁸⁹ probably calculated using the *Toledan Tables* (table 5.1), in which the position of Mars is clearly mistaken:

TABLE 5.1 The four elements and the corresponding four qualities

Planets	Text	Recomputed positions (<i>Toledan tables</i>)
Saturn	159°	158;25°
Jupiter	297°	297;34°
Mars	210°	195;13°
Sun	47°	47;32°
Venus	75°	77;30°
Mercury	42°	41;5°
Moon	180°–210°	204;19°

This booklet is an elementary introduction to Ptolemaic astronomy,¹⁹⁰ based on Petrus Gallecus' knowledge of Gerard of Cremona's translations of the

188 Edited in Martínez Gázquez, 2000, pp. 5–63. See also Martínez Gázquez, 1987; Martínez Gázquez, 2002.
189 Martínez Gázquez, 2002, pp. 62–63.
190 For an analysis of its astronomical contents see Samsó, 2000 a.

Almagest and of al-Farghānī's *Jawāmi' ʿilm al-nujūm*, as well as other sources like Ibn Rushd, al-Bīṭrījī, Martianus Capella and Macrobius. As a matter of fact, his direct use of the *Almagest* seems to be documented only in the introduction, and most of his Ptolemaic materials derive clearly from al-Farghānī's summary. Incidentally, he seems to have some indirect information about the observations sponsored by Caliph al-Ma'mūn (r. 813–833) because, when dealing with the obliquity of the ecliptic, after giving Ptolemy's value (23;51°) he states that “secundum uero consideracionem expertam, quam *Johannes filius Almansoris* considerauit in diebus *Maymonidis* et conuenit in ea numerus sapientium, est uiginti tres gradus et uiginti [instead of *triginta*] quinque minuta”.¹⁹¹ It is easy to identify *Johannes filius Almansoris* with Yaḥyā ibn Abī Maṣṣūr (fl. 830) and *Maymonidis* with al-Ma'mūn, although 23;35° (not 23;25° as in the text) is al-Battānī's value (not Ibn Abī Maṣṣūr's) of the obliquity of the ecliptic, also mentioned by al-Farghānī in Gerard of Cremona's translation.¹⁹² Petrus also seems to be aware of the trepidation theories accepted in al-Andalus and the Maghrib between the eleventh and the thirteenth centuries for, although he accepts Ptolemy's constant precession of 1° per century, he refers to the opinion according to which the stars' progression reaches a maximum of 11° (“et nisi sit quod dicitur quod non progrediuntur ultra undecimum gradum”),¹⁹³ which seems a rounding of 10;45°, the maximum value of the equation of trepidation in both the *Liber de motu octauae spere* and the *Toledan Tables* (6.2.1).

Certain passages of this book are clearly related to Ibn Rushd and al-Bīṭrījī's ideas. Thus, right at the beginning he discusses the problem of the speed of rotation of the celestial spheres for which he gives three different theories, the first being that of al-Bīṭrījī, according to whom the eighth sphere is the fastest one and the velocity of the spheres decreases as they approach the Earth. Interestingly, the terminology used by Petrus Gallecus insists, in his description, on the idea of the “delay” (*retardatur*) caused by the motion of the fixed stars and the planets in the opposite direction to that of the daily motion. Besides, this is the only part of the book in which our author mentions the existence of nine spheres; the usual number is eight,¹⁹⁴ derived from Ptolemy and al-Farghānī. A second opinion is that the fastest sphere is the Earth, due to its daily rotation, and that the velocity of the other eight spheres diminishes in accordance with their distance from the Earth. Finally, the third opinion is Ptolemaic, in which the Earth remains motionless in the centre of the

191 Martínez Gázquez, 2000, p. 48.

192 Farghānī, 1910, p. 74.

193 Martínez Gázquez, 2000, p. 56.

194 See for example Martínez Gázquez, 2000, p. 56.

universe. Another passage in which Ibn Rushd is mentioned appears immediately afterwards: there are eight spheres and they move against the starry sphere with their natural motion caused by their rational desire (*desiderium rationale* / *tashawwuq*).¹⁹⁵ A quotation from Ibn Rushd's commentary on Aristotle's *Physics* – in which the Andalusī philosopher insists on Ibn Bājjā's idea that science should be understood as a path of perfection – also appears in the introduction to the book.¹⁹⁶

A diagram appearing on fol. 56v of the Madrid manuscript introduces two extra spheres above the eighth (*sidereum*), which are called *cristallinum* (ninth) and *empireum* (tenth) but this addition does not necessarily correspond to Petrus Gallecus' own thoughts.¹⁹⁷ Whatever the case, it is clear that Petrus Gallecus is always thinking of nested spherical bodies: "Et forma quidem horum orbium est sicut forma spherarum quarum quedam sunt in concauitatibus aliarum".¹⁹⁸

5.5.1.2 Planetary Distances and Sizes

Another interesting point in the *Summa de astronomia* is its treatment of distances (Moon, Sun and Saturn) and the sizes (Moon, Sun, and fixed stars) of some celestial bodies. The corresponding passages, as shown by Martínez Gázquez,¹⁹⁹ can also be found in the *Historia Naturalis* of Juan Gil of Zamora, another Franciscan and a younger contemporary of our author. Both probably obtain their information from a third, unknown, source.

Petrus Gallecus begins with a reference to the size of the Earth, for which he gives two different measures: in his geographical description of the Earth he mentions a degree of the meridian of 66 miles and two thirds, corresponding to a full length of the meridian of 24,000 miles.²⁰⁰ Some sources ascribe this value to al-Ma'mūn's astronomers, although, in Nallino's opinion, it is an Arabic derivation from the Ptolemaic meridian, which uses Posidonius as its source, of 180,000 stadia, with a conversion factor of 1 mile = 7.5 stadia.²⁰¹

195 "Septem [sphere] in quibus sunt septem stelle errantes, qui mouentur contra celum stellatum motu naturali propter desiderium rationale, ut ait Auerrois, *Super librum de celo et mundo*" in Martínez Gázquez, 2000, p. 41. The references to al-Bīṭrījī's system appear in pp. 38–40.

196 "Ut ait Auerrois in *proemio Super librum phisicorum*: « Esse hominis in sua ultima perfectione est ipsum esse perfectum per ciencias speculatiuas »": Martínez Gázquez, 2000, p. 38.

197 On the meaning and origin of these two spheres see Martínez Gázquez, 2000, pp. 25–30.

198 Martínez Gázquez, 2000, p. 57.

199 Martínez Gázquez, 2000, pp. 24–25.

200 Martínez Gázquez, 2000, pp. 49–50.

201 Nallino, 1911, pp. 276–288.

Nevertheless, when Petrus Gallecus deals with planetary distances, he uses a length of the meridian of 12,000 leagues,²⁰² which corresponds to 36,000 miles applying the standard conversion factor of 1 league = 1 *farsakh* = 3 miles. This parameter is also well known and it was used in the Islamic world, in which it was attributed to Hermes. It corresponds to Eratosthenes' meridian of 252,000 stadia, using a conversion factor of 1 mile = 7 stadia.²⁰³ Whatever the case, this is the basic parameter that Petrus uses in the rest of the book.

As for the distances of the celestial bodies, Petrus' starting point is a reference to Martianus Capella (*De nuptiis* VIII, 858)²⁰⁴ according to whom the orb of the Moon is a hundred times greater than the orb of the Earth, which implies that the circumference of the lunar orb is 1,200,000 leagues. To this, he adds that the radius of the lunar orb, the geocentric distance of the Moon, is 180,000 leagues.²⁰⁵ This implies that he is using $\pi = 3 \frac{1}{3}$, instead of the more usual and more precise approximation of $3 \frac{1}{7}$, also mentioned by our author.²⁰⁶

In his calculation of the geocentric distances of the Sun and Saturn, Petrus Gallecus follows Martianus Capella (*De nuptiis* VIII, 461) who considers that all celestial bodies rotate in space at the same velocity. This implies that the solar orb is twelve times greater than the lunar orb and the orb of Saturn 360 times greater than the lunar orb, as the Moon makes a whole rotation in one month, the Sun in one year, and Saturn in thirty years. As a result of this assumption, he assigns a distance of the Sun of 1,800,000 leagues; this is a corrupt number in the manuscript, as it is ten (not twelve) times greater than the radius of the lunar orb.²⁰⁷ The correct distance should be 2,160,000 leagues, something we can confirm with the reference to a longitude of a great circle of the solar orb of 14,400,000 leagues, in the chapter of planetary sizes.²⁰⁸ Again, in the case of Saturn, the distance should be $180,000 \times 360 = 64,800,000$ leagues, but what we find in the text²⁰⁹ is 57,600,000 leagues (56,600,000 in Gil of Zamora) and 59,580,000 for its maximum distance, when the planet is at its apogee. If we assume that Gil of Zamora's 56,600,000 leagues corresponds to Saturn's mid-distance, we can check:

$$1,800,000 \times 30 = 54,000,000$$

²⁰² Martínez Gázquez, 2000, p. 57.

²⁰³ Nallino, 1911, pp. 274–275.

²⁰⁴ Dick, 1969.

²⁰⁵ Martínez Gázquez, 2000, p. 57.

²⁰⁶ Martínez Gázquez, 2000, p. 61.

²⁰⁷ Martínez Gázquez, 2000, p. 57.

²⁰⁸ Martínez Gázquez, 2000, p. 59. It is easy to check that $14,400,000 : (2,160,000 \times 2) = 3 \frac{1}{3}$, a value of π we have already found in our source.

²⁰⁹ Martínez Gázquez, 2000, pp. 57–58.

TABLE 5.2 Planetary distances

Planet	Petrus Gallecus	Ptolemy	al-Farghānī
Moon	100	max. 64 min. 33	max. 64;10 min. 33;33
Sun	1,200	max. 1,260 min. 1,160	max. 1220 min. 1120
Saturn	max. 33,100 min. [30,000]	max. 19,865 min. 14,187	max. 20,110 min. 14,405

which could, possibly, be Saturn’s minimum geocentric distance (at its perigee), as the arithmetic mean between the hypothetic minimum (54,000,000) and maximum distances (59,580,000) is 56,790,000 leagues, not far from Gil of Zamora’s figure of 56,600,000 leagues. Besides this, a marginal note in the manuscript gives a far greater size of the circumference of Saturn’s orb (432,000,000 leagues), which is 30 times the circumference of the Sun (14,400,000 leagues), a value found in another passage of the *Summa de astronomia*.²¹⁰

It seems interesting to compare Petrus Gallecus’ distances with the ones that appear in Ptolemy’s *Planetary Hypotheses* and in al-Farghānī’s astronomical compendium (table 5.2).²¹¹ This comparison shows a certain tendency to increase the size of the Universe, which we will also see in a fifteenth-century Hispanic source, the *Tratado de astrología* attributed to Enrique de Villena (5.6.2). In table 5.2 the unit is always the terrestrial radius (1,800 leagues) and the fractions are given in sexagesimal notation.

Concerning the sizes of planets, our author also starts with Martianus Capella’s “demonstration” that the apparent diameter of the Moon is $\frac{1}{600}$ of the circumference of the lunar orb. Using a terrestrial meridian of 12,000 leagues, a great circle of the lunar orb, according to Martianus Capella’s ratio, will measure 1,200,000 leagues and a diameter of the lunar body will be equivalent to:

$$1,200,000 : 600 = 2,000 \text{ leagues}$$

As for the size of the Sun,²¹² Petrus Gallecus begins by stating that the apparent diameters of the Sun and the Moon are equal, a remark derived from the

210 Martínez Gázquez, 2000, p. 60.
211 Swerdlow, 1968.
212 Martínez Gázquez, 2000, p. 59.

Almagest v,14; however, he specifies that the Moon should be in its apogee. From this he deduces that the true solar diameter should also be equivalent to $1/600$ of the circumference of its orb. As this circumference measures 14,400,000 leagues,²¹³ its true diameter will be:

$$14,400,000 : 600 = 24,000 \text{ leagues}$$

This parameter is confirmed twice in the *Summa*, where we read that the true diameter of the solar disk (24,000) is six times the diameter of the Earth (4,000)²¹⁴ and that the volume of the Sun is 216 times the volume of the Earth, which is also correct as $216 = 6^3$.²¹⁵

To this estimation, Petrus adds a second one, based on Macrobius' commentary to the *Somnium Scipionis* (1,20–26).²¹⁶ The latter author describes an "observation" made in an equinox with a hemispherical sundial (*scaphe*), which allowed him to determine the apparent diameter of the Sun, obtaining $1/216$ of a great circle of the solar orb, a value equivalent to:

$$360^\circ : 216 = 1;40^\circ$$

While, before, we had:

$$360^\circ : 600 = 0;36^\circ$$

The latter value is nearer Ptolemy's estimation (0;31,20°) while the former one can only be compared with the determination of Aristarchus of Samos (2°).²¹⁷

In order to obtain the diameter of the solar body expressed in leagues, Petrus Gallecus again uses the value of 14,400,000 leagues for the length of a great circle of the solar sphere and obtains a result of less ("ad minus") than 70,000 leagues, instead of:

$$14,400,000 : 216 = 66,666.66$$

²¹³ This value has already appeared in a marginal note to fol. 55r of the Madrid manuscript. See Martínez Gázquez, 2000, pp. 57 and 59. It is clearly incompatible with the diameter of the solar orbit of 1,800,000 leagues mentioned on p. 57 and it confirms that the correct value for this diameter is 2,160,000 leagues.

²¹⁴ In the chapter on distances, Petrus Gallecus stated that the length of the Earth's meridian is 12,000 leagues. 4,000 seems a rounded value. If we use $\pi = 3 \frac{1}{7}$, the correct value would be 3,818 leagues.

²¹⁵ Martínez Gázquez, 2000, pp. 58–59.

²¹⁶ Willis, 1970; see Neugebauer, 1975, II, p. 661.

²¹⁷ On these parameters see Neugebauer, 1975, I, p. 106 and II, p. 635.

As a matter of fact, Juan Gil of Zamora gives a better rounding (67,000 leagues). Whatever the case, he seems to realise that his result is too high, because he states that the real diameter of the solar disk is 16 times and “something more” (*et plus*) longer than the diameter of the Earth and

$$66,666 : 4,000 = 16.67$$

while

$$70,000 : 4,000 = 17.5$$

I should note, here, that these absolute values are totally independent of those obtained by Macrobius, whose great circle of the solar orb measures 30,170,000 stadia and a diameter of the solar disk of “almost” 140,000 stadia (the exact value is 139,675.9). If we use the usual conversion factors (7.5 stadia per mile and 3 miles per league) 140,000 stadia equal 6,222.22 leagues. This means that Petrus Gallecus conceives a diameter of the solar disk more than ten times greater than the figure proposed by Macrobius.

In contrast to his treatment of the planetary distances, here our author skips his discussion of the size of Saturn and passes on directly to that of the fixed stars. His argumentation is not the result of a rigorous analysis: as a great circle of Saturn's sphere is 30 times the great circle of the solar sphere, that of the sphere of the fixed stars should be (at least) several times greater. As the great circle of the solar sphere has a length of 14,400,000 leagues, that of Saturn should be:

$$14,400,000 \times 30 = 432,000,000$$

In spite of this, Petrus Gallecus mentions a great circle of the starry sphere measuring 420,000,000 leagues, smaller than that of Saturn's sphere.

To the apparent diameter of the stars he ascribes values between $1/_{20}$ and $1/_{24}$ of the apparent diameter of the solar disk. These are near the ones mentioned by Ptolemy in his *Planetary Hypotheses*: $1/_{20}$ (first magnitude) and $1/_{30}$ (sixth magnitude).²¹⁸

With these values and Macrobius' estimation that the solar disk has an apparent diameter $1/_{216}$ of a great circle of its orb, Gallecus calculates the real diameter of the smallest stars:

$$420,000,000 : (216 \times 24) = 81,018.52$$

²¹⁸ Goldstein, 1967 a.

which he rounds to 80,000 leagues. To this he adds that the diameter of a star is 21 times the diameter of the Earth ($4,000 \times 21 = 84,000$) and, after an arithmetical digression, he states that the ratio between the volumes of two spheres equals the cube of the ratio of their radii, and that, as he has “proved” (in a computation derived from Macrobius) that the diameter of the Sun is 16 times the diameter of the Earth, the volume of the Sun will be 4096 times (16^3) greater than that of the Earth. In the same way, as the diameter of a fixed star is 21 times that of the Earth, its volume should be $21^3 = 9261$, but the text gives as a result 80,841 (*occies milies octogesies quadragesies semel ad minus*), which is absurd.²¹⁹

A summary of Petrus Gallecus’ exposition of the sizes of the planetary bodies, compared to the values given by Ptolemy (*Planetary hypotheses*) and al-Farghānī, is shown in table 5.3 (diameters of the celestial bodies), in which the unit is always the terrestrial diameter (for Petrus Gallecus, 4,000 leagues). The results are given in sexagesimal notation. In the case of the Sun, as we have seen, our author has given two different estimations of the length of its diameter which will be found here as (1) and (2):

TABLE 5.3 Diameters of the celestial bodies

	P. Gallecus	Ptolemy	al-Farghānī
Moon	0;30	0;17,30	0;17,39
Sun	(1) 6 (2) 16 <i>et plus</i>	5;30	5;30
Stars			
1st magnitude	21	4;33	4;45
6th magnitude			

5.5.1.3 Conclusions

The analysis of Petrus Gallecus’ *Summa de astronomia* shows that *hay’a* was a topic of some interest to the scholars in the kingdom of Castile in the second half of the thirteenth century. In this work we find echoes of al-Bīṭrījī’s and Ibn Rushd’s books.

Although Petrus lived in the region of Murcia between 1250 and 1267, as Bishop of Cartagena, and although he worked together with Dominican monks in the foundation of a *Studium arabicum et latinum* associated with

219 Martínez Gázquez, 2000, pp. 59–61.

the translation of Arabic works,²²⁰ our author does not show any knowledge of Arabic. Martínez Gázquez has clearly proved that his quotations from the *Almagest*, al-Bīṭrūjī and Ibn Rushd derive from Latin Toledan translations, which he could have read in the library of the bishopric of Santiago de Compostela in his youth, or in Toledo where he later lived.²²¹

A good part of the *Summa* is dedicated to an attempt to quantify the distances and sizes of celestial bodies. This attempt does not rely on the Graeco-Islamic tradition and derives from late Latin sources (Marcianus Capella and Macrobius). Interestingly, in his distances and sizes, we find a tendency to enlarge the size of the Universe and that of the celestial bodies, which was not uncommon in the Late European Middle Ages. The clearest example of this tendency is to be found in the works of the Jewish astronomer Levi ben Gerson (d. 1344),²²² although, in the Iberian Peninsula in the fifteenth century, we have another clear case in the *Tratado de astrología* of pseudo-Enrique de Villena (5.6.2). My suspicion is that Petrus Gallecus' source for his treatment of this topic is not al-Farghānī, but Marcianus Capella and Macrobius, because the latter offer him a Universe which is far larger than Ptolemy's.

5.5.2 *The Alfonsine Translation of Ibn al-Haytham's Fī hay'at al-ʿālam*²²³

As we have already seen (5.4.2.1), two astronomical works by Ibn al-Haytham, *Fī hay'at al-ʿālam*²²⁴ and *Shukūk ʿalā Baṭlamīyūs*, circulated in al-Andalus from the late eleventh century onwards. The first of these books was the object of two Latin translations in the Iberian Peninsula: one of them is undated and anonymous and it is preserved in a manuscript copied towards the end of the thirteenth or beginning of the fourteenth century;²²⁵ the second is the result of a translation into Latin of a lost Alfonsine Castilian translation and should probably be dated in the second half of the thirteenth century.²²⁶ A previous translation of the same work was made in the mid-twelfth century, apparently in Antioch, by Stephen of Antioch or of Pisa, in a book entitled *Liber Mamonis*,²²⁷ and it is testimony to the existence of a link between Antioch

220 The interest in institutions of this kind appeared in the first half of the thirteenth century: see, recently, Martínez Gázquez & Ferrero, 2018.

221 Martínez Gázquez, 2000, pp. IX–XII.

222 Goldstein, 1986 a.

223 This section contains a revision of Samsó, 1990 a.

224 Edited and translated by Langermann, 1990 The Arabic edition is accessible through academia.edu/Tzvi Langermann.

225 Edited by Millás, 1942, pp. 285–312; see also pp. 206–208.

226 Edition in Mancha, 1990.

227 Edited and studied in depth in Grupe, 2013, which I have been able to read thanks to the generosity of its author. On the process of transmission of Arabic astronomy through Antioch-Pisa, see Burnett, 2000; Burnett, 2003 a; Burnett, 1999.

and Pisa which allowed the transmission to Europe of Eastern astronomical sources which never reached al-Andalus. Besides, five Hebrew translations of Ibn al-Haytham's book are extant: the earliest one was made by Jacob b. Mahir b. Tibbon in southern France ca. 1271 and it was later retranslated into Latin by Abraham of Balmes for Cardinal Grimani in the sixteenth century.

I will deal here with the second Latin translation which I have labelled "Alfonsine" in spite of the fact that, as E.S. Procter notes,²²⁸ there is no evidence that this Latin version was made by the collaborators of king Alfonso X, as is the case of the two Latin translations of the *Libro conplido* (3.2.2.2) and of Ptolemy's *Cuadripartito* (*Tetrabiblos*). We only know that there was a previous Castilian translation, made following the king's orders, which seems to have been lost. The Latin translation was made at an undetermined date but, in the present state of affairs, it is the only evidence left of one of the astronomical works of King Alfonso. In this section I will stress the clear relation existing between this Latin text and the rest of the Alfonsine astronomical production.

Our text begins with a prologue, due to the Alfonsine circle, which replaces the first chapter of Ibn al-Haytham's book and draws the reader's attention to the work done by the king's collaborators and to the interest of the book itself. On the one side, there are ideas found in the first chapter of the original work, such as the fact that Ibn al-Haytham replaces Ptolemy's imaginary circles by solid spheres:

Et ipse [= Ptolomeus] et omnes alij qui locuti fuerunt in scientia ista non fuerunt locuti in corporibus celestibus sed in circulis ymaginatis excepto eo qui edidit librum istum quem nos fecimus tran<s>ferri et ordinari. Vocabatur autem compositor huius libri Abulhazen Abnelaij tam [Abū l-Ḥasan ibn al-Haytham] et quod equidem dixit in hoc libro fuit secundum intentionem Ptolomei.²²⁹

References to *hay'a* and physical astronomy are not common in the works sponsored by Alfonso X, who was mainly interested in Uranography, astronomical instruments, tables, and astrology. The ninth sphere (first mover) is rarely mentioned in the Alfonsine astronomical works. I find one such reference in the *Libro de las taulas*, in a passage related to the trepidation of the equinoxes, in which I read the "çielo alto raso no estrellado" (the highest starless sphere,

²²⁸ Procter, 1945, pp. 20–22.

²²⁹ Mancha, 1990, p. 143. See Langermann, 1990, pp. 5–7. In the following pages I give the reference to the pages of Langermann's edition of the text.

ninth sphere) and “el cielo estrellado” (starry sphere, eighth sphere).²³⁰ A second mention appears in the book on the mercury clock (*Libro del reloj de la plata viva*) where the ninth sphere is the mover of the daily motion of the Universe, as it is “el que faze el día y la noche” (the one that produces day and night).²³¹ In other contexts, the ninth sphere seems to have religious connotations: the sphere of the first mover is also God’s abode in the *Estrellas fijas*²³² as well as in the *Setenario*.²³³ In this latter work the ninth sphere is used as a symbol of the Holy Trinity: the ninth sphere is the Father, its motion is the Son, and the work produced by the two is the Holy Ghost:

Onde, porque los antigos que cuydauan ser sabidores de los cuentos de los cielos e de los mouimientos dellos non pararon mientes al noueno cielo, que es el Padre, nin el mouimientos del, que es el Fijo, nin la obra que sale de amos, que es el Spiritu Santo, erraron en no conoscer a Dios nin creer en el como deuien.²³⁴

Obviously, the king’s main interests are not cosmological. This emphasises the importance of this translation in which, in an *Alfonsine interpolation*, we even find what seems to be a reference to a tenth sphere, which has no clear function:

Et de superiori ipsius superficie [i.e. superficie none spere] non sunt locuti sapientes istius scientie neque de eo quod est supre ipsam²³⁵

The prologue also includes information on the name of the translator and the characteristics of the Castilian translation:

Mandauimus magistro Abrache ebreo quod transferret librum istum de arabico in yspanum et quod ordinaret modo meliori quam ante fuerat ordinatus et quod diuideret in capitula. Et mandauimus de unaquaque res de qua locutus est auctor propiam ponere figuram adhoc ut melius intelligatur (...). Et mandauimus figurari unumquemquem modum circularum qui in isto libro continentur propijs coloribus ut melius cognoscatur.²³⁶

230 Chabás & Goldstein, 2003, pp. 89–90.

231 Ms. Villa Amil 156, fol. 191r; Rico, 1863–67, IV, p. 65.

232 Rico, 1863–67, I, pp. 10–11. This passage is missing in ms. Villa Amil 156.

233 Vanderford, 1984, pp. 66–67 (ley xxxv).

234 Vanderford, 1984, p. 60 (ley xxxvii).

235 Mancha, 1990, p. 165. See Langermann, 1990, p. 64, no. 375.

236 Mancha, 1990, p. 143.

Magister Abracha[*m*] *Ebreus* should be identified with Abraham, also called Don Abraham, one of Alfonso X's Jewish collaborators whose family name is unknown, but who is called "magister", "physicus", "fisicien" and "alfaquim" (*al-ḥakīm*). Together with Bernardo el Arábigo, he was the author of the second Alfonsine translation of Ibn al-Zarqālluh's treatise on the use of the *ṣafiha* (4.3.2), made in Burgos in 1277, as the earlier translation was not considered to be satisfactory. Before 1264 he had also translated the *Escala de Mahoma*, on the legend of the ascension of Prophet Muḥammad to heaven (*mi'rāj*).²³⁷ This is the only information available on this translator and the date of his Castilian translation of Ibn al-Haytham's book is also unknown.

It is also clear that, following the king's orders, Abraham not only translated the book but also restructured its contents. Alfonso X instructed the translator to add coloured illustrations in order to make the contents of the book easier to understand. The king's interest in restructuring the materials and adding figures already appears in the general prologue to the *Libros del Saber*:

Et fizo partir este libro en XVI partes, cada una con estos capitulos que muestran llanamente las razones que en ellas son.

Et fizolas otrossi figurar, porque los que esto quisiessen aprender lo podiessen mas de ligero saber, non tan solamente por entendimiento mas aun por vista.²³⁸

Other examples of the same kind can be found in the *Libro de las cruces* (3.2.1.1), in which one of the translators divided the book into chapters and added a list of the chapters at the beginning:

et porque este libro en el arauigo non era capitulado, mandolo capitular et poner los capitulos en compeçamento del libro, segont es uso de lo fazer en todos los libros por fallar mas ayna et mas ligero las razones et los iudizios que son en el libro, et esto fizolo maestre Johan a su servitio.²³⁹

As well as in the *Libro de la açafeha* (4.3.2), where careful instructions are given regarding the colours to be applied to the illustration appearing on fol. 112v of the royal manuscript (ms. Villa Amil 156): the interval between pairs of parallels (*al-madarāt*) is painted in saffron which, in principle, should be yellow,

²³⁷ Romano, 1971; Roth, 1990.

²³⁸ Not preserved in ms. Villa Amil 156. See Rico, 1863–67, I, p. 4.

²³⁹ Kasten & Kiddle, 1961, p. 1.

although, in the current state of the manuscript, it is now brownish. The meridians (*al-mamarrāt*) themselves are drawn in red.

Et que muchos ombres non podrien entender complidamente la manera de como se faz por las parabras que dixo este sabio que la compuso [i.e. Ibn al-Zarqālluh], mandamos figurar la figura della en este libro [...] Et otrosi porque sean estos cerculos [the *al-madārāt*] mas connoscidos et mas departidos de los otros, fiziemos tinnir lo que a entrell uno et ell otro dellos con açafra. Et mandamos fazer otrosi los çercos que son llamados en arauigo almamarrat [*al-mamarrāt*] que van de un polo del mundo al otro, con vermeion [...] Et porque se fazen muchos et se semeian los unos a los otros fiziemos los sennalar con colores departidas [...].²⁴⁰

This dual task of restructuring and illustrating was applied to the original Castilian translation of Ibn al-Haytham's *Hay'at al-ālam*. The illustrations can easily be justified, although only one of the two extant manuscripts of the Arabic original contains a figure, seven figures mentioned in the text can only be reconstructed from the Hebrew translations. As for the Alfonsine text, it contains references to 45 figures, 30 of which are incomplete or missing, while the extant ones are extremely elementary and Mancha has not reproduced them in his edition.²⁴¹ Obviously the original Alfonsine work added many new illustrations to those appearing in Ibn al-Haytham's *Hay'at al-ālam* and, as we have seen, the drawings were coloured ("Et mandauimus figurari unumquemquem modum circulorum qui in isto libro continentur propijs coloribus").

Explaining the changes introduced by the Alfonsine collaborators in the structure of the original book is more challenging. The *Fī hay'at al-ālam* is divided into 15 chapters, without any further subdivision. The Alfonsine version, for its part, contains two books. The first is an elementary introduction to spherical astronomy, reproducing the contents of chapters 2–8 of the Arabic text, with the only exception of materials related to the sublunar world (heavy and light bodies, and spheres of the four elements), described in chapter 2 of *Fī hay'at al-ālam*,²⁴² which, in the Alfonsine version, are dealt with in Book 2, chapters 13–17.²⁴³ There is a certain logic in this change, as the contents of the first book have a clearly static character and all topics related to the motion of the planets or of the falling bodies have their own place in the second book.

240 Ms. Villa Amil 156 fol. 112r; Rico, 1863–67, III, p. 147.

241 Mancha, 1990, p. 137.

242 Langermann, 1990, pp. 8–10.

243 Mancha, 1990, pp. 196–197. Chapters 13–15 are missing in the Oxford manuscript.

Besides, in the first book, the order of exposition is, usually, the same as in the Arabic text, with the only exception of the division of the Earth into climates and the treatment of geographical longitudes and latitudes which are dealt with in chapter 4 (whose title is precisely “Longitudes and latitudes”) in Ibn al-Haytham’s Arabic book.²⁴⁴ In the Alfonsine translation, these topics have been moved to the end of the first book (chapters 28–30).²⁴⁵ This reorganisation is more difficult to justify, as our text speaks about altitudes over the horizon, oblique ascensions and rising times without first explaining the concept of geographical latitude.

The changes introduced in the second book are even more notable and the rearrangement is more radical. The *Fī hay’at al-‘ālam* explains the planetary spheres in ascending order, with the sole exception of the Sun, which is dealt with first: Sun, Moon, Mercury, Venus, the upper planets (which are dealt with together), the fixed stars and the highest orb which corresponds to the first mover. This traditional order is probably influenced by the *Almagest*, which follows this order, although it introduces the treatment of fixed stars between the Sun and the Moon. In the Alfonsine translation the spheres are presented in descending order, as explained in the text itself:

Et post modum loquimur de unoquoque per se secundum quod descendunt ordinate²⁴⁶

In fact, our text deals first with the *celum majus* or *superius* and continues with the fixed stars, Saturn, Jupiter, Mars, Sun, Venus, Mercury and the Moon. This corresponds precisely to the order in which the spheres are mentioned in chapter 3 of the first book.²⁴⁷

The corresponding passage of the *Fī hay’at al-‘ālam* follows, obviously, the reversed order.²⁴⁸ This change in order may, possibly, be the result of the influence of al-Bīṭrījī’s *Kitāb al-hay’a* which, as we have seen (5.5.1.1), was known to Petrus Gallecus. The Andalusī author also applies the descending order although he locates the Sun between Mercury and Venus.

In other cases the changes are mere repetitions. As Ibn al-Haytham only intends to offer a qualitative explanation of the structure of the Universe, he deals with the three upper planets (Mars, Jupiter and Saturn) in the same

²⁴⁴ Langermann, 1990, pp. 16–20.

²⁴⁵ Mancha, 1990, pp. 163–165.

²⁴⁶ Mancha, 1990, p. 165.

²⁴⁷ Mancha, 1990, pp. 144–145.

²⁴⁸ Langermann, 1990, p. 11, no. 41.

chapter²⁴⁹ and does the same with certain partial aspects of the two lower planets (Mercury and Venus). This structure is not respected in the Alfonsine translation, in which each one of the three upper planets has an independent treatment in three parallel chapters.²⁵⁰ An explanation for this unusual characteristic can be found in an Alfonsine interpolation placed at the end of the third chapter of the second book, which explains Saturn's model:

Et quamvis celum Iouis et Martis et motus quos ipsi faciunt sint similes motibus Saturni de quibus locuti sumus, volumus tamen loqui de unoquoque ipsorum per se ut hic noster liber sit magis completus et propterea ut qui voluerit in eo motus horum planetarum inquirere inveniat in eo cercius quod querit [*sic*].²⁵¹

According to this passage, the purpose of the translator is to offer a more complete work and to make the task of the reader easier, allowing him to find information on a particular planet more rapidly.

The treatment of Venus and Mercury is similar. In *Fī hay'at al-ʿālam* we find Mercury²⁵² before Venus²⁵³ and, consequently, when it deals with the second planet, we often find a reference to Mercury so as not to repeat common characteristics (the latitude theory, for example). The Alfonsine version follows a different order, explaining Venus' model first²⁵⁴ and Mercury's model afterwards;²⁵⁵ due to the translator's interest in completeness, there are materials in his treatment of Venus borrowed from the ones explained by Ibn al-Haytham in his chapter on Mercury.

As the number of chapters has increased considerably in the Alfonsine translation (30 in the first book and 18 in the second) in relation to Ibn al-Haytham's book (15 chapters), most of the titles are additions by the Alfonsine adapter and there is only a total or partial coincidence when an Alfonsine chapter coincides with the beginning of another chapter of the *Fī hay'at al-ʿālam*. In this latter case the Alfonsine title is usually longer and more explicit, as we can see in the following examples:

249 Langermann, 1990, pp. 58–60.

250 Mancha, 1990, pp. 136–137.

251 Mancha, 1990, p. 170.

252 Langermann, 1990, pp. 47–54.

253 Langermann, 1990, pp. 55–57.

254 Mancha, 1990, pp. 178–182.

255 Mancha, 1990, pp. 182–188.

Arabic title	Latin title
Chapter 2: <i>Al-qawl fī jumlat al-‘ālam</i> (On the World as a whole)	Chapter 1.2: “Quod est quod dicitur philosopho nomen mundus et qualis est ipsius figure et in quot partes diuidantur generaliter ea que in ipso sunt”
Chapter 3: <i>Al-qawl ‘alā l-falak</i> (On the celestial sphere)	Chapter 1.3: “Quid dicitur per hoc nomen celum et in quot partes diuiditur et qualiter mouetur generaliter”
Chapter 5: <i>Al-qawl ‘alā falak al-burūj</i> (On the zodiac)	Chapter 1.14: “De circulo zodiaci et de quatuor temporibus anni”
Chapter 15: <i>Al-qawl ‘alā l-falak al-a‘lā</i> (On the supreme sphere)	Chapter 2.1: “De celo superiori maiori in quo sunt omnes alii celi”

Here, as in the rest of the Alfonsine astronomical production, we see a tendency to structure the contents in short chapters with explicit titles which allow the reader to find easily what he is looking for.

There are some other interesting features that link this Latin translation with the rest of the Alfonsine works. Mancha,²⁵⁶ for example, has drawn attention to two references in the Latin text (1.15 and 2.2) to the *Libro de las estrellas fixas*. It may be useful to repeat here a longer version of the second passage:

Et quia iste 1022 stelle fuerunt et sunt magis note, distinxerunt eas sapientes antiqui in quadraginta <et octo> partes **secundum quod ostendimus in primo libro Figurarum celi**. Et sunt in isto celo alie stelle valde minute quarum nullus potest accipere altitudinem et non numerantur inter istas 1022 de quibus locuti sumus.²⁵⁷

This passage is interesting for two reasons. On the one hand, the sentence “secundum quod ostendimus in primo libro Figurarum celi”²⁵⁸ is an obvious reference to the *Libro de las estrellas fixas* and it raises the question of whether Ibn al-Haytham’s Castilian translation was intended to be included in the Alfonsine *Libro del saber*. On the other, the second part of the sentence (from

²⁵⁶ Mancha, 1990, p. 136.

²⁵⁷ Mancha, 1990, p. 167.

²⁵⁸ In 1.15 (Mancha, 1990, p. 154) we find: “secundum quod ostendimus in libro in quo locuti sumus de ipsis”.

“Et sunt ...”) is an interpolation in Ibn al-Haytham's text which refers to the stars not included in Ptolemy's catalogue either because they are very small or because they are not visible in our latitudes.²⁵⁹ Our Alfonsine version seems to refer to the great number of stars smaller than those of the sixth magnitude, which are not included by Ptolemy in his catalogue but are mentioned by al-Šūfī in his *Kitāb šuwar al-kawākib al-thamāniya wa l-arbaʿin*, the main source of the *Libro de las estrellas fixas*.²⁶⁰ In this latter work the references to these small stars are generally omitted, although, at the end of the book, we find at least one allusion to the stars that Ptolemy did not mention, among which we find a great number of stars located outside the constellation of “el inflamado” (*al-multahab, Cepheus*) which are impossible to count.²⁶¹

There are also parallelisms between our text and the other Alfonsine astronomical sources with regard to the technical terms. Mancha has noted, for example, the use of *circulus zontinus* which always translates *al-dāʿira al-samtiyya* (vertical or azimuthal circle).²⁶² I can add the use of *rectificacio*²⁶³ meaning “observation” (*raṣd*), also found in other Alfonsine sources.²⁶⁴ Abraham Hebreus has a certain tendency to translate literally²⁶⁵ which leads him to translate *fī suṭūḥ al-rukhāmāt* (on the surfaces of sundials) by *in superficiebus marmorum*,²⁶⁶ in which it is difficult to recognise, in *marmor*, the technical meaning of the word *rukhāma*. Certain terms lack precision both in Ibn al-Haytham's text and in our Latin translation: the Arabic word *mayl* usually means the solar declination or the declination of a point of the ecliptic, while *buʿd ʿan muʿaddil al-nahār* (distance from the equator) is used to denominate the declination of a star or planet and, usually, this distinction is maintained in Castilian or Latin translations of Arabic astronomical texts. The title of chapter 1.22 is, however, *in demonstrando declinacionem stellarum fixarum et eciam planetarum*,²⁶⁷ which corresponds literally to the Arabic *fa ammā mayl al-kawākib al-saqqāra wa l-thābita*,²⁶⁸ where *mayl* is also used for the declination of a star or planet. Also, when Ibn al-Haytham speaks about the “second declination” (*al-mayl al-thānī*, although this denomination does not appear in

259 Langermann, 1990, p. 63, no. 374.

260 Comes, 1990; Samsó & Comes, 1988.

261 Ms. Villa Amil 156 fol. 25r; Rico, 1863–67, I, p. 143; Comes, 1990, pp. 58–59.

262 Mancha, 1990, p. 135.

263 Mancha, 1990, pp. 166 and 185.

264 Bossong, 1978, pp. 298–300.

265 Millás, 1933.

266 Langermann, 1990, p. 14, no. 72; Mancha, 1990, p. 149.

267 Mancha, 1990, p. 159.

268 Langermann, 1990, p. 28, no. 157.

the Arabic text) – the distance to the equator or a point of the celestial sphere, measured on a great circle passing through the poles of the ecliptic – there is also a certain lack of terminological precision, as the text uses both *mayl* and *ʿarḍ* (which normally means latitude).²⁶⁹ The Alfonsine translation only uses *latitudo*:

et similiter si iste circulus maior exierit a duobus polis zodiaci vocatur arcus supradictus latitudo stelle vel planete respectu circuli equatoris.²⁷⁰

Another example of the peculiar terminology used by Ibn al-Haytham is his use of *al-dawāʾir al-zamāniyya* (time circles) to name the declination parallels, an expression that is translated literally in the Alfonsine version (*circuli temporales*).²⁷¹ Ibn al-Haytham also uses *madārāt al-burūj* for the declination parallels corresponding to the beginnings of the zodiacal signs.²⁷² As for the Latin translation, it transliterates *almadarach* or gives a literal equivalence, in the title of the chapter, *de circulis circularibus*.²⁷³ This expression is also found in other Alfonsine astronomical sources like al-Battānī's canons in which *circulario* translates *madār*,²⁷⁴ or in the *Libro de la açafeha*, where we can read:

Et estos çercos que son llamados en arauigo almadarat dizen en castelano cerculos cerculares²⁷⁵

The most interesting of these technical terms appearing in our Latin translation is, probably, *figura pinee/figura pineata/pineatum*, meaning conical figure or cone. This use is somewhat surprising because Ibn al-Haytham's text has, in all cases, *makhrūt* (cone), which appears in three different contexts:

1.- When the Sun is at any point of the ecliptic which is not one of the equinoxes, the radius of the celestial sphere joining the centre of the Earth with the centre of the Sun generates, when it follows the Sun in its daily motion, a cone of revolution. Ibn al-Haytham applies this only if the Sun is in its perigee.²⁷⁶ Let us see the Latin source:

²⁶⁹ Langermann, 1990, p. 28, no. 157.

²⁷⁰ Mancha, 1990, p. 159.

²⁷¹ See, for example, Langermann, 1990, pp. 16–18; Mancha, 1990, pp. 149–151.

²⁷² Langermann, 1990, pp. 24–25.

²⁷³ Mancha, 1990, p. 156.

²⁷⁴ Bossong, 1978, pp. 167–168.

²⁷⁵ Ms. Villa Amil 156, fol. 112r; Rico, 1863–67, III, 147.

²⁷⁶ Langermann, 1990, p. 37, no. 205.

Et quando ymaginati fuerimus solem stantem in suo hadidiz [*ḥadīd*, perigee], hoc est depressione, et exire una linea recta a centro mundi ad centrum ipsius et ymaginamur solem mouentem se in directo terre, faciet illa linea superficiem pineatam²⁷⁷

2.- According to the theory of vision adopted by Ibn al-Haytham in this book, the text speaks about a cone of light whose vertex is in the human eye and whose base is the object seen.²⁷⁸ The corresponding Latin translation is the following:

Et lumen egreditur ab oculis hominum secundum figuram pinee et vocatur in arabico *mayhot* [*makhrūt*] et caput ipsius est punctum visus et basis fundamenti, que vocatur in arabico *thayda* [a corruption of *qā'ida*, base], erit in superficie eiusdem rei que videtur²⁷⁹

3.- Finally, the terms *figura pineata* and *pineatum* are used to name the shadow cone (*makhrūt al-ẓill*, *pineatum umbre*) projected by the Earth in a lunar eclipse.²⁸⁰

The use of these terms is related to the Arabic *ṣanawbar* (pine tree), but also cone.²⁸¹ At least one manuscript of the Arabic translation of Apollonius' *Conics* (British Museum 426, fol. 164v) bears the title *Kitāb fī l-ashkāl al-ṣanawbariyya* (On conical figures) and al-Šūfī, in his treatise on the astrolabe, uses *ṣanawbar* for the shadow cone in a lunar eclipse.²⁸² In the Alfonsine astronomical production we have already seen that the treatise on the construction of Ibn al-Zarqālluh's equatorium describes an approximate method for drawing Mercury's deferent, identified as an ellipse, and uses *figura pinnonata* (*shakl ṣanawbarī*) in its description of a conical section (4.5.3). Another instance of the use of this peculiar terminology is to be found in the *Libro de las estrellas fixas*, where, in its treatment of the constellation of *Corona meridional* (southern crown, *al-Iklīl al-janūbī*), the shape of several types of crowns is described, and we read:

277 Mancha, 1990, p. 178.

278 Langermann, 1990, p. 44, no. 255.

279 Mancha, 1990, p. 193.

280 Mancha, 1990, pp. 194–195; Langermann, 1990, pp. 44–45, no. 260–261.

281 Souissi, 1968, p. 221.

282 Kennedy & Destombes, 1983, p. 435.

Mas esta de mediodia no es dessa manera fecha, ca es toda redonda et cerrada, et suso aguda a manera de pinna, et por esso la llaman figura pinneal.²⁸³

This expression has an exact equivalent in the main source of *Estrellas fixas*: al-Šūfī's *Kitāb šuwar al-kawākib*, where the Eastern author, in his description of this constellation, says that it resembles a *shakl šanawbarī*.²⁸⁴

Our source contains a passage which has been the object of a recent discussion:

“Et vocatur arcus circuli transeuntis per caput Geminorum et Sagitarii declinacio Arietis et Thauri [*replaced by Taurus and Scorpio in Langermann's edition*]. Et nominatur arcus circuli transeuntis per caput Leonis et Aquarij declinacio Leonis et Virginis [*Cancer and Capricorn in Langermann*]. Et nominatur arcus circuli transeuntis per caput Virginis et Piscis declinacio Virginis” [*Leo and Aquarius in Langermann*]²⁸⁵

Obviously, in Langermann's interpretation,²⁸⁶ Ibn al-Haytham is alluding to zodiacal signs symmetrical about the equinoxes which have the same declination if one does not consider its positive or negative sign. Thus:

- δ (60°) [end of Taurus and beginning of Gemini] = δ (240°) [end of Scorpio and beginning of Sagittarius]
- δ (120°) [end of Cancer and beginning of Leo] = δ (300°) [end of Capricorn and beginning of Aquarius]
- δ (150°) [beginning of Virgo and end of Leo] = δ (330°) [beginning of Pisces and end of Aquarius]

Another interpretation has been proposed by Dirk Grupe, in his unpublished doctoral dissertation,²⁸⁷ following the commentary added by Stephen of Antioch to his translation of Ibn al-Haytham's book: the arc of the equatorial

283 Ms. Villa Amil 156, fol. 17v; Rico, 1863–67, I, p. 115.

284 Šūfī, 1954, p. 243; Schjellerup, 1874, p. 253.

285 Mancha, 1990, p. 157. I followed Langermann's interpretation in Samsó, 1990 a, p. 126. The anonymous Latin translation edited by Millás (1942, p. 294) agrees with the Alfonsine translation: “Arcus transiens per caput geminorum et per sagitarii uocatur declinacio arietis et tauri, arcus transiens per caput leonis et per caput aquarii, uocatur declinacio leonis et uirginis, arcus autem transiens per capud uirginis et per capud piscis uocatur declinacio uirginis”.

286 Langermann, 1990, p. 26 no. 146–148. The variants appearing in the two extant manuscripts as well as in the Latin and Hebrew translations forced the editor to choose an interpretation. See the critical apparatus in p. 110.

287 Grupe, 2013, pp. 37–39, 182.

meridian comprised between the beginning of Gemini and the equator equals the similar arc between the beginning of Sagittarius and the equator, and it is named “declination of Aries and Taurus”, using the equinoctial point as the beginning of the ecliptic arc to which the declination corresponds. He could also have added, in the case of Sagittarius, that it is named “declination of Libra and Scorpio”. This appears explicitly in Stephen's *Liber Mamonis*, but Ibn al-Haytham seems to have considered it superfluous and unnecessary. In the case of the great circle passing through the beginnings of Leo and Aquarius he considers the solstice (Cancer) as his starting point, and this is why he speaks of the declination of Leo and Virgo. Finally, in his reference to the beginnings of Virgo and Pisces the beginning of the arc is, once more, the equinox (Libra) and he refers to the declination of Virgo, considering (in the opposite direction to that of the signs) the arc Libra – Virgo.

I should add something about the errors made by Abraham Hebreus in his translation of Ibn al-Haytham's book. The most notable one appears in his descriptions of the planetary models for Saturn, Jupiter and Mars which, as we have already seen, are almost identical. In these three cases the equant point is placed in the midpoint between the centre of the Earth and the centre of the deferent. Thus, in the description of Saturn's planetary model, we read:

Et ponamus dyametrum spere epicycli in rectitudine linee longitudinis longioris, et quando mouetur celum deferens mouet secum celum epicycli et similiter mouetur dyameter ipsius a rectitudine linee supradicte. *Et tunc invenitur in directo alterius puncti quod non est centrum deferentis neque centrum mundi, sed in medio utriusque super lineam rectam cum ipsis et distat equaliter ab utroque.*²⁸⁸

This is the second time we have found mistake of this kind in the Alfonsine astronomical production: we have already seen it in the treatise on the construction of Ibn al-Samh's equatorium (4.5.2) which led us to suggest the possibility of Abraham Hebreus' being the translator of this latter text as well.

²⁸⁸ Mancha, 1990, p. 169. This is something we do not find in Ibn al-Haytham's book: the position of the equant point is not described in the chapter on the upper planets (Langermann, 1990, p. 59 no. 346) where we find a reference to the model for Venus (Langermann, 1990, p. 55 no. 326) in which the centre of the equant is located at a distance from the centre of the deferent equal to the distance between the centre of the deferent and the centre of the Universe. The anonymous Latin translation edited by Millás (1942, p. 309) agrees with Ibn al-Haytham's description.

Another unusual mistake, probably due to the manuscript of the *Fī hay'at al-ʿālam* that circulated in the Iberian Peninsula, is found in the chapter on altitudes, in which we read:

Et maior altitudo quam habeant stelle vel planete est sinus medietatis eius diei²⁸⁹

And in the anonymous Latin translation we also find:

Maxima ergo eleuationum stelle est sinus medietatis eius diei²⁹⁰

These quotations have little relation with Ibn al-Haytham's text; there, we find an explanation of the sine function followed by a sentence, not found in the Alfonsine translation, which states that "sines are often used as, from them, arcs of the circle can be obtained" to which the Arabic text adds "the maximum altitude of stars is their meridian altitude". All this suggests that the two translators were using a defective Arabic manuscript which neither was able to correct.

I will end this series of comments with a peculiar passage beginning with chapter 1.25, entitled *De eleuationibus*,²⁹¹ a term which translates *maṭālīʿ* (ascensions, both right and oblique).²⁹² Most of the chapter is, in fact, related to the rising times of zodiacal signs. Immediately afterwards we have chapter 1.26 (*De diuersitate eleuacionum propter diuersitatem orizontium*),²⁹³ whose final paragraph is an Alfonsine addition which develops an idea that has been already explained by Ibn al-Haytham: arcs of the ecliptic symmetrical about the equinoxes have the same rising times. Our Arabic source adds that the rising times of Aries and Pisces diminish when the geographical latitude increases, while the opposite occurs with the rising times of Virgo and Libra. The Alfonsine translation replaces this latter remark with:

et quando Virgo et Libra fuerint in uno orizonte secundum unam quantitatem, erunt eleuationes sue in alio orizonte quod est maioris latitudinis quam ipse minoris quantitatis

²⁸⁹ Mancha, 1990, p. 160.

²⁹⁰ Millás, 1942, p. 296.

²⁹¹ Mancha, 1990, pp. 129–130.

²⁹² Langermann, 1990, pp. 31–34.

²⁹³ Mancha, 1990, p. 162.

Langermann has noted this error made by the Alfonsine adapter. It is easy to check that, if the latitude increases, the rising times of Aries/ Pisces, Taurus/ Aquarius, and Gemini/ Capricorn diminish, while those of Cancer/ Sagittarius, Virgo/ Libra and Leo/ Scorpio increase.

To conclude, *hay'a* was not the kind of topic that interested King Alfonso x or the group of scholars who worked under his patronage. As we have already seen, only two works on *hay'a* written during his reign seem to have survived: Petrus Gallecus' *Summa de astronomia* (5.5.1) and the translation of Ibn al-Haytham's *Fī hay'at al-'ālam*, extant in a Latin translation of a previous lost Castilian text. This book by Ibn al-Haytham is not as important as his *Shukūk 'alā Baṭlamīyūs*, as it is merely a three-dimensional reinterpretation of Ptolemy's planetary models, without any significant innovation or criticism. Although we do not have the original Castilian text, my analysis of the Latin translation shows that it has all the characteristics of the Alfonsine astronomical sources: the book has been restructured and divided into smaller chapters; titles and illustrations have been added; there is a clear interest in making the book clearer and more accessible; the technical vocabulary has many points in common with the terminology used in other Alfonsine astronomical books,²⁹⁴ and there are translator's errors that are also found in the latter sources. On the whole, although I cannot prove it, I believe that this Latin translations is also Alfonsine.

5.6 Other *hay'a* Sources between the Twelfth and the Fifteenth Centuries

5.6.1 Jewish Sources

Jewish interest in *hay'a* in the Iberian Peninsula seems to have begun in the twelfth century with Abraham bar Ḥiyya's *Ṣurat ha-Areṣ* (1133)²⁹⁵ which, in spite of its title (description of the Earth), is not a treatise on geography but an elementary introduction to spherical astronomy, followed by a (more or less) detailed description of Ptolemy's planetary models. It also contains an analysis of the sphericity of the Earth and the heavens, a description of the Ptolemaic system of climes, and references to the measurement of a degree of the meridian made by the astronomers of the Abbasid Caliph al-Ma'mūn ca. 830. This is connected with the distances and sizes of planets, expressed in terrestrial radii, in which Bar Ḥiyya follows indirectly the tradition of Ptolemy's

²⁹⁴ Samsó, 2008–9.

²⁹⁵ Millás, 1956 a.

Planetary Hypotheses. To all this he adds references to cosmological topics, like the number of spheres existing in the cosmos and the order of the planetary spheres, focusing mainly on the problem of the position of Venus and Mercury above or beneath the Sun. As established by Millás, the main sources used by Bar Ḥiyya are al-Farghānī's *Kitāb fī l-ḥarakāt al-samāwīyya wa-jawāmī' ilm al-nujūm* and, to a lesser degree, al-Battānī's *zīj*. I believe that he may also have used other sources.²⁹⁶ My impression relies on his chapter on the precession of equinoxes,²⁹⁷ in which he may have used materials derived from Ibn al-Zarqālluh's book on the motion of the fixed stars (6.2.2).²⁹⁸

In spite of this beginning in the twelfth century, there is little information about the continuation of the process. Quite recently, however, a most important *hay'a* treatise, written by Joseph Ibn Naḥmias ca. 1400, has been the object of a critical edition, English translation and commentary by Robert G. Morrison.²⁹⁹ Practically nothing is known about the author, who seems to have belonged to an Aragonese family and who possibly was a practising astrologer, although his astronomical book does not contain any references to astrology.³⁰⁰ His cosmological work has been preserved in two versions: in Judeo-Arabic (*Nūr al-ʿālam* = Light of the world) and in Hebrew (*Or ha-ʿolam*), although the Hebrew recension is not only a strict translation of the Arabic text but, in a way, a second edition of the book carried out either by Ibn Naḥmias himself or by somebody else – probably with the collaboration of the author and during his lifetime.³⁰¹ The extant text seems unfinished or incomplete, for it deals only with the Sun, the Moon and the fixed stars, while the Arabic version contains the beginning of his treatment of the planets.

Nūr al-ʿālam follows al-Bīṭrūjī's line of thought (5.4.4) although the author is perfectly aware of the shortcomings of his predecessor's models and aspires to design new ones with the same predictive capacity as those of the *Almagest*, a source that he seems to know and understand much better than al-Bīṭrūjī. Among the important innovations that Ibn Naḥmias introduced, Morrison underlies the fact that he accepts the possibility of celestial bodies with apparent motions in opposite directions. As a matter of fact, when he deals with each

296 Fontaine, 2000.

297 Millás, 1956 a, pp. 119–121; Millás, 1959, p. 93; Millás, 1960 a.

298 Samsó, 2004 d, pp. 302–305.

299 Morrison, 2016 a.

300 Abraham Zacut reports that he wrote an astrological prediction for 1478: Morrison, 2016 a, pp. 5–6.

301 The significant insertions introduced in the Hebrew recension are translated in Morrison, 2016 a, pp. 241–262.

model, he offers two possibilities and two different solutions: an apparent motion in opposite directions (which he seems to prefer) and in only the direction of universal motion using, for the latter, al-Bīṭrūjī's idea of the lag (*taqṣīr*). In both cases, however, he considers that the transmission of motion from the first mover to the inner orbs is due to the desire (*tashawwuq*) of each orb to imitate the motion of the one above (5.4.4.3).

Another innovation introduced by Ibn Naḥmias is his replacement of the "polar epicycle" described by al-Bīṭrūjī (5.4.4.2) with another small circle, called *dā'irat mamarr al-markaz* (circle of the path of the centre), located near the equator of the corresponding orb. The pole of this small circle moves, in the case of the Sun, on a great circle (a circle carrying the circle of the path of the centre) with the solar mean motion in longitude, while the Sun rotates on this small circle, which accounts for the solar anomaly, as this great circle is inclined to the ecliptic by the maximum solar equation ($2;30^{\circ}$),³⁰² which is also the length of the radius of the circle of the path of the centre. The model tries also to keep the Sun on the ecliptic (and the Moon at a reasonable distance from it), an attempt that is unsuccessful. For both the Sun and the Moon, Naḥmias introduces a system of two small circles (a double epicycle) with the same radius; the inner one passes by the pole of the outer circle, while the latter's pole moves on the circle of the path of the centre. Both epicycles rotate in opposite directions and at different speed: the inner epicycle rotates at half the speed of the outer one. This system of a double epicycle is mathematically equivalent to the spherical couple of Naṣīr al-Dīn al-Ṭūsī,³⁰³ and this poses the problem of the possible access of our author to al-Ṭūsī's *Tadhkira*.

The aforementioned model gives fairly good results for the solar anomaly. The lunar model is obviously more complicated, and it is developed in two steps. The first one corresponds to the lunar positions at syzygies and is basically equivalent to the solar model, although Ibn Naḥmias does not give details about its dual epicycle system. In the second step he tries to justify the position of the Moon at quadratures and, in it, our author adds a small epicycle moving around the circle of the path of the centre, which is its deferent. The Moon rotates on this small epicycle at twice the speed of the pole of the circle of the path of the centre on its deferent great circle. There is no explanation, however, of how Ibn Naḥmias conceives the combination of this epicycle with the dual epicycle used to justify the lunar latitude. Furthermore, our author believes

302 $2;30^{\circ}$ is the Ptolemaic solar eccentricity.

303 This description corresponds to Ibn Naḥmias' second attempt to describe his system of double epicycles. See Morrison, 2016 a, pp. 302–4, 306–8; Saliba & Kennedy, 1991.

that this second correction also gives good results at the octants and does not introduce anything equivalent to Ptolemy's third lunar model. Another problem is the fact that he does not introduce any variation in the distance between the Moon and the Earth (which is impossible in homocentric astronomy) and he rejects the results of observations that have established the existence of changes in the apparent lunar diameter. According to Ibn Naḥmias, this variation may be due to parallax or to instrumental and/or observational errors. This scepticism is severely criticised by Profiat Duran (d. 1415).³⁰⁴

Finally, his treatment of the motion of the fixed stars is based on constant precession (for which he gives no parameter) which can be justified with a single orb. Ibn Naḥmias rejects variable precession (al-Biṭrūjī) as well as trepidation and is, once more, sceptical about observations justifying changes in the rate of precession, although he suggests that different orbs, each one of them for a pair of stars and moving at different speeds, may exist. What earlier scholars identified as trepidation was, in fact, the presence of different stars with different fixed rates of precession.

Apart from Ibn Naḥmias' book, we have no information about treatises on *hay'a* written by Iberian Jewish authors. We know that Judah ben Verga (fl. 1455–1480) wrote a *Zeh Sefer Toledot Shamayim ve-ha-Areṣ* and a commentary on al-Farghānī's book, but this source has not been studied. We are, however, aware of the existence of two texts, written in Hebrew and in Arabic, by two Jewish scholars living in the Christian kingdoms of the Peninsula. One of them is a brief text (only one page) discovered by Langermann in a Hebrew manuscript of the fourteenth or fifteenth century, written by a "Spanish hand".³⁰⁵ It is the beginning of a treatise on *hay'a*, and it analyses Ptolemy's eccentric solar model; the author considers it acceptable from a cosmological point of view, but he rejects the epicycles of the lunar and planetary theory and plans to describe planetary models which will consist of an eccentric only. His description of the solar eccentric is three-dimensional and he rejects the opinion of "later astronomers" who consider the two parts of the solar parecliptic as two separate orbs.

A more interesting text is found in a passage from the anonymous fourteenth-century *Kitāb al-ṭibb al-qashṭālī al-malūkī* (Royal Castilian medicine).³⁰⁶ This work contains a curious astronomical digression which

304 Morrison, 2016 a, pp. 395–398.

305 Langermann, 2001, pp. 325–327.

306 García Ballester & Vázquez de Benito, 1990; Vázquez de Benito, 2004; Vázquez de Benito, 2001.

shows that the interest in *hay'a* was still alive in the fourteenth century. The author deals with the problem of the amount of blood that should be extracted in a phlebotomy; he notes that, since Galen's time, this amount has diminished considerably and finds a possible explanation – that human bodies have also diminished in size, as it has been seen that the bones of men who lived in previous times are larger than those of more recent ones. Created beings change with time and the same happens in the world of celestial bodies, something which contradicts the Aristotelian principle according to which the supralunar world is invariable. This supralunar world is called the “intermediate world” (*al-ʿalam al-awsaʿ*) which the author identifies with the world of the celestial spheres (*ʿalam al-aflāk*) – something which indicates, in my opinion, the author's belief that there is an upper world (*al-ʿalam al-aʿlāʾ*?, that of the first mover?), and another inferior world (*al-ʿalam al-adnāʾ*?), located below the lunar sphere. The intermediate world changes (*mutaghayyir*). The first symptom of these changes is the length of the solar year, whose estimation has varied since the time of Hipparchus until the time the book was written. Sometimes it is longer and sometimes shorter, and nobody has been able to discover a system (*nizām*) which could justify these changes. Recent authors like Ibn al-Zarqālluh/Zarqiyāl (*al-Z.r.q.y.l*) have not succeeded in discovering a “real model” (*hay'a*) which could account for these variations, and have only described models which are imaginary and purely descriptive, with no physical reality. The same thing occurs with the changes in the obliquity of the ecliptic, the slow motion of the fixed stars (precession) and the mean motions of the planets. The anonymous author's attitude implies a serious criticism of Ibn al-Zarqālluh's mathematical models which try to explain and predict the variations of the obliquity of the ecliptic and the different estimations of the precession of equinoxes. These ideas may derive, ultimately, from Ibn al-Haytham's *Shukūk ʿalā Baṭlamīyūs* which, as we have seen, circulated in al-Andalus since, at least, the end of the eleventh century. The rest of the text also insists on the imaginary character of the planetary models, which have no real existence, and gives specific examples of the physical difficulties posed by Ptolemy's lunar model and by the inclination and oscillation of the deferents of Mercury and Venus, used by Ptolemy to justify their latitudes. These are the reasons why there are changes in the mean motions and equations of the planets, implying that *zīj*es are valid only for a limited amount of time and that the title *al-Amad ʿalā l-abad* (valid for eternity) of [Ibn al-Kammād's] *zīj* is absurd. As this intermediate world acts (*fāʿil*) on the inferior world, one can understand that the variations in the upper world are transmitted to the lower one and produce changes like those of the size of living beings.

5.6.2 *Pseudo-Enrique de Villena's Treatise on Astrology*

The *Tratado de astrología* is an elementary treatise dealing with astronomy, astrology, cosmography, meteorology, geography and chronology and it gives a clear reflection of a period of decline in astronomical knowledge in the Iberian Peninsula in the first half of the fifteenth century.³⁰⁷ The *incipit* of the only extant manuscript states that the author is “don Enrique de Aragón, señor de Yniesta”, better known as Enrique de Villena (ca. 1384–1434), an important humanist and scholar. Villena had some knowledge of astronomy, as he shows in his authentic works like his commentaries (“glosas”) on his translation of Virgil's *Eneida*³⁰⁸ and in his exposition of the psalm *Quoniam videbo*.³⁰⁹ In both sources we find cosmological references in which we read that the ninth sphere is the prime mover; it contains all the other spheres mentioned by astronomers, and they all receive from the ninth sphere their motion from East to West.

As for the *Tratado de astrología*, its author seems to have been called Andrés Rodríguez, who appears as the copyist of the manuscript and finished his task, according to the explicit of the manuscript, on 20 April 1428 in Segovia. 1428 should probably be replaced by 1438, as the text states that, while he was writing the book, his patron Íñigo López de Mendoza, the Marquis of Santillana (1398–1458) conquered the city of Huelma, an event which took place precisely on 20 April 1438.³¹⁰ Besides, when the text deals with questions related to ecclesiastical computation, it gives an example for the year 1439³¹¹ and mentions a solar eclipse in Libra,³¹² which might correspond to the partial eclipse (visible in Spain) of 19 September 1438.

The book is structured in two parts: the first one deals with the world below the sphere of the Moon, the four elements, and generalities on astrology. The second one describes the celestial bodies: the Sun, the Moon (with an explanation of the ecclesiastical luni-solar calendar), the *Draco* (lunar nodes), solar and lunar eclipses, the higher planets (Saturn, Jupiter and Mars), the lower planets (Venus and Mercury), the apogees and some general notions of planetary astronomy, and, finally, the fixed stars, including tables to determine the hour as a function of the solar altitude (2.4.1.1).

Very few aspects of the treatise could be interpreted as corresponding to what we usually find in *hay'a* texts, unless we accept Tzvi Langermann's interpretation of the meaning of the word. When speaking about Abraham bar

307 Cátedra & Samsó, 1983; Millás, 1943; Vera, 1930.

308 Cátedra, 1994; Cátedra, 2000.

309 Cátedra, 1986.

310 Cátedra & Samsó, 1983, pp. 11 and 132.

311 Cátedra & Samsó, 1983, pp. 186–187.

312 Cátedra & Samsó, 1983, p. 195.

Ḥiyya's *Ṣurat ha-Areṣ*, he states: “Bar Ḥiyya’s book belongs to a class of basic and largely non-technical astronomical literature whose purpose, as I understand it, was to familiarise those without any extensive mathematical training with the basic conceptions and fundamental knowledge current in astronomy”.³¹³ In a general way, when the treatise describes the planetary models, it uses spheres, orbs or circles in an indiscriminate way, although it speaks of the existence of twelve spheres: four of them correspond to the spheres of the four elements, while the other eight are those corresponding to the Sun, the Moon, the five planets and the fixed stars.³¹⁴

In spite of this, the treatise poses the problem of the opposite motions (from E to W and from W to E) of all celestial bodies and explains it with the metaphor of an ant moving in a rotating wheel in the opposite direction to that of the motion of the wheel. This simile resembles the one which appears in Ptolemy’s *Planetary Hypotheses*, from where it is repeated by Ibn Rushd (5.4.3). Besides, the treatise also states that, according to Ptolemy and Abū Ma’shar, the opposite motion of celestial bodies is due to the need to temper the excessive speed of the daily motion.³¹⁵ This reminds me of Ibn Rushd’s and al-Biṭrūjī’s interpretation of the contrary motion as a “delay” of the daily one (5.4.3 and 5.4.4), but the source is probably Isidore of Seville’s *De natura rerum* where the same idea appears.³¹⁶

From the point of view of this chapter, the most interesting part of the treatise is its treatment of the size of the Universe. Its starting point is the exposition of the system of four qualities (dry, humid, hot, cold) and their relation to the four elements (earth, water, air and fire). The treatise applies the Galenic scale of four degrees to each one of the qualities in the following way:³¹⁷

TABLE 5.4 The four elements and the corresponding four qualities

	Dryness (= Earth)	Humidity (= Air)	Heat (= Fire)	Coldness (= Water)
Earth	4°	2°	1°	3°
Water	2°	3°	1°	4°
Air	1°	4°	3°	2°
Fire	3°	2°	4°	1°

313 Langermann, 1999 a, p. 7.

314 Cátedra & Samsó, 1983, p. 133.

315 Cátedra & Samsó, 1983, pp. 69–70, 151, 154.

316 Fontaine, 1960: 12,6.

317 Cátedra & Samsó, 1983, pp. 30–31, 112.

As is underlined in the treatise, each one of the four elements contains ten degrees, distributed between the different qualities ($4^{\circ} + 3^{\circ} + 2^{\circ} + 1^{\circ} = 10^{\circ}$) and the number 10 will be particularly relevant in the quantification of the size of the Universe. To the aforementioned degrees corresponding to each quality, the treatise adds the notion of thickness, opposed to lightness: earth is ten degrees thicker than water, while water is ten degrees lighter than earth. This leads the author to establish that when an element is thicker, it occupies less space in the Universe: earth occupies ten times less space than water, water ten times less space than air, and air ten times less space than fire. Consequently, he applies a geometric progression with ratio = 10 to the distances of the consecutive spheres of the Universe, using, for the radius of the Earth, a value of 60° (*sic*), derived from the standard practice of assigning 60^{p} to the radius of each circle. Surprisingly, he mentions a radius of $56\frac{2}{3}$ miles,³¹⁸ although he does not use it. Consequently, we have:³¹⁹

TABLE 5.5 Radii of the spheres

<i>Radii of the sphere</i>	
Earth	60°
Water	$60^{\circ} \times 10$
Air	$60^{\circ} \times 10^2$
Fire	$60^{\circ} \times 10^3$
Moon	$60^{\circ} \times 10^4$
Mercury	$60^{\circ} \times 10^5$
Venus	$60^{\circ} \times 10^6$
Sun	$60^{\circ} \times 10^7$
Mars	$60^{\circ} \times 10^8$
Jupiter	$60^{\circ} \times 10^9$
Saturn	$60^{\circ} \times 10^{10}$
Fixed stars	$60^{\circ} \times 10^{11}$

This, obviously, implies an enormous increase in the size of the Universe if we compare it to Ptolemy's, where the distance of the first magnitude stars is 20,000 terrestrial radii.

318 Cátedra & Samsó, 1983, p. 137.

319 Cátedra & Samsó, 1983, pp. 31–34, 133–137.

This is not the only estimation of this topic found in our treatise. A second one is found, immediately afterwards,³²⁰ in an obscure passage which I interpret in the following way: it tries to establish the length of one degree of each sphere, using as a unit the length of one degree of the sphere of the Earth. Here he uses another geometrical progression in which the ratio is not 10, as before, but 60:

- 1° of the sphere of water = 60° of the terrestrial sphere
- 1° of the sphere of air = 60² degrees of the terrestrial sphere
- 1° of the sphere of fire = 60³ degrees of the terrestrial sphere

–

and so on until it reaches:

- 1° of the sphere of fixed stars = 60¹¹ terrestrial degrees.

5.7 Conclusions

Hay'ā in the Iberian Peninsula and the Maghrib does not seem to have reached the level of maturity it had in Eastern Islamic lands, especially from the time of Ibn al-Haytham (965–1041). Although Ibn al-Haytham's *Shukūk 'alā Baṭlamīyūs* and *Fī hay'at al-'ālam* were known in al-Andalus, their influence was limited and the extant sources which can be considered to deal with *hay'ā* are, usually, mere descriptions of three-dimensional Ptolemaic models without geometrical details, proofs, or mentions of numerical parameters. Even a first-rate scholar like Ibn Bājjā, when he refers to Ibn al-Haytham's *Shukūk*, does not seem to be aware of its importance and limits himself to a harsh censure of a mistake made by the Eastern scientist in his criticism of Ptolemy's model of Mercury.

In general terms, works dealing with *hay'ā* in the Maghrib (Dūnash ibn Tamīm), al-Andalus (Qāsim ibn Muṭarrif) and in the Christian kingdoms of the Iberian Peninsula (Petrus Gallecus, pseudo-Enrique de Villena) seem to have been written by authors with a limited knowledge of astronomy. They try to give general introductions to the subject intended for readers who want to acquire some knowledge of the Ptolemaic system of the world, and make no criticisms of any kind. The situation changes with “the Andalusian revolt against Ptolemy” undertaken by the great minds of twelfth-century al-Andalus (Ibn Bājjā, Ibn Ṭufayl, Ibn Rushd, Maimonides and al-Bīṭrūjī) who pose, for the first time, the problem of the inadequacy of Ptolemy's mathematical models, unable to describe a real world system and at odds with the kinds of physics

³²⁰ Catedra & Samsó, 1983, pp. 136–137.

(Aristotelian and Neoplatonic) known in their time. Whatever the case, and with the sole exception of al-Biṭrūjī, they were not able to design an alternative world system and they limited themselves to criticising. This enhances the importance of al-Biṭrūjī, who recovers ideas borrowed from Eudoxus' system of homocentric spheres and mixes them with new theories developed by Ibn al-Zarqālluh. Unfortunately, in spite of the acceptance of al-Biṭrūjī's system by some European Scholastic philosophers, his work is full of inconsistencies and has no predictive capacity to rival Ptolemy's. Only Ibn Naḥmias' *Nūr al-'alam* can be considered a serious attempt to overcome these shortcomings; it is the most important Iberian representative of a new *hay'a* which might have had some contacts with the ideas developed in the East, since the time of the thirteenth century and the Marāgha school. Apart from this isolated case, these developments do not seem to have reached Western Islam.

Astronomical Theory

6.1 Introduction

As we will see in chapter 7, the main link that joins Andalusi and Maghribi *zīj*es between the eleventh and the fourteenth centuries is that they all follow the innovations in astronomical theory introduced by Ibn al-Zarqālluh in the eleventh, and this allows us to speak of the existence of an Andalus-Maghribi school of *zīj*es. This astronomer's full name was Abū Ishāq Ibrāhīm b. Yaḥyā al-Naqqāsh al-Tujībī; he was known as Walad al-Zarqiyāl, whence the Hispanised form *Azarquiel*, al-Zarqālluh, al-Zarqāl or Ibn Zarqāl.¹ Al-Zarqālī (sometimes al-Zarqānī) and al-Zarqāla seem to be classicised Eastern forms.²

According to Isaac Israeli,³ Ibn al-Zarqālluh was an instrument maker in Toledo⁴ who worked for qādī Šā'id al-Andalusī (1029–1070), a distinguished scholar who patronised a team of astronomers that compiled the *Toledan Tables* (7.2.4).⁵ His patron lent him the books he needed to teach himself astronomy and he assumed a leading position in Šā'id's group. Ibn al-Zarqālluh also served King al-Ma'mūn of Toledo (1043–1075) and al-Mu'tamid ibn 'Abbād of Seville (1069–1091), the latter from 1048 onwards (4.3.3). Starting around 1050,⁶ he observed the Sun for around 25 years and the Moon for 37. He seems to have used a large-sized instrument for his observations in Toledo (see 5.2.3).

1 On the different forms of this name see Millás, 1943–50, pp. 15–17, Šā'id (Bū 'Alwān, 1985, pp. 180–1) names him as *Walad al-Zarqiyāl*; many years ago, John D. North made me realise that he was “the son (*walad*) of al-Zarqiyāl” and this led me to adopt the form Ibn al-Zarqālluh instead of al-Zarqālluh. The latter is the most common form of his name and we find it in Ibn al-Abbār (Codera, 1887, vol. 1, p. 120 (no. 358). As shown by Millás, it is the result of combining *zarq* (blue) with a Romance diminutive *-ello*. It was probably pronounced *Zarqello* as, in Andalusi Arabic, the ending *-uh* corresponds normally to a final vowel *-o*. The final *h* of *Zarqālluh* was interpreted as a *ta' marbūṭa* and this is the origin of the classicised form *Zarqāla*. *Al-Zarqālī* is not to be found in any contemporary historical source.

2 On Ibn al-Zarqālluh's works see Millás, 1943–50; an updated revision in Samsó, 2011, pp. 147–52, 166–240, 482–4, 487–95. See also R. Puig, *BEA*, pp. 1258–60; Calvo, 1997; Samsó, *EI* 2, vol. XI, pp. 461–2; Samsó, *BA*, vol. 6, pp. 257–64; Samsó, *EI* 3 (http://dx.doi.org/10.1163/1573-3912_ei3_COM_32310).

3 Isaac Israeli in Goldberg & Rosenkranz, 1846–48, book IV, chapter VII.

4 On the instruments designed by him, see 4.1.3.3, 4.3.2 and 4.5.3.

5 Richter-Bernburg, 1987.

6 According to Abū l-Ḥasan al-Marrākushī (Marrākushī 1984, I, p. 43) he was making observations in Toledo in 453/1061. French translation in Sédillot, 1834, p. 127.

Al-Zuhrī describes a water-clock which marked the date of the lunar month, built in Toledo by a certain Abū l-Qāsim b. ‘Abd al-Raḥmān known as al-Zarqāl: it is doubtful that he is referring to the same person.

Ibn al-Zarqālluh left Toledo and went to Córdoba either at the beginning of the reign of Yaḥyā II al-Qādir (1075–1085) or when Alfonso VI of Castile conquered the city in 1085. In Córdoba he was patronised by al-Mu‘tamid and he continued his observations until at least 480/1087–88. He died in that city on 8 Dhū l-Ḥijja 493/ 15 October 1100.⁷

In the field of astronomical theory, Ibn al-Zarqālluh is the author of several important works:

1. A lost work in which he summarises his 25 years of solar observations and which was probably written ca. 1075–1080. Its title was either *Fī sanat al-shams* (“On the solar year”) or *al-Risāla al-jāmi‘a fī l-shams* (“A comprehensive epistle on the Sun”). Its contents are known through secondary sources, both Arabic and Latin. In it Ibn al-Zarqālluh established that the solar apogee had its own motion of about 1° in 279 Julian years, and designed a solar model with variable eccentricity which became extremely influential both in the Maghrib and in Latin Europe until the time of Copernicus.

2. A treatise on the motion of the fixed stars, written ca. 476/1084–85 and extant in a Hebrew translation. According to al-Bīṭrījī, its title was *Maqāla fī ḥarakat al-iqbāl wa l-idbār* (5.4.2.2). It contains a study of three different trepidation models, in the third of which variable precession becomes independent from the oscillation of the obliquity of the ecliptic. Trepidation had already attracted the attention of Šā‘id and his team: the trepidation table extant in the *Toledan Tables* (which also appears in some manuscripts of pseudo-Thābit’s *Liber de motu octaue spere*) is probably due to the Toledan group and, according to Ibn al-Hā‘im, Abū Marwān al-Istijī, another member of the team, wrote a *Risālat al-iqbāl wa l-idbār* (“Epistle on accession and recession”).

3. We have indirect references to two other lost theoretical works: one is a *Maqāla fī ibṭāl al-ṭarīq allatī salaka-hu Baṭlīmūs fī istikhrāj al-bu’d al-ab‘ad li-‘Uṭārid* (“On the invalidity of Ptolemy’s method to obtain the apogee of Mercury”) mentioned by Ibn Bājja (d. 1139).⁸ Second, Ibn al-Hā‘im mentions reading, in Ibn al-Zarqālluh’s own writing (*bi-khatt yadi-hi*), a description of a modified lunar Ptolemaic model: the centre of the Moon’s mean motion in longitude was not the centre of the Earth but was placed on a straight line linking

7 On his biography, see the aforementioned Isaac Israeli’s *Jesod Olam*, Ibn al-Abbār’s *Takmila* and Šā‘id’s *Ṭabaqāt al-umam*.

8 ‘Alawī, 1983, p. 78.

the centre of the Earth with the solar apogee. The maximum correction in the lunar mean longitude amounted to 24'.

In this chapter I will concentrate on Ibn al-Zarqālluh's extant contributions to astronomical theory as well as its developments in Andalusī and Maghribī astronomy: his three models of trepidation, his discovery of the motion of the solar apogee, his models to account for the changes observed in the obliquity of the ecliptic and of the solar eccentricity and, finally, his correction of Ptolemy's lunar model.

6.2 The Motion of Accession and Recession of the Equinoctial Points (*al-iqbāl wa l-idbār*, Trepidation Theory)

6.2.1 *On the Introduction of Trepidation Theory in al-Andalus: the Liber de motu octaue spere*

The origins of medieval trepidation theory are extremely obscure. I will not deal here with Theon of Alexandria's formulation of the theory as it appears in his commentary of the *Handy Tables*, although it reappears (with errors) in the *Ghāyat al-ḥakīm* (*Picatrix*),⁹ a work on talismanic magic written in al-Andalus ca. 950.¹⁰ Nor will I be concerned with the formulations appearing in Indian astronomical texts.¹¹ I will limit myself to the analysis of geometrical models of trepidation which try to justify a variable rate of precession as well as a progressive decrease in the obliquity of the ecliptic. Among such models, the one described in the Latin *Liber de motu octaue spere* is well known.¹² This book has been ascribed to Thābit ibn Qurra, in spite of Duhem's serious arguments to the contrary;¹³ in fact, this attribution appears to have been dismissed today.¹⁴

The first reliable reference to a geometrical model of trepidation appears in qādī Šā'id's *Kitāb ṭabaqāt al-umam* ("Categories of nations"), a work dated in 1068, in which the author remarks that he was unable to understand trepidation

9 Ritter, 1933, pp. 78–79.

10 Fierro, 1996; Callataÿ, 2013.

11 Pingree, 1972.

12 The Latin text was edited by Carmody, 1960 as well as by J.M. Millás Vallicrosa; the latter edition was printed several times. I will quote it as it appears in Millás, 1943–50, pp. 496–509. Neugebauer, 1962 a, pp. 290–299, published an English translation and commentary. See also Goldstein, 1964; Dobrzycki, 2010, pp. 15–60 (English translation of a paper first published in Polish in 1965); North, 1967; North, 1976 a; Mercier, 1976–7; Mercier, 1996; Ragep, 1996; Comes, 1996; Comes, 2001.

13 Duhem, 1913–59, vol. II, pp. 246–259.

14 Ragep, 1993, pp. 400–408. Ragep's arguments (first formulated in his PhD thesis) have been accepted by Swerdlow & Neugebauer, 1984, I, 43. See also Morelon, 1987, pp. XVIII–XIX.

until he read the *Kitāb naẓm al-ʿiqd* written by the Eastern astronomer Ibn al-Ādamī and published by one of his disciples in 949.¹⁵ Ibn al-Ādamī's book is not extant and it is doubtful whether we will ever be able to establish the possible link between his theory of trepidation and the models explained by Ibrāhīm ibn Sinān (908–946) in his *Kitāb fī ḥarakāt al-shams*,¹⁶ or by Abū Jaʿfar al-Khāzin in his *Zīj al-ṣafāʾih*. In any case, Ibn al-Ādamī's book was probably the source that introduced a geometrical model of trepidation in al-Andalus which Ṣāʿid revised and changed in another lost work – his book on the correction of the motion of heavenly bodies (*Iṣlāḥ ḥarakāt al-nujūm / al-kawākib*). The problem of trepidation attracted the attention of Ṣāʿid's group of astronomers, since one of them, Abū Marwān ʿUbayd (or ʿAbd) Allāh al-Istijī, wrote an epistle on accession and recession (*Risālat al-iqbāl wa l-idbār*), which is mentioned by Abū Muḥammad ʿAbd al-Ḥaqq al-Ghāfiqī, known as Ibn al-Hāʾim al-Ishbīlī, in his *al-Zīj al-kāmil*, composed in 1204–5.¹⁷ Like the whole of the Western Islamic astronomical tradition, Ibn al-Hāʾim's *zīj* emphasises the role of the Toledan astronomers in the development of trepidation theory and we should note here that some time either before or after 1068, the date of the completion of the *Ṭabaqāt*, the *Toledan Tables* appear as the not particularly notable result of the team work undertaken by Ṣāʿid and his Toledan group. These tables include (without any reference to a source) the same set of trepidation tables which can be found in the *Liber de motu*.¹⁸ In my opinion, these tables are independent of the text of the *Liber de motu*, although related to it, and they should be attributed to the work of Ṣāʿid's Toledan team. Before clarifying my reasons for this assertion, I should first explain briefly the trepidation model described in this source (fig. 6.1).

Point A (mean equinox) is determined by the intersection of the equator QQ' and the mean ecliptic AC. Point B rotates, uniformly, on the small equatorial epicycle B₁B₂ with centre A, and is called “moving Aries”. With this motion, it drags a moving ecliptic, whose point of intersection with the equator (equinox) moves back and forth, keeping the common point C at a distance of 90° from A at all times. In the figure, we have two positions of the moving

15 Ṣāʿid, in Bū ʿAlwān, 1985, pp. 146–147; Blachère, 1935, p. 114.

16 See the edition by Saʿidān, 1983, pp. 274–304. This is the hypothesis suggested by Ragep, 1993, pp. 400–408, who seems to accept a certain participation of the Toledan team in the authorship of the *Liber de motu* which would be a reelaboration of Ibn al-Ādamī's text, whose ideas would derive from Ibrāhīm ibn Sinān.

17 Bodleian Library 11,2 ms. 285 (Marsh 618), fol. 4r. The date of this *zīj* is stated repeatedly throughout the manuscript: see for example fol. 88r. The little information given by Ibn al-Hāʾim on al-Istijī's *risāla* is carefully analysed by Comes, 2001, pp. 318–322. On al-Istijī, see Samsó in *BA* vol. 3, pp. 565–568.

18 Millás, 1943–50, pp. 507–8; Pedersen, 2002, vol. 4, pp. 1542–1545, 1552–1565.

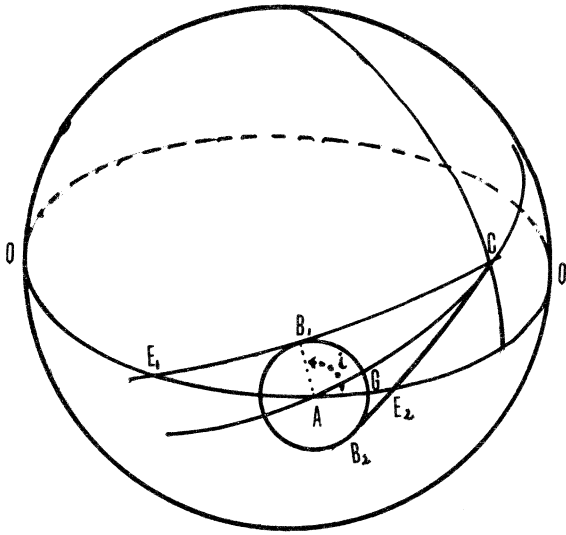


FIGURE 6.1
Trepidation model in the
Liber de motu

ecliptic: E_1B_1C and B_2E_2C , for which E_1 and E_2 will be the equinoxes corresponding to the moments at which point B occupies positions B_1 and B_2 , and the arcs of the moving ecliptic E_1B_1 and E_2B_2 will determine the increase in longitude due to precession/ trepidation at the two moments in question. Angle i is arc GB, measured from the equator which allows the determination of the position of point B on the equatorial epicycle. In this model there must necessarily exist a second equatorial epicycle, whose centre is located at a distance of 180° from point A, on which the beginning of moving Libra will rotate.

As we have seen, the text of the *Liber de motu* contains a set of two tables which allow the identification of the value of angle i (mean motion tables) and the value of EB (equation of trepidation). For the first table (mean motion),¹⁹ Pedersen has calculated a mean motion per day of $0;0,0,52,28,38,8^\circ$.²⁰ The radix position of the table is $1;34,2^\circ$, which corresponds to the beginning of the Hijra. It is easy to calculate the year in which angle $i = 0^\circ$, by dividing:

$$1;34,2^\circ : 0;0,0,52,28,38,8^\circ = 6,451 \text{ days}$$

If we assume that the civil date of the beginning of the Hijra is 16 July 622 (JDN 1,948,440):

$$1,948,440 - 6451 = 1,941,989$$

¹⁹ Millás, 1943–50, p. 507; Pedersen, 2002, vol. 4, p. 1545.
²⁰ Pedersen, 2002, vol. 4, p. 1544.

which corresponds to 16 November 604, eighteen years before the Hijra.²¹ We will see that earlier dates, with some kind of historical meaning, are chosen for $i = 0^\circ$, in the subsequent tradition of trepidation tables.

As for the second table (*equatio diuersitatis longitudinis capitis Arietis ab equatore diei* and *equatio dimidii diametri circuli*),²² it gives the value of the increase of longitude using the value of i as the argument, and with an interval of 5° between the values of i :

TABLE 6.1

i	Δ_λ	Equation of the radius
5°	0;55,52° 8 (−0;0,1°)	0;22,40° (+0;0,8°)
10	1;50,36 (−0;0,45)	0;44,31 (−0;0,22)
15	2;45,16 (−0;0,45)	1;6,45 (−0;0,11)
20	3;39,23 (−0;0,4)	1;27,20 (−0;1,5)
25	4;31,12 (−0;0,4)	1;48,4 (−0;1,11)
30	5;22,30 (+0;1,26)	2;9,21 (+0;0,5)
35	6;9,6 (+0;0,27)	2;28,6 (−0;0,12)
40	6;53,12 (+0;0,3)	2;45,55 (−0;0,17)
45	7;36,35 (+0;1,51)	3;2,38 (−0;0,13)
50	8;14,0 (+0;1,7)	3;17,45 (−0;0,22)
55	8;47,48 (+0;0,29)	3;31,40 (−0;0,12)
60	9;17,44 (−0;0,1)	3;43,46 ^a (−0;0,14)
65	9;43,53 (−0;0,3)	3;54,19 (−0;0,7)
70	10;5,30 (−0;0,10)	4;2,48 (−0;0,17)
75	10;22,47 (+0;0,1)	4;9,8 (−0;0,45)
80	10;35,1 (−0;0,4)	4;14,28 (−0;0,19)
85	10;42,13 (−0;0,17)	4;17,30 (−0;0,14)
90	10;45,0	4;18,43

a 3;44,46° in Millás's edition.

21 A possible explanation for the choice of this date can be found in Mercier, 1996, p. 307. See also Goldstein, 2011.

22 Millás, 1943–50, p. 508; Pedersen, 2002, vol. 4, p. 1554.

The *equatio diuersitatis longitudinis* has been recomputed according to the standard expression:

$$\Delta_{\lambda} = \arcsine (\sin i \sin 10;45^{\circ})$$

while for the *equatio dimidii diametri circuli*, I have used the expression suggested by Pedersen:²³

$$\delta = \arcsine (\sin i \sin 4;18,43^{\circ})$$

This table gives us the value of the radius of the equatorial epicycle B_1GB_2 (4;18,43) which corresponds to half of the diameter of the epicycle given in the text of the *Liber de motu* (8;37,26).²⁴ I interpret that the purpose of the table is to obtain the declination of the moving Head of Aries at a given moment. Although the application of this second table is not clear²⁵ in this context, it acquires full meaning in relation to Ibn al-Zarqālluh's third model of trepidation, in which a similar table appears and is used in an intermediate step of the computation of the obliquity of the ecliptic (6.2.2.5.2.2). We will see that in his *Maqāla fī ḥarakat al-iqbāl wa l-idbār* Ibn al-Zarqālluh does not include a table to compute the *equatio diuersitatis longitudinis*, although a table of this kind, which implies a fixed value of the obliquity of the ecliptic, is often found in later *zīj*es because it allows an easier computation of the value of precession. My impression is that this second table bears testimony to the Toledan origin of the tables of the *Liber de motu* and of an early conception of Ibn al-Zarqālluh's third model which he did not develop fully until a later stage of his life (6.2.2.5).

Having established this, I can now argue that the two aforementioned tables are independent of the text of the *Liber de motu*, for the following reasons:

1. The text states explicitly that the model aims to justify the secular decrease of the obliquity of the ecliptic; if we use the exact procedure established by Mercier²⁶ to calculate the values of ϵ , as well as the parameters of the text, on the basis of the text of the *Liber de motu*, we obtain satisfactory results for the time of Ptolemy as well as for the time of the observations sponsored by al-Ma'mūn (ca. 830) (fig. 6.2). On the other hand, the table *Equatio diuersitatis longitudinis capitis Arietis ab equatore diei*

²³ Pedersen, 2002, vol. 4, p. 1551.

²⁴ Millás, 1943–50, p. 498.

²⁵ Goldstein (1964, p. 234) believes that “it is an auxiliary table in that the value obtained from it must then be compared with the entries in a table of declination to see which longitude has this declination”.

²⁶ Mercier, 1996, p. 305.

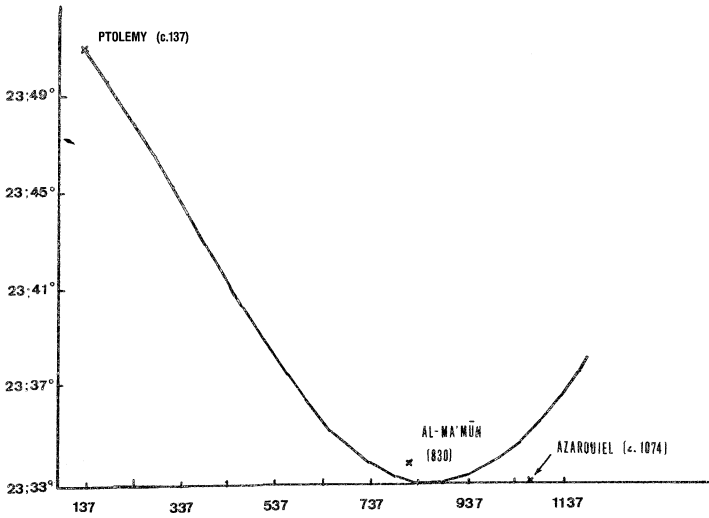


FIGURE 6.2 Secular variations of the obliquity of the ecliptic according to the trepidation model of the *Liber de motu*

is based, as we will presently see, on an approximate expression which assumes a constant obliquity.

2. The aforementioned equation table (as well as that of the *equatio dimidii diametri circuli*) is in very poor condition due, probably, to the accumulation of copying mistakes. The best results for its recomputation are obtained when using one of these two equivalent approximate formulae:

$$\begin{aligned}\sin \Delta_{\lambda} &= \sin i \sin \Delta_{\lambda \max} [1] \\ \sin \Delta_{\lambda} &= \sin i \tan r / \sin \epsilon [2]^{27}\end{aligned}$$

where

- Δ_{λ} is the equation, the amount of precession for a given moment,
- i is, as we have seen, the position of the moving head of Aries at a given moment on the small equatorial epicycle. It is the argument of our first table.
- $\Delta_{\lambda \max} = 10;45^{\circ}$ is the maximum value of Δ_{λ} both in the text and in the table.
- $r = 4;18,43^{\circ}$ both in the text and in the tables
- ϵ is the mean value of the obliquity of the ecliptic which, according to the *Liber de motu*, is $23;33^{\circ}$.

²⁷ Mercier, 1976–7, p. 205.

When using expressions [1] and [2], we should obtain, from the tabular values, a constant for :

$$\sin \Delta_{\lambda} / \sin i = \sin \Delta_{\lambda \max} = \tan r / \sin \varepsilon$$

This is, in fact, what we obtain in nine cases out of eighteen, which seem to be the “right” values of the tables (those having a difference with regard to the recomputation which attains a maximum of 4”: the entries for 5°, 20°, 25°, 40°, 60°, 65°, 75°, 80° and, of course, 90°). In all these cases we obtain:

$$\sin \Delta_{\lambda} / \sin i = 0.1865 = \sin 10;45^{\circ}$$

We should now check whether $0.1865 = \tan r / \sin \varepsilon$. It is easy to see that, for $r = 4;18,43$, ε should be the Ptolemaic 23;51° and not 23;33° as stated in the text of the *Liber de motu*.

My impression is, therefore, that if the *Liber de motu* predates the *Toledan Tables* (?), it reached the Toledan astronomers without tables; they must have computed a set of tables using most of the parameters of the text except for that of the mean value of the obliquity. These were the tables used in the *Toledan Tables*; no source had to be mentioned because they were the result of their own work. This is in clear agreement with Mercier’s remark that the mean motions of the *Toledan Tables* were the result of a complex interweaving of three components, the third of which is “the theory of accession and recession in a new and precise form, evidently due to Ibn al-Zarqāla”.²⁸

As for the author of the *Liber de motu*, after ruling out Thābit, I can only offer some hypotheses. One of the possible candidates is Ibrāhīm ibn Sinān, the grandson of Thābit b. Qurra and, apparently, the first astronomer to describe, in a purely qualitative way, a geometrical model of trepidation – not necessarily the same as the one in the *Liber de motu*. There are other possibilities, one of which would be Ibn al-Ādamī, who transmitted the theory to qāḍī Ṣā’id who could be, himself, the third candidate, as we know that he dealt with the topic and added original contributions to the received theories. The fourth one is al-Istijī, who wrote a *Risālat al-iqbāl wa l-idbār* (6.2.4), although we might also consider Ibn al-Zarqālluh himself as the author of the *Liber de motu* which might be a first draft of his book on the motion of the fixed stars.²⁹

28 Mercier, 1987, pp. 104–107.

29 This hypothesis has been defended by Mercier, 1996, pp. 321–325.

As we have seen, the tables seem to be independent of the text and, as Mercier has shown,³⁰ they are closely related to the *Toledan Tables*. In all probability, they were later incorporated to the *Liber de motu*.

6.2.2 *Ibn al-Zarqālluh's "Treatise on the Motion of the Fixed Stars"* (*Maqāla fī ḥarakat al-iqbāl wa l-idbār*)

6.2.2.1 Introduction

According to Abū l-Ḥasan al-Marrākushī, Ibn al-Zarqālluh was “the first of the modern [astronomers] who studied in detail (*ḥaqqāqa*) [this motion] and corrected the errors made [by his predecessors] ... He wrote a book on which other people (*al-nās*) base themselves in this matter”.³¹ He is probably referring to his book on the motion of the fixed stars written at a later stage of his life.

This book had an enormous influence on the development of astronomy in al-Andalus and the Maghrib, although it had a very limited diffusion in the Mashriq where we find only: 1) a brief and vague reference to Ibn al-Zarqālluh's trepidation model in a late author like al-Birjandī (d. 1525);³² and 2) S.M. Mozaffari (private communication) has recently found a manuscript of Muḥyī al-Dīn al-Maghribī's (d. ca. 1290) *Umdat al-ḥāsib*, copied ca. 1502–3, containing an interpolated passage which describes a trepidation model and attributes it to Ibn al-Zarqālluh. It uses Maghribī terminology (*al-mabda' al-dhātī* and *al-mabda' al-ṭabī'ī*), includes two tables (mean motion of the Head of Aries and equation of trepidation) and adds that the maximum value of the equation is 10;15°, a parameter unknown to me.

Ibn al-Zarqālluh's book is extant in a Hebrew translation, translated into Spanish by Millás.³³ In it he describes three geometrical trepidation models: the third one was first studied by Goldstein,³⁴ the second was briefly described by Neugebauer,³⁵ and the whole set was analysed by me in 1994.³⁶ The third model also appears in a second Arabic source: Ibn al-Hā'im's *zīj*, which was the object of an in-depth study by Mercè Comes.³⁷

Something should be said, first of all, about the date of composition of Ibn al-Zarqālluh's book: as we will see, 1071–72 appears mentioned twice, as a result of a reference to Maslama's stellar observations performed until 369/ 979–80,

30 Mercier, 1996, pp. 306 ff.

31 Marrākushī, 1984, p. 43; Sédillot, 1834, p. 127.

32 Ragep, 1993, pp. 406–407.

33 Millás, 1943–50, pp. 239–343 (it includes a facsimile of the Hebrew manuscript).

34 Goldstein, 1964.

35 Neugebauer, 1962, pp. 183–184.

36 Samsó, 1994 b. In this section I will give a revised version of this paper.

37 Comes, 2001.

“some 92 solar years before our time”;³⁸ another reference concerns a star observation by Ibn Bargūth in 441/1049–50, “some 22 years ago”.³⁹ On two other occasions 467/1074–75 appears mentioned as the date of a given position of the head of Aries in the small equatorial epicycle and as the *annus praesens*, which is 1266 of Alexander’s era.⁴⁰ Thirdly there is also a reference to 473/1080–81, the date of a stellar observation by Ibn al-Zarqālluh.⁴¹ 473 H is also the year in which Ibn al-Zarqālluh’s book on the construction of the equatorium was written (4.5.3) and in it we find two interesting details that imply that the book on the motion of the fixed stars was not yet finished: first, the author refers the reader to his book on the solar year (6.3), a source quoted in the book on the motion of the fixed stars, in a question that concerns precession; second, the computation of the longitudes of the planetary apogees is performed adding a value for precession calculated using either the trepidation tables of the *Liber de motu*⁴² or the first trepidation model described in his book on the motion of stars.⁴³ All this suggests that the book on the motion of the stars took a long time to write: Ibn al-Zarqālluh worked on it throughout the decade of the 1070s⁴⁴ but it was not finished until after 1081, probably towards the middle of the decade.

6.2.2.2 The Data of the Problem

Ibn al-Zarqālluh’s starting point for his study of precession is a series of indirect determinations of the longitude of *Qalb al-asad* (Regulus, α Leo),⁴⁵ which are summarized in table 6.2. Ibn al-Zarqālluh’s problem is that he cannot obtain enough observations of Regulus that give him exact positions of the star and allow him to measure its displacement due to precession. Most longitudes of this or another star based on an observation are expressed in multiples of ten minutes because no higher precision can be obtained with the naked eye. There is, however, one notable exception: Thābit ibn Qurra says that he has

38 Millás, 1943–50 pp. 310–312.

39 Millás, 1943–50, p. 309.

40 Millás, 1943–50, pp. 322 and 327.

41 Millás, 1943–50, p. 305.

42 Millás Vendrell, 1983.

43 Comes, 1991, pp. 90–92.

44 Or perhaps even earlier. Kunitzsch, 1980 published a star table computed for 459/1066–67 that can be attributed to Ibn al-Zarqālluh. The star longitudes are those of Ptolemy plus a precessional constant of 14;7°. We will see later that the difference of longitudes due to precession between the time of Ptolemy and 1074 amounts to 14;9° in Ibn al-Zarqālluh’s third model for trepidation.

45 Millás, 1943–50, pp. 296–299.

observed the longitude of Regulus and obtained $133;13^\circ$.⁴⁶ Ibn al-Zarqālluh feels extremely confident in Thābit's ability as an observer and relies on this longitude of Regulus for the rest of his calculations. An analysis of Ibn al-Zarqālluh's source, the book *On the solar year* attributed to Thābit, shows that here Ibn al-Zarqālluh has made an understandable mistake: first of all, Thābit does not mention an observation made by himself but by the astronomers of al-Ma'mūn in year 199 of Yazdijird/830–1 CE: they determined that the longitude of Regulus was slightly beyond 133° . Other passages of the work allow the interpretation that this "slightly beyond" amounts to $2'$, but Thābit's Arabic text is not always clear and $133;13^\circ$ could have been understood (indeed, this is the figure used by the Latin translator of this book).⁴⁷ In any case, 133° became a well-known value and it appears in the *zīj*es of Yahyā ibn Abī Manṣūr⁴⁸ and of Ḥabash al-Ḥāsib.⁴⁹ Two other real observations of Regulus are also mentioned by Ibn al-Zarqālluh, but they do not have for him the importance of the one he attributes to Thābit: Maslama al-Majrīṭī observed Regulus in 369/ 979–80 and obtained a longitude of $135;40^\circ$.⁵⁰ 'Abd Allāh ibn Bargūth made another observation of the same star in 441/1049–50, obtaining $136;20^\circ$.⁵¹

Let me now describe table 6.2: the first column, on the left, corresponds to the "observer" (Hipparchus, Ptolemy, al-Battānī, Ibn al-Zarqālluh) for whose time Ibn al-Zarqālluh aims to calculate the longitude of Regulus. Column [1] registers the displacement of the solar apogee between the time of Thābit and the time of the "observer". I do not know how the data in this column were obtained because they do not correspond to what one would expect for the longitudes of the solar apogee according to:

Hipparchus	$65;30^\circ$ (<i>Almagest</i> 111,4)
Ptolemy	$65;30^\circ$ (<i>Almagest</i> 111,4) ⁵²

46 Millás, 1943–50, pp. 295–296. In fact the text has, here, $133;14^\circ$ but $133;13^\circ$ is used in the following pages for all the computations.

47 On this problem see Morelon, 1987, pp. 30, 55, 56, 57 and footnotes in pp. 192, 205, 206, 207. Concerning the Latin translation see Neugebauer, 1962 a, pp. 267, 279, 280, 281. See also Moesgaard, 1974.

48 See the facsimile edition of the Escorial manuscript (Frankfurt, 1986) p. 188, as well as Mozaffari, 2016–17.

49 Manuscript Istanbul Yeni Cami 784 fol. 192r. See also Caussin, 1804, p. 144. On early Islamic observations of the longitude of Regulus, see Giahhi, 2016–17.

50 Millás, 1943–50, pp. 310–311; see also Kunitzsch, 1986, pp. 192–196.

51 Millás, 1943–50, p. 309.

52 Toomer, 1984, p. 153.

Thābit	80;45 ^{o53}
Battānī	82;17 ^{o54}
Ibn al-Zarqālluh	85;49 ^{o55}

With these data, column [1] should read:

Hipparchus	−15;15 ^o
Ptolemy	−15;15 ^o
Battānī	1;32 ^o
Ibn al-Zarqālluh	5;4 ^o

TABLE 6.2

“Observer”	[1] Δλ solar ap. between Thābit and “observer”	[2] Δλ of ap. own motion between Thābit and “observer”	[3] Rec. of [2]	[4] Difference between [1] and [2] (precession)	[5]= 133;13 ^o + [4] = λ _{Leo} in the time of “observer”
Hipparchus	−17;4 ^o	−3;30 ^o	−3;30 ^o (977 y)	−13;34 ^o	119;39 ^o
Ptolemy	−13;15 ^o	−2;28 ^o	−2;29 (692 y)	−10;47	122;26 ^o
Battānī	0;56 ^o	0;11 ^o	0;11 ^o (52y)	0;45 ^o	133;58 ^o
Ibn al-Zarqālluh	4;15 ^o	0;53 ^o	0;52 ^o (244y)	3;22 ^o	136;35

Column [2] gives the displacement due to the solar apogee’s own motion between the time of Thābit and that of the different “observers”. This column is, obviously, based on Ibn al-Zarqālluh’s theory according to which the motion of the solar apogee is faster than precession by an amount of about 1° in 279 years. My recomputation (column [3]) is based on a parameter of 0;0,0,2,7,10,38° per day, which is what we obtain from a table, obviously derived from Ibn

53 This is the longitude given in the book *On the solar year*, ascribed to Thābit, which is the source used by Ibn al-Zarqālluh. See Morelon, 1987, pp. 29–30.

54 According to observations performed in 1194 of Alexander/ 882–3 CE. See Nallino, 1899–1907, vol. 1, p. 44.

55 Observations performed in 1074–5. See Toomer, 1987, and Toomer 1969.

al-Zarqālluh, extant in the Tunisian recension of Ibn Ishāq's *zīj*⁵⁶ and in Ibn al-Bannā's *Minhāj* (1256–1321).⁵⁷

Column [4] is the difference between columns [1] and [2] and therefore gives the amount of displacement of the solar apogee due exclusively to precession between the time of Thābit and that of the different “observers”; it thus contains the basic data that will be used by Ibn al-Zarqālluh in the rest of the book. Finally, column [5] is the result of the addition of column [4] and 133;13° (the longitude of Regulus which Ibn al-Zarqālluh believes that Thābit observed) and thus gives us the longitude of Regulus for the times of the different “observers”. He can easily rely on these results because they agree with the “approximate” values he can find in the sources which he is obviously using: for Hipparchus’ time he gives 119;39° and he can read in the *Almagest* (VII,2), as well as in Thābit’s book *On the solar year*⁵⁸ that Hipparchus found that Regulus’ longitude was 119;50° in year 50 of the third cycle of Callipus (= –128/ –127). As for Ptolemy, Ibn al-Zarqālluh’s results give 122;26°, while the *Almagest* (VII,2) and Thābit⁵⁹ state that the Greek astronomer determined the position of Regulus in year 463 of Alexander /139 CE obtaining 122;30°. For the time of al-Battānī Ibn al-Zarqālluh gives us a longitude of Regulus of 133;58°, while the Syrian astronomer says, in his *zīj*,⁶⁰ that he observed this star in year 1191 of Alexander/ 879–80 AD, and obtained 134°. Finally, for his own time, Ibn al-Zarqālluh obtains 136;35° and states⁶¹ that, in an observation made in 467 H/1074–5 he determined the longitude of Regulus to be between 136;30° and 136;40°. Ibn al-Zarqālluh’s results depart from the observational data (in the case of Hipparchus) by a maximum of 11’; so they can be considered extremely satisfactory.

Let us now turn to the problem of the dates to which Ibn al-Zarqālluh ascribes these determinations of the longitudes of Regulus. They can be summarised as follows:

Ibn al-Zarqālluh: 1071–72 seems to be the starting point of his chronology. Ibn Barghūth’s observations, in 1049–50, took place “about 22 years ago”, while Maslama’s observation in 979–80 corresponds to “92 solar years before our time”. Thābit’s presumed observation of Regulus happened 241 years before Ibn al-Zarqālluh and this implies 830–31, which agrees with the date mentioned by

56 Ms. Hyderabad Andra Pradesh State Library 298. I owe photographs of this manuscript to the generosity of David King. See Díaz-Fajardo, 2005.

57 Published by Millás, 1943–50, p. 352.

58 Morelon, 1987, pp. 30, 55, 192 and 205.

59 Morelon, 1987, pp. 57 and 206–7.

60 Nallino, 1899–1907, I, p. 124.

61 Millás, 1943–50, p. 296.

Thābit in *On the solar year*, as well as with the interval of 149 years between Thābit and Maslama.⁶² In spite of all this, we will see that, for the longitude calculations corresponding to his own time, Ibn al-Zarqālluh is probably using 467/1074–75, the year of his observation of Regulus.⁶³

Al-Battānī: Ibn al-Zarqālluh places him in 882–83, 52 years after Thābit. This corresponds to his determination of the longitude of the solar apogee in 1194 of Alexander/882–83 CE.⁶⁴

Thābit: the date implied for him is, as we have seen, 830–1, a mistake that is easy to understand in view of the Arabic text of *On the solar year*, where we can read, in relation to the observation of Regulus: “We have found, in our time, slightly more than 13° of Leo”.⁶⁵

Ptolemy: 692 years before Thābit (138–39 CE),⁶⁶ or 693 years before the same author (137–8)⁶⁷ and 934 years before Ibn al-Zarqālluh⁶⁸ (1071–2 – 934 = 137–8). As we have seen, Ptolemy’s observation took place in 139, and this is the date for which I obtain the best results in the recomputations of Ibn al-Zarqālluh’s third model.

Hipparchus: 285 years before Ptolemy,⁶⁹ which implies –146. Ibn al-Zarqālluh is making a mistake of some 20 years, for Hipparchus’ observation took place in –127 but I imagine he had some trouble in interpreting the meaning of “year 50 of the third Callipic cycle”. He does not seem to have paid attention to the 957 years between Hipparchus and Thābit quoted in *On the solar year*,⁷⁰ which would have given him the correct result.

With the exception of the date for Hipparchus, Ibn al-Zarqālluh’s chronology is basically correct.⁷¹ We can now proceed to the analysis of his three trepidation models.

62 Millás, 1943–50, p. 312.

63 Millás, 1943–50, p. 296.

64 Nallino, 1899–1907, I, pp. 44 and 214.

65 Morelon, 1987, p. 30.

66 Millás, 1943–50, p. 299.

67 Millás, 1943–50, p. 314.

68 Millás, 1943–50, p. 314.

69 Millás, 1943–50, p. 299.

70 Morelon, 1987, pp. 55 and 205.

71 See another attempt to date the “observations” of Hipparchus (17 February –145), Ptolemy (4 March 140), al-Battānī (19 March 884) and Ibn al-Zarqālluh (28 March 1076) in Mercier, 1996, p. 328.

6.2.2.3 Ibn al-Zarqālluh's First Model

In this model⁷² (fig. 6.3), the small epicycle TGDZ, with centre A, is in the plane of the meridian P_NAP_S which passes through the poles of the equator P_N and P_S and though the mean equinoctial points A and W. Let E be the centre of the universe and EA the common radius of the equator HAU and of the meridian P_NAP_S. The epicycle rotates with uniform motion and is followed in its rotation by EMG, which, in modern terms, could be considered a vector of variable length. This vector will therefore have two extreme positions, ET and EZ, and its intersection with the meridian P_NAP_S will determine point M, the moving Head of Aries, which will oscillate along arc TZ and in its displacement will pull the moving ecliptic. Ibn al-Zarqālluh states explicitly that this model does not imply any variation of the obliquity of the ecliptic and, therefore, the successive positions of the ecliptic will be parallel to each other. The drawing represents three positions of the ecliptic: AF is the mean ecliptic, when M coincides with point A, which implies that this point is in this case both the mean and the true equinoctial point; a second position is HM, when moving Aries is at M and the true equinoctial point is at H; in its third position, the ecliptic lies on KU, the moving Aries is at K and the true equinox at U. The “equation of trepidation” (value of precession for a given time) will then be 0° in the first case, +HM in the second and –KU in the third.

Once he has described the model, Ibn al-Zarqālluh makes two successive attempts (*Ia* and *Ib*) to assign parameters to it – period of revolution, and radius of the epicycle TGDZ. In his first effort (*Ia*), he considers $i = 0^\circ$ in the “apogee” (point D of the epicycle placed at a maximum distance from the centre of the universe); in his second attempt $i = 0^\circ$ at B, the “perigee”. Fig. 6.4 shows how to calculate the value of the equation $\Delta_\lambda (= HM, \text{ in fig. 6.3})$, considering all triangles to be plane. In the following expressions α is angle GED, r the radius of the epicycle (AD) and R the radius of the universe ($EA = 60P$):

$$\begin{aligned}\tan \alpha &= GL / (EA + AL) = r \sin i / (R + r \cos i) = AM / R \text{ (fig. 6.4)} \\ \sin \varepsilon &= AM / HM = AM / \Delta_\lambda \text{ (fig. 6.3)} \\ \Delta_\lambda &= R r \sin i / \sin \varepsilon (R + r \cos i)\end{aligned}$$

Ibn al-Zarqālluh uses here $r = 3;54P$. Supposing $\varepsilon = 23;33^\circ$, a standard value for our author, $\Delta_{\lambda\max}$ will be $9;46,54^\circ$ (for values of i comprised between $93;25^\circ$ and $94;4^\circ$). We can check these parameters with the data (values of i and P) stated in the text by Ibn al-Zarqālluh for the times of Hipparchus, Ptolemy and al-Battānī as well as for his own time:

⁷² Millás, 1943–50, pp. 284–87 and 316–7.

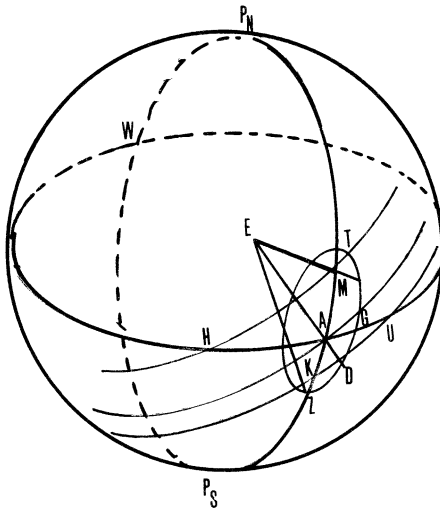


FIGURE 6.3
Ibn al-Zarqālluh's first trepidation model

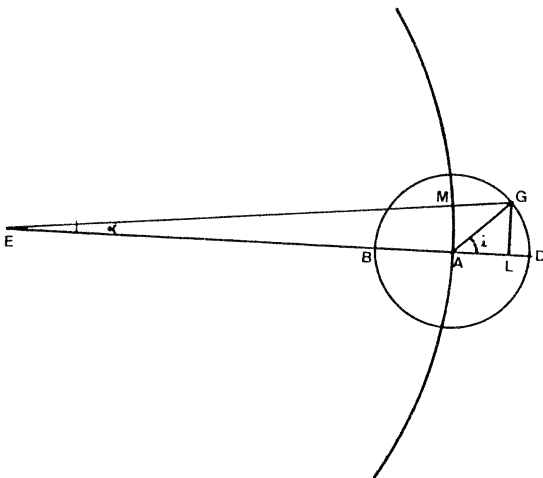


FIGURE 6.4
(Model Ia)

TABLE 6.3 (Model *Ia*)

Epoch	i (text)	Δ_λ (text)	Δ_λ (recomp.)
Hipparchus (−146)	−72;48°	−9°	−9;8,55°
Ptolemy (139)	−45;45°	−6;38°	−6;41,18°
Battānī (883)	27;15°	4;18°	4;13,31°
Ibn al-Zarqālluh (1075)	46;5°	6;45°	6;43,41°

The results in table 6.3 can be considered satisfactory and confirm the simplification we have assumed in the geometry of the model, as well as the parameters for the obliquity of the ecliptic and the radius of the epicycle. We will be less pleased with the second parameter in the text: a revolution of the epicycle in 3792 Julian years, a value which seems corrupt, as we can see in table 6.4:

TABLE 6.4 (Model *Ia*)

Epoch	Years since Hipparchus	Δ_i since Hipparchus (text)	Δ_i since Hipparchus (recomp.)
Ptolemy	285	27;3°	27;3,25°
Battānī	1029	100;3°	97;41,21°
Ibn al-Zarqālluh	1221	118;52°	115;55,4°

Ibn al-Zarqālluh makes a second attempt (*Ib*) with his first model assuming, as we have seen, that the “perigee” B is the point at which $i = 0^\circ$. In fig. 6.5 we can see that:

$$\Delta_\lambda = \text{HM (fig. 6.3)} = R r \sin i / \sin \varepsilon (R - r \cos i)$$

Here, Ibn al-Zarqālluh does not give us a new parameter for r . In table 6.5 we can see that, if we use 3;54^p, as before, the results obtained are clearly at odds

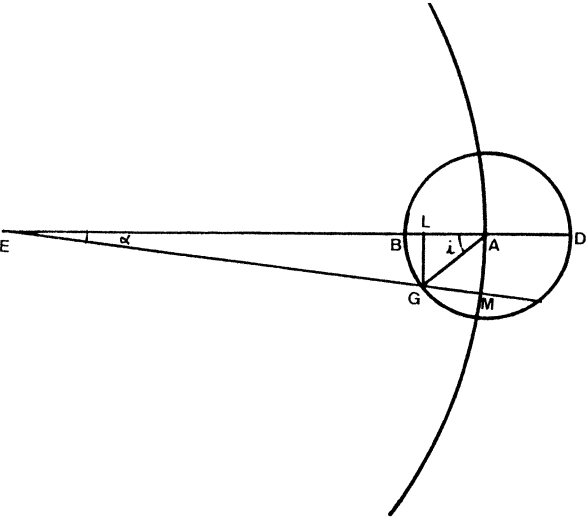


FIGURE 6.5
(Model *Ib*)

with those of the text. A clear improvement will be obtained with $r = 4;22^p$ (table 6.5). On the other hand, the text states that a revolution of the epicycle takes place in 4692 Julian years, a value that should be corrected to 4292 years. Table 6.6 shows the results of my checking of this latter parameter.

TABLE 6.5 (Model *Ib*)

Epoch	i (text)	Δ_λ (text)	Δ_λ recomp. ($r = 3;54^p$)	Δ_λ recomp. ($r = 4;22^p$)
Hipparchus (−146)	−63;45°	−10;6°	−9;0,49°	−10;7,40°
Ptolemy (139)	−39;45°	−7;19°	−6;34,12°	−7;24,9°
Battānī (883)	22;54°	4;38°	4;2,25°	4;33,30°
Ibn al-Zarqālluh (1075)	39;4°	7;20°	6;28,43°	7;18,1°

TABLE 6.6 (Model *Ib*)

Epoch	Years since Hipparchus	Δ_i since Hipp. (text)	Δ_i since Hipp. (recomp. 4692 y)	Δ_i since Hipp. (recomp. 4292 y)
Ptolemy	285	24°	21;52,1°	23;54,19°
Battānī	1029	86;39°	78;57,5°	86;18,34°
Ibn al-Zarqālluh	1221	102;49°	93;40,58°	102;24,50°

6.2.2.4 Ibn al-Zarqālluh's Second Model

In this second model (fig. 6.6) the small epicycle BMCN, the centre of which is at A, lies on the plane of the ecliptic EE', and OA is the common radius of the ecliptic and of the [mean] equator QQ'. The epicycle BMCN rotates with uniform motion and radius OB oscillates with it. OB can, again in modern terms, be considered a vector of variable length. The moving Aries point is determined by the intersection of OB with the ecliptic and oscillates along arc BC, the amplitude of which will be approximately equal to the diameter of the epicycle BMCN. Ibn al-Zarqālluh's text is not explicit here but I imagine that, as in the other cases, the obliquity of the ecliptic is kept constant. When the moving Aries point is in A, this will be both the mean and the true equinox and the value of precession will be 0°. The maximum positive and negative values of Δ_λ will be +AC and −AB, both approximately equal to the radius of the epicycle.

TABLE 6.7 (Model *Ila*)

Epoch	i (text)	Δ_{λ} (text)	Δ_{λ} (recomp. $r = 10;26,10^p$)	Δ_{λ} (recomp. $r = 9;53,10^p$)
Hipparchus (-146)	$-80;55^{\circ}$	$-9;35,30^{\circ}$	$-10;1,47^{\circ}$	$-9;30,53^{\circ}$
Ptolemy (139)	$-49;4^{\circ}$	$-6;43,40^{\circ}$	$-7;4,39^{\circ}$	$-6;44,28^{\circ}$
Battānī (883)	$34;4^{\circ}$	$4;52,0^{\circ}$	$5;6,35^{\circ}$	$4;52,22^{\circ}$
Ibn al-Zarqālluh (1075)	$55;33^{\circ}$	$7;26,20^{\circ}$	$7;50,6^{\circ}$	$7;27,26^{\circ}$

Ibn al-Zarqālluh also states that a complete revolution of the epicycle takes place in 3218 years, a parameter which seems to fit approximately the rest of the numerical data of the text:

TABLE 6.8 (Model *Ila*)

Epoch	Years since Hipparchus	Δ_i since Hipparchus (text)	Δ_i since Hipparchus (rec.)
Ptolemy	285	$31;51^{\circ}$	$31;52,59^{\circ}$
Battānī	1029	$114;59^{\circ}$	$115;6,54^{\circ}$
Ibn al-Zarqālluh	1221	$136;28^{\circ}$	$136;35,39^{\circ}$

In his second approach to this model (*Ilb*), Ibn al-Zarqālluh measures angle i from the perigee of the epicycle (N). Here we have (fig. 6.8):

$$AB = \Delta_{\lambda} = R r \sin i / (R - r \cos i)$$

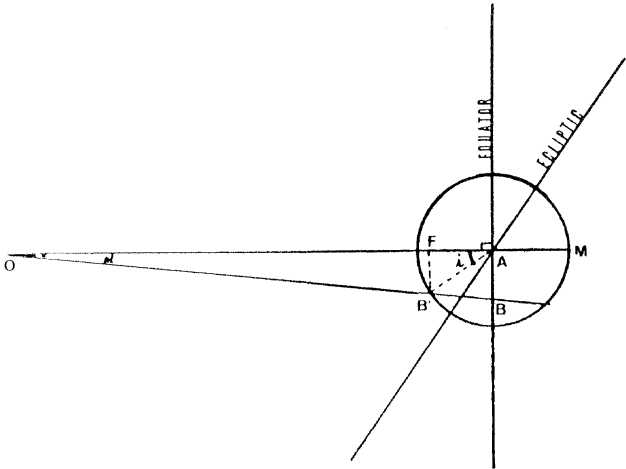


FIGURE 6.8
(Model *Ilb*)

The radius of the epicycle quoted in the text is 11;32^p which does not give good results. I have been unable to correct it successfully although a certain improvement is obtained, in some instances, with $r = 10;48^p$:

TABLE 6.9 (Model *IIb*)

Epoch	i (text)	Δ_{λ} (text)	Δ_{λ} (rec. $r = 11;32^p$)	Δ_{λ} (rec. $r = 10;48^p$)
Hipparchus (−146)	−53;46°	−9;45°	−10;29,44°	−9;44,55°
Ptolemy (139)	−33;50°	−6;57°	−7;38,30°	−7;4,13°
Battānī (883)	19;10°	4;37°	4;37,36°	4;16,20°
Ibn al-Zarqālluh (1075)	32;35°	7;11°	7;24,41°	6;51,21°

Finally, the period of revolution in the epicycle mentioned in Ibn al-Zarqālluh’s text (5150 years) seems to be fairly accurate:

TABLE 6.10 (Model *IIb*)

Epoch	Years since Hipparchus	Δ_i since Hipparchus (text)	Δ_i since Hipparchus (rec.)
Ptolemy	285	19;56°	19;55,20°
Battānī	1029	72;56°	71;55,48°
Ibn al-Zarqālluh	1221	86;21°	85;21,5°

6.2.2.5 Ibn al-Zarqālluh’s Third Model⁷³

6.2.2.5.1 *Description of the Model*

Here, Ibn al-Zarqālluh seems to use the model described in the *Liber de motu octaue spere* with one important correction: he states explicitly that the obliquity of the ecliptic remains constant in it, something which is in agreement with the tables (but not the text) of the *Liber de motu*. The parameters used are also new: 4;7,58^p for the radius of the equatorial epicycle (4;18,43^p in both the tables and the text of the *Liber de motu*) and a period of revolution of moving Aries of 3874 Julian years (4056.92 Julian years in the *Liber de motu*).

73 Millás, 1943–50, pp. 289–294, 304 ff.

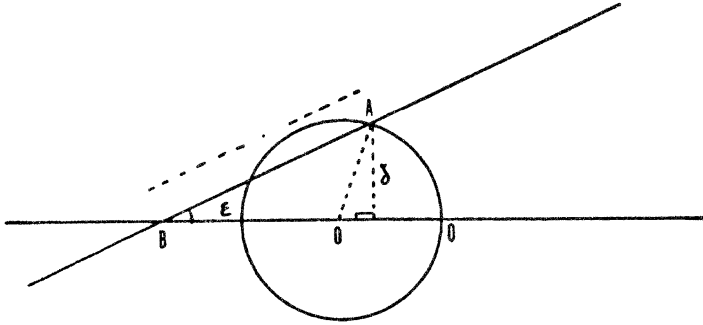


FIGURE 6.9
Computation
of the value
of precession
in Ibn al-
Zarqālluh's
third model

In spite of the equivalence of Ibn al-Zarqālluh's third model to the one in the *Liber de motu*, the procedure explained to calculate Δ_λ does not use a table like the *Equatio diuersitatis longitudinis capitis Arietis ab equatore diei* which, as we have seen, does not take into consideration the change in the obliquity of the ecliptic. Here, the procedure is indirect: once he has obtained the value of angle i , Ibn al-Zarqālluh reads δ in the table of the inclination of the beginning of Aries.⁷⁴

Once he has obtained δ in table 6.11, he calculates ϵ and, then:

$$\sin \Delta_\lambda = \sin \delta / \sin \epsilon$$

Δ_λ being arc BA of the ecliptic in fig. 6.9.

To compute ϵ , Ibn al-Zarqālluh uses a secondary model illustrated in fig. 6.10.⁷⁵ In it, O is the pole of the equator, C the pole of the ecliptic and AG is a parallel of declination with radius $OA = 23;43^\circ$. The pole of the ecliptic (C) rotates around a polar epicycle (ECB) the centre of which is A and radius $AB = 0;10^\circ$. A complete revolution of C around A takes place in 1850 Julian years and we have:

$$\epsilon_{\max} = OE = 23;43^\circ + 0;10^\circ = 23;53^\circ$$

$$\epsilon_{\min} = OB = 23;43^\circ - 0;10^\circ = 23;33^\circ$$

74 Millás, 1943–50, p. 336.

75 Both Ibrāhīm b. Sinān and Abū Ja'far al-Khāzin had explained (according to al-Bīrūnī) the variation of the obliquity of the ecliptic by a rotation of its poles about a point. See Mercier, 1996, p. 326; Bulgakof, 1962, p. 101; Ali, 1967, p. 70.

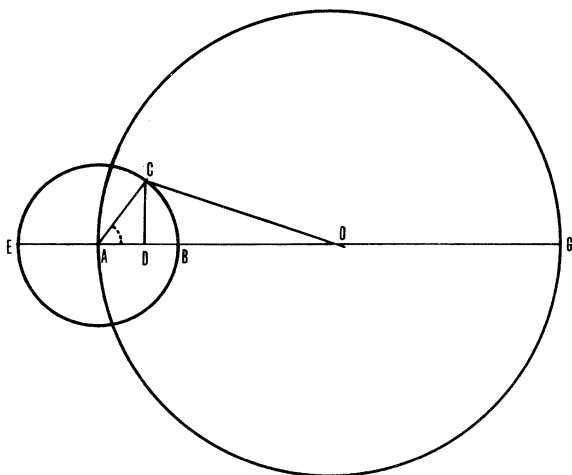


FIGURE 6.10
Ibn al-Zarqālluh's secondary
model for the variation of ϵ

6.2.2.5.2 The Tables

6.2.2.5.2.1 Mean Motion of the Head of Aries

Our author has computed three tables of the mean motion of the head of Aries, which are calculated for Arabic, Julian and Persian years⁷⁶ and have been analysed by H. Mielgo.⁷⁷ Mielgo has shown that the tables for Julian and Persian years are mutually compatible and use a mean motion per year of $0;5,34,32,16,36^{\circ}$ ($0;0,0,54,57,17,38^{\circ}$ / day), while the table for the Arabic years seems to be computed with a different parameter ($0;5,24,32,23,22^{\circ}$ / lunar year and $0;0,0,54,56,59,24^{\circ}$ / day). The radix values are:

- Arabic years: $3;51,11^{\circ}$ (*Hijra*). The same radix is used in Ibn al-Kammād's *Muqtabas zīj*,⁷⁸ while Abū l-Ḥasan 'Alī al-Marrākushī rounds this value to $3;51^{\circ}$.⁷⁹ Ibn 'Azzūz al-Qusanṭīnī (d. 1354) has $3;51,21^{\circ}$, while Ibn Iṣḥāq (fl. ca. 1193–1222) in the Tunisian recension, Ibn al-Bannā' (1256–1321) and Ibn al-Raqqām (d. 1315) increase this value and give $3;53,55^{\circ}$.⁸⁰
- Julian years: $277;12,55^{\circ}$ (Alexander's era)
- Persian years: $4;46,54^{\circ}$ (Yazdijird's era)

We can now calculate the date in which i and $\Delta_{\lambda} = 0^{\circ}$:

- Arabic years: $3;51,11 / 0;0,0,54,56,59,24^{\circ} \approx 15,146$ days before the Hijra. The calculated date is 26 January 581.
- Julian years: $360^{\circ} - 277;12,55^{\circ} = 82;47,5^{\circ}$

⁷⁶ Millás, 1943–50, p. 324.

⁷⁷ Mielgo, 1996.

⁷⁸ Chabás & Goldstein, 1994, p. 30. See also Chabás & Goldstein, 2015 a p. 596.

⁷⁹ Marrākushī, 1984, p. 44; Sédillot, 1834, p. 130.

⁸⁰ Samsó, 1997 p. 100; Samsó & Millás, 1994, p. 12.

$82;47,5^\circ / 0;0,0,54,17,38^\circ \approx 329,346$ days after Alexander's era, which corresponds to 14 June 591.

- Persian years: $4;46,54^\circ / 0;0,0,54,17,38^\circ \approx 19,023$ days before Yazdijird's era, which gives as a date 17 May 580.

Years 580 and 581 are interesting because they agree with a passage in Ibn al-Zarqālluh's book:

About 40 years before the Hijra, at the moment of the Prophet's birth because, at that time, there was an agreement between the positions of the Sun, Moon and stars calculated with the *Mumtaḥan* or with the Indian and Iranian systems which are similar.⁸¹

The chronology of the Prophet's birth is not at all clear and Ibn al-Zarqālluh seems to be misinterpreting a tradition according to which Muḥammad was about 40 years old when he began his mission: it is easy to identify this beginning with the Hijra and to forget his first years of religious activity in Mecca.⁸² This is corrected in Ibn al-Hā'im's *zīj* which states that the time at which $\Delta_\lambda = 0^\circ$ was 50 lunar years before the Hijra, which also coincided with the birth of the Prophet.⁸³ This seems to have been a fairly common belief both in al-Andalus and in the Maghrib and it has a link with the development of astrological cycles.⁸⁴ We can check this remark by calculating the initial moment of the cycle according to the radices and mean motion parameters in the *zīj*es of Ibn al-Kammād, Ibn 'Azzūz, Ibn Ishāq (in the Hyderabad recension), Ibn al-Bannā' and Ibn al-Raqqām:

- Ibn al-Kammād: $3;51,11^\circ / 0;0,0,54,56,58^\circ \approx 15,146$ days before Hijra, which corresponds to 24 January 581.
- Ibn 'Azzūz: $3;51,21^\circ / 0;0,0,54,57,18,17^\circ \approx 15,155$ days before Hijra, which corresponds to 15 January 581.⁸⁵
- Ibn Ishāq, Ibn al-Bannā' and Ibn al-Raqqām: $3;53,55^\circ / 0;0,0,54,57,17,35 \approx 15,323$ days before Hijra, corresponding to 2 August 580.

These identifications of years 580–81 with a beginning of the cycle of trepidation seem to follow a pure Indian tradition, represented by the *Brāhmasphuṭasiddhānta*, according to which the sidereal and tropical longitudes of the Sun were equal in 580.⁸⁶

81 Millás, 1943–50, p. 338.

82 F. Buhl and A.T. Welch, "Muḥammad" in *ET* 2, vol. 7, p. 361.

83 Comes, 200, pp. 331 and 373. This agrees with the horoscope of the birth of the Prophet in al-Battānī's astrological history, dated 20 April 571. See Kennedy *et al.* 2009–10, p. 93.

84 Comes, 2002, pp. 125–130.

85 Samsó, 1997, p. 108.

86 On this topic see also Goldstein, 2011.

6.2.2.5.2.2 Declination of the Head of Aries in the Equatorial Epicycle

This table⁸⁷ is calculated for intervals of 2° between 2° and 180°, disregarding the symmetry of the table: this is because, in a parallel column, we find the “equation of the diameter or the second accession” (*al-iqbāl al-thānī*), without this symmetry (6.2.2.5.2.3). It is the same kind of table as table 6.1 (*equatio dimidii diametri circuli*), for a radius of the equatorial epicycle of 4;7,58^p (obviously, much better calculated) with the expression:

$$\delta = \arcsine (\sin i \sin 4;7,58^\circ)$$

A sample of this table follows (table 6.11):

TABLE 6.11

Argument (<i>i</i>)	Δ
2°	0;8,39°
6	0;25,24
10	0;43,2 (+1")
14	0;59,[5]6 ^a
18	1;16,34
22	1;32,49
26	1;48,38
30	2;3,54
34	2;18,35
38	2;32,[3]4 ^b (−1")
42	2;45,51
46	2;58,18
50	3;9,[5]3 ^c
54	3;20,33
58	3;30,14
62	3;38,54

a 0;59,16° for argument 14°, but 0;59,56 for *i* = 166°.

b 2;32,54° for argument 38°; 2;32,34° for *i* = 142°.

c 3;9,13 for argument 50°; 3;9,33° for *i* = 130°.

87 Millás, 1943–50, p. 336.

TABLE 6.11 (*cont.*)

Argument (<i>i</i>)	Δ
66	3;46,29 (−1″)
70	3;52,59
74	3;58,20 (−1″)
78	4;2,34 (+2″)
82	4;5,[3]3
86	4;7,21 (−1″)
90	4;7,58

6.2.2.5.2.3 Equation of the Diameter or Second Accession (al-iqbāl al-thānī)
This table appears in a parallel column to that of the declination of the head of Aries and is also calculated for intervals of 2° between 2° and 180°. It seems to have been calculated using the expression:

$$4;7,58^{\circ} - 4;7,58^{\circ} \cos i$$

A sample of this table follows (table 6.12), in which I have corrected a few obvious copying mistakes due to confusions in reading the *abjad* numerals. Whatever the case, this table appears to have been computed less carefully than the previous declination table.

This table returns the value of the second accession (GD in fig. 6.9), the difference between the right ascension of the moving Head of Aries (B) and point D (the intersection of the equator with the equatorial epicycle). A table of this kind appears in many Andalusī and Maghribī *zīj*es and is used for the computation of the equation of time.

6.2.2.5.2.4 Mean Motion of the Pole of the Ecliptic
Ibn al-Zarqālluh’s book uses two kinds of tables for the computation of ϵ .⁸⁸ The first one is a mean motion table with which we can obtain the position of C (angle <BAC) on the polar epicycle ECB (fig. 6.10). As in the table of mean motion of the head of Aries, angle <BAC is calculated for Lunar, Julian and Persian

88 Samsó, 1987 b.

TABLE 6.12

Argument (i)	Equation of the diameter	Argument (i)	Equation of the diameter
2°	0;0,9°	94°	4;25,[1]5° (-1") ^a
6	0;1,22	98	4;42,[2]8 (-1") ^b
10	0;3,46	102	4;59,[3]2 (+1") ^c
14	0;7,28 (+6")	106	5;16,[1]7 (-2") ^d
18	0;12,10 (+2")	110	5;32.43 (-4")
22	0;18,5 (+2")	114	5;48,45 (-4")
26	0;25,8 (+2")	118	6;4,18 (-5")
30	0;33,16 (+3")	122	6;19,17 (-5")
34	0;42,27 (+3")	126	6;33,37 (-6")
38	0;52,38 (+4")	130	6;47,17 (-4")
42	1;3,46 (+5")	134	7;0,8 (-5")
46	1;15,48 (+5")	138	7;12,10 (-5")
50	1;28,39 (+4")	142	7;23,18 (-4")
54	1;42,19 (+6")	146	7;33,29 (-3")
58	1;56,39 (+5")	150	7;42,40 (-3")
62	2;11,38 (+5")	154	7;50,48 (-2")
66	2;27,11 (+4")	158	7;57,51 (-2")
70	2;43,13 (+4")	162	8;3,46 (-2")
74	2;59,40 (+3")	166	8;8,33 (-1")
78	3;16,[2]4 (-1") ^e	170	8;12,9 (-1")
82	3;33,28 (+1")	174	8;14,34
86	3;50,41 (+1")	178	8;15,47
90	4;7,58	180	8;15,56

a 4;25,35 in Millás's edition.
b 4;42,58 in Millás's edition, although, in this case, an error based on the *abjad* notation is not easy to justify.
c 4;59,22 in Millás's edition.
d 4;56,57 in Millás's edition.
e 3;16,34 in Millás's edition.

years⁸⁹ but, here, the three subtables are calculated for the same parameter (0;0,1,55,4,42,42°/day). The radices mentioned in Millás’s edition of the table are:

Alexander:	304;25,56°
Hijra:	295;14,33°
Yazdjird:	294;16,36°

We can easily realise that the radix position for Alexander’s era is not compatible with the other two. We should, first of all, note that there are two misprints in Millás’s edition of the table. One of them is little of importance, for it concerns the number of seconds in the radix for Alexander’s era which should be 55” (not 56”). The other is more significant: the number of degrees in Yazdjird’s radix is 297° instead of 294°.⁹⁰

Assuming that the Hijra radix is the correct one – as we will see below, it gives good results – we can calculate the two other positions, using as data the Julian Day Number of the beginning of each era and a mean motion per day of 0;0,1,55,4,42,42°:

TABLE 6.13

Eras	Δ JDN	Δ <BAC
Alexander-Hijra	340,701 days	181;30,56°
Hijra-Yazdjird	3,623 days	1;55,49°

From this we obtain:

Radix for Alexander’s era:	$295;14,33^{\circ} - 181;30,56^{\circ} = 113;43,37^{\circ}$
Radix for Yazdjird’s era:	$295;14,33^{\circ} + 1;55,49^{\circ} = 297;10,22^{\circ}$

This radical correction of the radix for Alexander’s era can be confirmed by a passage in Ibn al-Zarqālluh’s book where we read that, in year 1266 of Alexander/ 954–55 CE, the pole of the ecliptic reached its nearest position to the pole of the equator in the epicycle which regulates the variation of the

89 Millás, *Azarquiel* p. 329.

90 These two values were checked by my colleague Dr Teresa Martínez of the University of Barcelona. On this topic see also Mercier, 1996, pp. 329–330.

obliquity.⁹¹ This implies that point C was on point B, where $\angle BAC = 0^\circ$. Using our new radix, we can therefore calculate the value of $\angle BAC$ for the end of 1266 of Alexander:

Radix	113;43,37°
$\Delta \angle BAC$ in 1266 years	246;21,24°
$\angle BAC$	360;5,1°

6.2.2.5.2.5 Obliquity of the Ecliptic

The second table⁹² could have been calculated, assuming that the rectangular triangle OCD is plane, with the following expression (fig. 6.10):

$$\varepsilon = OC = (OD^2 + CD^2)^{1/2}$$

and we know that:

$$\begin{aligned} OD &= OB + BD = 23;33^\circ + 0;10^\circ \text{ vers } \angle BAC \\ CD &= 0;10^\circ \sin \angle BAC \end{aligned}$$

Consequently:

$$OC = [(23;33^\circ + 0;10^\circ \text{ vers } \angle BAC)^2 + (0;10 \sin \angle BAC)^2]^{1/2}$$

Another possibility is to use an exact expression of this kind:

$$\varepsilon = \arccos (\cos 0;10^\circ \cos 23;43^\circ + \cos 0;10^\circ \sin 23;43^\circ \sin \angle BAC)^{93}$$

The two formulas give similar results. Ibn al-Zarqālluh's table, recomputed using the exact formula mentioned above, which is slightly more accurate, follows:

91 Millás, 1943–50, p. 327. This passage was checked again by Dr Teresa Martínez, because Millás and Goldstein disagree on its interpretation.

92 Millás, 1943–50, p. 332.

93 See another exact method, based on spherical trigonometry, explained by Ibn al-Hā'im in Comes, 2001, pp. 351–2.

TABLE 6.14

<BAC	E	<BAC	E
10°	23;33,9°	100°	23;44,46° (-2")
20	23;33,36	110	23;46,27 (-2")
30	23;34,21 (-1")	120	23;48,2 (-1")
40	23;35,21 (-1")	130	23;49,27 (-1")
50	23;36,36 (-1")	140	23;50,40
60	23;38,2 (-1")	150	23;51,40
70	23;39,37 (-1")	160	23;52,24
80	23;41,18 (-2")	170	23;52,51
90	23;43,2 (-2")	180	23;53,0

With the result of this analysis of the two tables we can easily compute the values of the obliquity of the ecliptic for the time of Ptolemy (20.7.137, the date of Ptolemy's star catalogue) and for the time of Ibn al-Zarqālluh (31.12.1074, 467/1074–75 being the year probably used by our author for computations related to his own time):

20.7.137

$\epsilon = 23;52,21^{\circ}$

31.12.1074

$\epsilon = 23;33,49^{\circ}$

In fig. 6.11 we can see that Ibn al-Zarqālluh's model and tables are, more or less, adjusted to the values of the obliquity of the ecliptic determined by Ptolemy, the astronomers of al-Ma'mūn, and Ibn al-Zarqālluh himself.

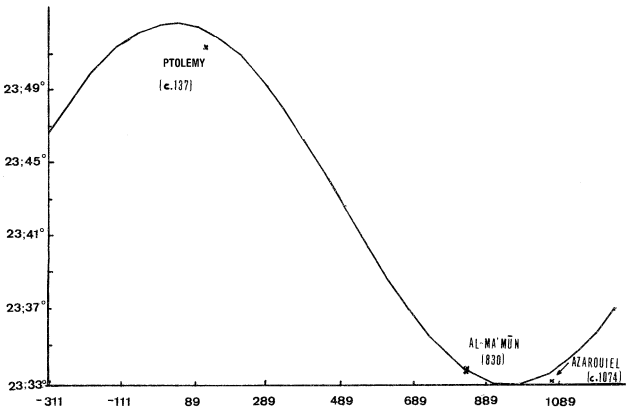


FIGURE 6.11
Secular variations of the obliquity of the ecliptic according to Ibn al-Zarqālluh's third precession model

6.2.2.6 An Evaluation of Ibn al-Zarqālluh’s Trepidation Models

It is now time to check the results obtained by Ibn al-Zarqālluh with his three trepidation models against the historical values of precession which he himself estimated through his indirect determinations of the longitude of *Qalb al-asad* (Regulus) for the time of Hipparchus (−146), Ptolemy (139), al-Battānī (883) and, finally, for his own time (1075). All these data have been summarised above in table 6.2 and we should add to them what he considers to be three real observations of the same star:

Thābit ibn Qurra (831)	133;13°
Maslama al-Majrīṭī (980)	135;40°
Ibn Barghūth (1050)	136;20°

An evaluation of his two first models appears in table 6.15, where the column Δ_λ *Regulus* corresponds to the differences in longitude of the star Regulus between the times of Hipparchus and those of Ptolemy (2;47°), al-Battānī (14;19°) and his own times (16;56°). The following columns correspond to the approximations to the same differences which Ibn al-Zarqālluh obtains with his two first models: I have noted in brackets the differences between the values quoted by Ibn al-Zarqālluh himself and those that I have recomputed, as explained above in 6.2.2.3 and 6.2.2.4. *A* and *B* denote the two variants described by the author for each one of his two first models. The unit used is always the degree.

TABLE 6.15

Δ_λ <i>Regulus</i>	Δ_λ Mod. 1A	Δ_λ Mod. 1B	Δ_λ Mod. 2A	Δ_λ Mod. 2B
2;47 (Pt)	2;22 (−6′)	2;47 (3′)	2;51,50 (5;25′)	2;48 (9′)
14;19 (Bt)	13;18 (−4′)	14;44 (3′)	14;27,30 (4;15′)	14;22 (21′)
16;56 (Zq)	15;45 (−8′)	17;26	17;1,50 (2′)	16;56 (40′)

The comparison has been extended slightly for the third model (table 6.16). I have used here all the data available corresponding to the authorities mentioned above and I have calculated the values of precession using not only Ibn al-Zarqālluh’s third model but also the tables for trepidation preserved in the *Liber de motu* and in the *Toledan Tables*. These computations have been made for 30 June of each of the years mentioned below.

TABLE 6.16

Epoch	Δ_λ "obs."	Δ_λ Mod. 3	Δ_λ <i>De motu</i>
Hipp. (−146)	−13;34°	−13;33.33°	−14;17,4°
Ptol. (139)	−10;47	−10;47,58	−11;14,1
Battānī (883)	0;45	0;47,19	0;49,59
Maslama (980)	2;27	2;9,49	2;18,6
Ibn Bargh. (1050)	3;7	3;3,37	3;16,39
Ibn Zarq. (1075)	3;22	3;21,29	3;36,22

The analysis of the numerical values in tables 6.15 and 6.16 shows that Ibn al-Zarqālluh is making a serious attempt to obtain results as close as possible to the data that constitute his starting point, and also that he has already obtained excellent approximations in his model 2B although we do not know exactly how he computed them. With our current state of knowledge, it seems clear to me that we understand the geometry of Ibn al-Zarqālluh's two first models fairly well but we are not sure of the numerical parameters, many of which seem to be corrupt in the Hebrew translation. The situation is not the same in the third model, for which recomputation gives us accurate results. One can conclude from them that Ibn al-Zarqālluh does not attach the same importance to all the determinations of the longitude of Regulus that he mentions in his text: his third model gives results which are consistent with the longitudes of Regulus related to Hipparchus, Ptolemy, Thābit, al-Battānī and himself, but there is no similar adjustment to the longitudes based on observations made by Maslama and Ibn Barghūth. This seems consistent with his attitude towards observations (see 6.2.2.2): he does not rely on them (the only exception is the one he ascribes to Thābit) but rather on the results of his own recomputation.

The last column, computed with the parameters of the tables of the *Liber de motu* and with the approximation $\sin \Delta_\lambda = \sin i \sin 10;45^\circ$ is particularly interesting, for the results obtained are as approximate as many of those computed with Ibn al-Zarqālluh's two first models. In fact, if we make a small correction to the dates corresponding to Hipparchus (−149, instead of −146), Ptolemy (137, instead of 139) and Thābit (830, instead of 831) and establish the differences in the value of precession between the standard epochs (30 June) mentioned by Ibn al-Zarqālluh and that of Thābit (3;39,41° for 830), we will obtain the following results (table 6.17):

TABLE 6.17

Epoch	Δ_{λ} "obs."	Δ_{λ} <i>De motu</i>
Hipparchus (−149)	−13;34°	−13;32,13°
Ptolemy (137)	−10;47	−10;45,26
Battānī (883)	0;45	0;48,47
Maslama (980)	2;27	2;13,17
Ibn Barghūth (1050)	3;7	3;9,31

The accuracy of the results obtained for the epochs of Hipparchus, Ptolemy and al-Battānī does not seem to be a matter of chance and it strengthens the arguments given above (see 6.2.1) in favour of the Toledan character of the trepidation tables extant in both the *Liber de motu* and the *Toledan Tables*. When Ibn al-Zarqālluh wrote his book on the motion of the fixed stars at a later stage of his life, he was actually developing a line of research which had started many years before in Toledo, and which was the result not only of his own work but also of the work of qāḍī Ṣāʿid, who probably wrote his *Iṣlāḥ ḥarakāt al-nujūm* or *al-kawākib* on the same topic. A second author who also wrote on trepidation was, as we have already seen, Abū Marwān al-Istijī, another member of the Toledan group (6.2.4). According to Ibn al-Hāʾim,⁹⁴ he stated in his *Risālat al-iqbāl wa l-idbār* that the value of precession for the time of Hipparchus was −9;38,40° and −6;50,40° for the time of Ptolemy. The difference between these two values is 2;48° and we have seen throughout this section that the difference between the longitudes of Regulus for the epochs of Hipparchus and Ptolemy was 2;47° according to Ibn al-Zarqālluh: if Abū Marwān had been relying on the *Almagest* (VII,2) he would have mentioned 2;40°, the increase in the longitude of Regulus between the observations of Hipparchus (−128/ −127) and those of Ptolemy (139).

6.2.3 Ibn al-Kammād on Trepidation

In chapters 1 and 3 (see especially 3.2.4), I often mentioned Abū Jaʿfar Aḥmad b. Yūsuf [b.] al-Kammād,⁹⁵ an astronomer who flourished in Córdoba towards the end of the eleventh century and during the first half of the twelfth. His chronology was established by A. Mestres who, in the Hyderabad recension of Ibn Ishāq’s *zīj*, discovered a reference to a nativity horoscope cast by him and

94 M. Comes, 2001, pp. 319 and 369.
95 M. Comes, in *BA* vol. 3, pp. 732–734; Comes, in *BEA*, pp. 559–560.

dated in 510/1116–17, followed by a quotation from the same author dealing with the eight indicators that should be considered in the analysis of a nativity horoscope.⁹⁶ Another date (480/1088) appears in the Madrid manuscript of the Latin translation of his *al-Zīj al-Muqtabas* (fol. 65v). Although we cannot be completely sure that the reference to this year really corresponds to the period of Ibn al-Kammād's activity, it fits the hypothesis that Ibn al-Kammād was a direct disciple of Ibn al-Zarqālluh, as established in a marginal note found in a Latin manuscript of the *Toledan Tables*;⁹⁷ indeed, he was probably the student who helped Ibn al-Zarqālluh in the observations he made in Córdoba.⁹⁸ Whatever the case, Ibn al-Kammād mentions Ibn al-Zarqālluh (d. 1100) several times and Ibn Mu'adh (d. 1093) only once.

Ibn al-Kammād is the author of three *zīj*es entitled *al-Kawr 'alā l-dawr*, *al-Amad 'alā l-abad* and the aforementioned *al-Muqtabas*, which seems to be a summary of the two previous ones⁹⁹ and was translated into Latin by Johannes de Dumpno in Palermo in 1262. This translation is extant in only one manuscript (Madrid Biblioteca Nacional ms. 10023). There is also a Hebrew translation of this *zīj*, and both versions have been analysed extensively by Chabás and Goldstein.¹⁰⁰ As for the other two *zīj*es, some information on *al-Kawr 'alā l-dawr* can also be obtained from ms. Madrid BN 10023: the canons of the *Muqtabas* seem to end in fol. 18v, but they are followed by an appendix, containing a new series of canons (also translated by Johannes de Dumpno) the first of which is “secundum cuiusdam libri tabulas dictas caur hale eddaur”. Furthermore, there are quotations from *al-Kawr* (as well as of the two other *zīj*es) in the anonymous Tunisian recension of Ibn Ishāq's *zīj*.¹⁰¹

Ibn al-Kammād's *Muqtabas* contains a set of trepidation tables in its Latin translation:¹⁰² the mean motion of the head of Aries is calculated with the table appearing in fol. 28v, in which the column for single lunar years (*mabsūṭa*) uses a parameter of 0;0,0,54,56,54°/ day, while the table of collected lunar years

96 Mestres, 1996, p. 404.

97 Millás, 1943–50, p. 14; Toomer, 1969, pp. 314–15.

98 Millás, 1943–50, p. 279. Millás translates “me ayudé con alumnos míos y personas técnicas de mi confianza” and, according to him, he was helped by several students. Dr. Teresa Martínez, who revised the Hebrew text of this passage, told me that the plural of “alumnos míos” should be replaced by a word in singular. This interpretation has been confirmed by B.R. Goldstein (private communication) who translates the passage in this way: “and I was helped in this by one of the scholars [*lit. men*] of investigation ...”.

99 According to the prologue of its canons: “Iste canon extractus a canonibus Elemed Hale Elebed et Caurhale Eddaur” (ms. BN Madrid 10023, fol. 1r).

100 Chabás & Goldstein, 1994; Chabás & Goldstein, 2015 a.

101 Mestres, 1996 pp. 385, 392, 420–426.

102 On these tables, see Mercier, 1996, pp. 330–337.

(*majmū'a*) is based on a slightly different parameter (0;0,0,54,56,58°/ day). Both parameters are very near the one used in Ibn al-Zarqālluh's third model, in his table calculated for Arabic years (0;0,0,54,56,59,24°/ day). Both tables are calculated to the precision of seconds and they reappear in Abū l-Ḥasan al-Marrākushī with a different distribution and with a precision of minutes, obtained by truncation of the seconds.¹⁰³

Chapter 12 of the canons (fol. 7v) gives a brief explanation of the use of the equation table appearing in fol. 35v (*Tabula directionis aduentionis capitis Arietis*). This brevity is due, according to the same chapter, to the fact that the topic was dealt with extensively in *al-Amad 'alā l-abad*.¹⁰⁴ This might be the source of a short text on trepidation preserved, in a Castilian translation, in ms. 115 of the Segovia Cathedral, part of which has been edited and commented by J.L. Mancha.¹⁰⁵ More information can be found in Ibn al-Hā'im's criticisms of Ibn al-Kammād's trepidation model, as explained in *al-Kawr 'alā l-dawr* and *al-Muqtabas*.¹⁰⁶ One of these criticisms is quite obvious: as Ibn al-Kammād's model is, as we will see, a variant of Ibn al-Zarqālluh's third model, how could he accept an equation table – valid for all times – such as the one appearing in the *Liber de motu*, which will necessarily be based on a fixed value of the obliquity of the ecliptic?¹⁰⁷

Like Ibn al-Zarqālluh's third model, Ibn al-Kammād's trepidation model uses an equatorial epicycle on which the moving head of Aries rotates, with radius 4^p (instead of the Zarqāllian 4;7,58^p). There is also a polar deferent whose pole is the pole of the equator and whose radius is 23;33° in the text, although this value should probably be replaced by 23;43°¹⁰⁸ (see fig. 6.10). A small polar epicycle, with radius 0;11° (Zarq. 0;10°), rotates on the polar deferent. So far,

103 Marrākushī, 1984, p. 44; Sédillot, 1834, p. 130.

104 "Sed nos iam tractavimus super expansione eius in canone Helmed Hale Elebed, et si intraverimus memorationem eius in hoc canone exiebamus a (termino) in transitus et brevitatis, et tendebamus ad prolixitatem et ad multitudines." (fol. 7v).

105 Mancha, 1998. The beginning of this text ("Este libro fizo maestre Yūçuf Benacomed sobre çircunferençia de moto sacado por tiempo seculo ...") seems, however, to refer both to *al-Kawr 'alā l-dawr* ("çircunferençia de moto") and to *al-Amad 'alā l-abad* ("por tiempo seculo").

106 Comes, 2001, pp. 306–318; Calvo, 2002, pp. 115–118.

107 Comes, 2001 pp. 308–9 and 366–7 (0.[3]). In spite of this, and in spite of the title of *al-Amad 'alā l-abad* which could be translated as "valid for ever", one should bear in mind the long quotation from the prologue of *al-Kawr 'alā l-dawr* reproduced in the anonymous Tunisian recension of Ibn Ishāq: in it Ibn al-Kammād gives a list of the errors found in *zīj*es after 40 years: most of them derive from erroneous values of precession and changes in the obliquity of the ecliptic. See Mestres, 1996, p. 410; Mestres, 1999, chapter 43 of the canons, pp. 296–297.

108 Mancha, 1998, p. 7, fn. 7.

this is an exact replica of Ibn al-Zarqālluh's third model with minimum changes in the numerical parameters. The great difference lies in the fact that Ibn al-Kammād considers that the period of rotation of the moving head of Aries on the equatorial epicycle, and the period of rotation of the pole of the ecliptic on the polar epicycle are the same. Consequently the head of Aries and the pole of the ecliptic would cover equal arcs in equal times. No information is given about the period of the two rotations, although this period is probably the same as the one implied by the table of the mean motion of the head of Aries of the *Muqtabas*: a mean motion per day of $0;0,0,54,56,58^{\circ}$ implies a complete rotation in, approximately, 3874 solar years, the same period used by Ibn al-Zarqālluh's third model for the rotation of the head of Aries. The divergence arises when Ibn al-Kammād considers that this same period of rotation can be applied to the motion of the pole of the ecliptic in its polar epicycle which, for Ibn al-Zarqālluh, is about 1850 years.

In Ibn al-Kammād's model, when the pole of the ecliptic is in a position that produces a mean value of the obliquity ($23;43^{\circ}$), implying that it is situated in one of the two intersections of the polar epicycle with the polar deferent (M or D in fig. 6.12), the head of Aries will also be in another intersection of the equatorial epicycle with the equator (A or O), where $\Delta_{\lambda} = 0^{\circ}$. The head of Aries will, consequently, move along 180° (arc AHO), where $\Delta_{\lambda} = 0^{\circ}$ in A and O, while the ecliptic pole will also move along 180° (arc DEM), where ε will attain its mid value in M and D, and will pass through its minimum value (E).¹⁰⁹

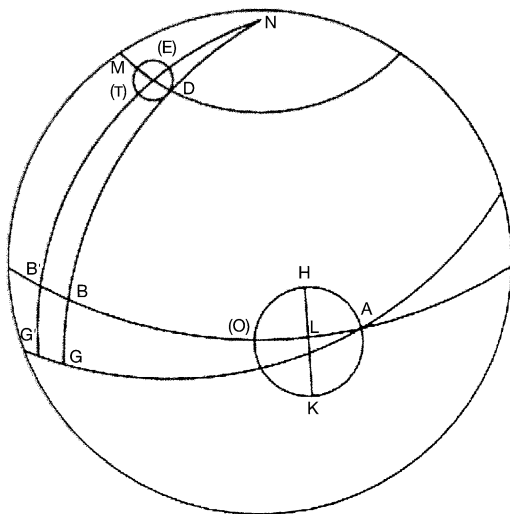


FIGURE 6.12
Borrowed from M. Comes

¹⁰⁹ Mancha, 1998, pp. 8–10.

Interestingly, Mercè Comes has shown that this model was probably known to Naṣīr al-Dīn al-Ṭūsī (1201–1274) who, in his *Tadhkira fī ‘ilm al-hay’a*, remarks that some astronomer/s considered that only one mover could justify the two differences (*ikhtilāfān*): precession and changes in the obliquity. Then, he describes a model that, with a small correction of one word (replacing *nuqṭa* by *quṭb*, two words that can be easily confused in Arabic script), corresponds exactly to Ibn al-Kammād’s model.¹¹⁰

This model is criticised by Ibn al-Hā’im because Ibn al-Kammād accompanied it with an equation table in which both $\Delta\lambda$ and ϵ were given for each value of angle i . This [double] equation table, considered by Ibn al-Kammād to be valid for any time, was given for arguments between 1° and 90° and it was considered to be symmetrical for the three other quadrants – something which is, obviously, not true for the function regulating the obliquity of the ecliptic. It is also evident, as Ibn al-Hā’im remarks, that, for arguments between 1° and 90°, the pole will not achieve all of its obliquities: if it starts at point D (mid obliquity), it will only reach point T (maximum obliquity), but not point E (minimum obliquity).¹¹¹

Finally, Ibn al-Hā’im considers that Ibn al-Kammād’s models and tables do not give good results for the basic data (*uṣūl*) adopted by Ibn al-Zarqālluh as the starting point for his models of trepidation (6.2.2.2). The *uṣūl* adopted by Ibn al-Hā’im,¹¹² derive clearly from Ibn al-Zarqālluh’s:¹¹³

TABLE 6.18

	Ibn al-Zarqālluh’s value of precession	Ibn al-Hā’im’s value of precession	Ibn al-Kammād’s value of precession
Hipparchus (–146)	–9;28,30°	–9;29°	–9;19°
Ptolemy (139)	–6;42,45°	–6;42°	–6;57°

Ibn al-Kammād’s values of precession for the times of Hipparchus and Ptolemy can be confirmed using the equation table of the *Muqtabas zīj*, which is not accompanied by a second table giving the obliquity of the ecliptic. It is

110 Comes, 2001, pp. 316–318; Ragep, 1993 vol. 1, pp. 124–125, and the commentary in vol. II, pp. 400–408.
111 Comes, 2001, pp. 310–311, 368 (0.[7]–0.[8]), 313–314, 374–376 (11.4.[4]–11.4.[7]).
112 Comes, 2001, pp. 311–2, 368–9 (0.[9]).
113 Millás, 1943–50, p. 318.

preserved in fol. 35v of ms. BN Madrid 10023 and it also appears in Abū l-Ḥasan al-Marrākushī's *Mabādī' wa ghāyāt*,¹¹⁴ Ibn 'Azzūz al-Qusantīnī's *Muwāfiq zīj*,¹¹⁵ the *Barcelona Tables*,¹¹⁶ and the Hebrew tables of Juan Gil of Castiello or of Burgos (fl. 1350–2).¹¹⁷

This table attains a maximum of 9;59° for $i = 90^\circ$. Chabás and Goldstein¹¹⁸ have made several attempts to recompute it and have concluded that the best results are obtained with the formula:

$$\Delta_\lambda = \arcsine (r \sin i / 60)$$

for $r = 10;24$, obtaining a maximum of 9;58,54°. This expression is based on the hypothesis that Ibn al-Kammād is computing a table following Ibn al-Zarqālluh's second model.

This value of the radius is contradicted by the Castilian translation edited by Mancha, where it is stated clearly that $r = 4$: this could be a rounding of the Zarqallian 4;7,58. Besides, it is clear that Ibn al-Kammād is using a modified version of Ibn al-Zarqālluh's third model. As a result of this, Mancha has suggested a second hypothesis, based on the third model, and recomputes the equation of trepidation using the following formulas:

$$\delta = r \sin i$$

$$\Delta_\lambda = \arcsine (\sin \delta / \sin \epsilon)$$

In which δ is the declination of the moving head of Aries (see table 6.11), $\epsilon = 23;33^\circ$ and $r = 4$.

A third possibility was suggested to me by Margarita Castells:¹¹⁹ with a maximum equation of 9;59° and a radius of 4°, the corresponding obliquity of the ecliptic should be 23;43°, the mean value of ϵ in Ibn al-Zarqālluh's obliquity model.

Table 6.19 shows a sample of Ibn al-Kammād's equation table, together with a recomputation of the corresponding values according to Chabás & Goldstein

114 Marrākushī, 1984 p. 44; Sédillot, 1834, p. 131.

115 Ms. D2461 of the Rabat National Library, p. 365; ms. 8772 of the Rabat Hasaniyya Library p. 11; Samsó, 1997, pp. 93–94.

116 Millás, 1962, pp. 194–5.

117 Goldstein, 1985, p. 237 ; Beaujouan, 1969, p. 11; Chabás & Goldstein, 1994, p. 24; Chabás & Goldstein, 2015 a, p. 598.

118 Chabás & Goldstein, 1994, pp. 24–27; Chabás & Goldstein, 2015 a, pp. 596–598.

119 Samsó, 2011, p. 323, fn. 31.

TABLE 6.19

<i>i</i>	Ibn al-Kammād	R ₁	R ₂	R ₃	<i>i</i>	Ibn al-Kammād	R ₁	R ₂	R ₃
5°	0;53°	0;52° (+1')	0;52° (+1')	0;52° (+1')	50°	7;4[3] ^{oa}	7;38° (+5')	7;41° (+2')	7;39° (+4')
10	1;45	1;43 (+2)	1;44 (+1)	1;44 (+1)	55	8;20	8;10 (+10)	8;13 (+7)	8;11 (+9)
15	2;41	2;34 (+7)	2;36 (+5)	2;34 (+7)	60	8;47 ^b	8;38 (+9)	8;42 (+5)	8;39 (+8)
20	3;25	3;24 (+1)	3;26 (-1)	3;24 (+1)	65	9;10	9;2 (+8)	9;6 (+4)	9;1 (+9)
25	4;19	4;12 (+7)	4;14 (+5)	4;11 (+8)	70	9;28	9;22 (+6)	9;27 (+1)	9;24 (+4)
30	5;7	4;58 (+9)	5;1 (+6)	4;59 (+8)	75	9;45	9;38 (+7)	9;43 (+2)	9;39 (+6)
35	5;52	5;42 (+10)	5;45 (+7)	5;44 (+8)	80	9;55	9;50 (+5)	9;54 (+1)	9;49 (+6)
40	[6];[3]3 ^c	6;24 (+9)	6;27 (+6)	6;24 (+9)	85	9;59	9;57 (+2)	10;1 (-2)	9;57 (+2)
45	7;14	7;2 (+12)	7;6 (+8)	7;4 (+10)	90	9;59	9;59	10;3 (-4)	9;59

a 7;58° in ms. Madrid BN; 7;43° in al-Marrākushī; 7;45° in Ibn ‘Azzūz (mss. Rabat NL and Ḥasaniyya).

b 8;45° in Ibn ‘Azzūz (ms. Rabat NL). With some hesitation I read 8;42° in the Ḥasaniyya ms.

c 7;53° in ms. Madrid BN; 6;53° in al-Marrākushī and in Ibn ‘Azzūz (mss. Rabat NL and Ḥasaniyya); 6;43° in the Barcelona tables. It is, however, quite possible that the *nūn* (50) is the result of a confusion with a *lām* (30) and that the correct reading is 6;33°.

(R₁), to Mancha (R₂) (using $r = 4$, while Mancha recomputes with $r = 3$; 58,19) and to my own third possibility ($r = 4$, $\varepsilon = 23,43^\circ$ and $\Delta_{\lambda\max} = 9;59^\circ$) (R₃).

In 6.19 I have not intended to give a critical edition of the table and I have only checked the four sources available to me in order to justify a few corrections of clearly corrupt values. Whatever the case, it seems clear that recomputation R₂ gives better results than the other two and it has the advantage of preserving both the characteristics of Ibn al-Kammād’s model and $r = 4$, a value clearly documented in the Castilian translation. However, it has a drawback:

it does not yield $9;59^\circ$ as the value for $i = 90^\circ$, and my experience is that these maxima are usually correct in astronomical tables. The frequent errors in the table cannot be ascribed to the Latin manuscript, as most of them also appear in the other four sources. My impression is that most of them are the result of Ibn al-Kammād's computing mistakes, something which seems to be a constant in the *Muqtabas zīj*.

6.2.4 *al-Istijī and the Muntakhab Zīj*

Ibn al-Hā'im gives us some new information about two lost works dealing with trepidation. One of them was written by Abū Marwān al-Istijī, one of the members of the Toledan team, and author of a short treatise on world astrology entitled *Risāla fī l-tasyirāt wa-maṭāriḥ al-shu'ā'āt wa ta'līl ba'd uṣūl al-ṣinā'a* (3.1.1.5.1),¹²⁰ dedicated to his patron qāḍī Ṣā'id al-Andalusī. He also wrote a *zīj* and a *Risālat al-iqbāl wa l-idbār*.¹²¹ As for the *zīj*, it could be identified with the *Zīju-nā al-muṣaḥḥaḥ* (our corrected *zīj*) mentioned in the *Risālat al-tasyirāt*.¹²² According to Ibn al-Hā'im, the mean motions of this *zīj*, together with al-Battānī's planetary equations and other erroneous information gathered from Ibn al-Kammād's *al-Kawr 'alā l-dawr*, were used in his own time (*fī zamāni-nā hādhā*, i.e., ca. 1205) by his contemporary, the anonymous author of the *Muntakhab Zīj*.¹²³ Neither the *Muṣaḥḥaḥ* nor the *Muntakhab Zīj* are extant and the only information we have about their treatment of trepidation relies on two short paragraphs of Ibn al-Hā'im's *al-Zīj al-Kāmil*.¹²⁴

Thus, Ibn al-Hā'im offers us some data on the value of trepidation for the times of Hipparchus and Ptolemy found in al-Istijī's *zīj* or *Risālat al-iqbāl* and in the *Muntakhab Zīj*, which we can compare to those of the *Liber de motu*, Ibn al-Zarqālluh and Ibn al-Kammād (table 6.20):

120 Edition, translation and commentary in Samsó & Berrani, 2005 (see pp. 176–178, 225). See also Samsó & Berrani, 1999.

121 Ibn al-Hā'im, *al-Zīj al-Kāmil*, ms. Oxford Bodleian Library Arab 285 (Marsh 618), fol. 4r.

122 Samsó & Berrani, 2005, pp. 211 and 238, 216 and 241. See also pp. 166–167.

123 Comes, 2001, pp. 309 and 367 (o.[4]). Ibn al-Hā'im (ms. Oxford B.L. 285), fol. 4r states that al-Istijī's time was 150 years before him. He therefore he places al-Istijī's activity ca. 1055.

124 Comes, 2001, pp. 318–323 and 369–370.

TABLE 6.20

	<i>De motu's</i> Δ_λ	Ibn al- Zarqālluh's Δ_λ	al-Istijī's Δ_λ	Ibn al- Kammād's Δ_λ	<i>Muntakhab's</i> Δ_λ
Hipparchus (-146)	$\Delta_\lambda = -9;51,11^\circ$ $i = 9^\circ 23;26,48^\circ$	$\Delta_\lambda = -9;28,30^\circ$ $i = 9^\circ 22;32,12^\circ$	$\Delta_\lambda = -9;38,40^\circ$ $i = 9^\circ 23;42^\circ$	$\Delta_\lambda = -9;19^\circ$ $i = 9^\circ 22;32,12^\circ$	$\Delta_\lambda = -9;45,30^\circ$ $i = 9^\circ 23;42^\circ$
Ptolemy (139)	$\Delta_\lambda = -7;3,58^\circ$ $i = 10^\circ 18;44,13^\circ$	$\Delta_\lambda = -6;42,45^\circ$ $i = 10^\circ 19;1,30^\circ$	$\Delta_\lambda = -6;50,40^\circ$ $i = 10^\circ 19;23^\circ$	$\Delta_\lambda = -6;57^\circ$ $i = 10^\circ 19;1,30^\circ$	$\Delta_\lambda = -6;55,15^\circ$ $i = 10^\circ 19;23^\circ$

In this table we can see, first of all, that both al-Istijī and the *Muntakhab zīj* use the same mean motion values for the head of Aries, and different values for the equations. Interestingly, Ibn al-Hā'im states elsewhere in the book that al-Istijī, like *qāḍī* Ṣā'id and Ibn al-Zarqālluh, did not compute an equation table for precession, due to the fact that such a table is only valid for a given obliquity of the ecliptic.¹²⁵ Besides, the two sources give different results from those of the *Liber de motu*, Ibn al-Zarqālluh's third model¹²⁶ and Ibn al-Kammād – even though, as noted by M. Comes,¹²⁷ the *Muntakhab* results are close to those mentioned by Ibn al-Zarqālluh for model *Iib* (Hipparchus $9;45^\circ$ and Ptolemy $6;57^\circ$) (6.2.2.4). The increase in longitude between the times of Hipparchus and Ptolemy is $2;48^\circ$ according to al-Istijī, and $2;50,15^\circ$ in the *Muntakhab zīj*. These values are identical, or very close, to those we can obtain from the text values of Ibn al-Zarqālluh's model *Iib* ($2;48^\circ$, see 6.2.2.4) and *Ib* ($2;47^\circ$, in 6.2.2.3). If al-Istijī had been relying on the *Almagest* (VII,2) he would have mentioned $2;40^\circ$, for this is the increase in the longitude of Regulus between the observations of Hipparchus ($-128/-127$) and those of Ptolemy (139).

Mercè Comes has also tried to obtain an approximation to the maximum values ($\Delta_{\lambda\max}$) of the equation used by al-Istijī and the *Muntakhab zīj*, with the expression:

$$\Delta_{\lambda\max} = \arcsine (\sin \Delta_\lambda / \sin i)$$

¹²⁵ Comes, 2001, pp. 346 and 382.

¹²⁶ Mercè Comes (2001, p. 319) adds that “neither Δ_λ nor i coincide with the date given by Azarquiel for his first and second models”.

¹²⁷ Comes, 2001, p. 320.

I have recomputed these values and obtained:

- al-Istijī:
 $\Delta_{\lambda\max} = 10;32,33^\circ$ (Hipparchus)
 $\Delta_{\lambda\max} = 10;32,54^\circ$ (Ptolemy)
- *Muntakhab zīj*:
 $\Delta_{\lambda\max} = 10;40,2^\circ$ (Hipparchus)
 $\Delta_{\lambda\max} = 10;40,1^\circ$ (Ptolemy)

I have never found a maximum equation of ca. $10;33^\circ$ in any Andalusī or Maghribī source, while $10;40^\circ$ is very near the value used by Ibn al-Raqqām in his *Qawīm zīj* ($10;40,13^\circ$),¹²⁸ while in his two other *zījes* (*Shāmil*¹²⁹ and *Mustawfī*¹³⁰), the tables are calculated for a maximum value of $10;24^\circ$. $10;40^\circ$ has also been found in the versified *zīj* of Abū l-Ḥasan ‘Alī al-Qusanṭīnī.¹³¹ This led M. Comes to suggest that the equation table of the *Mustawfī zīj* might have been preserved in the aforementioned *zījes*.¹³²

Finally I should say that, while Ibn al-Hā’im is critical of the treatment of trepidation in the *Muntakhab zīj*, he seems to be satisfied with the materials found in al-Istijī’s lost *zīj* or *risāla* on the same topic. Thus, he says that, using the mean planetary motions and al-Battānī’s equations, [and adding the value of precession obtained with the *Muntakhab* trepidation tables to the sidereal longitude obtained], there is a large difference between these computed values and those resulting from observations of meridian transits of the planets.¹³³

6.2.5 *Ibn al-Hā’im on Trepidation*

Ibn al-Hā’im’s *al-Zīj al-Kāmil fī l-ta’ālīm* (ca. 1205) is the oldest Arabic source extant in this language which describes in detail the Toledan theory of trepidation, represented mainly by Ibn al-Zarqālluh’s book. It has been preserved in a single manuscript (Oxford Bodleian Arab 285/ Marsh 618) and the relevant passages dealing with trepidation have been edited and exhaustively commented by Mercè Comes in the long article I have been quoting in the preceding pages and which I will follow in this section.

First of all we should note that Ibn al-Hā’im is using Ibn al-Zarqālluh’s third model, which he knows through his book on the motion of the fixed stars. His

¹²⁸ Ms. Rabat National Library 260, fol. 66v.

¹²⁹ ‘Abd al-Raḥmān, 1996, p. 231.

¹³⁰ Samsó, 2014 p. 314.

¹³¹ Kennedy & King, 1982; Comes, 1996, p. 360.

¹³² Comes, 2001, p. 322.

¹³³ Comes, 2001, pp. 323 and 370 (o.[11]).

treatment of this topic is critical, and he corrects some of Ibn al-Zarqālluh's numerical parameters (or accepts them in a more precise form) and has access to some non-extant Zarqāllian notes on trepidation. As he himself remarks:

When we examined the question in the pages (*ṣuḥuf*) of him (Ibn al-Zarqālluh) that were available to us related to this question [...] we found that they disagreed with the foundations (*uṣūl*) which were the basis of our knowledge. This compels us to review the whole question in order to correct him, especially in relation to the motion of the pole. The results calculated by him concerning the values of the obliquity for the times of the observations are different from those actually observed by him and by others. The difference is not small. Therefore, we cannot build our results on this basis, not even in an approximate way, for it produces important changes in the distance between the Head of Aries and the equinoctial point.¹³⁴

This is not the only passage of the *Kāmil zīj* in which he refers to Ibn al-Zarqālluh's "pages": Roser Puig has also found another comment of the same kind, related to the Toledan astronomer's observations of the motion of the Moon, in Ibn al-Zarqālluh's own hand (*'alā mā wajadnā la-hu bi-khaṭṭyadi-hi*).¹³⁵

Ibn al-Hā'im carefully describes the steps of the revolution of both the head of Aries and the pole of the ecliptic. Initially, some 50 years before the Hijra, the Head of Aries coincides with the equator (A in fig. 6.12), and the pole of the ecliptic will be at its mid distance from the pole of the equator (M) in the direction of its minimum distance (E). At a later stage, the head of Aries will move, approximately, to a midpoint between A and H, while the ecliptic pole will have rotated about 90° and will be in E, attaining minimum obliquity. The process continues until the head of Aries has made a full rotation on the equatorial epicycle, while the pole of the ecliptic has made almost two revolutions on the polar epicycle. There seems to be a connection between the two rotations, although no physical explanation is given.¹³⁶

The revolution periods accepted by Ibn al-Hā'im are those of Ibn al-Zarqālluh himself, although with greater precision:

1. Revolution of the head of Aries in its equatorial epicycle: 3874 Julian years + 3 and ca. ½ months, instead of the 3874 years given by Ibn al-Zarqālluh. This fits the parameter given by Ibn al-Hā'im himself: 0;0,0,54,57,30°/ day

¹³⁴ Mercè Comes's translation in Comes, 2001, p. 338; the Arabic text in p. 378.

¹³⁵ Puig, 2000, pp. 76–77 and 91.

¹³⁶ Comes, 2001, pp. 332–337.

and it is near the value obtained from Ibn al-Zarqālluh's table for Arabic years (0;0.0,54,56,59,24°/ day in 6.2.2.5.2.1).

2. Revolution of the pole of the ecliptic in its polar epicycle: 2032 years and ca.29 days, instead of Ibn al-Zarqālluh's 1850 Julian years, and in agreement with Ibn al-Hā'im's 0;0,1,44,49°/ day. This is clearly a correction of the Zarqāllian parameter (0;0,1,55,4,42,42°/ day in 6.2.2.5.2.4) and it corresponds to Ibn al-Hā'im's critical remark in the aforementioned quotation: "The results calculated by him concerning the values of the obliquity for the times of the observations are different from those actually observed by him and by others." If we look at fig. 6.11 (6.2.2.5.2.5), we can easily see that the obliquity of the ecliptic was increasing in Ibn al-Zarqālluh's time, contradicting the decrease shown by his own observations. We will see other corrections of the same kind in later Maghribī sources.

These two periods of revolution imply that the revolution period of the pole of the ecliptic is almost twice the revolution period of the head of Aries. The exact ratio (*nisba*) is the following:

$$\text{Revolution of the pole} / \text{revolution of the head of Aries} = 1;54,40 / 1$$

This is used by Ibn al-Hā'im to prove that it is impossible to construct a table to determine Δ_λ which would be valid for all times, as Ibn al-Kammād (as well as other astronomers contemporary of Ibn al-Hā'im) had sought to do by considering that the two periods were equal.

Ibn al-Hā'im also describes the tables needed to calculate both the obliquity of the ecliptic and the value of Δ_λ , although no such tables are preserved in the manuscript. As for the obliquity tables, they are the same as those used by Ibn al-Zarqālluh, although the mean motion table which allows the computation of the position of the pole of the ecliptic will necessarily be based on Ibn al-Hā'im's new parameter, and the radius of the polar epicycle is not the Zarqāllian 0;10°, but 0;10,20°. This is not the case for the tables used to calculate Δ_λ (*al-iqbāl al-awwal*, in his terminology), as our author uses here the same technique as the *Liber de motu*, Ibn al-Kammād, and all the following Maghribī tradition and computes a table which gives Δ_λ directly and is based on a fixed value of the obliquity of the ecliptic and on a radius of the equatorial epicycle of 4;8°, a rounded value of Ibn al-Zarqālluh's 4;7,58°. Ibn al-Hā'im had criticised Ibn al-Kammād for doing exactly the same thing, although, as noted by Comes, he accepts the use of such a table for a limited period of time when great accuracy is not needed.¹³⁷ In fact, Ibn al-Hā'im limits the validity of a *zīj* for a

¹³⁷ Comes, 2001, p. 348.

period of some 40 years, a figure we have already found in Ibn al-Kammād's *al-Kawr 'alā l-dawr* (6.2.3): Ibn al-Kammād, apparently, believes that the value of precession, the obliquity of the ecliptic or the solar eccentricity will not have changed significantly during this period of time. Besides, Ibn al-Hā'im also explains Ibn al-Zarqālluh's procedure for computing Δ_λ by establishing, first, the value of ϵ , for a given time, computing $\sin \delta$ (a table gives him $\sin \delta$, instead of δ as in the Zarqāllian table 6.11), and applying the formula:

$$\sin \Delta_\lambda = \sin \delta / \sin \epsilon$$

Finally, he explains how to calculate the *iqbāl al-thānī* (second accession) (6.2.2.5.2.3) which he calls *iqbāl al-'amūd fī l-quṭr* (accession of the perpendicular to the diameter), and its use for the computation of the equation of time.¹³⁸

All this is accompanied by careful trigonometrical proofs in which Ibn al-Hā'im explicitly uses the sine law and the cosine law. The same laws were used by Ibn al-Zarqālluh, something that led Mercè Comes to wonder about the source of the Toledan astronomer's knowledge of the new trigonometry: Ibn Mu'adh's *Kitāb majhūlāt qisī al-kura* does not seem to have circulated in Toledo in the eleventh century, given that it was not mentioned by *qāḍī* Ṣā'id in his *Ṭabaqāt al-umam*.¹³⁹

6.2.6 Trepidation in Maghribī zījēs

6.2.6.1 Introduction

In this section I will try to analyse the development of trepidation theory in Maghribī *zījēs* between the end of the thirteenth century and the beginning of the fifteenth, when it was finally replaced by the theory of constant precession as the result of the introduction of Eastern *zījēs*. The use of these *zījēs* by Maghribī astronomers signified the end of Ibn al-Zarqālluh's influence.

The *zījēs* I will consider have already been mentioned in 1.4.2 and will be analysed in depth in chapter 7. They begin with the unfinished *zīj* by Abū l-'Abbās ibn Ishāq al-Tamīmī al-Tūnisī (fl. Tunis and Marrakesh ca. 1193–1222). Apparently this astronomer compiled a set of tables for the computation of planetary longitudes, eclipses, equation of time, parallax and, probably, solar and lunar velocity, but did not accompany them with a complete collection of canons. Ibn Ishāq's *zīj* was the object of at least five different recensions, most of which contained Ibn Ishāq's numerical tables to which different canons were added. They are:

¹³⁸ Comes, 2001, pp. 356–360.

¹³⁹ Comes, 2001, p. 355.

1. The anonymous Tunisian recension extant in ms. Hyderabad Andra Pradesh State Library 298, compiled between 1266 and 1281.¹⁴⁰
2. Ibn al-Bannā's (1256–1321) *Minhāj al-ṭālib fī ta'dīl al-kawākib*.¹⁴¹
- 3–5. Three *zīj*es authored by Ibn al-Raqqām (d. 1315): *al-Zīj al-Mustawfī* (Tunis, after 1280–81),¹⁴² *al-Zīj al-Shāmil* (Bijāya, ca. 1290)¹⁴³ and *al-Zīj al-Qawīm* (Tunis, after 1280–81, revised in Granada).¹⁴⁴

To these five *zīj*es I will add another two which are not recensions of Ibn Ishāq's *zīj* but which show clear Andalusī influences. Both were compiled in the fourteenth century:

6. The *Muwāfiq zīj* of Abū l-Qāsim ibn 'Azzūz al-Qusanṭīnī (d. 1354).¹⁴⁵
7. An untitled *zīj*, with canons written in verse, by Abū l-Ḥasan 'Alī ibn Abī 'Alī al-Qusanṭīnī.¹⁴⁶

None of these *zīj*es contains important theoretical improvements on Ibn al-Zarqālluh's third model and in them we only find small changes in the numerical parameters.

6.2.6.2 Trepidation Tables

The radix of the mean motion of the head of Aries is 3;53,55 for the Hijra in the anonymous Tunisian recension, as well as in the *zīj*es of Ibn al-Bannā' and Ibn al-Raqqām (*Mustawfī*, *Shāmil* and *Qawīm*). The same radix is mentioned in the *zīj* of Abū l-Ḥasan 'Alī al-Qusanṭīnī, although the table of mean motion of the Head of Aries for years *majmū'a* implies a radix value of 2;53,4°, probably as a result of an error. Only Ibn 'Azzūz' *Muwāfiq zīj* uses a different radix: 3;51,21° (see 6.2.2.5.2.1).

- The mean motion tables for the revolution of the head of Aries are identical in the Tunisian recension, Ibn al-Bannā's *Minhāj*, the three *zīj*es of Ibn al-Raqqām and the *Muwāfiq zīj* of Ibn 'Azzūz. They are based on a daily mean motion of 0;0,0,54,57,17,35°/day $\pm$ 0;0,0,0,0,27°. This implies a complete revolution in 3874 Julian years (3921 years in Ibn al-Zarqālluh).

140 Mestres, 1996. The whole set of canons and a selection of the tables were edited and studied in Mestres, 1999.

141 Edition and Spanish translation of the canons in Vernet, 1952. See also Samsó & Millás, 1994; Samsó & Millás, 1998.

142 Samsó, 2014.

143 'Abd al-Rahmān, 1996.

144 Ms. Rabat National Library 260; an extract of this *zīj* is contained in an unnumbered manuscript of the Museo Naval of Madrid.

145 Samsó, 1997.

146 Kennedy & King, 1982. A copy of the only extant manuscript (Escorial 909, fols. 44v–63v) was generously offered to me by F. José Luís del Valle, director of the Escorial Library.

- Only Abū l-Ḥasan ‘Alī al-Qusanṭīnī uses a different parameter: 0;0,0,53,19, 3,34° / day (*majmū’a* years) and 0;0,0,53,19,7,11 (*mabsūṭa* years). Kennedy and King give 0;0,0,53,20,31° / day. These values imply a complete revolution of the head of Aries in 3,993 Julian years.

The maximum value of the equation of trepidation ($\Delta_{\lambda_{\max}}$) in the anonymous Tunisian recension, Ibn al-Bannā’, and Ibn al-Raqqām (*Shāmil*, *Mustawfi*) is 10;24°. This value of $\Delta_{\lambda_{\max}}$ derives from Ibn al-Zarqālluh’s $r = 4;7,58^\circ$ and $\varepsilon = 23;33^\circ$. In Ibn al-Zarqālluh’s table 6.11, we find for $i = 90^\circ$, $\delta = 4;7,58^\circ$, and then:

$$\Delta_{\lambda_{\max}} = \arcsine \left(\sin 4;7,58^\circ / \sin 23;33^\circ \right) = 10;23,29^\circ$$

The table of Δ_λ in these four *zīj*es is the same and it has been calculated with the expression:

$$\Delta_\lambda = \arcsin \left(\sin i \cdot \sin 10;24^\circ \right)$$

A sample of this table, as it appears in Ibn al-Bannā’s *Minhāj*, follows:

TABLE 6.21 Table of the equation of trepidation^a

<i>i</i>	Δ_λ	<i>i</i>	Δ_λ
5°	0;55 ^{ob} (+1')	50°	7;57°
10	1;48	55	8;30
15	2;40 (−1') ^c	60	8;59 ^d (−1')
20	3;32	65	9;25
25	4;23 ^e	70	9;46
30	5;11 ^f	75	10;2 ^g

a Samsó & Millás, 1994, p. 13.
b 0;55° in the Tunisian recension, Ibn al-Bannā’ and Ibn al-Raqqām’s *Mustawfi*. 0;54° in Ibn al-Raqqām’s *Shāmil*.
c 2;40° in the Tunisian recension, Ibn al-Bannā’ and Ibn al-Raqqām’s *Mustawfi*. 2;41° in Ibn al-Raqqām’s *Shāmil*.
d 8;59° in the Tunisian recension, Ibn al-Bannā’ and Ibn al-Raqqām’s *Mustawfi*. 9;0° in Ibn al-Raqqām’s *Shāmil*.
e 4;28° in the the Tunisian recension. A confusion between 3 (*jīm*) and 8 (*hā’*) is frequent in the *abjad* notation. All the other sources have 4;23°.
f 5;51° in the Tunisian recension. A confusion between 10 (*yā’*) and 50 (*nūn*) is frequent in the *abjad* notation. All the other sources have 5;11°.
g 10;2° in the Tunisian recension, Ibn al-Bannā’ and Ibn al-Raqqām’s *Mustawfi*. 10;3° in Ibn al-Raqqām’s *Shāmil*.

TABLE 6.21 Table of the equation of trepidation (*cont.*)

<i>i</i>	Δ_{λ}	<i>i</i>	Δ_{λ}
35	5;57	80	10;14
40	6;40	85	10;21 ^h (−1′)
45	7;21 ⁱ (+1′)	90	10;24

h 10;21° in the Tunisian recension, Ibn al-Bannā’ and Ibn al-Raqqām’s *Mustawfī*. 10;22° in Ibn al-Raqqām’s *Shāmil*.
i 7;20° in Ibn al-Raqqām’s *Shāmil*. The other sources have 7;21°.

Different values for $\Delta_{\lambda\max}$ appear in Ibn al-Raqqām’s *Qawīm* (10;40,13°)¹⁴⁷ as well as in the versified *zīj* of Abū l-Ḥasan ‘Alī al-Qusanṭīnī (10;40°).¹⁴⁸ We have already mentioned M. Comes’s suggestion that one of these two tables might derive from the lost *Muntakhab zīj* (6.2.4) The equation table of both *zīj*es is calculated with the expression:

$$\Delta_{\lambda} = 10;40^{\circ} \sin i.$$

A sample of this table follows:

TABLE 6.22 Abū l-Ḥasan al-Qusanṭīnī’s table of the equation of trepidation

Argument (<i>i</i>)	Δ_{λ}	Argument (<i>i</i>)	Δ_{λ}
5°	0;56°	50°	8;9° (−1′)
10	1;51	55	8;43 (−1′) ^a
15	2;45 (−1′) ^b	60	9;13 (−1′) ^c
20	3;38 (−1′) ^d	65	9;39 (−1′)
25	4;29 (−1′) ^e	70	10;1

- a 8;44° in the *Qawīm zīj*.
b 2;46° in the *Qawīm zīj*.
c 9;14° in the *Qawīm zīj*.
d 3;40° in the *Qawīm zīj*.
e 4;32° in the *Qawīm zīj*.

147 Ms. Rabat N.L. 260, p. 86.
148 Ms. Escorial 909 fol. 60r.

TABLE 6.22 Abū l-Ḥasan al-Qusanṭīnī's table of the equation of trepidation (*cont.*)

Argument (<i>i</i>)	Δ_{λ}	Argument (<i>i</i>)	Δ_{λ}
30	5;19 ^f	75	10;18
35	6;6 (−1') ^g	80	10;30 ^h
40	6;50 (−1') ⁱ	85	10;38 ^j
45	7;31 (−2')	90	10;40

f 5;24° in the *Qawīm zīj*.

g 6;10° in the *Qawīm zīj*.

h 10;29° in the *Qawīm zīj*.

i 6;51° in the *Qawīm zīj*.

j 10;37° in the *Qawīm zīj*.

The corresponding table of the *Muwāfiq zīj* has a maximum value of 9;59°. ¹⁴⁹ As we have already seen in 6.2.3, this table derives from Ibn al-Kammād's *Muqtabas*.

In the anonymous Tunisian recension ¹⁵⁰ and in Ibn al-Raqqām's *Shāmīl* and *Mustawfī*, ¹⁵¹ we also find two tables related to trepidation which have a clear Zarqāllian origin. One of them calculates the declination of the moving head of Aries (*Jadwal mayl awwal burj al-Ḥamal wa-huwa jadwal mayl al-mabda'*), which gives:

$$\delta(i) = 4;7,57 \sin i$$

This table is practically identical to table 6.11, although the latter is calculated for $r = 4;7,58^\circ$. The other table computes the second accession (6.2.2.5.2.3), here called *Jadwal iqbāl al-‘amūd fī l-quṭr wa-huwa tashrīq al-mabda'*, and it is the result of:

$$\Delta_{\lambda 2} = 4;7,57 - 4;7,57 \cos i$$

149 Ms. Rabat N.L. 2461 p. 365; Rabat Ḥasaniyya 8772 p. 12.

150 Table 55 in the Hyderabad manuscript (not edited by Mestres). A similar table, calculated for a different value of the radius of the equatorial epicycle, appears in the *Liber de motu* (table 6.1).

151 Ms. Tunis fol. 103r.

This table, in the three aforementioned sources, has a maximum value of $8;16^\circ$ (approximately the double of the radius of the equatorial epicycle, $4;7,57^\circ$) and it is the result of rounding to minutes table 6.12.¹⁵²

Finally, the Tunisian recension also contains a table for the motion of Qalb al-Asad (Regulus) (no. 11). It is a table of constant precession based on a parameter of 1° in 70;15 Persian years, the parameter being derived from Ibn Yūnus.¹⁵³ The text gives year 372 of Yazdajird (1003 AD) as a radix value for the table; this is the year in which Ibn Yūnus made an observation of Regulus which he appears to have used, together with the longitude of this star calculated by Hipparchus, to determine his constant of precession.

6.2.6.3 Obliquity of the Ecliptic

As we have already seen, all the aforementioned Maghribī *zīj*es contain a table allowing a direct computation of Δ_λ , following the tradition represented by the *Liber de motu*, Ibn al-Kammād, and Ibn al-Hā'im. In spite of this, the Hyderabad recension, as well as Ibn al-Raqqām's *Shāmīl* and *Mustawfī* *zīj*es, also contain tables to calculate the obliquity of the ecliptic, following Ibn al-Zarqālluh's model (6.2.2.5.2.4 and 6.2.2.5.2.5), although with a small change in the maximum ($\varepsilon = 23;52,30^\circ$) and minimum ($\varepsilon = 23;32,30^\circ$) values. The radius of the polar epicycle remains equal to $0;10^\circ$.

The mean motion tables which regulate the position of the pole of the ecliptic are the same in these three *zīj*es and use a Hijra radix $9^\circ 11;14,34$ (Ibn al-Zarqālluh: $9^\circ 15;14,33^\circ$) which, in the Hyderabad recension, is assigned to Toledo (longitude 28°), while the *Shāmīl* states that this value can be used for Bijāya and adds that $9^\circ 11;14,33$ corresponds to Arin. Obviously this difference of $1''$ seems excessive for a difference of longitudes of 56° (the time difference being $3;44^h$), given the fact that the mean motion parameter per day is only $0;0,1,18,54,14,40^\circ$ (Ibn al-Zarqālluh: $0;0,1,55,4,42,42^\circ$). This implies a complete revolution of the ecliptic pole in 2698 Julian years (Ibn al-Zarqālluh: 1850 years).

A second table calculates the obliquity of the ecliptic directly and can be found in the three *zīj*es with the title *Jadwal al-mayl al-kullī wa-daqa'iq ikhtilāf al-muyūl*.¹⁵⁴ The only difference between them is that, in Ibn al-Raqqām's *Shāmīl*, the argument is counted from point B (fig. 6.10) in which the obliquity

152 Table 57 in the Hyderabad manuscript (not edited by Mestres) and page 244 of ms. Rabat National Library 2461.

153 Mestres, 1999, pp. 201 and 167, no. 42/18; Caussin, 1804, p. 154, fn. 3; D.A. King, "Ibn Yūnus" in *DSB* vol. 14 p. 577.

154 Tunisian recension table 56 (not edited by Mestres); *Mustawfī*, ms. NL Tunis fol. 85r; ms. NL Rabat p. 272 (left).

reaches its minimum value (23;32,30°), while in the Tunisian recension and in the *Mustawfī zīj* it starts at point E (maximum obliquity, 23;52,30°). In table 6.23 we reproduce the values of the *Mustawfī zīj*, which are more accurate than those of the other two *zīj*es. The recomputation has been made using the following expression:

$$\epsilon = \text{acos} \left(\cos 0;10 \cos 23;42,30^\circ + \sin 0;10 \sin 23;42,30^\circ \cos i \right)$$

where *i* is the argument (angle <BAC in fig. 6.10). In a parallel column and with the same argument, the Tunisian recension and the *Mustawfī zīj* give the *daqā'iq ikhtilāf al-muyūl* (minutes of the variation of the obliquity), an interpolation function with a maximum of 60' (for maximum obliquity) and a minimum of 0' (for minimum obliquity).

With the two tables just mentioned, we can easily compute the obliquity of the ecliptic for the time of Ptolemy (20.7.137) and Ibn al-Zarqālluh (31.12.1074). I will also add a computation for Ibn Ishāq's time, for whom I assume

TABLE 6.23 Ibn al-Raqqām's *Mustawfī* table of the obliquity of the ecliptic

Argument (<BAC in fig. 6.10)	ε	Argument (<BAC in fig. 6.10)	ε
0°/180°	23;32,30°		
10/170	23;32,39 ^a	100°/80	23;44,14° (−2")
20/160	23;33,6 ^b	110/70	23;45,55 ^c (−2")
30/150	23;33,50 ^d (−1")	120/60	23;47,30 (−1")
40/140	23;34,50 ^e (−1")	130/50	23;48,56 (−1")
50/130	23;36,4 ^f (−1")	140/40	23;50,10
60/120	23;37,30 ^g (−1")	150/30	23;51,10
70/110	23;39,4 ^h (−3")	160/20	23;51,54
80/100	23;40,46 (−2")	170/10	23;52,21
90	23;42,30 (−2")	180/0	23;52,30

- a 23;32,34° in the Hyderabad recension; 23;32,37 (sic) in the *Shāmil zīj*.
- b 23;33,6° in the Hyderabad recension and in the *Mustawfī zīj*; 23;33,3° in the *Shāmil zīj*.
- c 23;45,56° in the Hyderabad recension and in the *Mustawfī zīj*.
- d 23;33,50° in the *Mustawfī zīj*; 23;33,45° in the *Shāmil zīj* and in the Tunisian recension.
- e 23;34,50° in the *Mustawfī zīj*; 23;33,44° in the *Shāmil zīj* and in the Tunisian recension.
- f 23;36,4° in the *Mustawfī zīj*; 23;35,56° in the *Shāmil zīj* and in the Tunisian recension.
- g 23;37,30° in the *Mustawfī zīj*; 23;37,21 in the *Shāmil zīj* and in the Tunisian recension.
- h 23;39,4° in the *Mustawfī zīj*; 23;38,55° in the *Shāmil zīj* and in the Tunisian recension.

31.12.1193, the year in which the beginning of his astronomical activity has been established:

ε (Ptolemy): $23;50,27^{\circ}$

ε (Ibn al-Zarqālluh): $23;33,1^{\circ}$

ε (Ibn Ishāq): $23;32,30^{\circ}$

Obviously, Ptolemy's obliquity is about one minute less than the attested value ($23;51,20^{\circ}$), while $23;33,1^{\circ}$ for Ibn al-Zarqālluh is very accurate, as the *Toledan Tables* ascribe to this astronomer figures of $23;33^{\circ}$, $23;33,8^{\circ}$ and $23;33,30^{\circ}$.¹⁵⁵ Finally, $23;32,30^{\circ}$ is precisely the value used in the declination tables of all the *zīj*es belonging to Ibn Ishāq's tradition. Interestingly, this parameter was established, according to Ibn Ishāq himself, by an anonymous astronomer from Miknāsa (Meknès) through an observation made in 602/ 1205–6; $23;32,30^{\circ}$ was also used in the declination table appearing in the Alfonsine *Libro del reloj de la piedra de la sombra*,¹⁵⁶ where the source seems to be Ibn Ishāq (7.6.2.2.1.4),¹⁵⁷ as well as in al-Sanjufīnī's *zīj*, compiled in Tibet in 1366.¹⁵⁸

As noted many years ago by Mercè Comes,¹⁵⁹ it is obvious that Ibn al-Zarqālluh's tables for the variation of the obliquity of the ecliptic gave inaccurate results for his own time: as we have seen in 6.2.2.5.2.5, the calculated value for 31.12.1073 is $23;33,49^{\circ}$, exceeding the amount obtained by Ibn al-Zarqālluh's observations. Besides, Ibn al-Zarqālluh's parameters implied that the function was increasing, while the reality observed required their adjustment to the value ($23;32,30^{\circ}$) used by Ibn Ishāq. This is the reason for the changes in the parameters of the model introduced by Ibn Ishāq. We will see, however, that $23;32,30^{\circ}$ as a minimum value will soon have to be rejected, and this will be one of the main reasons for the later abandonment of Ibn al-Zarqālluh's trepidation model.

6.2.6.4 A Strange Table in the Hyderabad Manuscript of the Tunisian Recension of Ibn Ishāq's *zīj*

Ibn Ishāq is not the only astronomer who tried to design a computational procedure to predict the value of the obliquity of the ecliptic as well as the longitude of the solar apogee (see 6.3). In the Hyderabad manuscript containing

¹⁵⁵ See my review of Fritz Pedersen's edition of the *Toledan Tables* in *Suhayl* 3 (2003), p. 250.

¹⁵⁶ Ms. Villa Amil 156, fol. 178r; Rico, 1863–67, vol. IV, p. 6.

¹⁵⁷ Comes, 1992 pp. 152–3.

¹⁵⁸ Kennedy, 1987–88; Kennedy & Hogendijk, 1988.

¹⁵⁹ Comes, 1992, p. 155.

the Tunisian recension of Ibn Ishāq’s *zīj* we find a peculiar table 5 which I will transcribe below in table 6.24 taken from Mestres’s edition,¹⁶⁰ adding an approximate equivalence of the dates before (BH) and after (H) Hijra and an attempt to date most of the astronomers mentioned. This table has the title *Jadwal li-asmā’ al-ruṣṣād al-mashāhīr li-awj al-shams wa-miqdār al-mayl al-kullī* (Table of the names of the famous observers of the solar apogee and of the value of the obliquity of the ecliptic):

TABLE 6.24

“Observers”	Location	Date	Solar apogee from Gemini	Obliquity of the ecliptic
M.w.ṭ.n wa-Uq.ṭ.mūn (fl. ca. 432 BC)	bi-Bābil	1090 BH/ 436 BC	11;7,55°	23;55,47°
Anāṭīs al-Ḥakīm	bi-Qūs	820 BH/175 BC	12;59,49°	23;55,3°
Sh.ṭ.j.jīr al-Yunānī	bi l-Iskandariyya	[7]30 BH/87 BC	13;40,42°	23;53,13°
Atastyūs	bi l-Iskandariyya	632 BH/9 AD	14;3,36°	23;50,59°
al-Nābk.l.h al-Hindī	bi-Bāshah	630 BH/11 AD	14;3,13° (?)	23;50,42°
Ibārkhūs al-Yunānī (ca. 190–120 BC)	bi-Anṭākiyya	763(?) BH/119 BC	1[4];53,32°	23;50,37°
J.r.mīn al-Hindī	bi-Sarandīb	732 BH/89 BC	1[4];14,14° (?)	23;50,5
M.n.lāwsh (ca. 70–ca. 120)	bi l-Ahrām	521 BH/117 AD	14;24,25°	23;47,58
Baṭlamiyūs (fl. 125–141)	bi l-Iskandariyya	480 BH/156 AD	14;32,8°	23;47,19°
Khawākhashb	bi-Khwarizm	220 BH/409 AD	15;21,2	23;43,13°
Khālīd b. Khālīd	bi-Barriyyat	195 H/810–11 AD	16;40,6	23;36,35°
al-Marwāzī (fl. 829)	Sinjār			
Ibn Abi Manṣūr	bi-Baghdād	209 H/824–5 AD	16;42,45°	23;36,23°
fī ayyām al-Ma’mūn				
Thābit ibn Qurra (836–901)	bi-Ḥarrān	211 H/826–7 AD	16;43,7°	23;36,21°
Ibrāhīm ibn Sinān (fl. 909–946)	bi l-Kūfa	216 H/831–2	16;43,53°	23;36,19°
Abū Mūsā ibn Shākīr (fl. first half of the 9th c.)	bi l-Baṣra	225 H/839–40 AD	16;45,35°	23;36,12°

160 Mestres, 1999, p. 185.

TABLE 6.24 (cont.)

“Observers”	Location	Date	Solar apogee from Gemini	Obliquity of the ecliptic
Muḥammad ibn Jābir al-Battānī (d. 929)	bi l-Raqqā al-Bayḍāʾ	235 H/ 849–50 AD	16;47,29°	23;35,45°
Abū ‘Amr al-Šūfī (= ‘Abd al-Raḥmān al-Šūfī, d. 983?)	bi-Baghdād	244 H/ 858–9 AD	1[6];49,10°	23;35,4°
al-Amīr ‘Abd Allāh al-Ḥākim (996–1021)	bi-Miṣr	293 H/ 905–6 AD	17;21,31°	23;33,31°
Ibrāhīm al-Naqqāsh (Ibn al-Zarqālluh, d. 1100)	bi-dākhil Ṭulayṭula	467 H/ 1074–5 AD	17;21,31°	23;32,31°
‘Alī al-Shajjār (= Ibn Khalaf, fl. ca. 1050) mā’a Ibn Wāfid (d. 1075)	bi-dākhil Ṭulayṭula	477 H/ 1084–5 AD	17;[2]3,35°	23;32,12°
al-Sharīf al-Aʿlam (Ibn al-Aʿlam d. 985)	bi l-Baṣra	484 H/ 1091–2 AD	17;34,15°	23;32,5°
al-Faḍl al-Qaṣṣī (Abū l-Faḍl al-Qaṣṣī, 1183–1251)	bi-dākhil Qaṣṣ	5[7]0 H/ 1174–5 AD	17;53,7°	23;30,33°
Ghiyām ibn Ruḡjār (r. 1154–1166)	bi-Šiqilliya	574 H/ 1178–9 AD	17;53,53°	23;30,7°
Abū l-Abbās ibn Iṣḥāq (fl. 1193–1222)	bi-madīnat Tūnis	589 H/ 1193 AD	17;56,32°	23;30,17°

This table merits a few comments. First of all, the identification of the pre-Islamic “observers” is, sometimes, extremely easy (Meton and Euctemon, Hipparchus, Menelaos and Ptolemy, for example). In other instances, however, it is impossible, although, in some cases, Mestres has tried to guess their identity.¹⁶¹

Anāṭīs al-Ḥākīm: Aratos?

al-Nābk.l.h al-Hindī: Tinkalluh and Bāshah, perhaps Bārah

¹⁶¹ Mestres, 1999, pp. 177–8.

J.r.mīn al-Hindī: Jamshīd?

Khwākhshb: Jāmasb?

Besides, in two cases at least, the list ascribes to a monarch the work of an astronomer at his service: apparently al-Amīr ‘Abd Allāh al-Ḥākim should be identified with the Fatimid caliph Abū ‘Alī al-Manṣūr al-Ḥākim bi-amr Allāh (996–1021) in spite of the fact that the date given in the table (293/905–6) is about one century off. The astronomer in his service, to whom the determination of the longitude of the solar apogee and of the obliquity of the ecliptic is ascribed, should be Ibn Yūnus (d. 1009). The second case is that of Ghiyām ibn Rujjār (William, the son of Roger). This name seems to be related to Ibn Khaldūn’s remark in the *Muqaddima*:¹⁶²

And the people of our time in the Maghrib use the *zīj* attributed to Ibn Ishāq, and they pretend that Ibn Ishāq used [his own] observations and that there was a Jew in Sicily who was an expert in *hay’a* and mathematics (*ta’ālīm*), that he had been interested in [making] observations, and that he had sent to him [Ibn Ishāq] what was convenient for him (*mā yaṣihhu la-hu*) related to the conditions (*aḥwāl*) and motions of the celestial bodies

Ghiyām ibn Rujjār is, no doubt William II (1154–1166), son of Roger II of Sicily, who probably had in his service an unnamed Jew who acted as his astrologer and who, at a later date, was an informant of Ibn Ishāq who provided him with a great deal of information on Andalusī astronomy and, particularly, on the works of Ibn al-Zarqālluh.

The list is given in a chronological order with a few mistakes (see Hipparchus, J.r.mīn al-Hindī, and al-Faḍl al-Qafṣī). As I have added approximate chronological indicators for the astronomers identified, the reader can see that the dates ascribed to each astronomer are, sometimes, more or less correct in the case of pre-Islamic identified authors (Meton and Euctemon, Hipparchus, Menelaus and Ptolemy), although we find major mistakes in their location: it seems surprising, for example, to place Meton and Euctemon in Babylon. As for astronomers in the Islamic period, Khālid al-Marwāzī and Yahyā b. Abī Manṣūr’s dates are adequate, Thābit ibn Qurra is attributed a date before his birth, Abū Mūsā b. Shākir, al-Battānī and ‘Abd al-Raḥmān al-Ṣufī seem to be placed about half a century earlier than their own time and Ibn al-A‘lam’s chronology is even

¹⁶² Ibn Khaldūn, *Muqaddima*. Ed. M. Quatremère, vol. III, pp. 107–8.

worse. In spite of some inconsistencies, the dates given by the author of the table seem to improve as they approach his own time.

When we check the values of the longitude of the solar apogee and the obliquity of the ecliptic, we can easily see that the solar apogee increases in longitude as time advances, with two errors that I have marked with a question mark. It is surprising to see a progression of this kind before the time of the Ma'mūnian observations which established that the solar apogee moves at the same rate as the precession of equinoxes, or even before Ibn al-Zarqālluh who seems to have been the first to discover that the solar apogee has its own motion. As for the obliquity of the ecliptic, it is always decreasing and reaches a minimum value of 23;30,17° in the time of Ibn Ishāq. This is interesting because it shows that the author of the table was aware that 23;32,30°, considered by Ibn Ishāq to be the minimum value of the obliquity, could not be regarded as such and that the obliquity continued stubbornly to decrease.

Besides, it is obvious that the table values ascribed to astronomers of the past for both the longitude of the solar apogee and the obliquity of the ecliptic do not bear a comparison with the documented values observed or used by those astronomers themselves. A few examples given by Mercè Comes¹⁶³ will suffice:

TABLE 6.25

Observers	Solar apogee (table 5)	Documented value of the apogee	Obliquity (table 5)	Documented value of the obliquity
Ptolemy	74;32,8°	65;30°	23;47,19°	23;51,20°
al-Ma'mūn's astronomers	76;42,45°	84;39° and 82;39°	23;36,23°	23;33°
Ibn al-Zarqālluh	77;31,21°	78;30°/ 77;55/ 77;50	23;32,31°	23;33°/ 23;33,30°
Ibn Ishāq	77;56,32°	78;47°	23;30,17°	23;32,30°

It seems, then, that the values in table 5 of the Hyderabad manuscript do not represent real values obtained in historical observations, but are the result of computations which were not made using the model of Ibn al-Zarqālluh and Ibn Ishāq and tables for the determination of the obliquity of the ecliptic, or

163 Comes, 1992, pp. 149–50.

using al-Zarqālluh's parameter for obtaining the longitude of the solar apogee. Unfortunately, I do not know how these values were computed.

6.2.6.5 Did Ibn al-Raqqām Use His Own Trepidation Tables?

We will soon see that Ibn Ishāq's trepidation tables were seriously criticised from the end of the fourteenth century onwards, because they did not agree with the reality observed. I have, however, the suspicion that Ibn al-Raqqām himself was aware of this disagreement. One indication in this regard is to be found in the *Shāmīl zīj*, which seems to have been compiled at a later date than the *Mustawfī* and the *Qawīm zījes*.¹⁶⁴ In it, Ibn al-Raqqām abandons the maximum value of $\Delta_\lambda = 10;24^\circ$, used in the *Mustawfī* and the *Qawīm*, as well as in the other *zījes* of the tradition of Ibn Ishāq, and replaces it with $10;40,13^\circ$.

Abū 'Abd Allāh al-Baqqār (fl. 1418) analyses the inconsistencies in Ibn al-Raqqām's treatment of trepidation in his *Kitāb al-adwār fī tasyīr al-anwār*.¹⁶⁵ He notes that Ibn al-Raqqām's *zījes* include a table of tropical longitudes of the stars for the end of 680/1282. In fact he is referring to the star table contained in the *Qawīm* and the *Mustawfī zījes*, which claims to be the result of observations (*marṣūda*) made in that year.¹⁶⁶ There is, however, a second star table in the *Shāmīl zīj*, dated in 687/1288–89. Our author calculates the value of trepidation for 1282 using Ibn al-Raqqām's own tables (those of the *Qawīm* or the *Mustawfī*) and obtains $9;26^\circ$.

Then, al-Baqqār subtracts $9;26^\circ$ from the tropical longitude of *Qalb al-asad* (Regulus, α Leo) in the star table of the *Qawīm* or *Mustawfī* ($139;16^\circ$), the difference being $129;50^\circ$, which should be the sidereal longitude of Regulus (*al-mawḍi' al-dhātī*), while the sidereal tables established by the ancients (*al-qudamā'*) give Regulus a longitude of $129;8^\circ$ or $129;18^\circ$.¹⁶⁷ This implies that the value of precession for the end of 680/ 10 April 1282 should be $10;8^\circ$ ($139;16^\circ - 129;8^\circ$), which is the amount one obtains with the *zījes* of Ibn Ishāq's school for the year 819/1416–7. Al-Baqqār studied the horoscope cast by Ibn al-Raqqām for the birth of his son in 685/1286–7, and saw that the value of trepidation used was 10° .¹⁶⁸

¹⁶⁴ Samsó, 2014.

¹⁶⁵ The astronomical part of this work was edited in Díaz-Fajardo, 2001: see, on this topic, pp. 55–57 and 91–92. On this work see also Vernet, 1971, pp. 188–189. On al-Baqqār, see 3.2.1.1 and 3.2.6.

¹⁶⁶ On this table see Parra, 2012 and Casulleras, 2012.

¹⁶⁷ In the sidereal star table of the *zījes* of Ibn al-Kammād, Ibn al-Bannā' and Ibn 'Azzūz, calculated for some 40 years before the *Hijra* (ca. 580), the longitude of *Qalb al-asad* is $129;8^\circ$.

¹⁶⁸ This comment is surprising because it implies that Ibn al-Raqqām was casting a tropical horoscope, something which is uncommon in the Andalusī-Maghribī tradition.

These values of trepidation can be checked for the two maxima of the equation values used by Ibn al-Raqqām: 10;24° and 10;40,13°. The recomputation can be found in table 6.26, in which I have used the last day of year 680 H. (29 Dhū l-Hijja/10 April 1282), as al-Baqqār states that the corresponding star table was calculated for the end of the year. As for the other two dates, I have computed the value of trepidation for the middle of the year: 1 Rajab 685 H./23 July 1286 and 1 Rajab 819/25 August 1416.

TABLE 6.26

Date	al-Baqqār's Δ_λ	Angle i	asin (sin 10;24° sin i)	asin (sin 10;40,13° sin i)
10.4.1282	9;26°	65;12,21°	9;25,56°	9;40,36°
23.7.1286	10°	65;36,43	9;27,47°	10;7,5
25.8.1416	10;8°	77;41,35	10;9,30	10;25,21

From these recomputations one can deduce that the value of Δ_λ given by al-Baqqār for the end of year 680 H. is correct, and that it has been calculated with the *Qawīm* or the *Mustawfī zīj*; the precession used in the nativity horoscope of Ibn al-Raqqām's son (1286) could have been calculated with the trepidation table preserved in the *Shāmil zīj* ($\Delta_{\lambda\text{max}} = 10;40,13^\circ$), in spite of the fact that this *zīj* was probably compiled in 689 H./ 1290.¹⁶⁹ As for the adscription of a precession of 10;8° for year 819/1416, this seems to have been an approximate value calculated with the *Qawīm* or the *Mustawfī*.

Whatever the case, it seems that al-Baqqār's treatment of the subject is correct. The tropical longitudes of *Qalb al-asad* in the star table of the *Qawīm* and the *Mustawfī*, compared to its sidereal longitude in the *zījes* of Ibn al-Kammād, Ibn al-Bannā' and Ibn 'Azzūz, give a value of precession which could not have been calculated with Ibn al-Raqqām's trepidation tables:

– *Qawīm/Mustawfī*: 139;16° – 129;8° = 10;8°

169 The date appearing in the prologue is 679 H./ 1280–81 ('Abd al-Raḥmān, 1996, p. 40), but there are reasons to believe that this is a mistake for 689: see Samsó, 2014, p. 298.

The same can be said of the *Shāmil*'s table of the stars in the lunar mansions in which *Qalb al-Asad* has a longitude of $139;20^\circ$, apparently for ca. 689 H./1290:¹⁷⁰

$$- \textit{Shāmil}: \quad 139;20^\circ - 129;8^\circ = 10;12^\circ$$

I suspect, therefore, that the values of precession used by Ibn al-Raqqām may have been calculated using a parameter of constant precession. The amounts I obtain from Ibn al-Raqqām's longitudes of *Qalb al-Asad* and those of Ptolemy's star catalogue are about $52''$ per year, or 1° in 69 years.

6.2.6.6 Decline and Fall of Trepidation Theory in the Maghrib

A few new sources dated between the fifteenth and the early seventeenth centuries have been analysed since the late 1990s.¹⁷¹ These sources shed some light on the progressive decline of the belief in the theory of trepidation in the Maghrib and on the reasons for the diffusion of Eastern *zīj*es in this part of the Islamic world.¹⁷² These sources are, on the one hand, the aforementioned *Kitāb al-adwār fī tasyīr al-anwār*, written by Abū 'Abd Allāh al-Baqqār in 821/1418) and, on the other, three commentaries on the poem on timekeeping written in 1391 by al-Jādirī (1375–1416):

- 1) The *Natā'ij al-afkār fī sharḥ Rawḍat al-azhār* ("Results of thoughts towards an explanation of the flower garden"). Its author is Abū 'Abd Allāh Muḥammad al-Ḥabbāk who worked in Tilimsān (Tlemcen) ca. 1515.¹⁷³ Our author mentions, among other sources (Ibn Ishāq, Ibn al-Bannā', Ibn al-Raqqām and Ibn Qunfudh, as well as Yaḥyā ibn Abī Manṣūr, the Banū Amājūr and al-Bīrūnī), the observations made in Barcelona by Abū Kursūm al-Yahūdī, who is none other than Jacob Corsuno (active in Barcelona between 1378 and 1380), the author of the final version of the *Tables of Barcelona* (7.6.3).¹⁷⁴ Interestingly, al-Ḥabbāk notes that Abū

¹⁷⁰ The table of the stars of the lunar mansions is dated (ms. Istanbul Kandilli 249, fols. 85v–86v) as corresponding to observations (*marṣūda*) made in 887/1482–3. I believe that this is a mistake for 687/1288–89. According to the introduction this *zīj* was compiled in Bijāya in 679/1279–80 which, again, I think is another mistake for 689/1290. The maximum value of the solar equation is $1;47,51^\circ$, a value calculated according to Ibn al-Zarqālluh's model of variable solar eccentricity (see 6.3.3.2) and it corresponds to 689/1290. See Samsó, 2014, p. 298.

¹⁷¹ Comes, 2002; Samsó, 1998; Samsó, 2001.

¹⁷² A similar phenomenon took place in Latin Europe from the second half of the thirteenth century onwards: see Nothaft, 2016.

¹⁷³ This source has been edited in Saidi, 2013.

¹⁷⁴ Millás, 1962; Chabás, 1996 a.

- Kursūm al-Yahūdī compiled a set of solar tables in Barcelona in 721/1321¹⁷⁵ which were in complete agreement with those of Ibn Abī l-Shukr.
- 2) A second commentary is the *Qatf al-anwār min Rawḍat al-azhār*, written by Abū Zayd ‘Abd al-Raḥmān b. ‘Umar b. Aḥmad al-Sūsī al-Jazūlī al-Bu‘aqlī, Ibn al-Muftī (d. 1611),¹⁷⁶ extant in MS Cairo K7584 (112v–155r).¹⁷⁷ The author lived in Marrakesh and both this city and Fez are mentioned in the text, but the main interest of Ibn al-Muftī was al-Sūs al-Aqṣā and, in particular, Tārūdant, for which he gives a latitude of 30;15° and computes a table of oblique ascensions.¹⁷⁸ Among the authorities quoted we should mention al-Battānī, [Ibn Hilāl] al-Sabtī, Ibn Ishāq, Ibn al-Raqqām (*Qawīm zīj*), Ibn al-Bannā’, Ibn Abī l-Shukr and Ibn Ḥabbāk.
 - 3) A third commentary was written by Abū Zayd ‘Abd al-Raḥmān al-Jānātī, known as al-Nafawī.¹⁷⁹ It has been preserved in ms. Cairo K7584 (fols. 78v–111r), dated 1257/1841,¹⁸⁰ and in ms. 80 of the Maktaba Ḥamzawiyya (Ayt ‘Ayāche). He seems to have been a late sixteenth-century author who worked in Fez (lat. 33;40°, sometimes rounded to 34°).¹⁸¹ He accepts a constant precession established by Ibn al-Rassām (?) and al-Sabtī of 1°/66 years. I have found two different statements on the value of precession for his own time: in one of them, precession has increased by 6° between the time of al-Jādirī and his own time (K7584, fol. 105r; Ḥamzawiyya 80, 315–316), which takes us to the end of the eighteenth century; in the other, which seems more reliable, the absolute value of precession for his time is 13°, 3° more than at the time of al-Jādirī (K7584 fols. 82v–83r; Ḥamzawiyya 80, 313). The authorities quoted are Ibn al-Haytham, Ibn al-Raqqām, Ibn al-Bannā’, Ibn Qunfudh, al-Ḥabbāk, Abū ‘Abd Allāh al-Harawī (?), Ibn Jaydūr (?), Muḥammad al-Sanūsī (d. 1490), the author of a commentary on al-Ḥabbāk’s *risāla* [*Bughyat al-ṭullāb*] and

175 This date is obviously wrong, as Jacob Corsuno’s presence in Barcelona is documented in 1380; previously (1376–8) he was in Seville. See Millás, 1962, pp. 70–71.

176 Brockelmann, *GALS*, vol. 11, p. 217; Renaud, 1932, pp. 179–180 (no. 535); Lamrabet, 2013, p. 228 (M142).

177 King, 1986 b, p. 142 (no. F43); King, 1981–86, vol. 1, p. 352.

178 K7584, fols. 122v, 124r and v. The value 30;15° for the latitude of Tārūdant (modern value 30;31°) does not appear in any source quoted by Kennedy & Kennedy, 1987, p. 335. See also fol. 139v.

179 This author is mentioned, though not identified, by Renaud, 1934, p. 94. Renaud reads al-Nafarī and is followed by Brockelmann, *GALS*, vol. 11, p. 218; Manūnī, 1963; Lamrabet, 2013, p. 273 (no. 578).

180 King, 1986 b, p. 139 (no. F26).

181 K7584 fol. 97v; Ḥamzawiyya 80, p. 290. 33;40° for the latitude of Fez seems to derive from Ibn Abī l-Shukr al-Maghribī: see Kennedy & Kennedy, 1987, pp. 117–118.

in relation to the question of the lunar mansions, early Eastern philologists such as Muḥammad ibn Kunāsa (d. 824), Ibn Qutayba (d. 889), Abū Ḥanīfa al-Dīnawārī (d. 895) and Abū Ishāq al-Zajjāj (d. 923).

These sources give some evidence that observations were made in the Maghrib in the thirteenth and fourteenth centuries. Ibn ‘Azzūz al-Qusanṭīnī speaks about his own observations made in Fez c. 1344 using with an armillary sphere in order to correct Ibn Ishāq’s mean motion tables,¹⁸² and al-Baqqār also talks, vaguely, about Fāsī observations (meridian altitudes of the Sun and other celestial bodies made with an astrolabe, a quadrant or even a large-sized instrument) and of a careful determination of the latitude of the city.¹⁸³ These observations raise doubts about the values of trepidation computed with the *zīj*es belonging to the Andalusī tradition. The *Natā’ij al-afkār*¹⁸⁴ states that al-Jādirī considered that the amount of precession for his time (1391–92) was 10°, an amount calculated with the trepidation table in Ibn Ishāq’s *zīj* which has a maximum of 10;24°. The author of the *Natā’ij* adds that, according to the observations made by Ibn Abī l-Shukr al-Maghribī in Damascus in 1259, precession at the end of the fourteenth century should be about 12°, or slightly more.¹⁸⁵ After Ibn Abī l-Shukr, a certain Abū l-Ḥasan ‘Alī b. Yūnus al-Balansī al-Ḥākimī (!) made observations in Cairo in 732/1331 and obtained a precession for that date of approximately 13°, while for al-Ḥabbāk’s own time (920/1514) it had reached 14° and some minutes.¹⁸⁶ As a conclusion, the *Natā’ij* states that the estimations of the *iqbāl* (accession) are incorrect and that there are disagreements between computation and observation when one tries to establish the meridian altitude of the Sun and other celestial bodies; there must be another motion in addition to accession and recession, although it is difficult to be more precise. This is why Muḥammad al-Ḥabbāk, following Ibn Abī l-Shukr, states that precession for his time is 13;40°¹⁸⁷ and that, using this amount, he obtains a good agreement with the observations, as happens with the table of solar tropical longitudes he has calculated for each day of the solar

182 Samsó, 1997; Samsó, 1999.

183 Díaz-Fajado, 2001, pp. 53–54, 89–90.

184 Saidi, 2013, pp. 44–48.

185 An unidentified scholar, al-Dalā’ili al-Tilimsānī transmits (*naqala*) the value 12;40°, in one of his works: Saidi, 2013, p. 45.

186 Saidi, 2013, p. 47. I have not been able to identify him. The implication seems to be the famous Ibn Yūnus (d. 958–59), who was also called Abū l-Ḥasan ‘Alī. King (“Ibn Yūnus”, *DSB*, vol. 14, p. 578) states that there are numerous later *zīj*es compiled in Egypt, Persia and Yemen which use materials derived from Ibn Yūnus’ *Ḥākimī Zīj*.

187 Saidi, 2013, p. 48.

year using Ibn Abī l-Shukr's tables.¹⁸⁸ Another passage in the commentary gives well-known information on constant precession ($1^\circ/100$ years according to Ptolemy, $1^\circ/80$ years according to the ancient Babylonians, $1^\circ/62$ years according to al-Battānī and $1^\circ/72$ years, which is the value used by Ibn Abī l-Shukr). To this he adds another estimation by Abū 'Abd al-'Azīz al-Rassām (?) and al-Sabtī, namely $1^\circ/66$ years, which is the amount accepted by al-Jādirī, while al-Ḥabbāk prefers Ibn Abī l-Shukr's parameter.

Similar critical remarks can be found in the *Qatf al-anwār*: precession in the time of al-Jādirī was 12° but the difference was, perhaps, imperceptible to that author. When the *Qatf* was written (ca. end of the sixteenth century?) precession reached 13° and some astronomers claimed that it was 14° .¹⁸⁹ Far richer in information is al-Baqqār's *Adwār* (1418) in which we find an explanation of the history of trepidation theory, emphasising the prominent role played by Abū Ishāq [ibn] al-Zarqālluh, whom he considers to be "the master of all scientists in relation to this topic" (*mu'allim al-'ulamā' li-hādhā l-sha'n*). According to al-Baqqār, Ibn al-Zarqālluh's maximum value of trepidation was 10° , a parameter close to that found in the latter's two first models, and it was Abū l-'Abbās ibn Ishāq who increased this value slightly by a few minutes.¹⁹⁰ Al-Baqqār computed the value of trepidation for his own time (821/1418) and obtained 10° , minus a few minutes, with Ibn al-Zarqālluh's tables, and $10;10^\circ$ with Ibn Ishāq's *zīj*. When this amount is used to obtain the tropical longitudes of the Sun or the planets using the (sidereal) tables of Ibn Ishāq and his followers, one can easily detect the presence of an error in the results: this is particularly noticeable when the Sun or the celestial body is near the equinoxes, and less so when its longitude is near one of the two solstices. If the results are checked against an observation of the meridian altitude of the Sun or of a planet, the difference, in the case of the Sun, amounts to 2° , as has been established in observations made in Fez, after a careful determination of the latitude of the city.¹⁹¹ Precession, for 1418, should be 12° and this raises doubts

188 This table (ed. Saidi, 2013, pp. 52–55) has been computed, according to the author, for the meridian of Tilimsān and for the year 1513 CE.

189 MS K7584, fol. 119r.

190 Díaz-Fajardo, 2001, pp. 48–50, 88. On pp. 50 and 52, Díaz-Fajardo shows disagreements between the values obtained by al-Baqqār and her recomputations using Ibn al-Zarqālluh's tables and suggests that al-Baqqār might have used Ibn al-Kammād's *zīj* which al-Baqqār ascribes to Ibn al-Zarqālluh himself. As we have seen in 6.2.6.2, the $10;24^\circ$ used by Ibn Ishāq and his school actually derives from Ibn al-Zarqālluh's parameters.

191 Díaz-Fajardo, 2001, pp. 53–55, 89–90.

about trepidation theory; obviously, one should either add a second motion to trepidation or calculate trepidation tables with a maximum higher than 12° .¹⁹²

Al-Baqqār also saw *zīj*es computed by Eastern astronomers who had made observations in Damascus and who disagreed with the theory of trepidation. Al-Baqqār was able to determine that the mean longitudes of the Sun for the end of year 810/1407–08 exceeded the solar mean sidereal positions, calculated in agreement with the observations of Ibn Ishāq, by $12;15,6^\circ$. One of these *zīj*es is the *Tāj al-azyāj* of Ibn Abī l-Shukr al-Andalusī,¹⁹³ who lived in Damascus, whose mean solar positions for 810/1407–08 exceed those of Ibn Ishāq by $12;15,6^\circ$.¹⁹⁴ All these data agree with Ibn Abī l-Shukr's tables of constant precession, from which al-Baqqār gives several numerical values. Values of precession calculated with these Eastern *zīj*es are of obvious interest to al-Baqqār, for they agree perfectly with his own estimations based on observations of meridian altitudes of the Sun and other celestial bodies, and our author has serious doubts about the truth of trepidation theory. Nevertheless he continues to use the sidereal tables of the Zarqāllian-Ibn Ishāq tradition because he needs them for astrological and magical purposes, but he adds 12° to the sidereal positions when an astronomical problem is involved.¹⁹⁵

Trepidation is obviously related to the secular diminution of the obliquity of the ecliptic.¹⁹⁶ The *Natāʾij al-afkār* mentions the standard historical values of the obliquity and quotes an obliquity of $23;31^\circ$ used by Ibn al-Shāṭir (fl. Damascus ca. 1350)¹⁹⁷ and by al-Mizzī (fl. Damascus, d. ca. 1350).¹⁹⁸ As for determinations made in the Maghrib, our source notes (see 6.2.6.3) that, according to Ibn Ishāq, a man from Miknāsa (Meknès) observed the obliquity in 602/1205–06 and obtained $23;32,30^\circ$, and that Ibn Hilāl in Ceuta¹⁹⁹ obtained the same result as al-Mizzī. Another observation was made in 704/1304–5 in Marrakesh (latitude $31;45,23$)²⁰⁰ by al-Ḥakīm al-Mirriḥ (?), who obtained $23;26,57^\circ$. Finally a certain Ibn al-Tarjumān said that the obliquity of the ecliptic, at the end of the seventh century (699 H/ 1299–1300) was $23;26^\circ$.²⁰¹ All

192 Díaz-Fajardo, 2001, pp. 54–55, 90–91.

193 On this *zīj* see Dorce, 2002–3.

194 Díaz-Fajardo, 2001, pp. 57, 93. For the recomputation of this value see also pp. 57–60.

195 Díaz-Fajardo, 2001, pp. 63–64, 95–96.

196 Saidi, 2013, pp. 72–84.

197 Ibn al-Shāṭir uses an obliquity of $23;31^\circ$ in his *al-Zīj al-Jadīd*: cf. Kennedy, 1956.

198 See Suter, 1900, p. 165 (no. 406); King, 1986 b, pp. 63–64 (C34); Saidi, 2013, pp. 74–75.

199 Identified by King (2007, p. 184) with Abū ʿAbd Allāh Muḥammad ibn Hilāl who lived in Ceuta in the first half of the fourteenth century. See also Lamrabet, 2013, M142, p. 182.

200 This value for the latitude of Marrakesh is not to be found in any of the sources quoted by Kennedy & Kennedy, 1987, p. 218. The modern value for this latitude is $31;49^\circ$.

201 R. Saidi, 2013, pp. 75–76.

these observations, made in the thirteenth century, have the obvious interest of invalidating the geometrical model designed by Ibn al-Zarqālluh in the eleventh century to calculate the value of the obliquity for a given date: this model had been used, with adaptations in the parameters, by all the Maghribī *zīj*es of Ibn Ishāq's school, but sooner or later it obliged an increase in the value of the obliquity that was at odds with the observations. Our author, therefore, decides to forget about the problem, accepts 23;30²⁰² (which he takes from Ibn Abī l-Shukr)²⁰³ and recommends anybody who wants to determine the precise value of the obliquity to observe it using the *libna* (mural quadrant) described by Ptolemy in the *Almagest*.²⁰⁴

6.2.6.7 Conclusions

Some information has been gathered over the last few years on the existence in the Maghrib of a group of *zīj*es of Andalusian descent, which appear to have exerted a major influence during the thirteenth and fourteenth centuries. To this one may add that the analysis of a few new sources made by Mercè Comes and myself shows that observations were made in the Maghrib during the same period and that the new evidence raised doubts about the belief in trepidation. In this situation, we find a new contact with the East which (at least in the case of al-Andalus) had been interrupted in the eleventh century, but now brought the *Tāj al-azyāj* of Muḥyī al-Dīn al-Maghribī (Ibn Abī l-Shukr) and the *al-Zīj al-Jadīd* of Ibn al-Shāṭir to late fourteenth-century Maghrib. The third important *zīj*, Ulugh Begh's *Zīj-i Jadīd*, does not seem to have reached the Maghrib until the end of the seventeenth century. These Eastern astronomical tables computed tropical longitudes, favoured constant precession and did not seek to predict the obliquity of the ecliptic for future times. As a result, the planetary positions computed with them were far more accurate than the ones that could be obtained from the *zīj*es of Ibn Ishāq's school. In spite of this, the Andalusian tradition stayed alive until the nineteenth century and coexisted with the "new" Eastern *zīj*es. The reason, clearly explained by the astrologer al-Baqqār, is that horoscopes were cast using sidereal longitudes. My impression is that astrologers used Ibn al-Bannā's *Minhāj* and other similar sources, while astronomers and conscientious *muwaqqits* preferred Eastern *zīj*es. I wonder, however, whether there were also astronomers who computed their

²⁰² R. Saidi, 2013, p. 74.

²⁰³ See Dorce, 2002–3, p. 139. 23;30° is also used by Ibn al-Muftī, the author of the *Qatf*: K7584, fol. 119v and 120r. Al-Baqqār, in his *Kitāb al-adwār* (fol. 241r) mentions 23;30° (Ibn Abī l-Shukr) and 23;31°, which he attributes to one of the unnamed *zīj*es made in Damascus (probably Ibn al-Shāṭir's).

²⁰⁴ R. Saidi, 2013, p. 76; *Almagest* 1,12. See Toomer, 1984, pp. 62–63.

horoscopes using tropical longitudes: the introduction of double-argument planetary equation tables in Sanjaq Dār's *Zīj al-Sharīf* seems to point in this direction.

6.2.7 *Trepidation in the Christian Kingdoms of the Iberian Peninsula*

6.2.7.1 Twelfth-Century Sources

The main channel through which trepidation theory was introduced in the Christian kingdoms of the Peninsula, as well as in the rest of Europe, was the Latin translation of the *Liber de motu octave spere* and that of the *Toledan Tables* which, as we have seen (6.2.1), contained the same set of trepidation tables. This diffusion also took place through the *Alfonsine Tables* (6.2.7.2). We will also see (6.2.7.3) that the corresponding tables of Ibn al-Kammād's *Muqtabas* reappear in the *Tables of Barcelona*, compiled under the patronage of King Pedro IV of Aragón (1336–1387).

Two twelfth-century sources analyse critically the theory of trepidation: Abraham ibn 'Ezra's *Liber de rationibus tabularum*,²⁰⁵ and a text without a title by the translator Johannes Hispanus or Johannes Hispalensis.²⁰⁶ The former studies the length of the tropical and sidereal solar years, the difference between them being the value of precession per year, for which the author gives Ptolemy's, al-Battānī's and al-Šūfī's estimations, followed by a brief account of trepidation according to Theon of Alexandria, Indian astronomers, and Ibn al-Zarqālluh himself, to whom he ascribes a maximum length of the arc of trepidation of 10;40°. This value is surprising because, while near the 10;45° of the *Liber de motu*, it corresponds exactly to the maximum $\Delta\lambda$ we have found in later sources like Ibn al-Raqqām's *Qawīm zīj* (10;40,13°) and in the versified *zīj* of Abū l-Ḥasan al-Qusanṭīnī (10;40°) (see 6.2.6.2). When dealing with the changes observed in the obliquity of the ecliptic, Ibn 'Ezra mentions and rejects Ibn al-Zarqālluh's model which justifies it; even though in fact he does not appear to understand it properly, because he seems to confuse the equatorial and the polar epicycles:

Dicit enim quemdam parvum circulum in capite arietis esse cuius dyametri medietas est 18 minutorum, unde diversitas illa contingere potuit, sed hec affirmatio frivola est et inutilis.²⁰⁷

²⁰⁵ Millás, 1947 a; Samsó, 2012.

²⁰⁶ Millás, 1949 b. The name of the author appearing in the Latin text edited by Millás is Iohannes Ispanus or Ispalensis (p. 245). The identification with Avendaut is unjustified. See also Robinson, 2007 pp. 64–69.

²⁰⁷ Millás, 1947 a, p. 93.

The radius of the equatorial epicycle is not $0;18^\circ$ in any of the extant sources, but this may be the result of a confusion with the $0;10^\circ$ of Ibn al-Zarqālluh's radius of the polar epicycle. Overall, Ibn 'Ezra rejects trepidation theory, as well as sidereal astronomy, and follows the lead of al-Battānī in his acceptance of constant precession. His attitude differs from Ibn al-Zarqālluh's in so far as the Toledan astronomer accepts the results of observations made by his predecessors which imply the existence of differences in the length of the tropical year and, consequently, a precession of the equinoxes which is not constant, as well as variations in the obliquity of the ecliptic. Ibn 'Ezra adopts a different attitude: many of these changes are due to observational errors or to imperfections of the astronomical instruments used in the observations.

As for the text of Johannes Hispanus or Hispalensis, its treatment of trepidation theory is more complete than that of Ibn 'Ezra's text.²⁰⁸ After explaining the model of the "ancient astrologers", as explained by Theon, it gives a numerical rule for computing the value of precession for a given date, using the Persian calendar and Yazdijird's era and states that, if the result is added to al-Khwārizmī's mean sidereal planetary motions, one obtains results equivalent to al-Battānī's mean tropical motions. This is followed by a reference to a previously mentioned book by [translated by?] Johannes Hispanus²⁰⁹ in which Ibn al-Zarqālluh himself explained this motion. The text continues with a summary of the model of the *Liber de motu* in which two of its characteristic parameters are mentioned (a diameter of the equatorial epicycle of $8;37,26^p$ and a maximum value of the equation of $10;45^\circ$). The passage ends with a paragraph stating that this trepidation model justifies the changes of the velocity of precession. The most interesting information contained in this source relies in the fact that Johannes Hispanus believes that the *Liber de motu* was written by Ibn al-Zarqālluh.²¹⁰

6.2.7.2 The Alfonsine Tables

As we will see in chapter 7, two entirely different versions of the *Alfonsine Tables* are extant: a set of Castilian canons which describe an unpreserved collection of astronomical sidereal tables, and another collection of tropical tables, with their headings written in Latin. The original canons of the Latin tables have not survived and they usually appear, both in the manuscripts and

208 Millás, 1949 b, pp. 281–287.

209 "In predicto autem libro nostro quem prefatus Azarchellus super hunc modum ordinavit exponens ita Albaten in hoc motu ita eum descripsit" (Millás, 1949 b, p. 283). As Millás notes, there has been no previous mention of Azarchellus or of the book.

210 See, on this passage, Mercier, 1996, pp. 322–323.

in the printed versions, accompanied by canons written by Parisian astronomers ca. 1320–30.

Both collections deal with trepidation, although in entirely different ways. The Castilian canons²¹¹ describe the use of a mean motion table with which the position of the moving Head of Aries in the equatorial epicycle will be obtained. The epoch used in this table is the beginning of the Alfonsine era which, in the Castilian canons, is 1 January 1252, the beginning of the year of Alfonso X's accession to the throne. The mean position of the Head of Aries will be used as an argument to enter an equation table which yields the value of precession for a given moment. No parameters are given and it is clear that the procedure described and the model involved is the same as in the *Liber de motu* and of most sets of tables described in the preceding pages.

As I have said, the Latin Alfonsine Tables²¹² are different. First of all, they compute tropical planetary longitudes and so precession is only needed if the user wants to obtain the corresponding sidereal positions. This implies that while the Castilian Alfonsine Tables – like the Toledan Tables and all the tradition of Andalusī – Maghribī sidereal tables – have a constant sidereal year and a variable tropical one, the Latin tables use a constant tropical year and a variable sidereal one.²¹³ The model used is also different, and is the result of a combination of constant precession and trepidation; it should be used to compute stellar longitudes and the positions of the planetary apogees for dates after the epoch used, as well as the sidereal planetary positions. This combination should not be regarded as new, because al-Battānī accepted it in his interpretation of Theon's trepidation model which implies a motion of accession and recession along an 8° arc with a velocity of 1° per 80 years, to which he would have added a constant precession moving 1° per 100 years.²¹⁴ This interpretation was also accepted by Andalusī astronomers like Ibn al-Zarqālluh²¹⁵ and al-Biṭrūjī.²¹⁶ This combined model of precession + trepidation is described in John of Lignères' *Priores astrologi* (1322)²¹⁷ and it reappears in the astronomical tables of John Vimond (1320), strongly influenced by the Alfonsine Tables.²¹⁸

211 Chabás & Goldstein, 2003, pp. 89–90, 217–220.

212 On Alfonsine trepidation see Chabás & Goldstein, 2003, pp. 256–266 and Chabás & Goldstein, 2012, pp. 48–52. See also Casanovas, 1987.

213 Chabás & Goldstein, 2003, pp. 255–256.

214 Ragep, 1996.

215 Millás, 1943–50, pp. 275–276.

216 Goldstein, 1971, I, pp. 89–91 and II, pp. 173–181.

217 Chabás & Goldstein, 2003, p. 257.

218 Chabás & Goldstein, 2004, pp. 263–267; Chabás & Goldstein, 2003, pp. 274–277.

The component of constant precession is based on a period of 49,000 years, usually interpreted as corresponding to the jubilar cycle (49 years). An explanation for this period has been given by Chabás and Goldstein: it is based on the difference between a fixed tropical year and a calendrical year of 365;15d.²¹⁹

$$360^\circ / [0;59,8^\circ \cdot (365;15d - 365;14,33,10d)] = 49,006 \text{ years}$$

The mean motion per day, as it appears in the *Tabula medii motus augium et stellarum fixarum*²²⁰ is 0;0,0,4,20,41,17,12°, and corresponds to a motion per tropical year (365;14,33,9,57,27,21,36 days) of 0;0,26,26,54,23,21,59°).

As for the periodical term, it must be calculated using two tables:

- a) The *Tabula medii motus accessus et recessus octave sphere*,²²¹ with which one calculates the position of the moving Head of Aries and the value of angle i (see fig. 6.1), as in the *Liber de motu*. In this case, the period of revolution is 7,000 years, interpreted as a reference to the sabbatical cycle of seven years (mean motion per day 0;0,0,30,24,49,0°). This cycle has been explained by Chabás and Goldstein as being associated with the difference between a variable sidereal year and the calendar year of 365;15 days.
- b) The *Tabula equationum motus accessus et recessus octave sphere*,²²² calculated for a maximum of 9°: a sample can be found in table 6.27, recalculated with the standard expression:

$$\Delta_\lambda = \arcsine (\sin i \sin 9^\circ)$$

Mercier has suggested another approximation,²²³ which I have already used in 6.2.1:

$$\sin \Delta_\lambda = \sin i \tan r / \sin \varepsilon$$

This expression allows us to try to establish the value of the radius of the equatorial epicycle and the obliquity of the ecliptic implied in the table. After a number of attempts, I have deduced that the best results in the recomputation of the table are obtained using $\varepsilon = 23;33^\circ$ and $r = 3;34,35$.

²¹⁹ Chabás & Goldstein, 1003, p. 258, and 2012, p. 49. An explanation of this kind for the 49,000-year period was already given by North, 1996 a, p. 460.

²²⁰ Poulle, 1984, p. 133.

²²¹ Poulle, 1984, p. 132.

²²² Poulle, 1984, p. 131.

²²³ Mercier, 1976–7, p. 205.

TABLE 6.27 The Alfonsine table of the equation of trepidation

Argument	Equation	Argument	Equation
1°	0;9,25° (+2")	50°	6;52,58° (+1")
5°	0;46,52°	55°	7;21,44°
10°	1;33,20° (-4")	60°	7;47,10°
15°	2;19,13° (-1")	65°	8;9,2° (-1")
20°	3;3,49° (-12")	70°	8;27,11°
25°	3;47,27°	75°	8;41,28° (+1")
30°	4;29,10°	80°	8;51,44°
35°	5;8,51° (-1")	85°	8;57,55° (-1")
40°	5;46,16°	90°	9;0,0°
45°	6;21,2° (-1")		

The radix positions of the moving Head of Aries of the different epochs used in the Alfonsine tables are given in the *Tabula prima radicum motuum hic positorum ad eras hic positas*.²²⁴ Mercier, like Dreyer²²⁵ and Price²²⁶ before him, has underlined the fact that the Alfonsine precession-trepidation model gives good results for the value of precession in the second half of the thirteenth century. To confirm this statement,²²⁷ we can calculate the position of the solar apogee for the beginning of the Alfonsine era which, in the Latin Alfonsine Tables corresponds to midday on 31 May 1252.²²⁸ The radix position of the solar apogee for that date is 80;37,0°, in which the term of constant precession has already been incorporated.²²⁹ To this value we should add the value of the trepidation term which is also given in the table of *Radices motus octave sphere* as 8;4,1°. However, there seems to be a small error in this latter value for, when computed with the corresponding table, it should be 8;3,6°. The position of the solar apogee at the beginning of the Alfonsine era is:

$$80;37,0^{\circ} + 8;3,6^{\circ} = 88;40,6^{\circ}$$

224 Poulle, 1984, p. 124.

225 Dreyer, 1920.

226 Price, 1955 a.

227 Samsó, 1987 c; Samsó & Castelló, 1988.

228 This corresponds to the precise date of Alfonso's accession to the throne of Castile (1 June 1252).

229 Poulle, 1984, p. 124.

This value can be compared to the longitude of the solar apogee for 16 May 16 CE which, as established by Mercier, is the date in which the value of the equation of trepidation is 0°. ²³⁰ For this latter date the tropical longitude of the apogee will only be affected by the term of constant precession, and we can easily obtain 71;32,10°. Therefore the increase in precession between years 16 and 1252 CE is:

$$88;40,6^{\circ} - 71;32,10^{\circ} = 17;7,56^{\circ}$$

From this value, which corresponds to the increase of precession in 1236 Julian years, we can obtain an excellent mean value of 0;0,49,54° per year. ²³¹ This rate is not far away from the value that can be obtained from John of Murs' *Expositio* (1321) which attributes to Alfonso X a precession of 1° in 72;30 years, corresponding to a motion of 0;0,49,39° per year. ²³²

17;7,56° can, on the other hand, be rounded to 17;8°, which is precisely the amount by which the longitudes of stars in Ptolemy's *Almagest* (VII,5) have been increased in the four *Libros de la Ochava Espera*, as well as in the star catalogue of the Latin Alfonsine tables. ²³³ This coincidence has been the object of much discussion because, obviously, it does not correspond to the increase of precession between the epoch of Ptolemy's star catalogue (the beginning of the reign of Emperor Antoninus, 20 July 137) and the Alfonsine era (31 May 1252). The attempts to explain the increase of 17;8° began in the fifteenth century when Giovanni Bianchini (d. after 1469) believed that 17 May 16 CE was the epoch of Ptolemy's star catalogue. ²³⁴ Slightly later, Abraham Zacut (1450–ca. 1510) believed that the Alfonsine Tables were completed in 1252 (the year of Alfonso's accession to the throne) and, four years later, in 1256 (date of the first version of the *Libros de la ochava espera*), King Alfonso abandoned his ideas on trepidation and adopted continuous precession, ²³⁵ using al-Battānī's parameter of 1° in 66 years, which was also used by 'Abd al-Raḥmān al-Šūfī in his *Uranography*, the main source of the Alfonsine *Ochava espera*. The same idea

²³⁰ Mercier, 1976–7, p. 59.

²³¹ As calculated by Chabás and Goldstein, 2003, p. 266, the modern value for the yearly rate of precession for epoch 1300 would be 0;0,50,7...°.

²³² Goldstein, 1994.

²³³ Kunitzsch, 1986. 17;8° is also the amount added to the longitudes in the *Almagest* in the star catalogue of the tables compiled by Prosdócimo de' Beldomandi of Padua (1424): see Chabás, 2007.

²³⁴ Chabás & Goldstein, 2012, p. 52.

²³⁵ Cantera, 1931; Cantera, 1935, pp. 242–44.

has been defended by Chabás and Goldstein, who also obtain $17;8^{\circ}$ for 1256.²³⁶ This hypothesis does not justify the difference of precession of $17;8^{\circ}$ between the significant date 16 May 16 CE (in which the periodic term has value 0°) and the Alfonsine epoch (31 May 1252) and I do not think that the Alfonsine astronomers would have used this insignificant year as an epoch for their star catalogue: in all probability, they would have reduced the star longitudes to the Alfonsine epoch (31 May 1252).

Another explanation was given by Emmanuel Poulle.²³⁷ Very briefly I will say that Poulle suggests that the Alfonsine astronomers used the trepidation model and tables of the *Liber de motu*, interpolating with a precision of minutes and making a mistake in the sign of the interpolated amount corresponding to the epoch of Ptolemy's star catalogue. With these assumptions, Poulle obtains exactly $17;8^{\circ}$ and this might be an explanation for the use of this increase in the star longitudes of the *Libros de la ochava espera*, but it does not clarify why the mean motion of the Head of Aries has a value of 0° for 16 May 16 CE or why this date seems to have been used as the epoch of Ptolemy's star catalogue. An increase in longitude of $17;8^{\circ}$ is, obviously, one of the links between the indisputable Alfonsine works (*Libros de la ochava espera* in this case) and the Latin Alfonsine Tables which, in Poulle's opinion, were not Alfonsine.

Years ago, I suggested a solution to the problem along the same lines as Giovanni Bianchini's: the Alfonsine astronomers were not necessarily aware of the epoch used by Ptolemy for his star catalogue and I wondered whether the king's collaborators had been using an Arabic or a Latin translation of the *Almagest*, or another source in which the name of Emperor Antoninus had been confused with that of Tiberius. To support this hypothesis, I relied on the fact that, although Tiberius began to reign in 14 CE, the Alfonsine *Primera crónica general* states erroneously that the beginning of his reign took place in year CE 16.²³⁸ The confusion of Antoninus/ Tiberius does not seem difficult to justify in Arabic script, but there is no evidence of such a mistake in the manuscripts of the Arabic translations of the *Almagest*,²³⁹ and, on the other hand, al-Šūfi's *Book on constellations* and al-Battānī's *Zīj* – both available to Alfonsine astronomers – date Ptolemy's star catalogue to year 886 of Bukht Naššar/138 CE instead of 885/137 CE in the same chronology. Whatever the case, dating Ptolemy's star catalogue to 16 CE seems an attempt to correct his

236 Chabás & Goldstein, 2003, pp. 234–237, 261–262.

237 Poulle, 1988.

238 Menéndez Pidal, 1977, vol. I p. III.

239 As I was told (private communication) by Paul Kunitzsch, author of a critical edition of the Arabic translations of Ptolemy's star catalogue (Kunitzsch, 1986 a). I have not explored the Latin translation.

star longitudes which contain an error of about 1.23° and would have been correct some 89 years earlier, ca. 48 CE.²⁴⁰

6.2.7.3 Pedro IV of Aragón and the Tables of Barcelona

As we have already seen in 1.5.3, King Pedro IV of Aragón (1336–1387) tried to follow the Alfonsine cultural model and was a keen patron of astronomical research. The results were a set of ephemerides giving tropical planetary (?) longitudes for the period 1361–1433, prepared by the king's astronomers Pere Gilbert (d. 1362) and Dalmau Sesplanes (d. 1383) between the beginning of 1360 and the end of 1366.²⁴¹ At the end of the introduction to the canons, the text states:

Quibus 5 planetis est addenda eciam motus axi octave spere quod est proquolibet mense 28 secunda 14 tertia. Sed quia in isto opere non sunt secunda neque tertia, adde in duobus mensibus unum minutum usque ad 20 menses. Quod facias per modum quod sit tota addicio 9 minuta sive plus et 24 secunda, 40 tertia ...²⁴²

As the tables of this set of ephemerides have not been preserved, we only know that the computed longitudes were tropical because, in the prologue to the Tables of Barcelona, we read that the “*Tabulas et Almanach*” were calculated “*secundum locum stellarum in spera nona in meridie diei in civitate predicta*” (i.e., Barcelona).²⁴³ If this remark is correct, it is surprising to read that the motion of the axis of the eighth sphere (i.e., the position of the Head of Aries) has to be added to the planetary positions given in the tables. Furthermore, the numerical values given in the text do not correspond to the increase of longitude due to precession or trepidation, but to the position of the Head of Aries in the equatorial epicycle, which corresponds to a mean motion per month of $0;0,28,14^\circ$, which, when multiplied by 20, yields a motion of $0;9,24,40^\circ$. This mean motion does not agree with any of the parameters seen in this chapter or with that given in the corresponding table of the Tables of Barcelona.

We saw in 1.5.3 that Pedro IV also wanted a set of proper astronomical tables to be compiled, and that he asked the Sevillian Jew Jacob Corsuno, who was in Barcelona at least between 1378 and 1380, to undertake this task. The

²⁴⁰ Neugebauer, 1975, I, 284. This explanation was suggested by North, 1996 a, pp. 467–469.

²⁴¹ Millás, 1962, p. 90.

²⁴² Millás, 1962, p. 93.

²⁴³ Millás, 1962, p. 142.

corresponding canons were written in Catalan, Latin and Hebrew²⁴⁴ and, in the introduction, we find a peculiar remark according to which Christian scientists cast their horoscopes (*iudicia stellarum*) “secundum eorum loca in spera nona” (tropical horoscopes), while many Jews and Arabs compute sidereal horoscopes (“tractant iudicia astrorum secundum eorum loca in 8^a spera”). The king, then, decided that the tables should be computed using his own epoch (midday on 28 February 1321),²⁴⁵ the Christian calendar, and should produce sidereal positions (“scire loca stellarum in octava spaera”).²⁴⁶ This justifies the presence of trepidation tables in which the Head of Aries rotates with a motion, according to Chabás, of 0;5,35,46° per Julian year,²⁴⁷ from which I obtain 0;0,0,55,9,24,16...° per day which, again, does not correspond exactly to any parameter analysed in this chapter. With the result obtained with this table we enter the table of the equation of the eighth sphere,²⁴⁸ with a maximum value of 9;59°, which is exactly the same table we have found in Ibn al-Kammād’s *Muqtabas* (6.2.3).

The Barcelona Tables also include a table of constant precession (“table of the motion of the fixed stars”) calculated for a motion of 50” per year. We have also found this kind of double system for the computation of precession in the Tunisian recension of Ibn Ishāq’s *zīj*, although here we find a surprising instruction in chapter 3 of the canons in which we are told that the result obtained with the trepidation tables should be added to the corresponding value of constant precession, as in the Latin Alfonsine Tables (“et postea trahe motum stellarum fixarum in illo tempore et iunge super equatum observatum, et quicquid aggregatum ex numero illo est illud quod est intra speram octavam et nonam”).²⁴⁹ This is obviously absurd, since 50”/year is enough to justify the value of precession.²⁵⁰

6.2.7.4 Abraham Zacut (1452–ca. 1515) and Precession Theory

We have already seen (6.2.7.2) that Abraham Zacut criticised Alfonsine trepidation and considered that the Alfonsine Tables were completed in 1252; Zacut

244 These canons, as well as the tables, were edited by Millás, 1962. See also Chabás, 1996 a, pp. 477–525; Chabás, 2004; Poulle, 1966 a.

245 This date has been calculated by Chabás, 1996 a, while Millás, 1962, p. 22 has the same day of the year 1320. Both dates seem strange because the text of the canons state that the epoch should correspond to the king’s birth (“et volumus facere istas tabulas secundum eram nativitatis nostre”); Pedro IV was born on 5 October 1319.

246 Millás, 1962, p. 143.

247 Millás, 1962, p. 190; Chabás, 1996 a, pp. 487–8.

248 Millás, 1962, pp. 194–195; Chabás, 1996 a, p. 488.

249 Millás, 1962, p. 146.

250 Millás, 1962, p. 48; Chabás, 1996 a, p. 488.

stated that the king changed his mind in 1256 and used, in his star catalogue (which, according to Zacut has 1256 as epoch) a constant precession derived from al-Battānī (1°/ 66 years), which he considers equivalent to 0;0,54,30° per year (more precisely 0;0,54,33°). His position is thus similar to that of Abraham ibn ‘Ezra (6.2.7.1).

His *ha-Hibbur ha-gadol* (“The great compilation”) contains a star table of 61 or, perhaps, 83 stars²⁵¹ giving their tropical longitudes for 1478.²⁵² According to the canons of this work his catalogue derives from a similar one compiled by Rabbi Judah ben Asher II of Burgos (fourteenth century),²⁵³ which gave sidereal longitudes of the stars.²⁵⁴ This is probably the source from which Zacut obtains the sidereal longitudes of two stars mentioned in the section of the canons dealing with precession:

Qalb al-Asad/ Regulus (α Leo): 9;8° of Leo
al-Simāk al-A‘zal/ Spica (α Vir): 3;18° of Libra

Interestingly, these two sidereal longitudes are the ones we find in star tables of the Andalusī-Maghribī tradition (Ibn al-Kammād, Ibn al-Bannā’, Ibn ‘Azzūz, etc.) which give their coordinates for the moment in which the value of trepidation was 0°. ²⁵⁵

Concerning the stars’ tropical longitudes in his catalogue, we should consider two criteria, mentioned by Zacut himself, using as examples the longitudes of the two aforementioned stars (Regulus and Spica):

1. The difference between tropical and sidereal positions is 13;52° for year 1478.
2. In his star catalogue, he has added 20;30° to the star longitudes of the *Almagest*.

²⁵¹ Chabás & Goldstein, 2000, pp. 68 and 145–7. On Zacut’s treatment of precession see also Mercier, 1996, pp. 337–338 and Nothaft, 2016.

²⁵² Chabás and Goldstein, 2000, p. 85.

²⁵³ Chabás & Goldstein, 2000, pp. 49–50.

²⁵⁴ I am using the Castilian translation of the *Hibbur* made by Juan de Salaya (with the help of Zacut himself) in 1481. This text was edited by Cantera, 1931. The edition appears on pp. 151–362 and his treatment of precession on pp. 192–195. Passages of the Hebrew original text were translated into Spanish by Cantera (1931, pp. 296–304). Bernard R. Goldstein has sent me translations of the relevant passages of the *Hibbur*, using the oldest Hebrew manuscript (Oxford MS Opp. Add. 8° 42, dated in 1489).

²⁵⁵ See Comes, 1991 a; Goldstein & Chabás, 1996; Samsó, 1997, pp. 107–110. The longitude of Spica is 3;38° in Ibn al-Kammād’s star table: 18 and 38 are easily confused in Arabic alpha-numerical notation.

As for the first criterion, 13;52° allows us to calculate (approximately) the date for which precession attains 0°:

$$\begin{aligned} 13;52^\circ: 0;0,54,30^\circ &= 915;57,37 \text{ years} \\ 1478-916 &= 562 \end{aligned}$$

This result is only ten years earlier than the date proposed by Ibn al-Hā'im (6.2.2.5.2.1) who considers that precession attains 0° some 50 years before the Hijra (572).²⁵⁶

Concerning the longitude of Regulus, Zacut states that it was Leo 19;38° according to the Alfonsine star catalogue, which he dates to 31 May 1256²⁵⁷ and that, “according to the equations of that book”, in 1478 it has reached Leo 22°. ²⁵⁸ The problem is the identification of “that book”, which might refer to the Alfonsine star catalogue or to al-Šūfi's *Kitāb šuwar al-kawākib* – both of which, in Zacut's opinion, use al-Battānī's constant precession of 1°/66 years. I suggest, however, a third possibility: Zacut may be using the Alfonsine precession + trepidation tables, something we can check by computing the difference between the positions of the solar apogee on 31.5.1256 and 31.5.1478:²⁵⁹

$$\begin{aligned} 90;58,33^\circ - 88;42,47^\circ &= 2;15,46^\circ \\ 19;38 + 2;15,46^\circ &= 21;53,46^\circ \approx 22^\circ \end{aligned}$$

However, Leo 22° does not agree with his estimation (13;52°) of the difference between the tropical and sidereal positions of the stars:

$$\text{Leo } 22^\circ - \text{Leo } 9;8^\circ = 12;52^\circ$$

which means that the longitude of Regulus should be ca. Leo 23°, and this is precisely the value that appears in the Oxford Hebrew manuscript of the

²⁵⁶ Zacut himself dates this event to the year 563 CE: see Cantera, 1931, p. 303, while the corresponding passage has been omitted in Juan de Salaya's translation (Cantera, 1931, p. 194). Another Iberian Jew, Solomon Franco (fourteenth c.) was also interested in determining the year in which precession reached 0°. His *zīj* confirms year 604 for the *Liber de motu* and the *Toledan Tables* and ascribes approximately year 580 to Ibn al-Kammād: see Goldstein, 2011.

²⁵⁷ Rico, 1863–67, vol. 1, illustration between pp. 70 and 71. The constellation of Leo has not been preserved in ms. Villa Amil 156.

²⁵⁸ Leo 23° in the Oxford Hebrew ms. My hypothesis here is based on assuming that Zacut's original text had 22° in this passage, although this is not certain.

²⁵⁹ I have used the computer programme ALFLN, designed by E.S. Kennedy and revised by H. Mielgo.

Hibbur, in which the canons give Leo 23° , while the star table has Leo $23;8^\circ$.²⁶⁰ My impression is that Zacut is using this as an argument to reject the Alfonsine trepidation theory which gives, as a result, a longitude for Regulus which is 1° off.

The use of $13;52^\circ$ as the difference between tropical and sidereal longitudes is confirmed by Zacut's observation, in 1474, of an occultation of Spica by the Moon and a computation of the lunar longitude (Libra $17;6^\circ$).²⁶¹ To obtain Spica's longitude for 1478, the epoch of his star catalogue, Zacut uses al-Battānī's precession parameter and:

$$0;0,54,30^\circ \times 4 = 0;3,38^\circ$$

Zacut seems to round this value and adds $4'$ to Spica's longitude in 1474, obtaining Libra $17;10^\circ$ which agrees with the estimated difference between the tropical and sidereal longitudes of a star in 1378 ($13;52^\circ$):

$$\text{Libra } 17;10^\circ - \text{Libra } 3;18^\circ = 13;52^\circ$$

His second criterion is based, as we have seen, on a difference of $20;30^\circ$ between the star longitudes of Zacut's catalogue and those of the star catalogue of the *Almagest*.

A difference of $20;30^\circ$ agrees with the aforementioned longitudes of Spica and Regulus:

$$\text{Libra } 17;10^\circ - \text{Virgo } 26;40^\circ{}^{262} = 20;30^\circ$$

$$\text{Leo } 23^\circ - \text{Leo } 2;30^\circ{}^{263} = 20;30^\circ$$

One should remark, however, that $20;30^\circ$ as the increase of precession between 137 CE and 1478 ($1478-137 = 1341$) is not in full agreement with a motion of $1^\circ/66$ years, but corresponds to a motion of $1^\circ/65;24,53$ years. Besides, the motion per year will be:

$$20;30^\circ : 1341 = 0;0,55,2^\circ$$

²⁶⁰ Chabás & Goldstein, 2000, p. 147.

²⁶¹ Libra $16;7^\circ$ in the Oxford Hebrew ms. (fol. 31a). B.R. Goldstein reminds me that 6 and 7 are easily confused in Hebrew script and that Juan de Salaya's text seems to offer a better reading.

²⁶² Toomer, 1984, p. 369.

²⁶³ Toomer, 1984, p. 367.

which does not correspond to Zacut's parameter ($0;0,54,30^{\circ}/\text{year}$).

An explanation is given by Zacut himself, aware of the fact that a precession of $1^{\circ}/66$ years does not justify an increase in longitude of $20;30^{\circ}$ in 1341 years.²⁶⁴ Zacut's explanation is based on his belief that Ptolemy's star catalogue derived from a previous catalogue compiled by Menelaos 41 years earlier,²⁶⁵ by an addition of $0;25^{\circ}$ to Menelaos' longitudes, which corresponds to Ptolemy's rate of precession ($1^{\circ}/100$ years).²⁶⁶ If we use Menelaos' observation of the longitude of Spica in 98 CE (Virgo $26;15^{\circ}$), $25'$ less than Ptolemy's (Virgo $26;40^{\circ}$)²⁶⁷ and $20;55^{\circ}$ less than Zacut's observation corrected for 1478 (Libra $17;10^{\circ}$), we can calculate:

$$20;55^{\circ} / (1341 + 41) = 0;0,54,29^{\circ}$$

which corresponds to the rate of precession per year used by Zacut ($0;0,54,30^{\circ}$). It seems therefore that Zacut obtained his parameter from Menelaos' observation of Spica, not from Ptolemy's and that he, like his Alfonsine predecessors, was aware of the existence of an error in the Ptolemaic star longitudes and tried to correct it.

6.3 Ibn al-Zarqālluh's Solar Model

6.3.1 Introduction

Ibn al-Zarqālluh followed the general tendency in Islamic astronomy of dedicating special attention to the study of solar motion and, after 25 years of solar observations, wrote a non-extant book which was probably entitled *Fī sanat al-shams* ("On the solar year"). I suggest this title as a result of a passage from Ibn al-Zarqālluh's own book on the construction of the equatorium, preserved in the Alfonsine translation (4.5.3), in which the author mentions "el libro que

264 I read in the Castilian translation (Cantera, 1931, p. 193): "yo añadido sobre el lugar dellas segun que estan en el almagesto que hiço ptholomeo en el año primero de antoninos [...] .20. grados e .30. minutos por cada .66. años un grado aunque entre el e nosotros ha .1341. años e no sale tanto dando a cada .66. años .1. grado ...".

265 Apparently Zacut makes a mistake in the time difference between Ptolemy and Menelaos: assuming that the date for Ptolemy's catalogue is 20 July 137 and that of Menelaos' observation 11 January 98, the number of years between these two dates is 39.52 years, not 41.

266 There is no evidence of Menelaos compiling a star catalogue, although the *Almagest* mentions two star observations made by him (Spica and β Sco): Toomer, 1984, pp. 336–338. Here Zacut is following al-Šūfi's idea: Šūfi, 1954, p. 24; Schjellerup, 1874, pp. 42–43. See also Neugebauer, 1975, I, 288.

267 Toomer, 1984, pp. 336–337.

yo fiz, que fabla en el anno del sol".²⁶⁸ Further evidence in the same sense can be found in Abraham ibn 'Ezra's *Fundamenta tabularum*, where we read: "Dicit Azarchel in libro de anno solari".²⁶⁹ A second possible title was suggested by G.J. Toomer²⁷⁰ who proposed *Al-risāla al-jāmi'a fī l-shams*, in agreement with Millás, who translates "Suma referente al movimiento del sol".²⁷¹

The contents of this lost book were carefully reconstructed by Toomer²⁷² from secondary sources which are, in chronological order:

- Ibn al-Zarqālluh's own book on the motion of the fixed stars (6.2.2).
- Ibn al-Kammād's *Muqtabas zīj*, already mentioned in 6.2.3.
- Abraham ibn 'Ezra's *Fundamenta tabularum*.
- Abū l-Ḥasan 'Alī al-Marrākushī's *Jāmi' al-mabādī wa l-ghāyāt*.
- The Franciscan Bernard of Virdun's (thirteenth c.) *Tractatus super totam astrologiam*.
- Regiomontanus' (1436–ca. 1476) *Epitome of the Almagest*.
- The echoes of this last work in Copernicus' *De revolutionibus*.

To this list of sources studied by Toomer we should add:

- Ibn al-Hā'im's (fl. 1205) *al-Zīj al-Kāmil fī l-ta'ālīm*, extant in a manuscript in the Bodleian Library in Oxford and already mentioned in 6.2.5.²⁷³
- The unfinished *zīj* of Ibn Ishāq al-Tūnisi (fl. 1193–1222) known through the five recensions mentioned in 6.2.6.1: the anonymous Tunisian recension preserved in a Hyderabad manuscript (ca. 1266–1281); Ibn al-Bannā's (1256–1321) *Minhāj al-ṭālib fī ta'dīl al-kawākib*,²⁷⁴ and Ibn al-Raqqām's (d. 1315) three *zīj*es.

The surprising diffusion in Latin Europe until the sixteenth century of this lost work by Ibn al-Zarqālluh led Toomer to suggest the possible existence of a Latin translation of the book. Hopefully, we may recover a direct or indirect version of the original book in the future. At present, I can only summarise the evidence collected from the aforementioned secondary sources.

We should remark, first of all, that Ibn al-Zarqālluh's book on the solar year was written at a late stage of his career, before 473/1080–1, which is the date mentioned as *annus praesens* in his treatise on the construction of the equatorium, in which, as we have seen (4.5.3), this book is mentioned. This work required a previous programme of observations which began in Toledo before

268 Ms. Villa Amil 152 fol. 166r; Rico, 1863–67, III, 273.

269 Millás, 1947 a, p. 79.

270 Toomer, 1969, pp. 315–6.

271 Millás, 1943–50, pp. 239–245.

272 Toomer, 1969, and 1987.

273 Calvo, 1998 a.

274 Samsó & Millás, 1994.

453/1061²⁷⁵ as, in the prologue of his book on the motion of the fixed stars (dated ca. 1085, see 6.2.2.1), Ibn al-Zarqālluh states that he made observations of the Sun, Moon and the fixed stars in Toledo, and later in Córdoba, for 25 years,²⁷⁶ after which he began to write his book on the solar year. On Ibn al-Zarqālluh's observations and on the instruments he used we have practically no information. We have already seen (4.1.3.3) that the anonymous Toledan author of a *Risālat al-hay'a*, who was a contemporary of our astronomer, reports that Ibn al-Zarqālluh made observations with an instrument which had problems of stability, something that seems to imply that it was quite large.²⁷⁷ Observations of stellar longitudes made in 455/1063 and 473/1080–1 are also mentioned in the treatise on the motion of the fixed stars.²⁷⁸ As for the Sun, in his *Yesod ha-'Olam*, Isaac Israeli gives some details of Ibn al-Zarqālluh's determination of the autumn equinox in Toledo which took place on 17 September 1075, 4;18 hours after noon, the modern calculated value being 17 September 1075, 5;30,40 h after noon.²⁷⁹ In table 6.24 we have already seen that the Tunisian recension of Ibn Ishāq's *zīj* ascribes to Ibn al-Zarqālluh determinations of the longitude of the solar apogee and of the obliquity of the ecliptic in 467/1074–5: although the parameters mentioned are not reliable, the date seems significant. Some of these observations were probably used to establish the mean motion parameter of the Sun used in the Toledan Tables (0;59,8,11,28,27°/ day) which, as we will see, was not changed in the final version of Ibn al-Zarqālluh's solar theory. The same year 1074–5 is mentioned in the treatise of the fixed stars as the year in which he determined the [solar] equation.²⁸⁰ This is probably the crucial year in the observations which led him to determine the parameters of his solar model. In his observations, according to Bernard of Virdun²⁸¹ (who must have read Ibn al-Zarqālluh's book on the Sun) Ibn al-Zarqālluh did not use the Ptolemaic method in which the passage of the Sun through the equinoctial and solstitial points are determined, but used instead the procedure designed by the astronomers of the Abbasid caliph al-Ma'mūn at the Shammāsiyya observatory in Baghdad in 830: observations of the passage of the Sun through the midpoints between equinoxes and solstices (Taurus 15°, Leo 15°, Scorpio 15° and Aquarius 15°).²⁸² This method improves upon the previous one because

275 Marrākushī, 1984, vol. 1, p. 43; Sédillot, 1834, p. 127.

276 Millás, 1943–50, p. 279.

277 Saliba, 1999.

278 Millás, 1943–50, p. 305.

279 Millás, 1943–50, pp. 244–5; Mercier, 1996, pp. 313–315.

280 Millás, 1943–50, p. 296.

281 Toomer, 1987, p. 334 fn. 71.

282 See Mozaffari, 2013 a, pp. 320–323.

it avoids the practical difficulty of determining the moment of the solstices exactly as, in them, the variation of the solar declination reaches a minimum. The *ma'mūnī* method is here documented for the first time in al-Andalus and it was used, again, in the thirteenth century by Alfonsine astronomers.²⁸³

6.3.2 *The Motion of the Solar Apogee*

Therefore, Ibn al-Zarqālluh's book was probably written between 1075 and 1080, and the 25 years of solar observations probably began between 1050 and 1055. The first important result obtained by the Toledan astronomer was probably the discovery of the solar apogee's own motion. While Ptolemy believed that the solar apogee was fixed and that its longitude was invariably $65;30^\circ$, solar observations made by Islamic astronomers in 829 and 831 led to two different values of its position: $82;39^\circ$ and $82;45^\circ$ respectively. The comparison of these results with Hipparchus' and Ptolemy's position destroyed the Ptolemaic dogmatic principle of the immobility of the solar apogee: the conclusion was that the apogee advanced 1° in 66 years, an identical parameter to the *ma'mūnī* estimation of the precession of equinoxes. This is the starting point of the belief widely held among Islamic astronomers that the solar and planetary apogees were sidereally fixed, which implies that each one of them was pointing to a particular star, moving slowly with it as a result of precession.²⁸⁴

In his solar observations of 1074–5, Ibn al-Zarqālluh determined that the longitude of the solar apogee was $85;49^\circ$, as documented by both Bernard of Virdun and Ibn al-Hā'im.²⁸⁵ The evidence furnished by Ibn al-Hā'im seems to confirm Toomer's intuition that Bernard of Virdun obtained his information from an extremely good source. This value agrees with the evidence we can obtain from other works by Ibn al-Zarqālluh: in his treatise on the construction of the equatorium we find that the longitude of Venus' apogee is $85;50^\circ$, and we must remember that early Eastern Islamic astronomers usually considered that the longitudes of the apogees of Venus and the Sun were the same.²⁸⁶ Besides, in his treatise on the construction of the *ṣafīḥa zarqālīyya*²⁸⁷ (4.3.2) the position of the solar apogee is 86° , a rounded value of $85;49^\circ$ or $85;50^\circ$.

According to Millás,²⁸⁸ Ibn al-Zarqālluh determined that the solar apogee moved 1° in 299 Julian years, which corresponds to 0;0,12,2,24... per Julian year.

283 Samsó, 1987 c.

284 Hartner & Schramm, 1963, pp. 208–9; on Eastern astronomers' knowledge of the solar apogee's own motion, see also Mozaffari, 2013 a, pp. 403–408.

285 Bodleian Library 11,2 ms. 285 (Marsh 618) fol. 5v.

286 Goldstein, 1977; Mozaffari, 2019.

287 Ms. B.N. Paris 4824, fol. 15r.

288 Millás, 1943–50, p. 241.

His source was Abū l-Ḥasan al-Marrākushī's *Mabādī' wa-ghāyāt*,²⁸⁹ in which this number of years is actually quoted, and it is said that it corresponds to a motion of 1' in five lunar years (actually 0;0,58,24,25°). When using this approximation there is an excess of 1' in 190 lunar years. This last number seems wrong: the correction should be about 5' in 190 lunar years (0;5,2,42°).

As established by Toomer, 1°/299 Julian years does not correspond to the evidence offered by Ibn al-Zarqālluh himself or by his followers. A first approximation can be found in Ibn al-Zarqālluh's treatise on the motion of the fixed stars, in which we can find evidence of the fact that the solar apogee advances 4;23° in the ecliptic in 1221 years, the period of time between Hipparchus (-146) and Ibn al-Zarqālluh (1075) (cf. 6.2.2.2). It is easy to calculate that this implies a motion of 1° in 278;33,18 Julian years.

1° in approximately 279 Julian years is, as Toomer established, the period of the revolution of the solar apogee and it can be obtained with the mean motion tables for the computation of the displacement of the solar apogee which are preserved in the collection of *zīj*es derived from Ibn Ishāq's astronomical tables.²⁹⁰ All these sources contain the same table and Millás edited it²⁹¹ using Ibn al-Bannā's *Minhāj* as his source, while Díaz-Fajardo has published a critical edition which takes into consideration all the evidence found in the aforementioned five *zīj*es.²⁹² The same table was also preserved in Abū l-Ḥasan 'Alī al-Qusānī's *zīj*, with versified canons.²⁹³ In all these sources the radix position for the beginning of the Hijra is 2° 16;44,17° and the calculated mean motion per day is 0;0,0,2,7,10,38±0;0,0,0,0,20°,²⁹⁴ which corresponds exactly to 1° in 279;0,3 Julian years. In my opinion, the table mentioned has a clear Zarqāllian origin.

A slightly different parameter can be obtained from Ibn al-Kammād's *Muqtabas zīj*: Chabás and Goldstein calculate 0;0,0,2,2,14,46...°/day from the value for 720 years and a radix for the beginning of the Hijra of 2° 16;45,21°. ²⁹⁵ This corresponds to 1° in 299 lunar years or 290 Julian years. A similar table appears in Pedro IV's Barcelona tables, calculated for a mean motion of

289 Marrākushī, 1984, vol. 1, pp. 45–47; Sédillot, 1834, pp. 132–3.

290 Mozaffari (private communication) tells me that the modern value of the sidereal motion of the solar apogee is 11.6" per year, or 1° in a little more than 310 years, not far from Ibn al-Zarqālluh's estimation.

291 Millás, 1943–50, p. 352.

292 Díaz-Fajardo, 2005.

293 Ms. Escorial 909, fol. 49r; Millás, 1943–50 p. 353; Kennedy & King, 1982, p. 16.

294 Parameter calculated by Mestres, 1999, p. 199.

295 Chabás & Goldstein, 1994 pp. 28 and 30; Chabás & Goldstein, 2015 a, pp. 602–3. The table appears in ms. BN Madrid 10023 fol. 28v: the values for 810 and 900 lunar years seem corrupt.

0;0,0,2,2,42,38°/day,²⁹⁶ equivalent to 1° in 289;9,23 Julian years. Finally, Ibn ‘Azzūz al-Qusanṭīnī’s *Muwāfiq zīj* contains a table of the motion of the solar apogee calculated with a parameter of 0;0,0,2,7,15°/day (1° in 278;50,28 Julian years) and a radix position for the beginning of the Hijra of 2^s 16;44,21°: both values could be the result of a small correction in the Zarqāllian parameters or, in the case of the radix, Ibn al-Kammād’s value with a copying mistake of –1.²⁹⁷

There is no evidence that Ibn al-Zarqālluh extended the motion of the solar apogee to that of the inferior planets, although there would have been a certain logic in doing so, especially in the case of Venus. We should now review what happens in the rest of the Andalusī-Maghribī tradition. Ibn al-Kammād, for example, states clearly that the motion of the solar apogee affects all the planets, and not only the inferior ones as in the case of Ibn al-Bannā’. Thus, in chapter 9 of the Latin translation of his *Muqtabas zīj* we read:

Differentia in aptatione augium. Cum volueris hoc extrahe singulos motus augis quem admodum praedictum est ad tempus quod volueris et *adde ipsos super auge cuiuslibet planete* qui augis planetarum positus est in capite tabularum cuiuslibet planete *ad principium annorum seductionis* et quod postea inde exierit erunt auges aptatos pro tempore illo pro quo numerasti.²⁹⁸

The same doctrine (i.e., keeping all the planetary apogees at a fixed distance from the solar apogee) is followed by Ibn al-Hā’im²⁹⁹ and by the Tunisian recension of Ibn Ishāq. The former does not state clearly which planets are affected by the motion of the solar apogee, but it defines the meaning of *awj mu’addal* (corrected apogee corresponding to Ibn al-Kammād *auges aptatos*):

Obtain the mean motion of the apogee for any moment, using any era (*ta’rīkh*) you wish. Add the result to the radix position for the beginning of the era. You have obtained, then, the corrected apogee (*awj mu’addal*) on the ecliptic, that is to say its distance from the point of the Head of Aries for that moment.

296 Millás, 1962, p. 190; Chabás, 1996 a, p. 486.

297 Samsó, 1997, pp. 81–83, 97, 102.

298 Ms. Madrid BN 10023 fol. 14v. I have used a provisional edition of this text prepared by Angel Mestres and other undergraduate students as a part of a course on Latin Paleography given by Dr Mercè Viladrich. The italicisation is mine.

299 Ms. Bodleian Marsh 618 fol. 34v.

The question becomes absolutely clear if we read the chapters in which Ibn al-Hā'im describes the procedure for calculating the true longitude of the planets: there, both for the superior (fol. 37r and v) and for the inferior planets (fol. 38r), he says that the *awj mu'addal* has to be subtracted from the mean longitude of the planet.

In the case of the Tunisian recension of Ibn Ishāq's *zīj*, there is clear evidence of the doctrine in chapter 14 of the canons,³⁰⁰ which is confirmed by the tables.³⁰¹ The same attitude is adopted by Ibn al-Raqqām in his *Shāmil zīj*, whose canons reproduce literally Ibn al-Hā'im's text,³⁰² as well as by Ibn 'Azzūz's *Muwāfiq zīj*³⁰³ and by the authors of the Barcelona Tables.³⁰⁴

The rest of the sources³⁰⁵ considered apply the motion of the solar apogee to the apogees of Venus and Mercury, while the apogees of Saturn, Jupiter and Mars are sidereally fixed. This is the case of Ibn al-Bannā's *Minhāj* and Ibn al-Raqqām's *Mustawfi* and *Qawīm zīj*es. In these three *zīj*es the apogees of the superior planets given at the top of the corresponding equation tables are not tropical apogees for the beginning of the Hijra, but fixed sidereal apogees calculated for the moment in which the value of precession was 0° (ca. 581 CE according to Ibn al-Zarqālluh: see 6.2.2.5.2.1).³⁰⁶ The *Mustawfi zīj* gives some more interesting information:

This was Ibn Ishāq's choice (*ikhtiyār*), as well as his method (*iṣṭilāḥ*), although it disagrees with the previous measurements and observations. This practice will be useful for a certain time until further observations confirm its validity³⁰⁷

This shows clearly that Ibn Ishāq considered that the motion of the solar apogee affected the apogees of Venus and Mercury, but not those of the superior planets. As for the second part of the sentence, it is not clear whether it is

300 Mestres, 1996, pp. 394–5 and 412; Mestres, 1999, pp. 39–42, and 62–63 of the Arabic edition.

301 Mestres, 1999, p. 199.

302 'Abd al-Raḥmān, 1996, pp. 58–59.

303 Samsó, 1997, pp. 82–83.

304 Millás, 1962, pp. 128, 147 and 190.

305 The only exception is the versified *zīj* of Abū l-Ḥasan 'Alī al-Qusanṭīnī. Although the canons do not clarify the author's position, the table of the mean motion of the solar apogee (ms. Escorial 909, fol. 49r) has a clear title: *Jadwal ḥarakat awj al-shams*. This, apparently, means that this motion is applied only to the solar apogee and not to the planetary apogees.

306 Samsó & Millás, 1998, pp. 267–269.

307 Díaz-Fajardo, 2005, pp. 22–23.

Ibn Ishāq or Ibn al-Raqqām himself who doubts the validity of the theory and awaits future observations to clarify the situation.

6.3.3 *Ibn al-Zarqālluh's Solar Model*

6.3.3.1 The Three Kinds of Solar Year

According to Abraham ibn 'Ezra,³⁰⁸ Ibn al-Zarqālluh distinguished three types of solar year: tropical, sidereal, and anomalistic. The same classification is confirmed by Ibn al-Hā'im's *Kāmil zīj*.³⁰⁹

As for the tropical year, it does not have a constant length because it is affected by variable precession, the motion of the solar apogee, and the motion of the centre of the eccentric (see 6.3.3.2). The only information on the tropical year adopted by Ibn al-Zarqālluh for his own time is provided by Ibn 'Ezra's *Fundamenta tabularum* in which he mentions a *revolutio anni* of 87;21°, ³¹⁰ which corresponds to a year of 365d 5;49,24h (= 365;14,33,30d).

The sidereal year is also not constant, as it is affected by the motion of the solar apogee and of the centre of the solar eccentric. The length established by Ibn al-Zarqālluh seems to correspond to an early stage of his life, because it does not imply any important change in the parameter used in the *Toledan Tables*. Let us compare the parameter extracted by Toomer³¹¹ and Pedersen³¹² from the Toledan solar mean motion tables with those obtained from the *zīj*es of Ibn Ishāq's school and from that implied in the table of the *revolutio anni* (92;24°) preserved in Ibn al-Kammād's *Muqtabas zīj* and attributed explicitly to Ibn al-Zarqālluh.³¹³

<i>Toledan Tables</i> :	0;59,8,11,28,27°/ day
Ibn Ishāq's school:	0;59,8,11,28,26,24°
Ibn al-Kammād = Ibn al-Zarqālluh:	0;59,8,11,28,25°

This mean motion parameter corresponds to a sidereal year of 6,5;15,24 days (= 365d 6;9,36h).

308 Millás, 1947 a, pp. 79–80; see also Toomer, 1969, pp. 317–320.

309 Calvo, 1998 a, pp. 54–56, 84–88.

310 Millás, 1947, p. 83.

311 Toomer, 1968, p. 44.

312 Pedersen, 2002, vol. III, p. 1140.

313 Ms. BN Madrid 10,023, fol. 57v (photograph no. 62); Chabás and Goldstein, 2015 a, p. 640. Abraham ibn 'Ezra attributes this *revolutio anni* to Thābit ibn Qurra (Samsó, 2012, p. 179). This value is not far from the *revolutio* given by Ibn al-Hā'im for the beginning of the seventh/thirteenth c. (Calvo, 1998, pp. 74, 109): 92;21,45° which corresponds to a year length of 365d 6;9,27h (= 365;15,23,37,30 d). The corresponding mean daily motion is 0;59,8,9,21,15°.

The third kind of solar year is the anomalistic year which Ibn al-Hā'im defines as the period of time elapsed between two returns of the Sun to the same point of its eccentric circle (*zamān al-ʿawda fī l-khārij al-markaz*). The length of this year is fixed, as it incorporates both the motion of the solar apogee and the motion of the centre of the solar eccentric.³¹⁴ We can obtain an approximation to Ibn al-Zarqālluh's anomalistic year by subtracting 0;0,0,2,7,10,38° from the mean sidereal motion per day in the *Toledan Tables*:

$$\begin{aligned} &0;59,8,11,28,27^{\circ} - 0;0,0,2,7,10,38^{\circ} = 0;59,8,9,21,16,22^{\circ} \\ \text{and} \quad &360^{\circ} / 0;59,8,9,21,16,22^{\circ} = 365;15,36,34,51,38\text{d} \end{aligned}$$

which is close to the length of the anomalistic year in Ibn al-Hā'im's *Kāmil zīj*: 365;15,36,35,0,8 days,³¹⁵ and to Ibn al-Kammād's anomalistic year of 365;15,36,24... d³¹⁶

6.3.3.2 Variation of the Solar Eccentricity

A second basic solar parameter is eccentricity. According to Bernard of Virdun, Ibn al-Zarqālluh determined an eccentricity of 1;58^p, which corresponds to a maximum solar equation of 1;52,42°. ³¹⁷ The latter is confirmed by multiple sources, such as the treatise on the motion of the fixed stars (1;53°), the *Almanac* (1;52,50°),³¹⁸ the equatorium (1;52,30°)³¹⁹ and Ibn al-Kammād's *Muqtabas zīj* (1;52,44°).³²⁰ As the maximum value of the solar equation in the *Toledan Tables* is 1;59,10°, ³²¹ it seems that 1;52,42° and an eccentricity of 1;58^p are the result of later observations made by Ibn al-Zarqālluh and which are the origin of his book on the Sun.

We have already seen (6.2.2.2) that Ibn al-Zarqālluh based his analysis of the values of precession on determinations of the longitude of the solar apogee made by Hipparchus, Ptolemy, Thābit ibn Qurra and al-Battānī. Consequently, he must have been aware of the fact that Ptolemy's and Hipparchus' eccentricity was 2;30^p (maximum equation 2;23°), Thābit ibn Qurra's 2;6,40^p (max eq. 2;1°), while al-Battānī and the *Toledan Tables* used an eccentricity of 2;4,46^p

³¹⁴ Calvo, 1998, pp. 54–55, 85.

³¹⁵ Calvo, 1998, pp. 55, 86–87.

³¹⁶ Chabás & Goldstein, 2015 a, p. 600. For other lengths of the solar year attributed to Ibn al-Kammād, see the aforementioned paper (p. 634) and Chabás & Goldstein, 1994, p. 35.

³¹⁷ Toomer, 1987, pp. 515–17.

³¹⁸ Boutelle, 1967.

³¹⁹ Samsó, 1987 d, p. 468.

³²⁰ Chabás & Goldstein, 2015 a, p. 605; Chabás & Goldstein, 1994, pp. 4–10.

³²¹ Pedersen, 2002, vol. IV, p. 1248.

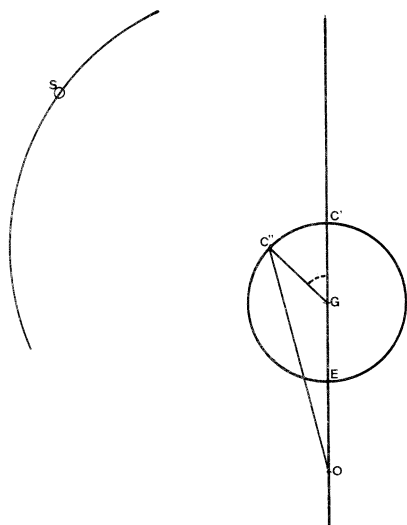


FIGURE 6.13
Ibn al-Zarqālluh's model to
justify the variation of the
solar eccentricity

(max. eq. $1;59,10^{\circ}$). As he considered the previous observations to be sufficiently precise, he reached the obvious conclusion that the solar eccentricity was variable; this led him to design an original solar model, although probably influenced by the Ptolemaic model for Mercury, and obviously similar to his model which seeks to justify the changes of the obliquity of the ecliptic (6.2.2.5.1).³²²

Using Toomer's own words,³²³ I will summarise what is known about Ibn al-Zarqālluh's solar model (fig. 6.13): "the Sun (S) rotates with uniform sidereal velocity about the centre (C) of its eccentric. C in turn rotates very slowly with uniform velocity about the centre of a small circle (CE), thus causing a slow secular variation in the Sun's eccentricity (OC'): O, the earth, lies outside the small circle CE. The sense of C's rotation is unknown and is irrelevant, since

³²² Mozaffari, 2013 a, pp. 398, 401–403.

³²³ Toomer, 1969, pp. 328–331. Toomer (p. 328) draws attention to a table preserved in some manuscripts of the *Toledan Tables*, with the heading *Tabula augmentans equationem solis*. Apparently this table would give the difference between the maximum equation for maximum eccentricity and the maximum equation for minimum eccentricity (see below), but the tabular maximum ($0;55^{\circ}$) seems too high if we compare it to the maxima of Ibn al-Kammād ($0;37,20^{\circ}$), the Tunisian recension, Ibn al-Raqqām and Ibn al-Bannā' ($0;36,51^{\circ}$), and Ibn 'Azzūz ($0;37,19^{\circ}$). Toomer believed this table to be "a remnant of al-Zarqāl's solar equation table". If his assumption proves to be true, it would imply that we have here an early sample of Ibn al-Zarqālluh's solar model with variable eccentricity. However, Pedersen (2002, vol. III, pp. 994–995) does not think that this table is connected to the solar equation theory, and states: "purpose unknown". Another edition of this table in Toomer, 1968, p. 57.

the effect of such rotation on the position of the apogee is neglected. The parameters of this system are unknown”.

I disagree with Toomer on two small points.³²⁴ First of all, the parameters of the system can be recovered from the later Zarqāllian tradition (mainly the collection of *zīj*es of Ibn Ishāq’s school); besides, the model, as described by Toomer according to the sources available to him at the time his article was written, produces a back and forth motion of the solar apogee and this is why he says that “the effect of such rotation on the position of the apogee is neglected”. This difficulty is avoided by considering the description of the model we find in Ibn al-Hā’im’s *Kāmil zīj*, in which the plane of the circle described by the centre of the eccentric is perpendicular to the plane of the eccentric itself, something which implies that the rotation of the centre of the eccentric will not produce a back and forth motion of the solar apogee, because it will move up and down.³²⁵ Ibn al-Hā’im’s words are clear enough: *fa l-takun dā’ira mawḍū’a ‘inda markaz al-khārij qā’ima ‘alā saṭḥ wa-markazu-hu yataḥarraku suflan wa ‘ulwan ‘alā muḥīṭi-hā* (“let us have a circle placed on the centre of the eccentric, perpendicular to [its] plane, and such centre moves down and up on this circle”).³²⁶

Computing the solar equation for a given date requires the use of the following tables:

1) Table of the mean motion of the centre of the solar eccentric (*Jadwal ḥarakat markaz al-falak al-khārij al-markaz li l-shams* [Ibn al-Bannā,³²⁷ Ibn al-Raqqām,³²⁸ Ibn ‘Azzūz³²⁹], *Jadwal ḥiṣṣat al-falak al-khārij al-markaz* [Tunisian recension],³³⁰ *Tabula motus centri circuli exeuntis centrum* [Ibn al-Kammād]),³³¹ These tables are computed for epoch Hijra and Arabic years and, consequently, the *anni collecti* (*majmū’a*) are presented in multiples of 30 years. The only exception is Ibn al-Kammād’s *zīj* which, in spite of its use of Arabic years, presents its collected years in multiples of 10 and 100 years, probably as the result of an intervention of the Latin translator. The parameters involved in these tables are:

324 Years ago (ca. 1995), after reading Samsó & Millás, 1994, Toomer wrote me a letter in which he said that he also believed in the preservation of the parameters.

325 Calvo, 1998 a, pp. 61–62, 94.

326 Calvo, 1998 a, p. 94.

327 Samsó & Millás, 1994, p. 18.

328 ‘Abd al-Raḥmān, 1996, pp. 178–181 for the *Shāmīl zīj*.

329 Ms. Rabat NL D/2461 p. 383.

330 Mestres, 1999, p. 198.

331 Ms. Madrid BN 10,023 fol. 65r.

- Position of point C (centre of the solar eccentric) at the beginning of the Hijra: $83;40,31^{\circ}$ (Ibn al-Kammād) and $83;40,53^{\circ}$ (Tunisian recension, Ibn al-Bannā', Ibn al-Raqqām). Ibn 'Azzūz al-Qusanṭīnī: $84;40^{\circ}$.³³² The differences between these positions cannot be attributed to the meridians used.
- Mean daily motion of point C: $0;0,1,3,41,21,50,8,52^{\circ}$ (Tunisian recension, Ibn al-Bannā', Ibn al-Raqqām), which corresponds to a revolution in 3,342.71 Julian years or 3,345 Persian years, as stated by Ibn al-Hā'im, who also records a mean daily motion of $0;0,1,3,41$. This value can also be extracted from Ibn al-Kammād's table, while Ibn 'Azzūz uses $0;0,1,5,36,5^{\circ}$.³³³

2) Two tables are used to compute the solar equation. They are called *Jadwal daqā'iq nisbat ikhtilāf al-markaz* (Ibn al-Bannā', *Jadwal daqā'iq al-nisba* in Ibn al-Raqqām's *Shāmīl*),³³⁴ table of the minutes of the proportion of the centre's variation, and *Jadwal ta'dīl al-shams al-majmū' al-kullī* (Ibn al-Bannā', Tunisian recension,³³⁵ Ibn al-Raqqām),³³⁶ table of the collected and total solar equation. The Latin titles in Ibn al-Kammād's *zīj* are *Tabula motus centri circuli exeuntis centrum in longitudine longiori et propinquiori a centro terre* and *Tabula directionis composite centri circuli solis exeuntis centrum de diuersitate centri eiusdem in longitudine propinquiori et longiori a circulo diuersitatis motus centri morantis tempus*.³³⁷

The table of the total solar equation (*ta'dīl al-shams al-majmū' al-kullī*) tabulates two functions for the same argument, which are called *ta'dīl al-ḥiṣṣa* (equation of the argument) and *ikhtilāf al-markaz* (variation of the centre). The *modus operandi* has been described by Toomer for Ibn al-Kammād³³⁸ and it consists of the following steps:

We calculate the mean longitude of the Sun, the longitude of its apogee and the centre (*markaz*, *L*, i.e., the solar mean longitude measured from its apogee). With this value as argument we enter the table of the total solar equation and we read the values corresponding to:

332 Samsó, 1997, p. 100.

333 Samsó, 1997, p. 97.

334 'Abd al-Rahmān, 1996, pp. 234–236.

335 The full title of the table in the Tunisian recension is *Jadwal ta'dīl al-shams al-majmū' al-kullī bi l-khums wa-nisfī-hi*, table of the collected and total solar equation with the fifth and its half (?). I have no explanation for the last part of the title. See Mestres, 1999, pp. 233 and 238–243 (edition of the table).

336 'Abd al-Rahmān, 1996, pp. 237–242.

337 Ms. Madrid BN 10,023 fols. 55v–56r.

338 Toomer, 1969, pp. 323–331; see also Samsó & Millás, 1994, which contains a detailed analysis of Ibn al-Bannā's tables.

- the equation of the argument (*ta'dil al-ḥiṣṣa*), which computes the value of the solar equation, using the standard eccentric model, for a minimum eccentricity ($\eta_{\min}$),
- the variation of the centre (*ikhṭilāf al-markaz*) which gives the difference between the solar equation for a maximum and a minimum eccentricity ($\eta_{\max} - \eta_{\min}$).

We then calculate the position (c) of the centre (C) of the solar eccentric with the mean motion table already mentioned and use c as argument in the table of minutes (*daqā'iq al-nisba*). We will obtain the value m and the solar equation (η') will be:

$$\eta' = \eta_{\min}(L) \pm m(c) \cdot (\eta_{\max} - \eta_{\min})(L)$$

As we have seen, the equation of the argument and the variation of the centre are clear enough and do not need an extra explanation. As for the *daqā'iq al-nisba*, it is an interpolation function designed to avoid the need of calculating the great number of values required by Ibn al-Zarqālluh's solar model in which the solar equation depends on two variables: the mean solar longitude from the apogee (L) and the position of the centre of the solar eccentric (c). The problem is analogous to the standard one of computing the equation of anomaly of a planet using astronomical tables based on a Ptolemaic model, and the solution adopted is also similar. We have seen a function which computes the equation of the centre for a minimum eccentricity and another function which establishes the difference between the equation of the centre for a maximum eccentricity and the same equation for a minimum eccentricity. We now need a third function which will take into account the actual value of the eccentricity at a given moment. This is the case of function $m(c)$ computed by Ibn al-Bannā' and Ibn al-Raqqām to the precision of seconds for each degree of the argument from 0° (m attains its maximum which is $0;60,00$) to 180° (m attains the value 0°). We obtain good results if we recompute the table using the expression:

$$m(c) = (e(c) - e_{\min}) / (e_{\max} - e_{\min})$$

where $e(c)$ is the value of eccentricity for a given c .

With the information gathered from the tables, we can calculate the parameters of the solar model (table 6.28): the maximum eccentricity ($e_{\max} = OC'$) corresponding to the maximum solar equation, the minimum eccentricity ($e_{\min} = OE$) for minimum solar equation and the radius ($r = GC'$) of the small circle on which the centre of the eccentric rotates:

TABLE 6.28 Parameters of Ibn al-Zarqālluh’s solar model with variable eccentricity

	e _{max}	e _{min}	r
Ibn al-Kammād	2;29,36 ^{Pa}	1;50,34 ^P	0;19,31 ^P
Ibn al-Hāʾim	2;29,30 ^{Pb}	1;51,30 ^P	0;19 ^P
Tun. rec. & Ibn al-Raqqām	2;29,30 ^P	1;50,56 ^P	0;19,17 ^P
Ibn al-Bannāʾ	2;29,33 ^P	1;50,59 ^P	0;19,17 ^P

- a Toomer, 1969, p. 327 obtains a maximum solar equation for maximum eccentricity 2;22,54° and the corresponding eccentricity should be 2;29,36^P (instead of 2;29,30^P).
- b Calvo, 1998 a, p. 59.

In this table we can see that the parameters involved are practically the same, and this is why I believe that they represent those of Ibn al-Zarqālluh himself. The tables preserved in the Tunisian recension and in Ibn al-Raqqām’s *Shāmil* and *Mustawfī zījes* probably correspond to Ibn al-Zarqālluh’s original tables. This hypothesis is based on the fact that the Hyderabad manuscript of the Tunisian recension contains two sets of tables which allow the computation of the solar equation: the first set corresponds to tables 28–29, to which I have referred in the previous pages; the second (tables 30–32) has the title *Jadwal ākhar bi-taʿdīl al-shams al-murakkab li-Ibn Ishāq bi-naṣṣ al-Zarqālluh lā bi-jadwali-hi* (“Another table containing the composite solar equation, [made] by Ibn Ishāq on the text of al-Zarqālluh, not on his table”).³³⁹ This seems to mean that tables 28–29 are by Ibn al-Zarqālluh himself.³⁴⁰

6.3.3.3

Two Maghribī Total Solar Equation Tables with Special Characteristics: Ibn al-Bannāʾ and Ibn ʿAzzūz

Ibn al-Bannāʾ’s *Minhāj*³⁴¹ contains total solar equation tables different from the rest of the sources, as he is, to the best of my knowledge, the first Western Islamic astronomer to use “displaced” equations, the purpose of which is to save the computer from having to consider, in each case, whether the equation has a positive or negative value. Tables of this kind appear in Eastern Islam in

339 Mestres, 1999, pp. 244–248. This is not a table for the “total” solar equation but a standard solar equation table which attains a maximum of 1;49,37°, the eccentricity being 1;54,46^P.

340 Mestres, 1999, p. 233.

341 This is a revised version of Samsó & Millás, 1994, pp. 18–30.

the ninth century³⁴² and in Latin Europe in the fourteenth century.³⁴³ In the case of Ibn al-Bannā', this is one of his attempts, in the *Minhāj*, to introduce new, more user-friendly, computational methods.

In the *ta'dīl al-ḥiṣṣa* (equation of the argument) of the *Minhāj*, the values are displaced 4° in both vertical and horizontal senses. The obvious purpose of the vertical displacement is to obtain a function that will always be positive: this also implies a horizontal displacement for, once we have obtained the value of L (mean centre), in order to obtain L_v (true solar longitude from the apogee), we have, if we use an ordinary, non-displaced, table of the solar equation (η):

$$L_v = L - \eta \quad [1]$$

an expression which is equivalent to

$$L_v = (L - 4^\circ) + [4^\circ \pm \eta(L)] \quad [2]$$

If we consider:

$$L' = L - 4^\circ \quad [3]$$

$$\text{and } \eta'(L') = 4^\circ \pm \eta(L) \quad [4]$$

we can obtain an expression equivalent to [2]:

$$L_v = L' + \eta'(L') \quad [5]$$

$\eta'(L')$ is the actual function tabulated by Ibn al-Bannā' and, in it, not only has η been displaced vertically by adding 4° to every value of the function in order to obtain η' (see [4]), but the new argument (L') is also the result of a horizontal displacement of the previous argument L (see [3]). This explains why Ibn al-Bannā's function reaches the value 4° for arguments 176° (instead of 180°) and 356° (instead of 360°).

Ibn al-Bannā's *ta'dīl al-ḥiṣṣa* is computed to minutes for each degree of the argument from 1° to 360°. The maximum value of the function is $\eta' = 5;46^\circ$ (for values of the independent variable from 260° through 269°) and its minimum value is $\eta' = 2;14^\circ$ (for values from 83° through 93°). These two values correspond to $\eta = +1;46^\circ$ (for values from 264° through 273°) and $\eta = -1;46^\circ$ (for values

342 See, for example, Kennedy & Salam, 1983; Kennedy, 1977; Kennedy, 1983 b; King, 1974 a; Jensen, 1971–2; Saliba, 1976; Saliba, 1978; Saliba, 1978; Saliba, 1977.

343 Chabás & Goldstein, 2011.

from 87° through 97°). From this minimum value of the maximum solar equation (min $\eta_{\max}$) we can obtain the corresponding value of the minimum eccentricity ($e_{\min}$) used in Ibn al-Bannā's solar tables (table 6.28):

$$e_{\min} = 60 \cdot \sin 1;46^{\circ} = 1;50,59^p$$

Having reached this stage, we can recalculate the whole table and observe that it is, in general, well computed, with the errors never exceeding a minute of arc. Table 6.29 exhibits recomputed values at intervals of 10°:

TABLE 6.29 *Ta'dīl al-ḥiṣṣa* in Ibn al-Bannā's *Minhāj*

L'	η'	L'	η'	L'	η'
10°	3;35°	140°	2;56°	260°	5;45° (−1')
20	3;18	150	3;12	264	5;46
30	3;2	160	3;30	270	5;45
40	2;48	170	3;49	280	5;42
50	2;36	176	4;0	290	5;36
60	2;26	180	4;8	300	5;26
70	2;19	190	4;26	310	5;15
80	2;15	200	4;44	320	5;1
88	2;14	210	5;1	330	4;45
90	2;14	220	5;15	340	4;28
100	2;16	230	5;27	350	4;11
110	2;22	240	5;36 (−1')	356	4;0
120	2;31	250	5;43	360	3;53
130	2;42				

The values in this table seem to be the result of a rounding of the corresponding tables in the Tunisian recension of Ibn Ishāq's $z\tilde{y}$ ³⁴⁴ where the function, which is not displaced, appears under the heading *al-ta'dīl al-mujarrab* and is calculated to seconds. The maximum value of the equation amounts to 1;45,46° (eccentricity 1;50,56^p). If, as suggested, this is Ibn al-Zarqālluh's value of the maximum equation for minimum eccentricity, it seems that Ibn al-Kammād introduced a small correction, because he uses a maximum value of 1;45,36°. ³⁴⁵

344 Mestres, 1999, pp. 238–243.

345 Toomer, 1969, p. 325.

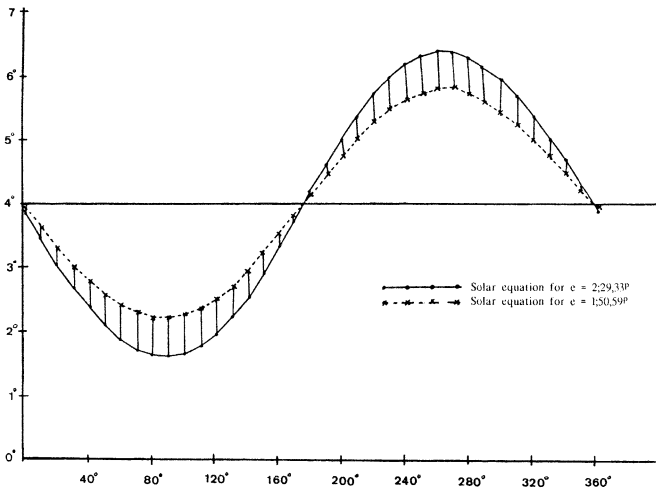


FIGURE 6.14 $\eta'_{\max}$ and $\eta'_{\min}$ in Ibn al-Bannā's *Minhāj*

The table for the *ikhtilāf al-markaz* (variation of the centre) is also displaced and gives $[\eta'_{\max} - \eta'_{\min}]$. It is computed, to the precision of seconds, for each degree of the argument from 1° to 360° . The function becomes negative for $0^\circ < L' < 376^\circ$ and for $356^\circ < L' \leq 360^\circ$, it acquires the value 0° when $L' = 176^\circ$ or 356° and it is positive elsewhere (see fig. 6.14 where I have plotted both $\eta'_{\max}$ and $\eta'_{\min}$)

As $\eta'_{\min} < 4^\circ$ when function $[\eta'_{\max} - \eta'_{\min}]$ is negative and $> 4^\circ$ when $[\eta'_{\max} - \eta'_{\min}]$ is positive, one can understand why Ibn al-Bannā' establishes that, in the expression

$$\eta' = \eta'_{\min}(L) \pm m(c) \cdot (\eta'_{\max} - \eta'_{\min})(L')$$

the second term should be added for $\eta'_{\min} > 4^\circ$ and subtracted for $\eta'_{\min} < 4^\circ$. $[\eta'_{\max} - \eta'_{\min}]$ reaches its maximum ($0;36,51^\circ$) for values of L' comprised between 87° and 90° and between 262° and 295° . This table seems to derive from Ibn Ishāq's non-displaced *al-zā'id al-mujarrad*, preserved in the Tunisian recension, which also attains a maximum of $0;36,51^\circ$ for L comprised between 91° and 94° .

We can now recalculate the whole table which is, like the previous one, well computed in general, with the errors amounting to a maximum of $4''$. Table 6.30 presents recomputed values at intervals of 10° :

TABLE 6.30 *Ikhtilāf al-markaz* in Ibn al-Bannā's *Minhāj*

L'	$\eta'_{\max} - \eta'_{\min}$	L'	$\eta'_{\max} - \eta'_{\min}$
10°	0;8,18° (−1")	200°	0;16,1°
20	0;14,1 (−1")	210	0;21,52 (−1")
30	0;19,25 (+1")	220	0;26,57
40	0;24,19 (+2")	230	0;31,5
50	0;28,34 (+1")	240	0;34,8
60	0;32,4 (+1")	250	0;36,4
70	0;34,41 (+1")	260	0;36,48 (−3")
80	0;36,18	262	0;36,51 (−2")
87	0;36,51 (+2")	265	0;36,51 (+2")
90	0;36,51 (−2")	270	0;36,31 (+1")
100	0;36,20 (+1")	280	0;35,5 (+1")
110	0;34,38 (+1")	290	0;32,40 (+2")
120	0;31,47	300	0;29,21 (+3")
130	0;27,53 (+2")	310	0;25,13 (+1")
140	0;22,57 (−1")	320	0;20,26 (+1")
150	0;17,14 (−1")	330	0;15,7 (−1")
160	0;10,53 (−1")	340	0;9,28 (−1")
170	0;4,9	350	0;3,34 (−1")
176	0;0,0	356	0;0,0
180	0;2,46	360	0;2,23 (−1")
190	0;9,33 (−1")		

The table contained in the *Muwāfiq zīj* of Ibn ‘Azzūz³⁴⁶ is unusual in a different way. Its title is *Ta’dil markaz al-falak al-khārij al-markaz li l-shams li-iṣlāḥ al-taqrib* (Equation of the centre of the solar eccentric for the correction of the approximation). This table computes two functions for arguments between 1° and 180°: the first is called *ta’dil* (equation) and reaches a maximum of 0;37,19° for an argument of 94°;³⁴⁷ and the second has the title of *daqā’iq mushtaraka* (common minutes) and decreases monotonically from 60’ (for an argument of 1°) to 0’ (for an argument of 180°). Obviously the *ta’dil* table is equivalent to Ibn al-Bannā’s and Ibn al-Raqqām’s *ikhtilāf al-markaz* and it gives $[\eta_{\max} - \eta_{\min}]$ for a given mean centre (L), while the *daqā’iq mushtaraka* correspond to the

346 This passage is a revision of Samsó, 1997, pp. 83–85.
347 Ibn al-Kammād has 0;37,20° for the same argument: ms. Madrid BN 10,023 fol. 56r.

interpolation function. The problem lies in the fact that the aforementioned *zīj* does not contain a table giving $\eta_{\min}$ and, therefore, it cannot be used to compute the value of the solar equation using Ibn al-Zarqālluh's model of variable eccentricity.

The answer can be found in the chapter on the *iṣlāḥ al-taqrīb*, which raises the problem of casting a horoscope for an anniversary when a long time has passed between the actual birth of the newborn or the conjunction and the moment of the anniversary: with Ibn al-Zarqālluh's solar model with variable eccentricity, the actual eccentricity may have changed when a long time span is involved and, consequently, the period of return of the Sun to the same true longitude may be different from its return to the same mean longitude. The text explains an approximate procedure for such a case, which is exactly the same as the one we find in a chapter of Ibn al-Kammād's *al-Kawr 'alā l-dawr*, preserved in Johannes de Dumpno's Latin translation.³⁴⁸ The correspondence is so close that I believe that Ibn 'Azzūz is giving us, here, the actual Arabic text of Ibn al-Kammād:

1. You should calculate the oblique ascension of the tropical ascendant (α_{φ}) for the time of the return of the Sun to the same mean longitude it had at the moment of the radical event: call it "the first preserved" (*al-mahfūz al-awwal*).
2. Then you obtain the value of c (i.e., the position of the centre of the solar eccentric) for the two moments involved, using the corresponding mean motion table, and look in the table the *daqā'iq mushtaraka* (m) for each one of the two values of c (c' and c''). Then you put:

$$\Delta m = m(c') - m(c'')$$

3. With the mean longitude of the Sun measured from its apogee (L) (for the moment of the anniversary) as argument, you enter the table of the *ta'dīl* and obtain:

$$\Delta\eta = \eta_{\max} - \eta_{\min}$$

348 Ms. Madrid BN 10,023 fols. 18v–19r. Interestingly, Ibn al-Kammād states (fol. 18v) that the approximate method should be, at least, applied after a period of 40 years. In the *Kāmil zīj* Ibn al-Hā'im says that the length of the sidereal year should be recalculated after approximately 40 years. This is explained by Calvo (1998 a, p. 75) by considering that in approximately 1671 years the eccentricity changes from its maximum to its minimum, which means that almost every 44 years the value of the eccentricity will be increased or decreased by 0.1°.

4. Then you multiply $(\Delta m \cdot \Delta \eta)$ and, thus, obtain the increase or decrease in the solar equation between the time of the radical event and the time of the anniversary due to the change of eccentricity.
5. The result should be divided by the solar velocity (*buht*, v_s) and we will obtain the time (t) required by the Sun to travel the distance represented by $(\Delta m \cdot \Delta \eta)$:

$$t = (\Delta m \cdot \Delta \eta) / v_s$$

t will be expressed in hours and fractions of an hour. By multiplying $t \cdot 15$, we express t in time degrees.

6. Ibn ‘Azzūz then divides the result by 60 (?)³⁴⁹ and calls it “the second preserved” (*al-mahfūz al-thānī*) ($\Delta \alpha_\varphi$). The oblique ascension of the ascendant at the moment of the anniversary will be:

$$\alpha_\varphi + \Delta \alpha_\varphi$$

where $\Delta \alpha_\varphi$ can be positive or negative. The result will allow us to obtain the longitude of the ascendant of the anniversary.

To conclude this section, one may wonder why Ibn ‘Azzūz showed such an interest in introducing a small correction in the computation of the ascendant of an anniversary while, as we will see, the value he uses for the maximum solar equation (1;52°) corresponds roughly to the end of the eleventh century, and not even remotely to the middle of the fourteenth century, if one uses a solar model of variable eccentricity and any collection of parameters documented in the Andalusī-Maghribī tradition.

6.3.3.4 A Conclusion

If, as suggested in the preceding pages, the parameters found in the Andalusī-Maghribī tradition are, with small corrections, those used by Ibn al-Zarqālluh in his treatise on solar theory, it is interesting to check the results against historical determinations of the value of solar eccentricity. A comparison of this kind can be found in fig. 6.15 in which I have plotted against time (from -178 to 1372 CE) the value of eccentricity according to the model and parameters used in the Tunisian recension and Ibn al-Bannā’. The figure shows an almost exact coincidence for the determination of the solar eccentricity made by Hipparchus (2;30^p for ca. -150; computation: 2;29,31°), good approximations to the eccentricities established by Thābit ibn Qurra (using

349 This division into 60 does not appear in Ibn al-Kammād’s Latin text.

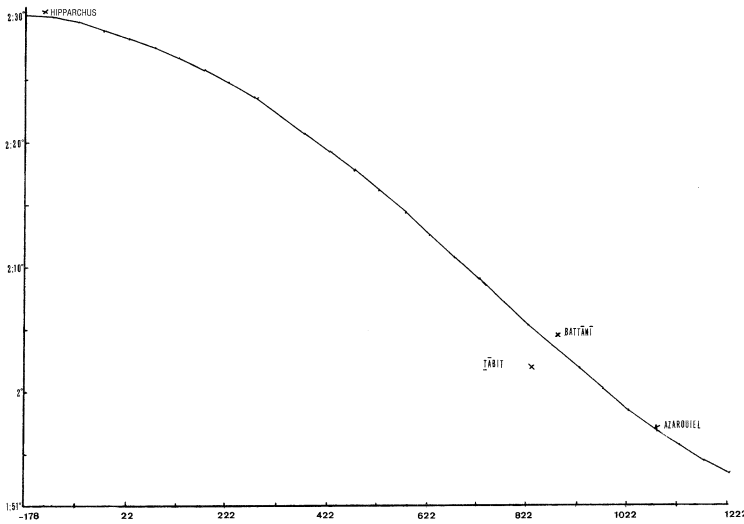


FIGURE 6.15 Historical values of solar eccentricity according to Ibn al-Zarqālluh's solar model

ma'mūnī observations made ca. 830–831 CE: 2;2,6^p; computation: 2;5,16^p for 831) and al-Battānī (2;4,5^p in 883 CE; computation: 2;3,26^p), precisely three sources which without any doubt had been used (indirectly in the case of Hipparchus) by Ibn al-Zarqālluh. The agreement is, again, excellent for the time of Ibn al-Zarqālluh: we know that he established the value of the maximum solar equation in 467 H./1074–75 CE, and the sources analysed by Toomer give values for this maximum solar equation comprised between 1;52° (eccentricity: 1;57,16^p) and 1;52,50° (eccentricity: 1;58,8^p). Computation with Ibn al-Bannā's parameters gives us 1;57,20^p for the end of 466 H. and 1;57,18^p for the end of 467 H. The conclusion is obvious: the model was definitely designed to yield the alleged changes in the solar eccentricity according to determinations made by previous astronomers whom Ibn al-Zarqālluh considered to be competent observers.

6.3.3.5 Tables of Partial Solar Equation

Although, as we have seen, the *zīj*es of Ibn al-Kammād, Ibn Ishāq (Tunisian recension), Ibn al-Bannā' and Ibn al-Raqqām (*Shāmil* and *Mustawfī*) contain tables of the total solar equation (*ta'dīl al-shams al-kullī*) which correspond to Ibn al-Zarqālluh's solar model of variable eccentricity, in them we often find another table of the standard "partial solar equation" (*ta'dīl al-shams al-juz'ī*) which, in some cases, may be the result of an eccentricity calculated with the "total" table (see table 6.31):

TABLE 6.31 Maximum solar equations and eccentricities in the Andalusī-Maghribī tradition

<i>Zījes</i>	Maximum eq.	Eccentricity
Toledan Tables	1;59,10 ^{oa}	2;4,45 ^p
Ibn al-Kammād	1;52,44 ^{ob}	1;58,2 ^p
Ibn Ishāq	1;49,[36],56,42 ^{oc}	1;54,46 ^p .
Tunisian rec. & Ibn al-Raqqām's <i>Mustawfi</i>	1;49,7 ^{od}	1;54,15 ^p
Ibn al-Raqqām's <i>Shāmil</i> & <i>Qawīm</i>	1;47,51 ^{oe}	1;52,55 ^o
Ibn al-Bannā'	1;51 ^{of}	1;56,13 ^p
Ibn 'Azzūz	1;52 ^{og}	
Abū l-Ḥasan 'Alī al-Qusanṭīnī	2;13 ^{oh}	2;19 ^p

a Pedersen, 2002, vol. IV, 1247–1249.
b Chabás & Goldstein, 1994, pp. 4–10.
c Mestres, 1999, pp. 244–248.
d Mestres, 1999, pp. 43–45, 236–237; Samsó, 2014, pp. 299, 302, 315.
e 'Abd al-Rahmān, 1996, pp. 232–233; Ms. Rabat BN 260 p. 107.
f Ms. BN Algiers 615 fol. 48r–48v; unnumbered ms. of the Madrid Museo Naval table 36.
g Samsó, 1997, p. 83.
h Ms. Escorial 909 fol. 56v.

This table deserves a few comments:

1. The solar equation of the *Toledan Tables* derives from al-Battānī's *zīj* and, consequently, is totally independent of the Zarqāllīan tradition.
2. The Tunisian recension of Ibn Ishāq's *zīj* contains two tables of the partial solar equation. One of them ($\eta_{\max} = 1;49,7^{\circ}$) has the inscription '*alā ra'y Ibn Ishāq*' (according to Ibn Ishāq's opinion) which probably means that it was calculated in agreement with Ibn Ishāq's table of the "total" solar equation. Mestres has calculated that an eccentricity of 1;54,15^p corresponds to year 619 H./1222 (if the tables of the total solar equation have been used) and that, according to a marginal note in manuscript Escorial 909 of Ibn al-Bannā's *Minhāj*, 1222 is the year in which Ibn Ishāq made the observations of the planets on which his *zīj* is based.³⁵⁰ The second table ($\eta_{\max} = 1;49,[36],56,42^{\circ}$) has been calculated to the precision of thirds (or even fourths in some cases) and has the title of *Jadwal ākhar bi-ta'dīl al-shams al-murakkab li-Ibn Ishāq bi-naṣṣ al-Zarqālluh lā bi-jadwali-hi* (another table of the composite solar equation by Ibn Ishāq according to the text of Ibn al-Zarqālluh's [canons] and independent from his tables).

350 Vernet, 1952, p. 21.

3. Ibn al-Raqqām's table of the partial solar equation preserved in the *Shāmil* and the *Qawīm zīj* ($\eta_{\max} = 1;47,51^{\circ}$, $e = 1;52,55^p$) may have been calculated for 689 H./1290, a year which fits his chronology well (d. 1315).
4. Ibn al-Bannā's *Minhāj* has no table of the partial solar equation and the canons only describe the use of the table of the total equation.³⁵¹ We can, however, obtain the value of the maximum solar equation that he used from the table of Venus's equation of the centre, where the maximum tabular value is $-2;9^{\circ}$. As the table is displaced by 4° , this corresponds to a maximum equation of $1;51^{\circ}$ ($e = 1;56,13^p$) which, if computed with Ibn al-Zarqālluh's solar model with variable eccentricity, corresponds to year 1113. If this proves to be true, Ibn al-Bannā' (1256–1321) probably copied this table from an older source.
5. Ibn 'Azzūz: $\eta_{\max} = 1;52^{\circ}$ derives from Ibn al-Kammād. Actually his table of the solar equation is the same as Ibn al-Kammād's by truncation of the seconds.
6. The versified *zīj* of al-Qusanṭīnī contains a solar equation table with a tabular maximum of $\eta_{\max} = 2;13^{\circ}$,³⁵² corrected by Kennedy & King into $2;14^{\circ}$,³⁵³ which derives from al-Khwārizmī's *zīj*.³⁵⁴

6.4 The Lunar Model

An interesting passage from Ibn al-Hā'im's *Kāmil zīj* contains the following information:

The apparent (*mar'ī*) mean motion of the Moon, in the tables of all *zīj*es, is calculated according to the assumption that the uniform motion of the centre of the lunar epicycle takes place around the centre of the ecliptic. When Abū Ishāq Ibrāhīm b. Yaḥyā, known as al-Zarqālluh – may God have mercy upon him – observed all the lunar motions in longitude and latitude, as well as its irregularities (*ikhtilāfāt*), and examined them in relation to many eclipses which took place in all parts of the ecliptic, during a long period of time of about thirty seven years, according to what we found [in a text] written by his own hand, [he reached results] which

351 Vernet, 1952, pp. 85–86 (Spanish tr.) and 29–30 (edition).

352 Ms. Escorial 909 fol. 56v.

353 Kennedy & King, 1982, p. 11.

354 Suter's edition (1914), pp. 132–137.

did not disagree at all with [what] the Ancients said on this topic, with the only exception of the mean [lunar] motion in longitude.

This means that he found that the mean position of the Moon, in the middle of eclipses, – according to what is required by observation and practice (*al-ʿamal*) – always [depends on the following considerations]: if the Moon advances from the solar apogee towards its perigee, its position is delayed in respect to the position calculated with tables, in the opposite sense to that of the zodiacal signs (*ilā khilāf al-tawālī*). If [however] the Moon moves from the [solar] perigee towards its apogee, [its position] is advanced in respect to the position calculated with tables, in the direction of the zodiacal signs (*ʿalā l-tawālī*). This advance and delay reaches a maximum of, approximately, twenty-four minutes [...],³⁵⁵ according to a measure in which ninety degrees corresponds to a right angle, and this takes place when [the Moon] is in quartile aspect to the [solar] apogee.

This obviously implies that the lunar mean motion in longitude does not really take place around the centre of the ecliptic, but around another centre which is away (*khārij*) from the centre of the ecliptic in the direction of the solar apogee and on the line that joins the solar apogee and its perigee.³⁵⁶

According to the description given by Ibn al-Hā'im and interpreted by Roser Puig,³⁵⁷ Ibn al-Zarqālluh's lunar model is as shown here:

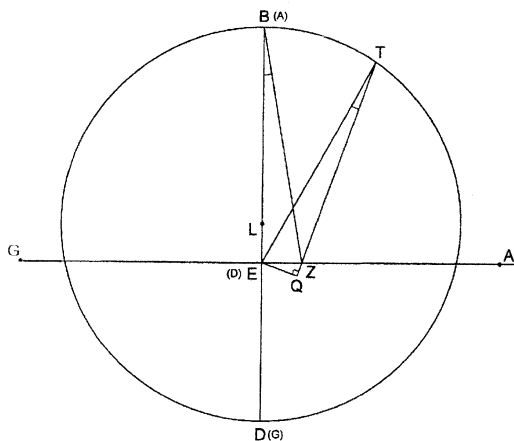


FIGURE 6.16
Ibn al-Zarqālluh's lunar model,
borrowed from R. Puig

355 Illegible word in the manuscript.

356 Arabic text edited by Puig, 2000 (see pp. 91–92).

357 Puig, 2000, pp. 92–93 (Arabic text) and 77–79 (Puig's interpretation).

In fig. 6.16 ABGD, with centre L, is the lunar deferent and diameter BLD is the lunar apsidal line. Point E is the centre of the ecliptic and AEG the solar apsidal line in which A points to the direction of the solar apogee. B corresponds to the position of the centre of the lunar epicycle in the syzygies, which are Ibn al-Zarqālluh's main interest in this case, as eclipses only occur in conjunctions and oppositions. Z is the centre of the lunar mean motion in longitude and, when we compute the lunar mean position with tables, the result will be angle $\angle BZA$ which should be corrected in order to obtain angle $\angle BEA$.³⁵⁸

When the centre of the lunar epicycle is in B we assume that $\angle BEA = 90^\circ$, and it is the true distance between the centre of the lunar epicycle and the solar apogee, while $\angle BZA$ is the distance obtained from the tables. It corresponds to the position in which the Zarqāllian maximum correction in mean lunar longitude is $\angle EBZ = 24' \approx EZ$.

Let us now consider that the centre of the lunar epicycle is point T. Ibn al-Hā'im joins T and E, as well as T and Z, and projects the perpendicular EQ from E to the prolongation of TZ. Now $\angle AET$ is the true distance between the centre of the lunar epicycle and the solar apogee ($\Delta\lambda_v$), while $\angle AZT$ is the distance obtained from the tables.

To obtain the value of angle $\angle ETQ$, the correction of the lunar mean position when the centre of the lunar epicycle is in T (c), Ibn al-Hā'im considers the small triangle $\triangle EQZ$ in which angle $\angle Q$ is right, $\angle EZQ$ is known, because it equals $\angle AZT$ and $EZ = 24'$. Consequently $\angle ETZ \approx EQ$ is also known.

Although Ibn al-Hā'im does not explain how to calculate EQ, he is probably applying the sine law:

$$\sin \angle EZQ / EQ = 60 / EZ$$

Consequently the correction ($c = \angle ETZ \approx EQ$):

$$c = 0;24^\circ / 60 \cdot \sin \angle AZT = 0;0,24^\circ \cdot \sin \angle AZT$$

and

$$\angle TEZ = \angle TZA - c$$

³⁵⁸ In Ibn al-Hā'im's terminology, $\angle BEA$ is called "the mean observed position" (*al-wasaf al-marṣūd*), while $\angle BZA$ is "the mean position obtained from the tables" (*al-mukharraj min al-jadāwil*).

Ibn al-Zarqālluh's solution reminds me of the "solution by sines" frequently applied in Indian astronomy.³⁵⁹ Besides, the Zarqāllian correction seems to imply the introduction of a kind of rotating equant point for the lunar mean motion, as the centre of such motion is placed on the solar apse line and it will rotate with the motion of the solar apogee.

Several sources explain the Zarqāllian procedure for correcting the lunar mean position. In Ibn al-Kammād's *Muqtabas zīj*, for example, we read:

Quod si volueris locum Lune verum pro eclipsi minue augem solis a centro et quod remanserit fac illud cordam et multiplica eum in duobus quintis unius minuti et duo quinti unius minuti sunt 24 secunda et divide quod inde colligetur per 60.³⁶⁰

where we see that Ibn al-Kammād is following al-Battānī in his use of *corda* (Ar. *watar*) to denote the sine and that at the end he adds a division by 60, which would be correct if the amount to be multiplied had been 0;24° instead of 0;0,24°. The same instructions, without Ibn al-Kammād's unnecessary division by 60, are given in the Tunisian recension of Ibn Ishāq's *zīj*,³⁶¹ and in Ibn al-Raqqām's *Shāmil zīj*. The latter reproduces Ibn al-Hā'im's text without the reference to Ibn al-Zarqālluh and without the geometrical proof.³⁶² Finally a similar correction appears in the Castilian canons of the Alfonsine Tables although, in this case, 0;0,24° is replaced by 0;0,5,30° (7.6.2.2.1.1).³⁶³

Three *zījes* belonging to Ibn Ishāq's tradition add tables giving the value of *c* using as argument the difference between the lunar mean position for a given moment and the position of the solar apogee: Ibn al-Bannā's *Minhāj*,³⁶⁴ Ibn 'Azūz's *Muwāfiq* (*Jadwal al-ṣarf*) and Ibn al-Raqqām's *Mustawfī* (*Jadwal ta'dīl al-qamar bi-bu'di-hi min awj al-shams*).³⁶⁵ Table 6.32 contains a sample of this table, which is the same in the three sources.³⁶⁶

359 Kennedy & Muruwwa, 1983.

360 Quoted by Puig, 2000, p. 79; ms. Madrid BN fol. 8r.

361 Mestres, 1999, pp. 50 (of the introduction) and 73–74 of the Arabic edition of the canons.

362 'Abd al-Rahmān, 1996, p. 65.

363 Chabás & Goldstein, 2003, pp. 37–38, 156.

364 Vernet, 1952, pp. 88–89 (of the Spanish translation) and 32 (of the Arabic edition).

365 Samsó, 2014, pp. 302 and 316.

366 I am using ms. Madrid Museo Naval without number, fol. 23r (*Minhāj*), Rabat NL 2461 (p. 391) & Rabat Ḥasaniyya 8772 (p. 45) (*Muwāfiq*) and ms. Tunis NL 11018 fol. 104r (*Mustawfī*).

TABLE 6.32

$\Delta\lambda_t$	$0;0,24^\circ \cdot \text{Sin } \Delta\lambda_t$	$\Delta\lambda_t$	$0;0,24^\circ \cdot \text{Sin } \Delta\lambda_t$
5°	0;2,6 ^{oa}	50°	0;18,23°
10	0;4,10	55	0;19,40 ^{ob}
15	0;6,13	60	0;20,47
20	0;8,12 (−1 ^o)	65	0;21,45 ^c
25	0;10,9 ^d	70	0;22,33 ^e
30	0;12,0	75	0;23,11 ^f
35	0;13,46	80	0;23,38
40	0;15,26	85	0;23,55 ^g
45	0;16,58	90	0;24,0

- a 0;2,5° in Ibn al-Bannā'.
b 0;19,39° in Ibn al-Raqqām.
c 0;21,46° in Ibn al-Bannā'.
d 0;10,8° in Ibn al-Raqqām.
e 0;22,32° in Ibn al-Raqqām.
f 0;23,10° in Ibn al-Raqqām.
g 0;23,54° in Ibn al-Raqqām.

With the aim of trying to understand why Ibn al-Zarqālluh introduced this correction in the lunar mean positions for the computation of eclipses, I have calculated the position of the Sun, Moon and lunar nodes for the date and hour in which a few total solar and lunar eclipses took place, using a computer programme created by Josep Casulleras with the parameters of Ibn Ishāq's tradition, which are basically Zarqāllian. The date and hour of maximum totality have been obtained in the NASA catalogue of eclipses between −1999 and 3000 (<https://eclipse.gsfc.nasa.gov>). In tables 6.33 and 6.34 I give examples of eclipses of the second half of the eleventh century which could have been observed by Ibn al-Zarqālluh himself, either in Toledo or in Córdoba. I have not introduced a correction between the GMT used by the aforementioned catalogue and the local time of Toledo or Córdoba.

TABLE 6.33 Total solar eclipse of 1 July 1079 visible in the south of the Iberian Peninsula

Date	Hour of max. eclipse	Computed solar long.	Computed lunar long.	Computed long. of the node
1.7.1079	14 h.	104;31,37°	104;53,21°	280;40,6°

TABLE 6.34 Examples of total lunar eclipses in the second half of the eleventh century

Date	Hour of max. eclipse	Computed solar long.	Computed lunar long.	Computed long. of the node
8.12.1052	22;45 h.	263;20,14°	83;52,40°	74;44,34°
3.3.1067	23;42 h.	348;45,40°	169;48,45°	159;16,42°
7.10.1074	21;44 h.	200;20,19°	20;6,11°	12;33,33°
30.1.1078	20;42 h.	316;55,11°	137;44,46°	308;3,5°

The results obtained with these samples show a good approximation to the lunisolar conjunction and oppositions and a distance from the lunar node which allows the possibility of an eclipse.

6.5 Conclusions

The contents of this chapter could have been included in chapter 7, which deals with *zīj*es. However, I prefer to give them an independent treatment because the original sources (both extant and non-extant) written by Ibn al-Zarqālluh were not astronomical tables, although they exerted a strong influence in later Andalusī and Maghribī *zīj*es until the beginning of the fifteenth century. Trepidation theory is undoubtedly Eastern in origin but its mathematical treatment seems to be the result of Ibn al-Zarqālluh's research as well as, probably, the research of his colleagues in the Toledan group (*al-jamā'a al-ṭulayṭulīyya*, in Ibn al-Hā'im's terminology) led by *qāḍī* Šā'id. As for the mean motion of the solar apogee, the solar model with variable eccentricity, and the small correction to Ptolemy's lunar model, they all seem to be original and to derive directly from Ibn al-Zarqālluh himself.

Trepidation and Ibn al-Zarqālluh's treatment of the solar model are the result of a careful analysis of historical observations known in al-Andalus in the eleventh century, as well as of his own work. His observations were recorded for periods of 25 years (for the Sun) and 37 years (for the Moon) but not a single detail about them has reached us. It would be extremely interesting to know precisely how Ibn al-Zarqālluh established the solar apogee's own motion (1° in 279 years); he believed this value to be extremely reliable, as he used it to obtain more precise values of the longitudes of Regulus, in historical times, which were the basis of his trepidation models. Information on his observations of eclipses would help us to understand his lunar correction about which we know very little at present.

Ibn al-Zarqālluh's new models had a clear influence on the development of Maghribī astronomy, although his parameters were the object of small corrections in an attempt to adjust the computational results to the reality observed. They were also influential in Latin European astronomy, though to a lesser degree. In the Mashriq, Ibn al-Zarqālluh's models were far less successful than other contributions of his such as the universal astrolabe (*ṣafīḥas shakkāziyya* and *zarqāliyya*), and in fact they remained practically unknown.

Astronomical Tables (*zīj*es)

7.1 Introduction: the Eastern Input in al-Andalus

Eastern *zīj*es seem to have been introduced in al-Andalus towards the beginning of the ninth century. We have already seen (1.5.1.2.2.1) that the historian Ibn Ḥayyān gives information about the books brought to Córdoba from Baghdad by the astrologer ‘Abbās b. Nāṣih (d. after 844) which includes a list of presumably astronomical tables called *al-Zīj*, *al-Qānūn*, *al-Sindhind*, and *al-Arkand*.¹ The *Sindhind* was probably al-Khwārizmī’s *Zīj*, whose use in al-Andalus is well documented in the tenth century, although one cannot rule out other possible sources (al-Fazārī, Ya‘qūb b. Ṭāriq); *al-Qānūn* refers probably to the *Handy Tables*, and *al-Arkand* is another set of Indian astronomical tables introduced in Baghdad in the eighth century. The reference to *al-Zīj* seems to be too vague to allow an identification. Whatever the case, it seems clear that the court-astrologers of ‘Abd al-Rahmān II (821–852) needed this kind of astronomical literature to cast their horoscopes and, also in 1.5.1.2.2.1, I mentioned that Ibn Ḥayyān quoted a poem dedicated to the emir Muḥammad (852–886) by ‘Abbās b. Firnās in which he complained that Ibn al-Shamir (or Shimr), another court astrologer, had had in his possession a book referred to as *al-daftar al-muḥkam* which ‘Abbās wanted to use.² This book probably contained another set of astronomical tables.³

Al-Khwārizmī’s *zīj* was, obviously, the collection of astronomical tables most commonly used in al-Andalus and it was the object of a serious revision made by Maslama al-Majrīṭī (see 7.2.1.2.2). This source seems to have been the cause of the predominance of sidereal astronomical tables both in al-Andalus and in the Maghrib, and its influence was strengthened by the circulation of other sidereal *zīj*es like Ibn al-Ādamī’s *Kitāb naẓm al-‘iqd* (published in 949) which, as we have seen (6.2.1), was the source of Šā‘id’s understanding of trepidation theory.⁴

1 Makkī, 2003, pp. 278, 525–527; Spanish translation in Makkī & Corriente, 2001, pp. 169–170. See also Forcada, 2004–2005, pp. 20–22.

2 Makkī, 1973, pp. 281–282.

3 Vernet, 1977, pp. 417–418.

4 Šā‘id, in Bū ‘Alwān, 1985, pp. 146–147; Blachère, 1935, p. 114.

Al-Battānī's (d. 929) *zīj* was also known in al-Andalus in the second half of the tenth century, and Maslama compiled a non-extant book in which he summarised the method to compute planetary positions according to al-Battānī's tables.⁵ Besides, Nallino's edition of this *zīj* contains half a dozen tables attributed to Maslama,⁶ although the attribution is not certain. This *zīj* represented the introduction in al-Andalus of Ptolemaic astronomy and it exerted a strong influence on the evolution of Andalusī astronomical tables, having been translated into Castilian at the court of Alfonso x.⁷

A third Eastern *zīj* which appeared in al-Andalus and the Maghrib is the *al-Zīj al-Mumtaḥan*, compiled by Yaḥyā ibn Abī Maṣṣūr on the basis of the observations made in Damascus and Baghdad ca. 830, under the patronage of caliph al-Ma'mūn,⁸ although, in this particular case, the references to *Mumtaḥan* could correspond to any of the *zīj*es calculated using these observations. The evidence is less clear for the circulation of sources of this kind. In 4.1.3.2.2 we saw that the Tunisian scholar Dūnash ibn Tamīm refers to the *Mumtaḥan* tables in the context of the computation of solar longitude. Another interesting source is the *Calendar of Córdoba* edited together with Gerard of Cremona's Latin translation by Dozy and Pellat.⁹ This text is the result of the fusion of an *anwā'* book due to 'Arīb b. Sa'īd (d. ca. 980) and a saints' calendar written by the Mozarab bishop of Iliberis Rabī' b. Zayd, compiled for the caliph al-Ḥakam II al-Mustanṣir (961–976). The former source has been identified¹⁰ with the text preserved in ms. Malik Milli 2049 in which the name of the author appears as *al-Kātib al-Andalusī* and the book seems to have been written in 973.

Both the *Calendar of Córdoba* and 'Arīb's *Kitāb al-anwā'* give the date of the entrance of the Sun in the twelve zodiacal signs according to the *Sindhind* and the *Mumtaḥan* traditions. These days appear in table 7.1 according to the printed edition of the *Calendar* and, in the cases in which there is disagreement, according to 'Arīb's *anwā'* book, they are separated by a /.¹¹ I have recalculated the solar positions for the day of each month of year 973 in which the Sun enters each sign according to the *Calendar*, using the parameters of al-Khwārizmī's

5 Ṣā'īd, in Bū 'Alwān, 1985, p. 169; Blachère, 1935, p. 130.

6 See, for example, Nallino, 1899–1907, vol. II, pp. 300–1.

7 Bossong, 1978.

8 Dalen, 2004.

9 Dozy & Pellat, 1961.

10 Forcada, 2000.

11 I have used an unpublished edition of ms. Milli 2049, prepared by Ahmad Alkuwayfi and sent to me by Miquel Forcada. My gratitude to both of them.

Sindhind zīj, Yahyā b. Abī Maṣṣūr’s *Mumtaḥan* zīj and al-Battānī’s zīj.¹² This latter zīj has been taken into consideration due to the fact that the Latin translator adds to the reference of the *Mumtaḥan* values the clarification “et est equatio Albateni”. In this table, the calculated solar positions that seem to disagree with the text values are marked with an asterisk (*).

TABLE 7.1 The Sun in the beginning of the zodiacal signs in the *Calendar of Córdoba*

Months	Solar positions	Dates <i>Sindhind</i>	Calc.solar positions for midday (year 973) (<i>Sindhind</i>)	Dates <i>Mumtaḥan</i>	Calc.solar positions for midday (973) (<i>Mumtaḥan</i>)	Calc.solar positions for midday (973) (Battānī)
January	Aqu 0°	20	299;42°	14	300;13°	300;16°
February	Pisc 0°	19/17	329;56°/ 327;56°(*)	13	330;26°	330;28°
March	Ari 0°	20	358;38° (*)	16	1;9° (*)	1;10° (*)
April	Tau 0°	?/20	/28;42°(*)	15	30;22°	30;22°
May	Gem 0°	22	59;15°	16	60;8°	60;7°
June	Canc 0°	23	89;36°	18	91;36° (*)	91;35° (*)
July	Leo 0°	24/22	119;7°/ 117;13° (*)	17/18	119;18°/ 120;15°	120;26°/ 120;14°
August	Virg 0°	23/22	148;4° (*)/ 147;5°(*)	18	150;10°	150;9°
September	Libr 0°	23	178;34° (*)	18	180;33°	180;34°
October	Sco 0°	22	207;40° (*)	17	209;28°	209;31°
November	Sagitt 0°	19	236;8° (*)	16/17 ^a	239;50°/ 240;51°	239;53°/ 240;54°
December	Capr 0°	21/20	268;51° (*)/ 267;50°(*)	17 ^b	271;27° (*)	271;31° (*)

a ‘Arīb b. Sa’īd adds here *wa huwa al-ta’dil al-ṣaḥīḥ* (“and this is the correct computation”).
b ‘Arīb b. Sa’īd gives this date for the entrance of the Sun in Capricorn, but notes that 15 December is the day of the winter solstice. For this date I obtain a solar position of 269;25° according to the *Mumtaḥan* and 269;28° with al-Battānī’s parameters.

12 I have used the computer programmes khwrln, plhrsc (for *Mumtaḥan*) and batln, designed by the late Prof. E.S. Kennedy and later revised by Honorino Mielgo.

A brief glance at table 7.1 shows that 1) the recomputation with the parameters of al-Khwārizmī's *zīj* is much worse than that obtained with the *Mumtaḥan* and al-Battānī's *zīj*; and 2) for our purposes, the results obtained with Yaḥyā ibn Abī Maṣṣūr's and al-Battānī's *zīj* are equivalent. This means that Gerard of Cremona's identification of the source with al-Battānī's *zīj* could be correct and that we might find here something that is quite common in later Maghribī sources in which *Mumtaḥan* means "tropical" astronomical tables, while *Sindhind* is equivalent to "sidereal" tables.

An important paper published by Mercè Viladrich some years ago points in a different direction.¹³ She used a table preserved in the two manuscripts of Ḥabash al-Ḥāsib's *zīj* which gives the mean solar positions in the moment in which the true Sun enters the zodiacal signs, as well as the values of the solar equation according to the parameters mentioned in the Ashrafi *zīj* for the *Mumtaḥan* tables, and the solar equation table in Yaḥyā ibn Abī Maṣṣūr's *zīj*. Using these tables, she tried "to find a specific year in which, according to the *Mumtaḥan* values, the dates of the entrance of the Sun in the zodiacal signs would either nearly coincide with the dates of the *Calendar of Córdoba*". Her conclusion was that "the authors of the *Calendar of Córdoba* were using the results of a computation, based on *Mumtaḥan* solar parameters, made towards the middle of the ninth century".¹⁴

It seems, therefore, that the *Calendar of Córdoba* does not provide sufficient evidence of the circulation of the *Mumtaḥan zīj* in tenth-century al-Andalus. It seems quite possible that the author of the calendar copied a table in which these positions had already been calculated one century earlier. Much later, however, it is clear, as shown by Chabás and Goldstein, that the *Mumtaḥan zīj* was well known to Ibn al-Kammād (fl. 1116) who, in the canons of the *Muqtabas zīj*, gives a maximum latitude of Venus of 8;36° (Yaḥyā b. Abī Maṣṣūr 8;56°, the *abjad* notation for 30 and 50 being easy to confuse) and one of 4;18° for Mercury, as in Yaḥyā's *zīj*;¹⁵ his *Tabula rectitudinum ad eclipses solares*, which computes the declination of midheaven as a function of the longitude of the ascendant, is closely related to Yaḥyā's *Jadwal al-samt li-ʿilm kusūf al-shams*, and the same table reappears in the *Barcelona Tables*;¹⁶ Ibn al-Kammād's *Tabula latitudinis solis* (difference between the lunar and the solar parallax in latitude, with a maximum of 0;48,32°), also in the *Barcelona Tables*, shares the

13 Viladrich, 1996.

14 Viladrich, 1996, pp. 260–261.

15 Chabás & Goldstein, 1994, p. 32 and 2015 a, pp. 616–7.

16 Chabás & Goldstein, 1994, pp. 14–17.

same errors with Yaḥyā's *Jadwal 'arḍ al-shams*.¹⁷ Other coincidences are the table of lunar latitude (max. 4;29°, Yaḥyā 4;30°),¹⁸ the table of solar eclipses which seems to derive from Yaḥyā,¹⁹ and an obliquity of the ecliptic of 23;33°, traditionally associated with the *Mumtaḥan* observations.²⁰ As a matter of fact this value of the obliquity is also attributed to Yaḥyā in the *Toledan Tables*, although Ibn al-Zarqālluh states that he confirmed it by observation.²¹

The *Mumtaḥan* tradition was also accessible to Abraham ibn 'Ezra in the twelfth century since, in his *De rationibus tabularum*, he gives a list of the *magistri probationum* (probably a translation of *aṣḥāb al-mumtaḥan*)²² which includes direct collaborators of caliph al-Ma'mūn such as Yaḥyā b. Abī Maṣṣūr and Khālid b. 'Abd al-Malik al-Marwarrūdī (fl. 800–850), as well as later scientists who used the results of the *Mumtaḥan* observations like the brothers Banū Mūsā b. Shākir (fl. 850–875), Thābit b. Qurra (d. 901) and al-Battānī. The problem lies in the fact that he also mentions Ibn al-Zarqālluh as one of the *magistri probationum* and this led Vernet²³ to interpret this expression as a reference to al-Ma'mūn's astronomers as well as to later scientists who computed tables based on original observations. Whatever the case, it is clear that the sources mentioned by Ibn 'Ezra correspond both to scholars who were well known in al-Andalus and to others whose circulation in this country is not documented, among whom we should mention 'Abd al-Raḥmān al-Šūfī, author of a *zīj*, lost in its Arabic original version, but preserved in Ibn 'Ezra's *Tabulae Pisanae*.²⁴ It seems, therefore, that the *De rationibus* cannot be considered reliable evidence of the knowledge, in al-Andalus, of the *Mumtaḥan* tables and that Ibn 'Ezra's knowledge of these sources could have been the result of his travels in Italy, France and, perhaps, England. His stay in Pisa (ca. 1142–45), where he compiled his *Tabulae Pisanae*, may have given him access to sources unknown in the Iberian Peninsula, due to the existence of a connection between Antioch and Pisa which could have favoured the arrival of Eastern scientific sources in the latter city.²⁵

17 Chabás & Goldstein, 1994, pp. 19–20.

18 Chabás & Goldstein, 1994, pp. 20–22.

19 Chabás & Goldstein, 1994, p. 23.

20 Chabás & Goldstein, 1994, pp. 29–30 and 2015 a, p. 587.

21 Pedersen, 2002, II, 410–11 and 612–13, where we find the remark: “quae [i.e. Yaḥyā's value] apud nos ducitur verior, quia primam novimus rumore, et hanc didicimus per considerationem” (“and among us this is considered truer, since we know the former from hearsay but have learnt the latter from observation”).

22 Millás, 1947 a, p. 76. On this topic see Samsó, 2012.

23 Vernet, 1956.

24 Mercier, 1987; Mercier, 1991.

25 Burnett, 2000 and 2003 a.

To sum up: al-Khwārizmī and al-Battānī seem to be the main Eastern contributors in Andalusī astronomical tables, and the influence of the *Mumtaḥan zīj* is attested only in Ibn al-Kammād's *Muqtabas zīj*.²⁶

7.2 A General Survey of Andalusī and Maghribī *zīj*es

7.2.1 *Al-Khwārizmī – Maslama*

7.2.1.1 The Sources

Al-Khwārizmī's original *zīj*, compiled in the first half of the ninth century, has not been preserved although we know that it might have been introduced in al-Andalus at an early date. In the tenth century it was the object of several recensions made by Maslama al-Majrīṭī (d. 1007) and by his disciples Ibn al-Ṣaffār (d. 1035) and Ibn al-Samḥ (d. 1035). Only a small part of Ibn al-Ṣaffār's recension (seven chapters) has been preserved in Arabic, written in Hebrew script, in ms. Paris BN heb. 1102.²⁷ We have no information about Ibn al-Samḥ's recension, although Maslama's has been preserved in two Latin translations made by Adelard of Bath (fl. 1116–1142) ca. 1126²⁸ and Petrus Alfonsi.²⁹

As Maslama made an important revision of the *zīj*, it is interesting that, according to Raymond Mercier, ms. Paris BN lat. 16208 contains another Latin version, independent from the two just mentioned and calculated for the meridian of Toledo, that could correspond to Khwārizmī's pre-Maslamian version³⁰ and that might, perhaps, be identified with Hermann of Carinthia's (ca. 1140) lost translation. This has been clarified by the critical edition (based on Paris 16208 as well as on four other mss) published by Fritz Pedersen,³¹ who has concluded that it is the most archaising version (*Cc* in Pedersen's terminology) of the three series of canons of the Toledan Tables. Part A of this text agrees perfectly with Ibn al-Muthannā's commentary (see below), which implies that this is a remnant of the pre-Maslamian al-Khwārizmī's *zīj*, while part B contains a series of additions which derive directly from al-Battānī.

26 Much later (ca. 1280), the anonymous compiler of Ibn Ishāq's Tunisian recension also uses the *Mumtaḥan Zīj* for astrological computations like that of the *tasyīr* of the sign of the *intihā'* and the ascendant of an anniversary horoscope: see Mestres, 1999, pp. 135–137, 150.

27 Ed. and comm. in Castells & Samsó, 1995.

28 Ed in Suter, 1914. Tr. and comm. in Neugebauer, 1962. A systematic revision of the contents of the *zīj* in Dalen, 1996. See also Pingree, 1996, Kennedy, 1956, pp. 26–29, 48–51 of the reprint (Philadelphia, 1989).

29 See Neugebauer, 1962, pp. 133–234; Casulleras, 1996 a.

30 Mercier, 1987, p. 101.

31 Pedersen, 2002, vol. II, pp. 571–736. This was preceded by another edition: Pedersen, 1992.

The seven extant chapters of Ibn al-Ṣaffār's recension contain the oldest Khwārizmian source preserved in Arabic. According to Ṣā'id's *Ṭabaqāt*,³² the title of this recension was *Zīj mukhtaṣar 'alā madhhab al-Sindhind* ("an abridged *zīj* according to the Sindhind school") and this is probably the work quoted by Ibn 'Ezra in the *De rationibus*,³³ who states that it was Ibn al-Ṣaffār who replaced the Persian calendar and the Yazdijird era, used by al-Khwārizmī, with the Muslim calendar and the Hijra era which appears in the Latin version of Adelard of Bath, although Ṣā'id attributed this change to Maslama.³⁴ Also according to Ibn 'Ezra, Ibn al-Ṣaffār corrected the position of the apogee of Saturn from $8^{\circ} 20';55''$ to $8^{\circ} 4';55''$, which he considered to be the correct longitude in al-Khwārizmī's *zīj*, stating that the translator (?)/ copyist (*ipsum qui transtulit*) had been misled by the similarity of the form of the 20 (*kāf*) and the 4 (*dāl*). Millás remarked that the confusion is more likely to occur in Hebrew than in Arabic script and, consequently, this could be Abraham ibn 'Ezra's own explanation. It is clear now that this correction was an error made by Ibn al-Ṣaffār,³⁵ which appears in Adelard of Bath's Latin translation.³⁶

Ibn al-Ṣaffār's extant chapters deal with the computation of lunar eclipses (chapter 33 of Adelard of Bath's canons which has strong similarities with a chapter of Ibn al-Zarqālluh's *Almanac*³⁷); the determination of the longitude of a locality from the base meridian of the centre of the Earth (with no equivalent in Adelard's translation); on this point the text remarks (see 7.2.1.2.2) that the time difference between Córdoba and Arin is 4;12 h; the computation of the lunar parallax in longitude and latitude (chapter 34 of Adelard's canons); solar eclipses (with a treatment which is more complete than what we find in Adelard's chapter 35); equation of time (with no exact parallel in Adelard's translation) and, finally, the determination of the ascendant and of the twelve astrological houses (also missing in Adelard's canons).

Adelard's translation is our most complete source. It was compiled ca. 1126 and, although there is no evidence that he ever visited the Iberian Peninsula (his knowledge of the Arab-Islamic world was restricted to southern Italy and Sicily, as well as Palestine) it is obvious that it was based on Maslama's recension of al-Khwārizmī's *zīj*. We may wonder how he obtained access to this Andalusī source. It seems that his translation was made in the west of England, where he stayed in the 1120s and had contact with two Lorraine clerics: Robert

32 Ṣā'id, in Bū 'Alwān, 1985, p. 171; trans. in Blachère, 1935, p. 131.

33 Millás, 1947 a, pp. 75, 109–10, comm. in pp. 23, 50–1.

34 Ṣā'id, in. Bū 'Alwān, 1985, p. 169; trans. in Blachère, 1935, p. 130.

35 See Pingree, 1968, p. 50; Mercier, 1987, p. 92.

36 Neugebauer, 1962, pp. 41, 99.

37 Millás, 1943–50, pp. 109–111 and 143–145.

Losinga, Bishop of Hereford, and Walcher, Prior of Malvern.³⁸ The latter was a student of Petrus Alfonsi/Alfonsus,³⁹ previously named Mosheh Sephardi, a Jewish scholar converted to Christianity and baptised in 1106 in the Cathedral of Huesca. King Alfonso I (1104–1134) of Aragón was his godfather, and this is why he adopted Alfonsus as his second name. Later he migrated to England where he became the royal physician of King Henry I (1100–1135) and was the predecessor of two more Jewish scholars, Abraham bar Ḥiyya and Abraham ibn ‘Ezra, in disseminating the new Arabic science in Europe. Around 1116, Petrus Alfonsi had prepared an unsuccessful first translation of al-Khwārizmī’s *zīj*, in which he had tried to adapt the mean motion tables to the Julian calendar, adjusted its radices to 1 October 1116 and, strangely, used 30-year periods for his tables for collected years. These periods are commonly used with a lunar calendar but cannot be used with a solar Julian calendar which requires periods containing a number of years that is a multiple of 4.

Scholars seem to doubt Petrus Alfonsi’s capacity to write in Latin [the same can be said of Abraham ibn ‘Ezra’s Latin works (4.2.4.2.1 and 6.2.7.1)], and have remarked that he uses a technical terminology also found in Bede’s chronological writings. This has led Charles Burnett to suggest the existence of a collaborator and a process of translation in two steps, also used by the Toledan translators: from Arabic to a common language (probably some kind of Romance) and a retranslation into Latin. The obvious choice is Walcher of Malvern, whose connections with Petrus Alfonsi are clearly documented.⁴⁰ The same could be said of Adelard’s knowledge of Arabic: it has been suggested that he had learnt spoken Arabic but that he could not read an Arabic text and, once more, a collaborator was needed and a similar translation technique could have been used. This time the collaborator could have been Petrus Alfonsi himself: he had awakened the interest in al-Khwārizmī’s *zīj*,⁴¹ had a copy of Maslama’s recension, and could have produced an oral translation which would be retranslated into Latin by Adelard of Bath.

In addition to these sources, one should remember that al-Khwārizmī’s *zīj* was the object of several commentaries by al-Farghānī, Ibn Masrūr and Ibn al-Muthannā and that the latter’s commentary circulated in Spain where it was the object of a Latin translation by Hugo Sanctelliensis⁴² and of two Hebrew versions, one of them by Abraham ibn ‘Ezra.⁴³ We have practically no

38 Gibson, 1987.

39 Millás, 1943 a.

40 Burnett, 1996; Burnett, 1990, pp. 103–106.

41 Mercier, 1987, pp. 99–100.

42 Millás Vendrell, 1963.

43 Goldstein, 1967.

information on this author apart from his full name (Aḥmad ibn al-Muthannā ibn ‘Abd al-Karīm) and the title of his book (*Ta’līl zīj al-Khwārizmī*); I believe that he was an Eastern author, as he is mentioned by Šā‘id al-Andalusī among Eastern astronomers of the late ninth century and first half of the tenth.⁴⁴

7.2.1.2 Materials in al-Khwārizmī’s *zīj*

7.2.1.2.1 *Indian and Persian Materials*

7.2.1.2.1.1 Introduction

According to Šā‘id, al-Khwārizmī “computed mean motions following the *Sindhind* but departed from it in the equations and the [solar] declination. His equations were based on the methods (*madhāhib*) of the Persians, while for the solar declination, he followed the school of Ptolemy”.⁴⁵ These remarks are basically correct: al-Khwārizmī’s *zīj*, as it has reached us, contains three different kinds of materials which correspond to the Indo-Iranian, Graeco-Arabic and Iberian traditions. A state of the art of the whole set of tables and a classification of them according to their origins has been made by van Dalen.⁴⁶ The mean motion tables are based on Indian parameters and derive from Brahmagupta’s (seventh century) *Brāhmasphuṭasiddhānta*, in the version transmitted by the Eastern astronomer al-Fazārī (ca. 750), as has been proved by Mercier.⁴⁷

7.2.1.2.1.2 The Sun and the Moon

The equation tables seem to be of Iranian origin, in agreement with Šā‘id’s remark. The maximum value of the solar equation is $2;14^\circ$ and it is attested in the Indian *Khaṇḍakhādya* as well as in the Iranian *Zīj al-Shāh*. This maximum value (tables 21–26)⁴⁸ corresponds to an argument of 90° , which implies that it was not calculated using a Ptolemaic method but according to the Persian “method of declinations”. For $e(c_m)$ = solar equation for a mean solar centre c_m (the difference between the mean solar longitude and the longitude of the apogee), $\delta(c_m)$ the corresponding solar declination and ε the obliquity of the ecliptic, here assumed to be a Ptolemaic $23;51^\circ$, we have:

44 Šā‘id, in Bū ‘Alwān, 1985, p. 146; tr. Blachère, 1935, p. 113.

45 Šā‘id, in Bū ‘Alwān, 1985, p. 132; tr. Blachère, 1935 pp. 102–3.

46 Dalen, 1996: see a summary of this classification in pp. 210–211.

47 Mercier, 1987, pp. 90–92.

48 Tables 21–26 (Suter, 1914, pp. 132–137) contain the tables of the solar and lunar equation, the solar declination (for a Ptolemaic maximum of $23;51^\circ$) and the lunar latitude. See Neugebauer, 1962, pp. 95–98.

$$e(c_m) = (e_{\max} / \varepsilon) \cdot \delta(c_m) = (2;14^\circ/23;51^\circ) \cdot \delta(c_m)^{49}$$

The Moon is more interesting for our purposes. The geometric model is either an eccentric or a concentric deferent with epicycle, with only one equation which is a function of the lunar anomaly (α_m). The maximum equation is $e_{\max} = 4;56^\circ$, as in the *Khandakhādya* and in the Iranian *Zīj al-Shāh*, for an argument of 90° and, once more, the equation table (tables 21–26) has been calculated using the “method of declinations”, $\delta(c_m)$ being the solar declination computed in column 5 of this table and α_m the lunar mean anomaly:

$$e(\alpha_m) = (e_{\max} / \varepsilon) \cdot \delta(c_m) = (4;56^\circ/23;51^\circ) \cdot \delta(c_m)$$

One should remark, first of all, that $4;56^\circ$ reappears as the maximum lunar equation of anomaly in the Latin *Alfonsine Tables*.⁵⁰ Besides, a simple lunar model, which uses an eccentric circle, appears in the first approximation to the lunar model in Ibn al-Samh’s equatorium (4.5.2).⁵¹ To justify this choice, the Alfonsine translation of Ibn al-Samh’s treatise on the equatorium refers to Ptolemy’s first lunar model as well as to Indian astronomers (“et segun ob-rauan los sabios de la India”) and adds that it gives adequate results for lunisolar conjunctions and oppositions but not for quadratures. The same simplified procedure appears, as an appendix to the Ptolemaic method, in canons *Ca* of the *Toledan Tables*.⁵²

Finally, something similar can be found in Ibn al-Zarqālluh’s *Almanac*. The Toledan astronomer was not aware of the existence of a lunar cycle allowing the computation of a perpetual set of lunar tables⁵³ and tried to simplify the calculation of the mean motions of the Moon in longitude (using the Metonic cycle of 19 years) and in anomaly (a cycle of 180 years), and added to this a standard set of tables of Ptolemaic equations.⁵⁴ This corresponds to what he calls “the *Mumtaḥan* method”. However, he also gives another approximate solution according to the ancient Indians, Persians and Greeks (*madhhab qudamā’ ahl al-Hind, wa l-Furs wa l-Rūm*) which consists of computing a single

49 Neugebauer, 1962, pp. 95–96; Kennedy & Muruwwa, 1958, pp. 118–9.

50 E. Poulle, 1984, p. 151; *editio princeps* (Ratdolt, 1483), e5v.

51 Comes, 1991, pp. 61–63.

52 Pedersen, 2002, vol. 1, 258.

53 This cycle (11325 days) seems to have been discovered in the first half of the fourteenth century and it was used by Ibn ‘Azzūz al-Qusanṭīnī (ca. 1340) in Morocco and by Jacob ben David Bonjorn in Perpignan (ca. 1361). See Chabás, 1988; Chabás, 1991; Samsó, 1997 pp. 86–88.

54 Millás, 1943–50, pp. 76–78, 117–9, 166–73.

lunar equation using, as argument, the mean lunar anomaly and obtaining the value of the equation of anomaly (c_4 in Neugebauer's terminology),⁵⁵ when the epicycle is at maximum distance from the observer. The technique is the same as in al-Khwārizmī's *zīj*, although Ibn al-Zarqālluh's table⁵⁶ derives from al-Battānī.⁵⁷

Finally the lunar latitude (β) has a maximum $i = 4;30^\circ$, which seems to be borrowed from Indian sources, and is calculated according to the precise relation:

$$\sin \beta = \sin i \cdot \sin \omega$$

in which ω is the argument of latitude (the distance of the Moon from the nearest node).

7.2.1.2.1.3 The Planets

7.2.1.2.1.3.1 *Introduction*

Al-Khwārizmī's planetary models are the result of an Indian adaptation of Greek pre-Ptolemaic planetary models which had neither equant nor a specific treatment for Mercury or the Moon. This adaptation aims to play about with the two equations (of the centre and of the anomaly) without the requirement of an eccentric deferent and, therefore, as if the centre of the epicycle rotated uniformly around the centre of the Earth: all this without losing precision in the final result of the computation. This is obtained by using an intricate set of computational procedures, explained by Neugebauer.⁵⁸ The refinements introduced led Kennedy to remark that "the inventor of the theory realised that the two equations are not independent; to make one affect the other he forced the planet's (deferent) apogee to oscillate with the period of the anomaly and an amplitude half that of the anomalistic equation".⁵⁹ To this, Mercier⁶⁰ has added that the procedure used "ensures that the final true longitude approximates significantly better to the equant model [...] than to a simple eccentric/epicycle model", and this could imply that the equant model preceded Ptolemy. All this will become clear by following the steps of the computation

55 Neugebauer, 1975, vol. 1, p. 94.

56 Millás, 1943–50, p. 173.

57 Millás, 1943–50, p. 118, fn. 1.

58 Neugebauer, 1962, pp. 23–30.

59 Kennedy, 1956, p. 26.

60 Mercier, 1987, p. 94.

which I will summarise at the end of this section. Let us begin, however, by saying something about the tables.

7.2.1.2.1.3.2 *Mean Motions and Positions at Epoch*

Like the mean motions for the Sun and the Moon, the planetary mean motions derive from Brahmagupta's *Brāhmasphuṭasiddhānta*, through al-Fazārī's revision. They are:⁶¹

Sun	0;59,8,10,21,31,6,37 ($\pm 0;0,0,0,0,4,29$)
Moon (longitude):	13;10,34,52,46,28,4°/ day ($\pm 0;0,0,0,0,0,7$)
(anomaly):	13;3,56,58,49,54,44,11°/day ($\pm 0;0,0,0,0,0,53$)
Saturn (long.):	0;2,0,22,57,34,57° ($\pm 0;0,0,0,0,0,7$)
Jupiter (long.):	0;4,59,9,8,39,16,4° ($\pm 0;0,0,0,0,0,1,30$)
Mars (long.):	0;31,26,28,6,46,6° ($\pm 0;0,0,0,0,0,6$)
Venus (anom.):	0;36,59,34,13,35,31° ($\pm 0;0,0,0,0,0,5$)
Mercury (anom.):	3;6,24,8,6,16,34° ($\pm 0;0,0,0,0,0,34$)
Asc. node:	0;3,10,48,22,58,9° ($\pm 0;0,0,0,0,0,5$)

The planetary positions for the beginning of the Hijra are:

Sun	83;25,48°
Moon	117;45,17
Saturn	117;58,37
Jupiter	330;10,49
Mars	210;25,15
Venus	46;0,3
Mercury	68;53,10

And the planetary apogees:

Sun	77;55°
Saturn:	244;55°
Jupiter:	172;32°
Mars:	128;24°
Venus:	81;15°
Mercury:	224;54°

61 Values established by Casulleras (1996 a, pp. 365–6) from the tables of collected years.

7.2.1.2.1.3.3 *Equations and Computation of the True Planetary Longitude*

As for the equation tables,⁶² after columns 1–2 for the arguments, we find the following functions:

Column 3 (c_3 , *sublimatio examinata/definita*). It calculates:

$$c_3 = \lambda_A - \frac{1}{2} c_4$$

in which λ_A is the longitude of the planetary apogee and c_4 the equation of anomaly

The equation of anomaly reaches value 0° for argument 180° and, consequently, $c_3 = \lambda_A$, which allows us to obtain directly the planetary apogees used in the table. One should remark here that, in contrast to most Islamic *zīj*es until ca. 1000, Venus's apogee does not coincide with the solar apogee (77;55°).⁶³ Besides, it is well known that Ptolemy's apogee (190°) for Mercury was inaccurate by about 30° in his own time.⁶⁴ Al-Khwārizmī's *zīj* gives a better value (224;54°) for the beginning of the Hijra (midday of 14 July 622).⁶⁵ although this does not seem to have affected the later development of Iberian and Maghribī *zīj*es which seem to have derived Mercury's apogees from Ptolemy's value.⁶⁶

Column 4 (c_4 , *examinatio argumenti/elheza* [*al-ḥiṣṣa*]):

It gives the equation of anomaly according to the simple eccentric model. A good approximation to the tabular values can be obtained with the expression:

$$\tan c_4 = r \sin \alpha / (R + r \cos \alpha)$$

62 Tables 27–56 in Suter, 1914, pp. 138–167; Neugebauer, 1962, pp. 98–103.

63 Mozaffari, 2019.

64 See Gingerich, 1993; Newton, 1977, pp. 278–279, reaches a similar conclusion when he says that the longitude of Mercury's apogee should be about 219° in Ptolemy's time instead of the 190° we find in *Almagest* 1x.7.

65 Neugebauer, 1962, pp. 41, 99; Mercier (1987, pp. 91–92) has proved the origin of this apogee longitude: with parameters of the *Brāhmasphuṭasiddhānta* he obtains 224;53,13° for the beginning of the Hijra.

66 A possible exception is Ibn al-Zarqālluh: Abū Bakr ibn Bājja ascribes to him a short non-extant treatise on the invalidity of the method used by Ptolemy to determine the position of Mercury's apogee (*Maqāla fī ibṭāl al-ṭariq allatī salaka-hā Baṭlimus fī istikhrāj al-bu'd al-ab'ad li-'Uṭārid*). Besides, Ibn al-Zarqālluh's *Almanac* uses a longitude of Mercury's apogee of 210° (instead of the Ptolemaic 190°). To this, one should add that Jābir ibn Aflaḥ severely criticised Ptolemy's method for Mercury, although he accepted its final results. See Samsó, 2001 b and 5.3.3.

in which R is the radius of the deferent, r the radius of the epicycle, and α the anomaly. The maximum values of c_4 and the radii of the epicycle⁶⁷ appear, according to al-Bīrūnī, in the *Zīj al-Shāh*⁶⁸ and these tables have, therefore, a Persian origin.⁶⁹

Column 5 (c_5 , *examinatio centri*, equation of the centre)

In this column, the maximum value of the equation is attained for an argument (c_m , the mean planetary centre) of 90° and the procedure used for the computation is the “method of sines”, in which:

$$c_5 = e \sin c_m$$

e being the eccentricity. The values of the eccentricities are obviously equal to those of the maximum equations and we can see that Venus’s eccentricity is the same as the solar one (even though the positions of their apogees are not equal). These maximum equations are the same as those of the *Zīj al-Shāh*, as documented by al-Bīrūnī, with differences of $1'$ (Saturn, Mars, Venus) or $2'$ (Mercury).⁷⁰ Table 7.2 summarises the values of the maximum equations and the radii of the epicycles:

TABLE 7.2 Solar, lunar and planetary equations

	Equations of the centre	Equations of anomaly	r (for $R = 60^P$)
Sun	2;14°	—	—
Moon	4;56°		
Saturn	8;36°	5;44°	6 ^P
Jupiter	5;6°	10;52°	11;20 ^P
Mars	11;13°	40;31°	39 ^P
Venus	2;14°	47;11°	44 ^P
Mercury	4;2°	21;30°	22 ^P

67 Neugebauer, 1962, p. 100.
68 With the sole exception of Mars, which is 41;30° instead of 40;31° in al-Khwārizmī’s *zīj*.
69 Kennedy, 1956, p. 50.
70 Kennedy, 1956, p. 50.

Following Neugebauer's analysis, the computation of the true longitude of a superior planet is made according to the following steps:

- 1) The mean anomaly of the planet

$$\alpha_1 = \lambda_s - \lambda$$

λ_s being the mean longitude of the Sun and λ the mean longitude of the planet

- 2) With α_1 as the argument we obtain the value of c_3 which, as we have seen is

$$c_3(\alpha_1) = \lambda_A - \frac{1}{2}c_4$$

- 3) Then we obtain a first value of the planet's centre (κ_1):

$$\kappa_1 = \lambda - (\lambda_A - \frac{1}{2}c_4) = \lambda - \lambda_A + \frac{1}{2}c_4$$

- 4) With κ_1 as argument we obtain $c_5(\kappa_1)$ and a corrected value of the centre (κ_2):

$$\kappa_2 = \kappa_1 + c_5(\kappa_1)$$

- 5) c_5 is also used to obtain a corrected value of the anomaly (α_2):

$$\alpha_2 = \alpha_1 - c_5$$

- 6) With α_2 as argument we obtain the value of c_4 which will be the equation of anomaly.

- 7) The final value of the centre (κ_3) (*al-ḥāṣil*) will be:

$$\kappa_3 = \kappa_2 + c_4$$

- 8) Finally, the true longitude of the planet (λ_v) will be:

$$\lambda_v = \kappa_3 + c_3 = \kappa_3 + (\lambda_A - \frac{1}{2}c_4)$$

7.2.1.2.1.4 The Table of Ascensional Difference

The table of right ascensions seems to be Ptolemaic in origin,⁷¹ but al-Khwārizmī's canons contain an interesting description of a method to obtain oblique ascensions by the use of ascensional differences and mention a table not extant in the *zīj*,⁷² although it has been preserved in the *Toledan Tables*⁷³ and in a late Latin manuscript (1428) containing astronomical computations for the latitude of Newminster.⁷⁴ The ascensional difference (Δ_α) is:⁷⁵

$$\Delta_\alpha = \alpha_o(\lambda) - \alpha_\varphi(\lambda)$$

for α_o = right ascension of a degree of the ecliptic (λ),

and α_φ = oblique ascension of the same degree of the ecliptic for a given latitude (φ).

It is common knowledge that:

$$\sin \Delta_\alpha(\lambda) = \tan \varphi \tan \delta(\lambda)$$

where $\delta(\lambda)$ is the declination of a degree λ of the ecliptic.

The instructions given by al-Khwārizmī state that the user should first know the noon solar shadow, for a given length of the gnomon g , when the Sun is in one of the equinoxes ($g \cdot \cot[90^\circ - \varphi] = g \cdot \tan \varphi$). Then, one enters the *Tabula differencie ascensionis universe terre*⁷⁶ using as argument the degree of the ecliptic whose oblique ascension we want to obtain and multiply the result by the noon solar shadow. The result of this product will be a sine and we will obtain the arcsine using a sine table.

As $\tan \varphi$ is assumed to be known, the *Tabula differencie ascensionis* should give $\tan \delta(\lambda)$ and it has been clearly established that the declination is computed assuming a radius of 150^ρ and an obliquity of the ecliptic of $23;51^\circ$, and that the shadow function (tangent) uses a gnomon of 12^d . Therefore, the table can be recomputed with the following expression:

$$150 \cdot \tan \delta(\lambda) / 12$$

71 Suter, 1914, table 59, pp. 171–173; Neugebauer, 1962, pp. 104–105.

72 Neugebauer, 1962, pp. 48–55.

73 Pedersen, 2002, vol. III, 986–990. See also vol. II, 414–6 and 616–9.

74 Neugebauer & Schmidt, 1952; Lesley, 1957, pp. 125–127.

75 Neugebauer, 1975, p. 36.

76 This is the title of the table both in the *Toledan Tables* and in the Newminster text.

in which a conversion factor $150/12 = 12;30$ is used. In Western Islamic *zīj*es, however, we often find auxiliary tables computing $5 \cdot \tan \delta (\lambda)$, in which the conversion factor is $60/12 = 5$, because the radius used is 60^p instead of $150'$. Thus, the *Toledan Tables* contain two tables of this kind: one of them is computed for $\epsilon = 23;51^\circ$ and $R = 60^p$, while the other is calculated for the same obliquity of the ecliptic but for $R = 150^p$.⁷⁷ This kind of table is usually called *Jadwal fuḍūl al-maṭālī* ('Table of ascensional differences'), *Jadwal nisbat al-tafāḍul* or *Jadwal nisbat al-ikhtilāf* (Table of the proportion of differences), or *Jadwal juyūb al-tafāḍul* (Table of the sines of the difference) and we can find it in Ibn al-Zarqālluh's *Almanac*,⁷⁸ the Hyderabad recension of Ibn Ishāq's *zīj*,⁷⁹ Ibn al-Bannā's *Minhāj*,⁸⁰ Ibn al-Raqqām's *Mustawfī*,⁸¹ *Shāmil*⁸² and *Qawīm*⁸³ and in the *Muwāfiq zīj* of Ibn 'Azzūz.⁸⁴

A sample of this table, as it appears in the *Toledan Tables* and in the Newminster text, is shown in table 7.3:

TABLE 7.3 *Tabula difference ascensionis universe terre*
(in parentheses: difference between the tabular
and recalculated values)

5°	0;26,27°
10	0;52,47 (−1)
15	1;18,55
20	1;44,44
25	2;10,4 (−1)
30	2;34,49
35	2;58,49
40	3;21,52
45	3;43,47
50	4;4,19 (−1)

77 Pedersen, 2002, Vol. III, pp. 986–990. This second table was edited and analysed by Neugebauer & Schmidt, 1952.
78 Millás, 1943–50, p. 225.
79 Mestres, 1999, pp. 279–281.
80 Vernet, 1952, pp. 46–48 (Ar. txt) and 109–111 (Sp. translation).
81 Ms. Rabat NL 2461, p. 285–286.
82 Ms. Kandilli 249, fol. 80v.
83 Ms. Rabat NL 260, p. 99.
84 Ms. Rabat NL D2461 p. 439 (with a maximum value of 2;10,44) and ms. Rabat HL 8772, p. 58 (max. 2;10,40), Another table of this kind is preserved in ms. Bodleian Canonici Misc. 27, within the Alfonsine *Tabulae resolutae*. Its maximum is 2;10,50 and the value of ϵ is either 23;33 or 23;33,30°: see Chabás, 1998, p. 171.

TABLE 7.3 *Tabula difference ascensionis universe terre (cont.)*

55	4;23,16(−1)
60	4;40,23
65	4;55,24
70	5;8,5 (+1)
75	5;18,12
80	5;25,35
85	5;30,4
8−1	5;31,34

7.2.1.2.2 *Maslama’s Intervention and Later Iberian Additions*

In spite of the Indian origin of the mean motion parameters, the formal presentation of the mean motion tables was the object of important changes ascribed by tradition to Maslama (according to Šāʿid) or Ibn al-Šaffār (Ibn ʿEzra), as we have already seen. The original tables used Persian years of 365 days, without fraction, and the radix for the computation of mean motions corresponded to the beginning of the reign of the last Sassanian king, Yazdijird III (16.6.632). Instead of this, the preserved tables use the Muslim lunar calendar of 354 or 355 days, and the radix values are calculated for the beginning of the Hijra (14.7.622).

The part of the *zīj* containing Ptolemaic materials raises doubts. On the one hand, al-Khwārizmī was a contemporary of the ʿAbbasid caliph al-Maʾmūn (813–833) and lived at a time when the Islamic East had a thorough knowledge both of the *Almagest* and of the *Handy Tables*. On the other, one sometimes has the impression, based on more or less clear evidence, that the original materials may have been altered or contain interpolations due to Maslama or some other author; we have already seen that this is the case of the tables of right ascensions. Maslama’s intervention, introducing Ptolemaic materials, may have taken place in the trigonometrical part of the *zīj*, which contains tables of sines⁸⁵ and shadows calculated for a 12-digit gnomon.⁸⁶ It is interesting to see that the sine table, calculated with an interval of one degree in the argument, uses a radius of 60^p, and that this table is the result of halving the values of the table of chords of the *Almagest*, in agreement with the expression $\sin A = \frac{1}{2} \text{chord } 2A$. However, we know from Ibn al-Muthannā’s commentary that al-Khwārizmī’s original sine table was calculated with $r = 150^p$ and an attempt

85 Suter, 1914, pp. 169–170.
86 Suter, 1914, p. 174.

to reconstruct it was made by J.P. Hogendijk,⁸⁷ although his results do not agree with a table of these characteristics preserved in the *Toledan Tables* and considered Khwārizmian by Pedersen.⁸⁸ The introduction of this Ptolemaic table could, therefore, be the result of Maslama's intervention.⁸⁹ Other Ptolemaic materials are also found in the tables for lunar and solar eclipses, equation of time and planetary stations.

Both the eclipse tables and the tables for the computation of planetary latitudes are considered to be Indian in origin, but, in both cases, there seems to have been an intervention on the part of Maslama; in the planetary latitude tables, it was not very successful, as certain changes introduced in the primitive rule (probably Khwārizmian) give results equal to double the correct latitude.⁹⁰

Maslama was probably responsible for the adaptation of certain tables of the *zīj* to the geographical coordinates of Córdoba. There are some tables of these characteristics, like the table for the computation of the conjunction and opposition of the Sun and the Moon.⁹¹ Tables of this kind were no doubt present in al-Khwārizmī's original tables since, although the extant table is adjusted to the meridian of Córdoba, the length of the synodic month (29;31,50,5,43,22... days) is Indian in origin. Something similar happens with the mean motion tables of the lunar ascending node,⁹² which contains a supplementary table for the midday of Córdoba and for a period comprised between years 360/960 and 570/1174.⁹³ Besides all this, many astrological tables contained in the *zīj* seem to be Maslama's additions. A clear example is represented by the tables for the equation of the houses and the tables for casting the rays (3.2.3.3). The latter represent one fifth of all the tabular material of the *zīj*; they are calculated for the latitude of Córdoba (38;30°) and are the result of Maslama's own work. This is easy to check, as al-Khwārizmī's original tables have been preserved in Ibn Hibintā's (fl. Baghdad ca. 850) *Mughnī* and they are totally independent from the tables of our Latin text.⁹⁴ The mathematical structure of Maslama's tables has been clarified by Hogendijk who emphasised Maslama's ability to solve a difficult trigonometrical problem, and added that Maslama's tables are a clear improvement on al-Khwārizmī's, as they are easier to use and give exact

87 Hogendijk, 1991.

88 Pedersen, 2002, vol. III, pp. 946–54.

89 Millás Vendrell, 1963, p. 63.

90 Pingree, 1976; Kennedy & Ukashah, 1969 a. See also G.J. Toomer's review of Neugebauer's translation and commentary of al-Khwārizmī's tables in *Centaurus* 10 (1964), 205–6.

91 Suter, 1914, pp. 183–6.

92 Suter, 1914, pp. 130–1.

93 Suter, 1914, p. XIX.

94 Kennedy & Krikorian-Preisler, 1972, repr. in Kennedy, *S.I.E.S.*, pp. 151–6.

results in certain parts of the computation for which the Eastern tables only gave approximations.⁹⁵

Maslama, together with his collaborators and, possibly, the later Iberian tradition, may have introduced certain changes in the *zīj* that allow us to speak of the existence of Iberian materials. This is the case of the use of the Hispanic Era (1.1.-37)⁹⁶ as well as the reference to the supplementary day in bissextile years which is added at the end of December. This custom is well documented in Hispanic sources, usually referring to Mozarabs. An example can be seen in the *Calendar of Córdoba*⁹⁷ where we read that it corresponds to the habits of the *ʿajam* (non-Arabs, speakers of a Romance language), unlike the Syrians who intercalate the extra day at the end of February. I do not believe that Abraham ibn ʿEzra's words ascribing this practice to "christiani in terra sarracenorum"⁹⁸ should be interpreted literally.

Another instance showing an intervention made, after Maslama, in the primitive Khwārizmian materials can be found in the mean motion tables in longitude, more specifically in the tables for collected years.⁹⁹ These tables are calculated for intervals of 30 years in the argument and reach a maximum of 570 years. Only two tables extend the computation until year 720 of the Hijra: those of the solar and mean lunar motions in longitude. Interestingly, the solar mean position for years 600–720 has been calculated using a different parameter from the one used for the collected years between 30 and 570. This implies that this addition is, probably, the result of a late post-Maslamian addition, probably made towards the end of the twelfth or beginning of the thirteenth century.

Another example of this later intervention was discovered by J.P. Hogendijk in the table for the computation of crescent visibility.¹⁰⁰ This table is based on an Indian visibility theory, but it is calculated for an obliquity of the ecliptic of 23;35° and a latitude of 41;35°, which is a good approximation to the 41;30° attributed in many Islamic sources to Zaragoza. Given the development of the exact sciences in this city in the eleventh century, one can consider (in agreement with Hogendijk) that this table is a late interpolation in the al-Khwārizmī – Maslama *zīj*.

95 Hogendijk, 1989.

96 Levi della Vida, 1943; Neugebauer, 1981.

97 Dozy-Pellat, 1961, pp. 16–17.

98 Neugebauer, 1962, pp. 11–13 and 224 fn. 25.

99 Suter, 1914, pp. 115–131.

100 Suter, 1914, p. 168; Hogendijk, 1988 a, pp. 32–35. This table had already been studied by Kennedy & Janjanian, 1965, as well as by King, 1987 a.

7.2.1.2.2.1 The Size of the Mediterranean Sea

An important feature of Maslama and Ibn al-Šaffār's revisions of al-Khwārizmī's *zīj* is that they both use a meridian of Córdoba which has a time-difference with the centre of the world (Arin) of 4;12h, corresponding to a difference in longitude of 63°. This implies that they are using the "meridian of water" located in the Atlantic Ocean at a distance of 27° to the west of Córdoba, instead of the 9;20° that is the standard longitude of Córdoba in sources related to the Ptolemaic geographical tradition, which uses as prime meridian the meridian of the Fortunate Islands. 4;12h is mentioned explicitly in the canons of Ibn al-Šaffār's *zīj*,¹⁰¹ where we read, in a passage on the computation of eclipses:

I have calculated the table of conjunctions and oppositions [of the Sun and the Moon] in this book on the basis (*aṣl*) of the meridian of Córdoba. When you finish computing the eclipse according to the meridian of Córdoba, add to the calculated time four equal hours and one fifth [of an hour] and the result will be the hour for the centre of the world (*wasat al-arḍ*).

Maslama's revision of al-Khwārizmī's *zīj* does not contain any such explanation, but we find in it a set of tables of mean luni-solar conjunctions and oppositions calculated for the meridian of Córdoba ("secundum medium diem Cordubae"), a remark we also find in the corresponding canons ("Haec autem omnia pro situ Cordubae dicta sunt").¹⁰² Neugebauer compared the results obtained with these tables with those calculated with the solar and mean lunar motion tables; he concluded that the latter were computed for a different meridian and that the time difference between the two results was exactly 4;12h, which he considered to correspond to the difference of longitudes between Córdoba and Baghdad.¹⁰³ This correction is also mentioned in a horoscope, dated 940 AD and added as an appendix to 'Umar ibn Farrukhān's *Liber Universus*, and also interpreted by Pingree as corresponding to the difference in longitude between Baghdad and Córdoba.¹⁰⁴ The horoscope in question was computed using al-Khwārizmī's *zīj* but it is difficult to establish whether it was cast before or after that date and, therefore, whether the correction in the difference of longitudes between al-Andalus and Arin should be ascribed to Maslama's school or to an earlier source.

101 Castells & Samsó, 1995, pp. 234 and 253.

102 Suter, 1914, p. 25 and tables 69–72, pp. 183–186.

103 Neugebauer, 1962, pp. 108–114 (see especially pp. 110–111).

104 Pingree, 1977.

The problem was clarified by Mercè Comes,¹⁰⁵ who offered a mass of evidence of the use of this meridian mainly by Andalusī, Maghribī and astronomers living in the Christian kingdoms of the Iberian Peninsula. Among the sources analysed by her we find the *Toledan Tables*, Ibn Mu'ādh's *Tabulae Jahen*, Ibn al-Kammād's *Muqtabas zīj*, Abraham bar Hiyya, Abū l-Ḥasan 'Alī al-Marrākushī, the Castilian and Latin *Alfonsine Tables*, Abraham Zacut, as well as by Muḥyī al-Dīn al-Maghribī in his *Tāj al-azyāj*, compiled in Damascus ca. 1258.¹⁰⁶

The use of the meridian of water had an interesting feature: the geographical tables found in western Islamic *zīj*es and similar sources gave the longitudes of Andalusī and Maghribī localities measured from the meridian of water, while the longitudes of eastern cities were usually calculated from the meridian of the Canary Islands. This meant that the length of the Mediterranean was considerably reduced and reached values similar to the modern ones.¹⁰⁷ This topic was also analysed in detail by Mercè Comes¹⁰⁸ who considered the differences in longitude between Tangier and Alexandria, Córdoba and Alexandria, Toledo and Damascus and Toledo and Alexandria and established, in all cases, that the error in the Ptolemaic geographical tradition, which used the prime meridian of the Canary Islands, lay between 16° and 19°; the correction introduced by al-Khwārizmī improved the situation, reducing the error to between 7° and 10°. Finally, the differences in longitude in sources using the meridian of water were between 0° and 2° away from the modern value.

We do not really know how the difference in longitude between Córdoba and Arin was established. A simultaneous observation of a lunar eclipse at both localities, followed by an exchange of information about the time of the eclipse, does not seem possible. Another possibility is mentioned in Ibn al-Ṣaffār's *zīj*: establishing the difference between the time of an observation of a lunar eclipse and the time of the same lunar eclipse calculated using the tables of a *zīj* computed for a locality of known longitude.¹⁰⁹ In our case, this implies an observation made in Córdoba and the computation using the solar and mean lunar motion tables of al-Khwārizmī's *zīj*, calculated for the meridian of Arin. The same technique was probably used by Abraham ibn 'Ezra¹¹⁰ and it is described in Ibn al-Raqqām's *Mustawfī zīj*.¹¹¹

105 Comes, 1992 a; Comes, 1995; Comes, 1997; Comes, 2000.

106 Comes, 2005.

107 On this topic see Robles, 2014.

108 Comes, 2000, pp. 129–132.

109 Castells & Samsó, 1995, pp. 234 and 253.

110 Samsó, 2012 pp. 186–188.

111 Samsó, 2014, p. 308.

The use of this method implies, in the case of Ibn al-Ṣaffār and his contemporaries, a reliance on the precision of the results obtained with al-Khwārizmī's *zīj*; in our case (historians working in the twenty-first century), we need a great deal of luck if we want to use this method to confirm the possibility of its use in the time of Maslama.

7.2.1.2.2.1.1 *The Lunar Eclipse of 15 September 973: Mean Opposition*

As an example of a method possibly used in tenth-century al-Andalus, both Mercè Comes¹¹² and myself have tried to compute lunar eclipse Oppolzer 3369¹¹³ which took place on 15 September 973/ 14 Dhū l-Ḥijja 362 H.¹¹⁴ This eclipse is mentioned in Ibn Ḥayyān's *Muqtabis*:¹¹⁵

At dawn of Monday 14 Dhū-l-Ḥijja there was a total lunar eclipse which ended before sunrise shone.

I begin computing the solar and mean lunar positions at midday of 14 Dhū l-Ḥijja 362 H., obtaining:

Sun	5 ^s 20;52,43°
Moon	10 ^s 29;44,30°

The difference between these two positions is 5^s 8;51,47°, and

$$6^s - 5^s 8;51,47^\circ = 21;8,13^\circ$$

The mean lunar and solar motion per hour being 0;32,56°/h and 0;2,28°/h:

$$\begin{aligned} 0;32,56^\circ - 0;2,28^\circ &= 0;30,28^\circ/\text{h} \\ 21;8,13^\circ / 0;30,28^\circ &= 41;37,35^h = 1^d 17;37,35^h \end{aligned}$$

¹¹² Comes, 2000, p. 138.

¹¹³ Oppolzer, *Canon*.

¹¹⁴ This equivalence, obtained with van Dalen's CALH, corresponds to a civil date based on the convention that 1 Muḥarram of year 1 H. is Thursday 15 July 622 (JDN 1948440). Al-Khwārizmī, however, adopts noon of Wednesday 14 July (JDN 1948438) as the beginning of the Hijra. See Neugebauer, 1962, pp. 10–11.

¹¹⁵ Ibn Ḥayyān, in Ḥajjī, 1965, p. 138. Spanish translation in García Gómez, 1967, p. 172.

The mean opposition will take place 1^d 17:37,35^h after midday of 14 Dhū l-Hijja 362 H. (civil 13), which corresponds to 15 Dhū l-Hijja (civil 14) 17:37,35^h after midday¹¹⁶ in which the solar and mean lunar positions will be:

Sun	5 ^s 22;35,18°
Moon	11 ^s 22;35,56°

These positions exceed the exact mean opposition by 38", which implies that the correct time should be 0;1,15^h earlier. We will adopt as the time of the mean opposition 5;37^h after midnight of 14 Dhū l-Hijja (astronomical date) and the longitudes:

Sun	5 ^s 22;35°
Moon	11 ^s 22;35°

7.2.1.2.2.1.2 *True Solar and Lunar Longitudes at Mean Opposition*

We now proceed to compute the solar and lunar true positions at the time of mean opposition:

For the Sun we have:

$$5^s 22;34,52^\circ - 2^s 17;55^\circ (\text{solar apogee}) = 3^s 4;39,52^\circ$$

The equation being for this value of the solar centre (tables 21–26): $-2;13,36^\circ$, and

$$5^s 22;34,52^\circ - 2;13,36^\circ = 5^s 20;21,16^\circ, \text{ which will be the true solar longitude at the moment of the mean opposition.}$$

As for the Moon, its mean anomaly for the aforementioned time is $3^s 5;38,50^\circ$, its equation (tables 21–26) being $-4;54,29^\circ$. The true lunar longitude for the moment of the mean opposition will be:

$$11^s 22;34,52^\circ - 4;54,29^\circ = 11^s 17;40,23^\circ$$

¹¹⁶ Both dates are astronomical: midday of 14 Dhū l-Hijja corresponds to civil 13 Dhū l-Hijja, while, obviously, midday of 15 Dhū l-Hijja is 14 Dhū l-Hijja.

7.2.1.2.2.1.3 *True Opposition*

The true elongation at the moment of the mean opposition will be:

$$11^s 17;40,23^o - 5^s 20;21,16^o = 177;19,7^o$$

$$\text{and } \Delta = 180^o - 177;19,7^o = 2;40,53^o$$

The time the Moon takes to attain its true opposition with the Sun (T) can be calculated using the following expression in which v_m is the true lunar velocity:

$$T = 1 / v_m (\Delta + \Delta/12) = 1 / v_m (2;40,53^o + 2;40,53^o/12) = 1 / v_m (2;40,53^o + 0;13,24^o) = 2;54,17^o / v_m$$

v_m can be obtained in tables 61–66, col. 4 (*buht, motus lunae in hora*) as a function of the mean lunar anomaly ($3^s 5;38,50^o$). We obtain $0;33,9^o$ /hour, and:

$$T = 2;54,17^o / 0;33,9^o = 5;15,27^h$$

The hour of the true opposition for Arin (t_a) will be:

$$t_a = 17;37,35^h + 5;15,27^h = 22;53,2^h$$

While the corresponding hour for Córdoba (t_c) will be:

$$t_c = 22;53,2^h - 4;12^h = 18;41,2^h$$

According to Oppolzer, the time for the full lunar eclipse (middle of totality) took place at $6;52^h$ TU (from midnight). Using modern coordinates (longitude of Córdoba $35;14^o$), the longitude difference between Greenwich and Córdoba is $4;46^o$ and the time difference between the two localities will be $0;19,4^h$. Therefore the local time for mid-eclipse will be $6;52^h - 0;19,4^h = 6;33^h$ after midnight, with a difference of about 8^m with the calculated time using Khwārizmī's *zīj*'s solar and mean lunar motion tables for Arin. A better result could have been obtained if our starting point had been tables 70–72 which give, directly, the time of the mean opposition, as well as the solar mean longitude, lunar mean anomaly and the argument of latitude. In that case we would have obtained $6;32,8^h$ (after midnight) for the time of the true opposition in Córdoba.¹¹⁷

¹¹⁷ This hypothesis is based on the assumption that if somebody calculated this eclipse with the purpose of establishing the difference in longitudes between Arin and Córdoba, he

This surprising result, which could justify the time difference of 4;12^h between Córdoba and Arin, is confirmed by the NASA Eclipse Web Site (<http://eclipse.gsfc.nasa.gov/lunar.html>, eclipse no. 7155), according to which the mid-eclipse took place at 7;27,56^h AT, 6;59,23^h UT and 6;40,23^h local time for Córdoba, which agrees with the hour (6;41,2^h) obtained using al-Khwārizmī's mean motion tables. There is, however, a serious contradiction with Ibn Ḥayyān's text (quoted at the beginning of this section), stating that the eclipse ended before sunrise, which took place in Córdoba at 5;51^h after midnight. Without giving the details here, the results of the rest of the computation of the eclipse follow:

Duration of the eclipse from its beginning until the commencement of totality: 1;7,7^h

Half duration of totality: 0;41,9^h

Half duration of the eclipse: 1;7,7^h + 0;41,9^h = 1;48,16^h

Hour of the beginning of the eclipse in Córdoba: 6;32,8^h - 1;48,16^h = 4;43,52^h

Hour of the end of the eclipse: 6;32,8^h + 1;48,16^h = 8;20,24^h

It seems, therefore, that Ibn Ḥayyān, or his source, was probably wrong to state that the eclipse ended before sunrise.

7.2.2 *Ibn Mu'adh's Tabulae Jahan*¹¹⁸

Another set of Western Islamic astronomical tables which follow the tradition of al-Khwārizmī's is the so-called *Tabulae Jahan*, compiled by the mathematician and astronomer Abū 'Abd Allāh Muḥammad b. Ibrāhīm b. Muḥammad b. Mu'adh al-Sha'bānī (d. 1093) who belonged to a well-known family of *qāḍīs* and *fuqahā'* and was, himself, *qāḍī* of the city of Jaén (Jayyān), which explains why he is mostly known as Ibn Mu'adh al-Jayyānī. We have no information about his life although we know that he probably also lived in Seville because he wrote a treatise on the solar eclipse of 1 July 1079, observed in Seville, which is only preserved in a Hebrew translation.¹¹⁹ Besides, his trigonometrical treatise, *Kitāb majhūlāt qisī al-kura*,¹²⁰ in which he seems to be perfectly aware of the new trigonometry which had appeared in the Mashriq towards the end of

would not have taken into consideration the equation of time which, for a solar longitude of 170;21,16° would imply a delay in the time of the eclipse of 0;23,40^h. On the equation of time in al-Khwārizmī's *zīj* see Dalen, 1996.

118 This section is a revised version of Samsó, 2011, pp. 152–166.

119 Goldstein, 1979.

120 Villuendas, 1979; Samsó, 1980 b; Samsó, 1996.

the tenth century and beginning of the eleventh, has led me to suggest that he had probably travelled to the East where he could have read trigonometrical sources that had not reached al-Andalus.

His *Tabulae Jahan* are mainly known through a Latin translation of its canons by Gerard of Cremona, a book printed in Nüremberg, 1549, under the title *Scriptum antiquum saraceni cuiusdam, de diuersarum gentium eris, annis ac mensibus et de reliquis astronomiae principiis*. To this text, one should add fragments of the canons and tables extant in the Tunisian recension of Ibn Ishāq's *zīj* dealing with lunar crescent visibility¹²¹ (chapter 19 of the Latin canons, see 2.3.2 and 2.3.4), astrological transits (chapter 11 of the Latin canons)¹²² and computation of the *qibla* using what was known as the "method of the *zījes*" (chapter 18, see 2.5.2.1).¹²³ Ibn Mu'ādh's treatment of transits was also reproduced by Ibn 'Azzūz al-Qusanṭīnī in his *Muwāfiq zīj*.¹²⁴ Finally, the Alfonsine *Libro del saber de astrología* contains multiple references to his procedure for dividing the houses of the ecliptic (3.2.3).

According to Hermelink,¹²⁵ the *Tabulae Jahan* are a simplified version of al-Khwārizmī's *zīj*, adapted to the needs of a *qāḍī*'s daily life. Apart from the fact that I do not see why a *qāḍī* might need a set of astronomical tables,¹²⁶ I believe that this judgement is unfair and that this *zīj* deserves a certain attention. It is, obviously, an adaptation of the *Sindhind* to the coordinates of Jaén, located 62° to the west of Arin (chapter 8), its latitude being 38° (chapters 4 and 22). A longitude 62° W of Arin implies that he is using the meridian of water; as for a latitude of 38°, the only earlier source which quotes this value is the Andalusī geographer Ishāq b. al-Ḥasan b. Abī l-Ḥusayn al-Zayyāt (late tenth century and early eleventh), who had a great influence on both Andalusī astronomers and geographers.¹²⁷

121 Mestres, 1999, pp. 150–1 of the Arabic edition and 87–88 of the introduction. The corresponding table (no. 155) was not edited by Mestres.

122 Mestres, 1999, pp. 253–257 of the Arabic edition and 150 of the introduction. See Samsó, 1996, pp. 601–612.

123 Mestres, 1999, pp. 272–275 of the Arabic edition and 159–160 of the introduction. See Samsó & Mielgo, 1994.

124 Samsó, 1997, pp. 91–92.

125 Hermelink, 1964.

126 There is, however, the peculiar case of the astrologer, poet and *qāḍī* 'Abbās ibn Nāṣiḥ (d. after 844) who did not deliver a judgement without previously casting the corresponding horoscope and who used astrological techniques to find the whereabouts of a lost ox. See Terés, 1962.

127 Kennedy & Kennedy, 1987, p. 158.

We have already seen (2.3.4.1) that Ibn Mu‘ādh’s treatment of the problem of lunar crescent visibility is different from that found in al-Khwārizmī-Maslama’s *zīj* and that the *Tabulae Jahen* contain the earliest known Western Islamic exact procedure for the computation of the *qibla* (2.5.2.1). This is one of the reasons for my belief that Ibn Mu‘ādh had privileged information on Eastern scientific sources which were not available in al-Andalus. This, as we have seen, is confirmed by his trigonometrical treatise, although his treatment of trigonometry in the *Tabulae Jahen* is far more elementary.

Chapter 1, on chronological matters, contains a few interesting features. While Al-Khwārizmī uses noon of Wednesday 14 July 622 (JDN 1948438) for the beginning of the Hijra, Ibn Mu‘ādh delays this beginning until midnight between Thursday 15 (JDN 1948439) and Friday 16 July (JDN 1948440) and this is the radix of his mean motion tables calculated for the meridian of Jaén (chapter 8, “ad illam horam extraximus radices secundum intentionem Indorum”). The eras he mentions, apart from Hijra, are Alexander’s era (1.10.-311), used by Greeks and Romans, Diocletian’s era (29.8.284), used by Egyptians, and Yazdjird’s era of the Persians (16.6.632, see 2.2). To these he adds a reference to the Hispanic era (*era aeris*) (see 7.2.1.2.2) and, interestingly, to the Christian era, 13,880 days after, which corresponds to 1 January of year 1 CE, and not to the Incarnation era (25.3.1 or 25.3.0).

The text gives the number of days between three of these eras and the Hijra (table 7.4). The results are correct with a difference of one day due to the change of the date of the Hijra era.

Following the Hispanic tradition, and as in al-Khwārizmī-Maslama’s canons, Ibn Mu‘ādh states that the extra-day in bissextile years is added at the end of December (“Porro gentes quidem nostrae terrae addunt hunc diem intercalarem in fine decembris”).

Ibn Mu‘ādh’s planetary theory is also Khwārizmian: the Moon has only one equation and it seems that the model implied is an eccentric circle. In chapter 39

TABLE 7.4

Eras	Days before or after Hijra
Alexander	-340,701
Diocletian	-123,410
Yazdjird	3,623

of the Tunisian recension and chapter 11 of the Latin translation, dealing with velocity sectors and planetary transits, we read:

وَأَمَّا صعود الكواكب وهبوطها فَإِنَّ لِكُلِّ كَوْكَبٍ مِنْهَا فَلَكَ أَوْجٌ ثَرَّ لِكُلِّ وَاحِدٍ مِنَ الْخَمْسَةِ
الْمُتَحَيِّرَةِ زِيَادَةً خَاصَّةً فَلَكَ تَدْوِيرُهُ.

and “Ad sciendum uero ascensionem stellarum, et earum descensionem, est notandum quod unicuique stellae earum est orbis augis, et unicuique 5 erraticorum est additio prima orbis reuolutionis suae”. This implies that the Sun, the Moon and the planets have an eccentric (*falak al-awj*) but only the *five* planets have an epicycle (*falak al-tadwīr*) (see 7.2.1.2.1).

In this chapter of the Tunisian recension we can also find information on the maximum solar and lunar equations. The text states that the Sun and the Moon do not retrograde but that they have a faster and slower motion. Mean velocity is attained at the end of the first and third sectors (*niṭāq*), when they are *mu’tadil al-sayr*, while their velocity is slow (*baṭī’ al-sayr*) in the first and fourth sectors, and fast (*sarī’ al-sayr*) in the second and third sectors. The limits of these sectors are:

- I. From the apogee to the position in which Sun and Moon reach their maximum negative equation. The solar apogee, mentioned in chapter 9 of the *Tabulae Jahen*, is the same as al-Khwārizmī’s: 77;55°
- II From maximum negative equation to the perigee,
- III From the perigee to maximum positive equation,
- IV From maximum positive equation to the apogee.

The Latin text gives explicitly the values of anomaly (*hiṣṣa* for the Sun, *hiṣṣa mu’addala* for the Moon) which correspond to the maximum positive and negative equation:

Sun: 92;14° and 267;46° (= 360° – 92;14°)
Moon: 94;56° and 265;4° (= 360° – 94;56°)

We can deduce two consequences from these values: 1) Ibn Mu’ādh is using al-Khwārizmī’s maximum equations, and 2) his tables for the solar and lunar equation were not calculated using the method of declinations which yields a maximum value for an argument of 90°, but with a Ptolemaic method. Besides, we can obtain some information about the mean solar motion from the value of the *revolutio anni* (93;2,15°) preserved in the table *Residuum ascensionum*

ad revolutiones annorum secundum Muhad Arcadius, of Ibn al-Kammād's *Muqtabas*,¹²⁸ which implies a sidereal year of 365;15,30,22,15 days: a table based on an identical parameter appears in al-Khwārizmī's *zīj*.¹²⁹

As for the planets, the method explained in chapter 9 of the *Tabulae Jahan* for calculating their true longitude is the same as al-Khwārizmī's (7.2.1.2.1.3). Some information about the maximum values of the planetary centre and anomaly equations can be gathered from Ibn 'Azzūz's *zīj*, in a passage in which the source is Ibn Mu'ādh:¹³⁰ to compute the astrological transit (*mamarr*) of one planet on top of the other if both are in conjunction (*iqtirān*) or aspect (*tanāzur*), we should first obtain the planetary "chord" (*watar*) or *aṣl al-mamarr* (root of transit) (*w*). Ibn 'Azzūz's *Muwāfiq zīj* transcribes the "Table of the minutes of transit" (*Jadwal daqā'iq al-mamarrāt*) (table 7.5)¹³¹ and, although it does not give any indication about how this table had been computed, I conjectured in a previous paper¹³² that $w = 60 / e_{\max}$ in Ibn Mu'ādh's procedures. This is confirmed by the table of the *Muwāfiq zīj*, with which we obtain, in most cases, al-Khwārizmī's maximum solar, lunar and planetary equations, as one could expect in the *Tabulae Jahan*. Without giving details about the complete procedure for establishing planetary transits, which is of no astronomical interest, I will only say that the compiler of this table was probably Ibn Mu'ādh himself:

TABLE 7.5 Ibn Mu'ādh's roots of transit and maximum planetary equations

Planet	Roots of the transit in the sphere of the apogee (<i>w</i>)	Calculated maximum equation of the centre	Roots of the transit in the sphere of the epicycle (<i>w</i>)	Calculated maximum equation of the anomaly
Sun	26;52'	2;14°	0'	—
Moon	12;10'	4;55,53° (Khw. 4 ;56°)	0'	—
Saturn	6;58' [6;58,36']	8;36,45° (Khw. 8;36°)	10;28'	5;43,57° (Khw. 5;44°)

128 Ms. Madrid NL 10,023 fol. 66r. Millás (1942, p. 246) identified *Muhad Arcadius* with [Ibn] Mu'ādh *al-qāḍī*.
129 Suter, 1914, p. 230; Neugebauer, 1962, p. 131.
130 This corresponds to chapter 11 of the Latin canons and to chapter 39 of the Tunisian recension. See Samsó, 1997, pp. 91–92.
131 Samsó, 1997, p. 106.
132 Samsó, 1996, pp. 606–608.

TABLE 7.5 Ibn Mu‘ādh’s roots of transit and maximum planetary equations (*cont.*)

Planet	Roots of the transit in the sphere of the apogee (w)	Calculated maximum equation of the centre	Roots of the transit in the sphere of the epicycle (w)	Calculated maximum equation of the anomaly
Jupiter	11;46’	5;5,57° (Khw. 5;6°)	5;31’	10;52,34° (Khw. 10;52°)
Mars	5;21’	11;12,54° (Khw. 11;13°)	1;29’	40;26,58° (Khw. 40;31°)
Venus	26;52’	2;14°	1;16’	47;22,6° (Khw. 47;11°)
Mercury	14;53’	4;1,53° (Khw. 4;2°)	2;47’	21;33,25° (Khw. 21;30°)

Although I cannot prove this, I believe that the tables of planetary equations in Ibn Mu‘ādh’s *zīj* were computed according to the Ptolemaic method and not according to al-Khwārizmī, in whose *zīj* the equation of the centre is calculated using the “method of sines”. My assertion is based on the evidence provided about the solar and lunar equations,

Chapter 11 of the Latin canons makes two more interesting remarks. One of them is that the *Tabulae Jahan* contained tables of the apparent sizes of planets (“Et nos quidam iam fecimus ad quantitatem corporum stellarum et 2 luminarium tabulas”), using as arguments the corrected anomaly of the planets (*portio secunda*)¹³³ and the solar and lunar anomaly (*portio*). He also mentions a star table with longitudes calculated for the beginning of the Hijra, as well as their latitudes and magnitudes, to which he adds a table of, apparently, constant precession, something which is missing in al-Khwārizmī’s *zīj*:

Stellis uero fixis iam feci tabulas, in quibus sunt loca quarumdam, que ex eis sunt famose ad initium repulsionis et adde desuper motum stellarum fixarum in annis et mensibus, et pone dies, mensem integrum non dimittas, si fuerint plures medietate mensis, tunc numera eos mensem

133 He probably means the anomaly of the planet corrected with the equation of the centre: $\alpha_2 = \alpha_1 - c_5$, in al-Khwārizmī’s procedure (see 7.2.1.2.1.3). If my interpretation is correct, I wonder why he does not take into consideration the distance of the planet from its apogee.

integrum, et quod aggregatur inde, est locus stellae ad horam quam uis, in tabulis eorum, et in tabulis illis sunt earum latitudines, et ipsarum partes, et earum quantitates ex ordinibus quos inueni super eas.

In chapter 11 we also find a reference to the daily planetary velocity (*ḥarakat al-kawkab al-mukhtalifa ... li-yawm wāḥid* in the Tunisian recension, or *motum stellae diuersum diei unius* in the Latin canons)¹³⁴ which may correspond to the standard method explained in chapter 29 of al-Khwārizmī's canons, which consists in computing the true longitude of a planet on two consecutive days, the difference being the daily velocity. Another possibility is, however, to consider that Ibn Mu'ādh's *zīj* might contain tables of daily planetary velocities like the ones we find in Ibn 'Azzūz's *Muwāfiq zīj*, which are the same as the ones in manuscripts of the Alfonsine tradition.¹³⁵ This seems rather unlikely, because we do not have here the same evidence as for the tables of solar and lunar velocity (chapter 12: "illud est quod intres cum porciones utriusque [i.e. solis et lunae] in tabulas motus diuersi in hora una"), although the fact that these tables appear in the *Muwāfiq zīj* (which is rarely original and, as we have seen, knows and copies the *Tabulae Jahen*) leads me to suggest that Ibn 'Azzūz's velocity tables might derive from Ibn Mu'ādh.

To sum up: it seems that the main source of the *Tabulae Jahen* is al-Khwārizmī's *zīj*, although Ibn Mu'ādh has introduced some important modifications that bear witness to his competence as a mathematician and as an astronomer. This is particularly conspicuous in his treatment of matters related to mathematical astrology such as the division of the houses and the projection of rays which are dealt with summarily in his *zīj*, where he seems to give an abridgement of his most important book on the subject: his *Risāla fī maṭraḥ al-shu'ā'āt*.¹³⁶

7.2.3 *The Versified zīj of Abū l-Ḥasan 'Alī b. Abī 'Alī al-Qusanṭīnī* (ca. 1359–61)

Dealing with this *zīj* here does not follow a strict chronological order, but it seems convenient to mention it here because, according to Kennedy and King, it "is the only known document extant in Arabic in which the planetary

134 In chapter 11 we also read: "Et compara in Venere et Mercurio motum eorum diuersum diei ad motum Solis medium in die".

135 Samsó, 1997, pp. 88–91; Goldstein, Chabás & Mancha, 1994.

136 See the edition, Spanish translation and commentary in Casulleras, 2010. More bibliography on this work is quoted in 3.2.3.0.

theory is essentially Indian rather than Ptolemaic".¹³⁷ We have no information about the author of this untitled *zīj* apart from the few data gathered from the only extant copy: ms. Escorial 909 (fols. 44v–63v),¹³⁸ where he is called *al-shaykh*, *al-faqīh* and *al-muʿaddil* and his *nisba* appears as *al-Qusunṭīnī*, instead of the standard form *al-Qusanṭīnī*. This implies that he was probably born in Constantine, in the central Maghrib, although, when he compiled his *zīj*, he was probably living in Fez: his mean motion tables have their radices calculated for the meridian of Fez (fols. 51r–55r) and the *zīj* contains a table of oblique ascensions for the latitude of Fez (fol. 61r). Besides, the *zīj* is dedicated to the Merinid sultan *al-Imām al-Mustaʿin bi-llāh Ibrāhīm*, identified by Renaud¹³⁹ with Abū Sālim Ibrāhīm who reigned only between 1359 and 1361. We know, therefore, the approximate date in which the *zīj* was compiled.

The fact that Abū l-Ḥasan ʿAlī is called *muʿaddil* probably implies that he was a *muwaqqit* working in one of the great mosques of Fez, and this hypothesis receives some support in the table for lunar crescent visibility of fol. 61v. This table is well known and it has been studied in detail by D.A. King,¹⁴⁰ who considers it to be “an early Andalusian crescent visibility table” and has found it in 14 different sources (among them, the Tunisian recension of Ibn Ishāq’s *zīj*) to which one should add the *Toledan Tables* (see 2.3.2).

In spite of this, the main interest of Abū l-Ḥasan ʿAlī in his *zīj* does not seem to be *mīqāt* but rather astrology. His versified canons and the corresponding numerical tables deal with chronology, the computation of solar, lunar and planetary longitudes, lunar (not planetary) latitude, planetary stations and direct and retrograde motions of the planets, eclipses and their colours, right and oblique (for Fez) ascensions, how to obtain the ascendant and its *tahwīl* (transfer), division of the houses of the horoscope according to the standard method (see 3.2.3.1.1.1), *al-faḍl al-muqawwam* (interpreted by Kennedy as “a measure of the amount by which the planet has passed the last cardine, perhaps for finding his house”),¹⁴¹ and *tasyīr* (see 3.2.3.2).¹⁴² This interest in astrological matters is probably to be expected in a *zīj* dedicated to a Merinid monarch.

137 Kennedy & King, 1982, pp. 3–4. This remark is absolutely correct, as the extant chapters of Ibn al-Ṣaffār’s *zīj* do not deal with the computation of planetary longitudes.

138 A copy of this manuscript was offered to me by Father José Luís del Valle, director of the Escorial library. Here, as in several other instances, I would like to thank him for his generous help.

139 Renaud, 1932, p. 172.

140 Kennedy & King, 1982, pp. 19–20; King, 1987 a.

141 Kennedy & King, 1982, p. 16.

142 Detailed table of contents in Kennedy & King, 1982, pp. 15–21.

As established by Kennedy and King,¹⁴³ the astronomical theory involved in the computation of solar, lunar and planetary longitudes has a clear Indian descent. The equation tables derive from al-Khwārizmī's and obtain the same maximum values (fols. 56v–59r). Only two functions are tabulated for each planet: equation of the anomaly (c_4 in al-Khwārizmī's $zīj$) and equation of the centre (c_5). The Khwārizmian column 3 (c_3), which, as we have seen, gives $c_3 = \lambda_A - \frac{1}{2} c_4$, is missing. This implies that the characteristic halving of the equation of anomaly is not considered in Abū l-Ḥasan 'Alī's $zīj$, and this is confirmed by the procedure described in the canons for the computation of the longitude of the superior planets,¹⁴⁴ which follows these steps, for which I have used the same symbols as in 7.2.1.2.1.3.3:

$$\alpha_1 = \lambda_s - \lambda$$

$$\kappa_1 = \lambda - \lambda_A$$

κ_1 is the argument for obtaining $c_5(\kappa_1)$, as well as corrected values of the centre (κ_2 , *markaz mu'addal*), the mean planetary position (λ' , *wasat mu'addal*) and the anomaly (α_2 , *hiṣṣa mu'addala*):

$$\kappa_2 = \kappa_1 + c_5(\kappa_1)$$

$$\lambda' = \lambda + c_5(\kappa_1)^{145}$$

$$\alpha_2 = \alpha_1 + c_5(\kappa_1) \text{ (algebraically)}$$

With α_2 as argument, he obtains c_4 (equation of the anomaly) and λ_v (the true longitude of the planet:

$$\lambda_v = \lambda' + c_4 \text{ (algebraically)}$$

It seems, therefore, that Abū l-Ḥasan 'Alī has simplified the Indian procedure used by al-Khwārizmī and that the results obtained will be less precise than those of the Khwarizmian method.

¹⁴³ Kennedy & King, 1982, pp. 11–15.

¹⁴⁴ Ms. Escorial 909 fol. 46r and v (versified passage translated by Kennedy and King, 1982, pp. 14–15). The same technique is described in a note at the top of the page (fol. 53r) which contains the table for the mean motion of Saturn.

¹⁴⁵ As noted by Kennedy & King, 1982, p. 15 the additive sign is incorrect here and it should be replaced by a negative sign. The same mistake appears in the note at the top of fol. 53r.

The solar and planetary apogees for the beginning of the Hijra appear in a table (fol. 50r) in which the positions of Saturn, Jupiter and Mars derive clearly from those used by the members of the school of Ibn Ishāq (the anonymous author of the Tunisian recension, Ibn al-Bannā' and Ibn al-Raqqām)¹⁴⁶ for the beginning of the Hijra. Mercury's apogee seems to have the same origin if we accept that 6^s 13;24° is probably a mistake for 6^s 18;24°. The apogees of the Sun and Venus (2^s 17;19°) are different from those accepted by Ibn Ishāq's school (2^s 16;44,17°),¹⁴⁷ although the latter value appears in the table for the motion of the solar apogee (fol. 49r):

Sun:	2 ^s 17;19°
Saturn	7 ^s 29;43°
Jupiter:	5 ^s 9;43°
Mars:	4 ^s 2;13°
Venus:	2 ^s 17;19°
Mercury:	6 ^s 13 ;24°

The influence of the Zarqāllian school is not limited to the positions of the apogees. We have already seen that Abū l-Ḥasan 'Alī's *zīj* contains a table for the motion of the solar apogee (fol. 49r, see 6.3.2), a table for the equation of trepidation (fol. 60r, see 6.2.4) and a table of the mean motion of the Head of Aries based on a daily parameter of 0;0,0,53,19,3,34^{ol48} (see 6.2.6.2), which is different from those I have found in the Andalusī-Maghribī tradition.

Another instance of this influence can be found in the mean motion parameters (table 7.6) in which we can see that the motion of the solar apogee coincides exactly with the parameter used in the *zīj*es of Ibn Ishāq's school, while those of the Sun (difference of +2^{iv}), lunar longitude and anomaly (+1^{iv}), Saturn (+4^{iv}), Jupiter (+2^{iv}) are practically identical. The parameters for Mars and Venus (+1ⁱⁱⁱ), as well as Mercury (+12^{iv}) show greater differences, and probably derive from a different source.

146 See Samsó & Millás, 1994, pp. 7–9.

147 Samsó & Millás, 1998, pp. 265–266.

148 Kennedy & King, 1982, obtain 0;0,0,53,20,31°.

TABLE 7.6 Mean motion parameters in Abū l-Ḥasan's versified *zīj* and in Ibn Ishāq's school

	Abū l-Ḥasan 'Alī ^a	Ibn Ishāq's school ^b
Sun	0;59,8,11,30,5,56 ^o	0;59,8,11,28,26,22,5,24,24,26 ^o
Solar apogee	0;0,0,2,7,11	0;0,0,2,7,11 ($\pm 20^v$)
Moon (long.)	13;10,34,52,48	13;10,34,52,46,51,38 ($\pm 26^{vi}$)
Moon (anom.)	13;3,53,56,19	13;3,53,56,17,52,4 ($\pm 21^v$)
Saturn (long.)	0;2,0,27,50,55	0;2,0,27,46,42,42 ($\pm 22^{vi}$)
Jupiter (long.)	0;4,59,7,37,54	0;4,59,7,36,25,11 ($\pm 23^{vi}$)
Mars (long.)	0;31,26,30,0,51	0;31,26,31,9,5,28 ($\pm 20^{vi}$)
Venus (anom.)	0;36,59,28,13,46,16	0;36,59,27,23,59,25 ($\pm 21^{vi}$)
Mercury (anom.)	3;6,24,7,55	3;6,24,7,42,40,49 ($\pm 20^{vi}$)
Head of Aries (trepidation)	0;0,0,53,19,3,34	0;0,0,54,57,17,35 ($\pm 27^{vi}$)

a Parameters calculated by Kennedy and King, 1982, p. 10.

b Parameters established by A. Mestres (1999, pp. 188–189, 200) from the anonymous Tunisian recension.

We should finally compare the epoch positions in Abū l-Ḥasan 'Alī's *zīj* with those found in the anonymous Tunisian recension (calculated for the meridian of Toledo) and in Ibn al-Bannā's *Minhāj*, which uses the meridian of Marrakesh. As we do not know the geographical longitudes used by Abū l-Ḥasan 'Alī to correct these positions,¹⁴⁹ I have used an indirect method to determine whether the differences between the positions of Abū l-Ḥasan 'Alī, the Tunisian recension and Ibn al-Bannā might correspond to a correction due to the differences in longitude. All these data can be found in table 7.7 in which we can read the differences between positions in parentheses, and the columns Δt give the corresponding time differences.

Obviously, Abū l-Ḥasan 'Alī must have been aware that the order of the three localities involved here is (from West to East): Marrakesh (-8° from Greenwich), Fez (-5°) and Toledo (-4°). Consequently the differences in longitude of the corresponding positions should be positive (Toledo – Fez) and negative (Marrakesh – Fez), while in table 7.7 we have the opposite in most

149 The Tunisian recension uses 28° for the longitude of Toledo (Mestres, 1999, p. 33), while Ibn al-Bannā gives 21° for the longitude of Marrakesh (Samsó & Millás, 1998, p. 263). Both *zījes* measure longitudes from the meridian of water.

cases. This probably means that Abū l-Ḥasan ‘Alī made a mistake when he adjusted his mean positions at epoch.

When we examine the longitude differences (Toledo – Fez) and the corresponding Δt , we can easily see that Δt records values between $9;10^m$ and $11;1^m$ for the Sun, lunar longitude and anomaly, Jupiter and Mars, the interval between $9;10^m$ and $9;37^m$ being the most common. This interval yields a difference of longitude between Toledo and Fez comprised between $2;17^\circ$ and $2;30^\circ$, while we would obtain $2;45^\circ$ with a time difference of $11;1^m$. This estimation could be considered acceptable, as a difference of 3° between Toledo and Fez is frequent in Western Islamic *zīj*es which use the meridian of water (Toledo 28° and Fez 25°).

Therefore, Abū l-Ḥasan ‘Alī may have used Ibn Ishāq’s radix positions (preserved in the Tunisian recension) calculated for the meridian of Toledo in the case of the Sun, Moon, Jupiter, Mars and the Head of Aries. The same can probably be said of the radix for Saturn which coincides exactly with Ibn al-Bannā’s value: the motion in longitude of this planet is too slow to give significant results in the computation of its radix position and there is, probably, an error in the value given by the Tunisian recension. As for the radix positions for the motion in anomaly of Venus and Mercury, they seem to derive from a different source, and the same could probably be said about the mean motion parameters of the inferior planets.

The comparison between the radices in Ibn al-Bannā’s *Minhāj* and in Abū l-Ḥasan ‘Alī’s *zīj* gives less consistent results except for the time difference of $18;40^m$ between Fez and Marrakech, which corresponds to a difference in longitude of $4;40^\circ$, near the 4° difference between Marrakech (21° from the meridian of water) and Fez (25°). Besides, the radix positions for Saturn and the Head of Aries coincide exactly in both sources and we have the confirmation that the radix positions for the inferior planets derive from a different source.

7.2.4 The Toledan Tables

7.2.4.1 Introduction

Two sets of astronomical tables produced in the Iberian Peninsula received wide acceptance in Medieval Europe: the *Toledan Tables* and the *Alfonsine Tables*. Both have very obscure origins. We should remember, first of all, that the Arabic original text of the *Toledan Tables* seems to have been lost and that they were the object of three Latin translations (or adaptations), one of which has been attributed to Gerard of Cremona. The number of manuscripts containing these translations is enormous: Toomer¹⁵⁰ listed 128 Latin manuscripts

150 Toomer, 1968 (list of mss. in pp. 160–170).

TABLE 7.7 Mean positions at Hijra

	Abū l-Ḥasan ‘Alī (Fez)	Tunisian recension (Toledo)	Δt	Ibn al-Bannā’ (Marrakech)	Δt
Sun	3 ^s 23;21,37 ^o	3 ^s 23;21,14 ^o (−0;0,23 ^o)	9;20 ^m	3 ^s 23;22,23 ^{oa} (+0;0,46 ^o)	18;40 ^m
Moon (long.)	4 ^s 0;37,39	4 ^s 0;32,32 (−0;5,7 ^o)	9;19 ^m	4 ^s 0;47,54 (+0;10,15 ^o)	18;40 ^m
Moon (anom.)	3 ^s 18;1[8],45 ^b	3 ^s 18;18,39 (−0;0,6 ^o)	11;1 ^m	3 ^s 18;23,54 (+0;5,9 ^o)	9;27 ^m
Saturn (long.)	3 ^s 26;17,17	3 ^s 26;17,15 (−0;0,2 ^o)	23;54 ^m	3 ^s 26;17,17	***
Jupiter (long.)	11 ^s 0;11,2	11 ^s 0;11 (−0;0,2 ^o)	9;37 ^m	11 ^s 0;11,5 (+0;0,3 ^o)	14;27 ^m
Mars (long.)	7 ^s 0;37,37	7 ^s 0;37,25 (−0;0,12 ^o)	9;10 ^m	7 ^s 0;35,31 (−0;2,6 ^o)	1;36,11 ^h
Venus (anom.)	1 ^s 15;10,18	1 ^s 14;30 (−0;40,18 ^o)	26;8,48 ^h	1 ^s 14;30,43 (−0;39,35 ^o)	25;41,14 ^h
Mercury (anom.)	2 ^s 13;18,24	2 ^s 13;25,29 (+0;7,5 ^o)	54;43 ^m	2 ^s 13;29,6 (+0;10,42 ^o)	1;22,40 ^h
Head of Aries (trepidation)	0 ^s 3;53,55	0 ^s 3;53,55		0 ^s 3;53,55	

a In Ibn al-Bannā’s *Minhāj* we actually read 3^s 19;22,23^o but, as the solar equation table is displaced by 4^o, I have added 4^o also to the position at epoch.
b 3^s 18;13,45^o in the ms.

containing the canons and/or the corresponding tables, and a similar list is given by Fritz Pedersen in his masterful critical edition of the corpus.¹⁵¹ As an example of the diffusion of this work, we should remember that the *Toledan Tables* were translated into Greek ca. 1330–40,¹⁵² and that Geoffrey Chaucer (1340?–1400), an author who knew and used the *Alfonsine Tables*, gives a peculiar description of the Toledan set in relation to the preparation of a magical act in his *Canterbury Tales*:

151 Pedersen, 2002.
152 Pingree, 1976 a.

His tables Toletanes forth he broght,
 Ful wel corrected, ne ther lakked noght,
 Neither his collect ne his expans yeres,
 Ne his rotes ne his othere geres,
 As been his centres and his arguments,
 And his proporcionels convenients
 For his equacions in every thing.¹⁵³

The *Toledan Tables* are usually considered to be the result of the collective work of a Toledan team (*al-jamā'a al-tulayṭulīyya* in Ibn al-Hā'im's terminology). The sources which bear witness to this interpretation are late. One of them is Isaac Israeli, the Jewish astronomer from Toledo, in his *Yesod ha-'Olam* (written in 1310), who stresses the astronomical research undertaken by a group of twelve astronomers, most of them Muslims although there were also a few Jews (?). Astronomical observations were made and the astronomer and instrument maker Ibn al-Zarqālluh (d. 1100) had an outstanding role within the team. However, this source does not mention that the compilation of a set of astronomical tables was the result of this research.

A second interesting source is a short passage, found in a fifteenth-century manuscript containing a translation, attributed to Johannes Hispanus, of the canons of the *Toledan Tables*. In it we find confirmation of an important part of Isaac Israeli's message: the tables are the result of a research group, led by Šā'id; inside this group, Ibn al-Zarqālluh was particularly important:

Has tabulas composuit Abensahet iudex regis Maymon Toletto, eius discipulus Arzachel et alii cum eo, sed Arzachel regebat instrumenta et dirigebat considerationes.¹⁵⁴

A third source is also mentioned by Richter-Bernburg, in his defence of the role played by Šā'id in the compilation of the tables: around 1150, Robert of Ketton used the Toledan canons attributed to Gerard of Cremona and stated that its author was "ebenzaet toletanus". All this evidence leads Richter-Bernburg to consider that Isaac Israeli's account, although embellished, is basically true and that Johannes Hispanus' Latin passage was probably found in the Arabic original. It also seems that Isaac Israeli's text intends to emphasise a certain parallelism between the teams of astronomers that made observations in

153 *Canterbury Tales*, ed. W.W. Skeat, repr. London, 1961, p. 451 (the passage corresponds to *The Franklin's Tale*).

154 Millás, 1943–50, pp. 12–13; Richter-Bernburg, 1987, pp. 374–5.

Baghdad (828–9) and Damascus (831–2), and finally compiled the *Mumtaḥan zīj*, and the Toledan team and tables. In both cases, astronomers who were also instrument-makers were important members of the research team.

The main difficulty with this interpretation lies in the fact that Šāʿid did not mention the *Toledan Tables* in his *Ṭabaqāt al-umam*, written in 1068, and that he died in July 1070. If our *qāḍī* was indeed the patron and promotor of the tables, these must have been compiled, hurriedly, between 1068–70. This is Richter-Bernburg's suggestion and the inner characteristics of our source do not contradict this hypothesis, as we will see below. Besides, we can also accept that observations were made before 1068 (see 6.1) and that the final compilation of the *zīj* could have been made after Šāʿid's death.

A different explanation was suggested in a paper published by Berrani and myself in 1999,¹⁵⁵ as a result of the new information gathered from Ibn al-Hāʾim's *Kāmil Zīj* (6.2.5 and 7.2.6) and al-Istijī's *Risāla fī l-tasyīrāt wa-maṭāriḥ al-shuʿāʾāt* (6.2.4). The former ascribes to al-Istijī an unnamed *zīj* in which the mean motions had been established by this author and were used, in Ibn al-Hāʾim's own time, by the anonymous author of the *Muntakhab zīj* together with al-Battānī's equations. This combination of apparently original mean motions and al-Battānī's equations coincides well with the contents of the *Toledan Tables*. As for the chronology of this *zīj*, Ibn al-Hāʾim believes that al-Istijī's *zīj* was compiled some 150 years before the *Muntakhab zīj*, written in Ibn al-Hāʾim's own time: assuming that he is using lunar years and knowing that the *Kāmil zīj* was finished ca. 601/1205, the date of al-Istijī's *zīj* must be about 451/1059.

Some more interesting information can be obtained in al-Istijī's *risāla* in which the author insists, in several passages, on the existence of "our corrected *zīj*" (*zīj-u-nā al-muṣaḥḥaḥ*) which allowed the Toledan astronomers to reach "a satisfactory and correct result" and to obtain what they were looking for "concerning the irregularities (*ikhtilāf*) in the motion of the planets and the fixed stars". Al-Istijī criticises other *zīj*es which are basically incorrect and cause errors in astrological predictions. Only the *Muṣaḥḥaḥ zīj* gives accurate positions for the planets and good results for lunar longitudes, due to the correctness of the lunar equations: as a result, it is the only *zīj* with which the results of the two animodars (calculated following the techniques of Ptolemy and Hermes) (3.2.4) coincide. As he says towards the end of his *risāla*: "in the animodar and *tasyīr* (3.2.3.2), there are indications that confirm the correctness of the *zīj* we are using".

155 Samsó & Berrani, 1999, pp. 296–8, and Samsó & Berrani, 2005, pp. 166–167.

As a result of this information, our problem is to decide whether the unnamed *zīj* of al-Istijī and “our corrected *zīj*” are one and the same source. Mercè Comes decided in favour of this identification and rejected our hypothesis in which we suggested that *zīj-u-nā al-muṣaḥḥaḥ* is a reference to the *Toledan Tables* (see 6.2.4). Her main argument lies in the fact that the few data given by Ibn al-Hā'im on the values of trepidation used by al-Istijī in his *Risālat al-iqbāl wa l-idbār* disagree with those computed with the Toledan trepidation tables. Therefore, she concludes that the tables of al-Istijī's *risāla* must be the same as those found in al-Istijī's *zīj* and, consequently, in the *Muṣaḥḥaḥ zīj*, and denies the identification of the latter *zīj* with the *Toledan Tables*.¹⁵⁶

In spite of her argument, I still believe that *zīj-u-nā al-muṣaḥḥaḥ* might refer to the *Toledan Tables* and that there is no conclusive reason for its identification with al-Istijī's lost *zīj*. Besides, the use of the first person plural (*zīj-u-nā* = our *zīj*) in a *risāla* dedicated to *qāḍī* Ṣā'id implies that the latter participated actively in the preparation of the work, something which refers us to the *Toledan Tables*. See, however, 7.2.4.2.2.1, where I suggest the possibility of a revision made some years later: the *Muṣaḥḥaḥ zīj* could have been a first draft of the tables completed when Ṣā'id was still alive.

7.2.4.2 The Contents of the *Toledan Tables*

Millás made a preliminary analysis of the tables based mainly on its canons, and concluded that they were strongly influenced by al-Khwārizmī-Maslama's *zīj* and, to a lesser degree, by al-Battānī's *al-Zīj al-Ṣābi'*.¹⁵⁷ A deeper study, in particular of the numerical tables, was later undertaken by Toomer in his *Survey*. He begins by stating that the manuscripts give, under the title of *Toledan Tables*, a patchwork of tables containing many additions to the primitive set. Giving up all hope of being able to determine the composition of the original tables with any precision, he undertakes a description of the extant tables including an analysis of its sources, leaving aside the problem of their authors and origin.

Our understanding of this set of tables has been considerably improved by Fritz Pedersen's great critical edition of the corpus. Although the manuscript tradition is a pure Latin one,¹⁵⁸ the Arabic originals being apparently lost, Pedersen has found traces of a revision of the tables which can be dated ca. 1110 (some ten years after the death of Ibn al-Zarqālluh in 1100) in an early

¹⁵⁶ Comes, 2001 pp. 318–322.

¹⁵⁷ Millás, 1943–50, pp. 37–71.

¹⁵⁸ Pedersen, 2002, I, p. 11.

Latin copy:¹⁵⁹ two horoscopes probably for the latitude of Toledo dated 1110 and 1106¹⁶⁰ and a star table¹⁶¹ with an increment of 14;55° on Ptolemaic longitudes and in which the date is 1422 Alexander/1110–11 (Table 13A). I will discuss this latter topic below but, given the fact that Toledo was conquered by Alfonso VI in 1085, the existence of horoscopes cast for that latitude at a time when the city was no longer under Muslim authority poses the very interesting problem of the survival of Islamic astronomy in Toledo after the Castilian conquest.

7.2.4.2.1 *The Canons*

Pedersen has established the existence of three sets of canons:

- 1) *Ca* (“Scito quod annus”),¹⁶² based on al-Battānī, carrying a plausible ascription to Ibn al-Zarqālluh.¹⁶³
- 2) *Cb* (“Quoniam cuiusque actionis”), the “vulgate”, a revision of *Cc* with some Christian adaptations: a previous edition of this text had been published by Pedersen himself in 1987.¹⁶⁴ This collection has been attributed to Ibn al-Zarqālluh as the author and Gerard of Cremona as its translator, although, according to Pedersen, the Latin original text does not justify these attributions.¹⁶⁵
- 3) *Cc* (“the archaic version”), modelled on al-Khwārizmī’s *Sindhind*, but also strongly influenced by al-Battānī. In 1992 Pedersen published an edition of a passage of the canons (*Cc* 123–212)¹⁶⁶ (7.2.1.1) and showed that *Cc* contains two parts: the first one depends on a version of al-Khwārizmī’s rules which corresponds to fragments of Ibn al-Muthannā’s commentary, i.e., a version of the *zīj* independent of Maslama’s revision, while the second draws on al-Battānī and, in some cases, perhaps on Ibn al-Zarqālluh’s *Almanac* or similar texts.¹⁶⁷

Cb and *Cc* derive ultimately from the same Arabic exemplar, *Cb* being a thorough stylistic revision from the Latin text of *Cc*.¹⁶⁸ The attribution of the

¹⁵⁹ Pedersen, 2002, I, p. 15 (fn. 1) and III, p. 759.

¹⁶⁰ North, 1995.

¹⁶¹ See on this kind of tables Pedersen, 2002, IV, pp. 1489–1508.

¹⁶² Edition and English translation in Pedersen, 2002, I, pp. 216–311.

¹⁶³ Several manuscripts have the heading: “Incipiunt lectiones tabularum secundum Arzachel” (Pedersen, 2002, I, p. 216). On the attribution of the Latin translation of canons *Ca* to John of Spain, see Pedersen, 2002, I, pp. 194–5.

¹⁶⁴ Pedersen, 1987.

¹⁶⁵ Pedersen, 2002 II, p. 331.

¹⁶⁶ Pedersen, 1992.

¹⁶⁷ Pedersen, 2002, I, p. 18.

¹⁶⁸ Pedersen, 2002, II, p. 337.

authorship of *Cb* to Ibn al-Zarqālluh and of the Latin translation to Gerard of Cremona seems tenuous.¹⁶⁹ The earliest dated reference to the Latin Toledan Tables corresponds to 1141,¹⁷⁰ but the oldest MSS of the three versions date from the late twelfth or early thirteenth century, canons *Cb* dominating the scene in the late thirteenth century. They were still being copied in the fifteenth century and from around 1320, they entered into competition with the *Alfonsine Tables*.

The three sets of canons share characteristics showing their Hispanic origin: in the chronological chapters the Christian era is used,¹⁷¹ as well as the Hispanic era in canons *Cc*;¹⁷² the extra day in bissextile years is inserted at the end of December;¹⁷³ the radices are calculated for the meridian of Toledo, placed at a distance of $4\frac{1}{10}$ hours from the middle of the world (i.e., Arin): this amount is equivalent to $61;30^\circ$ from Arin and $28;30^\circ$ from the meridian of water.¹⁷⁴ Neither of them contains any reference to the changes introduced by Ibn al-Zarqālluh in solar and lunar theory (see chapter 6) and canons *Ca* omit the treatment of trepidation, which appears in canons *Cb*¹⁷⁵ and *Cc*,¹⁷⁶ in agreement with the model described in the *Liber de motu* (6.2.1). In spite of the strong influence of al-Khwārizmī in *Cb* and *Cc*, we often find procedures related to both Indian and Ptolemaic traditions: thus, to obtain sines, versed sines and declinations, we find descriptions of kardagas of 15° ($R = 150'$)¹⁷⁷ and tables computed for each degree and for $R = 60P$;¹⁷⁸ planetary latitude theory derived from al-Khwārizmī¹⁷⁹ is followed by al-Battānī's treatment of the same topic.¹⁸⁰ In one case we find an attempt to compare the results of the computations of luni-solar conjunctions and oppositions using al-Battānī's or al-Khwārizmī's methods:¹⁸¹

169 Pedersen, 2002, II, pp. 331 and 338.

170 Pedersen, 2002, III, p. 754.

171 Pedersen, 2002, I, p. 230; II, pp. 384–7, 393–5, 396, 398, 400. For the tables see III, pp. 878–883, 897–8, 900–1, 907–10, 912–921, 928–9, 940–3.

172 Pedersen, 2002, II, pp. 598–602. For the tables see III, pp. 901–2, 918.

173 Pedersen, 2002, I, p. 222; II, pp. 480–2; III, p. 936.

174 Pedersen, 2002, I, p. 250; II, p. 430. Same values in an appendix to canons *Cc* (Pedersen, 2002, II, p. 730) which also gives a latitude of $39;54^\circ$ for Toledo.

175 Pedersen, 2002, II, pp. 478–80, 540.

176 Pedersen, 2002, II, pp. 602–609, 664–71, 686–7.

177 See tables of sines and declinations for $R=150$ in Pedersen, 2002, III, pp. 950–2, 967–8.

178 Pedersen, 2002, II, pp. 404–408, 486–494. For the tables see III, pp. 957–964.

179 Pedersen, 2002, II, pp. 634–637.

180 Pedersen, 2002, II, pp. 678–683.

181 Pedersen, 2002, II, pp. 690–691.

“And when you have computed the conjunction or opposition by means of this book, which is by Albattani, then, if you compute it by means of the tables of Alkhwarizmi, and one of the results is one hour or more later than the other, then repeat <the computations> you suspect of being in error. But if there is less than one hour between them, it will be <a> possible <result>, if God wills” (Pedersen’s translation).

In all cases the computation of planetary longitudes follows the Ptolemaic procedures.¹⁸²

7.2.4.2.2 *Numerical Tables*

The numerical tables appear in vols. III and IV of Pedersen’s edition and they are classified¹⁸³ into different kinds corresponding to: A, chronology; B, trigonometry and spherical astronomy; C, mean motions of the Sun, the Moon, node and planets; D, apogees, nodes, daily mean arguments; E, equations of the Sun, the Moon and planets; F, planetary latitudes; G, mean syzygies; H, parallax; J, eclipses; K, visibility of the lunar crescent; L, fixed stars; M, geographical; N, projections of rays; O, planetary visibility and retrogradation; P, eighth sphere; Q, revolution of years; R, astrology; S, almanacs and ephemerides; T, calendars and computus; U, various auxiliary tables. In relation to category S, it is interesting to remark that the term *almanac* is not always applied to a perpetual almanac such as Ibn al-Zarqālluh’s but also to a set of ephemerides calculated for a lunar or a solar year (7.7.2).¹⁸⁴

Altogether, the detailed analysis of sources made by Toomer and Pedersen establishes clearly that most tables derive from al-Battānī’s or al-Khwārizmī’s *zīj*es, and, consequently, that the *Toledan Tables*, in spite of their reputation and diffusion, are an almost complete fiasco. There are, however, a few instances in which we might find the result of original work made by the Toledan team of astronomers. Thus, we have a table of oblique ascensions calculated for an obliquity of the ecliptic of 23;33° or 23;33.30° and a local latitude of 39;54° (Toledo),¹⁸⁵ a table for the division of the houses of the horoscope computed also for the same parameters,¹⁸⁶ and a table of the length of daylight through-

182 Pedersen, 2002, II, pp. 438–442.

183 Pedersen, 2002, I, pp. 18–20.

184 Pedersen, 2002, II, pp. 542–6.

185 Pedersen, 2002, III, pp. 997–1004.

186 Pedersen, 2002, III, pp. 1093–1100. This table was analysed in North, 1986, pp. 16–7. Another table for the division of the houses is also attributed to Toledo, although the computed latitude lies between 38;22° and 38;43° and, therefore, corresponds to Córdoba: cf. Pedersen, 2002, III, pp. 1078–92.

out the solar year in Toledo.¹⁸⁷ We also find a table of syzygies for Toledo, in which the radices and the velocity constants agree with the Toledan mean motions (see 7.2.4.2.2.1 and 7.2.4.2.2.3), and a parallax table for the same city using a latitude of 39;54° but a Ptolemaic obliquity (23;51°).¹⁸⁸ Finally a table giving eclipse digits, minutes of immersion and half delay in totality appears under the title *Tabula Toletana* or *Tabula Azarchelis* and is also preserved in Ibn al-Zarqālluh's *Almanac*.¹⁸⁹

In the following pages I will limit myself to an analysis of the tables used for the computation of solar, lunar and planetary longitudes.

7.2.4.2.2.1 Mean Motions

Already in 1968 Toomer¹⁹⁰ made an accurate study of the sources used for the compilation of the *Toledan Tables* and established that only the mean planetary motions could be considered original, while the rest of the materials were the result of hasty adaptations of the corresponding tables in the *zīj*es of al-Khwārizmī and al-Battānī. This general idea was fully confirmed by Pedersen¹⁹¹ who states that the *Toledan Tables*, in a strict sense, comprise the planetary mean motions and the syzygy tables.¹⁹² Although Theon's *Handy Tables* ("Zaiun Alexandrinum")¹⁹³ are mentioned, their influence (quite obvious in the planetary equation tables) was indirect and was exerted through al-Battānī's *zīj*.¹⁹⁴ On the other hand, Pedersen also confirms Mercier's discovery¹⁹⁵ that only the mean solar motion can be considered original in the *Toledan Tables*, for "the differences between the tropical longitudes of the Sun and planets in the *zīj* of al-Battānī are respectively equal to the difference between the sidereal longitudes of the Sun and planets in the *Toledan Tables*". This idea can also be checked, as Pedersen does,¹⁹⁶ by a direct comparison of the mean motion parameters in both *zīj*es. I have done this in table 7.8 in which I use Pedersen's Toledan parameters¹⁹⁷ compared to those of al-Battānī, as calculated by E.S. Kennedy.¹⁹⁸

187 Pedersen, 2002, III, pp. 1125–7.

188 Pedersen, 2002, IV, pp. 1353–8.

189 Pedersen, 2002, IV, pp. 1475–8; Millás, 1943–50, p. 231.

190 See Toomer, 1968, p. 44.

191 Pedersen, 2002, III, pp. 1139 ff.

192 Pedersen, 2002, I, pp. 16–17.

193 Pedersen, 2002, II, p. 521.

194 Pedersen, 2002, I, p. 47.

195 Mercier, 1987, pp. 104–7; Mercier, 1996, pp. 300, 305–13.

196 Pedersen, 2002, III, p. 1140.

197 Very similar values in Toomer, 1968, p. 44.

198 These parameters were established by Kennedy during one of his stays in Barcelona, for use in his computer programme TOLLN, which calculates planetary positions according to the *Toledan Tables*.

TABLE 7.8 Toledan mean motions

Planet	Mean motion (Tol.)	Mean motion (Bat.)	Batt – Tol
Sun	0;59,8,11,28,27,30 ^o	0;59,8,20,46,54,56,55,12	0;0,0,9,18,27,26,55,12
Moon long.	13;10,34,52,48,47,8	13;10,35,2,7,15,30	0;0,0,9,18,28,22
Moon anom.	13;3,53,56,17,58,12	13;3,53,56,17,51,14	-0;0,0,0,0,6,58
Node	0;3,10,46,42,33,17	0;3,10,37,23,58,31	-0;0,0,9,18,34,46
Saturn	0;2,0,26,35,17	0;2,0,35,50,27,26	0;0,0,9,15,10,26
Jupiter	0;4,59,7,37,19,23	0;4,59,16,54,58,16	0;0,0,9,17,38,53
Mars	0;31,26,32,15,17,1	0;31,26,40,15,13,57	0;0,0,7,59,56,56
Venus anom.	0;36,59,29,27,29,29	0;36,59,29,27,43,15	0;0,0,0,0,13,46
Mercury anom.	3;6,24,7,39,31,26	3;6,24,7,45,13	0;0,0,0,5,41,34

In Table 7.8 one can easily see that there is a difference of approximately 0;0,0,9,18° between the mean motion per day obtained from al-Battānī's *zīj* and from the *Toledan Tables*. This difference affects the motion in longitude of the Sun, Moon, lunar nodes and planets, with the sole exception of Mars which is either the result of an error or derives from a different source. As we will see in table 7.9, the same exception is found in the position of Mars at the beginning of the Hijra. As the difference of 0;0,0,9,18° seems to be the result of precession, mean motions in anomaly (the Moon, Venus and Mercury) seem to be nearer al-Battānī's parameters and do not show constant differences.

Some more information on the mean solar motion can be obtained from the values of the *revolutio anni* mentioned in the *Toledan Tables*. We have already seen that Ibn al-Kammād attributes a *revolutio anni* of 92;24° to Ibn al-Zarqālluh (6.3.3.1) which yields a daily mean motion of 0;59,8,11,28,25°, very near the parameter obtained from the mean solar motion tables.

Several other values are also given in Pedersen's edition,¹⁹⁹ although only one seems to be related to the tradition of the *Toledan Tables*: CG11, which gives 92;20,55...°, equivalent to 6;9,23,43...^h or 0;15,23,29,17...^d. The corresponding mean motion per day (0;59,8,11,28,27,35°) is very near the aforementioned value squeezed from the Toledan mean solar motion tables. It is interesting to note that similar values can be found in a set of tables ascribed to Ibn al-Hā'im in the Hyderabad MS of Ibn Ishāq's *zīj*:²⁰⁰ here the *revolutio anni* is 92;20,56,40,12°, 6;9,23,46,40,48^h, or 0;15,23,29,26,42^d. This is not the only value

199 Pedersen, 2002, IV, p. 1567.

200 'Abd al-Rahmān, 1996 a, pp. 372–5.

quoted by Ibn al-Hā'im who, in the text of his canons, says that the length of the solar year for the beginning of the seventh/thirteenth century²⁰¹ was 365;15,23,37,30^d (which fits the values of the *revolutio anni* extant in the same text, 92;21,45^o and 6;9,27^h), very close to the value ascribed to Ibn al-Zarqālluh (365;15,24^d). Finally, in Pedersen's edition,²⁰² we find a set of tables of the *revolutio mensium*, which are the result of the division of a tropical year of 365^d 5;47,30^o into 13 equal "months".

7.2.4.2.2.2 Trepidation

As we have already seen in 6.2.1, the *Toledan Tables* contain the same set of trepidation tables found in the *Liber de motu*.²⁰³ It is interesting to note that canons *Ca* contain no allusion to precession/trepidation, except for one canon on solar declination:²⁰⁴ "intra cum loco solis aequato, cuius initium est a capite arietis". If one takes this expression seriously, it implies the declination which corresponds to a sidereal longitude of the Sun, measured from the [moving] Head of Aries. Pedersen is not happy with this interpretation, for he translates:²⁰⁵ "from the vernal point (= head of Aries)". Trepidation is dealt with in canons *Cb* and *Cc*.²⁰⁶ In the former we find a peculiar expression which seems to show the influence of Andalusī astronomical terminology: at the end of the computation of the solar longitude,²⁰⁷ we read "et tunc habebis locum solis certissime cuius initium erit a initio arietis [in 8'a sphaera]". In this context *certissime* makes me think of an Arabic *dhātīyya* [= sidereal]. It is also interesting that canons *Cb*²⁰⁸ refer to a tropical ascendent ("ascendens cum motu 8'vi circuli"), in disagreement with the standard tradition of Andalusī-Maghribī astrology which tends to use sidereal ascendants.

It is legitimate, therefore, to use these tables in order to justify a difference in longitude of 0;0,0,9,18^o between al-Battānī's tropical mean motion parameters and the Toledan sidereal ones by computing the value of trepidation for a reasonable date comprised between 1060 and 1080. After several attempts, a satisfactory result has been obtained for year 1078 and, specifically, for 14 July 1078,

201 'Abd al-Rahmān, 1996 a, pp. 370–1.

202 Pedersen, 2002, IV, pp. 1586–89.

203 Pedersen, 2002, IV, pp. 1542–1566.

204 Pedersen, 2002, I, 232–3.

205 Pedersen, 2002, I, p. 233, fn. 2.

206 Pedersen, 2002, II, pp. 478–9 and 686–7.

207 Pedersen, 2002, II, pp. 436–437.

208 Pedersen, 2002, II, p. 533.

456 Julian years after the Hijra.²⁰⁹ The Julian Day Numbers for the two dates involved are:

$$\begin{array}{ll} 14.7.622 & 1.948.438 \\ 14.7.1078 & 2.114.992 \end{array}$$

The difference between these two dates amounts to 166.554 days. Using the parameters of the *Liber de motu*, which are also found in the *Toledan Tables*²¹⁰ (see 6.2.1):

$$\begin{aligned} i &= 0;0,0,52,28,38,8^{\circ} \cdot 166.554 + 1;34,2^{\circ} = 42;1,54^{\circ} \\ & \text{(1;34,2}^{\circ} \text{ being the value of } i \text{ for the beginning of the Hijra)} \\ \Delta\lambda &= \sin^{-1}(\sin 42;1,52^{\circ} \cdot \sin 10;45^{\circ}) = 7;10,27^{\circ} \\ & \text{which is the value of precession for 1078, and} \\ 7;10,27^{\circ} : 166.554 &= 0;0,0,9,18,14^{\circ} \end{aligned}$$

1078 is, approximately, eight years after the death of *qāḍī* Šā'id and it implies that our *zīj* might have been finished, or revised, after 1070. This leads me to modify my previous hypothesis about the identification of the *Toledan Tables* with the *Muṣaḥḥaḥ Zīj* mentioned by al-Istijī. As I have already suggested, the *Muṣaḥḥaḥ zīj* could have been a first draft of the *Toledan Tables*, later modified by Šā'id's team of Toledan astronomers.

To sum up, there is nothing new in the mean motion values of the *Toledan Tables* and we cannot even be sure of the originality of their mean solar motion tables, in spite of the 25 years of solar observations carried out by Ibn al-Zarqālluh which led him to introduce modifications in the Ptolemaic solar model (6.3.3). Besides, we have already seen (6.3.3.1) that the parameter of the mean solar motion was not changed by Ibn al-Zarqālluh in the later stages of his life and that it was adopted, with minimum changes, by the later Maghribī tradition.

Trepidation may also be connected with the precessional increments of star longitudes. Pedersen²¹¹ edits several sets of star tables which seem to correspond to early Toledan (or derived) material. In them, two main different increments on Ptolemaic longitudes are used:

209 This approximate date is close to that of Ibn al-Zarqālluh's observation of the autumnal equinox in 1075 (see 6.3.1), which is also the year calculated by Mercier (1996, pp. 306–321).

210 Pedersen, 2002, IV, pp. 1545 and 1554.

211 Pedersen, 2002, IV, pp. 1489–1508.

1) $14;7^{\circ}$ in table LA11, of which a close Arabic cognate was published by Kunitzsch,²¹² the latter being dated in 459/1066–67. This date makes sense, for it is confirmed by the “corrected” longitudes of Qalb al-Asad used by Ibn al-Zarqālluh in his treatise on the motion of the fixed stars, to establish the accurate values of precession (6.2.2.2), which are $122;26^{\circ}$ for the time of Ptolemy (139) and $136;35^{\circ}$ for his own time (1075), the difference being $14;9^{\circ}$.

2) $14;55^{\circ}$ in tables LA12, LA13 and LA13a, although LA12 also has $15;7^{\circ}$ in 12 cases out of 35. One of the manuscripts containing LA12 gives 577H/1181–1182 as a date, while table 13a is dated 1422 Alexander/1110–11. Following a suggestion by Kunitzsch, Pedersen proposes that 1110–11 is the date to which an increase of Ptolemaic longitudes of $14;55^{\circ}$ corresponds, while 577H could be corrected to 527/1132–33, a date to which an increase of $15;7^{\circ}$ could be assigned. Surprisingly, table LA13 includes columns showing the maximum altitude of the star and its half daily arc, implying a latitude of $39;54^{\circ}$ (Toledo). This latitude is unusual when related to a date ca. 1110 (later than 1085, the year Toledo was conquered by Alfonso VI). However, the suggested dates (1110–11 and 1132–33) could fit Ibn al-Kammād, who was probably Ibn al-Zarqālluh’s disciple and who was active in Córdoba in 1116–17. He might have corrected the star longitudes in a Toledan table but did not bother to do the same with the maximum altitude of stars or their half daily arcs.

My impression is, however, that the increment of $14;55^{\circ}$ may correspond to 527H/1132–33. The longitude of Qalb al-Asad in table 13A is Leo $17;25^{\circ}$ (Ptolemy, Leo $2;30^{\circ}$: dif. $14;55^{\circ}$). The longitude of this star in Western Islamic tables for precession 0° is Leo $9;8^{\circ}$ or $9;18^{\circ}$.²¹³ The absolute value of precession implied is, therefore, $8;17^{\circ}$ or $8;7^{\circ}$. For the beginning of year 527 H and using Ibn al-Zarqālluh’s tables based on his third model of trepidation, I obtain a value of $8;4,4^{\circ}$, not far from $8;7^{\circ}$.

3) There is, finally, a star table (LA14) in which the star longitudes do not seem to be related to the Ptolemaic ones by adding a constant of precession. Some of them (Qalb al-Asad, for example, the longitude of which is Leo $9;10^{\circ}$) seem to derive from a table which computed star longitudes for precession 0° . A column includes values of the half daily arc for each star for a latitude between $33;30^{\circ}$ and 34° (Fez?).

7.2.4.2.2.3 Epoch Positions

The longitude radices for Hijra are more difficult to justify. They are comprised between $0;23,45^{\circ}$ and $0;24,18^{\circ}$ in relation to those of al-Battānī, as we can see

²¹² Kunitzsch, 1980.

²¹³ See Díaz-Fajardo, 2001, p. 56.

TABLE 7.9 Radix positions at epoch in the *Toledan Tables* and al-Battānī's *zīj* (the latter corrected for the meridian of Toledo)

Planet	Radix Toledan Tables	Radix Battānī (Toledo)	Battānī – Tol. Tables
Sun	113;41,11°	114;5,29°	0;24,18°
Moon long.	120;58,18	121;22,35	0;24,17
Moon anom.	108;8,39	108;8,39	0;0
Node	234;9,55	233;45,42	-0;24,13
Saturn	115;51,15	116;15	0;23,45
Jupiter	331;39,37	332;4	0;24,23
Mars	211;24,59	211;52	0;27,1
Venus anom.	45;28,37	45;12	-0;16,37
Mercury anom.	73;46,18	73;46	-0;0,18

in table 7.9, where al-Battānī's radices have been corrected for the longitude of Toledo assuming a time difference of three hours between Raqqa and Toledo.²¹⁴ However, I have not been able to obtain 0;24° for the beginning of Hijra with the trepidation tables extant in the Toledan collection (considered by Pedersen to be probably the result of the work of the Toledan team) with which the calculated value amounts to 0;17,32°. Other attempts, made with al-Istijī's parameters (see 6.2.4) and with the different models described by Ibn al-Zarqālluh in his treatise on the motion of the fixed stars (6.2.2), have also been unsuccessful. Whatever the case, we can also see that the radix for the longitude of Mars increases the difference to 0;27° instead of 0;24°. The epoch positions for the anomaly of the Moon and Mercury are identical or very near those of al-Battānī, while that of Venus could derive from a different source.

7.2.4.2.2.4 Apogees

The Toledan positions of the solar and planetary apogees, compared to those found in al-Khwārizmī's and al-Battānī's *zīj*es, are reproduced in table 7.10, where we can see that the Toledan Tables and al-Khwārizmī give these positions for the beginning of the Hijra, while al-Battānī's correspond to year 879 AD. We can easily see that the Toledan solar (77;50°) and Jupiter's apogee (164;28°) are very near al-Khwārizmī's (77;55° and 164;30°). The differences between al-Battānī's positions for 879 AD and the Toledan ones are 4;24° or 4;23° for the Sun, Venus and Saturn, which leads me to infer that these Toledan

²¹⁴ Pedersen, 2002, III, p. 1141.

apogees derive from al-Battānī with a correction for precession.²¹⁵ To check this hypothesis, using a JDN 1.948.438 for 14.7.622, and a JDN 2.042.307 for 14.7.879, we can calculate a difference of 93.969 days, and

$$i = 93.869 \cdot 0;0,0,52,28,38,8^{\circ} + 1;34,2^{\circ} \text{ (i at Hijra)} = 24;22,22^{\circ}$$
$$\Delta\lambda = \sin^{-1} (\sin 24;22,22^{\circ} \times \sin 10;45^{\circ}) = 4;24,53^{\circ}$$

I have no explanation for the origin of the apogees of Mars and Mercury and I should also say that the sidereal solar apogee of the *Toledan Tables* (77;50°) is unrelated to the tropical apogee determined by Ibn al-Zarqālluh through observations made in 1074–5 (85;49°, see 6.3.2). The difference between the two positions is 7;59°, while a difference due to precession (7;7,58°, with the tables of the *Liber de motu*) and to the motion of the solar apogee (1;37,21°) yields a result of 8;45,19°. ²¹⁶

TABLE 7.10 Solar and planetary apogees^a

Planet	Toledan apogee	Khwārizmī's apogee	Battānī's apogee (879 AD)	Diff. Batt – Toledan
Sun	77;50 ^{ob}	77;55 ^o	82;14 ^{oc}	4;24 ^o
Saturn	240;5	244;55	244;28 ^d	4;23
Jupiter	164;30	172;32	164;28 ^e	–0;2
Mars	121;51 ^f	128;24	126 ;58 ^g	5;7
Venus	77;50	81;15	82;14 ^{oh}	4;24
Mercury	197;30	224;54	201;28 ^{oi}	3;58

- a Toledan apogees in Pedersen, 2002, III, p. 1226; Toomer, 1968, p. 45.
- b 77;50° in *Ca* and *Cb* (Pedersen, 2002, I, p. 256 and II, pp. 434–6); 77;55° in an appendix of canons *Cc* (II, p. 724); 77;50,10° in the Toulouse Tables (Pedersen, 2002, III, p. 1223).
- c Nallino, 1899–1907, I, pp. 44 and 214.
- d Nallino, 1899–1907, II, p. 108.
- e Nallino, 1899–1907, II, p. 114.
- f Variants: 121° and 121;50°: Pedersen, 2002, III, p. 1224.
- g Nallino, 1899–1907, II, p. 120.
- h Nallino, 1899–1907, II, p. 126.
- i Nallino, 1899–1907, II, p. 132.

215 Some mss. ascribe Toledan apogees to al-Battānī, but this is ruled out by Pedersen (2002, III, pp. 12252 ff.).

216 Three mss. contain tables for the motion of the solar (?) apogee (“Tabula motus augis in circulo signorum”) with a mean motion parameter which agrees with Ibn al-Zarqālluh’s 1°/279 years: see Pedersen, 2002, III, pp. 1228–30.

7.2.4.2.2.5 Solar, Lunar and Planetary Equations

As established by Toomer²¹⁷ and Pedersen, the equation tables derive from al-Battānī and, indirectly, from Ptolemy’s *Handy Tables*. The column giving the planetary first station derives from al-Khwārizmī’s *zīj*, although the underlying theory is Ptolemaic and one might also assume a derivation from al-Battānī.²¹⁸ In spite of the lack of originality of these tables, here I will reproduce the basic parameters in order to complete the survey of the Toledan tables for the computation of planetary longitudes. Interestingly, the parameters of the solar eccentricity and maximum equation of the centre disagree with those established ca. 1075 by Ibn al-Zarqālluh, as the result of his solar observations (see 6.3.1, 6.3.3.2, 6.3.3.4 and 6.3.3.5): $e = 1;58^p$ and maximum equation $= 1;52,42^\circ$.

TABLE 7.11 Solar, lunar and planetary equations

	Eccentricity	Max. centre eq.	Radius of epicycle	Max. anom. eq. (apogee)	Max. anom. eq. (perigee)
Sun ^a	2;4,45 ^p	1;59,10 ^o	—	—	—
Moon ^b	10;19	13;9	5;15 ^p	5;1 ^o	7;40 ^o
Saturn ^c	3;25	6;31	6;30	5;53	6;38
Jupiter ^d	2;45	5;15	11;30	10;35	11;35
Mars ^e	6;0	11;24	39;30	36;45	46;59
Venus ^f	1;15 ^g	1;59	43;10	44;49	47;15
Mercury ^h	3;0	3;2	22;30	19;2	23;53

a Pedersen, 2002, IV, pp. 1245–9.
b Pedersen, 2002, IV, pp. 1250–8.
c Pedersen, 2002, IV, pp. 1265–73.
d Pedersen, 2002, IV, pp. 1273–81.
e Pedersen, 2002, IV, pp. 1281–9.
f Pedersen, 2002, IV, pp. 1289–97.
g Pedersen, 2002, IV, p. 1290, although the values of the equation of the centre are calculated for an eccentricity of 2;4,45^p, like that of the Sun.
h Pedersen, 2002, IV, pp. 1298–1306.

217 Toomer, 1968, pp. 56–68.
218 Pedersen, 2002, IV, p. 1259.

7.2.5 *The zījēs of Ibn al-Kammād*

7.2.5.1 Ibn al-Kammād

I mentioned Abū Jaʿfar Aḥmad b. Yūsuf [b.] al-Kammād²¹⁹ on many occasions in chapters 1, 2, 3 and 6 (see especially 3.2.4.2, on his treatise on the animodar, and 6.2.3 on trepidation), an astronomer who flourished in Córdoba towards the end of the eleventh century and during the first half of the twelfth. His chronology was established by A. Mestres who, in the Tunisian recension of Ibn Ishāq's *zīj*, discovered a reference to a nativity horoscope cast by him in Córdoba and dated 510/1116–17, followed by a quotation from the same author dealing with the eight indicators that should be considered in the analysis of a nativity horoscope.²²⁰ Another date (480/1088) appears twice in the Madrid manuscript of the Latin translation of his *al-Zīj al-Muqtabas* (fol. 65v). Although we cannot be completely sure that the reference to this year really corresponds to the period of Ibn al-Kammād's activity, it fits the hypothesis that he was a direct disciple of Ibn al-Zarqālluh, as established in a marginal note found in a Latin manuscript of the *Toledan Tables*.²²¹ He is probably the student who helped Ibn al-Zarqālluh in the observations he made in Córdoba.²²² Whatever the case, Ibn al-Kammād mentions Ibn al-Zarqālluh (d. 1100) several times and Ibn Muʿādh (d. 1093) only once. Other influences are al-Khwārizmī, al-Battānī, the *Toledan Tables* and, surprisingly (see 7.1), the *Mumtaḥan zīj*.

7.2.5.2 The *zījēs*: Sources

7.2.5.2.1 *Muqtabas zīj*

Ibn al-Kammād is the author of three *zījēs* entitled *al-Kawr ʿalā l-dawr*, *al-Amad ʿalā l-abad* and the aforementioned *al-Muqtabas*, which seems to be a

219 Comes, *BA* vol. 3, pp. 732–734 and *BEA*, pp. 559–560. The different names of this astronomer mentioned in the sources were collected in Vernet, 1949, pp. 72–73: ʿAbd al-Šadiq Sufyān b. al-Kamad, Abū l-ʿAbbās Aḥmad b. al-Kammād, and Abū Jaʿfar/ Abū l-ʿAbbās Aḥmad b. Yūsuf al-Kammād.

220 Mestres, 1996, p. 404; Mestres, 1999, pp. 128 and 202 (paragraph 35/32).

221 After mentioning *qāḍī* Šāʿid and Ibn al-Zarqālluh, and the latter's migration to Córdoba after the conquest of Toledo by Alfonso VI of Castile, the text (ms. NL Paris 7281, fol. 30r) adds "Post uenit Alcamet discipulus et fecit tabulas persianas et 4 principia tabularum": see Millás, 1943–50, p. 14; Toomer, 1969, pp. 314–15.

222 Millás, 1943–50, p. 279. Millás translates "me ayudé con alumnos míos y personas técnicas de mi confianza" and, according to him, he was helped by several students. Teresa Martínez, who revised the Hebrew text of this passage, told me that the plural of "alumnos míos" should be replaced by a word in singular. This interpretation has been confirmed by B.R. Goldstein (private communication) who translates the passage as follows: "and I was helped in this by one of the scholars [*lit. men*] of investigation ...".

summary of the two previous ones.²²³ It was translated into Latin by Johannes de Dumpno, who finished the translation in Palermo on Friday 4August 1262, 19 Ramaḍān 658 H²²⁴ (fol. 18v). This work is extant in a single manuscript, Madrid NL ms. 10023.²²⁵ There is also a Hebrew translation of this *zīj*, by Solomon Franco (fl. 1375),²²⁶ and both versions have been analysed extensively by Chabás and Goldstein,²²⁷ who, in their second paper especially, carried out a thorough study of Ibn al-Kammād’s influence in later Iberian astronomical sources.²²⁸

Chapter 28 of *al-Muqtabas* on the projection of rays has been preserved, in Arabic, in mss. Algiers NL 1454²²⁹ and Escorial 939.²³⁰ To this, I should add that the anonymous Tunisian recension of Ibn Ishāq’s *zīj*, preserved in the Hyderabad manuscript, contains a certain number of quotations of the Arabic text of the *Muqtabas zīj*. I will give a few examples here:

TABLE 7.12

	Hyderabad	<i>Muqtabas</i>
Chronology	Chapter 2 ^a Ch. 3 ^b Ch. 6 ^c	Passages of chapters 2, 3 Ch. 4 Ch. 7, 8

a Mestres, 1999, pp. 14–7 (Arabic edition).
b Mestres, 1999, pp. 18–9 (Arabic edition).
c Mestres, 1999, pp. 25–8 (Arabic edition).

223 According to the prologue of its canons: “Iste canon extractus a canonibus Elemed Hale Elebed et Caurhale Eddaur” (ms. BN Madrid 10023, fol. 1r). According to the same prologue, the *Muqtabas* is a “canon abreviatum”, containing brief canons except in the treatment of solar and lunar eclipses and parallax, in which brevity could lead to confusion.

224 These two dates do not agree: 4 August 1260 was a Wednesday and the equivalent lunar date is 24 Sha’bān 658. Taking the Muslim date as the correct one, 19 Ramaḍān 658 corresponds to Saturday 28 August 1260.

225 See a description of this manuscript in Millás, 1942, pp. 231–247.

226 On this astronomer see Goldstein, 2013.

227 Chabás & Goldstein, 1994 and 2015 a. On Ibn al-Kammād’s method for finding the difference in time between mean and true syzygies and its influence on later *zīj*es, see Chabás & Goldstein, 1997, pp. 42–46.

228 Chabás & Goldstein, 2015 a, pp. 582–4.

229 Vernet, 1949, pp. 74–78, translates the Arabic text into Catalan and compares it to Johannes de Dumpno’s Latin translation.

230 See M. Comes’s entries on Ibn al-Kammād in *BA* and *BEA*.

TABLE 7.12 (cont.)

	Hyderabad	Muqtabas
Trigonometry and spherical astronomy	Ch. 19 ^d	Ch. 17
	Ch. 22 ^e	Ch. 20
	Ch. 23 ^f	Ch. 21
Houses of the horoscope	Ch. 27g	Ch. 24

d Mestres, 1999, pp. 96–9 (Arabic edition).
e Mestres, 1999, pp. 108–9 (Arabic edition).
f Mestres, 1999, pp. 110–4 (Arabic edition).
g Mestres, 1999, pp. 131–4 (Arabic edition).

7.2.5.2.2 *al-Kawr ‘alā l-dawr*

As for the other two *zīj*es, some information on *al-Kawr ‘alā l-dawr* can also be obtained from ms. Madrid NL10023: chapter 30 is the last canon of the *Muqtabas*, ending on fol. 18v, but it is followed by an appendix containing a new series of canons, also translated by Johannes de Dumpno and dated 1262 (fol. 23r). In spite of the explicit in fol. 23r, the text does not end here, because it is followed (fols. 23r–24v) by another chapter (“Ad inveniendos annos Xristi per annos arabum et econverso”) which ends with a couple of examples of calendar conversion for the years 1266 and 1268.

The first chapter of the appendix contains an introduction followed by the translation of chapter 60 (ms. Madrid NL, fols. 18v–19r), the last chapter of *al-Kawr ‘alā l-dawr*, dealing with the procedure to obtain the revolution of a nativity or a world-year, taking into account the change in solar eccentricity produced by Ibn al-Zarqālluh’s solar model (6.3 and see below). This topic had already been analysed in chapter 30 of the *Muqtabas* (ms. Madrid NL, fols. 18r and v), although in this case we have only simple instructions for the use of a table of *revolutio anni* without any reference to the changes in solar eccentricity. Chapter 60 is the only explicit quotation from *al-Kawr*, although the following chapter in this series (*Ad dirigendum solem*, ms. Madrid NL fols. 19r and v) could have the same origin, as it deals with the computation of solar eccentricity using the method described by Ibn al-Zarqālluh in his book “in anno solari”.

The rest of the chapters are more conventional and seem to be the result of the interest of the translator in obtaining information on topics which have not been treated in the *Muqtabas* and are mainly related to problems of spherical astronomy (degree of mediation of a star or planet, degree of the ecliptic which rises or sets together with a star or planet, day and night arcs, night hour

obtained from a star's altitude, etc.). Other chapters are astrological: length of pregnancy (animodar, ms. Madrid NL fol. 20r) (see 3.2.4.2) and planetary transits (Madrid NL, fol. 22v–23r).

The manuscript Catedral de Segovia 115 contains a brief Castilian text that begins “Este libro fizo maestre Yuçuf Benacomed sobre çircunferençia de moto sacado por tiempo seculo ...” which seems to refer both to *al-Kawr ‘alā l-dawr* (“çircunferençia de moto”) and to *al-Amad ‘alā l-abad* (“por tiempo seculo”).²³¹ In ms. Leiden 468, which contains the *Kanz al-yawāqīt*, a treatise on *mīqāt* written by an anonymous Egyptian *muwaqqit* from the beginning of the fourteenth century, we find a description of a trepidation table (fols. 207r and v) and the remark that Muḥammad b. Yūsuf al-Kammādī/ al-Kammād followed this system in his *zīj al-Kawr ‘alā l-dawr*. Furthermore, there are quotations from *al-Kawr* in the Hyderabad manuscript containing the Tunisian recension: chapter 43, paragraph 9 contains an abridgement of the prologue of *al-Kawr* in which Ibn al-Kammād explains Ibn al-Zarqāllūh's theories on the variation of solar eccentricity, changes of the obliquity of the ecliptic, motion of the solar apogee, and trepidation;²³² this passage coincides with the Castilian translation edited by Mancha; chapter 16, paragraph 6, of the Hyderabad ms. reproduces a passage from *al-Kawr* describing the procedure for calculating the true longitude of a planet;²³³ chapter 36, paragraphs 13–15, of the same source deals with the computation of anniversary horoscopes according to *al-Kawr*, although its contents do not agree with chapter 60 of the Madrid NL manuscript mentioned above.²³⁴ To this I should add that there are tables of the Hyderabad ms. attributed to *al-Kawr*: this is the case of table 78 on planetary stations and retrogradations²³⁵ which is also found in the *Muqtabas zīj*.²³⁶ Finally, chapter 32, paragraph 20, mentions a table for calculating a solar eclipse from *al-Kawr*.²³⁷

7.2.5.2.3 *Al-Amad ‘alā l-abad*

We have less information about this *zīj*, which must have dealt at length with the theory of trepidation, since chapter 12 of the *Muqtabas* justifies its concise description of the use of the equation table for trepidation by the fact that the topic was dealt with extensively in *al-Amad ‘alā l-abad*: “Sed nos iam

²³¹ A passage from this source has been edited and analysed in Mancha, 1998.

²³² Mestres, 1999, p. 172 (introduction), 296–7 (Arabic edition).

²³³ Mestres, 1999, pp. 52–3 (introduction), 78 (Arabic edition).

²³⁴ Mestres, 1999, pp. 131, 134–5 (introduction), 217 (Arabic edition).

²³⁵ Not edited by Mestres.

²³⁶ Chabás & Goldstein, 1994, p. 33 and 2015 a, pp. 617–8; Mestres, 1996, p. 422 and 1999, p. 53. The table appears in the Madrid NL ms. fol. 46r.

²³⁷ Mestres, 1999, p. 103 (introduction) and 169 (Arabic edition).

tractavimus super expansione eius in canone Helmed Hale Elebed, et si intraverimus memorationem eius in hoc canone exiebamus a (termino) in transitus et brevitatis, et tendebamus ad prolixitatem et ad multitudines.” (ms. Madrid NL fol. 7v). Besides, Abū l-Ḥasan ‘Alī al-Marrākushī says “If you want to obtain the precise value of the solar longitude, you should use a *zīj* based on reliable observations such as the one authored by Aḥmad b. Yūsuf al-Kammād and named *al-Amad ‘alā l-abad*, which is most precise”.²³⁸ To this I can only add that the Hyderabad recension (chapter 36, paragraphs 16–18)²³⁹ deals with the month transfer using aphetic months (*shuhūr intihā’yya*)²⁴⁰ and refers to *al-Amad*. According to this table (no. 175) the year transfer is 92;21,0,0,2° and the sidereal year has 365^d 6;9,24,5^h.²⁴¹

7.2.5.3 The Contents of the *Muqtabas zīj*

7.2.5.3.1 Introduction

We have already seen that Ibn al-Kammād follows Ibn al-Zarqālluh’s ideas on trepidation (6.2.3) (albeit with important differences in the model, severely criticised by Ibn al-Hā’im), the motion of the solar apogee which he extends to all planets (6.3.2), the variable solar eccentricity (6.3.3.2)²⁴² and the lunar correction²⁴³ (6.4). Leaving these topics aside, I will try to summarise the contents of the *Muqtabas*, following the previous studies of Chabás and Goldstein.

This *zīj* was obviously calculated for Córdoba: it contains a table of the oblique ascension (fols. 49r–51r)²⁴⁴ and another for half the duration of daylight for this city (fol. 47v):²⁴⁵ in both cases the local latitude used was 38;30°, while the obliquity of the ecliptic was the Ptolemaic 23;51° in the table of oblique

238 Marrākushī, 1984, I, 48; Sédillot, 1834, I, 135.

239 Mestres, 1999, p. 218 (Arabic edition).

240 A solar year has 13 such months. See Goldstein & Chabás, 2013.

241 Table 175 is explicitly attributed to *al-Amad ‘alā l-abad* in the Hyderabad ms. See Mestres, 1999, pp. 134–5, 139. The table is edited in Goldstein & Chabás, 2013, p. 472.

242 Although chapter 30 of the canons has a clear reference to a variable solar eccentricity, a good description of the method to calculate it appears in the chapter “Ad dirigendum solem” (ms. Madrid NL fols. 19r and v) which belongs to the previously mentioned appendix. Besides, the two corresponding tables appear in fols. 55r–56r and belong to a group of tables preserved in ms. Madrid NL, but not in the Hebrew translation (see Chabás & Goldstein, 1994, p. 36 and 2015 a, pp. 637–8). It seems, therefore, that the *Muqtabas zīj* omitted a clear reference to this Zarqāllian theory and only explained how to compute the solar longitude using a standard solar equation table calculated for Ibn al-Kammād’s own time. The full procedure and the tables seem to derive from *al-Kawr* or *al-Amad*.

243 The computational procedure to obtain the lunar correction is explained in chapter 13 (ms. Madrid NL fols. 7v–8v).

244 Chabás & Goldstein, 1994, p. 31 and 2015 a, pp. 589–91.

245 Chabás & Goldstein, 1994, p. 30 and 1015 a, p. 592.

ascensions and $23;33^\circ$ in the other table. The latter value is the one used in the solar declination table (fol. 35v)²⁴⁶ and it is related to Yahyā b. Abī Manṣūr (fl. ca. 830) and Ibn al-Zarqālluh (6.2.6.3).²⁴⁷ Other references to Córdoba appear in chapters 9 (Madrid NL fols. 6r and v),²⁴⁸ 10 (fol. 6v) and 11 (fols. 6v–7v). An interesting Iberian characteristic is the mention of the Spanish Era (*ta'rikh al-ṣufr, anni Saphr*), beginning 1.1.-37 (JDN 1707544), in chapter 5 (fol. 5r).

Another interesting table which may derive from Ibn al-Zarqālluh is found in fol. 48r: it is an auxiliary function (“directio arcus luminis”) giving $5 \cdot \tan(x)$ in which 5 is the result of dividing $60/12$ and the tangent function is calculated for $R = 1$.²⁴⁹ A similar table appears in Ibn al-Zarqālluh’s *Almanac*,²⁵⁰ under the title *Jadwal anṣāf fuḍūl zuhūrāt al-kawākib* (table of half the excess of appearance of stars). We find similar tables in the Tunisian recension of Ibn Ishāq’s *zīj*, and in the three *zījes* of Ibn al-Raqqām.²⁵¹ Chabás and Goldstein have established a relation of this kind of table and the computation of ascensional differences (see 7.2.1.2.1.4) although it may also have other applications.

7.2.5.3.2 Mean Motions and Apogees

Table 7.13 contains the solar and planetary apogees, radix positions for the beginning of the Hijra and the mean motion parameters for the Sun, Moon and planets, as calculated by Chabás and Goldstein. The radix positions given in the *Muqtabas* correspond to the positions of the *centrum* (*markaz*) of the Sun

246 Chabás & Goldstein, 1994, pp. 29–30 and 2015 a, pp. 586–7.

247 In the *Toledan Tables* we find $23;33^\circ$ (Pedersen, 2002, I, p. 69) and $23;33,30^\circ$ (Pedersen, 2002, I, pp. 67 and 69; II, p. 508; III, pp. 961–64) ascribed to Yahyā b. Abī Manṣūr and/or to Ibn al-Zarqālluh. Canons *Cb* (Pedersen II, 410–11) and *Cc* (Pedersen II, 612–13) add the remark “quae [i.e. Yahyā’s value] apud nos ducitur verior, quia primam novimus rumore, et hanc didicimus per considerationem” (“and among us this is considered truer, since we know the former from hearsay but have learnt the latter from observation”). In this relation we find (in Pedersen, 2002, III, pp. 765, 967) a declination table with a maximum $23;33,8^\circ$, a parameter which, until recently, was only known through another declination table ascribed to Abraham ben ‘Ezra: the situation changed radically with the publication of a paper by Saliba (1999, p. 11) which describes how a Toledan anonymous astronomer, who was a contemporary of Ibn al-Zarqālluh, ascribes to him an obliquity of the ecliptic of $23;33,8^\circ$, obtained, probably, through observation.

248 The longitude of Córdoba (27° from the Western meridian, 153° from the Eastern one), using the meridian of water, is mentioned in this chapter. A different meridian is used in the chapter “Ad dirigendum solem” (ms. Madrid NL fols. 19r and v), as it gives 81° (from the East, here meaning Arin) and 9° from the West.

249 Chabás & Goldstein, 1994, p. 36 and 2015 a, pp. 592–3.

250 Millás, 1943–50, p. 226.

251 Samsó, 2014, p. 323.

TABLE 7.13

	(1) Apogee	(2) Radix	(1) + (2)	Daily motion
Sun	76;45,21 ^o	36;35,9 ^o	113;20,30 ^o	0;59,8,9,21,15 ^o
Moon (long.)	–	120;34,42	–	13;10,34,52,46
Moon (anom.)	–	108;11	–	13;3,53,56,19
Saturn	238;38,30	236;52	115;30,30	0;2,0,24,24
Jupiter	158;21,30	171;58	330;19	0;4,59,5,31
Mars	119;41,30	91;46	211;27	0;31,26,28,47
Venus (anom.)	76;45,21	45;21	–	0;36,59,29,21
Mercury (anom.)	198;21,30 ^a	74;1	–	3;6,24,7,19

a In ms. Madrid NL 10023, the apogees for Jupiter (158;21,0°), Mars (119;41,0°) and Mercury (198;21,0°) show a difference of 0;0,30° with respect to those of the Hebrew translation of the *Muqtabas zīj*, which I have reproduced in table 7.14. The latter are confirmed by the *Muwāfiq zīj* of Ibn ‘Azzūz (Samsó, 1997, p. 102). See Chabás & Goldstein, 2015 a, p. 599.

and the three superior planets (col. 2), indicating their mean distance from the corresponding apogees (col. 1), while the third column is the result of adding (1) + (2) and corresponds to the radices from the beginning of Aries.

The sidereal longitude of the solar apogee seems to derive from Ibn al-Zarqālluh’s determination of its tropical position, in 1074–5, in which he obtained 85;49° (6.3.2).²⁵² I cannot find any explanation for the longitudes of the other apogees and for the radix positions, which do not seem to derive from the corresponding ones in al-Khwārizmī’s *zīj*²⁵³ or from the *Toledan Tables*. Table 7.14 shows the differences with respect to the Toledan values, which cannot be considered as the result of a correction due to the difference in longitude between Toledo and Córdoba: Ibn al-Kammād’s table of geographical longitudes ascribes to Córdoba 27° and to Toledo 28°,²⁵⁴ which implies a time difference of only four minutes.

252 Samsó & Millás, 1994, pp. 7–9.

253 The case of Mercury (224;54° in al-Khwārizmī’s *zīj*) is particularly significant.

254 Chabás & Goldstein, 2015 a, p. 632.

TABLE 7.14

	Apogee	Ap. Toledan – Kammād	Radix	Rad. Toledan – Kammād
Sun	76;45,21 ^o	–1;4,39 ^o	113;20,30 ^o	0;20,41 ^o
Moon (long.)	–	–	120;34,42	0;23,36
Moon (anom.)	–	–	108;11	0;2,21
Saturn	238;38,30	1;26,30	115;30,30	0;20,45
Jupiter	158;21,30	6;8,30	330;19	1;20,37
Mars	119;41,30	2;9,30	211;27	–0;2,1
Venus (anom.)	76;45,21	–1;4,39	45;21	0;7,37
Mercury (anom.)	198;21,30	0;51,30	74;1	–0;14,21

The mean motion parameters deserve a few comments: the mean motion in longitude of the Moon derives from al-Khwārizmī's *zīj*, while those of the anomaly of the Moon, Venus and Mercury have minimal differences with respect to the parameters of the *Toledan Tables*: 1^{iv} in the case of the lunar anomaly, 6^{iv} in that of Venus and 21^{iv} in Mercury. The mean motions in longitude of the Sun and the three superior planets present a difference of 2ⁱⁱⁱ (Sun, Saturn and Jupiter), and almost 4ⁱⁱⁱ for Mars.²⁵⁵ On the whole, these daily motion parameters seem to be the result of small corrections to the values of the *Toledan Tables* which may be due to Ibn al-Kammād himself or to his collaboration with Ibn al-Zarqālluh.

In the case of the Sun, a mean motion per day of 0;59,8,9,21,15^o corresponds to a year of 365;15,36,35^d but this is not the only value of the solar year attributed to Ibn al-Kammād. The information available was collected by Chabás and Goldstein²⁵⁶ and I have summarised it in Table 7.15:

²⁵⁵ Chabás & Goldstein, 1994, pp. 28–30 and 2015 a, pp. 600–3.

²⁵⁶ Chabás & Goldstein, 1994, pp. 35 and 2015 a, pp. 634–5.

TABLE 7.15 Solar year lengths according to Ibn al-Kammād

Source	Year length in days	Year length in days and hours
(1) 0;59,8,9,21,15° (mean motion parameter)	365;15,36,35 ^d	365 ^d 6;14,38 ^h
(2) <i>revolutio anni</i> (fol. 54v of the Madrid NL ms.): 93;36 ^{oa} = 6;14,24 ^h	365;15,36,0,30 ^d	365 ^d 6;14,24,12 ^h
(3) <i>revolutio anni</i> 2 (fol. 54v of the Madrid NL ms.): 6;14 ^h or 93;30 ^o	365;15,35,58,30 ^d .	365 ^d 6;14,23,24 ^h
(4) Canons fol. 2v of the Madrid NL ms.	365;15,36,19,34,12 ^d	365 ^d 6;14,31,49,40,48 ^h
(5) Fol. 65v of the Madrid NL ms. attributed explicitly to Ibn al-Kammād	365;15,36,19,35,22 ^d	365 ^d 6;14,31,50,8,48 ^h
(6) Table of astrological months (<i>al-shuhūr al-intihā'īyya</i>) in <i>al-Amad 'alā l-abad</i> according to the Tunisian recension of Ibn Ishāq's <i>zīj</i> ^b	365;15,23,29,26,45 ^d	365 ^d 6;9,23,46,41,56 ^h

a The tabular value for one year is 92;36°, an obvious mistake shown by the rest of the table. This length of the solar year (365^d 6;14,24^h) reappears in Marrākushī, 1984, I, 19; Sédillot, 1834, p. 82, and in ms. Leiden 468 (*Kanz al-yawāqūt*) fol. 206v.

b Mestres, 1999, p. 15 fn. 2, 134–5, 139. See the table in ‘Abd al-Raḥmān, 1996 a, p. 376, and in Goldstein & Chabás, 2013, p. 472. A note in the manuscript states that the original table was computed to the precision of seconds.

Value number (1) has been explained by Goldstein and Chabás,²⁵⁷ who have established that the mean solar motion of 0;59,8,9,21,15° has a clear Zarqāllian origin as it corresponds to the difference between the mean motion of the Sun in the Toledan Tables (0;59,8,11,28,27,30°) and Ibn al-Zarqālluh’s mean motion of the solar apogee (0;0,0,2,7,10,38°):

257 Chabás & Goldstein, 1994, p. 28 and 2015 a, p. 600.

$$0;59,8,11,28,27,30^0 - 0;0,0,2,7,10,38^0 = 0;59,8,9,21,16,52^0$$

It comes as a surprise to discover that Ibn al-Kammād is using Ibn al-Zarqālluh's value of the motion of the solar apogee, instead of his own parameter ($0;0,0,2,2,14,46^0$)²⁵⁸ which gives much worse results:

$$0;59,8,11,28,27,30^0 - 0;0,0,2,2,14,46^0 = 0;59,8,9,26,12,44^0$$

This could mean that $365;15,36,35^d$ may be ascribed to Ibn al-Zarqālluh's parameter for the length of the anomalistic year. This is confirmed by Ibn al-Hā'im, who gives the same length of the anomalistic year in the *Kāmil zīj* ($365;15,36,35,0,8^d$) as well as the same daily mean solar motion ($0;59,8,9,21,15^0$)²⁵⁹

Concerning the rest of the values, Ibn al-Hā'im seems to have been the first to complain about their diversity. Thus, the first of his 25 criticisms of Ibn al-Kammād's errors in the *Muqtabas* and *al-Kawr*²⁶⁰ is that the length of the solar year mentioned in the first part of the book ([4] of the list in table 7.15) disagrees with the mean solar motion appearing in the corresponding tables (1).²⁶¹ He also remarks that his *faḍalāt al-maṭālī'* in the *Muqtabas zīj* (no. 2 in table 7.11) do not agree with his *faḍalāt al-tawārikh* (no. 3 in the same table).²⁶² According to him, Ibn al-Kammād calculates the return of the Sun to the same point of the solar eccentric (*zamān al-ʿawda fī l-khārij al-markaz*), instead of obtaining its return to the same point of the ecliptic (*zamān al-ʿawda fī falak al-burīj*).²⁶³ This implies that Ibn al-Kammād is using anomalistic years instead of sidereal years for astrological computations.

This idea can be applied to the rest of the lengths of the solar year given in table 7.15.

We can calculate the mean solar motion per day and obtain:

- (2) $0;59,8,9,26,50,8^0$
- (3) $0;59,8,9,27,9,34^0$
- (4) $0;59,8,9,23,44,54^0$
- (5) $0;59,8,9,23,44,43^0$
- (6) $0;59,8,11,28,26^0$

258 Chabás & Goldstein, 1994, p. 28 and 2015 a, p. 601.

259 Calvo, 1998 a, pp. 55–56, 86–87.

260 Calvo, 2002.

261 Ms. Bodleian Library O 285 (Marsh 618), fol. 5r.

262 Ms. Bodleian Library O 285 (Marsh 618), fol. 8r.

263 Ms. Bodleian Library O 285 (Marsh 618), fols. 8r, 70v–71r. See Calvo, 2002, pp. 113–114.

Parameter (6) is the nearest to the Zarqāllian value and, therefore, it corresponds to a sidereal year, while the other four, all of them beginning with 0;59,8,9..., seem to correspond to anomalistic years and could be explained in the same way as number (1). By adding Ibn al-Zarqālluh's or Ibn al-Kammād's daily parameter of the motion of the solar apogee, we will obtain values 0;59,8,11... as in (6).²⁶⁴ A good example is (2) which, when added to Ibn al-Kammād's mean motion of the apogee, yields a value similar to the parameter of the *Toledan Tables*:

$$0;59,8,9,26,50,8^{\circ} + 0;0,0,2,2,14,46^{\circ} = 0;59,8,11,29,4,54^{\circ}$$

As a matter of fact, there is an explanation of the different values of the *revolutio anni* in the appendix to Johannes of Dumpno's translation of the *Muqtabas zīj*. This appendix begins with a translation of chapter 60 of *al-Kawr 'alā l-dawr*, introduced by these words:²⁶⁵

Sciendum est quod in tabulis istis que sunt in capite huius libri que vocantur *elmuctebis* accidit diversitas in revolucionibus annorum mundi et nativitatum, et hoc propter reversionem solis ad locum suum que quandoque fit ante tempus suum et quandoque post in dictis tabulis propter quod quesivimus aptacionem huius diversitatis in tabulis que vocantur *caur hale eddaur* et invenimus huius rei aptacionem super veritate in porta 60a que est ultima porta dictarum <fol. 19r> tabularum quam studui hic notare. Scias quia cum multiplicabuntur anni nativitatis aut anni revolucionis coniunctionis erit tempus reversionis solis ad locum suum aptatum plus tempore aut minus ex tempore reversionis eius ad locum mediatu sui cursus prope diversitatem motus centri.

One should know that in these tables, found in the beginning (?) of this book called *al-Muqtabas*, there is a divergence in the revolutions of the world years and of nativities. This is due to the return of the Sun to the position it occupied, something which sometimes takes place before, and at other times later, in these tables, because we made an effort to adapt its

²⁶⁴ Interestingly, Ibn al-Kammād tabulates the mean motion in longitude of Mercury for which the daily parameter is 0;59,8,11,23...° (ms. Madrid NL fols. 34r and v) which does not agree with the solar parameter 0;59,8,9,21,15...°. It seems, therefore, that the parameter for Mercury is sidereal, while that of the Sun is anomalistic. The radix positions for Hijra are also extremely different: 274;58° (Mercury) and 113;20,30° (Sun).

²⁶⁵ Ms. Madrid NL, fols. 18v–19r. The following passage, as well as the passage from chapter 30 quoted below, have been edited by Millás, 1942, pp. 238–239.

diversities in the tables named *al-Kawr 'alā l-dawr*. We found this adaptation to reality in chapter 60, which is the last chapter of the aforementioned tables, that can be studied here. You should be aware that when the years of a nativity or the years of the revolution of a conjunction are many, the time of the return of the Sun to its [previous] position can be more or less than the time of its return to the [same] place with its mean motion, due to the unequal motion of its centre.

This implies that the values of Ibn al-Kammād's *revolutio anni* ([2] and [3] in table 7.15) are affected not only by the motion of the solar apogee but also by the motion of the centre of the solar eccentric which is the cause of changes in solar eccentricity (6.3). One may wonder whether the different values of the length of the solar year are the result of computations made by Ibn al-Kammād for different dates.

A further clarification appears in chapter 30 of the *Muqtabas*:²⁶⁶

Et scias quia accidit propinquitas in revolucionibus annorum in hoc opere post annos 40 vel partes eius in annis nativitatis vel in revolutione anni ex annis mundi in quantitate tercii unius decimi diei ita tamen quod sunt dies cum nocte eius centum partes quia reversio solis ad caput arietis vel ad loca eius in radice nativitatis per tempora est plus tempore et minus tempore a tempore reversionis eius ad centrum radicale ex parte centri circuli solis exeuntis centrum.

You should know that, in this work, after forty years or parts of it, there is an approximation in the revolutions of the years in the years of nativities or in world years, which amounts to a third of a tenth of a day, considering that a day with its night contains one hundred parts, because the return of the Sun to the beginning of Aries or to the position it had at the radix date of the nativity will take place in a longer or shorter time, due to the time of the return of the centre of the solar eccentric at the position it had at the radix.

This passage deserves two comments. First, the amount of the error accumulated in 40 years, due to the motion of the centre of the solar eccentric, is $1/30$ of a day, equivalent to 48 minutes, which implies a change of the length of the solar year of about $0;1,12^h$ or $0;0,3^d$; this seems excessive and does not justify any of the differences in the length of the year in table 7.15. Second,

²⁶⁶ Ms. Madrid NL, fol. 18v.

this seems to be the earliest mention of the 40-year period, later identified by Ibn al-Hā'im²⁶⁷ and others as the period during which a *zīj* is considered to be valid, mainly due to the cycles introduced by Ibn al-Zarqālluh for the motion of the apogees, the obliquity of the ecliptic, the changes in solar eccentricity and the motion of trepidation.

7.2.5.3.3 *Solar, Lunar and Planetary Equations*

The planetary equations are the same as those in al-Battānī and in the *Toledan Tables* (see table 7.11) and they derive, indirectly, from the *Handy Tables*. This is not the case of the solar equation, which reaches a maximum of 1;52,44° (e = 1;58,2^p). This value derives from Ibn al-Zarqālluh who, according to Bernard of Virdun, determined, ca. 1075, an eccentricity of 1;58^p, which corresponds to a maximum equation of 1;52,42° (6.3.3). The small correction of 2" is probably due to the fact that Bernard of Virdun is giving a rounded value of the maximum solar equation. This value of the solar eccentricity is not transferred to the table of Venus's centre equation, which has a maximum of 1;59°, as in al-Battānī's *zīj*.²⁶⁸

7.2.5.3.4 *Ibn al-Kammād's Star List*

The *Muqtabas zīj* contains a table of 30 stars giving the names of each star, its magnitude, ecliptic coordinates and astrologically associated planets. It has been studied in detail by Goldstein and Chabás,²⁶⁹ who underlined its diffusion in medieval Iberian Latin and Hebrew sources, as well as in some Maghribī *zīj*es like Ibn al-Bannā's *Minhāj*.²⁷⁰ It corresponds precisely to Kunitzsch *Typ. xv*²⁷¹ and the stellar longitudes show an increase of 6;38° on the Ptolemaic longitudes. These features led Goldstein and Chabás to consider that this star table was computed for the beginning of the Hijra.

Years ago I found the same table in the *Muwāfiq zīj* of Ibn 'Azzūz al-Qusanṭīnī (d. 1354),²⁷² in which the difference with respect to Ptolemaic longitudes amounts to 6;48° or 6;49°, instead of the 6;38° of Ibn al-Kammād. The analysis of the two tables led me to conclude, following Mercè Comes's previous study of Ibn al-Bannā's star table, that the epoch of the table was not the beginning of Hijra, but a time at which the value of precession was 0° (see 6.2.1, 6.2.2.5.2.1).

267 Ms. Bodleian Library O 285 fols. 26r, 70v, 71v.

268 Chabás & Goldstein, 1994, pp. 4–10, 32–3 and 2015 a, pp. 605–12.

269 Goldstein & Chabás, 1996; see also Chabás & Goldstein, 1994, pp. 34–5, and 2015 a, pp. 630–1.

270 On the *Minhāj*'s star table, see Comes, 1991 a.

271 Kunitzsch, 1966, pp. 99–102.

272 Samsó, 1997, pp. 107–110.

My main argument is based on the terminology used in the heading of the table in the Latin translation (ms. Madrid NL fol. 47r): *Tabula longitudinum stellarum fixarum a puncto capitis Arietis in circulo signorum ab initio essentiali*. A *puncto capitis Arietis* corresponds to Arabic *min nuqṭat Ra's al-Ḥamal*, found in Ibn 'Azzūz's table,²⁷³ and it implies "distance from the [moving] Head of Aries" in a trepidation model. Besides, *ab initio essentiali* is a translation of *min al-mabda' al-dhātī* which appears in Ibn al-Bannā's table.²⁷⁴ *Al-mabda' al-dhātī* (*initium essentiali*) and *al-mabda' al-ṭabī'ī* (*initium/principium naturale*, often found in Ibn al-Kammād's Latin text) are standard terminology in Andalusī-Maghribī astronomy and they mean, respectively, sidereal (starting in the moving Head of Aries) and tropical (starting in the spring equinoctial point). This is clearly explained in the canons of the *Minhāj* of Ibn al-Bannā', which talk of *darājāt dhātīyya* and *darājāt ṭabī'īyya*,²⁷⁵ as well as in the Tunisian recension of Ibn Ishāq's *zīj*,²⁷⁶ and in Abū l-Ḥasan 'Alī al-Marrākushī's *al-Mabādī' wa l-ghāyāt*, where we read:²⁷⁷

To obtain the distance between the *mabda' dhātī* and the spring equinoctial point (*nuqṭat al-i'tidāl al-rabī'ī*) for any time you wish: precise observations have established that the point of the *mabda' dhātī* of the [circle of the] signs (*li l-burūj*) is not the same as the *mabda' ṭabī'ī*, which is the point of the vernal equinox ...

If an epoch has to be assigned to both Ibn al-Kammād's and Ibn 'Azzūz's tables, it must be a date in which the value of precession was 0°, which corresponds to an argument (position of the moving Head of Aries) 0° or 180°. In the case of Ibn al-Kammād we will have, as we have already seen in 6.2.2.5.2.1:

$$n = 3;51,11^{\circ} / 0;0,0,54,56,58^{\circ} \approx 15,146 \text{ days}$$

in which 3;51,11° is the position of the moving Head of Aries at the beginning of the Hijra in the *Muqtabas zīj*, while 0;0,0,54,56,58° is its mean daily motion. The date is, therefore 15,146 days before the Hijra and it corresponds to 24.1.581 (26.1.581 if we use Ibn al-Zarqālluh's parameters in the table of Arabic years).

In the case of Ibn 'Azzūz, the result is:

273 Ms. Rabat National Library D 2461/6 p. 427.

274 Comes, 1991 a, p. 93.

275 Vernet, 1952, p. 46 of the Arabic edition: the passage is missing in the Spanish translation.

276 Mestres, 1999, pp. 295–296 of the Arabic edition (chapter 43, paragraph 8).

277 Marrākushī, 1984, I, pp. 42–43; Sédillot, 1834, p. 126.

$3;51,21^{\circ} / 0;0,0,54,57,18,17^{\circ} \approx 15,155$ days before Hijra, which corresponds to fifteenth January 581.

To this I should add that Ibn al-Kammād's star-longitudes are those of Ptolemy's star catalogue increased by $6;38^{\circ}$ and this seems to be related to Ibn al-Zarqālluh's first approach to his first trepidation model (6.2.2.3) in which we can see that he assumes that the value of precession in the time of Ptolemy was $-6;38^{\circ}$. Ibn al-Kammād seems to be using this Zarqāllian value here, which is not the value he would have obtained using his own mean motion of the Head of Aries:

If we accept that Ibn al-Zarqālluh used year 139 CE "for the time of Ptolemy" (6.2.2.2), which corresponds to a determination of the longitude of Regulus made by the Alexandrian astronomer precisely in that year,²⁷⁸ we can calculate the value of precession, with Ibn al-Kammād's tables, for that year. Using 30.6.139 (JDN 1,772,008) as the exact date, we will see that 161,284 days will pass until 24.1.581 (JDN 1,933,292). The argument will be:

$$161,284 \cdot 0;0,0,54,56,58^{\circ} = 41;1,47^{\circ}$$

In Ibn al-Kammād's table for the equation of trepidation (6.2.3), the value for an argument of 41° is $7;57^{\circ}$, which is obviously corrupt; both the *Tables of Barcelona*²⁷⁹ and the *Muwāfiq zīj*²⁸⁰ contain a variant reading ($6;57^{\circ}$) which is much better. We have, therefore, a value of precession for Ptolemy's time of $-6;57^{\circ}$, instead of $-6;38^{\circ}$. $-6;57^{\circ}$ for the time of Ptolemy also appears in Ibn al-Zarqālluh's second approach to his second model of trepidation (6.2.2.4). This value is confirmed by Ibn al-Hā'im, within his criticisms of Ibn al-Kammād's treatment of trepidation, who remarks that, according to Ibn al-Zarqālluh (?), Ptolemy's *iqbāl* was $-6;42^{\circ}$, while the value obtained with this book (Ibn al-Kammād's?) was $6;57^{\circ}$, with an excess of 15' on the value adopted (by Ibn al-Zarqālluh?) as a base (*aṣl*).²⁸¹

It seems, therefore, that Ibn al-Kammād's star table is related to Ibn al-Zarqālluh's first model of trepidation (-6.38), while his trepidation table gives a precession of $-6;57^{\circ}$, for the time of Ptolemy, which appears in the second model: this seems to confirm Chabás and Goldstein's hypothesis that Ibn

²⁷⁸ *Almagest* VII,2: Toomer, 1984, p. 328.

²⁷⁹ Chabás & Goldstein, 1994, p. 25.

²⁸⁰ Ms. Rabat National Library 2461, p. 365; Rabat Ḥasaniyya Library 8772 p. 11.

²⁸¹ Comes, 2001, pp. 258–259.

al-Kammād's table would have been calculated with Ibn al-Zarqālluh's second model (6.2.3).

7.2.5.3.5 *Other Tables*

Little new is found in the rest of the *zīj*, in which we find a mixture of Ptolemaic traditions with others of Indian origin derived from early Islamic sources such as Ya'qūb b. Ṭāriq (eighth century), al-Khwārizmī and Yaḥyā b. Abī Maṣṣūr (ninth century). The only possible exceptions are the tables for syzygies for which Chabás and Goldstein do not ascribe any source; interestingly, the table for computing the time between mean and true syzygy is a double argument table.²⁸²

The radix positions of the planetary nodes are the same as those found in al-Khwārizmī's *zīj* and the *Toledan Tables*,²⁸³ and the same can be said of the tables of planetary stations.²⁸⁴ The tables of solar and lunar velocities are also Khwārizmian,²⁸⁵ while the tables for solar eclipses derive from Yaḥyā ibn Abī Maṣṣūr's *Mumtaḥan zīj*;²⁸⁶ the Madrid NL ms. contains two sets of tables for lunar eclipses, of which the second is ascribed to Ya'qūb b. Ṭāriq.²⁸⁷ The tables for the planetary latitudes are clearly Ptolemaic (*Almagest* and *Handy Tables*),²⁸⁸ while there are two tables of lunar latitude: one of them Ptolemaic (with a maximum value of 5°) and another one of Indian origin, based on a maximum of 4;29° (al-Khwārizmī and Yaḥyā ibn Abī Maṣṣūr have 4;30°) and calculated using the method of sines.²⁸⁹ Finally, the parallax tables also derive from the Indian tradition, with certain changes in the parameters: the horizontal parallax reaches a maximum of 0;48,32°, a parameter also used by Yaḥyā b. Abī Maṣṣūr (al-Khwārizmī 0;48,45°)²⁹⁰ and this value is used to calculate the component in latitude by multiplying it by the sine of the zenith distance of a point of the ecliptic 90° away from the ascendant; the longitude component reaches

282 Chabás & Goldstein, 1994, p. 14 and 2015 a, pp. 624–5.

283 Chabás & Goldstein, 2015 a, pp. 612–3.

284 Chabás & Goldstein, 1994, p. 33 and 2015 a, pp. 617–8.

285 Chabás & Goldstein, 1994, pp. 10–13 and 2015 a, pp. 613–4.

286 Chabás & Goldstein, 1994, pp. 14–17, 23 and 2015 a, pp. 626–8.

287 Chabás & Goldstein, 1994, pp. 17–8 and 2015 a, pp. 628–9, 639. The Tunisian recension of Ibn Ishāq's *zīj* contains two sets of tables for lunar eclipses derived from Ibn al-Kammād's *al-Kawr 'alā l-dawr*.

288 Chabás & Goldstein, 1994, pp. 31–2 and 2015 a, pp. 616–7.

289 Chabás & Goldstein, 1994, pp. 20–2 and 2015 a, pp. 614–6.

290 Kennedy, 1956 a, pp. 44–46; Kennedy & Fares, 1970.

a maximum of 1;36^h in both al-Khwārizmī and Ibn al-Kammād, although the former uses degrees for the argument and the latter uses hours.²⁹¹

7.2.6 *Ibn al-Hā'im's al-Zīj al-kāmil fi l-ta'ālīm*

In the early thirteenth century we find a new *zīj* which, together with Ibn al-Kammād's three *zīj*es, bears witness to the survival of Zarqāllian astronomy in al-Andalus.²⁹² The author's name is Abū Muḥammad 'Abd al-Ḥaqq al-Ghāfiqī al-Ishbīlī, known as Ibn al-Hā'im, and no biographical information about him is given, except for the fact that he wrote, ca. 1204–5,²⁹³ his *al-Zīj al-kāmil fi l-ta'ālīm* ("The perfect *zīj* based on mathematical principles")²⁹⁴ which he dedicated to the Almohad caliph Abū 'Abd Allāh Muḥammad al-Nāṣir (1199–1213).²⁹⁵ As a matter of fact we do not know for sure whether Ibn al-Hā'im was active in al-Andalus (Seville?) or in the Maghrib, but there is no doubt that he was one of the earliest introducers of Zarqāllian astronomy in the other side of the Strait of Gibraltar. This *zīj* has only been preserved in the Bodleian Library (Oxford) manuscript 11,2, no. 285 (Marsh 618), which seems to be incomplete, because the Tunisian recension of Ibn Ishāq's *zīj* contains a method, attributed to Ibn al-Hā'im, which describes how to compute the true conjunction of the Sun and the Moon from their mean conjunction. The corresponding passage is not extant in the Bodleian manuscript.²⁹⁶

The dedication of the book to al-Nāṣir shows that, in spite of the rejection of the study of philosophy and logic by the Almohad caliph Abū Yūsuf al-Manṣūr (1184–99) after the battle of Alarcos (1195), the situation seems to have changed under his successor, al-Nāṣir, who agreed to be the dedicatee of Ibn Mun'im's (d. 1228–9) *Fiqh al-ḥisāb*, as well as of Ibn al-Hā'im's *Kāmil zīj*. This second work is particularly significant because, after all, the main purpose of a *zīj* is the computation of horoscopes and, in his book, Ibn al-Hā'im does not hide his interest in topics related to mathematical astrology.

The *Kāmil zīj* is a fairly long text (173 pp.) containing a collection of chapters, some of them standard canons of a set of astronomical tables, although many

291 Kennedy, 1956 a, pp. 49–51. On Ibn al-Kammād's parallax tables see Chabás & Goldstein, 1994, pp. 19–20, 22–3 and 2015 a, pp. 618–24.

292 It was Mercè Comes who attracted my attention to this *zīj*.

293 The text mentions on several occasions (Bodleian ms. fols. 36r, 71r, 72r, 88r) that Ibn al-Hā'im's book was written at the beginning of the seventh century of the Hijra. Most of these references are given in relation to the 40-year period for which a *zīj* is valid.

294 At the end of fol. 9v of the Bodleian ms. Ibn al-Hā'im justifies his title, particularly his use of the word *kāmil*.

295 Mentioned as the Caliph Abū 'Abd Allāh on fols. 2r and 4v of the Bodleian manuscript.

296 Mestres, 1999, pp. 93–95.

others are not: they deal extensively with problems of astronomical theory and are always accompanied by careful geometrical demonstrations. Besides this, the *Kāmil* does not contain numerical tables, and this led Ibn al-Raqqām (d. 1315) to use a selection of such canons (without the proofs) to which he added his own tables, the result being his *al-Zīj al-Shāmīl fī tahdhīb al-Kāmil*. As the end of each page in the Bodleian manuscript is badly damaged (and often illegible), it is interesting that in her edition of the chapters related to trepidation, Mercè Comes used the extremely literal quotations from Ibn al-Raqqām's *Shāmīl zīj* to complete many passages.

However, the text of *Kāmil zīj* includes some references to tables that should accompany these canons, specifically a table of the sines of the declinations of the Head of Aries (6.2.2.5.2.2 and 6.2.5), a "table for the amount of accession (*iqbāl*) only" computed for a fixed value of the obliquity of the ecliptic, a table of geographical longitudes, tables of the equation of anomaly of the six planets and tables of the excess of revolution for the computation of anniversaries, as well as of the times of the *dawriyya* and *maṭla'īyya* months. Only the tables of the excess of revolution have been preserved in the Tunisian recension of the *zīj* of Ibn Ishāq and I will deal with them later in this section.²⁹⁷ The existence of such tables is an unsolved problem unless new evidence appears, although it is clear that the copy of the *zīj* which reached Ibn al-Raqqām did not have any tables.

The book is divided into an introduction and seven *maqālas* of which only the very interesting parts related to trepidation, solar and lunar motion have been properly edited and studied.²⁹⁸ I will try to summarise the contents of this *zīj* here:

Introduction (fols. 1v–10r): it contains Ibn al-Hā'im's 25 criticisms of Ibn al-Kammād's *Muqtabas* and *al-Kawr 'alā l-dawr zīj*es,²⁹⁹ which are numbered in the margins of the manuscript. He seems to have made a careful study of Ibn al-Kammād's *zīj*es for, in his criticism no. 17, he notes that he has found the same mistake in all the copies of Ibn al-Kammād's *zīj* available to him.³⁰⁰ I have already referred to his treatment of trepidation (no. 2, see 6.2.3),³⁰¹ and to the length of the solar year (no. 1, see 7.2.5.3.2). To these I can add solar theory (nos. 3–5: errors in the position of the solar apogee for the beginning of the Hijra and the Yazdijird era, as well as in the computation of the equation

297 'Abd al-Raḥmān, 1996 a.

298 Comes, 2001; Puig, 2000; Calvo, 1998 a.

299 Calvo, 2002.

300 Ms. Bodleian Library 285 fol. 7v.

301 Comes, 2001, pp. 307–311, 366–371.

of time), arcs of visibility of stars and planets (no. 6), trigonometry (no. 7, errors in the method of interpolation used to obtain the sine of an arc and vice versa), obtaining the degree of the mediation of a star (no. 8), timekeeping (nos. 9–10, solar altitude as a function of the hour of the day and vice versa), use of approximate procedures for the equalisation of the houses of the horoscope (no. 11), luni-solar conjunctions and oppositions (no. 12), parallax in longitude and latitude (nos. 13–16), lunar and solar eclipses (nos. 17–21), Ibn al-Kammād’s method for obtaining the excess of revolution and length of astrological months from the return of the Sun to its position in the ecliptic instead of its position in the eccentric circle (nos. 22–23), errors in the computation of the *tasyīr* (no. 24) and computation of the solar equation using “abridged” tables (*jadāwil mukhtaṣara*) which do not take into consideration the changes in solar eccentricity (no. 25, see 6.3.3.2). On fols. 8v–9v he deals, again, with trepidation and gives a list of historical determinations of the longitude of Regulus (*Qalb al-asad*) which are necessary to establish the *uṣūl* (radices) that both Ibn al-Zarqālluh (6.2.2.2) and he himself use to design the model of trepidation and the corresponding parameters. This list is full of errors in the Bodleian manuscript (fol. 9v), although the chronology is slightly better than Ibn al-Zarqālluh’s (see table 7.16 with my additions and comments in square brackets):

TABLE 7.16

Observer	Date	Longitude (Bodl. ms.)	Longitude (original source: Hipparchus, Ptolemy etc.)
Hipparchus	[–128/7]	100;29,50°	119;50°
Ptolemy	[139] 287 [instead of 267] yrs after Hipparchus	100;2,30	122;30 ^{oa}
Battānī	[879] 941 yrs after Hipparchus	130;14	134 ^{ob}
Ibn Barghūth	[1050] 171 yrs after Battānī	136;20°	136;20 ^{oc}

a For Hipparchus and Ptolemy see *Almagest* VII.2 (Toomer, 1984, p. 325).
b Nallino, 1899–1907, I, p. 124; III, p. 188.
c The source is Ibn al-Zarqālluh’s treatise on the motion of the fixed stars: see Millás, 1943–50, p. 309.

Maqāla 1 (fols. 10r–23r) contains 16 chapters and deals with chronology. In chapter 1, Ibn al-Hā'im gives (fol. 11r) his value for the mean lunar month (29;[30],44,3.47,49,25,36^d)³⁰² and the mean lunar year (354;8,48,[45],33,53,7,12^d).³⁰³ In chapter 2 (fols. 12v–13v) he mentions “the well known eras used by all nations living in the north-western part of the inhabited world, although most of them are not commonly used in our time”: Adam, Flood, Nabonassar, Şufr (= Hispanic era), Jesus Christ, Diocletian (*Qilṭiyānush al-Qibṭī*), Hijra and Yazdijird (see 2.2). Chapters 3–13 deal with date conversion between different calendars, most of them by computation (*bi l-ḥisāb*), although chapters 11 (fols. 19v–20r) and 12 (fols. 20r–20v) mention the use of tables of two kinds: tables structured (*murakkaba*) in years, months and days, and others which only use days (*al-murakkaba min al-ayyām faqat*). The latter chapter seems to allude to a set of tables structured sexagesimally in four ranks (*martaba*) which reminds me of the Latin *Alfonsine Tables*. In chapter 14 (fols. 21r–22r) we find, once more, a reference to tables allowing the identification of the day of the week corresponding to the beginning of a year or a month. Finally chapters 15 (fols. 22r–v) and 16 (fols. 22v–23r) deal with the procedure to establish the time difference between two localities of known geographical longitude. Interestingly, chapter 15 says *Fa in kāna dhālika l-balad min al-bilād al-mashhūra 'inda-nā wa l-mawḍū'a aṭwālu-hā fī hādhā l-kitāb aw fī ghayri-hi min al-kutub al-musta'mala 'inda ahl hādhihi l-nawāḥi al-shimāliyya al-gharbiyya ...* (if this locality is one of the places well known to us, whose longitudes are recorded in this book, or in other books used by the people of this region of the north-west ...). This is a reference to a table of geographical coordinates which Ibn al-Hā'im was planning to include in his book.

Maqāla 2 (fols. 23r–35r) contains 11 chapters, of which chapters 1–4 deal with trepidation.³⁰⁴ Chapter 5 (fols. 29v–32r)³⁰⁵ contains an explanation of the three kinds of solar year (6.3.3.1) and gives a length of the anomalistic year of 365;15,36,35,0,8^d which seems to be a Zarqāllian parameter (7.2.5.3.2), obtained by comparing Hipparchus' observations with those of Ibn al-Zarqālluh himself.³⁰⁶ This is followed by a digression on the differences of astronomical parameters which do not imply scientific (*'ilmī*) disagreements, but have a purely practical (*'amalī*) character and may be due to different causes such as

302 29;12,44... in the Bodleian ms. fol. 11r.

303 354;8,48,40,21,33... in the Bodleian ms. fol. 11r.

304 These chapters have been edited and analysed by Comes, 2001 pp. 371–385 (ed.) and 329–346 (comm.). See also 6.2.2, 6.2.3, 6.2.4 and 6.2.5.

305 Edited by Calvo, 1998 a, pp. 84–88 (commentary pp. 54–56).

306 Calvo, 1998 a, pp. 86–87.

observational errors, instruments used, different periods of time between pairs of observations, and so on. As an example, Ibn al-Hā'im imagines a discussion between two scholars about the height of the atmosphere: one of them considers that it amounts to three miles, while the other believes that its height is 51 miles. Here Ibn al-Hā'im seems to allude to Ibn Mu'adh al-Jayyānī who, in his *Mā al-fajr wa l-shafaq* (*Liber de crepusculis*) obtained a value of about 50 miles.³⁰⁷

The following chapters are canons of the standard type: Ibn al-Hā'im's mean motion tables are sidereal and are calculated for the lunar calendar (Hijra epoch), Persian solar calendar (Yazdijird) and *rūmī* solar calendar (Alexander). The radices are computed for the meridian of Arin (chapter 6, fols. 32r and v). The rest of the topics are: equal and unequal hours (ch. 7, fols. 32v–33r); how to calculate the mean position of a celestial body using mean motion tables (ch. 8, fols. 33r–34r); how to correct the aforementioned mean position for a meridian different from that of Arin (ch. 9, fols. 34r and v); how to obtain the longitude of the apogee of a celestial body with the table of the mean motion of the apogees (ch. 10, fol. 34v); how to interpolate in an equation table if we enter with an argument given in degrees and minutes.

Maqāla 3 (fols. 35r–49v) contains 13 chapters. Several of them give information on Zarqāllian theories, like chapter 1 (fol. 35v) on the procedure for obtaining the obliquity of the ecliptic using a table of the total obliquity (*al-mayl al-kullī*) (see 6.2.2.5.2.4 and 6.2.2.5.2.5) and chapter 2 (fols. 35v–36r), on how to obtain the value of trepidation with a standard equation table which gives approximate results for arguments between 0° and 180°. The approximation will be worse for an argument greater than 180°. These two chapters have been edited by M. Comes.³⁰⁸

Chapter 3 (fols. 36r–36v) deals with the computation of the true longitude of the Sun using a standard table of the solar equation (*al-ikhtilāf al-juzʿī*, the partial equation): the procedure is also approximate, as it does not take into account the changes in the solar eccentricity, and it will be valid for some 40 years from the date of the *zīj*. This is followed by a brief explanation of the method used to obtain the solar eccentricity, and the corresponding solar equation for a given date, using a table of *taʿdīl al-shams al-kullī* (the total solar equation) based on the Zarqāllian model of variable solar eccentricity (6.3.3.2).³⁰⁹

307 Sabra, 1967; Goldstein, 1976 a; Goldstein, 1977 a; Mark Smith, 1992; Mark Smith & Goldstein, 1993.

308 Comes, 2001, pp. 386–388 (edition) and 346–348 (commentary).

309 Chapter edited in Calvo, 1998 a, pp. 89–90. See pp. 56–58 (commentary).

This chapter is complemented by chapter 8 (fols. 38v–41r),³¹⁰ in which a detailed description of the model and the corresponding parameters are given.

Chapter 4 (fols. 36v–37r)³¹¹ describes concisely the procedure for obtaining the true lunar longitude following the standard steps, although Ibn al-Hā'im adds the Zarqāllian correction according to which the mean lunar motion in longitude takes place not around the centre of the ecliptic but around another point placed on the line that joins the centre of the ecliptic with the solar apogee (6.4). In this case, as well as in others in which Ibn al-Hā'im introduces one of the Zarqāllian corrections, he labels it with the expression *al-ziyāda al-mustadraka 'alā l-qudamā'* (the correction added to the [model/ theory] of the ancient astronomers). Apart from this, the steps followed in the computation are standard and the only interesting thing, as noted by R. Puig,³¹² is the terminology used by Ibn al-Hā'im: *inḥirāf al-quṭr* is the equation of the centre, *daqā'iq al-nisba* corresponds to the interpolation function, *ta'dīl al-bu'd al-aqrab* is the difference between the epicyclic equation at the perigee and at the apogee, *ta'dīl al-ḥiṣṣa* is the equation of anomaly at the apogee and the final result (equation of the anomaly for a given double elongation and true anomaly) is called *al-ta'dīl al-murakkab*. This chapter is complemented by chapter 9 (fols. 41v–43v),³¹³ in which Ibn al-Hā'im explains the Ptolemaic lunar model in detail, shows how to calculate the lunar equations with the corresponding geometrical proofs, and gives a full justification of the Zarqāllian correction.³¹⁴

Chapters 5 (fol. 37r), 6 (fols. 37r–37v) and 7 (fols. 38r–38v) contain standard explanations about how to compute the position of the lunar node and the true longitudes of the upper and lower planets. Planetary theory continues in chapters 10 (fols. 43v–45v), 11 (fols. 45v–47r), 12 (fols. 47r–48r) and 13 (fols. 47r–49v) which deal with computational methods for obtaining the equation of the centre of Mercury (ch. 10), the superior planets and Venus (ch. 11), the distance between the centre of the epicycle and the centre of the ecliptic (ch. 12) and the equations of anomaly of the Moon and planets (ch. 13). In all these cases the procedure explained is purely computational, without any reference to tables, and is followed by a geometrical proof introduced by the sentence *wa l-illa fī dhālika*. The terminology used is also interesting: the centre equation is called *al-ikhtilāf al-wasaṭī*, and the epicyclic equation is *al-ikhtilāf*

310 Edited in Calvo, 1998 a pp. 90–96. See pp. 58–62 (commentary).

311 Edited in Puig, 2000, pp. 88–89.

312 Puig, 2000, p. 73.

313 Edited in Puig, 2000, pp. 89–93.

314 Puig, 2000, pp. 74–79.

min qibal falak al-tadwīr. These chapters contain the necessary parameters to perform the computation:

TABLE 7.17

Planet	Eccentricity (fols. 41v, 46r, 47r)	Radius of the epicycle (fol. 48r, 60r, 57r)
Moon	10;19 ^P	5;10 ^P
Saturn	3;25	6;30
Jupiter	2;45	11;30
Mars	5;57	39;30 ^a
Venus	1;2	43;10
Mercury	3;6	22;30

a 36;30^P in fol. 60r, which seems an obvious mistake.

The epicycle radii are Ptolemaic and they are the same used in al-Battānī’s *zīj* and in the Toledan Tables, with the sole exception of the lunar radius (5;10^P instead of the Ptolemaic 5;15^P). The eccentricities of Saturn, Jupiter and the Moon are Ptolemaic, but the other three have undergone slight corrections: 5;57^P for Mars (instead of 6;0^P), 1;2^P for Venus (1;15^P) and 3;6^P for Mercury (3;0^P).

Although at first sight one might believe that the new parameters used for Mars, Venus and Mercury could be the result of the observations made by Ibn al-Zarqālluh towards the end of his life, the available evidence does not confirm this: Ibn al-Zarqālluh uses the Ptolemaic eccentricity for Mars and 2;51,26^P for Mercury in his treatise on the equatorium (4.5.3). The case of Venus is interesting, for it could imply a solar eccentricity of 2;4^P if we assumed a reflection of the solar eccentricity in the eccentricity of Venus. 2;4^P could have been the result of a computation, made by Ibn al-Hā’im, of the solar eccentricity for the beginning of the thirteenth century, using Ibn al-Zarqālluh’s solar model. This does not seem a valid hypothesis, as we know that Ibn al-Zarqālluh obtained an eccentricity of 1;58^P in observations made ca. 1075 (see 6.3.3.2) and his solar model implies a decrease of the eccentricity from Ibn al-Zarqālluh’s time onwards (see 6.3.3.4). I have no explanation for these new planetary parameters of the *Kāmil zīj*.

Maqāla 4 (fols. 49v–65r) contains eleven chapters and deals with the solar declination, latitude theory, and planetary stations and retrogradations.

Chapter 1 (fol. 50r): how to use a standard table of declination. The text notes that the obliquity of the ecliptic changes due to the motion of its pole around a polar epicycle (6.2.2.5.1, 6.2.2.5.2.4, 6.2.2.5.2.5).

Chapter 2 (fols. 50r–51r): on lunar latitude³¹⁵ which, according to Ibn al-Hā'im, should not be calculated using as the argument the difference between the lunar longitude on the ecliptic and the longitude of the nearest node. The lunar position should be taken on its own orbit (*al-falak al-mā'il*), and for this purpose he gives instructions to use the *Jadwal tafāḍul ḥiṣaṣ 'arḍ al-qamar* (table of the differences of the arguments of the lunar latitude), which reaches a maximum of 7'. No such table is extant in the *Kāmil zīj*, but this kind of correction appears already in Yahyā b. Abī Maṣṣūr's *Mumtaḥan zīj* and, in an Andalusī-Maghribī context, in Ibn al-Raqqām's *Shāmil zīj*. Both *zīj*es, as well as (probably) the *Kāmil zīj*, use the Ptolemaic 5° as the maximum lunar latitude, and Ibn al-Raqqām's table of this correction (*Jadwal faḍl mā bayn al-ḥiṣṣatayn*, the table of the difference between the two arguments) reaches a maximum of 6;35'.³¹⁶

Chapters 3 (fols. 51r–52v), 4 (fols. 52v–53v), 5 (fols. 53v–54v), 6 (fols. 54v–57r), 7 (fols. 57r–58v), 8 (fols. 58v–59v), and 9 (fols. 60r–61v) are concerned with planetary latitudes. Chapter 3 explains how to compute the latitude of the three superior planets giving instructions for the use of the tables, as well as a geometrical description with the corresponding demonstration. The superior planets are also described in chapter 9, which deals with the computation of the inclination (*mayl*) of each degree of the epicycle of the superior planets in respect to the ecliptic. The parameters mentioned in the text are Ptolemaic.

Chapters 4–8 are concerned with Venus and Mercury, for which we have instructions for the use of tables in chapters 4 and 5, where we find values of the maximum latitude of Venus (8;36° in fol. 53v), and Mercury (4;38° in fol. 54v) which are not the same as those in the *Almagest* (6;22° and 4;5° respectively).³¹⁷ The manuscript values seem to be corruptions of the 8;56° and 4;18° of the *Handy Tables* and al-Battānī's *zīj*.³¹⁸ Chapter 6 contains the corresponding geometrical description, while chapters 7 and 8 respectively explain how to compute the inclination (*mayl*) and the *inḥirāf* (slant) of each degree of the epicycle of Venus and Mercury in respect to the ecliptic with the corresponding geometrical proof.

315 Chapter edited in Puig, 2000, pp. 93–94, and 82–84.

316 See an edition of this table in Puig, 2000, p. 97.

317 Neugebauer, 1975, I, p. 226.

318 Neugebauer, 1975, II, p. 1014; Nallino, 1899–1907, I, p. 116, and III, p. 175.

Chapter 10 explains how to compute the periods of time in which a planet is moving with direct or retrograde motion. It contains simple instructions for the use of tables. Chapter 11 deals with the detailed explanation of the model, with a geometrical proof. On fol. 62v we find the following approximate values for the distance, in degrees, of the motion of the centre of the epicycle on its deferent, between the first station and the *muqāṭara* (explained on fol. 63v as the midpoint between the two stations):

Saturn 2;20°
 Jupiter 5°
 Mars 19°³¹⁹
 Venus 20;30°
 Mercury 11°

Maqāla 5 (fols. 65r–72r) contains three chapters edited and explained by Emilia Calvo.³²⁰ Again, it deals with the Sun and the length of the solar year (6.3.3.1).

Chapter 1 (fols. 65r–70v) establishes the procedure for obtaining the length of the anomalistic year (*zamān al-ʿawda fī khārīj al-markaz*), by comparing the results of two solar observations at either the spring or the autumn equinox, one of them as old as possible and the other contemporary. For the computation one has to consider the increase in longitude of the Sun due to precession, the motion of the solar apogee and the change in the value of the solar equation due to the variation of the solar eccentricity, since the purpose is the return of the Sun, in the second observation, to the same point of the ecliptic and to the same distance from its apogee. The details of the computation are not the same if the two observations are made at the spring equinoxes or at the autumn equinoxes and, in both cases, the computation should take into account 1) whether the solar equation has the same value in both observations, 2) whether the solar equation in the recent observation is greater than the one in the ancient observation, or 3) whether the recent observation has a smaller value of the solar equation than the ancient one. Six cases, therefore, have to be considered and Ibn al-Hāʾim gives careful instructions, accompanied by geometrical figures, for each one.

Chapter 2 (fols. 70v–71v) calculates the length of the sidereal year (*zamān al-ʿawda fī falak al-burūj*), used for astrological purposes. The procedure is far

319 These values agree, approximately, with those of the *Almagest* for Saturn (2;21,18°) and Jupiter (5;6,35°), but not in the case of Mars (20;58,21°). See Pedersen, 1974, p. 350.

320 Calvo, 1998 a, pp. 96–111 (edition) and 62–75 (commentary).

more simple: one has to calculate the solar equation for midday of a given day, as well as its mean longitude and the position of the solar apogee. Then, one adds to the solar mean longitude and to the longitude of the apogee their motion in 40 Persian years and 10 days and determines their corresponding true longitude by adding or subtracting the value of the solar equation calculated for the previous date.³²¹ Then one adjusts the solar equation, in order to obtain the same true longitude as in the first computation, by adding the necessary number of hours, minutes and seconds. This excess in time will be divided by 40, obtaining fractions of an hour as a result. These fractions will be added to 365 days and 6 hours to obtain the value of the sidereal year.

According to Ibn al-Hā'im, the value of the sidereal year should be used for astrological purposes such as the revolutions of the years (*taḥāwīl al-sinīn*) of the nativities or of the world; one should not make the same mistake as Ibn al-Kammād, who used the anomalistic year for that purpose. In this chapter, he mentions a *revolutio anni* of 92;21,45° equivalent to 6;9,27^h, which yields a sidereal year of 365^d 6;9,27^h, or 365;15,23,37,30^d, a length of the year confirmed in chapter 3. As a matter of fact a different value is attributed to Ibn al-Hā'im in the table of the *fuḍūl al-ṭawālī* preserved in the Tunisian recension of Ibn Ishāq's *zīj*,³²² which yields a value of the *revolutio anni* of 92;20,56,40,12° from which we obtain a sidereal year of 365^d 6;9,23,46,40,48^h or 365;15,23,29,26,42^d. This length of the sidereal year is valid for a period of about 40 years, or slightly more (*aw azyad qalīlan*) and it should be recomputed afterwards, as the position of the solar apogee will have moved and the solar eccentricity will have changed.

Chapter 3 (fols. 71v–72r) deals with two kinds of astrological months named *dawrī* and *maṭla'ī*. The former is equivalent to the *shahr intihā'ī* (7.2.5.3.2), as it is the result of dividing the length of the sidereal year by 13 and, in chapter 2, Ibn al-Hā'im has already established that a *dawrī* month contains 28^d 2;19,51^h, which implies a sidereal year of:

$$28^{\text{d}} 2;19,51^{\text{h}} \cdot 13 = 365^{\text{d}} 6;18,3^{\text{h}} = 365;15,45,7,30^{\text{d}}$$

which does not correspond precisely to any of the values of the sidereal year previously mentioned.

321 This is clear in the Arabic text: *Thumma qawwim-hā ayḍan bi-tilka al-ta'ādīl al-ūlā anfuṣi-hā wa-hiya allatī bi-hā qawwamta-hā awwalan* (then establish its true position using the same equation as in the first computation). This seems surprising because the solar apogee has changed and, therefore the equation as well.

322 'Abd al-Raḥmān, 1996 a, pp. 374–5.

The *maṭlaʿ* month can be obtained by dividing the sidereal year by $15;4,44^\circ$. In the Oxford manuscript we read, in chapter 2, that the length of this month is 24 days and a fraction impossible to read. Emilia Calvo³²³ computes it from the two parameters given by Ibn al-Hāʾim:

$$365;15,23,37,30 / 15;4,44^\circ = 24^d 5;21,11^h = 24;13,22,57^d$$

Maqāla 6 (fols. 72r–80r) contains 17 chapters. With the sole exception of chapter 17, they are standard canons giving brief instructions for the use of tables and solving problems of spherical astronomy: obtaining right ascension from ecliptic longitude and vice versa (chapters 1 – fols. 72v–73r – and 2 – fol. 73r); right descension from longitude and vice versa (ch. 3 – fol. 73v – and 4 – fols. 73v–74r); oblique ascension from longitude and vice versa (ch. 5 – fols. 74r–74v – and 6 – fol. 74v); oblique descension from longitude and vice versa (ch. 7 – fols. 74v–75r – and 8 – fol. 75r); half the day arc of a given degree of solar longitude from its oblique ascension (ch. 9 – fols. 75r–75v); obtaining the time degrees of a seasonal day or night hour: in this chapter he mentions a table of oblique ascensions in which a column gives the number of degrees of a seasonal day hour, as a function of the solar longitude and the local latitude (ch. 10 – fol. 75v); number of equal hours of a day or night (ch. 11 – fols. 75v–76r); time degrees of a seasonal hour of the day or night from the day or the night arc (ch. 12 – fol. 76r); number of equal hours of a day or night from the day or the night arc (ch. 13 – fols. 76r–76v); number of seasonal or equal hours of a particular day since sunrise or sunset (ch. 14 – fol. 76v); converting equal hours with their minutes and seconds into minutes, seconds and thirds of a day (ch. 15 – fol. 77r); and determining the number of equal hours of the longest day and night of the year (ch. 16 – fols. 77r–77v).

Chapter 17 (fols. 77v–80r) is far more interesting, dealing as it does with the equation of time. It does not contain a full treatment of the topic, but concentrates on the effect that the motion of trepidation has on its computation. The whole chapter deals with the amount to be added to or subtracted from the difference in solar right ascension between the two dates considered, in order to obtain the unequal (*mukhtaliḥ*) time from the equal (*mustawī*) time or the other way round. This amount is the “second accession” (*al-iqbāl al-thānī*), and it corresponds to the difference between the right ascension of the moving Head of Aries and the intersection of the equator with the equatorial epicycle (equatorial arc GZ in fig. 7.1 below). The second accession is tabulated in Ibn al-Zarqālluh’s treatise on the motion of the fixed stars (6.2.2.5.2.3) and

323 Calvo, 1998 a, p. 74.

it reappears in many Andalusī and Maghribī *zīj*es (6.2.6.2). This is, in fact, the only chapter related to trepidation in Ibn al-Hā'im's *zīj* that was not edited and analysed by Mercè Comes.³²⁴

Maqāla 7 (fols. 80r–87v) contains 9 chapters, of which chapters 1–4 were edited and studied by Mercè Comes.³²⁵

Chapter 1 (fols. 80r–81r) explains in detail how to compute the minimum value of the obliquity of the ecliptic from an observed obliquity. In this chapter, Ibn al-Hā'im's treatment of the trigonometrical problem is extremely careful and he only uses exact procedures. In his description, he shows his knowledge of the cosine law formulated both in Ibn Mu'adh's *Kitāb majhūlāt qisī al-kura* and in Jābir b. Aflah's *Iṣlāḥ al-Majisṭī*. His radius of the polar epicycle is 0;10,20^p instead of Ibn al-Zarqālluh's 0;10^p.

Chapter 2 (fols. 81r–81v) deals with the computation of the value of the obliquity of the ecliptic for a given time. Ibn al-Hā'im's treatment of the topic is similar to that in chapter 1 but in the reverse order. He uses, again, a radius of the polar epicycle of 0;10,20^p as well as the cosine law. As noted by Comes, this chapter is closely related to the corresponding passage in Ibn al-Zarqālluh's treatise on the motion of the fixed stars.³²⁶

Chapter 3 (fols. 82r–82v) uses Ibn al-Zarqālluh's procedure³²⁷ (6.2.2.5.1) for computing the value of trepidation Δ_λ (the first accession, *al-iqbāl al-awwal*) for a known obliquity of the ecliptic (ϵ) and the declination of the moving Head of Aries (δ), by applying the sine law, which seems also to have been known by Ibn al-Zarqālluh:

$$\sin \Delta_\lambda = \sin \delta / \sin \epsilon$$

Interestingly, Ibn al-Hā'im seems to use a table that gives the sine of the declination of the head of Aries directly, while Ibn al-Zarqālluh needs two tables: a table of declination (6.2.2.5.2.2) and a table of sines. Besides, Ibn al-Hā'im considers that the first accession is “perceptible by the senses” (*maḥsūs*) while, in chapter 4, the second accession is only “perceptible by the intellect” (*ma'qūl*). This idea also derives from Ibn al-Zarqālluh, although the latter considers the second accession to be the “true” one.³²⁸

³²⁴ Comes, 2001.

³²⁵ Comes, 2001, pp. 388–398 and 348–360.

³²⁶ Comes, 2001, p. 352 and Millás, 1943–50, p. 330.

³²⁷ Millás, 1943–50, p. 330.

³²⁸ Comes, 2001, p. 353.

Chapter 4 (fols. 82v–83v) deals with the computation of the “second accession” (*al-iqbāl al-thānī*), which Ibn al-Hāʾim calls *iqbāl al-ʿamūd fī l-quṭr* (“accession of the perpendicular to the diameter”). In fig. 7.1 this second accession is GZ, an arc of the equator comprised between G (intersection of the equator with the equatorial epicycle) and Z (foot of the perpendicular BZ, dropped from point B, the moving Head of Aries). It is clear, therefore, that while the “first accession” (BD) is an arc of the ecliptic, the second accession (ZG) is an arc of the equator and it is defined, in the title of the chapter, as the right ascension of each degree of the equatorial epicycle with respect to the meridian; it seems to refer to the arc of the equator between the meridian transits of two points of the equatorial epicycle.³²⁹ We have already seen that, in chapter 17 of *maqāla* 6, Ibn al-Hāʾim uses the *iqbāl al-thānī* to calculate the equation of time.

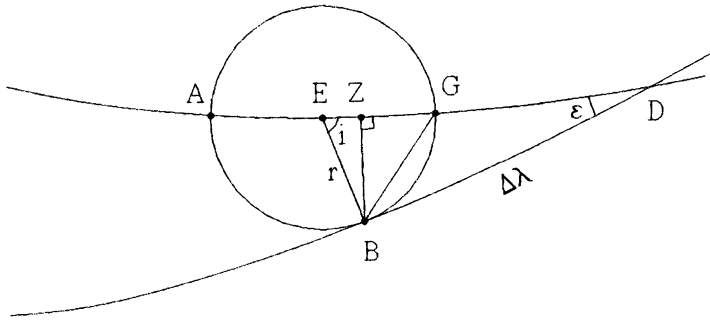


FIGURE 7.1 Borrowed from Mercè Comes

Interestingly, Ibn al-Hāʾim gives instructions for obtaining the second accession using tables but adds a description of the computational procedure, for which he uses a radius of the equatorial epicycle of 4;19,26^p, which is not the same as the 4;8^p (a rounded value of Ibn al-Zarqālluh's 4;7,58^p) mentioned in chapter 1 of the second *maqāla* (see 6.2.5).³³⁰ For this purpose he obtains BG (4;19,26^p · chord of angle i) and BZ (4;19,26^p · sine of $[180^\circ - i]$), considering that both BG and BZ are arcs of a great circle, and determines ZG (second accession), by an application of the cosine law to triangle BZG.

The rest of the chapters of this *maqāla* are less interesting, although they bear witness to Ibn al-Hāʾim's knowledge of the new trigonometry. They deal with the following issues:

³²⁹ Comes, 2001, p. 357.

³³⁰ Comes, 2001, pp. 371–377.

Chapter 5 (fols. 83v–84r): computation of the value of the solar declination if the solar longitude is known, after computing the obliquity of the ecliptic for a given time. A geometrical explanation is added, in which Ibn al-Hā'im shows that he is using the sine law.

Chapter 6 (fols. 84r–84v): computation of the lunar latitude using the same procedure as in the previous chapter, knowing that its maximum value is always 5° . This is followed by another rule for obtaining the difference between the lunar argument of latitude (*al-ḥiṣṣa al-ḥaḳīqīyya*) and the arc of the ecliptic comprised between the lunar node and the foot of the perpendicular to the ecliptic traced from the Moon (*ḥiṣṣat al-tūl*). This is the *tafāḍul ḥiṣaṣ al-ʿarḍ* (difference between the arguments of latitude), for which Ibn al-Hā'im has mentioned a table in *maqāla* 4, chapter 2.

Chapter 7 (fols. 84v–85v): obtaining the right ascension of a degree of the ecliptic if one knows its declination, which depends on the variable obliquity of the ecliptic: this explains the title of the chapter which states that the procedure is valid “for any time” (*fī kull zamān*).

Chapters 8 (fols. 85v–87r) and 9 (fol. 87v): obtaining sines, chords, versed sines and cosines of given arcs and vice versa. At the end of chapter 8, Ibn al-Hā'im draws attention to a mistake in Ibn al-Kammād's *Muqtabas zīj*.

7.3 Maghribī zījēs: the School of Ibn Ishāq

7.3.1 Introduction: Ibn Ishāq's Lost zīj

Ibn al-Hā'im's *Kāmil zīj* seems to be the last work of this kind produced in al-Andalus (?). Years ago, I found that this tradition had a continuation in thirteenth- and fourteenth-century Maghrib (see 1.4.2). Before this, the only reference to a *zīj* compiled in the Maghrib is Ibn Abī l-Rijāl's *Ḥall al-ʿaqad wa bayān al-raṣad* (“Untying of knots and explanation of observations”), apparently lost and mentioned repeatedly in his great astrological handbook *Kitāb al-bārī fī aḥkām al-nujūm*. In 3.2.2.3 I have tried to gather together the information on this *zīj* contained in the *Bārī*.

Maghribī *zījēs* reappear with the figure of Abū l-ʿAbbās Aḥmad b. ʿAlī b. Ishāq al-Tūnisī, who flourished in Tunis and Marrakesh ca. 1193–1222, and began the compilation of an unfinished *zīj*. This is confirmed by Ibn al-Bannā in the prologue to his *Minhāj*, where he states that his book is based on Ibn Ishāq's *zīj*, which he discovered in the collection of tables noted down on cards on which he based his motions of the planets and their equations (*baʿda wuqūf ʿalā mā khalafa-hu muqayyad^{an} fī-baṭāʾiqi-hi mim mā iʿtamada-hu fī l-ḥarakāt*

wa l-ta'ādīl).³³¹ This is why three later Maghribī astronomers compiled five “editions” of these tables, containing different sets of canons and respecting the contents of Ibn Ishāq’s “cards”. I am referring to the anonymous compiler of the Tunisian recension (fl. ca. 1266–1281), Ibn al-Bannā’s (1256–1321) *Minhāj*, and the three *zīj*es of Ibn al-Raqqām (d. 1315).

According to Ibn Khaldūn (1332–1382)³³² (cf. 6.2.6.4), the *zīj* attributed to Ibn Ishāq was widely used in the Maghrib of his time and it was the result of Ibn Ishāq’s own observations as well as the information on the conditions (*aḥwāl*) and motions of the celestial bodies he received from a Sicilian Jew, who was also an observer. The identity of this Sicilian Jew was the object of many discussions at the University of Barcelona, as a result of the discovery of ms. Hyderabad Andra Pradesh State Library 298, a copy of which was sent to us years ago by David King. In this manuscript we found a strange table with the names of astronomers who made observations that allowed them to determine the position of the solar apogee and the value of the obliquity of the ecliptic (6.2.6.4). In this table, the name of a Sicilian observer, ca. 574H/1178–9, appears as Gh.yām b. Y.ḥār.³³³ As this date might correspond to that of the mysterious informant of Ibn Ishāq, we tried, unsuccessfully, to identify him until we realised that a small correction of a *rāʾ* instead of a *yāʾ* (R.ḥār) and a diacritic mark under the *ḥāʾ*, resulting in a *jīm*, gave the reading Ghiyām ibn Rujjār, William son of Roger. This person could be identified with the Norman King William II (1166–89), son of William I (1154–66), who was the son of Roger II of Sicily,³³⁴ and who probably had in his service an unnamed Jew who acted as his astrologer and who, at a later date, was Ibn Ishāq’s informant. Mestres considers that the information given to Ibn Ishāq by our anonymous Jewish astronomer consisted mainly in the transmission of Andalusī materials derived from Ibn al-Zarqālluh.³³⁵ This was probably the channel through which Andalusī astronomy exerted its strong influence in the Maghrib for a couple of centuries.

The aforementioned dates in which Ibn Ishāq was, apparently, active (ca. 1193–1222), have been obtained by Angel Mestres. The first date (1193) appears in the table of the historical observations of the solar apogee and obliquity of the ecliptic, where 589H/1193 is considered to be the year of Ibn Ishāq’s

331 Vernet, 1952, pp. 13 (Arabic edition) and 57 (Spanish translation).

332 Ibn Khaldūn, in Quatremère, 1858, vol. III, pp. 107–8.

333 Mestres, 1996, p. 388.

334 Mestres, 1999, p. 4.

335 Mestres, 1996, pp. 387–391.

observations.³³⁶ As for the second year (1222), it is mentioned in a marginal note in ms. Escorial 909, which contains Ibn al-Bannā's *Minhāj*: "Ibn Ishāq observed the motion of the planets, upon which this *zīj* is based, in year 619[/1222] of the Hijra".³³⁷ According to Ibn al-Raqqām's *Mustawfi zīj*, Ibn Ishāq observed two lunar eclipses which allowed him to determine the geographical longitude of Tunis (41;45° from the meridian of water) and established, after multiple observations, that the latitude of this city was 36;37°. ³³⁸ Besides, Ibn Ishāq uses Ibn al-Zarqālluh's solar model with variable eccentricity (cf. 6.3.3.2) and obtains a maximum solar equation of 1;49,7°, corresponding to an eccentricity of 1;54,15^p. This value corresponds precisely to the eccentricity Mestres obtains for year 619/1222 (see 6.3.3.5), ³³⁹ and it was also used in Ibn al-Raqqām's *Mustawfi zīj*, attributed to Ibn Ishāq. ³⁴⁰

We have already seen some aspects of the Zarqāllian influence in the *zīj*es belonging to Ibn Ishāq's tradition, particularly concerning trepidation (6.2.6.2), the obliquity of the ecliptic (6.2.6.3), the motion of the solar apogee (6.3.2), the solar model with variable eccentricity (6.3.3.2, 6.3.3.3 and 6.3.3.5) and the correction of Ptolemy's lunar model (6.4). To these we can add Ibn Ishāq's method for predicting lunar crescent visibility (2.3.2, 2.3.4.2.2, and 2.3.4.2.3). Here I will try to give an overview of other interesting aspects of this group of *zīj*es. Among them, I will begin with the tables related to the computation of the true longitudes of the Moon and the planets. Ibn al-Bannā's statement concerning the "cards" seems to imply that these documents contained planetary mean motions and equations, to which one should add certain other topics like the equation of time. ³⁴¹ It seems that Ibn Ishāq's cards also contained eclipse tables, because the anonymous compiler of the Tunisian recension notes the existence of a difference between Ibn al-Kammād's and Ibn Ishāq's tables for the computation of lunar eclipses, although the latter have not been preserved. ³⁴²

As the parameters used by the five extant *zīj*es belonging to Ibn Ishāq's tradition both for mean motions and equations are the same, we can conclude that they correspond to Ibn Ishāq's original *zīj*.

336 Mestres, 1996, pp. 388–9.

337 Vernet, 1952, p. 21.

338 Samsó, 2014, p. 301.

339 Mestres, 1999, pp. 43–44.

340 Samsó, 2014, p. 302.

341 Mestres, 1996, p. 385.

342 Mestres, 1999, pp. 106 (commentary) and 173 (Arabic text).

7.3.2 *The Five zījēs of Ibn Ishāq's School*

7.3.2.1 The Anonymous Tunisian Recension

This source was discovered by David A. King in 1978 in ms. Hyderabad Andra Pradesh State Library 298, and was compiled between 1266 and 1281.³⁴³ This is a manuscript written in a clear Eastern script, although most of the ciphers used are Maghribī and Fez-numbers are also found.³⁴⁴ It is divided into two volumes, containing the canons (43 chapters) and the numerical tables respectively, dated 13 February 1317 and 8 June 1317. The name of the copyist appears at the end of the tabular section: 'Alī ibn Shadhān b. Mūsā b. 'Abd Allāh al-Ḥasanī, in the city of Ḥimṣ (Syria).

The name of the author of the compilation is unknown but some information about the period in which he lived can be gathered from the many examples that are worked out, all of them located in Tunis and dated in the following years: 1267, 1268, 1278, 1281, 1282, and 1283. In his preface, our unknown author asserts that he found Ibn Ishāq's tables, without canons, and that he tried to complete them using mainly Andalusī sources: the canons of the three *zījēs* of Ibn al-Kammād (7.2.5) and of Ibn al-Hā'im's *Kāmil zīj* (7.2.6). To these, Mestres adds the *Calendar of Cordoba*, Ibn Mu'ādh's *zīj* (7.2.2), the *Sindhind* of al-Khwārizmī-Maslama (7.2.1), the *Toledan Tables* (7.2.4), the *zīj* of a certain Ibn al-Bayṭār, the astrolabe treatise of Ibn al-Ṣaffār (4.2.1), Ibn al-Zarqālluh's treatise on the *shakkāziyya* (4.3.2.2), the astrological *Kitāb mafātiḥ al-asrār* by Ibn al-Kammād (3.2.4) and the *Ghāyat al-ḥakīm (Picatrix)*, as well as the Maghribī *Kitāb al-bārī* of Ibn Abī l-Rijāl (3.2.2) and a *zīj* of al-Sabtī, compiled in Tlemcen. Among the Eastern sources mentioned we find the *Mumtaḥan zīj*, Ibn Yūnus and al-Battānī, as well as astrological sources like *al-Kitāb al-Kāmil* of Mūsā b. Nawbakht, Abū Ma'shar's *Kitāb al-milal wa l-duwal* and other works by Abū 'Alī al-Khayyāt, Bishr al-Isrā'īlī, al-Rukānī and Abū Hanīfa al-Dīnāwārī.

This anonymous recension of Ibn Ishāq's lost *zīj* is a most interesting compilation of mainly Andalusī sources, some of which are not extant, dealing with all the aspects of a standard *zīj*. It also contains a complete survey of everything needed for the practice of astrology from a mathematical point of view (*ṣinā'at al-nujūm*), excluding the treatment of predictions based on the data of the horoscope (*ilm aḥkām al-nujūm*).³⁴⁵

343 King, 2007, pp. 181–184; Mestres, 1996. The whole set of canons and a selection of the tables were edited and studied in Mestres, 1999.

344 Comes, R., 2002–3.

345 All the information given in this section derives from Mestres, 1999, pp. 1–8 and 1996, pp. 384–387.

7.3.2.2 Ibn al-Bannā's *Minhāj*

A second recension of Ibn Ishāq's *zīj* is the *Minhāj al-ṭālib fī ta'dīl al-kawākib* (Method a student should use in order to calculate the true positions of celestial bodies). Its author is the well-known mathematician of Marrakesh Abū l-'Abbās Aḥmad b. Muḥammad b. 'Uthmān al-Azdī, known as Ibn al-Bannā' al-Marrākushī, born 9 or 10 Dhū l-Ḥijja 654/29 or 30 December 1256, and died 5 Rajab 721/31 July 1321.³⁴⁶ This author was a polymath interested in both the exact and the religious sciences; he is better known as a mathematician than as an astronomer, although he was also a cultivator of astrology, probably during the early stages of his scholarly life. As we have seen (6.3.3.3) and we will again see later (7.3.3.5), his mathematical ability led him to design methods that help the computer to calculate solar and planetary positions.

According to Jabbār and Aballāgh, Ibn al-Bannā's *Minhāj* was compiled ca. 1290–1301, and it contains a standard set of tables and a collection of canons which are very different from those of the Tunisian recension.³⁴⁷ While the latter tried to accumulate the information available deriving, basically, from Andalusī sources, Ibn al-Bannā's *zīj* contains simple canons of a practical nature. This is the main reason for its popularity: it has been preserved in a great number of manuscripts³⁴⁸ and Ibn Khaldūn states that it is a summary and bears witness to this diffusion, when he says "Ibn al-Bannā' summarised it (*lakhkhaṣa-hu*) [Ibn Ishāq's *zīj*] in another [*zīj*] called "the method" (*al-minhāj*). People loved this book because it was easier to work with and because it was needed to obtain the positions of the celestial bodies in the sphere in order to build astrological predictions on them".³⁴⁹ It is a fact that this *zīj*, based on sidereal mean motions, was used by astrologers (until the nineteenth century),³⁵⁰ when Ibn Ishāq's astronomical tradition had been discarded in the Maghrib and replaced by Eastern *zīj*es that used tropical mean motions (7.5).

7.3.2.3 Ibn al-Raqqām's Three *zīj*es

Abū 'Abd Allāh Muḥammad b. Ibrāhīm b. 'Alī b. Aḥmad (Muḥammad?) b. Yūsuf al-Mursī al-Andalusī al-Tūnisī al-Awsī, known as Ibn al-Raqqām, belonged to "the people of Murcia" (*min ahl Mursiya*) and died at an advanced age in Granada on 2 Ṣafar 715 (26 May 1315). He lived in Tunis and Bijāya and migrated

346 On his biobibliography see Jabbār & Aballāgh, 2001. A detailed summary, in English, of its contents can be found in my review of this book published in *Suḥayl* 3 (2002–3), 251–256.

347 The canons were edited and translated into Spanish in Vernet, 1952.

348 See Jabbār & Aballāgh, 2001, pp. 112–6.

349 Ibn Khaldūn in Quatremère, 1858, III, p. 108.

350 Vernet, 1980 b.

from Bijāya to Granada during the reign of Muḥammad II (1273–1302), probably after 1288–89.³⁵¹

He compiled three recensions of Ibn Ishāq's *zīj*:

1. *Al-Zīj al-Shāmīl fī tahdhīb al-Kāmil*: extant in only one manuscript (Istanbul, Kandilli 249).³⁵² It was compiled in Bijāya in 679/1279–80, according to the introduction. The table of the lunar mansions, however, is dated 887 (*sic*, a mistake for 687/1288–89?) and it adds 16;50° to the longitudes of the star catalogue in the *Almagest*. The maximum tabular value of the solar equation is 1;47,51°, which corresponds to year 689/1290, due to the fact that Ibn al-Raqqām uses a Zarqallian model with variable solar eccentricity. It is possible to imagine that 679 (written in words in the introduction) could also be an error for 689. The *Kāmil* mentioned in the title is *al-Zīj al-Kāmil fī ʾl-taʾālīm*, compiled by Ibn al-Hāʾim (fl. 1204–5), summarised in 7.2.6. Neither the Bodleian Library MS nor the copy available to Ibn al-Raqqām contained any tables, in spite of the fact that the canons mention a few of them. So Ibn al-Raqqām copied the canons in the *Kāmil* of a more practical character, removed the mathematical proofs, and completed them with Ibn Ishāq's numerical tables. To this he added some 60 chapters on problems related to spherical astronomy and astrology, which seem to be unrelated to the *Kāmil*.

2. *Al-Zīj al-Qawīm fī funūn al-taʾdīl wa ʾl-taqwīm*: The complete text is extant in MS Rabat NL 260, and a fragmentary copy can be found in an unnumbered MS at the Museo Naval in Madrid, where we also find the *Minhāj* of Ibn al-Bannā'.³⁵³ It was finished in Tunis (longitude 41;45°, latitude 36;37°),³⁵⁴ and it was obviously revised in Granada, after Ibn al-Raqqām's arrival in the city, for it contains a table for establishing the visibility of the crescent calculated for a latitude of 37;10°. This is, precisely, the modern value for the latitude of that city, with no precedents before Ibn al-Raqqām, who must have made a very careful determination of this parameter. What is more, Kennedy³⁵⁵ studied the procedures described by Ibn al-Raqqām to determine crescent visibility in the *Shāmīl* and in the *Qawīm*, and showed that the method explained in

351 See Ibn al-Khaṭīb, in 'Inān, 1973–77, vol. III, pp. 69–70, 334. A summary of what we know about this author can be found in Casulleras, "Ibn al-Raqqām", in *BEA* 1, pp. 563–564; and Samsó, in *BA*, vol. 4, pp. 440–444.

352 A partial edition of the canons and tables of this *zīj* was prepared by 'Abd al-Raḥmān, 1996. Besides, 'Abd al-Raḥmān left a provisional transcription of the rest of the canons in the section of Arabic Studies of the University of Barcelona, which I have found particularly useful.

353 See Vernet, 1952.

354 In King's survey of tables for timekeeping calculated for Tunis, the latitudes for this city are 36;40° and 37°, but not 36;37°: King, 2004, pp. 64–65, 110, 119, 157–158, 428–431.

355 Kennedy, 1997, pp. 38–41.

this second *zīj* could only be applied to the latitude of Granada. It contains a table of stellar coordinates “observed” (*marṣūda*) in 680/1280–81 [MS Rabat 260, 100–103].³⁵⁶ The *Qawīm zīj* must have been finished after 1290, because it contains the table of the solar equation that we find in the *Shamil*. This *zīj* is, apparently, the least ambitious of the three and the canons are the usual ones in works of this kind: namely, mere instructions for the use of the tables, with very few theoretical materials.

In 1997, I spent a week at the Frankfurt Institut für Geschichte der Arabisch-Islamischen Wissenschaften exploring the collection of microfilms compiled by Prof Fuat Sezgin. Among them I found a microfilm of MS Rabat National Library 2461, which proved to be a late copy of a third *zīj* by Ibn al-Raqqām. Prof Sezgin generously provided me with an enlargement of this microfilm and I (as well as two other colleagues at the University of Barcelona) have used it in four papers,³⁵⁷ including a general description of this third *zīj*, which I will summarise here:

3. *Al-Zīj al-Mustawfī*³⁵⁸ *li-mā ḥāza min al-baṣṭ wa l-ḥaẓẓ al-awfār wa 1-qisṭ al-awfā* (*Zīj* with which everything acquired through donation and generous fortune in plentiful amount is fully returned): extant in MSS 534K (discovered by M.J. Parra), 2461 and 4157 of the National Library in Rabat, which also preserves MS 4156, containing a fragment of the same work related to the equation of time. Other MSS are Tunis National Library 11018 (Aḥmadiyya collection, old number 5584)³⁵⁹ and Cairo National Library DM 718,2 (only Chapters 50–51 of the canons). It was compiled in Tunis after 680/1280–81 (the date of the incomplete star table calculated with an increase of 16;46° on the longitudes of the *Almagest*). From the column of meridian altitudes of the stars, one can calculate that the latitude of the place of observation is 36;37° (Tunis). This agrees with the coordinates given in the *Qawīm Zīj* and with the *radice* for mean motions, calculated for the meridian of Tunis for which a longitude of 41;45° is given, with the comment that it was established by Ibn Ishāq as a result of the observation of two lunar eclipses. The table of the solar equation is of little

356 On this table see Parra, 2012; Casulleras, 2012.

357 Díaz-Fajardo, 2005; Samsó, 2008 b; Bellver & Samsó, 2008; Samsó, 2014.

358 This title contains a possible rhyme and this led me to believe that the first word was *Mustawfā* (rhyme with *al-awfā*) which both Díaz-Fajardo and I have used in several publications. George Saliba suggested to me that *Mustawfī* was a better option and I follow his advice here.

359 I have been able to analyse the Tunis ms. thanks to the generosity of my friend Chedli Guesmi.

use for establishing a date: it was copied from Ibn Ishāq's *Zīj*, and the original table was calculated for year 619/1222. Of the three *zīj*es by Ibn al-Raqqām, the *Mustawfī* is the one that spread the most widely in the Maghrib. We know this from the number of extant manuscripts as well as from the fact that the *Mustawfī* was used by Maghribī *muwaqqits*. It is frequently quoted and used by two commentators of the well-known *urjūza* (written in Fez by al-Jādirī in 1391–92), *Rawḍat al-azhār fī 'ilm waqt al-layl wa l-nahār*: these are al-Habbāk, the author of the *Natā'ij al-afkār* (Tlemcen ca. 1514–15) and Ibn al-Muftī (fl. Marrakesh, Fez and Tarudant, d. 1611) (see 6.2.6.6). This *zīj*, like the *Shāmīl*, is clear proof of Ibn al-Raqqām's interest in spherical astronomy, something that is quite rare in Andalusian and Maghribi astronomy.

From the information available we might think that the three *zīj*es were compiled between ca. 1280 and ca. 1290, before Ibn al-Raqqām's departure for Granada, where he arrived before 1302, although additions were made to the *Qawīm* after the astronomer's arrival in the city. The constants of precession used in the star tables of the *Mustawfī*, the *Qawīm* (16;46°), and the *Shāmīl* (16;50°) lead me to believe that the *Mustawfī* and the *Qawīm* were compiled before the *Shāmīl*. On the other hand, the table of the solar equation in the *Mustawfī* (copied from Ibn Ishāq), suggests that this *zīj* was written before the *Shāmīl* and the *Qawīm* (table calculated for year 689/1290). As a result, I suggest the following order: 1st *Mustawfī* (Tunis), 2nd *Qawīm* (Tunis, with a later revision in Granada) and 3rd *Shāmīl* (Bijāya). If this suggestion proves to be true, it implies that Ibn al-Raqqām began his professional life in Tunis and, at a later date, moved to Bijāya.

7.3.3 *An Attempt to Recover Some Materials from Ibn Ishāq's Original Tables*

In chapter 6, I already tried to analyse the common features found in the five aforementioned *zīj*es, derived from Ibn al-Zarqālluh's astronomical theories and related to trepidation and obliquity of the ecliptic (6.2.6), the motion of the solar apogee and the solar model with variable eccentricity (6.3) and, finally, Ibn al-Zarqālluh's correction to Ptolemy's lunar model (6.4). The original source from which the authors of our *zīj*es derived their ideas was, clearly, Ibn Ishāq's lost work. In this section, I will try to give some information on the computation of the longitudes of celestial bodies, as well as eclipses, equation of time, parallax and, probably, solar and lunar velocity. My hypothesis is: when the same tables reappear in the five extant *zīj*es, it means that they share a common source, which should be identified with Ibn Ishāq's *zīj*.

7.3.3.1 Mean Motions³⁶⁰

TABLE 7.18

	Ibn Ishāq	Difference Ibn Ishāq – <i>Toledan Tables</i>
Sun	0;59,8,11,28,26,22,5,24,24,26°	0;0,0,0,0,1, 8°
Moon (longitude)	13;10,34,52,46,51,38 ± 0;0,0,0,0,0,26	0;0,0,0,1, 55,30
Moon (anomaly)	13;3,53,56,17,52,4 ± 0;0,0,0,0,0,21	0;0,0,0,0,0,50
Moon (node)	0;3,10,46,41,0,4 ± 0;0,0,0,0,0,20	–0;0,0,0,1,33,13
Saturn (longitude)	0;2,0,27,46,42,42 ± 0;0,0,0,0,0,22	0;0,0,1,11,26
Jupiter (longitude)	0;4,59,7,36,25,11 ± 0;0,0,0,0,0,23	0;0,0,0,0,54,12
Mars (longitude)	0;3,1,26,31,9,5,28 ± 0;0,0,0,0,0,20	–0;0,0,1,6,11,33
Venus (anomaly)	0;36,59,27,23,59,25 ± 0;0,0,0,0,0,21	–0;0,0,2,3,30,4
Mercury (anomaly)	3;6,24,7,42,40,49 ± 0;0,0,0,0,0,20	0;0,0,0,3,9,23

The comparison between Ibn Ishāq's mean motion parameters and those of the *Toledan Tables* (see table 7.8 in 7.2.4.2.2.1) shows that the mean solar motion, as well as the mean motion of the lunar anomaly, are almost the same. The parameters for the mean solar motion and for the mean lunar motion in anomaly seem to have a Zarqāllian origin; it seems that Ibn al-Zarqālluh did not change his mean solar motion when, at a later stage in his life, he wrote his book on the Sun (see 6.3.3).³⁶¹ The rest of the mean motion parameters are either original or derive from a different source.

As remarked by Mestres,³⁶² one interesting characteristic of the mean motion tables in the Tunisian recension, Ibn al-Bannā's *Minhāj* and Ibn al-Raqqām's *Shāmīl* is the presence of a subtable, with arguments from 1 to 60, giving the corresponding mean motions in hours. The same values, divided by 60, will correspond to mean motions in minutes.

³⁶⁰ Mean motion parameters squeezed by Mestres, 1999, p. 189.

³⁶¹ See also Samsó & Millás, 1994, pp. 3–4.

³⁶² Mestres, 1999, pp. 27, 188.

7.3.3.2 Epoch Radices³⁶³

The source for Ibn Ishāq’s positions for the beginning of the Hijra is the anonymous Tunisian recension. They are calculated for the meridian of Toledo, to which the anonymous compiler adds a second set of positions, apparently calculated by himself, for the meridian of Tunis, with results similar to those of Ibn al-Raqqām for the same meridian. Besides, in his *Shāmīl zīj*, Ibn al-Raqqām gives a set of radices for the meridian of Arin that is consistent with those for Bijāya.

TABLE 7.19

	Ibn Ishāq (Toledo)	Toledan Tables	Ibn al-Bannā’ (Marrakesh)	Ibn al-Raqqām (<i>Mustawfī</i> , <i>Qawīm</i> , Tunis)	Ibn al-Raqqām (<i>Shāmīl</i> , Bijāya)
Sun	113;21,14°	113;41,11°	113;22,23°	113;18,58°	113;20,14°
Moon (longitude)	120;32,32	120;58,18	120;47,54	120;2,20	120;19,21
Moon (anomaly)	108;8,39	108;8,39	108;23,54	10[7];38,34 ^a	107;55,35
Moon (node)	234;19,37	234;9,55	234;19,41	234;19,30	234;19,30
Saturn (longitude)	116;17,15	115;51,15	116;17,17	116;17,9 ^b	116;17,13
Jupiter (longitude)	330;11	331;39,37	330;11,5	330;10,49 ^c	330;10,55
Mars (longitude)	210;37,25	211;24,59	210;35,31	210;36,13	210;36,54
Venus (anomaly)	44;30	45;28,37	44;30,43	44;28,34 ^d	44;29,24
Mercury (anomaly)	73;25,29	73;46,18	73;29,6	73;18,22	73;22,23

a 107;38,43° in the Tunisian recension. My recomputation gives 107;36,32°.
b 116;17,10° in the Tunisian recension, in agreement with my recomputation.
c 330;10° in the Tunisian recension. My recomputation gives 330;10,48°.
d 44;28,30° in the Tunisian recension. Rec. 44;28,29°.

An analysis of the positions for the beginning of the Hijra in the five *zīj*es considered shows that the differences are due to corrections for the following longitudes, all of them calculated from the meridian of water (7.2.1.2.2.1):

363 Mestres, 1999, pp. 191–2 (Ibn Ishāq); Samsó & Millás, 1994, pp. 4–5, and Samsó & Millás, 1998 (Ibn al-Bannā’); Samsó, 2014 (Ibn al-Raqqām, *Mustawfī*); ‘Abd al-Raḥmān, 1996 (Ibn al-Raqqām, *Shāmīl* and *Qawīm*).

Toledo: 28^{o364}
Marrakesh: 21^{o365}
Tunis: 41;45^o as established by Ibn Ishāq himself³⁶⁶
Bijāya: 34^{o367}

The sole exception is the position of Mars, for Marrākush, in Ibn al-Bannā’s *Minhāj* (210;35,31^o), which is obviously wrong, as it is smaller than the corresponding position in Ibn Ishāq’s *zīj* for Toledo. One should also note the exact coincidence between Ibn Ishāq and the *Toledan Tables* in the lunar position in anomaly at the beginning of the Hijra (108;8,39^o).

7.3.3.3 Solar and Planetary Apogees

TABLE 7.20

	Tunisian recension ^a and Ibn al-Raqqām’s <i>Shāmīl</i> ^b	<i>Toledan</i> <i>Tables</i>	Ibn al-Bannā ^c	Ibn al-Raqqām (<i>Mustawfī</i> , <i>Qawīm</i>) ^d
Sun	76;44,17 ^o	77;50 ^o	76;44,17 ^o	76;44,17 ^o
Saturn	237;44,17	240;5	239;42,45	239;43
Jupiter	157;44,17	164;30	159;42,45	159;43
Mars	119;54,17	121;51	122;12,45	122;13
Venus	76;44,17	77;50	76;44,17	76;44,17
Mercury	198;24,17	197;30	198;24,17	198;24,17

a Mestres, 1999, pp. 192–3.
b ‘Abd al-Raḥmān, 1996, pp. 57–8.
c Samsó & Millás, 1994, pp. 7–9; Samsó & Millás, 1998, pp. 265–70.
d Samsó, 2014, p. 317; ‘Abd al-Raḥmān, 1996, p. 58. The same list also appears in another table of the Tunisian recension: the position of Saturn’s apogee is 239;42,45^o, as in the *Minhāj*, while the apogees of Jupiter and Mars are rounded to the nearest minute, as in Ibn al-Raqqām’s *zīj*es. The apogee of Mercury (194;43^o) seems to be corrupt. See Mestres, 1999, p. 194.

364 ‘Abd al-Raḥmān, 1996, p. 328.
365 Vernet, 1952, pp. 14 (edition) and 57 (Sp. translation).
366 Samsó, 2014, pp. 298–9.
367 ‘Abd al-Raḥmān, 1996, p. 328.

7.3.3.3.1 *The Position of the Solar Apogee*

All the sources mentioned above agree that the position of the solar apogee at the radix date is $76;44,17^\circ$. This value is very near the position in Ibn al-Kammād's *Muqtabas* ($76;45,21^\circ$, cf. table 7.13 in 7.2.5.3.2) and, years ago, I suggested a derivation of Ibn al-Zarqālluh's determination of its position ($85;49^\circ$) in 1074–5 (6.3.1).³⁶⁸ Here I will try to determine whether the same hypothesis can be applied to the position of the solar apogee in Ibn Ishāq's tradition:

$$85;49^\circ - 76;44,17^\circ = 9;4,43^\circ$$

This difference may correspond to the increase in longitude, in 1074–75, due to precession and the motion of the solar apogee ($0;0,0,2,7,10,38^\circ$ / day) (see 6.2.6.2 and 6.3.2). I have made several unsuccessful attempts to obtain this difference assuming that $76;44,17^\circ$ was the position of the solar apogee at the beginning of the Hijra, as stated by Ibn al-Bannā' in the *Minhāj*.³⁶⁹ The best results, however, are obtained if we consider that the radix date is not the beginning of the Hijra, but the moment in which precession was 0° .

To obtain the value of precession, I assume that the position of the solar apogee was $85;49^\circ$ on 1 January 1075 (JDN 2,113,702) and that precession was 0° on 2 August 580 (JDN 1,933,117) (see 6.2.2.5.2.1). This date has been calculated from Ibn Ishāq's value according to which $i = 3;53,55^\circ$ at the beginning of Hijra (16 July 622, JDN 1,948,440) and $\Delta i = 0;0,0,54,57,17,35^\circ$ / day (cf. 6.2.6.2). As,

$$\begin{aligned} 2,113,702 - 1,933,117 &= 180,585, \\ i &= 0;0,0,54,57,17,35^\circ \cdot 180,585 = 45;56,40^\circ \end{aligned}$$

The value of precession for 1 January 1075 is:

$$\Delta\lambda_p = \arcsin(\sin 45;56,40^\circ \cdot \sin 10;24^\circ) = 7;27,15^\circ$$

And the displacement of the solar apogee will be:

$$180,585 \times 0;0,0,2,7,10,38^\circ = 1;46,20^\circ$$

and

$$7;27,15^\circ + 1;46,20^\circ = 9;13,35^\circ$$

³⁶⁸ Samsó & Millás, 1994, pp. 7–9.

³⁶⁹ Samsó & Millás, 1998, p. 268.

The result shows an excess of about 9' with respect to the previously calculated difference (9;4,43°).

7.3.3.3.2 *Planetary Apogees in the Tunisian Recension and in Ibn al-Raqqām's Shāmīl zīj*

All planetary apogees are affected by the motion of the solar apogee. This implies that they are always kept at a fixed distance from the solar apogee and, consequently, their positions at epoch are:

Sun/ Venus:	76;44,17°
Saturn:	76;44,17° + 161° = 237;44,17°
Jupiter:	76;44,17° + 81° = 157;44,17°
Mars:	76;44,17° + 43;10° = 119;54,17°
Mercury:	76;44,17° + 121;40° = 198;24,17°

I cannot find any source for this set of differences between the apogee of each planet and the Sun. No similar differences can be found in the *zīj*es of al-Khwārizmī and al-Battānī or the *Toledan zīj*. It seems, therefore, that the Tunisian recension and Ibn al-Raqqām's *Shāmīl* follow the tradition represented by Ibn al-Kammād and Ibn al-Hā'im (6.3.2). This is obvious in the case of the *Shāmīl zīj*, whose canons derive directly from Ibn al-Hā'im.³⁷⁰

7.3.3.3.3 *Planetary Apogees in Ibn al-Bannā' and Ibn al-Raqqām (Qawīm and Mustawfī zīj*es)

The positions of the superior planets seem to derive from al-Battānī's *zīj* (7.2.4.2.2.4)

Saturn:	244;28° - 239;43° = 4;45°
Jupiter:	164;28° - 159;43° = 4;45°
Mars:	126;58° - 122;13° = 4;45°

This systematic difference of 4;45° seems to correspond to the difference in precession between the epoch used in the four aforementioned *zīj*es and al-Battānī's time of the determination of the longitudes of the apogees (879 CE). This can be justified by the fact that, with his third model of trepidation (6.2.2.5), Ibn al-Zarqālluh calculates that precession for the time of

³⁷⁰ 'Abd al-Raḥmān, 1996, p. 57.

al-Battānī (883 CE according to Ibn al-Zarqālluh's chronology) is $4;52,56^{\circ}$,³⁷¹ a value obtained from the moment in which precession was 0° . $4;52,56^{\circ}$ is not far from our $4;45^{\circ}$, and it seems, therefore, that the apogees of the superior planets, in our four *zīj*es, have also been calculated for a radix date in which the value of precession was 0° (580 or 581 AD), as we have seen in the case of the solar apogee. In other words, these apogees are strictly sidereal, and are not tropical apogees for the beginning of the Hijra.

Mestres³⁷² has established that these apogees are also placed at a fixed distance from the solar apogee which is the same as that established in the Tunisian recension (with the sole exception of Mars, in which case we have $43;30^{\circ}$, instead of $43;10^{\circ}$) plus a fixed amount of $1;58,28^{\circ}$ in the case of the superior planets:

Saturn	$76;44,17^{\circ} + 161^{\circ} + 1;58,28^{\circ} = 239;42,45^{\circ}$
Jupiter	$76;44,17^{\circ} + 81^{\circ} + 1;58,28^{\circ} = 159;42,45^{\circ}$
Mars	$76;44,17^{\circ} + 43;30^{\circ} + 1;58,28^{\circ} = 122;12,45^{\circ}$

This implies that, in spite of the evidence that the motion of the solar apogee is not applied to the superior planets in Ibn al-Bannā's *Minhāj* and in Ibn al-Raqqām's *Mustawfī* and *Qawīm zīj*es (6.3.2), the aforementioned planetary apogees are also placed at a fixed distance from the solar apogee. Besides, the constant $1;58,28^{\circ}$ added to their longitudes must be the result of precisely the motion of the apogee between the radix, used in the Tunisian recension and the *Shāmil*, and a later date which we can calculate:

$$1;58,28^{\circ} : 0;0,0,2,7,10,38^{\circ} = 201,206 \text{ days}$$

Assuming that the radix date is 2 August 580 (JDN 1,933,117), we should have:

$$1,933,117 + 201,206 = 2,134,323 \text{ (17th June 1131)}$$

or if, as assumed by Mestres, the radix date is the beginning of the Hijra (JDN 1,948,440):

$$1,948,440 + 201,206 = 2,149,646 \text{ (6th June 1173)}$$

³⁷¹ Samsó, 1994 b, p. 25.

³⁷² Mestres, 1999, pp. 192–3; Mestres, 1996, pp. 411–2.

In the latter case, we would have a date that might correspond to Ibn Ishāq's period of activity (ca. 1193–1222).

This correction has not been applied to the apogees of the Sun/Venus and Mercury, which have the same longitudes as in the Tunisian recension and the *Shāmīl zīj*. We do not know the origin of the longitude of Mercury's apogee (198;24,17°), although it is very near Ibn al-Kammād's position (198;21,30°).

7.3.3.4 Solar, Lunar and Planetary Equations

I will not dwell here on the tables of the solar equations, as they have already been analysed in chapter 6. All the *zīj*es considered here use Ibn al-Zarqālluh's solar model with variable eccentricity and contain the corresponding mean motion and equation tables described in 6.3.3.2. I have also added (6.3.3.3) an analysis of Ibn al-Bannā's solar tables which have special characteristics and I have, finally, given some information about tables of the partial solar equation (6.3.3.5), which use a specific value for the solar eccentricity, calculated for a given date, and which simplify the task of computing the solar equation for a short period of time, during which the solar eccentricity will not change substantially.

Our sources contain standard Ptolemaic lunar tables, with a few exceptions, among which I should mention that the Tunisian recension, Ibn al-Raqqām's *Mustawfī zīj*³⁷³ and Ibn al-Bannā's *Minhāj* contain a table which computes Ibn al-Zarqālluh's correction to the mean lunar position, as a function of its distance to the solar apogee (6.4). The same sources and Ibn al-Raqqām's *Shāmīl* also contain a table of the equation of the lunar inclined sphere which computes the difference between the lunar position in its inclined sphere and its position on the ecliptic. The maximum tabular value is 0;6,39° for arguments 45°–46° (*Mustawfī*) and 0;6,35° for argument 45° (*Shāmīl*). Tables of this kind appear often in Eastern Islamic *zīj*es, at least since Yaḥyā ibn Abī Maṣṣūr's *Mumtaḥan zīj*, and we find a reference to them in Ibn al-Hā'im's *Kāmil zīj* (7.2.6).³⁷⁴

The rest of the tables are basically Ptolemaic with the sole exception of the table of the equation of anomaly at the syzygies, when the centre of the lunar epicycle is placed at its maximum distance from the Earth (apogee): its

373 The computational procedure is also described in Ibn al-Raqqām's *Shāmīl*: see 'Abd al-Raḥmān, 1996, p. 65.

374 Puig, 2000, pp. 87 and 97; Samsó, 2014, p. 316.

maximum is $4;55,59^{\circ}$ or $4;56^{\circ}$ ($5;1^{\circ}$ in the *Almagest*, the *Handy Tables*, al-Battānī and the *Toledan Tables*). This parameter is Indian in origin and it appears in the *Khandakhadyaka* and other similar sources,³⁷⁵ in the *Zīj al-Shāh* and, within Islamic astronomy, in al-Khwārizmī (for a double elongation 90°), Abū Ma'shar, Māshā'allāh, Ibn Hibintā etc.³⁷⁶ The same maximum value, in a table calculated with a Ptolemaic method, can be found in Ibn Mu'ādh's *Tabulae Jahan* (7.2.2), as well as in the *Alfonsine Tables*.³⁷⁷

As for the planetary tables, those of the equation of the centre for Saturn, Jupiter and Venus seem to be based on new parameters, while those of Mars and Mercury are Ptolemaic,³⁷⁸ and derive from the tradition represented by the *Handy Tables*, al-Battānī and the *Toledan Tables* (see 7.2.4.2.2.5). All the tables of the equations of anomaly are also Ptolemaic.

In the case of Venus, the equation of the centre of this planet is the same as that of the Sun. The maximum value, $1;51^{\circ}$, used by Ibn Ishāq's school, seems to be a slight correction of the maximum solar equation calculated by Ibn Ishāq ($1;49,35,56,42^{\circ}$), using Ibn al-Zarqālluh's solar model with variable eccentricity.³⁷⁹ In spite of this, $1;51^{\circ}$ for the maximum equation of the Sun and Venus is a problematic parameter. The corresponding eccentricity is $0;58,8^p$ for Venus, and twice that amount ($1;56,16^p$) for the Sun: if we calculate the corresponding date with Ibn al-Zarqālluh's model, we will obtain 1113 – a date which cannot correspond to Ibn Ishāq but fits the time of Ibn al-Kammād, whose *Muqtabas zīj* uses a maximum solar equation of $1;52,44^{\circ}$. We should acknowledge, finally, that we know nothing of the origin of the new eccentricities used by our Maghribī astronomers for Saturn and Jupiter.

375 Cf. Haddad, Kennedy & Pingree, 1981, pp. 167, 210, 220, 300, 308, 313.

376 Pingree, 1968, pp. 47–48; Kennedy, *SIES*, pp. 329 and 349.

377 *Editio princeps* (Ratdolt 1483), e5v; Poulle, 1984, p. 151.

378 Samsó & Millás, 1998, pp. 272–273; Mestres, 1999, p. 234.

379 This table of the solar equation (Mestres, 1999, pp. 244–248) bears the following title: *Jadwal ākhar bi-ta'dīl al-shams al-murakkab li-Ibn Ishāq bi-naṣṣ al-Zarqālluh lā bi-jadwali-hi* ("Another table of the solar equation, calculated by Ibn Ishāq, following [Ibn] al-Zarqālluh's instructions, not his tables"). Another table of the same *zīj* (Mestres, 1999, pp. 236–7), calculated "according to the opinion of Ibn Ishāq" (*'alā ra'y Ibn Ishāq*) attains a maximum value of $1;49,7^{\circ}$.

TABLE 7.21

Planets	Maximum equations of the centre	Max. eq. of the anomaly (apogee)	Max. eq. of the anomaly (mean distance)	Max. eq. of the anomaly (perigee)
Saturn	5;48° (arg. 90°–94°)	0;21° (arg. 100°–107°) ^a	6;13° (arg. 91°–99°) ^b	0;25° (arg. 102°–111°)
Jupiter	5;41° (arg. 89°–97°)	0;30° (arg. 107°–118°) ^c	11;3° (arg. 99°–102°)	0;33° (arg. 109°–120°)
Mars	11;25° (arg. 93°–96°)	5;38° (arg. 153°–156°)	41;9° (arg. 130°–132°) ^d	8;3° (arg. 159°) ^e
Venus	1;51° (arg. 85°–89°) ^f	1;42° (arg. 161°–162°) ^g	45;59° (arg. 135°–136°)	1;52 ^{oh} (arg. 161°–163°)
Mercury	3;2° (arg. 93°– 97°)	3;12° (arg. 129°–132°) ⁱ	22;2° (arg. 111°–112°)	2;1° (arg. 130°–136°)
Moon	13;9° (arg. 114°)	4;55.59° (arg. 95°)	-----	2;40° (arg. 103°–105°) ^j

- a Arg. 100°–105° in Ibn al-Raqqām's *Shāmil*; 98°–105 in the *Qawīm*.
b Arg. 94°–99° in Ibn al-Raqqām's *Shāmil*, *Mustawfī* and *Qawīm*.
c Arg. 106°–118° in Ibn al-Raqqām's *Shāmil*.
d 41;10° for arg. 131°–132° in Ibn al-Raqqām's *Shāmil*.
e 8;59° for arg. 159° in the Tunisian recension, which seems a corrupt value.
f Arg. 86°–89° in Ibn al-Raqqām's *Shāmil* and *Mustawfī*.
g Arg. 161°–163 in Ibn al-Raqqām's *Qawīm*.
h 1;55° in the *Mustawfī*; 1;51° (arg. 86°–89°) in the *Qawīm*.
i Arg. 129°–131° in Ibn al-Raqqām's *Shāmil* and *Mustawfī*.
j Arg. 103°–109° in Ibn al-Raqqām's *Qawīm*.

From the tables of the centre equation, Mestres³⁸⁰ and ‘Abd al-Raḥmān³⁸¹ have calculated the following eccentricities:

Saturn	3;2,30 ^P
Jupiter	2;59 ^P
Mars	6 ^P
Venus	0;58 ^P

380 Mestres, 1999, pp. 234, 253–262.
381 ‘Abd al-Raḥmān, 1996, pp. 256–319.

Mercury	3 ^P
Moon	10;19 ^P (R=49;41 ^P)

The Ptolemaic epicycle radii and eccentricities calculated from the tables of the equation of anomaly are:

Saturn	$r = 6;30^P$	$e = 3;25^P$
Jupiter	$r = 11;30^P$	$e = 2;45^P$
Mars	$r = 39;30^P$	$e = 6^P$
Venus	$r = 43;10^P$	$e = 1;15^P$
Mercury	$r = 22;30^P$	$e = 3^P$
Moon	$r = 5;15^{P*}$	$e = 10;19^P$ (R= 49;41 ^P) *table for quadratures, perigee
	$r = 5;9,41^{P**}$	** table for syzygies, apogee.

7.3.3.5 Innovations in Ibn al-Bannā's Procedures to Compute Planetary Equations³⁸²

7.3.3.5.1 *Equation of the Centre*

In 6.3.3.3, I already explained that Ibn al-Bannā seems to have been the first Maghribī astronomer to use “displaced” equations of the centre (η), a computational technique well known in the Islamic East from the ninth century onwards. This technique allowed the preparation of tables in which the equation was always positive, simply by adding a constant ($k \geq \eta_{max}$) to each tabular value in the equation. To compensate for this addition, the same constant k was subtracted from the mean longitude of the centre of the epicycle at epoch. To the vertical displacement of the equation, Ibn al-Bannā’ also adds an horizontal displacement of the argument, both in the case of the Sun and in that of the planets (see the explanation in 6.3.3.3). Here the constant k is subtracted from the value of the argument. All this implies that the function is no longer symmetrical about 180° , but about $180^\circ - k^\circ$.

In table 7.22 we present the values of the maximum equations of the centre (both displaced and not displaced), of the minimum equations (displaced), and the value of the displacement used by Ibn al-Bannā’. As in the other four *zīj*es considered here, the maximum equations are Ptolemaic in the cases of Mars, Mercury and the Moon, while those of Saturn, Jupiter and Venus seem to be new.

³⁸² In this section I reproduce, almost literally, Samsó & Millás, 1998, pp. 270–285.

TABLE 7.22

Planet	Max. eq.	Min. eq.	Displacement	Max. eq. without displacement	Eccentricity
Saturn	11;48° (261°–264°)	0;12° (83°–88°)	6°	5;48°	3;2,30 ^P
Jupiter	11;41° (257°–265°)	0;19° (83°–91°)	6°	5;41°	2;59 ^P
Mars	23;25° (253°–255°)	0;35° (81°–83°)	12°	11;25°	6 ^P
Venus	5;51° (265°–270°)	2;9° (82°–85°)	4°	1;51°	0;58 ^P
Mercury	7;2° (259°–263°)	0;58° (89°–93°)	4°	3;2°	3 ^P
Moon	26;18° (114°)	0;0° (216°)	13;9°	13;9°	10;19 ^P (R = 49;41°)

7.3.3.5.2 *Equation of Anomaly*

7.3.3.5.2.1 The “Planetary” Method

We calculate the equation of anomaly (γ) with a Ptolemaic table which follows the tradition of the *Handy Tables* according to the method outlined below, which I denominate *planetary*.³⁸³

a) When the true centre (c_v , mean longitude of the centre of the *epicycle* from the planetary apogee) is comprised between 0° (apogee) and the position in which the centre of the epicycle is at the mean distance from the centre of the Earth, i.e., this distance equals the deferent radius (R), then the equation of anomaly is:

$$\gamma(\alpha_v, c_v) = c_6(\alpha_v) - c_5(\alpha_v) \cdot c_{4a}(c_v) [1]$$

In this expression:

α_v is the true anomaly of the planet.

³⁸³ Pedersen, 1974, pp. 291–2, 309, 325–8; Neugebauer, 1975, I, pp. 183–90, and II, pp. 1002–4.

$c_6(\alpha_v)$ (column 6 in a general table of equations in the *Almagest*) is the equation of anomaly for the mean distance which we can also symbolise by γ_R .

$c_5(\alpha_v)$ (column 5) is the difference between $c_6(= \gamma_R)$ and the equation of anomaly when the centre of the epicycle is on the apogee (maximum distance between the centre of the epicycle and the centre of the Earth equal to $R + e$, e being the eccentricity). This latter function can, thus, be represented by γ_{R+e} . We have, then:

$$c_5(\alpha_v) = \gamma_R - \gamma_{R+e} [2]$$

Thus, c_5 is a function of just one variable: α_v , the true anomaly.

c_{4a} is an interpolation function for intermediate positions of the centre of the epicycle between the apogee and the mean distance from the centre of the Earth. Its argument is the true centre of the planet. We calculate this function with the following expression, the result being given in minutes:

$$c_{4a}(c_v) = [\max\gamma_R - \max\gamma(c_v)] / [\max\gamma_R - \max\gamma_{R+e}] [3]$$

b) When c_v is comprised between the mean distance and 180° (perigee):

$$\gamma(\alpha_v, c_v) = c_6(\alpha_v) + c_7(\alpha_v) \cdot c_{4b}(c_v) [4]$$

an expression in which $c_6(\alpha_v)$ is defined as before. As for the rest of the functions:

$c_7(\alpha_v)$ (column 7) tabulates the difference between the equation of anomaly for the minimum distance, i.e., the perigee (γ_{R-e}) and the equation of anomaly for mean distance (γ_R). Therefore:

$$c_7(\alpha_v) = \gamma_{R-e} - \gamma_R [5]$$

$c_{4b}(c_v)$ is the second half of the interpolation function, for positions of the centre of the epicycle located between the mean distance from the centre of the Earth and the perigee; it is calculated with the following expression, and the result is also given in minutes:

$$c_{4b}(c_v) = [\max\gamma(c_v) - \max\gamma_R] / [\max\gamma_{R+e} - \max\gamma_R] [6]$$

In his *Minhāj*, Ibn al-Bannāʾ uses the aforementioned procedure for the computation of the equation of anomaly of Mars, Venus and Mercury (planets with large epicycles). It is interesting, however, to analyse his terminology:

c_m is called *al-markaz al-awwal* (“the first centre”)

c_v is called *al-markaz al-muʿaddal* (“the corrected centre”)

$c_6 (\alpha_v)$ corresponds to the column entitled *al-mufrad* (“singular”)

$c_5 (\alpha_v)$ is *al-buʿd al-abʿad* (apogee)

$c_7 (\alpha_v)$ is *al-buʿd al-aqrab* (perigee)

$c_{4a} (c_v)$ and $c_{4b} (c_v)$ are called *daqāʾiq nisbat inḥirāf al-kawkab* (“minutes of the ratio of the planet’s deviation”). This column is divided into two parts – marked *abʿad* (“far”) and *aqrab* (“near”) – which correspond exactly to the two functions described.³⁸⁴

We should mention here that the functions *mufrad* and *buʿd* are together in a single table entitled *Jadwal al-taʿdīl al-...* (“Table of the equation of ...”). As these functions are not displaced, Ibn al-Bannāʾ uses the standard symmetries and tabulates only 180 values for each function. The *daqāʾiq*, on the other hand, appear in the table of the equation of the centre (*taʿdīl al-markaz*), which – as we have seen – is a displaced function. For that reason, Ibn al-Bannāʾ has to give 360 values and repeats the series of *daqāʾiq* twice (in spite of the fact that this function is not displaced). This has a certain utility for checking erroneous values in the manuscript.

7.3.3.5.2.2 The “Lunar” Method

7.3.3.5.2.2.1 The Moon

The tradition of the *Handy Tables* uses a different procedure for the calculation of the equation of anomaly of the Moon.³⁸⁵ The final computation is, in all cases:

$$\gamma (2f, \alpha_v) = c_4 (\alpha_v) + c_5 (\alpha_v) \cdot c_6 (2f) [7]$$

Where $2f$ is the double mean elongation

α_v is the true lunar anomaly

³⁸⁴ For all this terminology see Vernet, 1952, p. 34, for example (Mars).

³⁸⁵ Pedersen, 1974, pp. 195–9; Neugebauer, 1975, I, pp. 93–8, and II, pp. 988–9. As mentioned before, the symbols c_4 , c_5 , and c_6 correspond to the columns in the *Almagest*.

$c_4(\alpha_v)$ is the equation of anomaly at the syzygies (γ_R), in which the centre of the epicycle is placed at its maximum distance from the Earth (R).

$c_5(\alpha_v)$ is the difference, for a given α_v , between the equation of anomaly for the quadrature (γ_{R-2e}) – where the centre of the epicycle is placed at its minimum distance (R–2e) from the Earth – and the equation of anomaly for the syzygies (γ_R).

$c_6(2f)$ is the interpolation function which corresponds to the expression:

$$c_6(2f) = [\max\gamma(2f) - \max\gamma_R] / [\max\gamma_{R-2e} - \max\gamma_R] [8]$$

the result given in minutes.

This method is also used by Ibn al-Bannā', who adds certain corrections that appeared in Eastern Islamic astronomy as early as the ninth century and were introduced by Ibn al-Zarqālluh in the eleventh.³⁸⁶ The terminology used by our author is the same as in the case of Mars and the inferior planets although, obviously, Ibn al-Bannā' adapts the meaning of the technical terms he is using. The function $c_4(\alpha_v)$ is called *al-mufrad*, while $c_5(\alpha_v)$ receives the name of *al-bu'd al-aqrab*. These two functions appear together in the same table and the number of calculated values is 180, as these functions are not displaced. The interpolation function $c_6(2f)$ is called *daqā'iq al-nisba* and it appears in the same table as the equation of the centre, which has been displaced: here, as in the case of Mars, Venus and Mercury, we have 360 values and, consequently, the series of values of *daqā'iq* appears repeated twice.

The underlying parameters are Ptolemaic with the sole exception of the column *al-mufrad* which, as we have seen (7.3.3.4), has a maximum value of 4;56°, a parameter of an Indian origin.

7.3.3.5.2.2.2 *Saturn and Jupiter in the Minhāj*

Ibn al-Bannā' uses the Ptolemaic lunar method for the computation of the equation of anomaly of Saturn and Jupiter.³⁸⁷ In this method an expression like [7] has a general value and the calculator does not need to consider the position of the mean centre (c_m) in order to choose between expressions [1] and [4] for the computation of the equation of anomaly, as he does with the "planetary" method. On the other hand, there is a good reason for using this method for Saturn and Jupiter and not for the rest of the planets: their epicycles are smaller than those of Mercury, Venus and Mars and the resulting inaccuracies of the method will be slighter.

³⁸⁶ See Vernet, 1952, pp. 31–2 (Arabic) and 87–9 (Spanish translation). See also 7.3.3.4.

³⁸⁷ See, for example, for Saturn, Vernet, 1952, pp. 33–5 (Arabic) and 91.3 (Spanish translation).

For these two planets, *al-mufrad* (m) corresponds to the equation of anomaly at the apogee (γ_{R+e}), while the *daqā'iq al-bu'd* (b) correspond to the difference between the equation of anomaly at the perigee and at the apogee ($\gamma_{R-e} - \gamma_{R+e}$). These two functions appear together in the same table, which contains 180 calculated values.

If we remember the definitions of the “planetary” functions c_5 , c_6 , and c_7 (see 7.3.3.5.2.1), we can easily establish:

$$\begin{aligned} m &= c_6 - c_5 = \gamma_R - (\gamma_R - \gamma_{R+e}) = \gamma_{R+e} [9] \\ b &= c_5 + c_7 = (\gamma_R - \gamma_{R+e}) + (\gamma_{R-e} - \gamma_R) = \gamma_{R-e} - \gamma_{R+e} [10] \end{aligned}$$

It is also easy to see the Ptolemaic character of the underlying parameters of the two columns. Ibn al-Bannā' has calculated them by adding or subtracting columns c_5 , c_6 , and c_7 in a table derived from the tradition of the *Handy Tables*. In tables 7.23 and 7.24 we can see samples of m and b for Saturn and Jupiter compared to c_5 , c_6 , and c_7 which we copy from the *zīj* of al-Battānī, to which we have added the variant readings of the Tunisian recension. When these variant readings have an asterisk, this implies that the values of the Tunisian recension agree much more closely with the m and/or b of Ibn al-Bannā' than with those of al-Battānī. An examination of tables 7.23 and 7.24 leads us to the conclusion that Ibn al-Bannā' used an equation table derived from Ibn Ishāq's, or a similar one, for Saturn. We cannot, however, conclude the same for Jupiter.

Ibn al-Bannā's tables of the equation of anomaly also contain an interpolation function resembling the lunar c_6 (2f). This function, expressed in minutes, is called *daqā'iq nisbat al-ikhṭilāf* (d) (“minutes of the ratio of the variation”) and it appears in the same table as the equation of the centre of Saturn and Jupiter which, as we have already seen, are displaced. As in other similar cases, Ibn al-Bannā' repeats this table and provides us with 360 values. The final formula (*al-murakkab*) of the equation of anomaly of these two planets is, according to his canons:

$$\gamma = m + b d [11],$$

whence we can easily conclude that, as in the case of the lunar c_6 (2f) [8], we have, here:

$$d(c_v) = [\max \gamma(c_v) - \max \gamma_{R+e}] / [\max(\gamma_{R-e} - \gamma_{R+e})] [12]$$

and we can use the following values:

– For Saturn:

$$\max\gamma_{R+e} = \max m = 5;53^\circ$$

$$\max\gamma_{R-e} = \max (m + b) = 6;37^\circ$$

– For Jupiter:

$$\max\gamma_{R+e} = \max m = 10;34^\circ$$

$$\max\gamma_{R-e} = \max (m + b) = 11;35^\circ$$

If we recompute both d tables with [12] and the parameters just mentioned, the results obtained can be considered correct, but not remarkable. On the other hand I cannot imagine Ibn al-Bannā' painstakingly calculating $\max \gamma$ (c_v) with Ptolemaic parameters when this value is implied in the “planetary” functions c_{4a} and c_{4b} . As we have already seen (7.3.3.5.2.1):

$$c_{4a}(c_v) = [\max\gamma_R - \max\gamma(c_v)] / [\max\gamma_R - \max\gamma_{R+e}] [3]$$

Substituting in expression [3] the parameters just mentioned and also:

$$\max\gamma_R = 6;13^\circ \text{ (Saturn)}^{388}$$

$$\max\gamma_R = 11;3^\circ \text{ (Jupiter)}^{389}$$

we will obtain:

$$c_{4a}(c_v) = [6;13^\circ - \max\gamma(c_v)] / 0;20^\circ [13] \text{ (Saturn)}$$

and

$$\max\gamma(c_v) = 6;13^\circ - 0;20^\circ c_{4a}(c_v) [14] \text{ (Saturn)}$$

By substituting in expression [12], we will obtain:

$$d(c_{4a}) = [0;20^\circ - 0;20^\circ c_{4a}(c_v)] / 0;44^\circ = [0;20^\circ (1 - c_{4a}(c_v))] / 0;44^\circ [15] \text{ (Saturn)}$$

388 It is the standard value in the tradition of the *Handy Tables*. If we used Ibn al-Bannā's tables, we would have:

$$(6;37^\circ + 5;53^\circ) / 2 = 6;15^\circ$$

I do not use this value because the results obtained are clearly worse.

389 As in the case of Saturn, it is the *standard* value of the *Handy Tables* tradition. With the *zīj* of Ibn al-Bannā', we would have:

$$(10;34^\circ + 11;35^\circ) / 2 = 11;4,30^\circ.$$

Here, once again, the results obtained would be worse with this value.

This solves the problem of the computation of the first part of $d(c_v)$ (between the apogee and the middle distance) as a function of $c_{4a}(c_v)$ which can be read directly in a Ptolemaic table. On the other hand, it is easy to see that [15] implies an arithmetical progression with an increase of $0;27,16$ ($0;20^0/0;44^0$) every time that $c_{4a}(c_v)$ increases by one unit.

For Jupiter, we can find the equivalent expressions:

$$\begin{aligned}\max\gamma(c_v) &= 11;3^0 - 0;29^0 c_{4a}(c_v) \text{ [16] (Jupiter)} \\ d(c_{4a}) &= [0;29^0 - 0;29^0 c_{4a}(c_v)] / 1;1^0 = [0;29^0 (1 - c_{4a}(c_v))] / 1;1^0 \text{ [17] (Jupiter)}\end{aligned}$$

This implies an arithmetical progression with a constant increase of $0;28,31$ ($= 0;29^0/1;1^0$) every time that c_{4a} is increased by one unit.

As for the second part of the function (between the middle distance and the perigee) our starting point will be $c_{4b}(c_v)$. Substituting the corresponding values in expression [6], we will obtain:

$$c_{4b}(c_v) = [\max\gamma(c_v) - 6;13^0] / 0;24^0 \text{ [18] (Saturn)}$$

and

$$\max\gamma(c_v) = 6;13^0 + 0;24^0 c_{4b}(c_v) \text{ [19] (Saturn)}$$

Finally

$$d(c_{4b}) = [0;20^0 + 0;24^0 c_{4b}(c_v)] / 0;44^0 \text{ [20] (Saturn)}$$

This time, the underlying arithmetical progression has a ratio of $0;32,44$ ($= 0;24^0/0;44^0$) for an increase of one unit in c_{4b} .

For Jupiter:

$$\begin{aligned}\max\gamma(c_v) &= 11;3^0 + 0;32^0 c_{4b}(c_v) \text{ [21] (Jupiter)} \\ d(c_{4b}) &= [0;29^0 + 0;32^0 c_{4b}(c_v)] / 1;1^0 \text{ [22] (Jupiter)}\end{aligned}$$

which implies an arithmetical progression with a ratio $0;31,29$ ($= 0;32^0/1;1^0$) for every increase of one unit in the function $c_{4b}(c_v)$.

Tables 7.25 and 7.26 contain a sample of functions $d(c_{4a})$ and $d(c_{4b})$ for Saturn and Jupiter, as they appear in the *zīj* of Ibn al-Bannā'. Minutes inside round brackets correspond to the differences between tabular and recomputed values using the following ratios:

- Saturn: $d(c_{4a}) (+ 0;27^{\circ})$ and $d(c_{4b}) (+0;33^{\circ})$
- Jupiter: $d(c_{4a}) (+ 0;28^{\circ})$ $d(c_{4b}) (+0;31^{\circ})$

Tabular values of c_{4a} and c_{4b} have been taken from al-Battānī's *zīj* and I have noted the variant readings which appear in the Tunisian recension (TR).

TABLE 7.23 *Mufrad* and *bu'd* for Saturn

	Mufrad (c_6-c_5)	c_6	c_5	c_7	Bu ^c d (c_5+c_7)
1	0;6°	0;6°	0;0°	0;0°	0;0°
5	0;28	0;30	0;2	0;2	0;4
10	0;57	1;0	0;3	0;3	0;6
15	1;23	1;28	0;5	0;6	0;11
20	1;51	1;57	0;6	0;8	0;14
25	2;17	2;24	0;7	0;10	0;17
30	2;42	2;50	0;8	0;11	0;19
35	3;6	3;15	0;9	0;13	0;22
40	3;30	3;40	0;10	0;15 (TR 0;14)	0;25
45	3;52	4;3	0;11	0;16	0;27
50	4;13	4;26	0;13	0;18	0;31
55	4;32	4;46	0;14	0;19	0;33
60	4;49 (+1')	5;5 (TR *5;4)	0;15	0;20	0;35
65	5;6	45;22	0;16	0;20	0;36
70	5;20	5;37	0;17	0;21	0;38
75	5;31	5;49	0;18	0;21	0;39
80	5;41 (+1')	5;59	0;19 (TR *0;18)	0;22	0;40 (-1')
85	5;48	6;7	0;19	0;22	0;41
90	5;52 (-1')	6;12	0;19	0;23	0;42
95	5;53°	6;13°	0;20°	0;24°	0;44°
100	5;51	6;12	0;21	0;24	0;45
105	5;48	6;9	0;21	0;25	0;46
110	5;42	6;2	0;20	0;24 (TR *0;25)	0;45 (+1')
115	5;33	5;53	0;20	0;24	0;44
120	5;22	5;41	0;19	0;23	0;42
125	5;7	5;25	0;18	0;22	0;40
130	4;49	5;6	0;17	0;21	0;39 (+1')
135	4;29 (+1')	4;45	0;17 (TR *0;16)	0;20	0;36 (-1')
140	4;6	4;21	0;15	0;18	0;33

TABLE 7.23 *Mufrad and bu'd for Saturn (cont.)*

	Mufrad (c_6-c_5)	c_6	c_5	c_7	Bu ^d (c_5+c_7)
145	3;41	3;55	0;14	0;16	0;30
150	3;13	3;25	0;12	0;14	0;26
155	2;45	2;55 (TR 2;54)	0;10	0;12	0;22
160	2;15	2;23	0;8	0;10	0;18
165	1;42 (−1')	1;48	0;5 (TR *0;6)	0;8	0;14 (+1')
170	1;8 (−1')	1;13	0;4 (TR *0;5)	0;6	0;11 (+1')
175	0;35 (−1')	0;38	0;2 (TR *0;3)	0;3	0;6 (+1')
180	0;0	0;0	0;0	0;0	0;0

TABLE 7.24 *Mufrad and bu'd for Jupiter*

	Mufrad (c_6-c_5)	c_6	c_5	c_7	Bu ^d (c_5+c_7)
1	0;10 ⁰	0;10 ⁰	0;0 ⁰	0;0 ⁰	0;0 ⁰
5	0;47	0;49	0;2	0;2	0;4
10	1;34	1;37	0;3	0;3	0;6
15	2;19	2;24	0;5	0;6	0;11
20	3;4	3;11	0;7	0;8	0;15
25	3;49	3;57	0;8	0;9	0;18 (+1')
30	4;32	4;42	0;10	0;11	0;21
35	5;14	5;26	0;12	0;13	0;25
40	5;56	6;9	0;13	0;14	0;27
45	6;34	6;49	0;15	0;16	0;31
50	7;11	7;28	0;17	0;19	0;34 (−2')
55	7;46	8;4	0;18	0;20	0;38
60	8;17	8;37	0;20	0;22	0;42
65	8;46	9;8	0;22	0;24	0;46
70	9;13	9;36	0;23	0;25	0;48
75	9;35	10;0	0;25	0;27	0;52
80	9;55	10;21	0;26	0;29	0;55
85	10;12 (+1')	10;38	0;27	0;30	0;57
90	10;24	10;51	0;27	0;31	0;58
95	10;33	11;1 (TR 11;0)	0;28	0;31	0;59
100	10;34 ⁰	11;3 ⁰	0;29 ⁰	0;32 ⁰	1;1 ⁰

TABLE 7.24 *Muḡrad* and *buʿd* for Jupiter (*cont.*)

	Muḡrad (c_6-c_5)	c_6	c_5	c_7	Bu ^c d (c_5+c_7)
105	10;33	11;2	0;29	0;32	1;1
110	10;25	10;55	0;30	0;33	1;3
115	10;12	10;42	0;30	0;33	1;3
120	9;54	10;23	0;29	0;33	1;2
125	9;31	10;0	0;29	0;32	1;1
130	9;2	9;30	0;28	0;31	0;59
135	8;27 (−1ʹ)	8;55 (TR 8;57)	0;27	0;30	0;57
140	7;48	8;13	0;25	0;29	0;54
145	7;3	7;26	0;23	0;25	0;48
150	6;13	6;34	0;21	0;22	0;43
155	5;18	5;36	0;18	0;19	0;37
160	4;20	4;35 (TR 4;34)	0;15	0;16	0;31
165	3;18	3;29	0;11	0;12	0;23
170	2;14 (+1ʹ)	2;21	0;8 (TR *0;7)	0;9	0;16 (−1ʹ)
175	1;6	1;11	0;5	0;5	0;9 (−1ʹ)
180	0;0	0;0	0;0	0;0	0;0

TABLE 7.25 Interpolation function (*d*) for Saturn

c_v	c_{4a}	$d(c_{4a})$	c_v	c_{4b}	$d(c_{4b})$
5°	60ʹ	0ʹ	88°	1ʹ	28 ^a
10	59	0	90	3	30 (+1ʹ)
15	57	1	95	8	32
20	55	2	100	13	34
25	53	3	105	17	36 (+1ʹ)
30	51	5 (+1ʹ)	110	21	40 (+1ʹ)
35	48	6 (+1ʹ)	115	25	41
40	46 TR 45(?)	8 (+2ʹ)	120	30	44
45	42	10 (+2ʹ)	125	33	45
50	38 TR 39	12 (+2ʹ)	130	37	47 (−1ʹ)
55	34 TR 35	13 (+1ʹ)	135	42	49 (−1ʹ)
60	30	14	140	45	52

a My starting point for this column is the value of $d(c_{4b})$ calculated exactly with expression [20], for $c_{4b}=1$. The result is 27;49ʹ.

TABLE 7.25 Interpolation function (d) for Saturn (*cont.*)

c_v	c_{4a}	$d(c_{4a})$	c_v	c_{4b}	$d(c_{4b})$
65	25	16	145	49	54
70	20	20 (+2')	150	51	56(+1')
75	14	22 (+1')	155	54	57
80	8	25 (+1')	160	57	58(-1')
85	3	26	165	58	59
87	1	27	170	59	60
			175	60	60
			180	60	60

TABLE 7.26 Interpolation function (d) for Jupiter

c_v	c_{4a}	$d(c_{4a})$	c_v	c_{4b}	$d(c_{4b})$
5°	60'	0'	88°	1'	29' ^a
10	60	0	90	2	30
15	59	1 (+1')	95	6	32
20	57	2 (+1')	100	11	35 (+1')
25	55	2	105	16	37
30	53	3	110	21	39
35	50 (TR 49)	5	115	25	41
40	47	6	120	30	44
45	43	8	125	35	46 (-1')
50	39	9 (-1')	130	39	49
55	34 (TR 35)	12	135	43	51
60	30	14	140	46	53
65	25	17 (+1')	145	49	54
70	19	19	150	52	56 (+1')
75	14	21	155	54	57
80	8	24	160	56	58
85	3	26 (-1')	165	58	59
88	0	28	170	59	60
			175	60	60
			180	60	60

a My starting point for this column is the value of $d[c_{4b}]$ calculated exactly, with expression [22], for $c_{4b} = 1$. The result is 29;3'.

7.3.3.6 Equation of Time³⁹⁰

Two of our sources (Tunisian recension³⁹¹ and Ibn al-Raqqām's *Mustawfi*)³⁹² have preserved a table of the equation of time attributed explicitly to Ibn Ishāq: *Jadwal ta'dil al-ayyām bi-layālī-hā bi 'amal Abī l-Abbās Ibn Ishāq li-ʿasri-nā hādhā* (Table of the equation of time according to the practice of Abū l-Abbās ibn Ishāq, computed for our time). Both the reference to “our time” and to “the practice of Ibn Ishāq” are confirmed in the canons where we read that the method for computing the equation, explained by Ibn al-Hāʾim in the *Kāmil zīj*, “agrees with the table used in our time [calculated] according to the opinion of Ibn Ishāq” (*yuwāfiqu al-jadwal al-mustaʿmal fī zamāni-nā ʿalā raʾy Ibn Ishāq*).³⁹³ This makes me think that the table was not necessarily computed by Ibn Ishāq himself, and may be the result of a later adaptation.

This table has been calculated for each degree of the argument (360 values) and the entries are given in hours and minutes. Mestres analysed and recalculated the table extant in the Tunisian recension and established that it is based on the following parameters:

Obliquity of the ecliptic:	23;32,30°
Solar eccentricity:	1;56°
Solar apogee:	88°
Constant:	1;12
Displacement	9;26°

The tabular maximum is 24;20^h (for arguments 199°–217°) and its minimum is 23;49^h (for 306°–317°).

We can easily check the consistency of three of these parameters with what might have been expected from Ibn Ishāq's unfinished *zīj*:

An obliquity of the ecliptic of 23;32,30° can be found in the solar declination tables extant in two of the sources we are analysing.³⁹⁴ Ibn al-Raqqām uses 23;32,40° in his *Qawīm* and *Shāmil*.³⁹⁵ As a matter of fact, this parameter corresponds to the value of ϵ calculated for the end of 1193, the year in which it has been established that Ibn Ishāq began his astronomical activity. In spite of this, it seems that it might have an observational origin. According

390 On the equation of time in Islamic astronomy, see Kennedy, 1988; Dalen, 1996.

391 Mestres, 1999. pp. 31–32, 266–270.

392 Ms. Rabat NL 2461 p. 273; ms. Tunis NL 11018 fol. 80v. A note above the table states: generated by the two motions of the Sun and of trepidation.

393 Mestres, 1999, p. 41.

394 Mestres, 1996, p. 418; Mestres, 1999, p. 277; Samsó, 2014, p. 320.

395 Ms. Rabat NL 260 p. 154; ʿAbd al-Rahmān, 1996, p. 322.

to Abū ‘Abd Allāh Muḥammad al-Ḥabbāk (d. after 1562), Ibn Ishāq himself stated that 23;32,30° had been obtained by an anonymous astronomer from Miknāsa (Meknès) through an observation made in 602/ 1205–6.³⁹⁶ This obviously means that Ibn Ishāq believed that the obliquity of the ecliptic had reached its minimum value in his own time and it implies that the declination table extant in the two aforementioned sources is due to Ibn Ishāq himself and belongs to his primitive *zīj*. The same parameter appears in al-Sanjufīnī’s *zīj*, compiled in Tibet in 1366,³⁹⁷ as well as in Alfonsine sources (7.6.2.2.1.4).

The solar eccentricity of 1;56° agrees with Venus’s eccentricity of 0;58° (see 7.3.3.5.1).

The tropical longitude of the solar apogee can be calculated by assuming that its sidereal position (76;44,17°) corresponds to an epoch 580 AD in which the value of precession was 0°:

$$\begin{aligned} &2,167,394 \text{ (JDN of 1.1.1222)} - 1,933,117 \text{ (JDN 2.8.580)} = 234,277 \\ &\text{Motion of the solar apogee: } 234,277 \times 0;0,0,2,7,10,38^\circ = 2;17,56^\circ \\ &\Delta\lambda \text{ (trepidation): } \arcsin(\sin 59;36,18^\circ \times \sin 10;24^\circ) = 9;20,33^\circ \\ &\text{and } 76;44,17^\circ + 2;17,56^\circ + 9;20,33^\circ = 88;22,46^\circ \end{aligned}$$

The computation of the equation of time is described in the canons of the Tunisian recension³⁹⁸ beginning with a long quotation from Ibn al-Hā’im’s *al-Zīj al-Kāmil*, which explains how to calculate it. The same quotation also appears in Ibn al-Raqqām’s *Shāmīl*.³⁹⁹ This is followed by a very brief allusion to Ibn Ishāq’s table, and the text states that, according to this astronomer, the argument is the *sidereal* longitude of the Sun – an unusual practice in the time of the compiler. In fact, he adds that, in the source from which he is copying, he has found a canon for the aforementioned table (*risālat al-‘amal bi l-jadwal fī l-aṣl al-muntasakh min-hu hādhā l-kitāb*) according to which the argument that should be used to enter the table is the *tropical* solar longitude. This proves the previous existence of canons to be used in connection with Ibn Ishāq’s tables, although it does not seem that these canons were due to Ibn Ishāq himself.

396 Saidi, 2012.

397 Kennedy, 1987–88; Kennedy & Hogendijk, 1988.

398 Mestres, 1999, pp. 40–46 (Arabic text) and 29–32 (introduction).

399 ‘Abd al-Raḥmān, 1996, pp. 107–108. Similar instructions are found in Ibn al-Bannā’s *Minhāj* (Vernet, 1952, pp. 52–3 (Arabic text) and 117–8 (Spanish translation). The computation of the equation of time is also described in Ibn al-Raqqām’s *Mustawfī* (ms. Rabat NL 2461, p. 170) which introduces the correction in right ascension called the *al-iqbāl al-thānī* in the trepidation tables belonging to the Zarqāllian tradition (6.2.2.5.2.3).

The corresponding canons in Ibn al-Raqqām's *Mustawfī zīj* also state that one should enter the table with the solar tropical longitude.⁴⁰⁰

In the sources considered we find two other tables of the equation of time, calculated in degrees. One of them appears in the *Mustawfī zīj*:⁴⁰¹ the values are given to the precision of degrees and seem to be the result of truncating those given in al-Battānī's table to the nearest degree.⁴⁰² The second table is to be found in the *Qawīm zīj*:⁴⁰³ it is also calculated in degrees with values given to the precision of minutes, and it reaches a maximum of 8;30° for arguments 217°–218°, and a minimum of 0° for 321°–322°.

7.4 Other Maghribī zījēs

7.4.1 *Ibn 'Azzūz and His Muwāfiq zīj*⁴⁰⁴

7.4.1.1 Introduction

Abū l-Ḥasan 'Alī ibn Abī 'Alī al-Qusanṭīnī (7.2.3) is not the only Constantinian astronomer active in Fez in the fourteenth century. Ahmed Djebbar⁴⁰⁵ drew attention to the existence of a new *zīj*, entitled *al-Zīj al-Muwāfiq*, extant in manuscript D 2461 of the Rabat National Library (hereafter RNL). According to a note written by a certain *faqīh* and *mu'addil* Sīdī Muḥammad ibn Abī Yaḥyā al-Ḥabbāk,⁴⁰⁶ which appears on page 355 of this manuscript⁴⁰⁷ the author of the work appears to be Abū l-Qāsim ibn [al-Ḥajjāj] 'Azzūz [al-Qusanṭīnī, d. Constantina [755/ 1354],⁴⁰⁸ an astronomer already mentioned in 3.1.1.8. A second copy of the same *zīj* is also extant in manuscript 8772 of the Rabat Ḥasaniyya Library (hereafter RHL) in which the name of the author has been

400 Ms. Rabat NL 2461 pp. 169–70.

401 Ms. Rabat NL 2461, p. 273.

402 Nallino, 1899–1907, II, pp. 61–64.

403 Ms. Rabat NL 260, p. 104.

404 This section is an extract, with corrections, of Samsó, 1997.

405 Djebbar, 1988, pp. 67–68.

406 A mathematician and astronomer from Tlemcen who died in 867/1463: see Suter, 1900, no. 435, p. 177. See also King, 1986b, p. 140, F28; Lamrabet, 2013, p. 207, no. M219.

407 The manuscript is paginated, not foliated. The *Muwāfiq Zīj* occupies pp. 355–463.

408 The only source that mentions him appears to be Abū l-'Abbās Aḥmad b. Ḥasan b. 'Alī b. al-Khaṭīb, known as Ibn Qunfudh al-Qusanṭīnī, *al-Wafayāt*, ed. by 'Ādil Nuwayhid (Beirut, 1978), p. 358, no. 755. See also Nuwayhid, 1980, p. 231. Ibn Qunfudh states that he was a *faraḍī*, a *Mālikī faqīh* and an expert in other sciences. He wrote a summary (*mukhtaṣar*) on *Farā'id* (partition of inheritances) as well as other works. I owe this information to the kindness of Prof. Fuat Sezgin.

omitted.⁴⁰⁹ Al-Ḥabbāk's note in RNL also states that Ibn 'Azzūz had written another *zīj*, called *al-Zīj al-Kāmil*, in 718/1318–19, and Djebbar adds that the Ṣubayḥiyya Library in Salé preserves a third work by Ibn 'Azzūz entitled *Risāla fī adwār al-nayyirayn* (ms. 509/2). On the other hand, the second book (*al-maqāla al-thāniya*) of a fourth astronomical work, the *Kitāb al-Fuṣūl fī jam' al-uṣūl* is extant in ms. 1110 of the Ḥasaniyya Library:⁴¹⁰ this second book is concerned with year transfers (*taḥwīl al-sinīn*), conjunctions and oppositions, and it contains ten chapters (*fuṣūl*) and a conclusion. Chapter 3 of this same *maqāla* is also preserved in ms. D 2128 of the National Library in Rabat (pp. 42–44): it deals with year transfers and what they imply for kings, citizens and their situations.⁴¹¹

The prologue of the *Muwāfiq zīj*⁴¹² contains a certain amount of interesting information about its author, who reminds us of the important observations made in his time by the “unique” *faqīh* Abū l-Abbās Aḥmad ibn Ishāq al-Tūnisī. Most of his contemporaries use his cycles of planetary motions (*adwār ḥarakāt al-kawākib*) in [the mean motion tables of] their *zīj*es. Modern astronomers have, however, observed with their own eyes (*bi-l-mushāhada al-ʿiyāniyya*) the existence of disagreements and errors [with the amounts calculated with the tables] which affected the times of the conjunctions of the superior planets, the velocity of the motion of Mars, and the stations and retrogradations of Venus and Mercury. As we have already seen in 3.1.1.8, Ibn 'Azzūz himself established the existence of these disagreements when he calculated the *tasyīr* of the signifiers (*adilla*) of conjunctions, eclipses and their ascendants to determine the time of events of the past (*fī l-mudād al-khāliya*) and, especially, the great battle that took place in the days of the *Amīr al-muʿminīn* and Caliph

409 Khaṭṭābī, 1983, pp. 288–289. A photocopy of this manuscript was sent to me, years ago, by Prof. Aḥmad Shawqī Bīnbin, Keeper of the Ḥasaniyya Library. The manuscript contains 51 folios and has been paginated, starting with the second page, using the *Rūmī* or Fez ciphers. Mònica Rius (Arabic Department, University of Barcelona) drew my attention to Mannūnī, 1963 (pp. 155–157) who has identified the existence of excerpts from the first chapters of this same *zīj* (geographical coordinates, chronology, solar model) in ms. 80 of the Zāwiya Ḥamzawiya of Ait Ayache (pp. 198–202). An incomplete microfilm is extant in the General Library in Rabat: The late Keeper of the RNL, Prof. Muḥammad b. Sharīfa kindly provided me with a copy of it. On this manuscript see Kunitzsch, 1994 & 1997.

410 Khaṭṭābī, 1983, pp. 367–368. See also Lamrabet, 2013, pp. 187–8, no. M159.

411 Prof. Aḥmad Tawfiq, who years ago was Keeper of National Library in Rabat, sent me photocopies of ms. D 2461. During a short stay in Rabat, Mònica Rius obtained for me a photocopy of p. 355 of this same ms. and discovered the short text by Ibn 'Azzūz in ms. RNL D 2128. On Ibn 'Azzūz' astrological works see the publications of Montse Díaz-Fajardo quoted in 3.1.1.8.

412 See RNL p. 357; RHL p. 2 is practically illegible in my photocopy.

of the Lord of the Worlds (*khalīfat Rabb al-‘ālamīn*)⁴¹³ in Faḥṣ Ṭarīf [battle of El Salado] in the month of Jumādā I of 741 H. [7 Jumādā I 741/ 30 October 1340]. The times calculated did not correspond to historical reality and he considered that the tables in use were responsible for the mistake. Therefore, in order to correct the divergence (*ikhtilāf*) existent in the famous *zīj*, he based his correction on extremely careful and precise observations made with the instrument called the armillary sphere (*dhāt al-ḥalaq*). He then divided the *ikhtilāf* [i.e., the difference between the calculated and the observed positions] by the number of “star revolutions” (*adwār nujūmiyya*) between the observation of Ibn Ishāq and the beginning of year 745 H. [1 Muḥarram 745/ 15 May 1344].⁴¹⁴ Ibn ‘Azzūz used geometrical demonstrations, reasonable measurements and numerical ratios (*bi-barāhīn handasiyya wa-maqāyīs ‘aqliyya wa-nisab ‘adadiyya*), and found that the result obtained from the division by the revolutions of the motions (*mā kharaja la-nā min adwār al-ḥarakāt bi-l-taqṣīm*) was consistent with the result obtained from the observation “between the two of them” (?) (*muwāfiqan li-mā kharaja la-nā bi l-raṣad mā bayna-humā [sic]*)⁴¹⁵ except for a small approximation which has no importance in this “art” (*ṣinā‘a*). Then, taking into account the differences obtained, he cast the horoscopes of the conjunctions and total eclipses according to the system used in this *zīj* (*‘alā madhhab waḍ‘ ḥādhā l-zīj*) and, again using *tasyīr* techniques, obtained the times of known historical events such as the fall (*intiqāl*) of the Almohads, the establishment of the Kingdom of the Naṣrids [in Granada] and of the Banū Marīn [in Fez] as well as other events that took place in this period. The author himself explained all this in his book (*maqāla*) entitled *al-Ḥurūz fī sharḥ al-rumūz* (“Protections to interpret signs”). The *radices* (*uṣūl*) [of his mean

413 Abū l-Ḥasan ‘Alī, sultan (731/1331–752/1351) of the dynasty of the Banū Marīn of Fez.

414 The date 17 Rabī‘ I 745 appears quoted in an example for the calculation of the solar longitude (RHL p. 9; RNL p. 363, Ḥamzawiyya pp. 201–202). A note in the margin of p. 357 of ms. RNL states that this was the year of a conjunction of the two inferior planets (*qirān al-suflīyyayn*) in the sign of Aquarius, according to what was said by the author himself in his book *al-Ḥurūz fī sharḥ al-rumūz* (see below). Tuckerman, 1964 does not give any conjunction of Venus and Mercury in 745/1344–45, but there was a conjunction of Saturn and Jupiter in Aquarius 19° on 17 Dhū l-Qa‘da 745/ 22 March 1345. I obtain the same result with the *Alfonsine Tables*.

415 The expression *muwāfiqan li-mā kharaja la-nā bi l-raṣad* (“in agreement with what we obtained from observation”) appears to explain to some extent the incomplete title of the *zīj*: Ibn ‘Azzūz states, at the end of his introduction (RNL p. 358; RHL p. 3), that he has named his work *al-Zīj al-Muwāfiq* and adds *wa-Allāhu al-muwāfiq li-l-ṣawāb* (“and God gives success to what is right”).

motions] were established for the meridian of Fez, the latitude of which is $33;40^{\circ}$ while its longitude from the western meridian is 25° .⁴¹⁶

This passage from Ibn ‘Azzūz’s introduction, which I have summarised here, has the obvious interest of describing a most peculiar “experimental” method used to correct the tables of Ibn Ishāq: our author used these latter tables to calculate the expected time of a known historical event using *tasyīr* techniques, the main purpose of which is precisely the prediction of the moment when a future event will take place. As he seems to be concerned only with important events, such as the battle of Faḥṣ Ṭarīf, he worked with [Saturn-Jupiter] conjunctions and with eclipses. The results obtained were not satisfactory and Ibn ‘Azzūz made a series of observations with an armillary sphere, probably in Fez c. 1344, the purpose of which must have been to determine the longitudes of the Sun, Moon and planets at the moment of the observation and the comparison between the observed and the computed positions using the tables of Ibn Ishāq. The difference (*ikhtilāf*) thus obtained was, then, reduced to time and the resulting time difference was divided by the number of revolutions undertaken by each planet between the time of Ibn Ishāq’s observations (Ibn ‘Azzūz does not give a precise date) and the year 1344. The result of this whole process must have been a small correction in the mean motion parameters used by Ibn Ishāq and by the majority of Maghribī astronomers: we will see that no correction has been introduced in the equation tables. The new mean motion tables thus established were used to calculate the time of past historical events, this time obtaining a good agreement between prediction and historical fact.

7.4.1.2 Mean Motion Parameters

We should look for confirmation of my interpretation of the prologue of the *Muwāfiq zīj* in its mean motion tables. As the tables for *majmū’a* and *mabsūṭa* years are often incompatible, the parameters corresponding to both sets of values have been noted in table 7.27.⁴¹⁷ For the sake of comparison, the column “Other sources” gives the mean motion parameters used in the *Toledan Tables*,

416 Cf. Kennedy & Kennedy, 1987, p. 118: this pair of coordinates appears only in two *zīj*es by Muḥyī al-Dīn al-Maghribī and in an anonymous list, probably from the Ottoman period, preserved in a manuscript in the Zāhiriyya Library in Damascus. Mercè Comes reminded me that these two coordinates appear in the geographical table of Ibn al-Raqqām’s *al-Zīj al-Shāmil*.

417 RHL p. 59, RGL p. 408 (Saturn); RHL p. 84, RGL p. 412 (Jupiter); RHL p. 89, RGL p. 415 (Mars); RHL p. 90, RGL p. 419 (Venus); RHL p. 99, RGL p. 423 (Mercury). The parameters have been obtained using Honorino Mielgo’s programme for the recomputation of mean motions: see Mielgo, 1996. This programme gives the maximum and minimum possible values for the parameter: in table 7.27 I give the mean between these two extreme values.

the tables of Ibn al-Kammād, Ibn Ishāq, and, in two instances, in the *al-Zīj al-Kāmil* by Ibn al-Hā'im. My conclusions are:

- a) The mean motions of the head of Aries, of the Sun, of the Moon (longitude) seem to derive from a common Andalusian tradition. The mean solar motion implied in the table for the excess of revolution (92;20,45) is slightly different from that of the table for the mean motion of the Sun and is very near one of the values used by Ibn al-Hā'im (92;21,45).
- b) No conclusions can be drawn from the table of the mean motion of the centre of the solar eccentric, due to the corrupt state of the table for the *majmū'a* years.
- c) The mean motion of the lunar nodes seems related to those established by Ibn al-Kammād or by Ibn Ishāq.⁴¹⁸
- d) The mean motions of the apogees, Saturn, Jupiter and Mars (longitude), as well as Venus (anomaly) could be original, or the result of a small correction of the parameter used by Ibn Ishāq.
- e) The mean motions in anomaly of the Moon and Mercury seem to derive from Ibn al-Kammād.

TABLE 7.27 Mean motion parameters in Ibn ‘Azzūz’s *Muwāfiq zīj*

	Majmū'a	Mabsūṭa	Other sources
Apogees	0;0,0,2,7,15°	Corrupt table	Kammād: 0;0,0,2,2,14,46° Ibn Ishāq: 0;0,0,2,7,10,38° (±20 ^{vi})
Head of Aries	0;0,0,54,57,16,30°	0;0,0,54,57,18,17,30°	Toledo/De motu: 0;0,0,52,28,38,8° (±1 ^v) Ibn al-Zarqālluh: 0;0,0,54,57,17,38 Ibn al-Kammād: 0;0,0,54,56,58° Ibn Ishāq: 0;0,0,54,57,17,35°

⁴¹⁸ Years ago J. Chabás suggested to me that the fact that Ibn al-Kammād and Ibn ‘Azzūz use the same radix position for the lunar node (see table 7.28) seems to imply that the actual source used by Ibn ‘Azzūz was, probably, Ibn al-Kammād.

TABLE 7.27 Mean motion parameters in Ibn 'Azzūz's *Muwāfiq zīj* (cont.)

	Majmū'a	Mabsūṭa	Other sources
Sun: mean motion	0;59,8,11,28,19°	0;59,8,11,28,26,30°	Toledo: 0;59,8,11,28,27,30° Kammād: 0;59,8,9,21,15° Ibn Ishāq: 0;59,8,11,28,26,22,5,24,24,26°
Sun: Excess of revolution (92;20,45°)			Ibn al-Zarqālluh: 92;24° Ibn al-Kammād: 93;36° and 93;30° (7.2.5.3.2) Ibn al-Hā'im: 92;21,45° ^{aa} Ibn al-Raqqām: 92;33,22,30°
Centre of the solar eccentric	0;0,1,5,36,5°	0;0,1,3° (table in a very bad shape)	Ibn al-Hā'im: 0;0,1,3,41,21,48,25° Ibn Ishāq: 0;0,1,3,41,21,50,8,52°
Moon: longitude	13;10,34,52,47,21,33°	13;10,34,52,46,52,30°	Khwarizmī: 0;13,10,34,52,46,28,4° (7 ^{vi}) Toledo: 13;10,34,52,48,47,8° Ibn al-Kammād: 13;10,34,52,46° Ibn Ishāq: 13;10,34,52,46,51,38° (±26 ^{vi})
Moon: anomaly	13;3,53,56,18,53,29°	13;3,53,56,18,53,29°	Toledo: 13;3,53,56,17,58,12° Ibn al-Kammād: 13;3,53,56,19° Ibn Ishāq: 13;3,53,56,17,52,4° (±21 ^{vi})

a On the different values ascribed to Ibn al-Hā'im see 'Abd al-Rahmān, 1996 a.

TABLE 7.27 Mean motion parameters in Ibn ‘Azzūz’s *Muwāfiq zīj* (cont.)

	Majmū‘a	Mabsūṭa	Other sources
Moon: nodes	0;3,10,46,41,3,30°	0;3,10,46,40,58,30°	Toledo: 0;3,10,46,42,33,17° Ibn al-Kammād: 0;3,10,46,41° Ibn Ishāq: 0;3,10,46,41,0,4° (± 20 ^{vi})
Saturn: longitude	0;2,0,26,31,14°	0;2,0,26,30,46,42,30°	Toledo: 0;2,0,26,35,17° Ibn al-Kammād: 0;2,0,25,36° Ibn Ishāq: 0;2,0,27,46,42,42° (± 22 ^{vi})
Jupiter: longitude	0;4,59,7,37,35°	0;4,59,7,37,57°	Toledo: 0;4,59,7,37,19,23° Ibn al-Kammād: 0;4,59,6,43° Ibn Ishāq: 0;4,59,7,36,25,11° (± 23 ^{vi})
Mars: longitude	0;31,26,30,53,59,10,30°	0;31,26,30,53,51,30°	Toledo: 0;31,26,32,15,17,1° Ibn al-Kammād: 0;31,26,31,40° Ibn Ishāq: 0;31,26,31,9,5,28° (± 20 ^{vi})
Venus: anomaly	0;36,59,29,25,21°	0;36,59,29,25,27,33°	Toledo: 0;36,59,29,27,29,29° Ibn al-Kammād: 0;36,59,29,21° Ibn Ishāq: 0;36,59,27,23,59,25° (± 21 ^{vi})

TABLE 7.27 Mean motion parameters in Ibn ‘Azzūz’s *Muwāfiq zīj* (cont.)

	Majmū‘a	Mabsūṭa	Other sources
Mercury: anomaly	3;5,2,51,2,17°	3;6,24,7,20°	Toledo: 3;6,24,7,39,31,26° Ibn al-Kammād: 3;6,24,7,19° Ibn Ishāq: 3;6,24,7,42,40,49° (± 20 ^{vi})

7.4.1.3 Mean Positions and Apogees at Epoch

Table 7.28 collects all the information in the *Muwāfiq zīj* concerning the mean positions at the beginning of the Hijra. These positions have been compared to those appearing in the rest of the *zīj*es that correspond to the same tradition. The differences have been corrected to take into account the differences in geographical longitude in those cases in which the motion is fast enough to affect the radix position when there is a change of the meridian. The meridians used for this correction are:

- Muwāfiq*: Fez (25°)⁴¹⁹
- Toledo: Toledo (28;30°) (Hour. diff. with Fez: +14 min.)
- Ibn al-Kammād: Córdoba (27°) (H.d.F.: + 8 min.)
- Ibn Ishāq: Toledo (28°) (H.d.F.: + 12 min.)

If we consider the third column of table 7.28. we will see that the positions are not identical, with the possible exception of the mean lunar positions in longitude which could be the same for the *Muwāfiq zīj* and for the tables of Ibn al-Kammād and Ibn Ishāq. The longitude of the lunar node is practically the same as Ibn al-Kammād’s. Minimal corrections seem to have been introduced in the Sun’s position at epoch (very near Ibn Ishāq’s), while the longitude of Saturn and Jupiter seem to be more or less related to Ibn al-Kammād’s position. The influence of Ibn al-Kammād is very clear in the positions of the solar and planetary apogees in the *Muwāfiq zīj*, which are the same as in Ibn al-Kammād’s *Muqtabas* (see table 7.13): there is only a difference of 1’ in the

419 Interestingly, practically all the mean motion tables extant in both manuscripts state that they have been computed for the *latitude* of Fez which is 33;40° (!).

solar apogee (76;44;21° instead of 76;45,21°), an obvious error, as 76;45,21° appears as the apogee of Venus.

TABLE 7.28 Mean positions at epoch in the *Muwāfiq zīj*

	Aṣl (1)	Other sources (2)	Diff.(1)–(2)+ corr. hour diff. = ()
Head of Aries	3;51,21°	Zarqālluh/Kammād: 3;51,11°	+0;0,10°
		Ibn Ishāq: 3;53,55°	–0;2,34°
Sun	113;21,56	Toledo: 113;41,11°	–0;19,15 –0;0,34(–0;19,49)
		Kammād: 113;20,30°	+0;1,26 –0;0,20(+0;1,6)
		Ishāq: 113;21,14	+0;0,42 –0;0,30(+0;0,12)
Centre of the solar eccentric	84;40,0	Kammād: 83;40,31°	0;59,29
		Ishāq: 83;40,53°	0;59,7
Moon: longitude	120;39,5	Toledo: 120;58,18°	–0;19,13 –0;7,41(–0;26,54)
		Kammād: 120;34,42°	+0;4,23 –0;4,24(–0;0,1)
		Ishāq: 120;32,32°	+0;6,33 –0;6,35(–0;0,2)
Moon: anomaly	108;8,4	Toledo: 108;8,39°	–0;0,35 –0;7,37(–0;8,12)
		Kammād: 108;11°	–0;2,56 –0;4,21(–0;7,17)
		Ishāq: 108;8,39°	–0;0,35 –0;6,32(–0;7,7)
Moon: nodes	233;30	Toledo: 234;9,55°	–0;39,55 –0;0,2(–0;39,57)
		Kammād: –126;30° (=233;30°)	0 –0;0,1 (–0;0,1)
		Ishāq: 234;19;37°	–0;49,37 –0;0,2(–0;49,39)
Saturn: longitude	115;32,2	Toledo: 115;51,15°	–0;19,13 –0;0,1(–0;19,14)
		Kammād: 115;30,30°	+0;1,32 –0;0,1(+0;1,31)
		Ishāq: 116;17,15°	–0;45,13 –0;0,1(–0;45,14)
Jupiter: longitude	330;20,18	Toledo: 331;39,37°	–1;19,19 –0;0,3(–1;19,22)
		Kammād: 330;19°	+0;1,10 –0;0,2(+0;1,8)
		Ishāq: 330;11°	+0;9,18 –0;0,3(+0;9,15)
Mars: longitude	211;7,57	Toledo: 211;24,59°	–0;17,2 –0;0,18(–0;17,20)
		Kammād: 211;27°	–0;19,3 –0;0,11(–0;19,14)
		Ishāq: 210;37,25	+0;30,32 –0;0,18(+0;30,14)
Venus: anomaly	45;22,53	Toledo: 45;28,37°	–0;5,44 –0;0,22(–0;6,6)
		Kammād: 45;2°	+0;1,53 –0;0,12(+0;1,41)
		Ishāq: 44;30°	+0;52,53 –0;0,18(+0;52,35)
Mercury: anomaly	73;54,57	Toledo: 73;46,18°	+0;8,39 –0;1,48(+0;6,51)
		Kammād: 74;1°	–0;6,3 –0;1,2(–0;7,5)
		Ishāq: 73;25,29°	+0;29,28 –0;1,33(+0;27,55)

7.4.1.4 Solar, Lunar and Planetary Equations

Nothing new appears in the tables of the lunar and planetary equations, which follow the tradition of the *Handy Tables* and use Ptolemaic parameters with the sole exception of Venus, whose maximum equation of the centre is $1;59^{\circ}$, like that of al-Battānī. The case of the Sun is somewhat different: the *zīj* contains a standard table of the solar equation,⁴²⁰ approximated to minutes, with a maximum of $1;52^{\circ}$, apparently lower than Ibn al-Zarqālluh's ($1;52,42^{\circ}$) or Ibn al-Kammād's ($1;52,44^{\circ}$) values, but higher than those used by Ibn Ishāq ($1;49,7^{\circ}$) and Ibn al-Raqqām's *Shāmil* ($1;47,51^{\circ}$). A quick check shows, however, that this table derives from the solar equation table in Ibn al-Kammād's *Muqtabas*, but that the tabular values have been truncated, not rounded. We are, therefore, in the same situation as in the *Tables of Barcelona* or in the *zīj* of Ibn al-Waqār.⁴²¹ Interestingly, like Ibn al-Kammād, Ibn 'Azzūz uses a maximum solar equation that is different from the maximum equation of the centre of Venus, and he also follows al-Battānī in using $1;59^{\circ}$ for the maximum equation of the centre of Venus.⁴²² So it seems quite possible that the actual source that Ibn 'Azzūz used for his equation tables was Ibn al-Kammād's *Muqtabas*.⁴²³

7.4.1.5 Almanac Cycles

Ibn 'Azzūz seems interested in the rapid computation of solar, lunar and planetary longitudes. For this purpose he explains the use of *goal-years*, cycles which, in the case of the planets, comprise an entire number of Julian years in which the planet makes an entire number of revolutions in longitude and in anomaly (7.7.3). These cycles were mentioned in the *Almagest*⁴²⁴ with the exception of the period for Jupiter which is 83 Julian years instead of 71 years as stated in the *Almagest*: this period of 83 years was also used by Ibn al-Zarqālluh for the calculation of his *Almanac*⁴²⁵ and the Tunisian recension of the *zīj* of Ibn Ishāq contains tables which use these cycles for the computation of the longitudes of the three superior planets.⁴²⁶ Like Ptolemy, for each planet Ibn 'Azzūz aims to give the correction (*ikhtilāf*) to be introduced, in each cycle,

420 RHL p. 11; RNL p. 365.

421 Chabás, 1996 a.

422 Chabás & Goldstein, 1994, pp. 32–33; Chabás & Goldstein, 2015 a, p. 609.

423 We have already seen (6.3.3.3) that the *Muwāfiq zīj* contains a second table, related to the solar equation, which bears the title *Ta'dil markaz al-falak al-khārij al-markaz li l-shams li-islāh al-taqrib* (Table of the equation of the centre of the solar eccentric for the correction of the approximation).

424 Neugebauer, 1975, I, 150–152.

425 See Boutelle, 1967; Samsó, 2011, pp. 166–171. Both Jupiter periods (71 and 83 years) appear mentioned in Babylonian goal-year texts: see Neugebauer, 1975, I, pp. 441 ff.

426 Mestres, 1996, pp. 429–430.

for its motion in longitude and in anomaly, as well as the displacement of its apogee and the correction of the solar longitude.⁴²⁷ All the pertinent data are summarised in table 7.29, where the “corrections” quoted in the *zīj* have been recalculated (recomputed values appear inside square brackets []) using the mean motion parameters squeezed from the *majmū’a* years. Only in two instances (the increase in anomaly of Mars and Mercury, both marked with an asterisk *) has the mean motion obtained from the *mabsūta* years been used. In spite of several errors, my impression is that Ibn ‘Azzūz calculated these *ikhṭilāfāt* using his own mean motion tables, approximating to the precision of seconds and truncating the results.

The case of the Moon stands apart. No cycle of this kind seems to have been known in Antiquity and when Ibn al-Zarqālluh deals with the problem of the computation of the lunar longitude in his *Almanac*, he tries only to simplify the calculation of the mean motions of the Moon in longitude (using the Metonic cycle of 19 years) and in anomaly (a cycle of 180 years), and adds to this a standard set of tables of Ptolemaic equations.⁴²⁸ In his astronomical tables, Jacob ben David Bonjorn (= Yomṭob) of Perpignan (fl. 1361)⁴²⁹ states that his predecessors were able to establish cycles allowing the determination of the true longitude of a planet which could be read directly on the table taking care only to add or subtract an equation after the completion of the first cycle of each planet. As Bonjorn remarked, no such effort had been made for the easy computation of true syzygies, and he gave a table of *true syzygies* for the period March 1361–February 1391. This table gives the hour and minute of the conjunction or opposition for Perpignan and uses a period of 31 Egyptian years, 9 days, 23;34,11 hours (almost 11325 days) which is equivalent to 383.5 synodic months and thus contains 767 consecutive syzygies. The discovery of this period had been attributed to Bonjorn himself until new evidence was furnished in the *Muwāfiq zīj* of Ibn ‘Azzūz.⁴³⁰ Ibn ‘Azzūz talks about a lunar cycle of 11324 days equivalent to

31 lunar years, 11 months and 14 days (= 11323.8667 d.)

or 31 solar years and 1 day (= 11323.75 d.)

427 See RHL p. 37 and RNL p. 364 (Sun); RHL p. 37, 75 and RNL p. 387 (planets); RHL p. 37, RNL p. 388 (lunar node).

428 Millás, 1943–50, pp. 76–78, 117–119, 166–173.

429 Chabás, 1988; Chabás, 1991; Chabás, 1992; Chabás, 1992.

430 RHL p. 38; RGL pp. 386–389.

The *ikhtilāf* in lunar longitude in this period amounts to $6^s 2;13,33^o$ (RNL p. 386) or $6^s 2;0,33^o$ (RHL p. 38; RNL p. 389). It is easy to see that we cannot obtain either of these results using the two parameters computed from the mean motion tables in longitude for *majmū'a* or *mabsūṭa* years (see table 7.27):

$$\begin{aligned} 13;10,34,52,47,21,18^o \text{ (majmū'a)} \cdot 11324 &= 360 \cdot n + 5^s 2;59,5,28^o \\ 13;10,34,52,46,52^o \text{ (mabsūṭa)} \cdot 11324 &= 360 \cdot n + 5^s 2;57,33,18^o \end{aligned}$$

According to the text, the corresponding *ikhtilāf* in lunar anomaly amounts to $0;55,45^o$ and in solar longitude $2;1^o$. As before, neither of these results can be obtained with a cycle of 11324 days, but a perfect agreement in two cases is obtained if we use a cycle one day longer (11325 days) which will give us:

- *ikhtilāf* in lunar longitude: $13;10,34,52,47,21,18^o \cdot 11325 = 360 \cdot n + 6^s 2;13,33,0^o$
- *ikhtilāf* in lunar anomaly: $13;3,53,56,18,53,14^o \cdot 11325 = 360 \cdot n + 0;55,54,25^o$
- *ikhtilāf* in solar longitude: $0;59,8,11,28,19^o \cdot 11325 = 360 \cdot n + 2;1,5,19^o$

The small difference in the amount corresponding to the lunar anomaly can easily be solved if we restore the text to $0;55,[5]5^o$.

As a conclusion: Bonjorn's cycle appears attested in a Maghribi source dated approximately 741/1340–41, while his tables use 1361 as the *radix* date. It seems clear to me that Ibn 'Azzūz did not discover the cycle but rather took it from somebody else, copied the number of entire days and forgot about a fraction which amounts to almost another day. It is impossible to guess who discovered the cycle. Bonjorn appears to have been the first to make an extensive use of this cycle for the computation of his table of syzygies, and thus contributed to its diffusion in Europe.

7.4.1.6 Solar, Lunar and Planetary Velocities

The *Muwāfiq zīj* has tables of solar⁴³¹ and lunar⁴³² velocities (*buht*) which contain nothing new: they are the same as in the *zīj*es of al-Khwārizmī and of Ibn al-Kammād, among other sources.⁴³³ Far more interesting is the fact that Ibn

431 RHL pp. 13, 78–79; RNL pp. 365, 446–447.

432 RHL p. 62, 66–68; RNL p. 399, 442–444.

433 Chabás & Goldstein, 1994, pp. 10–13; Chabás & Goldstein, 2015 a, pp. 613–4. See also Goldstein, 1980; Goldstein, 1992; Chabás & Goldstein, 1992; Goldstein, 1996.

TABLE 7.29 Almanac cycles

Planet	Cycle	Increase in the planet's longitude	Increase in longitude of the apogee	Increase in solar longitude	Increase in anomaly
Saturn	21550 d. (59 Jul. yrs.)	0;58,45° [0;58,45,18°]	0;12,41° [0;12,41,44°]	11 ^s 29;52,0° [11 ^s 29;52,0,20°]	11 ^s 28;53,15° [11 ^s 28;53,15,2°]
Jupiter	30317 d. [mss. 30316] (83 Jul. yrs.)	11 ^s 29;4,48° [11 ^s 29;3,59,35°]	0;17,[5]1° [0;17,51,37°]	0;41,52° [0;41,52,34°]	1;47,4° [1;37,53,0°]
Mars	28855 d. (79 Jul. yrs.)	0;56,29° [0;56,30,12°]	0;16,[5]9° [0;16,59,56°]	11 ^s 29;[4]4,16° [11 ^s 29;44,17,2°]	0;1,12,13° [11 ^s 28;47,48,51 [*] , equivalent to -1;12,11]
Venus	2920 d. (8 Jul. yrs.)	11 ^s 27;[5]8,[3]7° [11 ^s 27;58,38,18°]	0;1,43° [1;43,13°]	*****	0;15,13° [0;15,11,54°]
Mercury	16802 d. (46 Jul. yrs.)	0;11,47° [0;11,48,32°]	0;9,53° [0;9,53,54°]	*****	11 ^s 2[8];47,3° [11 ^s 28;47,1,34 [*]]
Sun	1461 d. (4 Jul. yrs.)	11 ^s 29;58,27° [11 ^s 29;58,27,21°]	0;0,51 [0;0,51,39°]	*****	*****
Moon	1132[5] d. (mss. 11324) (31 Jul. yrs. + [3] d.)	6 ^s 2;13,33° [6 ^s 2;13,33,0°]	*****	2;1° [2;1,5,19°]	0;55,45° [0;55,54,24°]
Lunar node	6793 d. (18 Jul. yrs. + 7 months + 7 d.)	11 ^s 29;59,15° [11 ^s 29;59,15,25°]	*****	*****	*****

‘Azzūz repeatedly mentions⁴³⁴ tables for planetary velocities (*buht*) used to calculate the exact moment of a conjunction. His *zīj* contains tables of daily velocities (*al-ḥaraka al-mukhtalifa*) for each planet.⁴³⁵ These tables tabulate two functions:

434 RHL pp. 74, 88, 94, 99; RNL pp. 407, 414, 418, 422, 426.
435 RHL p. 82, RNL p. 411 (Saturn); RHL p. 87, RNL p. 414 (Jupiter); RHL p. 92, RNL p. 418 (Mars); RHL p. 98, RNL p. 422 (Venus); RNL p. 426 (Mercury).

- a) the velocity of the centre of the epicycle on its deferent using as an argument the true centre (*markaz*) (v_1).
- b) the velocity of the planet on its epicycle as a function of its true anomaly (*ḥiṣṣa*) (v_2).

These planetary velocity tables are the same as those edited by Goldstein, Chabás and Mancha⁴³⁶ on the basis of manuscripts of the Alfonsine tradition (7.6.2.2.1.2): the tables also appear described in the Castilian canons of the first version of the *Alfonsine Tables* as well as in the canons of Jean de Lignères. Even the terminology used coincides: the *mouimientos diuersos* of the Alfonsine text and the *motus diuersus* of Jean de Lignères correspond exactly to the *ḥaraka mukhtalifa* of Ibn ‘Azzūz. All variants from the edition by Goldstein, Chabás and Mancha (even those which are obvious errors) will be found in table 7.30, where it is clear that the two manuscripts form a group that is independent from the two other groups found in Latin manuscripts by the three aforementioned authors. The authors of that remarkable study stated that “No other table of planetary velocity has been located in a medieval astronomical text in Arabic, Hebrew, Greek or Latin”,⁴³⁷ this is absolutely true as the table extant in the *Muwāfiq zīj* is the same one. They also suggested “an ultimate Arabic (or possibly Hebrew) origin” of the table⁴³⁸ and the new evidence given here supports this idea: obviously Ibn ‘Azzūz’s *Muwāfiq zīj* is a later source than the *Alfonsine Tables*, but the astrologer from Fez is here, as often elsewhere, copying from an earlier source. A further study of the Andalusī-Maghribī tradition of *zīj*es will, in the future, shed some light on the problems posed by the *Alfonsine Tables*.⁴³⁹ I can also provide some limited evidence showing that the aforementioned set of planetary velocity tables is not necessarily an isolated case:

1. A passage from Ibn Mu‘ādh’s *Jayyān* (= Jaén) *zīj*, preserved in the Tunisian recension of the *zīj* of Ibn Ishāq, dealing with velocity sectors and transits in thickness mentions the possibility of considering the planetary daily velocity (*ḥarakat al-kawkab al-mukhtalifa ... li-yawm wāḥid*) in order to solve the problem of the slow or fast character of the motion of the planet

436 Goldstein, Chabás & Mancha, 1994; Chabás & Goldstein, 2003, pp. 170–182.

437 Goldstein, Chabás & Mancha, 1994, pp. 61, 62.

438 Goldstein, Chabás & Mancha, 1994, p. 63.

439 ‘Abd al-Rahmān, 1996, pp. 158–9 shows, for example, that chapter 52 of the *Libro de las Taulas* (Chabás & Goldstein, 2003, pp. 91–92) on the projection of rays seems to be a literal translation of chapter 113 of the *Shāmīl*. The ultimate source of both texts is al-Battānī’s *zīj*. See also Chabás & Goldstein, 2003, p. 222, who indicate similarities with the corresponding passage in Ibn al-Bannā’s *Minhāj*.

when, for example, the centre of the epicycle is on a fast sector and the planet is on a slow sector of the epicycle (7.2.2).⁴⁴⁰

2. In his *al-Zīj al-Kāmil fī-l-taʿālīm*, Ibn al-Hāʾim (fl. 1205) deals carefully with the determination of the points of station for each planet and, in order to calculate the displacement of the centre of the epicycle in a given period of time, suggests multiplying “days and hours by the unequal motion of the centre in one day corresponding to that place in the eccentric” (*ḥarakat al-markaz al-mukhtalifa li-yawm wāḥid fī dhālika l-mawḍiʿ min al-khārīj*).⁴⁴¹ Ibn al-Raqqām copies this passage in chapter 47 of his *al-Zīj al-Shāmil* and insists on the idea of planetary velocity in chapter 126 of the same *zīj*, using the standard procedure which consists in calculating the velocity of the Sun, the Moon or the planets by computing their true position for midday of one particular date and, then, for midday of the following date. The difference between the two positions will be the “unequal motion” (*al-masīr al-mukhtalif*) of the planet for that particular date.

In both instances the references do not seem to make much sense unless a table of planetary velocity is involved. In addition, when Ibn al-Hāʾim refers to the unequal motion of the centre of the deferent he seems to be alluding to a table similar to the one we already know.

TABLE 7.30 Planetary velocities: Variant readings in mss. RNL (N) and RHL (H) in relation to the edition prepared by Goldstein, Chabás and Mancha in “Planetary and Lunar Velocities” pp. 90–92. When a variant reading is not accompanied by N or H the manuscripts agree. An asterisk (*) implies the absence of variant readings. Erroneous values are marked with (+).

Saturn			Jupiter ^a		Mars		Venus		Mercury	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
6	*	5:37	*	9:0	*	*	*	*	*	51:15
12	*	5:30	4:33	*	*	*	*	*	*	*
18	*	*	4:34	*	*	*	*	*	*	50:35
24	*	*	*	8:13	*	*	57:16	*	*	*
30	*	*	4:37	8:6	*	*	*	*	*	*
36	*	*	*	*	*	*	*	*	*	*

a Both mss. omit the indication of retrograde motion (*rājiʿ*) in the case of Jupiter.

440 Samsó, 1996, p. 604.
441 Ms. Bodleian Library Marsh 618 fol. 63 r.

TABLE 7.30 Planetary velocities: Variant readings in mss. RNL (N) and RHL (H) (*cont.*)

Saturn			Jupiter ^a		Mars		Venus		Mercury	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
42	*	*	4;40	*	*	10;36	*	*	56;56	47;50
48	*	*	4;42	7;14	*	10;23	57;44	*	*	45;50
54	*	3;48	*	*	*	10;15	*	*	57;20	*
60	*	*	*	*	*	10;3	*	14;34	*	*
66	*	*	*	*	*	*	*	*	*	*
72	*	2;52	*	*	*	*	58;23	*	*	*
78	*	*	*	4;15	*	*	*	*	58;34	*
									(+)	
84	*	*	*	3;10	*	*	*	13;15	58;58	*
									(+)	
90	*	*	*	H:2;10 N:2;1	*	*	59;5	12;40	*	24;0
96	*	*	*	*	*	*	*	*	*	*
102	*	*	*	*	*	*	59;31	*	*	*
108	*	-1;14	*	*	32;50	*	*	*	*	*
114	*	-1;54	*	*	*	*	*	*	59;2	-3;0
									(+)	
120	*	-2;30	*	*	34;10	*	*	*	*	*
126	*	*	*	*	*	*	*	5;0	*	*
132	*	-3;30	*	*	35;30	*	H:60;33 N:60;43	-2;0	*	*
138	*	*	*	*	36;6	H:-3;18 N:-3;58	*	H:-4;5 N:-3;5	*	*
144	*	-5;10	*	*	*	*	60;47	*	*	-58;0
150	*	-5;30	*	*	*	*	*	-13;10	*	-21;0
									(+)	
156	*	-5;55	*	*	*	-20;20	*	-20;50	61;36	*
162	*	-6;20	*	*	*	*	*	*	*	*
168	*	*	*	*	*	*	*	*	*	*
174	*	*	*	*	*	*	*	*	*	*
180	*	-7;10	*	13;10	38;30	H:.* N:-53;30	*	*	62;0	*

7.4.1.7 Other Tables Showing an Andalusī Influence

Many tables other than those mentioned so far seem to have an Andalusian origin or to be related to al-Khwārizmī's *zīj*, the influence of which was predominant in Andalusian astronomy. Here I will try to give a short summary of a few facts of this kind which I have been able to establish:

1. al-Khwārizmī: I have already mentioned the tables for the computation of solar and lunar velocities (7.4.1.6) – although these tables also appear in Ibn al-Kammād's *zīj* which could, therefore, be the actual source used by Ibn 'Azzūz, and Ibn Mu'ādh's tables to calculate the “minutes of transit” (table 7.5). One should add here the tables for the computation of planetary latitudes⁴⁴² which are the same as the ones that appear in al-Khwārizmī's *zīj*.⁴⁴³
2. Ibn al-Zarqālluh: I have already mentioned the solar model with variable eccentricity (6.3.3.3) and the almanac period of 83 years for Jupiter (7.4.1.5). We also have:
 - The value for the obliquity of the ecliptic ($23;33^{\circ}$) used, for example, in a table which gives ($90^{\circ} \pm \delta$) for each degree of longitude in order to facilitate the computation of the solar meridian altitude.⁴⁴⁴ The same obliquity is used in the table of solar declination.⁴⁴⁵
 - Ibn al-Zarqālluh's correction of the Ptolemaic lunar model (6.4) attaining a maximum of $0;24^{\circ}$ also appears in the *Muwāfiq zīj*.⁴⁴⁶
 - The table for the computation of the ascensional difference for any latitude (7.2.1.2.1.4): the maximum value is $2;10,40^{\circ}$ in RHL (pp. 59–60) and $2;10,44^{\circ}$ in RNL (p. 439). Ibn al-Zarqālluh⁴⁴⁷ has the same table with a maximum of $2;10,46^{\circ}$.
3. Ibn al-Kammād: his influence has already appeared in the tables of the mean motions in anomaly of the Moon and Mercury (7.4.1.2), the mean position in longitude of the Moon at epoch and the positions of the solar and planetary apogees at epoch (7.4.1.3), the solar, lunar and planetary equations (7.4.1.4) as well as the table of the equation of the centre of the solar eccentric for the “correction of the approximation” and the actual procedure for this correction (6.3.3.3). Apart from these, one should add:

442 RHL p. 83, RNL p. 411 (Saturn); RHL p. 88, RNL p. 414 (Jupiter); RHL p. 93, RNL p. 418 (Mars); RHL p. 97, RNL p. 422 (Venus); RNL p. 426 (Mercury); this latter table is missing in RHL.

443 Suter, 1914, pp. 138–167.

444 RHL p. 13; RNL p. 440.

445 RHL pp. 15–26; RNL pp. 366–377.

446 RHL p. 45; RNL p. 391. See Samsó, 2011, pp. 218–219.

447 Millás, 1943–50, p. 225.

- The table for the computation of the “equation of trepidation”⁴⁴⁸ with a maximum of $9;59^\circ$ as in the *al-Zīj al-Muqtabas*,⁴⁴⁹ Abū l-Ḥasan ‘Alī al-Marrākushī’s *al-Mabādī’ wa l-ghāyāt*,⁴⁵⁰ and the *Tables of Barcelona*⁴⁵¹ among other sources (6.2.3 and 6.2.6.2).
 - Tables of stations and retrogradations (*al-istiḳāma wa l-rujū’*) of planets:⁴⁵² only four values are given for each planet, as in Ibn al-Kammād’s *zīj*⁴⁵³ although, as Chabás and Goldstein state, the same values appear in al-Khwārizmī’s *zīj* and in the *Toledan Tables*.
 - Table giving the coordinates (longitude, latitude and the planet related to it due to its *mizāj* or temperament) of 30 stars⁴⁵⁴ (7.2.5.3.4).
4. Ibn al-Raqqām:
- RNL (pp. 438–439) contains two tables of oblique ascensions and length of seasonal hours for the latitude of Granada (the name of the city is explicitly mentioned) which is $37;10^\circ$. This extremely precise value for the latitude of Granada seems to have been established by Ibn al-Raqqām after his arrival in that city:⁴⁵⁵ his *al-Zīj al-Qawīm* preserves exactly the same two tables copied by Ibn ‘Azzūz.⁴⁵⁶

7.4.1.8 Tables Computed for the Latitude of Fez

A certain number of tables depend on the geographical coordinates of Fez which implies that they are the result of Ibn ‘Azzūz’s own work or that they were copied from another Fāsī source. Obviously, the mean motion tables have radices calculated for Fez (see 7.4.1.3). Other examples are:

- A table giving the solar altitude and shadow projected by a gnomon of 12° for hours 1–6 corresponding to the entry of the Sun in the twelve signs of the Zodiac.⁴⁵⁷ The values corresponding to the solar meridian altitude for Aries 0° (56°), Cancer 0° (80°) and Capricorn 0° (32°) seem to imply a latitude of 34° and an obliquity of the ecliptic of 24° .

448 RHL p. 11; RNL p. 365.

449 Chabás & Goldstein, 1994, pp. 24–27; Chabás & Goldstein, 2015 a, pp. 617–8.

450 Marrākushī, 1984, p. 44; Sédillot, 1834, p. 131.

451 Millás, 1962, pp. 194–195.

452 RHL p. 82, RNL p. 409 (Saturn); RHL pp. 86–87, RNL p. 413 (Jupiter); RHL p. 92, RNL p. 418 (Mars); RHL p. 97, RNL p. 422 (Venus); RNL p. 426 (Mercury, table missing in RHL).

453 Chabás and Goldstein, 1994, p. 33.

454 RNL p. 427. This table does not appear in RHL.

455 Samsó, 2011, pp. 422–423.

456 Ms. 260 of the Rabat General Library pp. 87 (length of the seasonal hours for the latitude of Granada) and 164–165 (oblique ascensions).

457 RHL p. 12; RGL p. 366.

- A series of 12 tables (one per each zodiacal sign) which give (apart from right ascensions, solar declination and the equation of time), oblique ascensions, length of a seasonal hour, half the day arc and the division of the astrological houses II–VI as a function of the degree of the ascendant.⁴⁵⁸ these latter tables depend on a latitude of 33;40° for Fez, which is quoted explicitly. RNL also contains a second set of twelve tables for the computation of the projection of rays, according to the single hour line method (3.2.3.3.5), for the latitude of Fez.⁴⁵⁹ they seem to be the only known example of numerical tables according to this method. This collection includes the same tables of oblique ascensions and length of seasonal hours mentioned above.

7.4.1.9 Conclusions

The *Muwāfiq zīj* of Ibn ‘Azzūz al-Qusanṭīnī is an astronomical source which has, in my opinion, an obvious interest and deserves a detailed study. Its first point of interest is the prologue which explains a most unusual and unorthodox method used by the author to correct the mean motions in the *zīj* of Ibn Ishāq: the results obtained by the recomputation of the mean motion parameters seem to confirm the words of Ibn ‘Azzūz for, in the case of the three superior planets (precisely the ones that would attract the attention of an astrologer interested in important events) they are unrelated to Ibn Ishāq’s mean motions and therefore imply a break with the Maghribī tradition represented by the compiler of the Tunisian recension, Ibn al-Bannā’ and Ibn al-Raqqām.

However, like his predecessors, Ibn ‘Azzūz preserves Andalusian materials: in several cases I have noted echoes of the astronomical works of Ibn al-Kammād, which appear in a proportion which seems to be higher than customary in Maghribī astronomy. Sometimes the information brought by the *Muwāfiq zīj* is particularly interesting: such is the case of the set of planetary equations of Ibn Mu‘ādh (7.2.2), for it is the first time we have confirmation of the not unexpected fact that he used al-Khwārizmī’s maximum equations. Still more important are the tables of planetary velocities and the new lunar cycle: the fact that the former were used by the Alfonsine astronomers points in the direction of an Andalusian source, and the same could probably be said of the lunar cycle, although in this latter case I am less sure.

458 RHL pp. 15–26; RGL pp. 366–377.

459 RGL pp. 428–437. See Casulleras, 2007.

7.5 The Introduction of Eastern *zīj*es in the Maghrib

In 6.2.6.6. I have already explained that the Zarqāllian theories related to trepidation and cyclical variation of the obliquity of the ecliptic were heavily criticised and were later abandoned in the Maghrib in the second half of the fourteenth century, mainly because the longitudes computed using the Zarqāllian Maghribī *zīj*es were in clear disagreement with the positions obtained by observation.⁴⁶⁰ This seems to be the main reason for the introduction of Eastern *zīj*es in the Maghrib from the fourteenth century onwards. In them, mean motions were tropical, trepidation was replaced by constant precession and there were no tables to calculate the obliquity of the ecliptic. This implied a complete change of mentality: a change which had already taken place in the Iberian Peninsula towards the end of the thirteenth century with the Latin version of the *Alfonsine Tables*. As for the Eastern *zīj*es which became known in the Maghrib, one should underline the role played by those of Ibn Abī l-Shukr – frequently mentioned because of his estimation of the precession of equinoxes – Ibn al-Shāṭir, and Ulugh Beg. The Maghribī manuscripts of these *zīj*es show evidence of their adaptations for use in specific cities of the Maghrib. In spite of this, the Andalusī tradition remained alive until the nineteenth century and coexisted with the “new” Eastern *zīj*es. The reason is clearly explained by the astrologer al-Baqqār: horoscopes continued to be cast using sidereal longitudes, and my impression is that astrologers used Ibn al-Bannā’s *Minhāj* and other similar sources, while astronomers and conscientious *muwaqqits* preferred Eastern *zīj*es.⁴⁶¹

7.5.1 *Ibn Abī l-Shukr al-Maghribī*

Muḥyī al-Dīn Ibn Abī l-Shukr al-Maghribī (d. 1283) is the author of at least two *zīj*es of which only one, the *Tāj al-azyāj*, seems to have been known in the Maghrib. I know of three manuscripts of this *zīj*, all of them written in Maghribī script: Escorial ár. 932, Dublin ar. 4129 and a third one preserved in the library of the University of Barcelona.⁴⁶² In the Escorial ms., at the end of the canons (fol. 57v), we find a note stating that the author compiled this *zīj* in Damascus

460 See Comes, 2002; Samsó, 1998, 2001 and 2003. This section is a revised summary of these three papers of mine.

461 Díaz-Fajardo, 2001, pp. 64, 95–6. On the use of Ibn al-Bannā’s *Minhāj* in the nineteenth century, see Vernet, 1980 b.

462 It was probably bought in Morocco by the late Prof. J.M. Millás Vallicrosa. The manuscript has no title and does not mention the author’s name: it was identified by Mercè Comes. See Dorce, 2002–3, p. 20 and Comes, 2005.

in 656/1258⁴⁶³ and that the copy was made in Tunis by Muẓaffar b. ‘Abd Allāh, for his own use, in 797/1394, under the supervision of ‘Abd al-‘Azīz b. Mas‘ūd b. ‘Abd al-‘Azīz al-Tilimsānī al-Mālikī, who had been *imām* and *muwaqqit* in the cities of Fez, Tunis, Jerusalem and Damascus.⁴⁶⁴ This note gives information about one of the channels through which the *Tāj al-azyāj* may have been introduced in the Maghrib, and the chronology (end of the fourteenth century) fits well with the remark made by the author of the *Natā’ij al-afkār* (al-Ḥabbāk al-Tilimsānī al-Ḥafīd, fl. 1514–1563) that “in the time of the author [*mu‘allif* = al-Jādīrī (1375–1416)] this *zīj* [the *Tāj*] was well known and it had been diffused through the rest of the countries of the Maghrib.⁴⁶⁵ This manuscript seems to have been used in Tunis and Cairo.⁴⁶⁶

The manuscript of the *Tāj*, preserved in the library of the University of Barcelona, bears witness to its diffusion in the Maghrib (Tilimsān and Marrakesh), and in fact this *zīj* was still in use in the nineteenth century. The copy is unfortunately not dated, but it contains quite a few marginal notes written in a script different from that of the copyist, which have a certain interest: the Syriac names of the months of the solar year need translation, and a marginal note in fol. 4r states that *Tishrīn al-awwal* is *Uktūbar*, *Kānūn al-Awwal* is *Dujanbar* and *Kānūn al-Thānī* is *Yannayr*. The author of the notes also explains that the *Tāj* gives tropical positions calculated from the equinox (*al-mabda’ al-ṭabī’ī*), which are not the same as the sidereal ones, computed from the first point of Aries (*al-mabda’ al-dhātī*) (fol. 15r). The radices of mean motions are given for the beginning of the Hijra for Damascus and Tilimsān (written by the copyist of the manuscript himself) and notes written at the end of each page give the difference between the two mean positions. There are tables of oblique ascensions calculated for Damascus (latitude 33;20°) (fols. 90v–91v), but also for latitudes 35° (fols. 86v–87r) (Tilimsān), 30° (fols. 92r–93r) and 36° (fols. 93v–94v) (Tunis?): all of them use Ibn Abī l-Shukr’s value for the obliquity

463 According to the *Natā’ij al-afkār* (see 6.2.6.6 and Saidi, 2013, p. 45) this *zīj* is based on the observations made by this astronomer in Damascus in 657/1259. The star table is calculated for the first year of the reign of Hulāwūn (= Hulaghu) which is 627 Yazdijird/1259. See Saliba, 1983.

464 This is the author of the *Risālat al-Sayb fī ‘amal al-jayb*, extant in ms. Escorial ár. 918: see King, 1986 b, p. 66 no. C46. The original work was written in Cairo in 795/1393 and it contains a precession table copied from the *Tāj* (fol. 91r). This *risāla* is the subject of a doctoral dissertation (Aguar, 1995); see Aguair, 1996. He is also the author of an astrological tract already mentioned in 3.1.1.8.

465 Saidi, 2013, p. 45.

466 The mean motion tables have their *uṣūl* calculated for the beginning of the Hijra in Damascus but, in the case of the Sun, two interlinear notes give the *aṣl* for Tunis (fol. 71v) and Cairo (fol. 80v).

of the ecliptic (23;30°). The length of daylight is also given for latitudes 33;20°, 30° and 36° (fols. 95r, 95v, 96r). Marginal notes also give radices for the Sun, the Moon (longitude and anomaly) and the lunar nodes for the end of 948/1542 in Tilimsān (fol. 49v, 50v, 51v, 52v), and for the solar apogee for year 1281/1864 (fol. 50r). The manuscript includes tables of the solar and lunar velocity copied from Ibn al-Bannā's *Minhāj* and from the *zīj* of al-ʿAṭṭār (?) (fol. 90r). Finally, a note at the end of the canons, in a script different from that of the copyist and the author of most of the marginal notes, states that the manuscript was the property of the *muwaqqit* (*al-muqaddam ʿalā l-tawqīt*) ʿAbd Allāh al-Ṣanhājī al-Dādisī in Marrakesh.

7.5.2 *Ibn al-Shāṭir*

Our information about the introduction of the *Jadīd zīj* of Ibn al-Shāṭir (fl. Damascus, d. 1375) in the Maghrib is less precise, mainly because the sources available are undated and are probably from a later period. Years ago, King said that there is evidence that this *zīj* was known in Tunis in the late fourteenth century⁴⁶⁷ and I have the impression that it was known in other parts of the Maghrib as well: al-Baqqār (fl. 1418) quotes an obliquity of the ecliptic of 23;31° which he ascribes to an unnamed Damascus astronomer, and this same parameter is mentioned in relation to Ibn al-Shāṭir and al-Mizzī by the author of the *Natāʾij al-afkār* (ca. 1514). We also know of the existence of a late recension of the *Jadīd Zīj* for Algiers.⁴⁶⁸ This work seems to have circulated mainly through recensions and abridgements made by various authors, such as those made by Shihāb al-Dīn Aḥmad b. Ghulām Allāh, called al-Kawm al-Rishī (d. 1432) for the coordinates of Cairo,⁴⁶⁹ or by Muḥammad b. ʿAlī b. Ibrāhīm, called Ibn Zurayq al-Khayrī or al-Jizī (d. 1400), in Damascus.⁴⁷⁰ In other in-

467 King, "Ibn al-Shāṭir", *DSB* XII, p. 362b.

468 King, 2007, p. 133. The recension for Algiers is extant in MS Cairo DM 533 (ca. 1150/1737): see King, 1986 b, p. 62 (C30) and p. 145 (F66).

469 See King, 1986 b, p. 65 (C41). This work, entitled *Kitāb al-Lumʿa fī ḥall al-sabʿa*, is extant in an undated Maghribī manuscript from the library of the late scholar Ḥasan Ḥusnī ʿAbd al-Wahhāb, now in the National Library in Tunis: Manšūr, 1975, ms. 18158 (pp. 393, 398). The Ḥasaniyya Library in Rabat possesses an undated Maghribī manuscript entitled *Nuzhat al-khāṭir fī talkhiṣ Zīj Ibn al-Shāṭir* (ms. 2723) which Khaṭṭābī (1983, n° 467, pp. 382–383) also attributes to al-Kawm al-Rishī.

470 *al-Rawḍ al-ʿāṭir fī Talkhiṣ Zīj Ibn al-Shāṭir*. On this author see Suter, 1900, p. 173, no. 426; Brockelmann, *GALS*, II, 157. This work seems to have been quite popular in the Maghrib, but the dated mss. were copied in the nineteenth century: see, for example, Rabat General Library 1347D; Rabat Ḥasaniyya 1576, dated in 1255/1839 (Khaṭṭābī, 1983, no. 334, pp. 273–276); Rabat Ḥasaniyya 6292, undated (Khaṭṭābī, 1983, no. 335, pp. 276–277); Rabat Ḥasaniyya 5366, dated in 1265/1848 (Khaṭṭābī, 1983, no. 336, pp. 277); Rabat Ḥasaniyya

stances, Ibn al-Shāṭir's *zīj* was used for timekeeping: an anonymous author used it to compute conjunctions and oppositions of the Sun and the Moon for the meridian of Tetuan,⁴⁷¹ and, in 1214/1799 a certain Muḥammad b. al-Ṭayyib b. al-Makkī al-Ajbarī al-Ḥasanī wrote a treatise with two tables in which he explained how to determine the hours using the *zīj*es of Ibn al-Shāṭir and Ulugh Beg: he states that his method is original and that nobody has used it before him, because his starting point was the tropical longitude of the Sun (*mawḍi' al-shams al-ṭabī'i*) (!).⁴⁷²

7.5.3 *Ulugh Beg*

The last Eastern *zīj* introduced in the Maghrib was Ulugh Beg's (1393–1449) *Zīj-i Jadīd*. I have no evidence that this *zīj* was known in the Maghrib before the end of the seventeenth century, but it is obvious that it became very popular in the following two hundred years. The Arabic translation of the introduction to this *zīj*, made by Ḥasan b. Muḥammad, known as Qāḍī Ḥasan al-Makkī (fl. seventeenth century) reached Morocco, for the Ḥasaniyya Library preserves a manuscript of this work dated 1291/1874.⁴⁷³ Far more interesting is, however, the fact that there were at least two Tunisian recensions of this *zīj*, prepared by Muḥammad al-Sharīf, called Sanjaq Dār al-Tūnisī (before 1633)⁴⁷⁴ and by 'Abd Allāh Ḥusayn Quṣ'a b. Muḥammad b. Ḥusayn al-Ḥanafī al-Tūnisī.⁴⁷⁵ Both authors are undated but ms. 16650 of the Tunis National Library (Khaldūniyya collection) contains a copy of Quṣ'a's recension – under the title *Ghunyat al-ṭālib fī taqwīm al-kawākib* – where it is stated that the work (not the copy) was finished on Thursday 22 Shawwāl 1091/15 November 1680 (which was a

2802, dated in 1262/1845 (Khaṭṭābī, 1983, no. 337, pp. 277–278). Another Maghribī copy in Cairo TR 235,1 (King, 1986 b, 164, C116).

471 Rabat Ḥasaniyya 981, undated (Khaṭṭābī, 1983, no. 195, p. 167).

472 Rabat Ḥasaniyya 8873, dated in 1214/1799 (Khaṭṭābī, 1983, pp. 223–4, no. 261). Other tabular materials derived from Ibn Abī l-Shukr, Ibn al-Shāṭir and other astronomers can be found in an anonymous and undated compilation extant in Rabat Ḥasaniyya 1433 (Khaṭṭābī, 1983, pp. 407–8, no. 496).

473 Khaṭṭābī, 1983, pp. 193–5, no. 227; see also King, 1986 b, p. 164, G99. Mss. 11420 and 12616 of the Cairo National Library contain versions of Ulugh Beg's *zīj* for the coordinates of Cairo (long. 55° from the western meridian).

474 Lamrabet, 2013, no M332, p. 233.

475 Ms Cairo DM 814 is a copy of this recension dated in 1325/1907–8: cf. King, 1986 b, 142, F46. A third (?) anonymous recension is preserved in two mss. of the Ḥasaniyya Library in Rabat: 2650 (undated) and 2148 (dated 1218/1803). See Khaṭṭābī, 1983, pp 335–8, nos. 406–8. Ihsanoğlu, 1997, p. 314 (no. 175) and pp. 347–8 (no. 213).

Friday), and that the tables have been adapted to the longitude of Tunis (41;45° from the western meridian).⁴⁷⁶

As for Sanjaq Dār's *Zīj al-Sharīf* – also called *al-Zīj al-mukhtaṣar fī 'ilm al-ta'dīl wa l-taqwīm* – I have been able to study two manuscripts of this work: ms. 1816 of the National Library in Tunis (ms. Ta hereafter),⁴⁷⁷ and 18104 of Ḥasan Ḥusnī 'Abd al-Wahhāb's library, now also in the Tunis National Library⁴⁷⁸ (ms. Tb hereafter). Ms. Tb is dated 1107/1695–6 (fol. 62r).⁴⁷⁹ In this *zīj* the mean positions of the Sun, Moon and planets are given, in intervals of 30 years, between 1080/1669–70 and 1290/1873–74; in addition, the positions of the lunar mansions are given, according to the observations of Ulugh Beg, for the end of year 1090/1679–1680 (Ta fol. 36r, Tb fol. 90r) and the day, hour and minute of the spring equinox is given for each year between 1090/1679 and 1205/1790. Another manuscript of the same work, also extant in Tunis,⁴⁸⁰ is dated 1107/1695–96.⁴⁸¹ All this evidence points to a work produced in the late seventeenth century⁴⁸² and this agrees with the presumed date for Quṣ'a's *Ghunya*. Commentaries on the introduction of this *zīj* were written towards the end of the seventeenth century and the beginning of the eighteenth. One of them was written by Aḥmad b. Muḥammad Bū Daydaḥ al-Qayrawānī (fl. before 1150/1737–8), who, in his introduction, praised the importance of Sanjaq Dār's *zīj* but added that this work was difficult to understand and to use. For that reason, commentaries had been written by his masters *al-ḥājj* Abū 'Abd Allāh Muḥammad al-Qalī and, before him, by Abū l-Ḥasan 'Alī b. Māmī al-Ḥanafī, known as Karbāšo (or Karbāša) (before 1163/1749–50).⁴⁸³ Mss. of these commentaries are preserved in Cairo NL and the copies are dated 1150/1737–38, 1163/1749–50, 1169/1755–56.⁴⁸⁴

476 Another copy of the same *zīj* is extant in ms. 5990 of the same library. The copy was finished in 1229/1814 by Muḥammad b. 'Abd Allāh al-Khayyārī.

477 Copy dated 1242/1826. Mercè Comes obtained a microfilm of this manuscript and drew my attention to it.

478 Manṣūr, 1975, p. 398.

479 See also King, 1986 b, p. 143 (F53) who lists other manuscripts of the same work: ms. Cairo Ṭal'at Riyāda 319, 1 (pp. 1–16, ca. 1300/18831) contains the mean motion tables of this *zīj*. A complete copy is in ms. Paris B.N. ar. 2536 (48 fols., ca. 1150/1737).

480 Ms. 18104 of the library of Ḥasan Ḥusnī 'Abd al-Wahhāb: Manṣūr, 1975, p. 398.

481 See also King, 1986 b, p. 143, F53. King also lists other manuscripts of the same work.

482 The incomplete copy extant in ms. Tb (fol. 104r) gives an example of the computation of the solar longitude and the date used is the sixteenth Jumādā 1 1099/19.3.1688.

483 Lamrabet, 2013, p. 233 (M333) states that he was alive in 1074/1663 and that his *Ḥāshiya 'alā zījāt Sanjaq Dār* is extant in ms. NL Tunis 3050 and Salé Ṣabīḥiyya 38/2. Another copy (dated 1163/1749–50) is extant in Cairo NL (King, 1986 b, p. 143, F54).

484 King, 1986 b, 143–4, F53–55. The commentary authored by Bū Daydaḥ al-Qādirī is also extant in Tunis, Ḥ. Ḥ. 'Abd al-Wahhāb's collection, no. 18104, dated 1301/1883–4: see Manṣūr, 1975, p. 397.

Sanjaq Dār's *zīj*, like Quṣ'a's *Ghunya*, is computed for longitude $41;45^\circ$ and contains tables of oblique ascensions for a latitude of $36;50^\circ$ (Ta fol. 28r, Tb fol. 82r), id. for the division of the houses for the same latitude (Ta fols. 28v–31r, Tb fol. 82v–85r), id. for half daylight for the same latitude (Ta fol. 33v, Tb fol. 87v), lunar longitude at sunset for latitude $36;40^\circ$ (Ta fol. 35r, Tb fol. 89r), lunar parallax in longitude and latitude also for $36;40^\circ$ (Ta, fol. 36v, Tb fol. 90v) and ascendants of anniversaries for the same latitude (Ta fol. 45v, Tb fol. 98r): $36;50^\circ$ is the modern value for the latitude of Tunis and it is not attested in any other historical source, while $36;40^\circ$ is one of the three values for the latitude of Tunis given by al-Khwārizmī,⁴⁸⁵ and it is systematically used by the compiler of the Tunisian recension of Ibn Ishāq's *zīj*. Apart from this, it is interesting to note that the prologue of the *Zīj al-Sharīf* states that he is going to follow the *ḥabṭaq* method, which means that planetary equations are computed in a simplified way using double-argument tables. This type of table is documented in the East from the time of Ibn Yūnus (d. 1009),⁴⁸⁶ but I do not know of any instance in which they appear in the Maghrib before the *Zīj al-Sharīf*, for the computation of lunar or planetary equations. In spite of this, Ibn al-Kammād (fl. Córdoba 1116, see 7.2.5) used double-argument tables for the computation of the time from mean to true syzygy,⁴⁸⁷ the calculation of solar eclipses, and visibility of the new Moon,⁴⁸⁸ and the Tunisian recension of Ibn Ishāq's *zīj* contains a set of double argument planetary latitude tables, based on Ptolemaic parameters, attributed to an otherwise unknown Ibn al-Bayṭār.⁴⁸⁹

In the equation tables of the *Zīj al-Sharīf* one enters the table vertically with the mean anomaly of the Moon or the planet (tabulated from 0° to 360° with 6° intervals) and horizontally with the *zodiacal sign* of the *markaz* (the double elongation in the case of the Moon, and the mean longitude of the planet computed from the apogee): the total equation (centre + anomaly) will be read directly at the intersection of the two and is to be added to the mean longitude of the planet. In this part of the *zīj*, the solar equation is standard, although displaced $1;56^\circ$ (which is also the maximum value of the undisplaced equation)⁴⁹⁰ and computed for 6° intervals (Ta fol. 10r, Tb fol. 63v). The author adds to this

485 Kennedy & Kennedy, 1987, pp. 362–3.

486 King, 1974 a. For other double argument tables for the computation of lunar and planetary equations see King, 1975, pp. 44–45, 55; Jensen, 1971–2; Tichenor, 1967; Saliba, 1976; Saliba, 1978.

487 See a similar table in the *Zīj al-Sharīf*: ms. Ta fol. 31v, Tb fol. 85v.

488 Chabás & Goldstein, 1994, pp. 14 and 23; Chabás & Goldstein, 2015a, pp. 624–5, 627 and 638.

489 Mestres, 1999, pp. 264–5.

490 We have already seen Ibn al-Bannā's use of displaced equations in 6.3.3.3 and 7.3.3.5.

a set of displaced equations of the centre for all the planets, also computed for 6° intervals (fol. 19v). This simplified set of tables computes mean motions to the precision of minutes, but the *zīj* also contains a second set of mean motion tables for the Sun (Ta fol. 23r, Tb fol. 78r) and the Moon (Ta fols. 24v–25r, Tb fols. 78v–79r), computed to the precision of thirds, as well as a second table of the (displaced) solar equation (Ta fol. 24r, Tb fol. 78r) computed for each degree of the argument and to the precision of thirds. There is also a set of lunar equations (Ta fols. 25v–27r), computed to seconds, which is not standard.⁴⁹¹

7.6 *Zījes* in the Christian Kingdoms of the Iberian Peninsula

7.6.1 *Abraham bar Ḥiyya and Abraham ibn Ezra*⁴⁹²

Having seen the main lines of the development of *zījes* in al-Andalus and the Maghrib, we should now continue to analyse the kinds of astronomical tables produced in the Christian kingdoms of the Iberian Peninsula. These *zījes*, which appear between the twelfth and the fifteenth centuries, share two main characteristics. One of them is that their authors were mainly Jewish astronomers: as the Kingdom of Castile and the Crown of Aragon advanced towards the south in their conquests of the lands under Muslim domination, Muslim scholars and scientists usually migrated southwards where, either in the Peninsula or on the other side of the Strait of Gibraltar, they were able to continue to live in Islamic countries. For their part, Jewish scholars could not find shelter there and preferred to live under the new Christian conquerors, where they capitalised on their knowledge of Arabic and their scientific training which provided them with a source of livelihood. Jewish astronomy in the later Middle Ages is an obvious continuation of Islamic astronomy.

The second common characteristic is an evolution towards tropical astronomy which followed the ideas of Ptolemy's *Almagest* and of the development of Eastern Islamic astronomy after the time of the Baghdad and Damascus *Mumtaḥan* observations ca. 830. One of the main influences was al-Battānī, whose *zīj* had been translated into Latin by Plato of Tivoli (fl. 1132–1146), with the collaboration (?) of Abraham bar Ḥiyya (ca. 1065–1136).⁴⁹³ Abraham bar Ḥiyya was also the author of a set of canons (*Heshbon mahalakhot ha-kokhavim*)

491 Samsó, 2003, pp. 294–298.

492 This section has been revised by Shlomo Sela, to whom I would like to express my gratitude.

493 Rosa Comes (who is preparing a critical edition of Plato's translation) tells me that none of the extant manuscripts contains any evidence of a collaboration of this kind.

for the use of his *zīj* (*Luḥot ha-nasi'*), strongly influenced by al-Battānī, as well as by al-Farghānī and by Ptolemy's *Almagest*,⁴⁹⁴ and, in 3.1.2.1.2.1, I have already referred to Bar Ḥiyya's *Sefer tiqqun ha-tequfot min ha-luḥot ha-'eleh*,⁴⁹⁵ in which this author uses astrological months which are the result of dividing the solar year into 13 months (1 month = 28^d 2;18^h) and 15 months (1 month = 24^d 8;23,40^h). These two parameters imply a solar year of 365^d 5,54^h and 365^d 5;55^h respectively and they are clearly related to Ptolemy's solar year of 365^d 5;55,12^h.⁴⁹⁶ All this implies that the tropical motions of the Sun and the planets were also applied to astrology, in clear contradiction to the standard practices in the Andalusi and Maghribi traditions.

The same mixture of Ptolemaic and Battanian materials can be found in Abraham bar Ḥiyya's *Luḥot ha-nasi'*, analysed by Raymond Mercier,⁴⁹⁷ who remarks that the solar, lunar and planetary mean motions have been adapted from the *Almagest*, due to Bar Ḥiyya's belief that Ptolemy's length of the year was the same as that of the Jewish calendar. The radices (for 21 September 1104), however, were determined from al-Battānī's positions. The solar equation is Ptolemaic, but the maximum planetary equations derive from al-Battānī.

In fact, Bar Ḥiyya's actual source is not al-Battānī's Arabic *zīj*, but an anonymous Hebrew translation extant in two manuscripts.⁴⁹⁸ A revised Latin recension of the *Luḥot*, which uses a different epoch (28 September 1110) is also extant.⁴⁹⁹ Bar Ḥiyya states that he uses the meridian of reference of Jerusalem (long. 35;14° from Greenwich) but the results obtained are not particularly good; according to Mercier, the best fit is to be found at the meridian 39;36° and for year 1214.

Far more interesting are the astronomical works of Abraham ibn 'Ezra (ca. 1089–1167).⁵⁰⁰ In 6.2.7.1 I have already mentioned his *De rationibus tabularum*, in which he rejects trepidation and sidereal astronomy.⁵⁰¹ This book was written in two different recensions: the first one in Pisa (1142–45) and the second in Angers (1154).⁵⁰² Ibn 'Ezra was also the author of three non-extant recensions of his *Luḥot* (tables) written in Lucca (1142–45), Narbonne (1148–53) and

494 See Millás, 1959; Sela, 2016–17.

495 Sela, 2013 a.

496 Pedersen, 1974, p. 423.

497 Mercier, 2014.

498 Munich heb. 343, fols. 182v–200v; Munich heb. 386, fol. 15v. ff.: see Mercier, 2014, p. 181.

499 Cambridge UL Hh.6.8 vol. 1, fols. 41–77: Mercier, 2014, p. 165.

500 This section is a revised version of Samsó, 2012. I have tried to correct quite a few errors I made in that earlier paper.

501 See Millás, 1947 a.

502 Sela & Freudenthal, 2006. The extant text is the 1154 recension: see Millás, 1947 a, p. 99.

Rouen (1154). Fortunately a Latin version has been preserved in two sets of tables computed for the coordinates of Pisa and London,⁵⁰³ and a third non-extant text was calculated for the coordinates of Winchester.⁵⁰⁴ The mean motion tables for collected years give positions for 20-year intervals, starting in 1149, and Millás suggested that the tables were composed in 1148,⁵⁰⁵ although Mercier⁵⁰⁶ changes this date to 1143. This year fits the period (1142–5) in which Ibn ‘Ezra lived in Lucca and Pisa, where he wrote the first version of his *Luḥot* and of *De rationibus*, and it precedes 1154,⁵⁰⁷ the date of the extant recension of *De rationibus*, where we find references to the previous tables.⁵⁰⁸ Apart from the fact that Ibn ‘Ezra states that the Pisan tables are based on al-Šūfī’s *zīj*, Mercier has shown that the mean motion and positions at the radix date do not derive from observations made in Pisa but are the result of a correction, of a set of tables calculated for Baghdad or for some other place near this city that takes into account the difference in longitudes.⁵⁰⁹

It has commonly been considered that the *De rationibus* contains a set of canons designed to accompany Ibn ‘Ezra’s astronomical tables. As a matter of fact, the *De rationibus* does not have the character of a set of canons (a standard collection of instructions of the procedures that should be followed for the use of the numerical tables), although it could be conceived as a broad introduction to one, in which the author explains the main astronomical problems of his own time and the reasons that have led him to choose the kind of tables he is presenting. Several recensions of canons for these tables, which belong to the standard type, are extant.⁵¹⁰

The *De rationibus* is, on the one hand, an elementary introduction to astronomy, following the lines of al-Farghānī’s (ca. 833–861) *Kitāb fī l-ḥarakāt al-samāwiyya wa jawāmī’ ‘ilm al-nujūm*, a compendium of Ptolemaic astronomy without too many mathematical details.⁵¹¹ It also belongs to the kind of

503 Mercier, 1987; Mercier, 1991.

504 Sela *et al.*, 2020.

505 Millás, 1947 a, p. 63.

506 Mercier, 1987, p. 111.

507 Millás, 1947, p. 99.

508 Millás, 1947, pp. 83 (“de compositione tabularum quas fecimus secundum probationem predictorum virorum sibi consentientium ...”), 84 (“He tabule quas composuimus utiles sunt ...”), 87–88, 89 (“si ... vis operari secundum tabulas meas ...”).

509 Mercier, 1991, pp. 10–11.

510 Millás, 1947, pp. 59–66; Mercier, 1987, pp. 110–2; Mercier, 1991, pp. 13–21.

511 The edition prepared by Golius (Amsterdam, 1669) has been the object of a facsimile reprint which appeared in Frankfurt, 1986, together with Golius’ Latin translation. The Latin translation by Johannes Hispanensis (*sic*) atque Lunensis was published in Carmody, 1943. Gerard of Cremona’s version was edited in Campani, 1910. Shlomo Sela has shown

astronomical books called, in Arabic, *Kitāb ‘ilal al-zījāt* (About the reasons or causes of astronomical tables), and our *De rationibus tabularum* seems to be an accurate translation of these titles. A work of this kind is ‘Alī ibn Sulaymān al-Hāshimī’s (fl. ca. 890) book with the same title which, like *De rationibus*, analyses the differences between Indian and Ptolemaic astronomy.⁵¹² We should not forget that Ibn ‘Ezra was also the translator into Hebrew of Ibn al-Muthanna’s commentary on al-Khwārizmī’s astronomical tables, a work that deals with the same subject matter.⁵¹³

In his critical analysis of the differences between sidereal and tropical astronomy, Ibn ‘Ezra deals with the length of the solar year, obliquity of the ecliptic, precession of the equinoxes and motion of the solar and planetary apogees. His sources are “the Indians”, “the Persians”, the *magistri probationum* (probably a translation of the *aṣḥāb al-mumtaḥan*), among whom he mentions the astronomers of Caliph al-Ma’mūn ca. 830 (Yaḥyā b. Abī Maṣṣūr, Khālīd b. ‘Abd al-Malik al-Marwarrūdhī), the Banū Mūsā b. Shākīr (fl. 850–875), Thābit b. Qurra (d. 901), al-Faḍl b. Hātim al-Nayrīzī (d. ca. 922), al-Battānī (d. 929), al-Ṣūfī (903–998), and Ibn Yūnus (d. 1009).⁵¹⁴ These *magistri probationum* are all followers of Ptolemaic tropical astronomy, and this reinforces their identification with the *aṣḥāb al-mumtaḥan* in the Andalusī and Maghribī meaning of the term which identifies “the school of the Indians” with sidereal astronomy, while “the *Mumtaḥan* school” corresponds to those who calculate tropical motions.⁵¹⁵ The main problem of this interpretation is that Ibn ‘Ezra mentions

that both Abraham bar Ḥiyya and Abraham ibn ‘Ezra knew al-Farghānī’s work through another version that circulated in the Iberian Peninsula: see Sela, 2016.

512 Ḥaddād, Kennedy & Pingree, 1981. See other works with the same title, by ‘Abd Allāh al-Naṣrānī (ninth century) and ‘Abd al-‘Azīz al-Qabiṣī (eleventh century) in Rosenfeld & Ihsanoğlu, 2003, pp. 57 and 85.

513 Goldstein, 1967.

514 Millás, *Fundamentos* p. 76.

515 A very recent paper (Sela *et al.*, 2020) offers new evidence about the meaning of the word *probatio* = observation in Ibn ‘Ezra: “Loca quidem enim planetarum inuenta sunt per instrumenta probationum, in quibus secunda designari non poterant.”; “Et quilibet experiri potest per umbram necnon et per instrumenta probationum”; “Nam bene apparet quod Ptolomeus in tempore suo inuenit per instrumenta probationum Solem intrare caput Arietis ...” This seems to confirm Vernet’s hypothesis (Vernet, 1956) according to which the *Tabulae probatae* were tables based on observation. Another passage in the same source points to my interpretation (*tabulae probatae* = *mumtaḥan* = tropical tables): “Alii quoque magistri probationum, perscrutatores intimi, motum stellarum fixarum inuenerunt in septuaginta annis unius gradus esse”; “et hoc tempus subtrahendum est de tabulis, que facte sunt secundum uiam probationis, 8 gradus, quadraginta quinque minuta et triginta secunda”.

Ibn al-Zarqālluh among the *magistri probationum*,⁵¹⁶ possibly because our author ascribes to him a tropical *revolutio anni* of 87;21^o,⁵¹⁷ which corresponds to a year of 365d 5;49,24h (= 365;14,33,30d) (see 6.3.3.1). Ibn ‘Ezra attributes the same parameter to the Banū Shākir, al-Šūfī and Ibn Yūnus.

The list of sources mentioned by Ibn ‘Ezra poses another problem: it does not seem likely that he could have read the astronomical works of Yaḥyā ibn Abī Maṣṣūr (see, however, 7.1), al-Marwarrūdhī, the Banū Mūsā, Thābit b. Qurra,⁵¹⁸ al-Nayrīzī, al-Šūfī (*zīj*), and Ibn Yūnus in any city in al-Andalus. My impression, when I consider this list, is that Ibn ‘Ezra had access to sources which had not reached al-Andalus; this may be explained by one of the two following hypotheses:

- a) Isaac, Ibn ‘Ezra’s son, studied in Baghdad with the philosopher Abū l-Barakāt al-Baghdādī, to whom he dedicated a poem in 1143.⁵¹⁹ He may have brought Eastern books to his father when he returned to al-Andalus.
- b) A more probable channel of transmission corresponds to the connections between Antioch and Pisa analysed by Charles Burnett.⁵²⁰ This may have been the way through which Ibn ‘Ezra became acquainted with al-Šūfī *zīj*, as well as with the other Eastern sources he mentions.

As for the Pisa tables, later adapted to London (probably by Robert of Chester), Winchester and Paris,⁵²¹ Mercier⁵²² has established that they were not the result of observations made in Pisa,⁵²³ but in Baghdad. or in some locality near this city, ca. 981. From the data gathered by Mercier,⁵²⁴ one can extract the following mean motion parameters:

⁵¹⁶ See, for example, Millás, 1947, p. 76.

⁵¹⁷ Millás, 1947, p. 83. Ibn ‘Ezra has read Ibn al-Zarqālluh’s book on the solar year (6.3.3) and says that “Azarchelis probatio fuit in anno 482 ab helligera ... et in eodem anno fuerint completi 1089 anni Christi” (Millás, 1947, p. 87).

⁵¹⁸ With the exception of the book *On the solar year* which can be attributed to the Banū Mūsā or to Thābit.

⁵¹⁹ Pines, 1979.

⁵²⁰ Burnett, 2000; Burnett, 2003 a.

⁵²¹ Pedersen, 2007.

⁵²² Mercier, 1991, pp. 5–12.

⁵²³ In spite of the fact that Abraham ibn ‘Ezra performed observations in Lucca and Pisa. See Sela, 2010, II, &15: 1–25, 164–7.

⁵²⁴ Mercier, 1991, pp. 22–25.

TABLE 7.31 Mean motion parameters in Ibn ‘Ezra’s *Tables of Pisa*

Planet	Motion in 20 Julian years	Motion per day
Sun	20 ^r 0;9°	0;59,8,19,42,45,6°
Moon (long.)	267 ^r 133;[3]3,34°	13;10,35,1,8,30,4°
Moon (anom.)	265 ^r 39;4[1],30	13;3,53,56,3,27°
Node	−1 ^r − 26;49,17°	−0;3,10,37,50,23,25°
Saturn	0 ^r 244;40,2°	0;2,0,34,30,48,3°
Jupiter	1 ^r 247;16,25°	0;4,59,16,20,41,53°
Mars	10 ^r 228;19,10°	0;3,1,26,38,46,4,40°
Venus (anom.)	12 ^r 183;43,18°	0;36,59,29,35,36,21°
Mercury (anom.)	63 ^r 14;28,50°	3;6,24,8,17,44,29°

These mean motions deserve a couple of comments when we compare them to the information gathered from the *De rationibus*:

- a) The solar year corresponding to the aforementioned mean solar motion should be 365;14,32,36,24,33,16^d or 365^d 5,49,2,33,49,18,30^h. In the *De rationibus*, Ibn ‘Ezra ascribes to the Banū Shākir, Ibn Yūnus, al-Šūfī and Ibn al-Zarqālluh a *revolutio anni* of 87;21^o52⁵ which yields a solar year of 365;14,26,2,30 or 365^d 5;46,25^h.⁵²⁶ In spite of this, the *De rationibus* confirms that the Sun performs, in 20 Julian years, 20 complete revolutions and 0;9°, in agreement with the *Tables of Pisa*.⁵²⁷
- b) From the mean solar and lunar motions in longitude we can extract a lunar month of 29;31,50,8,6,49,32^d or 29^d 12;44,3,14,43,48,39^h. This parameter is fairly near the value mentioned by Maimonides and Ptolemy (29^d 12^h (793/1080)^h = 29;31,50,8,20^d = 29^d 12;44,3,20^h). In the *De rationibus* we find, however, a lunar month⁵²⁸ of

$$29\text{d } 12^{\text{h}} + (44/1080)^{\text{h}} + (1/18)^{\text{m}} = 29;30,6,15^{\text{d}} (= 29^{\text{d}} 12;2,30^{\text{h}})$$

525 Millás, 1947 a, p. 83.

526 Ibn Yūnus ascribes to al-Šūfī a solar year of 365;14,32,29,22,32^d or 365^d 5;48,59,45,0,47,52^h: see Mercier, 1991, p. 1.

527 Millás, 1947 a, p. 97, lines 1–2. As stated by Mercier (1991, p. 14), the correct reading is 0;9° (as it appears in ms. O7) and not 0;2°, chosen by Millás. Ibn Yūnus says that al-Šūfī’s mean motion of the Sun in a Persian year of 365 days was 359;45,40,2°. This implies that the motion, in 20 years, will be 20^r 0;9,2,19° (Mercier, 1987, p. 111).

528 Millás, *Fundamentos* p. 99.

which is clearly wrong, because it does not agree with what we obtain from the lunar year also mentioned by Ibn ‘Ezra:⁵²⁹

$$354^d 8^h + 0;48^h + [2/3 (1/1080)]^h = 354;22,0,5,33,20^d$$
$$354;22,0,5,33,20^d / 12 = 29;31,50,0,27,46,40^d = 29^d 12;44,0,11,6,40^h$$

This value is close to Ptolemy’s and to the length of the lunar month used in the Islamic calendar (29;31,50^d), also mentioned in another passage of the *De rationibus*.⁵³⁰

As for the planetary equations, Mercier⁵³¹ gives the following values, with the maximum equations of anomaly corresponding to the mean distance:

TABLE 7.32 Maximum equations in Ibn ‘Ezra’s *Tables of Pisa*

Planet	Max. centre eq.	Max. anom. eq.
Sun	1;59,10°	—
Moon	13;8°	5;1°
Saturn	6;31°	6;13°
Jupiter	5;15°	11;3°
Mars	11;24°	41;9°
Venus	1;59°	45;59°
Mercury	3;2°	22;2°

These values are the same as those found in the *De rationibus*, with the sole exception of Mars’ maximum equation of anomaly which is 41;55° instead of 41;9°, probably due to a copyist’s mistake.⁵³² In the case of the Sun, its maximum equation is 1;59° and a few minutes according to the *magistri probationum*:⁵³³ actually, 1;59° is the parameter used by Yahyā ibn Abī Manṣūr and Ḥabash al-Ḥāsib, while the 1;59,10° of the *Tables of Pisa* derives from al-Battānī.⁵³⁴ A rounded value is used for Venus’s maximum equation of anomaly (1;59°). The lunar and planetary equations are the same as in the *Almagest*⁵³⁵ with a differ-

529 Millás, *Fundamentos* p. 98.
530 Millás, *Fundamentos* p. 107.
531 Mercier, “Lost zīj” p. 26.
532 Millás, *Fundamentos* pp. 146–7.
533 Millás, *Fundamentos* p. 78.
534 Nallino, *Battānī* II, p. 81.
535 Toomer, 1984, pp. 549–553.

ence of 1' in the case of the Moon's and Jupiter's maximum equation of the centre (13;8° instead of 13;9° for the Moon⁵³⁶ and 5;15° instead of 5;16° for Jupiter) and two important corrections, derived from the tradition of the *Handy Tables*, in Mars's and Mercury's centre equation (11;24° instead of 11;32° for Mars,⁵³⁷ and 3;2° instead of 2;52° for Mercury).⁵³⁸

7.6.2 *The Alfonsine Tables*⁵³⁹

7.6.2.1 Introduction

My purpose in this section is to underline the fact that the *Alfonsine Tables* (AT hereafter) both in the Castilian canons and in the Parisian Latin numerical tables (PAT), are an authentic *zīj*, strongly influenced by the Andalusī-Maghribī tradition we are analysing in the present chapter.

Many astronomical works patronised by King Alfonso are based on Arabic sources. This is obvious in the case of the *Libro del saber de astrología*.⁵⁴⁰ Establishing links between Arabic *zīj*es and the AT is less easy, although I made an attempt along these lines many years ago.⁵⁴¹ Most of the documented sources are Andalusī, as they were easily available to the king's astronomers. It would be interesting, however, to find evidence of their use of Eastern sources, as we know that Alfonso X maintained diplomatic relationships with Egypt: Castilian embassies were sent to Cairo, and we know of an Egyptian embassy, sent by the sultan Baybars, which arrived in Seville in 1261.⁵⁴² In relation to the latter, a late source, Ortiz de Zúñiga, in his *Anales eclesiásticos y seculares* (Madrid, 1677) states that "The king writes, in the *Libro del candado* [...], that, being aware of the existence in Egypt of a great astrologer, he sent an embassy to search for him".⁵⁴³ Although the source is not reliable, the king's interest in having Arabic scientists in his service has precedents: one of the translators at his service, Bernardo el Arábigo, was a Muslim convert (though he did not become a very useful collaborator); besides, after the conquest of Murcia in 1266, the king tried unsuccessfully to incorporate the polymath Muḥammad

536 Toomer, 1984, p. 238.

537 11;25° in the *Handy Tables* (edition by Abbé Halma, vol. II, Paris, 1822, p. 165) and in al-Battānī (Nallino, 1899–1907, II, p. 123).

538 As in the *Handy Tables* (ed. Halma II, 189) and in al-Battānī (Nallino, 1899–1907, II, p. 135).

539 This section has been revised and commented by B.R. Goldstein and J. Chabás, to whom I would like to express my gratitude. This does not mean, of course, that they fully agree with my conclusions.

540 Samsó, 1987. See also chapter 3 of this book.

541 Samsó, 1987 c.

542 See Martínez Montávez, 1962; Ballesteros, 1963, pp. 305–7.

543 The passage is quoted by Martínez Montávez, 1962, p. 346, and Ballesteros, 1963, p. 306.

b. Aḥmad al-Riqūṭī al-Mursī into his team of scholars.⁵⁴⁴ I would not be at all surprised if King Alfonso had cherished the idea of bringing one of the important Mamlūk astronomers active in Cairo, for example the well-known Abū l-Ḥasan ‘Alī al-Marrākushī, to his court.⁵⁴⁵ We also know that the king sent embassies to the lord of the fortresses of the Ismā‘īlīs in 664/1265–66,⁵⁴⁶ an act that led Juan Vernet⁵⁴⁷ and Mercè Comes⁵⁴⁸ to suggest a possible link between Alfonsine astronomers and those of the Marāgha observatory, which began to be built in 1259.

The first set of *Alfonsine Tables* was prepared between 1262 and 1272. Their authors were the two main Jewish scientific collaborators of King Alfonso (see 1.5.2): Yehudah ben Mosheh (Yehuda fi de Mose fi de Mosca) and Ishāq b. Sīd (Rabiçag Aben Cayut).⁵⁴⁹ Following the king’s orders, these two astronomers built the instruments mentioned by Ptolemy in the *Almagest*, namely the “armillas” (armillary sphere?) and other devices and made observations in Toledo. The prologue to the Castilian *AT* gives the following details:⁵⁵⁰

- Solar observations during an unidentified year, made at the equinoxes and solstices, as well as at the midpoints between them. This implies that they followed both the Ptolemaic method of observing the Sun in its passage through the equinoctial and solstitial points, and the method used by the astronomers of Caliph al-Ma’mūn at the Shammāsiyya observatory of Baghdad in 830, based on the passage of the Sun through the midpoints between equinoxes and solstices (Taurus 15°, Leo 15°, Scorpio 15° and Aquarius 15°). As we have already seen (6.3.1), this second method was also used by Ibn al-Zarqālluh.
- Observations of planetary conjunctions with other planets and the fixed stars.
- Many solar and lunar eclipses: some details of these observations have been transmitted by Isaac Israeli of Toledo in his *Yesod ha-‘Olam* (ca. 1310). According to this source, the lunar eclipses of 24.12.1265 (true opposition at 3;20^h after midnight), 19.6.1266 (3;7^h after midnight), and 13.12.1266 (6;37^h after noon), as well as the solar eclipse of 5.8.1263 (true conjunction at about

544 Samsó, 1981 a.

545 King, 1983.

546 Martínez Montávez, 1962, pp. 355–7.

547 Vernet, 1981 a.

548 Comes, 2003.

549 All the information available on Yehudah has been compiled by Hilty, 1955. Judah b. Asher II ascribes the leadership of the research project to Ishāq b. Sīd together with his colleagues under the command of the king: see Chabás & Goldstein, 2003, p. 226.

550 Chabás & Goldstein, 2003, pp. 19–20, 136–43.

2^h after noon), were observed. Chabás & Goldstein have compared the hour given by Isaac Israeli with the Greenwich hour obtained by modern sources, and I have reduced these hours to the meridian of Toledo (−6;24^m), assuming that the longitude of this city is 4.023W. The differences between the time of the maximum eclipses given by Isaac Israeli and modern computations are −20^m, −23^m, −9^m and +13^m respectively. Chabás & Goldstein have also calculated the time of the mean oppositions and conjunctions with both the *Alfonsine* and the *Toledan Tables*. The hour calculated with the *AT* show differences with respect to Isaac Israeli's reported time of −6^m, −3^m, +8^m and −16^m.⁵⁵¹

According to the prologue, the *AT* were the result of these observations.

The problem is that we do not know precisely what the *AT* were. As we saw in 6.2.7.2, we have two entirely different versions of this work. The first is a set of canons without tables, written in Castilian, which describe tables giving sidereal longitudes, and are strongly influenced by the *Toledan Tables*.⁵⁵² The second is a collection of numerical tables, with their headers in Latin, which lack canons and are usually accompanied by Latin canons written in Paris, ca. 1320–30, by astronomers like John Vimond (fl. c. 1320), John of Lignères (fl. 1320–35), John of Murs (fl. 1310–1348), John of Saxony (fl. 1321–1327). These tables (*PAT* hereafter) can be used to calculate tropical positions.

The main sources which should be taken into consideration in this early Parisian period are:⁵⁵³

- John Vimond's tables (radix 1320 according to the text, although Chabás & Goldstein (2004) believe that they were compiled before this year).
- John of Murs' *Expositio Regis Alfonsii circa tabulas ejus*, written c. 1321;⁵⁵⁴ tables for epoch 1321 and canons "Si vera loca planetarum per presentes tabulas ..."; canons *Patefit ex Ptolomei disciplinis in libro suo*, epoch 1321; canons "Prima tabula docet differentiam unius ere ..." (1339); *Tabulae permanentes*.
- John of Lignères' tables of 1322 with two sets of canons "Cujuslibet arcus" and "Priores astrologi"; *Tabule magne* (c. 1325), associated with canons "Multiplicis philosophie variis radiis"; canons "Quia ad inveniendum loca planetarum" (after 1322) apply to tables of the type described by John of Saxony.

551 Chabás & Goldstein, 2003, pp. 141–3.

552 Edition and commentary by Chabás & Goldstein, 2003.

553 Poulle, 1987 a; Poulle, 1988; Chabás & Goldstein, 2009 a; Chabás & Goldstein, 2003, pp. 266–294.

554 Edited in Poulle, 1980 b; Chabás & Goldstein, 2003, pp. 278–9 show doubts about this date.

- John of Saxony’s canons “Tempus est mensura motus” (1327) printed in the *editio princeps* together with the standard tables.

The Latin *PAT* had enormous success and were widely used in France,⁵⁵⁵ Italy,⁵⁵⁶ England,⁵⁵⁷ Germany⁵⁵⁸ and Poland.⁵⁵⁹ Their use is not documented in the Iberian Peninsula until the end of the fourteenth century (ca. 1396–1400), when they appear in ms. 4238 of the Madrid NL, which contains John of Saxony’s canons and a set of tables derived from the Latin *AT*, using radices for the era of the Incarnation and for the longitude of Morella.⁵⁶⁰ Already in the fifteenth century, a copy of the *PAT*, apparently without canons, appears in the inventory of the royal library of the Crown of Aragon dated in 1410,⁵⁶¹ and there is a vague reference to them in the *Tratado de astrología* attributed to Enrique de Villena, dated in 1438–9.⁵⁶² In the mid-fifteenth century, a set of the *PAT*, with the structure of the *Tabulae resolutae* (see below), are calculated for the coordinates of Salamanca with 1460 as radix. As this kind of table had become very popular in central Europe and, especially, in Poland, its author was probably Nicolas Polonio (i.e., from Poland), the first holder of the chair of Astrology in Salamanca (before 1464) (see 3.1.2.2.3).

7.6.2.2 On the Original Corpus of Tables

This spectacular diffusion of the *PAT* had an obvious consequence: each scholar who made a recension of the tables added new tabular material to them. This was pointed out by John North, who remarked “the Alfonsine tables which circulated in such large numbers outside Alfonso’s kingdom owed almost as much to their editors as the original revisions had apparently owed to Alfonso and his scholars”, and “a full set of conventional tables like Rede’s [fl. 1341–4], for example, might contain 40,000 digits or more, while a full set of Killingworth’s tables [ca. 1461] might contain a quarter of a million”.⁵⁶³ Similar remarks were made by Chabás & Goldstein: the *editio princeps* contains more than 51,000 numbers of one or two digits. Bianchini’s (fl. Ferrara, 1440–1464)

555 Poule, 1987; Poule, 1988; Poule, 1980 b; Husson, 2011; Nothaft, 2015 a; Chabás & Goldstein, 2003 a; Chabás & Goldstein, 2004 Chabás & Goldstein, 2009 a; Chabás & Goldstein 2012 a; Chabás & Goldstein, 2013.

556 Chabás, 2007; Chabás, 2016; Chabás & Goldstein, 2009; Brummelen, 2018.

557 North, 1977; Chabás & Goldstein, 2016; Chabás & Goldstein, 2018; Chabás, 2017.

558 Goldstein & Chabás, 2008.

559 Dobrzycki, 1987; Kremer, 2007.

560 Chabás, 2000. See also Chabás & Goldstein, 2003, pp. 292–3: the fourteenth century ms. Madrid NL 7856 contains the Latin Alfonsine tables with the canons of John of Saxony.

561 Chabás, 1992, p. 30.

562 Cátedra & Samsó, 1988, p. 157.

563 North, 1977, pp. 269/327 and 290/348.

tables have 315,000 numbers of one or two digits.⁵⁶⁴ Many of these derived sources introduced important new methods already known in Islamic astronomy (displaced equations, tables with double argument etc.), which simplify the computer's task.⁵⁶⁵

As a result of this accumulation of materials, it is extremely difficult to establish a list of tables which can be considered genuinely Alfonsine. Emmanuel Poulle made a selection based on the canons of John of Saxony, which were printed, together with a standard set of tables, in the *editio princeps* (Erhard Ratdolt, 1483).⁵⁶⁶ Unfortunately, he was not always consistent in applying this criterion and his *PAT* do not include the table for the equation of time, mentioned by John of Saxony in canon 22.⁵⁶⁷ Poulle also excluded the tables of eclipses used by John of Murs:⁵⁶⁸ John of Saxony mentions the eclipse limits for both solar and lunar eclipses,⁵⁶⁹ which are the same as in the Castilian canons, the *Toledan Tables*, and chapter 33 of John of Lignères' canons *Priores astrologi* (c. 1325).⁵⁷⁰

7.6.2.2.1 *The Castilian Canons*

Another possible criterion is to follow the list of tables mentioned in the Castilian canons which can give us some hints regarding this identification, at least in the case of tables independent of the large-scale changes introduced in the second (Latin) recension of the *AT*.

As I have already said, the Castilian canons describe a set of sidereal tables for the computation of solar, lunar and planetary longitudes. They contain, however, a few hints of an evolution towards the use of tropical longitudes in the practice of astrology: the longitude of the ascendant and of the beginning of each house is tropical,⁵⁷¹ and the *revolutio anni*, used for the computation of the revolutions of nativities or world years, is defined in the text as either the entry of the Sun in the vernal equinox (tropical) or the beginning of Aries (sidereal).⁵⁷² This is in clear contradiction to the principles of Western Islamic astrology, but we have seen precedents already in the works of Abraham bar

⁵⁶⁴ Chabás & Goldstein, 2009, p. 11.

⁵⁶⁵ An example can be found in Chabás & Goldstein, 2013 a.

⁵⁶⁶ Poulle, 1984.

⁵⁶⁷ Poulle, 1984, pp. 86–7.

⁵⁶⁸ He compared his observations of the solar eclipses of 3.3.1337 and 10.3. 1344 with the results obtained with the *Alfonsine Tables*: see Beaujouan, 1975.

⁵⁶⁹ Poulle, 1984, pp. 60–61. These inconsistencies in Poulle's edition of the tables have been underlined by Chabás & Goldstein, 2003, p. 284.

⁵⁷⁰ Chabás & Goldstein, 2003, pp. 66–8, 193–5.

⁵⁷¹ Chapter 28: Chabás & Goldstein, 2003, pp. 52–3, 183–5.

⁵⁷² Chapter 48: Chabás & Goldstein, 2003, pp. 88–9, 216–7.

Ḥiyya and Abraham ibn ‘Ezra (7.6.1) and, in 3.1.2.2.1, we saw that the horoscope cast for determining the propitious moment for the beginning of the Castilian translation of Ibn Abī l-Rijāl’s *Bārī* used planetary longitudes computed with the *Toledan Tables*, but corrected for precession.

Unlike the Latin *PAT*, the Castilian canons do not refer to chronological and mean motion tables based on a strict sexagesimal system. The chronological tables use the eras of Alexander, Caesar, Hijra, Yazdijird and Alfonso, the latter corresponding to the beginning of the year in which King Alfonso X ascended the throne (31 December 1251). As we will see, for the Alfonsine era, the Latin tables use the exact date of his accession to the throne (31 May 1252). In both sets of *zīj*es the radices for mean motions are given for the meridian of Toledo, and the Castilian canons mention some other tables calculated precisely for the coordinates of that city (longitude 28° from the meridian of water, and latitude, 39;54°).⁵⁷³ those of the division of the houses,⁵⁷⁴ conjunctions and oppositions of the Sun and the Moon,⁵⁷⁵ a star table which indicates the degrees of mediation, rising, setting and meridian altitude of each star,⁵⁷⁶ length of daylight,⁵⁷⁷ time degrees of a seasonal hour and oblique ascensions.⁵⁷⁸ All this points to a certain local character of the tables described in the Castilian canons, in contrast to the Latin tables.

Both the chronological and mean motion tables are computed for years, months, days and hours and the mean motions use multiples of 20 years and single years, which implies that, unlike the Latin tables, they depend on specific calendars. They use zodiacal signs of 30° instead of the signs of 60° of the tables in the *editio princeps*.

The Castilian canons also follow several doctrines characteristic of the Andalusī-Maghribī tradition, such as trepidation according to the model of the *Liber de motu* (6.2.1), computed with a table of the mean motion of the head of Aries and another table of the corresponding equation.⁵⁷⁹ We also find a reference to Ibn al-Zarqālluh’s model of variable obliquity (6.2.2.5.2.4 and 6.2.2.5.2.5) including the two corresponding tables,⁵⁸⁰ although the use of the computed obliquity to obtain the value of trepidation for a given date is not explained.

⁵⁷³ Chabás & Goldstein, 2003, p. 22.

⁵⁷⁴ Chabás & Goldstein, 2003, p. 53.

⁵⁷⁵ Chabás & Goldstein, 2003, p. 58.

⁵⁷⁶ Chabás & Goldstein, 2003, p. 84.

⁵⁷⁷ Chabás & Goldstein, 2003, p. 86.

⁵⁷⁸ Chabás & Goldstein, 2003, p. 86.

⁵⁷⁹ Chapter 49: Chabás & Goldstein, 2003, pp. 89–90, 217–20.

⁵⁸⁰ Chapter 50: Chabás & Goldstein, 2003, pp. 90, 220–1.

Neither the Castilian canons nor the Latin tables contain any reference to the solar apogee's own motion or its application to the planetary apogees (6.3.2), or to Ibn al-Zarqālluh's theory about the variation of solar eccentricity (6.3.3.2), although it is clear that the former was in some way adopted by the Alfonsine astronomers, as we will see from the evidence given by John Vimond (7.6.2.2.2.4).

Apart from all this, the tables mentioned by the Castilian canons are more or less standard in the Ptolemaic tradition: solar, lunar and planetary equations, trigonometric functions, lunar and planetary latitudes, stations and retrogradations, equation of time, parallax, eclipses, timekeeping, and so on. Unfortunately, very few numerical parameters are given. Among them we find the maximum northern and southern latitudes of Venus and Mercury of 8;36° and 4;38° respectively: the former appears in Ibn al-Kammād's *zīj* and in the tables of Judah b. Asher II of Burgos, while the second was only used by Judah b. Asher II. Chabás and Goldstein suggest that they might be the result of a corruption of the 8;56° and 4;18° of the *Handy Tables*, as a result of the similarity of letters in the *abjad* notation.⁵⁸¹ Another parameter is the maximum equation of time (7;48°, equivalent to 0;31,15^h).⁵⁸² Interestingly, this canon also gives the sidereal mean longitude (276;57°) and true longitude (277;29°) of the Sun for the beginning of the Alfonsine era (31.12.1251), and Chabás & Goldstein⁵⁸³ have shown that these positions have been calculated with the Toledan Tables, though using al-Battānī's solar apogee (82;17°) instead of the Toledan one. Whatever the case, it is clear that these positions were not computed with the Latin *PAT* and that a solar equation of 0;32° (277;29° – 276;57°) does not seem to agree with a maximum equation 2;10°, used in the *PAT*.

A few other chapters of the Castilian canons deserve attention:

7.6.2.2.1.1 Ibn al-Zarqālluh's Lunar Correction

Interestingly, the Zarqāllian correction of the mean lunar position also appears also in chapter 17,⁵⁸⁴ although with an unexpected value of the maximum correction. As we saw in 6.4, Ibn al-Zarqālluh recommends correcting the mean lunar position using the expression:

$$l_v = 0;0,24^\circ \sin l_t$$

⁵⁸¹ Chabás & Goldstein, 2003, pp. 41–3, 163–5.

⁵⁸² Chabás & Goldstein, 2003, pp. 55, 186.

⁵⁸³ Chabás & Goldstein, 2003, p. 188.

⁵⁸⁴ Chabás & Goldstein, 2003, pp. 37–8, 155–7.

where l_t is the difference between the mean lunar position (as obtained from the mean motion tables) and the position of the solar apogee, while l_v will be the corrected position in longitude from the solar apogee.⁵⁸⁵ The maximum ($0;24^\circ$) will be attained for $l_t = 90^\circ$, as $\sin l_t = 60$.

The Castilian AT give the same expression but use $0;5,30^\circ$ instead of $0;24^\circ$: the text instructs the multiplication of $0;0,5,30^\circ$ by $\sin l_t$ to obtain l_v . To explain this parameter I can only offer a rather bold hypothesis. I suggest that the text is a result of an amalgamation of two different sets of instructions: one of them described Ibn al-Zarqālluh's correction, while the other corresponded to a very old Iranian procedure to compute the solar equation, using the "method of declinations",⁵⁸⁶ as in al-Khwārimī's *zīj*:

$$\eta(\lambda) = [\eta_{\max} / \varepsilon] \cdot \delta(\lambda)$$

in which $\eta(\lambda)$ is the solar equation for a value of λ (solar *centre* = *markaz*); $\eta_{\max}$ is the maximum solar equation; ε is the obliquity of the ecliptic; and $\delta(\lambda)$ is the solar declination for λ .

The original text, mixed with Ibn al-Zarqālluh's correction, probably said that the mean solar position should be subtracted from the solar apogee. The solar declination corresponding to this difference in longitude was, then, multiplied by $0;5,30^\circ$ (instead of $0;0,5,30^\circ$) and the result obtained would be the solar equation. If we assume that $\eta_{\max} = 2;12^\circ$ and $\varepsilon = 24^\circ$, we have:

$$2;12^\circ / 24^\circ = 0;5,30^\circ$$

A maximum solar equation of $2;12^\circ$ does not seem to be far from the Alfonsine parameter ($2;10^\circ$). A better result ($2;9,31,30^\circ$) can be obtained using an obliquity of $23;33^\circ$, instead of 24° .

7.6.2.2.1.2 Solar, Lunar and Planetary Velocities

Another interesting item can be found in the tables for planetary velocities, described in chapter 27.⁵⁸⁷ According to this description, the hourly velocity of each planet (there are references to the lunar velocity) is obtained by adding two entries computed as a function of the equated planetary centre and the equated equation of anomaly. This description is also found in John of Lignères' canons: this text has many strong correlations with the Castilian

⁵⁸⁵ Puig, 2000.

⁵⁸⁶ Kennedy & Muruwwa, 1958, pp. 118–9.

⁵⁸⁷ Chabás & Goldstein, 2003, pp. 47–52, 170–182.

canons that suggest (at the very least) a common origin. Tables which agree with the Alfonsine and John of Lignères' canons have been discovered in 42 Latin (eight of them independent of John of Lignères' tradition), two Hebrew and two Arabic manuscripts (see 7.4.1.6), many of them from the thirteenth century. The table also appears in five printed editions in Latin. There is a table for the velocity of the Moon, but no explicit table of the solar velocity, although this can be obtained from the entry whose argument is Venus's equated centre, since Venus and the Sun share the same eccentricity and the same apogee.⁵⁸⁸ My impression is, therefore, that the planetary velocity tables were included in the original set of *AT*, although their origin was probably Andalusī or Maghribī.

As for the table of lunar velocity, the situation is not so clear. The *editio princeps* of the *AT* (1483) contains tables of solar and lunar velocities which might correspond to the original set.⁵⁸⁹ The solar velocity table (max. v. 0;2,34°/h; min. v. 0;2,23°/h) can be recomputed using an eccentricity of 2;14^p, which yields a maximum equation of 2;8°, not far from the maximum solar equation (2;10°) in the Latin equation table (7.6.2.2.2.5). Most entries in the table of lunar velocity (max. v. 0;36,25°/h; min. v. 0;30,21°/h) seem to have been computed for an eccentricity of 5;10^p, which corresponds to a maximum equation of 4;56° in a simplified lunar model, precisely the value of the maximum equation of anomaly in the Latin tables (7.6.2.2.2.5). The last 20 entries, however, seem to have used a different eccentricity and appear to be the result of an error.⁵⁹⁰

7.6.2.2.1.3 The Castilian Canons and Their Islamic Context

Chabás and Goldstein have carefully examined the contents of the Castilian canons and have shown the existence of many coincidences between this source and the *Toledan Tables*. Apart from the reference to the standard tables found in many *zīj*es we find two chapters that make sense only in an Islamic context: one of them is chapter 53⁵⁹¹ in which there is a procedure for determining the time of the beginning of dawn and end of twilight, which correspond to the moment in which the Sun reaches the altitude of -19°, as in Ibn Mu'ādh's *Liber de crepusculis*.⁵⁹² The interest in this topic is due to the fact that the early morning prayer (*ṣubḥ*) takes place between the beginning of dawn

588 Goldstein, Chabás & Mancha, 1994; Chabás & Goldstein, 2003, pp. 170–182.

589 A table of solar velocity is mentioned in chapter 48 of the Castilian canons (Chabás & Goldstein, 2003, pp. 88–9, 216–7). In chapter 32 we find a reference to a table of lunar velocity in elongation (Chabás & Goldstein, 2003, pp. 60–6, 191–3).

590 Goldstein, 1980; see also Goldstein, 1996.

591 Chabás & Goldstein, 2003, pp. 92–3, 222–3.

592 Goldstein, 1976 a; Goldstein, 1977a; Mark Smith & Goldstein, 1993.

and sunrise, while the time for the sunset prayer (*maghrib*) is between sunset and the end of twilight.

A second instance of a clear Islamic context is chapter 36, on lunar crescent visibility⁵⁹³ (see 2.3.4.1). The initial criteria, based on lunar elongation, are the same as those adopted by the *Toledan Tables*, Maimonides, Ibn al-Raqqām's *Shāmīl*, Ibn al-Bannā', and so on. The computation for the doubtful cases introduces two more corrections to the lunar longitude and latitude, which have already been corrected in parallax (λ'_L and β'_L):

- With λ'_L as argument, one enters a table⁵⁹⁴ which calculates $0;24 \cos \lambda'_L$ and returns a value m . Then:

$$\beta'_L \times m$$

and $\lambda'_L \pm \beta'_L \times m = \lambda''_L$

- The second correction is based on the difference in oblique descensions ($\Delta\alpha_\varphi$) between λ''_L and the position of the Sun:

$$\Delta\alpha_\varphi = \alpha_\varphi (\lambda''_L + 180^\circ) - \alpha_\varphi (\lambda_S + 180^\circ)$$

$$\Delta\alpha_\varphi \pm \tan \varphi \cdot \beta'_L = \text{arco de la vista (qaws al-ru'ya)}$$

The first of these corrections can be found in Ibn Mu'ādh, the *Toledan Tables*, and Ibn al-Raqqām's *Qawīm*, but the second one is only found in the *Toledan Tables*.⁵⁹⁵ Maimonides uses the same procedure as the *Toledan zīj*, but his second correction is different. It seems, therefore, that the source of this chapter is the *Toledan Tables*.

7.6.2.2.1.4 The Declination Table

The Castilian canons mention the parameter $23;33^\circ$ for the obliquity of the ecliptic,⁵⁹⁶ the value used by Ibn al-Zarqālluh, and dedicate a chapter to the use of a declination table.⁵⁹⁷ Surprisingly, the three editions of the Latin *Alfonsine Tables* that I have been able to examine (1483, 1524 and 1553) do not contain a table of the solar declination, and nor does the 1492 edition. We do, however, have evidence of the use of another value for the obliquity of the ecliptic: in

593 Chabás & Goldstein, 2003, pp. 70–2, 199–200.

594 *Tabula reflexionis Lunae* in the *Toledan Tables* (Pedersen, 2002, IV, p. 1486); Mestres, 1999, p. 287; ms. 260 of the Rabat National Library p. 105. In the Castilian *Alfonsine Tables* this table is called “tabla del desviamiento de la Luna”.

595 Pedersen, 2002, II, pp. 638–41.

596 Chabás & Goldstein, 2003, p. 60.

597 Chabás & Goldstein, 2003, pp. 40, 160–2.

the Alfonsine treatise on the *Cuadrante para rectificar*,⁵⁹⁸ Ishāq b. Sid mentions a value of the ecliptic of $23;32,29^\circ$ (“for our time”) as well as declination values for 30° and 60° :

30°	$11;31,11^\circ$
60°	$20;14,13^\circ$ (rec. $20;14,14^\circ$)

This value of ϵ reappears, rounded into $23;32,30^\circ$ in another Alfonsine text, the *Libro del reloj de la piedra de la sombra*, where we find a complete declination table⁵⁹⁹ calculated for this parameter. This table seems to be related to Ibn al-Zarqālluh’s model for the computation of ϵ , although with the parameters used by the *zīj*es of Ibn Ishāq’s school (anonymous Tunisian recension, Ibn al-Bannā’ and Ibn al-Raqqām), in which $23;32,30^\circ$ is the minimum value of the obliquity of the ecliptic.

As a matter of fact, this parameter corresponds to the value of ϵ calculated for the end of 1193, the year in which the beginning of Ibn Ishāq’s astronomical activity has been established. In spite of this, it seems that it might have an observational origin. According to Abū ‘Abd Allāh Muḥammad al-Ḥabbāk (d. after 1562), Ibn Ishāq himself stated that the value of $23;32,30^\circ$ had been obtained by an anonymous astronomer from Miknāsa (Meknès) through an observation made in 602/1205–6.⁶⁰⁰ Obviously, this means that Ibn Ishāq believed that the obliquity of the ecliptic had reached its minimum value in his own time, and it implies that the declination table reproduced in Table 1 is due to Ibn Ishāq himself and belongs to his primitive *zīj*. The same parameter appears in al-Sanjufīnī’s *zīj*, compiled in Tibet in 1366.⁶⁰¹

In 1992, Mercè Comes remarked that the solar declination table (no. 54 of the Hyderabad ms.), of the anonymous Tunisian recension of Ibn Ishāq’s *zīj* is the same as the one that appears in the *Libro del reloj de la piedra de la sombra*, as both share the same small computational errors.⁶⁰² My own edition and recomputation of Table 7.33 shows that she was absolutely right. A comparison between the tables shows only two important differences, although they seem to be mere copying mistakes:

598 Ms. Villa Amil 156 fol. 171v; Rico, 1863–67, III, p. 296.

599 Ms. Villa Amil 156 fol. 178r; Rico, 1863–67, IV, p. 6.

600 Saidi, 2012, p. 75.

601 Kennedy, 1987–88; Kennedy & Hogendijk, 1988.

602 Comes, 1992, pp. 152–3.

40° 14;22,36° (Alf.) instead of 14;52,36° (Ibn Ishāq)
48° 17;17,2° (Alf.) instead of 17;16,2° (Ibn Ishāq)

TABLE 7.33 Solar declination in the Alfonsine *Libro primero de la piedra de la sombra* (Ms. Villa Amil 156 fol. 178r; Rico, 1863–67, IV, p. 6). In parentheses (text – recomputation).

1°	0;23,57° (–1)	31°	11;52,17°	61°	20;26,49 (+1)
2	0;47,54 (–1)	32	12;13,11 (+1)	62	20;39,2 (+1)
3	1;11,54 (+2) ^a	33	12;33,53 (+1)	63	20;50,52 (+1)
4	1;35,47 (–1)	34	12;54,22 (+1)	64	21;2,19 (+1)
5	1;59,41 (–1)	35	13;14,38	65	21;13,22
6	2;23,33 (–1)	36	13;34,42 (+1)	66	21;24,2
7	2;47,24	37	13;54,32 (+1)	67	21;34,19 (+1)
8	3;11,13 (–1)	38	14;14,8 (+1)	68	21;44,10
9	3;34,58 (+2)	39	14;33,29	69	21;53,39 (+1)
10	3;58,37 (–1)	40	14;22,36 ^b	70	22;2,42 (+1)
11	4;22,14 (–1)	41	15;11,27 (–1)	71	22;11,20 (+1)
12	4;45,47 (–2)	42	15;30,4 (–1)	72	22;19,32
13	5;9,17 (–1)	43	15;48,26	73	22;27,20 (+1)
14	5;32,43 (–1)	44	16;6,32 (+1)	74	22;34,43 (+2)
15	5;56,3 (+2)	45	16;24,20 (+1)	75	22;41,39 (+1)
16	6;19,15	46	16;41,52 (+2)	76	22;48,7 (–1)
17	6;42,22	47	16;59,6 (+2) ^c	77	22;54,12
18	7;5,23 (–1)	48	17;17,2 ^d	78	22;59,51 (+1)
19	7;28,18	49	17;32,40 (+1)	79	23;5,3 (+1)
20	7;51,6	50	17;49,0	80	23;9,48 (+1)
21	8;13,46	51	18;5,1	81	23;14,7 (+2)
22	8;36,18	52	18;20,44 (+1)	82	23;17,57 (+1)
23	8;58,42 (–1)	53	18;36,7 (+1)	83	23;21,21
24	9;[20],59 (+1)	54	18;51,10 (+1)	84	23;24,18
25	9;[43],6 (+1) ^e	55	19;5,53 (+1)	85	23;26,49 (+1)

a 1;11,51° in the Tunisian recension. See Mestres, 1999, p. 277.
b 14;52,36° in the Tunisian recension, which is the correct value.
c 16;59,4° in the Tunisian recension.
d 17;16,2° in the Tunisian recension which, with an excess of (+1), is the correct value.
e 9;43,5° in the Tunisian recension.

TABLE 7.33 Solar declination in the Alfonsine (*cont.*)

26	10;[5].3 ^f	56	19;20,15	86	23;28,52 (+1)
27	10;26,50	57	19;34,16 ^g	87	23;30,27
28	10;48,28	58	19;47,57	88	23;31,36 (+1)
29	11;9,55	59	20;1,16	89	23;32,17 (+1)
30	11;31,12	60	20;14,14 (+1)	90	23;32,30

f For arguments 24–26, the minutes are covered by an inkblot.
g 19;34,17° in the Tunisian recension.

All this implies that Ibn Ishāq’s *zīj* (ca. 1193–1222) had reached the Alfonsine milieu. This hypothesis will be confirmed below (7.6.2.2.2.3) in my analysis of the mean motion parameters of the Latin *PAT*. To the best of my knowledge, there is only another instance of the diffusion of Ibn Ishāq’s *zīj* in the Iberian Peninsula: in 1357, the Jewish astronomer Yosef ibn Waqār compiled a *zīj* for the coordinates of Toledo, using as a radix date the end of 720H/1320–1.⁶⁰³ In it he gives two sets of the mean solar, lunar and planetary positions for that radix according to *qāḍī* Ibn Šāʿid of Toledo and Ibn Ishāq, and Castells has proved that these positions were calculated with the *Toledan Tables* and with Ibn Ishāq’s *zīj*.⁶⁰⁴

7.6.2.2.2 *The Latin Tables (PAT)*

7.6.2.2.2.1 Introduction

According to Emmanuel Poulle,⁶⁰⁵ the corpus of Latin tables usually called Alfonsine shares the following characteristics:

- 1) Full sexagesimalisation, which implies the use of “physical” signs of 60°
- 2) Precession with two components
- 3) Tropical motions
- 4) New values for the maximum solar equation (2;10°), centre equation of Venus (2;10°), lunar equation of anomaly (4;56°) and centre equation of Jupiter (5;57°).
- 5) Radices calculated for Toledo

603 The mean positions in longitude for the Sun, Moon and the three superior planets are calculated for 28 Dhū l-Ḥijja 720/29 January 1321, while the mean positions of anomaly of Venus and Mercury correspond to two days later: 1 Muḥarram 721/31 January 1321.
604 Castells, 1996.
605 Poulle, 1987 a and 1988.

I have already dealt with item 2 in 6.2.7.2. Besides, there is no difficulty in accepting the validity of items 3–5, while item 1 deserves discussion. Poulle himself (1987 and 1988), for example, considers that the early Parisian Alfonsine texts use the meridian of Paris and signs of 30° , and that the meridian of Toledo and signs of 60° appear later. The actual situation is not so clear: signs of 30° appear in John Vimond's tables, in John of Murs' *Patefit*, in John of Lignères' tables for 1322, as well as in his canons *Cujuslibet* and *Multiplicis philosophie variis*. They will continue to be used in William Batecombe's Oxford tables (1348) and in the fifteenth-century *Tabulae resolutae* (Dobrzycki, 1987; Chabás, 1998). Signs of 60° appear in several works of John of Murs generally, but not systematically: in his *Expositio* he uses signs of both kinds. John of Lignères' *Quia ad inveniendum* uses signs of 60° .

Full sexagesimalisation appears in John of Saxony's *Tempus est mensura motus*. This structure has the obvious advantage of making mean motion tables independent of calendars, as the basic unit is the day, common to all of them. It is obvious that the Parisian astronomers had difficulty in assimilating this new concept, and tables for *anni collecti* (for multiples of 20 years), *expansi*, months and days continued to be used in several sources like John of Lignères in his tables of 1322 and in his canons, as well as in Batecombe's tables and, obviously, in the *Tabulae resolutae*.

The question is to ascertain whether the sexagesimal structure of the Alfonsine mean motion tables was conceived by the Alfonsine astronomers or is the result of an adaptation made in Paris in the 1320s. Many years ago, both G.J. Toomer (January 1984) and Owen Gingerich (June 1984) expressed their doubts, in private conversations, about the Parisian origin of this structure, due to the fact that it simplified the problem of the use of two (at least) different calendars in Castile, in the thirteenth century, something which did not make much sense in France.⁶⁰⁶ I will say something more about this in the next section.⁶⁰⁷

606 Poulle himself seems to be aware of this difficulty: "l'adoption faite par les astronomes parisiens d'une structure chronologique universelle, non liée à un calendrier particulier, paraissait une complication bien inutile pour des gens qui n'opéraient que dans le calendrier chrétien" (Poulle & Savoie, 1998, p. 202). The same idea reappears in Noel Swerdlow's review of Chabás & Goldstein's *Alfonsine Tables of Toledo* in *JHA* 35 (2004), 479–484 (see p. 483). José Chabás does not consider full sexagesimalisation as a distinctive feature of the tables compiled by Alfonsine astronomers (private communication).

607 Samsó, 1987 pp. 31–2.

7.6.2.2.2.2 Sexagesimal Structure of the Calendar Tables

This most interesting feature of the Latin *Alfonsine Tables*, as it appears, for example, in the *editio princeps*, is the use of a strict sexagesimal structure (the zodiacal signs of 30° are replaced by signs of 60°) in both the chronological and the mean motion tables. In this section I will try to underline the Islamic precedents of this sexagesimal structure as well as the existence of a relation between one of the chronological tables and an authentic Alfonsine work.

Sexagesimal mean motion tables appear at the beginning of Islamic astronomy. As I remarked, many years ago,⁶⁰⁸ the eighth-century astronomer al-Fazārī computed mean motion tables from 1 to 60 *saura* (solar) days, from 1;0 to 6;0 *saura* days, from 1 to 60 years and, probably, for a certain number of 60-year intervals.⁶⁰⁹ A *saura* day is the mean time required for the Sun to travel one degree: therefore, 6;0 *saura* days is a solar year.

Evidence of mean motion tables based on the number of days since epoch in Arabic sources is rare; however, on this issue, I should mention that Mestres⁶¹⁰ has drawn attention to one interesting characteristic of the mean motion tables in the Tunisian recension of Ibn Ishāq's *zīj*, Ibn al-Bannā's *Minhāj* and Ibn al-Raqqām's *Shāmīl*: the presence of a subtable, with arguments from 1 to 60, giving the corresponding mean motions for hours. The same values, divided by 60, will correspond to mean motions in minutes.

A structure of this kind is fairly common, however, in the case of chronological tables. Ibn al-Muthannā's commentary on the tables of al-Khwārizmī⁶¹¹ states that the original chronological tables were based on days, expressed sexagesimally in four orders (60⁰, 60¹, 60², and 60³); this led Pedersen to suggest that the calendar tables in al-Khwārizmī's *zīj* should, perhaps, be ascribed to Maslama's revision.⁶¹² This characteristic appears often in the *Toledan Tables*,⁶¹³ as well, as we have seen, in the Castilian *Alfonsine Tables*.⁶¹⁴ Besides, al-Bīrūnī's *Chronology* contains a table (named *al-ṭaylasān al-muḍaʿaf*) which, like the Alfonsine *Tabula differentiarum* (table 7.34), gives the differences in days (expressed sexagesimally) between eras.⁶¹⁵

608 Samsó, 1987, p. 32.

609 Pingree, 1970; Haddad. Kennedy & Pingree, 1981, pp. 97 and 220.

610 Mestres, 1999, pp. 27, 188.

611 Goldstein, 1967, pp. 24–25.

612 Pedersen, 2002, III, p. 877.

613 Pedersen, 2002, III, pp. 878–901.

614 Chabás & Goldstein, 2003, pp. 28–35, 148–151, 249–50.

615 Bīrūnī, *Athār*, p. 132; Sachau, 1879, p. 133; see also Craddock, 1974, p. 376.

New evidence of this sexagesimal structure of the chronological tables can be found in Ibn al-Hā'im al-Ishbīlī's *Kāmil zīj* (7.2.6). In chapter 12 of the first book (*maqāla*), Ibn al-Hā'im describes the procedure for the conversion of a date, expressed in years, months and days, of a given calendar (A) into another (B), using tables structured only in days (*al-murakkaba min al-ayyām faqat*). These tables are computed for *majmū'a* (*collecti*), and *mabsūṭa* (simple) years, months and days and, for each value of the argument, we find a number of days expressed sexagesimally, according to four orders (*marṭaba*). The number of days since the beginning of the corresponding era (A) is called the *basis* (*aṣl*). Then, one enters the tables of the difference of eras (*jadāwil tafāḍul al-tawārīkh*) and obtains the number of days elapsed between era A and era B. This difference will be added to or subtracted from the *basis*, and one obtains the *requested basis* (*al-aṣl al-maṭlūb*). With this value one enters another table, based on calendar B, and obtains the corresponding date expressed in years, months and days. Ibn al-Hā'im's description agrees very well with the similar one we can find in chapter 9 of the Castilian canons,⁶¹⁶ which deals with the transformation of a date of a particular calendar into another date of another calendar using a table of only days ("por las tablas compuestas de los dias no mas"). For each calendar there is a table of collected ("años colectos") and *expansi* ("años bueltos") years, as well as months giving the equivalent number of days expressed sexagesimally in four orders called *offiçios* (*marṭaba*). The result obtained is called the *padrón* (*aṣl*).

Both these sources agree with the Latin Alfonsine chronological tables, whose main purpose is to obtain the sexagesimal number of days elapsed from the beginning of the era, although they can also be used for calendar conversion. The *Tabula differentiarum unius regni ad aliud ...* corresponds to Ibn al-Hā'im's *jadāwil tafāḍul al-tawārīkh*, although, in the case of the Latin *Alfonsine Tables*, the difference between eras is repeated, as the *radix*, at the beginning of each table used for the extraction of one date from another one (*Tabula extractionis unius ere ex altera*). A similar reference can be found in chapter 10 of the Castilian tables ("De la diversidad de los tiempos entre cada era") and in the *Toledan Tables*.⁶¹⁷ A remnant of this kind of table, partially expressed in sexagesimal notation, appears in the appendix to Ibn al-Kammād's *Muqtabas zīj*.⁶¹⁸

⁶¹⁶ Chabás & Goldstein, 2003, pp. 28–30, 148.

⁶¹⁷ Pedersen, 2002, III, pp. 899–903.

⁶¹⁸ Ms Madrid NL 10023, fol. 58r; Chabás & Goldstein, 2015 a, pp. 640–1.

This *Tabula differentiarum* has another precedent in a source which is clearly Alfonsine. The table gives the number of years and days elapsed between each couple of the ten eras used in the Latin tables. These eras are: the Flood, Nabuchodonosor (= Nabonassar), Philippus, Alexander, Caesar (Spanish Era), Incarnation, Diocletian, Hijra, Yazdijird and Alfonso (midday of 31 May 1252). This list of eras, with the obvious exception of Alfonso's, and the addition of the World era or era of Creation, can be found in most Andalusi and Maghribi *zīj*es (see 2.2). In the Castilian *Alfonsine Tables* we only find the eras of Alexander, Caesar, Hijra, Yazdijird and Alfonso, although the Alfonsine era is located at the beginning of the year of his accession to the throne (midday of 31 December 1251).⁶¹⁹ To this one should add that chapter 10 of the same Castilian source⁶²⁰ also gives the difference in days between the epochs of different eras (Alfonso-Yazdijird, Alfonso-Hijra, and Yazdijird-Hijra), expressed in Persian, lunar and Julian years, as well as in days given in decimal and *sexagesimal* form.⁶²¹

Interestingly, the Alfonsine legal code entitled *Partidas*, compiled in 1256–1265, contains a chronological note, at the end of the first part of the general prologue, which gives the difference in days between the Alfonsine era and the same nine eras⁶²² of the *Tabula differentiarum*.⁶²³ It is worth mentioning that this note uses the same radix date as the Latin *Alfonsine Tables* for the beginning of the Alfonsine era (31 May 1252). Table 7.34 contains a comparison between this number of days in the *Partidas* and in the Castilian (only for the Hijra and Yazdijird eras)⁶²⁴ and Latin *PAT*, and we can easily see that there is a strict correspondence between columns 2 and 4 of the table, and that column 3 presents a difference of –152 days, as the result of the change of era.⁶²⁵

619 Chabás & Goldstein, 2003, pp. 20–22, 143–145.

620 Chabás & Goldstein, 2003, pp. 30–31, 148–150.

621 In Chabás, 2019, José Chabás remarks that “In a number of manuscripts and in the second edition (Venice, 1492), for each time interval we are also given its equivalence in Julian years and days”.

622 As well as the world era or era of Creation, which took place, according to the text, 5011 Jewish years + 287 days before the Alfonsine era.

623 Craddock, 1974.

624 Chabás & Goldstein, 2003, pp. 30–31, 148–50.

625 The Castilian Alfonsine Tables add that, between Hijra and Yazdijird eras, there are 10 lunar years + 80 days = 9 Persian years + 339 days = 9 Julian years + 336 days = 3,624 days = 1,0,24 days.

TABLE 7.34

1. <i>Tabula differentiarum</i> : number of days before Alfonso's era	2. <i>Partidas</i> (epoch: 31.5.1252)	3. <i>Castilian Alfonsine Tables</i> (epoch: 31.12.1251)	4. <i>Parisian Latin Alfonsine Tables</i> (epoch: 31.5.1252)
Flood	$4353^y + 105^d = 1,590,038^d$	–	$7,21,40,38^d$
Nabuchodonosor	$1998^y + 96^d = 729,865^d$	–	$3,22,44,25^d$
Philippus	$1574^y + 202^d = 575,105^d$	–	$2,39,45,5^d$
Alexander	$1562^y + 244^d = 570,764^d$	–	$2,38,32,44^d$
Caesar	$1289^y + 152^d = 470,959^d$	–	$2,10,49,19^d$
Incarnation	$1251^y + 152^d = 457,079^d$	–	$2,6,57,59^d$
Diocletian	$967^y + 277^d = 353,473^d$	–	$1,38,11,13^d$
Hijra	$629^y + 32[2]^d \text{ a} =$ $= 649 \text{ lunar yrs} + 80^d =$ $= 230,064^d$	$648 \text{ lun. yrs} + 282^d =$ $= 629 \text{ Pers. yrs} + 327^d =$ $= 629 \text{ Jul. yrs} + 169 \frac{3}{4}^d =$ $= 229,912^d =$ $= 1,3,51,52^d$	$1,3,54,24^d$
Yazdijird	$619^y + 350^d =$ $= 620 \text{ Persian yrs} + 140 =$ $= 226,440$	$619 \text{ Pers. yrs} + 353^d =$ $= 619 \text{ Jul. yrs} + 198 \frac{5}{8}^d =$ $= 638 \text{ lun. yrs} + 200^d =$ $= 226,288^d =$ $1,2,51,28^d$	$1,2,54,0^d$

a 321 days in all manuscripts: see Craddock pp. 386–7.

7.6.2.2.2.3 Mean Motion Tables

One of the obvious advantages of the sexagesimal structure of the *PAT* is that one has no need to “squeeze” the underlying parameters, for they are already given in the tables themselves. It is interesting to compare these parameters to those underlying the tables of the tradition of Ibn Ishāq (Tunisian recension, Ibn al-Bannāʾ, Ibn al-Raqqām) (7.3.3.1). Those corresponding to the mean motions in anomaly of Venus and Mercury (not affected by precession) are practically identical, although the same cannot be said of the mean lunar motion in anomaly. The mean motions in longitude of the Sun, Moon and planets show similar differences which vary between $0;0,0,7,31^o$ and $0;0,0,8,34^o$ (table 7.35). As we have already seen that Ibn Ishāq’s mean motion parameters seem to derive from those of the *Toledan Tables* (7.3.3.1), we might think that the source used by Alfonsine astronomers was the latter *zīj*. I explored this possibility and

obtained much poorer results for the differences in the mean motion in anomaly of Venus (0;0,0,2,3,29,58°) and Mercury (0;0,0,0,3,9,26°).

TABLE 7.35

Planet	Mean motion (Alf.)	Mean motion (Ish.)	Alf – Ish
Sun	0;59,8,19,37,19,13,56°	0;59,8,11,28,26,22,5°	0;0,0,8,8,52,51,51°
Moon long.	13;10,35,1,15,11,4,35	13;10,34,52,46,51,38	0;0,0,8,28,19,26,35
Moon anom.	13;3,53,57,30,21,4,13	13;3,53,56,17,52,4	0;0,0,1,12,29,0,13
Node	0;3,10,38,7,14,49,10	0;3,10,46,41,0,4	-0;0,0,8,33,45,14,50
Saturn	0;2,0,35,17,40,21	0;2,0,27,46,42,42	0;0,0,7,30,57,39
Jupiter	0;4,59,15,27,7,24	0;4,59,7,36,25,11	0;0,0,7,50,42,13
Mars	0;3,1,26,38,40,5	0;3,1,26,31,9,5	0;0,0,7,31
Venus anom.	0;36,59,27,23,59,31	0;36,59,27,23,59,25	0;0,0,0,0,0,6
Mercury anom.	3;6,24,7,42,40,52	3;6,24,7,42,40,49	0;0,0,0,0,0,3

It seems, therefore, that the Alfonsine mean motions derive from the tradition of Ibn Ishāq which is also the source of the declination table (7.6.2.2.1.4). This is, in my opinion, extremely important, because it is hard to imagine that Ibn Ishāq’s *zīj* was available to the Parisian astronomers and it shows clearly the Alfonsine origin of the mean motion parameters. The correction of, approximately, 0;0,0,8° for the mean motions in longitude corresponds to the value of precession which allows the conversion of mean sidereal motions into mean tropical motions. We can easily compute this value:

According to the *PAT*, precession 0° was attained on 16.5.16 (JDN 1,727,038) (see 6.2.7.2). As the Alfonsine epoch used in the Latin *AT* is on 31.5.1252 (JDN 2,178,502):

$$2,178,502 - 1,727,038 = 451,464 \text{ days}$$

- 1) Constant precession: 0;0,0,4,20,41,17,12°/ day

$$0;0,0,4,20,41,17,12^{\circ} \cdot 451,464 = 9;4,52^{\circ}$$

- 2) Periodic term: the position of the head of Aries on 31.5.1252 is:

$$0;0,0,30,24,49,0^{\circ} \cdot 451,464 = 63;34,4^{\circ}$$

$$\Delta\lambda = \arcsin(\sin 63;34,4 \cdot \sin 9^\circ) = 8;3,9^\circ$$

($\Delta\lambda$ being the increase in longitude due to the trepidation term)

- 3) Addition of the two terms:

$$\Delta\lambda' = 9;4,52^\circ + 8;3,9^\circ = 17;8,1^\circ$$

(where $\Delta\lambda'$ is the full increase in precession between the two dates considered)⁶²⁶

- 4) Daily increase of $\Delta\lambda'$:

$$17;8,1^\circ : 451,464 = 0;0,0,8,12^\circ 627$$

A mean solar motion per day of $0;59,8,19,37,19,13,56^\circ$ yields a tropical solar year of $365;14,33,9,57,4,26,5\dots^d$, equivalent to $365^d 5;49,15,58,49,46,26\dots^h$. John of Murs gives a tropical year of $365;14,33,9,59,20,7,30^d 628$ and Copernicus, in the *Commentariolus*, uses a solar year of $365^d 5;49^h$ which Swerdlow considers to be a rounding of the Alfonsine value.⁶²⁹ The tables for the *revolutio anni*, contained in the *editio princeps* of the *PAT* and not included in Poulle's edition, give a series of approximations to the length of the tropical year, of which the most accurate is found in the *Tabula reuolutionis mensium*. It is interesting to see that these tables contain values of the *revolutio anni* close to $87;21^\circ$, attributed by Abraham ibn 'Ezra to Ibn al-Zarqālluh (6.3.3.1).

- The *Tabula reuolutionis annorum in suis fractionibus* gives, for one year, a length of $365^d 5;49,15,43^h$. From the tabular value for 16 years ($24 \cdot 3 + 21;8,11,39^h$) I obtain a year of $365^d 5;49,15,43,41,15^h$.
- In the *Tabula reuolutionis ascendentium annorum*, the value for one year is $87;18,55,55^\circ$ and, from the value for 16 years ($360 \cdot 3 + 317;2,54,58^\circ$), I obtain a *reuolutio anni* of $87;18,55,55,30^\circ$ and a year of $365^d 5;49,15,43,42^h$.
- The *Tabula reuolutionis mensium* gives, directly, a year of $365^d 5;49,15,59,34,3^h$. The corresponding *reuolutio anni* will be $87;18,55,53,30,45^\circ$.

626 As we have seen in 6.2.7.2, $17;8^\circ$ is the amount by which the longitudes of stars in Ptolemy's *Almagest* have been increased in the four *Libros de la Ochava Espera*, as well as in the star catalogue of the *PAT*.

627 Part of the information appearing in this section has been published in Samsó, 2019.

628 Goldstein & Chabás, 2001.

629 Swerdlow, 1977, pp. 204, 206–208. Swerdlow, 1974, considers the Alfonsine parameter to be the origin of the length of the Gregorian civil year.

We can finally use the lunar and the mean lunar and solar motion to obtain the length of the mean synodic month, which will be 29;31,50,7,37,27...^d (=29^d 12;44,3,2,58,51^h). The same value is also found in John of Murs' tables for 1321 (29^d 12;44,3,3^h)⁶³⁰ and is not far from the one used by John Vimond (29;31,50,7,44,35^d = 29^d 12;44,3,6^h).⁶³¹

7.6.2.2.2.4 Mean Positions at Epoch and Apogees
The Latin *PAT* give mean positions for the ten different epochs⁶³² mentioned in 7.6.2.2.2.2. In order to check whether these mean positions are also derived from Ibn Ishāq's *zīj*, I have compared the corresponding mean positions for the Hijra. The negative results are shown in table 7.36:

TABLE 7.36 Mean positions at Hijra in the *AT* and in Ibn Ishāq's *zīj*

	Radix Alf.	Radix Ibn Ishāq	Dif. Alf – Ish
Sun	114;52,2,0,51	113;21,14	1;30,48,0,51
Moon (long.)	122;1,16,23,53	120;32,32	1;28,44,23,53
Moon (anom.)	107;21,27,42,28	108;8,39	-0;47,11,17,32
Node	233;20,35,21	234;19,37	-0;59,1,39
Saturn	118;21,0,3	116;17,15	2;3,45,3
Jupiter	331;43,9,52	330;11	1;32,9,52
Mars	212;42,3,32	210;37,25	2;4,38,32
Venus (anom.)	47;41,17,16	44;30	3;11,17,16
Mercury (anom.)	73;26,14,31	73;25,29	0;0,45,31

Similar negative results are to be found for the positions of apogees at Hijra. Those of the *PAT* are given, in the Latin version, *sine motu octave spere*, which means that they only include the term of constant precession, but not the trepidational one. As for Ibn Ishāq's tradition, we should consider that the Tunisian recension and Ibn al-Raqqām's *Shāmil* share the same values for the superior planets (7.3.3.3.2), while Ibn al-Bannā's *Minhāj* and Ibn al-Raqqām's *Qawīm*

630 Chabás & Goldstein, 2009 a, p. 305.
631 Chabás & Goldstein, 2004, p. 214.
632 Pouille, 1984, pp. 124–126.

and *Mustawfī* show major differences which, as we have seen, are due to their derivation from al-Battānī’s positions (7.3.3.3). I have obtained similar negative results by a comparison of the Alfonsine apogees with those of the *Toledan Tables* and Ibn al-Kammād’s *Muqtabas*.

TABLE 7.37 Apogees at Hijra in the *PAT* and in Ibn Ishāq’s *zīj*

Planet	Latin <i>AT</i>	Tun. rec. & <i>Shāmīl</i> / <i>Qawīm</i> & <i>Mustawfī</i>	Diff. Alf-Ishāq
Sun	75;59,21°	76;44,17°	−0;44.56°
Saturn	237;57,40,58	237;44,17°/239;43°	0;13,23,58/−1;45,59,2
Jupiter	158;10,58,58	157;44,17°/159;43°	0;26,41,58/−1;32,1,2
Mars	119;46,11,58	119;54,17°/122;13°	−0;8,5,2/−2;26,48,2
Venus	75;59,21	76;44,17°	−0;44.56°
Mercury	195;13,31,58	198;24,17°	−3;10,45,2

One should bear in mind, however, that both the Latin *PAT* and Ibn Ishāq’s tradition (represented here by the Tunisian recension and Ibn al-Raqqām’s *Shāmīl zīj*) keep the planetary apogees at a fixed distance from the solar apogee, as they are all affected by the double-term precession, or, in the case of Ibn Ishāq, by the motion of the solar apogee and the trepidational term. Interestingly, John Vimond’s tables (1320) take into consideration a motion of the solar apogee of 0;0,0,11,35°/day (Ibn al-Zarqālluh 0;0,0,2,7,10,38°), a value that includes precession.⁶³³ Chabás & Goldstein suggest the use of Battānī’s precessional constant, 1° in 66 years, which is approximately equivalent to 0;0,0,9°/day, and 0;0,0,9° + 0;0,0,2° = 0;0,0,11°. This motion affects the planetary apogees, as in Ibn Ishāq’s tradition (see 6.3.2). All this could suggest that the original *AT* accepted Ibn al-Zarqālluh’s theory of the motion of the solar apogee, which was also shared by the planetary apogees.

The distances of the planetary apogees from the solar apogee in the Latin *PAT* are very near those mentioned by the tradition of Ibn Ishāq. John Vimond’s values are the result of rounding the differences between the planetary apogees and the solar apogee in Alfonso’s epoch.

633 Chabás & Goldstein, 2004, pp. 221, 232–3.

TABLE 7.38 Distance between the planetary apogees and the solar apogee

	Ibn Ishāq	<i>PAT</i>	John Vimond
Saturn	161°	161;58,20°	162°
Jupiter	81°	82;11,38°	82°
Mars	43;10°	43;46,51°	44°
Mercury	121;40°	119;14,11°	119°

Besides all this, we should also consider that the Alfonsine position of the solar apogee at the beginning of the Alfonsine era (88;40,6°)⁶³⁴ is not far away from the value we obtain with Ibn al-Zarqālluh’s determination of its longitude (85;49°, see 6.3.2) in 467H/ 1074–75, by correcting this position with the increase in longitude due to the motion of the solar apogee (0;0,0,2,7,10,38°/ day) + the Alfonsine constant precession (0;0,0,4,20,41,17,12°/ day) + the trepidational term. With this assumption I obtain:

85;49° + 0;38,9° (motion of the apogee) + 1;18,12° (constant Alfonsine precession) + 0;44,24° (trepidational motion) = 88;29,45°, not far from the Alfonsine 88;40,6°.⁶³⁵

7.6.2.2.2.5 Solar, Lunar and Planetary Equations

The tables of equations in the Latin *PAT* derive mainly from the tradition of the *Handy Tables*, represented here by al-Battānī’s *zīj* and the *Toledan Tables*. There are, however, three exceptions to this rule: a maximum solar equation of 2;10°, and maximum equations of the lunar anomaly of 4;56° and 5;57° for Jupiter’s maximum equation of the centre. These three values are well documented in the Parisian Alfonsine tradition.

I do not know the origin of Jupiter’s parameter for the maximum equation of the centre which is different from al-Battānī’s 5;15°, or from the 5;41° used by Ibn Ishāq’s school (7.3.3.4). The other two deserve some comment.

634 80;37,0° + 8;3,6° = 88;40,6° (Poulle, 1984, p. 124). 8;3,6° corresponds to the trepidation term: the table has 8;4,1°, a small mistake which appears already in John of Murs’ *Expositio* (Poulle, 1980 b, p. 261).

635 With Ibn al-Zarqālluh’s trepidation model and parameters, and an obliquity of the ecliptic 23;33°, the result is worse: 88;16,59°. Using the parameters of the *Liber de motu* or the *Toledan Tables*, I obtain 88;22,9°. In both cases to the value of the trepidation I have added 0;38,9° (Ibn al-Zarqālluh’s motion of the apogee).

The maximum equation of the lunar anomaly of $4;56^\circ$ is a well-known parameter, used as the maximum lunar equation in the *Zīj al-Shāh*, Abū Ma'shar, Māshā'allāh, Ibn Hibintā, al-Khwārizmī, and many other Eastern *zīj*es.⁶³⁶ It is also an Indian parameter, appearing in the *Khandakhādya*.⁶³⁷ In our Andalusī-Maghribī context it appears in Ibn Mu'adh (7.2.2), Ibn al-Bannā's *Minhāj* and Ibn al-Raqqām's *Qawīm* and *Shāmīl* (7.3.3.3), while $4;59,59^\circ$ is the maximum equation used in the Tunisian recension and in the *Mustawfi zīj*.⁶³⁸ Besides, as we have seen (7.6.2.2.1.2), this parameter was also used for the computation of the lunar velocity table of the *editio princeps* of the *PAT*.

The maximum solar equation of $2;10^\circ$, with a corresponding eccentricity of $2;4,10^p$, does not seem to be related to Ibn al-Zarqālluh's solar model, since, in observations made in 1074–5, the Toledan astronomer obtained a maximum solar equation of $1;52,42^\circ$ corresponding to an eccentricity of $1;58^p$ (6.3.3.2), and Ibn al-Zarqālluh's model of variable solar eccentricity predicted a decrease which would reach Alfonso's time.⁶³⁹ As a matter of fact, this value seems to be related to other values documented in early Islamic astronomy: $2;13^\circ$ and $2;14^\circ$ in the *Zīj al-Shāh*,⁶⁴⁰ and $2;14^\circ$ in al-Khwārizmī's *zīj*.⁶⁴¹ On the other hand we also know that, in the eighth century, al-Fazārī used $2;11,15^\circ$ and $2;14^\circ$, and al-Bīrūnī ascribes to the *Sindhind* tradition the use of $2;10,46,40^\circ$.⁶⁴²

This maximum solar equation of $2;10^\circ$ has been the object of some scholarly discussion. John of Murs analyses it in his *Expositio*⁶⁴³ and seems to derive this parameter from two observations of the autumnal equinox made by Ptolemy on 25.9.132 and by "Alfonso" on 16.9.1252. This interpretation has been criticised by John North,⁶⁴⁴ and also by Chabás & Goldstein.⁶⁴⁵ While Ptolemy's observation of year 132 is mentioned in the *Almagest* III,6, there is no record of the presumed Alfonsine observation at the very beginning of his reign: the solar true and mean longitude at the moment of the autumn equinox of 1252 could be one of the solar observations mentioned in the prologue to the Castilian

636 Kennedy, 1958, p. 256.

637 Kennedy & van der Waerden, 1963 p. 326.

638 See also Chabás & Goldstein, 2015 a, p. 608.

639 Chabás & Goldstein, 2012, pp. 66–67, note that $2;10^\circ$ is the mean value between the maximum ($2;29,30^\circ$) and minimum ($1;50,30^\circ$) values of the maximum solar equation in Ibn al-Kammād's tables of variable solar eccentricity.

640 Kennedy, 1958 p. 259.

641 Neugebauer, 1962, pp. 95–96.

642 Pingree, 1970 and Pingree, 1968 a.

643 Poulle, 1980 b, pp. 255 and 263.

644 North, 1996 a.

645 Goldstein & Chabás, 2001.

canons (7.6.2.1) or, more probably, the result of a computation.⁶⁴⁶ Surprisingly, John of Murs mentions Ibn al-Zarqālluh's maximum equation of $1;59,10^{\circ}$, actually the parameter of the *Toledan Tables*,⁶⁴⁷ and states that the increase in the solar equation made by the Alfonsine astronomers is due to the fact that Ibn al-Zarqālluh's motions were sidereal, while those of Alfonso are tropical.⁶⁴⁸

My impression is that $2;10^{\circ}$ was a maximum solar equation obtained by the Alfonsine astronomers after the completion of the Castilian canons in which, as we have seen (7.6.2.2.1), we find the mean and true sidereal longitude of the Sun for the date of King Alfonso's accession to the throne (31.5.1252), calculated with the *Toledan Tables*; the result is incompatible with a maximum of $2;10^{\circ}$.

One might conjecture that $2;10^{\circ}$ is the result of the Alfonsine observations of equinoxes and solstices and of the mid points between them mentioned in the prologue of the Castilian canons. There is, furthermore, strong evidence that this parameter was established before 1277: Chabás notes that the undated *Libro del astrolabio llano* reproduces a table at the end of the text giving the tropical longitudes of the Sun for every day of an unidentified common year. The solar longitudes agree with the recomputed values (using the Latin tables) for year 1278.⁶⁴⁹ I obtained similar results⁶⁵⁰ from another table found in the *Libro del cuadrante para rectificar* (dated in 1277) which could have been computed using al-Battānī's *zīj* or the *Alfonsine Tables*.

Apart from these remarks on the lunar and solar maximum equations, the rest of the equation tables are standard and seem to derive either from al-Battānī's *zīj* or from the *Toledan Tables* and, ultimately, from the Ptolemaic *Handy Tables*. Interestingly, John Vimond's tables have a different structure to the tables of the equation of planetary anomaly, which follow the model adopted by Ibn al-Bannā' (1256–1331) for Saturn and Jupiter (7.3.3.5.2.2).⁶⁵¹ A direct influence of Ibn al-Bannā' on the work of Alfonsine astronomers does not seem to be possible due to the chronology, but his method of dealing with planetary anomalies is not necessarily a result of his own original work; the method could have been a reinvention of Vimond himself.

646 Actually the Latin *PAT* yield a true solar longitude of $180;0,2^{\circ}$ and a mean longitude of $182;9,59^{\circ}$ the 15.9.1252 (astronomical date) two hours after noon.

647 As we have seen in 6.3.3.2, according to Bernard of Virdun, in observations made c. 1075, Ibn al-Zarqālluh obtained a solar eccentricity of $1;58^{\circ}$ and a maximum solar equation of $1;52,42^{\circ}$.

648 Poulle, 1980 b, p. 263: "si Arzachel talem [i.e. $2;10^{\circ}$] non ponat hoc est pro tanto quia revolutiones posuit in 8^a spera".

649 Chabás, 2003.

650 Samsó, 1987, p. 28.

651 Chabás & Goldstein, 2004, pp. 248–9.

7.6.2.2.2.6 Other Tables in the *editio princeps* of the *Alfonsine Tables*

In addition from the tables mentioned so far, I will try to add here a few comments on other tables preserved in the *princeps* edition.⁶⁵² This does not mean that I consider this edition to be a faithful representative of the original *AT*: we will not be able to make a serious attempt to reconstruct them until the whole corpus of the Alfonsine work has been revised, with special emphasis on the versions produced in Paris in the early 1320s.

- Lunar latitude: computed for a maximum of 5° . A similar table is described in the Castilian canons (chapter 21) but no parameters are mentioned.⁶⁵³ Tables of this kind and with the same maximum value are found in various manuscripts in the Alfonsine corpus.⁶⁵⁴
- Planetary latitudes: the tables agree with the description in chapter 22 of the Castilian canons.⁶⁵⁵ They derive from the *Almagest* XIII,5 through either al-Battānī⁶⁵⁶ or the *Toledan Tables*.⁶⁵⁷
- Right ascensions and equation of time: the tables could derive either from al-Battānī⁶⁵⁸ or from the *Toledan Tables*.⁶⁵⁹ The errors in the column for the equation of time for arguments 225° – 240° , 247° – 254° are easier to justify by imagining a displacement of one row when copying from the *Toledan Tables*. The maximum equation of time ($7;54^\circ$ for arguments 218° – 219°) does not coincide with the parameter mentioned in chapter 29 of the Castilian canons ($7;48^\circ$).⁶⁶⁰
- Oblique ascensions and time degrees for one seasonal hour for each of the seven climates. They derive from a set of tables of unknown origin incorporated into the *Toledan Tables* and calculated for an obliquity of $23;51,20^\circ$ or $23;51^\circ$.⁶⁶¹
- Parallax tables for the seven climates: the tables corresponding to climates I, V and VII derive from al-Battānī,⁶⁶² while those for climates II, III, IV and VI are the same as those in a set of tables of unknown origin included in

652 More information on these tables, as well as on those in other editions and manuscripts of the *AT*, will be found in Chabás, 2019.

653 Chabás & Goldstein, 2003, pp. 40–41.

654 Chabás & Goldstein, 2003, pp. 162–3.

655 Chabás & Goldstein, 2003, pp. 41–43, 163–5.

656 Nallino, 1899–1907, II, pp. 140–1.

657 Pedersen, 2002, IV, 1325.

658 Nallino, 1899–1907, II, pp. 60–64.

659 Pedersen, 2002, III, pp. 972–975.

660 Chabás & Goldstein, 2003, pp. 55, 186.

661 Toomer, 1968, pp. 42–3; Pedersen, 2002, III, pp. 1032–1070.

662 Nallino, 1899–1907, II, pp. 95–101.

the corpus of the *Toledan Tables*.⁶⁶³ The maximum horizontal parallax is 54' instead of 50.8' in al-Battānī's tradition.⁶⁶⁴ Pedersen has recomputed these tables with an obliquity of 23;51° and considers that they were calculated assuming that the Moon is in its apogee and without subtracting the solar parallax. Certain errors can be attributed to the *abjad* notation, which leads Pedersen to suggest an Arabic source for climates II and III (not for climates VI and VI).⁶⁶⁵ The information on the seven climates is compiled in table 7.39.

TABLE 7.39

Climates	Latitude <i>PAT</i>	Maximum length of daylight <i>PAT</i>	Latitude <i>Tol.</i> <i>Tables</i>	Max length <i>Tol. Tables</i>	Latitude Battānī	Max length Battānī
I	16;39°	13h			16;32°	13 ^h
II	24;15	13;28h	24	13;28		
III	30;38	14	30;48	14		
IV	36;24	14;27	36;24	14;27		
V	41;20	15			41;15	15
VI	45;24	15;30	45;53	15;28		
VII	48;40	16			48;13 (corr. 48;53)	16

- *Tabula attacium* (*al-taqwīm*) can be found in al-Battānī's *zīj*,⁶⁶⁶ Ibn al-Zarqālluh's *Almanac*,⁶⁶⁷ and the *Toledan Tables*.⁶⁶⁸ It contains three subtables: 1) with maximum 60', used for interpolation between two tables of eclipses calculated for the lunar position placed in the apogee and perigee of the epicycle, as in the *Almagest* VI,8; 2) with maximum 12' to correct the parallax when the Moon is not in the apogee of its epicycle; 3) with maximum 32' to correct the parallax when the centre of the lunar epicycle is not in the deferent's apogee.

663 II (Pedersen, 2002, IV, p. 1365), III (IV, p. 1367), IV (IV, pp. 1370–1), VI (IV, pp. 1378–9); Toomer, 1968, pp. 97, 100–1, 105 and Pedersen, 2002, IV, pp. 1361ff.
664 Pedersen, 2002, IV, pp. 1361–2.
665 Pedersen, 2002, IV, pp. 1364, 1366, 1369, 1377.
666 Nallino, 1899–1907, II, p. 89.
667 Millás, 1943–50, p. 233.
668 Pedersen, 2002, IV, p. 1440.

- Colours of solar and lunar eclipses: two small tables show the colours of solar eclipses as a function of the distance to the node (from 1° to 12°), and of lunar eclipses (as a function of the lunar latitude, the argument being minutes of latitude, with intervals of $10'$ until $60'$). The colour of lunar eclipses is a topic dealt with in chapter 35 of the Castilian canons,⁶⁶⁹ where two criteria are used: the lunar latitude (as in the Latin table) and the distance to the apogee. A rule based on the same criteria can be found in the *Toledan Tables*.⁶⁷⁰
- *Tabula proportionis augmentata per duos gradus*: arguments from 2° until 180° with intervals of 2° . The entries are the *minuta proportionalia* which increase monotonically from $0;2'$ until $60;0'$. The same table can be found in al-Khwārizmī's *zīj*,⁶⁷¹ and in the *Toledan Tables*,⁶⁷² and it derives from the *Almagest* VI,8. In some manuscripts the title is the same as in the *PAT*.
- Tables of solar and lunar eclipses: for the solar eclipse, the same table can be found in al-Khwārizmī's *zīj*⁶⁷³ and in the *Toledan Tables*.⁶⁷⁴ The tables for lunar eclipses in the apogee and perigee of the epicycle can also be found in the same two sources.⁶⁷⁵
- Table of geographical coordinates of localities: 68 entries, most of them European. The base meridian is that of the Fortunate Islands (Toledo's longitude is 11°), and not the meridian of water as in the Castilian canons (7.6.2.2.1).
- Star catalogue: as we have already seen (7.6.2.2.1) the Castilian canons describe a star table giving not only the stellar longitudes and latitudes, but also other values which are latitude-dependent, calculated for Toledo. This kind of table does not correspond to the star catalogue preserved in the *editio princeps*, as well as in many manuscripts and printed editions. Here we have Ptolemy's star catalogue in the translation of Gerard of Cremona, with the interesting characteristic that Ptolemy's star longitudes have been increased by $17;8^\circ$,⁶⁷⁶ as in the four Alfonsine books of the "Ochaua espera" (6.2.7.2).⁶⁷⁷ Most probably, this star catalogue is unrelated to the primitive core of the *AT*.

669 Chabás & Goldstein, 2003, pp. 70, 196–9.

670 Pedersen, 2002, II, pp. 652–3.

671 Suter, 1914, pp. 187–9 (tables 73–75); Neugebauer, 1962, pp. 117–8.

672 Pedersen, 2002, IV, p. 1446.

673 Suter, 1914, p. 193 (table 78).

674 Pedersen, 2002, IV, pp. 1461–2.

675 Suter, 1914, pp. 187–90 (tables 73–76); Pedersen, 2002, IV, pp. 1469–70.

676 Kunitzsch, 1986. On this parameter see 6.2.7.2.

677 Comes, 1990.

7.6.2.2.3 *Conclusions*

In 1987, Emmanuel Poulle claimed that the Latin Alfonsine Tables were totally independent of the work undertaken by the astronomers of King Alfonso x and that they had in fact been produced by Parisian astronomers (John of Lignères, John of Murs and John of Saxony) in Paris between 1320 and 1330. His hypothesis was formulated clearly, with the following words: “We must therefore definitively conclude that the Latin Alfonsine Tables were truly and totally independent of Alfonso’s original tables”.⁶⁷⁸ Since then, a great deal of research has been carried out, mainly by Chabás and Goldstein, who have systematically refuted Poulle’s theory. This rejection was supported by Noel Swerdlow, who, in his review of Chabás & Goldstein’s *The Alfonsine Tables of Toledo*,⁶⁷⁹ summarises clearly his three main reasons for accepting the Alfonsine origin of the *AT*:

1. The Parisian tables are called *Alfonsine Tables*: Poulle’s contention that they were named so in order to give these tables a distinguished pedigree cannot be accepted; according to Swerdlow, the standard practice was “to copy earlier tables and say nothing of their origin”.
2. The radix positions are given for the meridian of Toledo. If the tables had been produced in Paris, they would have used the meridian of Paris, which we find mentioned in several early recensions.
3. The variety of eras had no practical application for Parisian astronomers, who only used the Julian calendar and the Incarnation era; in contrast, they would have been useful in multicultural Castile.

As a conclusion, Swerdlow states: “Without any apparent motivation for composing such tables in Paris, in the fourteenth century, and I can see none, does it not seem, not just more likely, but certain that they were adapted from something earlier, which can only be the Alfonsine tables from Castile for the meridian of Toledo of an identical or similar form?”

I fully agree with Swerdlow’s arguments, and the only problem left is to try to establish the extent of the Parisian adaptation. When John of Murs wrote his *Expositio*, he had in front of him a set of *Alfonsine Tables* but no canons,⁶⁸⁰ and this is why the Parisian astronomers began to write canons containing instructions for the use of the tables. We should now ask about the kind of tables John of Murs used:

1. Tropical or sidereal longitudes? In principle, the evidence found in the Castilian canons, as well as the fact that the mean motions, in John

⁶⁷⁸ Poulle, 1988, p. 105.

⁶⁷⁹ Swerdlow, 2004, pp. 483–4.

⁶⁸⁰ Poulle, 1980 b, p. 250.

Vimond's tables, are sidereal, suggests that they were sidereal. This is a possibility that should not be neglected, although the existence of an Iberian Jewish astronomical tradition that favoured computing tropical positions, even for astrological purposes, leads me to suggest that Yehudah b. Mosheh and Ishāq b. Sīd could have followed the school of Abraham bar Ḥiyya and Abraham ibn ʿEzra in their preparation of the second set of *Alfonsine Tables*, after abandoning their first project based on the *Toledan Tables*, and represented by the Castilian canons. This hypothesis receives a certain support in the horoscope cast by Yehudah b. Mosheh to determine the propitious moment for beginning the Castilian translation of Ibn Abī l-Rijāl's *Al-bārī fī aḥkām al-nujūm* (3.1.2.1.1.2): the planetary positions are tropical. This attitude may also have been influenced by the Castilian translation, in the Alfonsine circle, of al-Battānī's canons,⁶⁸¹ extant in manuscript 8322 of the Arsenal Library of Paris, a royal codex produced in the Alfonsine *scriptorium*. This translation is not dated, but Laura Fernández has suggested that the translation of al-Battānī was made before 1278, the date of the completion of the Alfonsine *Libro del Saber*, which contains the book on the sundial (*Libro del reloj de la piedra de la sombra*, see 2.4.2.1.3.2) that derives from al-Battānī's description of the instrument.⁶⁸²

2. Did they contain tables for the solar equation with a maximum of $2;10^{\circ}$, lunar maximum equation of anomaly $4;56^{\circ}$ and Jupiter's maximum equation of the centre $5;57^{\circ}$? The question is easy to answer in the case of the solar equation, as Chabás has shown that this table had been used for the computation of the table of solar positions for each day of the year 1278, contained in the *Libro del astrolabio llano*. The same is probably true of the lunar table of the equation of anomaly, as a maximum of $4;56^{\circ}$ is well documented in Andalusī and Maghribī *zīj*es. However, I can see no argument in favour of Jupiter's maximum equation of the centre (7.6.2.2.2.5).
3. Were the chronological and mean motion tables structured sexagesimally? Assuming a sexagesimal structure of the chronological tables is easy, as they often appear in this way in Arabic *zīj*es. Besides this, I have shown (7.6.2.2.2.2) that the chronological note inserted in the Alfonsine *Partidas* uses the same set of eras, and the same number of days between each era and the Alfonsine era (also 31.5.1252), as in the Latin *AT*. As for the mean motion tables, their sexagesimal structure allows the use of the only unit of time (the day) which is common to all calendars and,

681 Bossong, 1978.

682 Fernández, 2013, p. 327; Fernández, 2010–11.

consequently, they seem to be better suited to multicultural Castile than for Paris (7.6.2.2.2.3).

4. Did the *AT* that reached Paris contain tables for precession with two components, constant precession + trepidation? (6.2.7.2) The Castilian canons seem to use only the trepidation term, with tables similar to those of the *Liber de motu*, although North has suggested that the mean motions of the original tables may have included the precessional term.⁶⁸³ I have no serious arguments in support of this point, although the fact that a combination of precession + trepidation was mentioned by al-Battānī, Ibn al-Zarqālluh and al-Bīṭrūjī seems to support an Iberian origin. Besides, the use of a period of 7,000 years for the revolution of the head of Aries (the argument of the trepidational term) and another period of 49,000 years for constant precession, both periods apparently corresponding to the sabbatical (7 years) and jubilee (49 years) cycles could, perhaps, be related to the fact that the authors of the tables were Jews.

My hypothesis is, therefore, that there were two different versions of the original *AT*. The first one, represented by the Castilian canons, followed the model of the *Toledan Tables*, and we do not know whether the corresponding numerical tables were ever compiled. The second recension corresponds to the Latin tables which, apparently, reached Paris without canons.

We do not know whether this second recension was written in Latin or in Castilian. If it was written in Castilian, this could explain why the corresponding canons disappeared: the copyists, who were unable to understand the canons, simply omitted them. On the other hand, the use of Latin in scientific texts is not uncommon in the Alfonsine circle, since Latin retranslations of Alfonsine works were made by Pietro de Reggio and Egidio Tebaldi of Parma, who arrived at Alfonso's chancery ca. 1271.⁶⁸⁴ A Latin version of the *AT* could be the result of Alfonso's cultural policy at the time when he sought to become emperor of the Holy Empire (1256–1275).

A Castilian text is, however, more probable. Ms. Paris Arsenal 8322 contains the Castilian translations of al-Battānī's *zīj* and Ibn al-Zarqālluh's *Almanac*, as well as the Alfonsine booklet on the *Quadrante sennero* (4.4). This manuscript seems to be incomplete, with around a hundred folios missing, and Laura Fernández has suggested that the missing pages may have contained the *AT*.⁶⁸⁵

683 North, 1996 a, p. 462.

684 Procter (1945 pp. 25–6) has suggested that the arrival of these two collaborators, as well as two other Italians, was related to Alfonso's attempts to become emperor. Laura Fernández (2010–11, p. 257) remarks that, in the printed editions (with the sole exception of the edition of 1524) the king is called "romanorum regis et castelle".

685 Fernández, 2013, pp. 326–9.

This hypothesis is, however, unrelated to the arrival of the tables in Paris, because the Arsenal manuscript was in the library of King Manuel I of Portugal, in 1496, and did not reach France until after 1701, where it is recorded in the library of the Augustinian monks of Lyon.⁶⁸⁶ Whatever the case, it is clear that an illuminated manuscript of the *AT* was in Seville in the sixteenth century, in the library of Gonzalo Argote de Molina (1548–1596).⁶⁸⁷

As for the *AT*'s arrival in Paris, there is little information. We know that many embassies were sent by the King of Castile to France and that, in one of them (1280), Pietro de Reggio was one of its members; he may have introduced Alfonsine astronomical materials.⁶⁸⁸ Both John North⁶⁸⁹ and C. Philipp E. Nothaft⁶⁹⁰ have drawn attention to passage of the *Expositio tabularum Alfonsii* of 1347, in which the anonymous⁶⁹¹ author writes:

a book on the fixed stars written in Spanish, which contains the *radices* of the fixed stars in the same manner as the tables. This book was taken from King Alfonso's book cabinet [*de armario regis Alfontiū*], as I was told by the person who administered its removal. I have also seen the fixed stars placed in this manner on a solid sphere made for Alfonso himself.

It seems, therefore, that even the Castilian scientific works reached France. It is also known that Charles V of France (r. 1364–1380), also called "Le Sage", had a beautifully illuminated French translation of the *Alfonsine Tables*.⁶⁹²

7.6.3 *Pedro IV of Aragon and the Tables of Barcelona*

7.6.3.1 Introduction

In 6.2.7.3 I already mentioned the treatment of precession and trepidation in the Tables of Barcelona patronised by King Pedro IV of Aragon (1336–1387). It seems clear to me, that King Pedro was trying to imitate the Alfonsine cultural model; this required, among other things, the production of another

686 Fernández, 2013, pp. 333–334.

687 According to the inventory of his library in ms. Madrid NL 18,554: "Las Tablas Alfonsies originales que mando escreuir don Alonso el sabio escriptas en pergamino y luminadas". See Fernández, 2013, pp. 334–5.

688 Fernández, 2010–11, p. 254.

689 North, 1977, pp. 289–90.

690 Nothaft, 2015 a.

691 Usually identified with Geoffrey de Meaux (fl. 1310–ca. 1348).

692 Fernández, 2013, pp. 341–2. As Laura Fernández remarks to me (private communication) there is evidence of the existence, in Charles V's library in the Louvre, of two original Alfonsine manuscripts: the *Libro de astromagia* and *El libro de las formas et las ymagenes*. This second book was also translated into French.

set of astronomical tables adapted to the second half of the fourteenth century and calculated for the geographical coordinates of Barcelona,⁶⁹³ just as the Alfonsine Tables were calculated for the coordinates of Toledo. The *AT* are mentioned in the prologue to Pedro's *Almanac*, also written by order of the king,⁶⁹⁴ and it is well known that the royal library had a copy of the *AT*, translated into Catalan in Majorca,⁶⁹⁵ according to the inventory of the library compiled after the death of King Martin the Human in 1410. In spite of this, the influence of the *AT* in the Barcelona tables is limited to the correction in precession of the mean planetary motion using two terms: a constant precession of about 50" per year, and a trepidational term with a maximum of 9;59° (6.2.7.3). Chabás's detailed study has only been able to add one more item of influence in the tables of the colours of solar and lunar eclipses.⁶⁹⁶

The king's astronomers were obviously aware of the fact that a set of astronomical tables was only valid for a limited period of time,⁶⁹⁷ and this was the reason for the computation of the new tables. Three astronomers were involved in the process which began with *magister* Petrus Gileberth (= Pere Gilbert, d. 1362) and his disciple *magister* Dalmacius Planes (= Dalmau ses Planes, d. 1383), who undertook a programme of observations between 1360 and 1366 of the true positions of planets and fixed stars, after building 13 observational instruments, some of which must have been of a considerable size.⁶⁹⁸ The first result of these observations was, according to the prologue to the

693 Longitude 33° (from the meridian of water) and latitude 41°: Millás, 1962, pp. 124, 142–3.

694 "Et idem fecit fieri istis temporibus novissimis rex Castelle Alphonsus, unde et eius tabule Alphonsine adhuc apellantur": Millás, 1962, p. 90.

695 Chabás, 2004, p. 509.

696 Chabás, 1996 a, pp. 512–14 and 522.

697 "Et nos invenimus multorum philosophorum in ista scientia libros infinitos compositos qui assignaverunt loca planetarum et stellarum secundum quod eis apparuerunt in tempore eorum, et propter multitudinem temporis quod est intra eos et nos invenimus aliquam diversitatem et permutationem oculatam et manifestum ad oculum": Millás, 1962, p. 142.

698 "Quiquidem mandatum regum exequentes de libris veterum non confisi, cum armillis pro maximis et aliis magnis, multis et sumptuosis instrumentis numero 13, quarum quedam 16 cubitos et ultra in dyametrum, quedam et quedam minus habeant, per multa et magna tempora, nocte et die inspectantes diligentissime, loca vera dictorum planetarum et stellarum fixarum non sine multis laboribus et vigiliis invenerunt et per modum infra contentum notaverunt"; "Et magister Petrus predictus sollicitavit se iuxta suum posse cum instrumentis diversis factis per scientiam geometrie, sicut est instrumentum circulorum magnorum, parvorum et mediorum, et cum quadrante oculorum, et cum multis aliis ab istis sicut posuit sapiens Ptholomeus in suo libro maiori vocato *Almagesti* ...": Millás, 1962, pp. 90 and 142. As we have seen in 7.6.2.1, the same reference to Ptolemy's instruments appears in the prologue to the Castilian *AT*.

canons of the Barcelona Tables, to determine the positions of the fixed stars and the building of a celestial sphere (finished in December 1361), with a diameter of seven spans (about 1.40 metres), on which the stellar positions were recorded. Besides this, the two aforementioned astronomers compiled a work, called *Tabulas et Almanach*, of which only a long fragment of the introduction and canons is preserved.⁶⁹⁹ Apparently it contained a set of ephemerides giving the tropical longitudes of the planets for the period 1361–1433.

There is no evidence that the stellar positions appearing in several star tables, associated with the astronomers of Pedro IV, were based on observations. Chabás has reviewed all the available evidence about these star tables: a list of 30 stars giving the magnitude and ecliptic coordinates of each star, which is the same as the list appearing in Ibn al-Kammād's star list (7.2.5.3.4), a second list of 20 stars in which the coordinates are mediation and meridian altitude (for a latitude of 40°),⁷⁰⁰ and, most interestingly, a third list of 88 stars, dated in 1362, bearing the following title:⁷⁰¹

Iste stelle fuerunt correcte anno Domini 1362 in civitate Barchinone, cum 2 magnis armillis, per unum magnum astrologum quem dominus rex Aragonum pentionabat pro hoc faciendo. Iste stelle ascendunt cum istis gradibus signorum in horizonte Barchinone.

Chabás identifies this “magnus astrologus” with Dalmacius Planes, as Petrus Gileberth died in this same year (1362). For each star, the table gives the longitude of the point of the ecliptic that rises, in the horizon of Barcelona, together with the star, as well as its magnitude. Chabás has shown that the coordinates of these stars do not derive from observations, because, for example, the star Suhayl (Canopus, α Car) is not visible in Barcelona and the recomputation of the rising degree of the ecliptic, for a sample of ten stars, gives good results using an obliquity of 23;33° and a local latitude of 41°. A fourth list of 31 stars is also related to Ibn al-Kammād and only gives the names of the planets associated with each star.

The third astronomer who participated in the creation of the Barcelona tables was a Sevillian Jew named Jacob Corsuno, who was in Barcelona at least between 1378 and 1380, although, as we have seen (6.2.6.6), a late Maghribī source states that a certain Abū Kursūm al-Yahūdī made observations and compiled a solar table in Barcelona in 1321 (!). To this we can add that an otherwise unknown Aḥmad ibn Aḥmad ibn Salāma al-Ṣ.ṇ[...]*jānī* al-Ḥājirī made a

699 Millás, 1962, pp. 87–93.

700 Chabás, 1996 a, pp. 517–520.

701 Chabás, 2015.

copy of Ishāq ibn Ḥunayn's Arabic translation of the *Almagest* in Zaragoza in 1381. The scribe wrote it for his master and teacher Abū Ishāq Ya'qūb ibn Ishāq ibn Ya'qūb, known as Ibn al-Qursunūh al-Isrā'īlī, astronomer (*munajjim*) of King Pedro IV of Aragon.⁷⁰²

According to the introduction, Jacob Corsuno's task was to compile a set of astronomical tables giving sidereal planetary longitudes, because many Jewish and Arab scholars cast sidereal horoscopes – unlike Christian scholars, who preferred tropical horoscopes.⁷⁰³ This latter remark contains perhaps, a reference to the *Alfonsine Tables*. The result of his work were the *Tables of Barcelona*, whose canons, edited by Millás,⁷⁰⁴ were preserved in three languages (Latin, Catalan and Hebrew).

Like Alfonso X, Pedro IV has his own era which, according to the introduction, corresponds to his date of birth ("et voluimus facere istas tabulas secundum eram nativitatis nostre et annorum nostrorum latinorum in meridie omnis diei in ista civitate").⁷⁰⁵ This is quite surprising because King Pedro was born on 5 September 1319,⁷⁰⁶ while the era of the tables, as shown by Chabás, begins at midday of Saturday 28 February 1321, corresponding to the beginning of year 1321, assuming the convention that the year begins on 1 March. I have no explanation for this inconsistency. The chronological tables deal with only two eras and two calendars: the Hijra and the lunar calendar, as well as the era of King Pedro and the Julian calendar, in which the year begins in March.

7.6.3.2 Mean Motions, Radix Positions and Apogees

Far more interesting are the tables of mean motion and equation of the centre of the Sun, Moon and planets.⁷⁰⁷ As remarked by Poulle,⁷⁰⁸ the equation tables are not presented in the usual way and they offer several important formal novelties which Millás considered to have been inherited from the almanac tradition.⁷⁰⁹ Whatever the case, these tables represent an attempt to simplify the computation of longitudes which can be easily exemplified for the case of the Sun: the argument is the solar mean longitude (λ_m) and the entries give directly its true longitude (λ_v), which is the result of the algebraic addition of λ_m and the solar equation (η). In this way the user of the table can save himself the trouble of an addition or a subtraction and does not have to concern himself

⁷⁰² Kunitzsch, 2001, pp. 32–33.

⁷⁰³ Millás, 1962, p. 143.

⁷⁰⁴ Millás, 1962, pp. 101–156.

⁷⁰⁵ Millás, 1962, p. 143.

⁷⁰⁶ Tasis, 1991, p. 5; Hillgarth, 1992–93, p. 8.

⁷⁰⁷ Millás, 1962, pp. 162–189.

⁷⁰⁸ Poulle, 1966.

⁷⁰⁹ Millás, 1962, pp. 47–49.

about the positive or negative value of the equation. The same remark applies to the Moon and the planets, for which the table gives directly the result of the addition or subtraction of the mean longitude and the equation of the centre. However, the procedure restricts the attainment of reasonable results of a long period of time, when using the table, because the position of the corresponding apogee is fixed and its motion will introduce changes in the value of the centre equation.

It is interesting to see the radix positions of the celestial bodies for the era of King Pedro IV (beginning of 1321) in table 7.40, their mean motions per day calculated by Chabás (table 7.41), as well as the positions of the apogees (table 7.43).⁷¹⁰

TABLE 7.40 Planetary positions the 28.2.1321^a

Sun	335;48 ^o
Moon (long.)	343;59
Moon (anom.)	351;39
Saturn	15;50
Jupiter	293;17
Mars	9;42
Venus (anom.)	43;52
Mercury (anom.)	103;17
Ascending node	284;15

a Millás, 1962, pp. 162–3; Chabás, 1996 a, p. 483.

TABLE 7.41 Daily mean motions^a

Sun	0;59,8,19,10,13 ^o
Moon (long.)	13;10,35,1,23,16
Moon (anom.)	13;3,53,54,51,29
Saturn	0;2,0,38,26,21
Jupiter	0;4,59,13,30,40
Mars	0;31,26,40,28,51
Venus (anom.)	0;36,59,29,27,29
Mercury (anom.)	3;6,24,7,54,5
Ascending node	–0;3,10,38,55,53

a Chabás, 1996 a p. 486.

710 Chabás, 1996 a, p. 483.

As noted by Chabás, these mean motions seem tropical,⁷¹¹ unlike the radix positions which are sidereal.⁷¹² Neither of them derives from any known source. Therefore, they could be the result of Petrus Gileberth's and Dalmacius Planes' observations made in the period 1360–66. In the case of the Sun, a mean solar motion per day of 0;59,8,19,10,13^o yields a solar year of 365;14,35,57,21^d or 365^d 5;50,22,56,18^h, which is a tropical year. Some evidence about a parameter for the sidereal year can be found in the table of the *revolutio anni*,⁷¹³ which I reproduce here (table 7.42):

TABLE 7.42 *Revolutio anni*

Years	Rev. anni in hours	Rev. anni in degrees	Degrees corresponding to hours
1	6;1 ^h	90;15 ^o	–
2	12;2	180;30	–
3	18;3 (+1)	270;45	–
4	0;3	0;45	–
5	6;4	91;0	–
6	12;5	181;15 (+3')	–
7	18;6 (+1)	271;30 (+6)	–
8	0;6	1;30 (–6)	–
9	6;7	91;45 (–3)	–
10	12;8	182;0	–
20	0;16	4;0	–
30	12;24 (+1)	186;0	–
40	0;32 (+1)	8;0	–
50	12;40 (+1)	1[9]0 ^a	–
60	0;48 (+1)	12;0	–
70	12;55	1[9]3;56 ^b	193;45
80	1;3 (+1)	15;56	15;45
90	13;11 (+1)	197;56	197;45
100	1;18	19;56 (+1)	19;30

a 100 in Millás's edition.
b 103;56 in Millás's edition.

711 Chabás, 1996 a, p. 485.
712 Chabás, 1996 a, p. 484.
713 Millás, 1966, p. 238; Chabás, 1996 a, pp. 514–515.

The values of the *revolutio anni* expressed in hours and degrees are mutually consistent, with the exception of those for years 70–100, for which the values in degrees corresponding to the hours appear in the third column. Two obvious mistakes (for arguments 50 and 70) have been corrected between brackets. The first column (hours) seems to have been computed with a parameter 6;0,46,48^h/year, which yields a sidereal year of 365^d 6;0,46,48^h = 365;15,1,57^d. The second column uses three different parameters. The differences expressed in parentheses have been calculated using the corresponding parameter for each case:

- Years 1–4: 90;15°/year. Year: 365^d 6;1^h = 365;15,2,30^d
- Years 5–60: 90;12°/year. Year: 365^d 6;0,48^h = 365;15,2^d
- Years 70–100 use a more precise value: 90;11,57 (± 30ⁱⁱⁱ). Year: 365^d 6;0,47,48 = 365;15,1,59,30^d.

As for the positions of the apogees at epoch (table 7.43):

TABLE 7.43 Apogees

	Barcelona tables	Diff. with Ibn al-Kammād ^a	Diff. planetary ap. – solar ap.	Diff. planetary ap. – solar ap. in Ibn al-Kammād
Sun	79;25,55 ^{ob}	2;40,34 ^{oc}	–	
Saturn	241;23 ^{od}	2;44,30 ^o	161;57	161;53,9 ^o
Jupiter	161;3 ^{oe}	2;42 ^o	81;57	81;36,9
Mars	122;23 ^f	2;42 ^o	42;57	42;56,9
Venus	79;25 ^{og}	2;39,39 ^o	–	–
Mercury	201;3 ^{oh}	2;42 ^o	121 ;37	121;36,9

a Chabás, 1996, pp. 496–7.
b Millás, 1962, pp. 192–3; Chabás, 1996, p. 489. This value appears in ms. Ripoll 21 fol. 136v, and Chabás states that ms. Vat. heb. 356 has 79;30°.
c 2;40,34° according to the value found in ms. Ripoll 21 and in Millás’s edition; the difference will be 2;44,39° if we use the apogee longitude in ms. Vat. heb. 356.
d Millás, 1962, pp. 200–201.
e Millás, 1962, pp. 204–5.
f Millás, 1962, pp. 208–9. This value is near the one we find in al-Qusantīnī’s versified *zīj* (122;13°): see 7.2.3.
g Millás, 1962, pp. 212–13.
h 6° 21;3°. 3° 21;3° in Millás, 1962, pp. 216–7, which is the value appearing in ms. Vat. heb. 356: see Chabás, 1996, p. 496.

These apogees seem to be sidereal, although affected by the motion of the apogees (6.3.2)⁷¹⁴ and, as established by Chabás, the differences between the longitude of the apogees in the Barcelona Tables (for 28.2.1321) and those of Ibn al-Kammād (for the beginning of the Hijra) have a certain regularity.⁷¹⁵ I cannot give a convincing explanation for this more or less constant difference.⁷¹⁶ Besides, as a result of the motion of the apogees, the planetary apogees are always kept at a fixed distance from the solar apogee and we can easily see that these distances are more or less the same as those we find in Ibn al-Kammād (7.2.5.3.2), Ibn Ishāq's family of *zīj*es (7.3.3.3.2) and the *Alfonsine Tables* (7.6.2.2.4).

7.6.3.3 Equations

There is nothing original in the equations of the Barcelona Tables, which are highly dependent on Ibn al-Kammād's *Muqtabas zīj* (7.2.5.3.3). The maximum solar equation is 1;52°, instead of 1;52,44° in the *Muqtabas*, and Chabás has shown that the whole table derives from Ibn al-Kammād by truncating (not rounding) the result to minutes.⁷¹⁷ The same happens with the rest of the lunar and planetary equation tables which also derive directly from Ibn al-Kammād, and, ultimately, from the *Almagest* and the *Handy Tables*. The only exception is the maximum centre equation of Venus (1;59°, instead of the 1;52° in the case of the Sun), which derives from al-Battānī and the *Toledan Tables*.⁷¹⁸

7.6.3.4 Other Tables

Chabás has shown many other instances in which the Barcelona tables seem to depend on Ibn al-Kammād's *Muqtabas*:⁷¹⁹ for example, the declination table (calculated for a maximum of 23;33°), right ascension (in which $\epsilon = 23;35^\circ$), the two tables of lunar latitude (with maximums of 5;0° and 4;30°), equation of time, planetary stations, eclipse tables, solar and lunar velocities, parallax in

⁷¹⁴ Millás, 1962, p. 128.

⁷¹⁵ Chabás, 1996, pp. 496–7.

⁷¹⁶ The displacement of the solar and planetary apogees during 255,174 days (between the 14.7.622 and the 28.2.1321) is 2;24,58°, if we use Ibn al-Kammād's parameter: 0;0,0,2,2,42,38°/day.

⁷¹⁷ Samsó, 1987 d: in pp. 472–3 I tried to edit the solar equation table of the Barcelona *zīj*, but my recomputation is no longer valid as my hypothesis (at that time I had not seen Ibn al-Kammād's *Muqtabas*) was that the table was computed according to the “method of declinations”. The matter was definitively clarified by Chabás, 1996, pp. 489–494.

⁷¹⁸ Chabás, 1996, pp. 495–499; Chabás & Goldstein, 2015 a, pp. 605–612.

⁷¹⁹ Chabás, 1996, pp. 499–514.

longitude,⁷²⁰ and so on. Some tables, which are latitude-dependent, have been calculated for Barcelona and show anomalies: the table of equal and seasonal hours are not mutually consistent, and the same is true of the table of unequal hours and the table of oblique ascensions.

7.6.3.5 Conclusions

As we have seen, the only original part of the Barcelona Tables seems to be the set of mean motion tables and the radix positions for the Sun, Moon and planets, which could be the result of the observations made by King Pedro's astronomers between 1360 and 1366. It is, however, quite surprising to realise that mean motions are tropical, and that sidereal values are only to be found in the radix positions, in the positions of the apogees at the beginning of the king's era, and in the table of the *revolutio anni*. This seems at odds both with the king's wish to have a set of sidereal tables for astrological purposes and with the explicit statement in this sense that we find in the canons: "Et positi sunt medii motus in spera octava, in loco in quo figura et signa apparent ad oculum visibiliter".⁷²¹ It seems, therefore, that Jacob Corsuno is also following the tendency we have already found in other Jewish astronomers (Abraham bar Ḥiyya, Abraham ibn 'Ezra and, according to my hypothesis, the Alfonsine astronomers) to calculate tables giving tropical positions and to abandon the Khwārizmian tradition which was predominant both in al-Andalus and in the Maghrib.

King Pedro also boasts that the computation of planetary positions with his tables demands less work and can be done more quickly (" [Jacob Corsuno] compilavit istas tabulas quibus cum pauco labore et breviter potest habere homo quando vult scire loca planetarum").⁷²² With these words, he is obviously referring to the peculiar characteristic of the mean motion tables which give the combination of the mean planetary position and the equation of the centre directly, and save the user a certain amount of computation, although it has the obvious disadvantage of limiting the space of time during which the tables can be used, due to the displacement of the apogee.

There is very little really interesting in the rest of the tables: unlike the Latin *Alfonsine Tables*, the combination of constant precession and trepidation makes no sense in the Barcelona Tables and a large part of them are mere copies of Ibn al-Kammād's tables; they mix materials in which different numerical

720 The table of the parallax in latitude, in spite of the coincidences with the corresponding tables of the *Muqtabas*, seems to derive from an earlier source.

721 Millás, 1962, p. 145.

722 Millás, 1962, p. 143.

parameters are used, for example, for the obliquity of the ecliptic, and in which the table of oblique ascensions for the latitude of Barcelona does not agree with the number of degrees corresponding to a seasonal hour throughout the year. It seems to me that Jacob Corsuno arrived in Barcelona with a copy of the *Muqtabas zīj* or another of Ibn al-Kammād's *zīj*es, did a rushed job, and abused the king's trust.

7.6.4 *Other Iberian zījes of the Fourteenth and Fifteenth Centuries*

7.6.4.1 Introduction

Astronomical activity and the production of *zīj*es in the fourteenth century were mainly restricted to the practices of Jewish astronomers (see 1.5.3) until the pogrom of 1391. As a result, there are no important names or works that deserve mention in the first half of the fifteenth century, although, in the second half, we have two outstanding figures: Judah ben Verga (fl. Lisbon, 1455–1480) and, very especially, Abraham Zacut (Salamanca, Lisbon, the Maghrib and Jerusalem, 1452–1515). To this, I should add a set of tables in Castilian produced by Alfonso de Córdoba (b. 1458) and dedicated to Queen Isabel I of Castile.⁷²³ Many of these *zīj*es are only concerned with the motions of the Sun and the Moon in order to compute syzygies and predict eclipses,⁷²⁴ although others also deal with the planets.

A quick survey of the evidence gathered by the research of Bernard Goldstein and José Chabás shows the existence of two astronomical traditions: on the one hand, a school that computed tropical positions following Abraham bar Hiyya, Abraham ibn 'Ezra, the Alfonsine Tables and the Tables of Barcelona; and on the other a second school influenced by al-Khwārizmī's *zīj* and Ibn al-Zarqālluh, which favours the computation of sidereal longitudes.

7.6.4.2 Sidereal versus Tropical Astronomy

Sidereal *zīj*es were computed by the following astronomers:

- Juan Gil de Castiello, author of a set of Hebrew tables for Burgos with radix date 1310, which seems to derive from the Toledan Tables, although they contain a severe criticism of the tables of Ibn al-Kammād.⁷²⁵
- Joseph ibn Waqār (fl. Toledo 1357–1395), a member of a family of physicians from Seville and author of a set of tables in Arabic and for the latitude of Toledo (c. 1357, revised c. 1395 with canons in Hebrew). We have already

⁷²³ A general survey in Chabás, 2002, pp. 81–92. On Alfonso de Córdoba's tables see Chabás, 2004 a.

⁷²⁴ See for example Goldstein & Chabás, 2019a.

⁷²⁵ Goldstein, 2013, p. 176.

seen (7.6.2.2.1.4) that he knew and used the *zīj* of Ibn Ishāq, as well as those of al-Khwārizmī and Ibn al-Kammād. Besides, a medical manuscript containing astronomical and astrological notes bears witness to the survival of this tradition among several members of the Banū Waqār family living in Toledo and Guadalajara during this century.⁷²⁶

- Solomon Franco (fl. 1375), disciple of Joseph ibn Waqār and author of a *zīj* clearly derived from Ibn al-Kammād's *Muqtabas*.⁷²⁷
- Judah ben Asher II (fl. Burgos, d. 1391): his tables (*Huqqot shamayim*, Statutes of the heavens) were mentioned by Zacut. In the canons we find the date 1364 and the city of Burgos, although the epoch for mean motions is 1400; this seems to imply that not all the tables in this *zīj* are due to Judah ben Asher II.⁷²⁸
- Isaac al-Ḥadib, a disciple of Judah ben Asher II in Burgos in the 1370s, who migrated to Sicily no later than 1396. He compiled a *zīj* entitled *Oraḥ selulah* (The paved way), in which he mentioned al-Battānī, Ibn al-Kammād and, surprisingly, Ibn al-Raqqām, giving us more evidence of the knowledge of the *zīj*es of the school of Ibn Ishāq by Jewish astronomers of the fourteenth century (see 7.6.2.2.1.4 and 7.6.2.2.2.3).⁷²⁹

Several astronomers of the fourteenth century compiled **tropical *zīj*es**:

- Isaac Israeli (fl. Toledo, 1310–30) who is the author of three works containing astronomical tables: *Yesod 'Olam* (1310), *Sha'ar ha-shamayim* (The gate of the heavens, 1320) and *Sha'ar ha-millu'im* (The supplementary gate, 1330). His mean motions are near (though not identical in all cases) the Ptolemaic ones and Ptolemy is also the source used in the eclipse tables. Other influences are al-Battānī (through Abraham bar Ḥiyya) and the *Toledan Tables*. Surprisingly, although Isaac Israeli gives information about Ishāq b. Sīd's observations of eclipses in his *Yesod 'Olam*, he seems to be totally unaware of the contents of the *Alfonsine Tables*.⁷³⁰
- Jacob b. David Yomtov (= Bonjorn) (fl. 1361), an astronomer from Perpignan who was in the service of King Pedro IV of Aragon⁷³¹ and used a new lunar cycle of 31 Egyptian years, 9 days, 23 hours, 34 minutes and 11 seconds, equivalent to 383.5 synodic months (767 consecutive syzygies), later used by Zacut (see 1.5.3).

⁷²⁶ Castells, 1991; Chabas & Goldstein 1997, pp. 95–96; Chabas & Goldstein, 2012, p. 67.

⁷²⁷ Goldstein, 2004; Goldstein, 2011.

⁷²⁸ Goldstein, 1998, pp. 179–80; Chabás & Goldstein, 2000, pp. 49–50.

⁷²⁹ Goldstein & Chabás, 2006; Goldstein & Chabás, 2019a.

⁷³⁰ Goldstein & Chabás, 2017.

⁷³¹ Chabás, 1992; Chabás, 1988; Chabás, 1991.

- Immanuel ben Jacob Bonfils (fl. 1340–1365) who compiled a set of tables for the coordinates of Tarascon in Provence based on al-Battānī's *zīj*, which he seems to have known through Abraham bar Ḥiyya's tables (7.6.1).⁷³²

In spite of the fact that the *Alfonsine Tables* were known in Iberia towards the end of the fourteenth century,⁷³³ their influence (7.6.2.1) was not felt until the second half of the fifteenth century and the reintroduction of the *AT* in Salamanca by Nicolas Polonius (1460), This influence is clear in the Castilian tables with epoch 1460,⁷³⁴ and in the two Hebrew translations of the *AT*, in one of which two epochs are used: the Alfonsine era (31 May 1252) and 31 December 1472.⁷³⁵ It is also felt in the astronomical tables of Moses Farissol Botarel of Avignon, whose master, Moses b. Abraham of Nîmes, is one of the translators of the Latin *AT* into Hebrew. More or less at the same time, Zacut used the *AT* for the computation of planetary positions in his almanac (7.7.3.4), as well as in his *Tabule verificate* and in the tables for 1513. The *AT* had a very limited influence on the *zīj* of Judah ben Verga (only the mean motion of the Moon and of the lunar node seem to use an Alfonsine parameter), although they reappear in Alfonso de Córdoba's *Tabule astronomice Elisabeth Regine*.

- Moses Farissol Botarel (fl. Avignon c. 1481), who wrote commentaries on the *AT* (epoch 1368), and compiled his own set of tables (*Nofet šufim*, “Drippings of a honeycomb”), which deal only with the Sun and the Moon and combine elements from the Parisian *AT*, the tables of Levi ben Gerson, as well as the tables of Bonfils and Bonjorn. He uses the Julian calendar (epoch 31 March 1481) and signs of 30°. ⁷³⁶
- Judah ben Verga (fl. Lisbon 1455–1480), a very important astronomer who made observations and compiled a set of tables under the title of *Ḥuqot šamayim* (Ordinancies of the heaven).⁷³⁷ His mean motions are near those of the *Almagest* (Saturn, Jupiter, Mars, anomaly of Mercury), Levi ben Gerson (Sun, lunar anomaly), and we see some influence of the *AT* in his mean motions of the Moon and of the lunar node.
- Abraham Zacut (1452–1515), the most important astronomer of the fifteenth century in the Iberian Peninsula. His fame relies, above all, on his perpetual almanacs in Hebrew (*Ḥibbur*) and in Latin and Castilian (*Almanach Perpetuum*), based on the *AT* (see 7.7.3.4). Here I will limit myself to his, more

⁷³² Goldstein & Chabás, 2016.

⁷³³ Or even earlier, as the prologue of Pedro IV's *Almanac* (written before 1373) mentions the *AT*; see Millás, 1962, p. 90, line 6, and Chabás, 1992, p. 18.

⁷³⁴ Chabás & Goldstein, 2000, pp. 37–47.

⁷³⁵ Chabás & Goldstein, 2000, pp. 22–23; Chabás, 2019.

⁷³⁶ Goldstein & Chabás, 2016–17.

⁷³⁷ Goldstein, 2004; Goldstein, 2001; Langermann, 1999 a, pp. 19–25.

or less, standard astronomical tables like the *Tabule verificate*, a set of tables for the computation of eclipses which, strangely enough, use 1 January 1461 as an epoch, when Zacut was only nine years old.⁷³⁸ To this I should add that fragments of two more sets of tables are extant and we know that they were compiled somewhere in the Maghrib (Fez, Tlemcen or Tunis) in 1501, using the Julian calendar and the meridian of Salamanca, and in Jerusalem in 1513 (Jewish calendar and coordinates of Jerusalem).⁷³⁹ The 1513 tables dealt only with the computation of syzygies and solar and lunar eclipses.

- Alfonso of Córdoba (born in Seville, 1458), the *Hispalensis* mentioned by Copernicus in the *Commentariolus*, is the only non-Jewish astronomer who deserves mention.⁷⁴⁰ Between 1496 and 1503 he lived in Italy, where he was in the service of the Borgia family. At a later stage of his life he compiled the *Tabule astronomice Elisabeth Regine*, printed in Venice, 1503 and Venice, 1524. He dedicated the first edition of this work to Isabel I of Castile (1451–1504) and Fernando II of Aragon (1452–1516), and used noon 24 December 1474 as the epoch, the date of Isabel's accession to the throne. This *zīj* derives clearly from the Latin *Alfonsine Tables*.

In this section, I do not aim to analyse the contents of these *zīj*es in detail because I am unable to add anything original. My main purpose here is to establish the connections existing between them and the Iberian-Maghribī tradition which exerted, without any doubt, a major influence on this school of (mainly) Hebrew astronomical tables. A clear example is the use of the meridian of water as the prime meridian for geographical coordinates, by Bar Ḥiyya, Isaac Israeli, Judah ben Verga, the Tables of Barcelona and Zacut. Al-Battānī,⁷⁴¹ the *Toledan Tables*,⁷⁴² Ibn al-Kammād,⁷⁴³ and, in particular, the *Alfonsine*

738 Goldstein & Chabás, 2018.

739 Goldstein, 1999; Goldstein, 2013 a; Goldstein, 2009; Goldstein, 1981; Goldstein & Chabás, 2018.

740 Chabás, 2004 a.

741 Isaac Israeli's and Isaac b. al-Ḥadib's tables of the solar equation table (max. 1;59,10°) derive from al-Battānī. Isaac Israeli's tables of the lunar and planetary equations have the same origin. Immanuel ben Jacob Bonfils' tables are also based on al-Battānī.

742 The *Toledan Tables* seem to be the main source used by Juan Gil de Castiello, as well as by the compiler of the miscellaneous tables in ms. Madrid NL3349 (Coimbra c. 1339): Chabás & Goldstein, 2010. In the tables of Isaac ibn al-Ḥadib, the mean motion in lunar longitude (13;10,34,52,40,30°/d) and anomaly (13;3,53,55,56,18°/d) are very near the Toledan parameters (13;10,34,52,48,47°/d. and 13;3,53,56,17,58°/d respectively). As for Judah ben Verga, his mean motion in anomaly of Venus (0;36,59,29,44°/d) is also near the Toledan value (0;36,59,29,27,29°/d).

743 Ibn al-Kammād's influence is clear in the case of Solomon Franco, whose *zīj* reproduces the tables of the *Muqtabas*. His master, Joseph ibn Waqār, and Judah ben Asher II use Ibn al-Kammād's maximum solar equation of 1;52,44°, while Judah ben Verga rounds this

Tables are sources used by these astronomers, who seem to know them mainly through Hebrew translations or adaptations like the one made by Abraham bar Ḥiyya of al-Battānī's *zīj*. Hebrew astronomical works written in southern France by authors like Bonjorn (fl. 1361) and Levi ben Gerson (1288–1344) were also extremely influential, and Levi's highly original contributions seem to have been the main cause of the departure of some of these astronomers from the tradition I am analysing in this book. Non-Jewish authors were rarely mentioned, the main exceptions being King Alfonso X, John of Lignères or the revision of the *Alfonsine Tables* made by Nicholaus de Heybech (fl. c. 1400).⁷⁴⁴ The latter source was used in Zacut's *Tabule verificate* and in his tables compiled in Jerusalem in 1515.⁷⁴⁵ Many of these Jewish astronomers knew Arabic, and this explains Isaac Israeli's information regarding Ibn al-Zarqālluh's origins and the rest of the authors of the *Toledan Tables* (6.1), Joseph ibn Waqār's knowledge of Ibn Ishāq's *zīj* (7.6.2.2.1.4), Isaac ibn al-Ḥadib's reference to Ibn al-Raqqām,⁷⁴⁶ and Judah b. Asher 11's double entry tables of planetary velocity which are also found in Ibn 'Azzūz and in many Latin sources (7.4.1.6). Besides, it comes as quite a surprise to discover that the chronological tables in the anonymous Hebrew *zīj*, with epoch 1400, deal only with calendar conversion between the Julian and the Muslim calendar.⁷⁴⁷

7.6.4.3 Astronomical Observations

Two of the astronomers just mentioned are particularly interesting because their works contain information on their astronomical observations: Judah ibn Verga and Abraham Zacut. Ibn Verga compared Ptolemy's observation of Regulus in 132 AD (actually 139 AD), in which he obtained 122;30° for the star's longitude, with his own in 1456 (longitude 142°). The difference between the two positions is an increase in longitude, due to precession, of 19;30° in 1324 years, which yields a precessional constant of 0;0,53° per year, equivalent to a

value to 1;53°. Ibn Waqār has a table of the *revolutio anni* based on 92;25,34°, which yields a sidereal solar year of 365;15,24,15,40d and a mean solar motion per day of 0;59,8,11,20,57,0°. This parameter, as noted by Castells (1996), seems to derive from the anomalistic one in Ibn al-Kammād, 0;59,8,9,21,15 by adding to it 0;0,0,2° (see 7.2.5.3.2), which gives as a result 0;59,8,11.... The same table also appears in the notes on a medical manuscript analysed by Castells (1991). Finally, Ibn Waqār slightly modifies Ibn al-Kammād's method for calculating the time between mean and true syzygies.

744 Chabás & Goldstein, 1992; Goldstein & Chabás, 2008.

745 Goldstein, 2009; Chabás & Goldstein, 2000, pp. 23–36; Goldstein & Chabás, 2018, pp. 23–33.

746 Goldstein & Chabás, 2006.

747 Goldstein, 2003, pp. 155–6.

motion of 1° in nearly 68 years.⁷⁴⁸ Besides, Ibn Verga estimated that total precession in 1468 was about 13° , a value exceeding all the known maxima in the trepidation tables. 13° allows us, however, to obtain the year in which precession was 0° :

$$13^\circ: 0;0,53^\circ = 883 \text{ years}$$

which implies that precession 0° was attained in $1468 - 883 = 585$, not far from years 580 or 581 obtained from Ibn al-Zarqālluh, and from the *zīj*es of Ibn al-Kammād, Ibn Ishāq, Ibn al-Bannā', Ibn al-Raqqām and Ibn 'Azzūz (6.2.2.5.2.1). Zacut's year for precession 0° is 562 (6.2.7.4). This implies a certain continuity with the Andalusī-Maghribī tradition.

Ibn Verga also calculated the length of the solar year by comparing:

- His observation, using a large instrument divided at intervals of 10 minutes of arc, of the autumnal equinox on 13 September 1456 (complete) + 7 PM (which Goldstein suggests it should be 5 PM) (JDN 2,253,119).
- Ptolemy's observation of the same equinox (25 September 132 at 2 PM) (JDN 1,769,539)
- The time between the two equinoxes is: 1324 years (of 365 days) + 320 days + 3 hours (using Goldstein's correction) = 483,580d + 3h.
- As the difference in longitude between Alexandria and Lisbon is 40° , equivalent to 2;40h, he adds this to the number of hours: $3 + 2;40 = 5;40$ h which Goldstein amends, again, to 6h, obtaining 483,580d + 6h.
- He then divides $483,580\text{d} + 6\text{h}$ by 1324 solar cycles and obtains a tropical year of $365;14,30,46,13\text{d} = 365\text{d } 5;48,18,29\text{h}$, the value recorded in one of the manuscripts being $365\text{d } 5;48,18,31\text{h}$. This agrees with a *revolutio anni* of $87;4,37,20^\circ$ ($87;4,38^\circ$ in the text) and a mean daily solar motion of $0;59,8,20,1$, also given in the text.⁷⁴⁹

Abraham Zacut, for his part, states in the *Hibbur* that he observed an occultation of the star al-Simāk al-A'zal/ Spica (α Vir) by the Moon in Salamanca in 1474, both celestial bodies being near midheaven. No specific date is given but modern computation yields the night between 26/27 May 1474, shortly after 22h UT. This observation was used by him to support al-Battānī's constant of precession (1° in 66 years, see 6.2.7.4). Besides, in the *Sefer Yuḥasin*

⁷⁴⁸ Goldstein, 2001, pp. 231–2.

⁷⁴⁹ Goldstein, 2001, pp. 232–234.

(Book of genealogies), he mentions an observation of a total solar eclipse on 29 July 1478,⁷⁵⁰ which had been predicted by Judah ibn Verga.⁷⁵¹

Far more interesting is his observation of an occultation of Venus by the Moon. This occultation is only described in an Oxford ms. (Bodleian MS Opp. Add. 8° 42, dated 1489) containing the canons, but not the tables, of the *Ḥibbur*. It dates the event 24 July 1476 at 8;30h after [mean] noon in Salamanca. It corresponds to 8;42h after true noon. Modern computation establishes that the hour was 8;43h after true noon in Salamanca (21;11h UT). After the observation, Zacut used the results to try to check the validity of the *Alfonsine Tables*. His computations were not entirely successful because they led to a difference between the lunar latitude and the latitude of Venus of 1;46°, which made an occultation impossible, as the apparent size of the lunar diameter is only 0;30°; but he finally suggested that the nodal line for Venus had not moved with the motion of Venus's apogee, which allowed him to correct the latitude of Venus and to obtain a result in agreement with the observation.⁷⁵²

7.6.4.4 Trepidation and Precession

Apparently, these Jewish astronomers of the fourteenth and fifteenth centuries had reached the same conclusions as their contemporaries in the Maghrib concerning the problem of trepidation versus constant precession (6.2.6.6). Estimations of precession in their time exceeded the values computed by any one of the known trepidation tables: 13° according to Judah ibn Verga and 13;52° in Zacut's *Ḥibbur* (6.2.7.4). Similar values are also documented in the Maghribī tradition.⁷⁵³ This led these scholars to estimate a precessional constant of 1°/68 years (Ibn Verga), 1°/66 years (Zacut), 1°/67 years (Bonjorn)⁷⁵⁴ or 1°/68.5 years (Farissol). Surprisingly, Alfonso de Córdoba's *zīj* contains a star list in which the longitudes are those of the star catalogue of the *PAT* (and of the *Alfonsine Ochava Espera*) increased by 2;15°, which corresponds to a precession of 1°/100 years for the period of 222.5 years between the epochs of King Alfonso and Queen Isabel – this in spite of the fact that Alfonso de Córdoba

⁷⁵⁰ Cantera, 1931, pp. 194, 300–1; Goldstein, 1998, p. 177 and fn. 7. A total solar eclipse took place on this date at about 1 PM and the path of totality crossed central Spain. See <https://eclipse.gsfc.nasa.gov/SEsearch/SEsearchmap.php?Ecl=14780729>.

⁷⁵¹ Goldstein, 2004, p. 63.

⁷⁵² Goldstein & Chabás, 1999.

⁷⁵³ See also, for the Latin astronomical tradition since the thirteenth century: Nothaft, 2016.

⁷⁵⁴ Chabás, 1992, pp. 79, 95–97. This value seems to derive from Levi ben Gerson's estimation of precession: 1°/67 Egyptian years, 123 days and 16;40 hours: see Goldstein, 1975.

used the Alfonsine precession + trepidation model to determine the position of the apogees.⁷⁵⁵

Apart from this, the references to trepidation are scarce in this collection of *zīj*es: one of them is to be found in Solomon Franco's *zīj*, whose tables are the same as in Ibn al-Kammād's *Muqtabas*. Here, Franco states that trepidation was 0° 18 years, 4 months and 7 days before the Hijra according to Šā'id, and 42 years, 7 months and 7 days, also before the Hijra, in the *Muqtabas zīj*. The reference to Šā'id must correspond to the *Toledan Tables*, where the trepidation tables are the same as in the *Liber de motu* and, according to my own computation (6.2.1), precession 0° was attained 6,451 days before the Hijra, equivalent to 18 lunar years, 2 lunar months and 13 days.⁷⁵⁶ As for the *Muqtabas*, I obtained (6.2.2.5.2.1) 15,146 days, or 42 lunar years, 8 months and 26 days before the Hijra.

We have another reference to trepidation in an anonymous Hebrew *zīj* with epoch 1400 in which we find an equation table for trepidation with a maximum of 10°, probably derived from Ibn al-Kammād (max. 9;59°). There is no table for the computation of the mean position of the head of Aries. According to this source, the motion in access was 9;56° in 1435, 10° in 1505 (which leads Goldstein to consider that this was the date of the compilation of the *zīj* or of this particular table), and value 0° was attained in 805 (581 according to Ibn al-Kammād, see 6.2.2.5.2.1) and will reach the same value in 2205.⁷⁵⁷

7.6.4.5 Motion of the Apogees

Another interesting point is related to the motion of the solar and planetary apogees. The longitude of the solar apogee in Joseph ibn Waqār's *zīj* is 79°, and 80° in Judah ben Asher's tables; it has been suggested that these positions could derive from Ibn al-Kammād, whose sidereal position of the solar apogee is 76;45,21°. ⁷⁵⁸

The influence of Ibn al-Kammād is seen in the ascription of the same motion to the planetary apogees as that of the solar apogee (1° in 279 years, according to Ibn al-Zarqālluh or 1° in 290 years in Ibn al-Kammād, see 6.3.2). In the *Yesod*, Isaac Israeli states that the proper motion of the solar apogee is 1°/280 years, and the *Sha'ar ha-shamayim* (1320) contains a table of the tropical positions of the planetary apogees which seems to derive from Ibn al-Kammād's positions with an increase of about 12° for c. 1310. The apogee for Venus is a

⁷⁵⁵ Chabás, 2004 a, pp. 201–213.

⁷⁵⁶ Goldstein (2011, p. 79) obtains 18 years and 3 months using a mean motion of the Head of Aries of 0;5,9° per year.

⁷⁵⁷ Goldstein, 2003 pp. 158–9.

⁷⁵⁸ Chabas & Goldstein, 2012, p. 67; Goldstein, 1998, p. 179.

round number (90°) which seems to derive from Ibn al-Zarqālluh's position of the tropical solar apogee (85;49° in 1075) with an increase of 0;50,21° (motion of the apogee) + 3;20,39° (precession).⁷⁵⁹ Besides, Bonjorn seems to use Levi ben Gerson's parameter for the motion of the solar apogee, equivalent to 0;1,22,30°/year,⁷⁶⁰ a value which must be the result of adding precession (1°/67 years = 0;0,53,44°/year)⁷⁶¹ to the apogee's own motion (0;1,22,30° – 0;0,53,44° = 0;0,28,46), although the result obtained (1°/125 years) is different from that of the Zarqāllian tradition.⁷⁶²

Ibn al-Kammād's influence is patent in the anonymous Hebrew *zīj* with epoch 1400.⁷⁶³ The sidereal apogees are 2;47,24° or 2;47,54° more than Ibn al-Kammād's and this corresponds to the motion of the apogees (1°/ 277 years)⁷⁶⁴ in 778 years (778 = 1400 – 622, 622 being “the [assumed] epoch of Ibn al-Kammād's *zīj*”):

TABLE 7.44 Sidereal apogees in the anonymous Hebrew *zīj* (1400)

	Hebrew <i>zīj</i>	Ibn al-Kammād	Diff.
Saturn	241;25,24°	238;38,30°	2;47,24°
Mars	122;28,54	119;41,0	2;47,54
Sun and Venus	79;32,45	76;45,21	2;47,24
Mercury	201;8,54	198;21,0	2;47,54

Similar remarks can be made about the tropical apogees in Ibn Verga's *zīj*, in which we have only rounded values which show an increase of 10° on the also (and sometimes incorrectly) rounded values appearing in the headings for the planetary equation tables of the anonymous Hebrew *zīj*.⁷⁶⁵

759 Goldstein & Chabás, 2017, pp. 362–363.

760 Chabás, 1992, pp. 85, 87; Goldstein, 1974, pp. 94, 146; Goldstein, 1985 b, p. 113.

761 Chabás, 1992, p. 94.

762 According to Levi ben Gerson, Venus's apogee moves with the same velocity as the Sun, while the superior planets have different motions: 1° in 44 Julian years (Saturn), 1° in 60 Julian years (Jupiter) and 1° in 66 Julian years (Mars): Goldstein, 1974, p. 113.

763 Goldstein, 1998, p. 184, fn. 22; Goldstein, 2003, pp. 160–1.

764 There is a table for the motion of the apogees and the header states that they move 1° in 200 years and Goldstein (2003, p. 160) believes that this number should be replaced by 277 years.

765 Goldstein, 2001, pp. 235–6.

TABLE 7.45 Tropical apogees in Ibn Verga's *zīj*

	Ibn Verga	Kammād	Diff. with regard to K's rounded values
Saturn	252°	238;38,30°	13°
Jupiter	172	158;21,30	14
Mars	133	119;41,30	13
Sun & Venus	90	76;45,21	13
Mercury	212	198;21,30	14

Finally I should underline that these apogees, as well as those of the anonymous Hebrew *zīj*, show the standard differences in longitude with regard to the solar apogee that we found in Ibn al-Kammād (7.2.5.3.2), Ibn Ishāq's family of *zīj*es (7.3.3.3.2), the *Alfonsine Tables* (7.6.2.2.2.4) and the Tables of Barcelona (7.6.3.2):

Saturn	162°
Jupiter	82°
Mars	43°
Mercury	122°

7.7 **Almanacs and Ephemerides**

7.7.1 *Introduction*

Almanacs, as well as ephemerides for a particular year or for a number of years, appear in response to the astrologers' need to obtain the solar, lunar and planetary positions for a date and hour in order to cast a horoscope. This task, which requires a lot of computation when using a *zīj*, becomes easier when the astrol-
o-ger has access to a collection of tables that give the aforementioned positions directly and so he does not need to calculate them. Astronomical instruments like the equatoria (4.5) were designed with exactly this purpose in mind.

Calendars containing astronomical ephemerides had been developed in Eastern Islam at least since the tenth century, and their computation provided astronomers with a way to earn a living. This kind of document received the name of *taqwīm*, a word which usually refers to any kind of information in

the form of a table,⁷⁶⁶ transliterated as *tacuinum* or *attacium* in Latin, with the same meaning or, sometimes, with the meaning of “correction” (see 7.6.2.2.2.6).⁷⁶⁷ Towards 1029, al-Bīrūnī describes ephemerides of this kind based on the Persian calendar, as well as others compiled in Kashmīr using the Hindu calendar and used throughout the Indian subcontinent. He also refers to similar tables made in Khwārizm which used the Islamic, Persian or Greek calendar and gave solar, lunar and planetary positions, as well as the longitude of the ascending node for each noon of a particular locality.⁷⁶⁸

The Iberian-Maghribī tradition seems to have preferred the so-called “perpetual almanacs” which used an old Hellenistic technique of Babylonian origin, based on the use of cycles called goal-years, which are specific for each planet. These cycles are characterised by the fact that they contain an integer number of solar years during which a planet makes an integer number of synodic and zodiacal revolutions. They were known by Babylonian astronomers, Ptolemy (*Almagest* IX,3) and even by Isidore of Seville.⁷⁶⁹ These cycles, as they appear in the *Almagest*, are the following (table 7.46):

TABLE 7.46 Goal-years in the *Almagest*

Planet	Solar years	Synodical rev.	Zodiacal rev.
Saturn	59	57	2
Jupiter	71	65	6
Mars	79	37	42
Venus	8	5	8
Mercury	46	145	46

These goal-years have an obvious advantage for the astronomer because, theoretically and assuming that no great precision is required, they allow the computation of planetary positions for a whole cycle (eight years in the case of Venus), and, in the following cycles, these positions will be repeated on the same dates of the year. Continuing with the example of Venus, the longitude of this planet will be the same on 1 January of the first year of the cycle, 1 January

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Thorndike, 1928; Sarton, 1928.

767

Ibn Khaldūn (Quatremère, 1858, III, p. 107) considers *taqwīm* to be synonymous to *taʿdīl* (computation of the true position of a celestial body).

768

Bīrūnī, *Taʿfīhīm*, pp. 156–191.

769

Samsó, 1979 a.

of year 9 (the first year of the second cycle) and 1 January of year 17 (first year of the third cycle) etc. Consequently, this will be a good theoretical basis for the computation of ephemerides which will provide an almanac “valid for all times”.

7.7.2 *Yearly Ephemerides*

Did yearly ephemerides exist in the Iberian Peninsula and in the Maghrib? It seems that they did, although the evidence is scarce. The earliest (though doubtful) one is a treatise on the almanac, extant in two Latin manuscripts which also contain Abraham ibn ‘Ezra’s *De rationibus tabularum* (7.6.1), attributed by Millás to Ibn ‘Ezra himself.⁷⁷⁰ There is no edition of this Latin text and, according to Millás’s summary, this book describes the numerical tables contained in the almanac which give the positions of the Sun, Moon and five planets in two-day intervals. The reference to the Moon may be significant for establishing that this almanac is not a perpetual almanac of the kind described below, because, in the twelfth century, no lunar cycle that could be used for that purpose seems to have been known. Besides, this work does not contain any reference to the planetary cycles, with the sole exception of the solar bissextile cycle of four years. In order to compute the solar position for one year, the text states that it should be calculated for six-day intervals, and gives some rules to simplify the task of obtaining the three other years of the cycle.⁷⁷¹ In the case of the Moon, the text computes its position for three-day intervals and suggests the possibility of using al-Khwārizmī’s simplified lunar model with only one equation (7.2.1.2.1.2). For Saturn, the author seems to calculate its position once a month and adds some observations which take the planetary stations and retrogradations into account.

Another interesting reference appears in Ibn ‘Ezra’s *De rationibus*:

Sed vere sunt tabule que singulis diebus docent coequare planetas vel a tempore determinato dant rationes componendi almanac, id est tabulas per quas semel factas per totum annum planetas coequatos habebis⁷⁷²

Here, the tables which “a tempore determinato dant rationes componendi almanac” may be auxiliary tables using the “goal-years” and allowed the computation of yearly ephemerides, which correspond to the definition of an *almanac* given in the text (“tabulas per quas semel factas per totum annum

⁷⁷⁰ Millás, 1946; Samsó, 2012, pp. 191–2.

⁷⁷¹ Millás, 1946, pp. 427–8.

⁷⁷² Millás, 1947 a, p. 119.

planetas coequatos habebis"). Auxiliary tables to calculate yearly ephemerides are known in Eastern Islam from the eleventh century onwards.

A second, clearer piece of evidence has been incorporated in the canons of the *Toledan Tables*,⁷⁷³ which contain two sets of notes on the computation of planetary positions. The first one uses the Islamic calendar, which seems to imply that it derives from an Arabic original. There is no reference to the goal-year cycles and it contains a most interesting remark ("In anno, in quo almanac facere volueris") which seems to refer to the computation of a set of ephemerides for a given year. This text contains instructions for the computation of mean positions of the Sun, Moon and five planets with different intervals, and there are no references to the equations. The brief second text uses both the Islamic and the Julian calendar; there is no doubt that it refers to the compilation of ephemerides for a given year because it describes a set of tables with columns, the first of which gives the corresponding weekday of a given day of the month according to both calendars, the corresponding position for each planet, and an extra column which can be added with their latitude.

In spite of this, the manuscripts containing the *Toledan Tables* do not seem to have preserved a set of planetary ephemerides for a given year. Pedersen's edition only contains several collections of solar positions for each day of a given year, or for the beginning of each month, as well as a lunar almanac.⁷⁷⁴ Solar tables of this kind are fairly common, especially in astrolabe treatises, and we have already found them (7.6.2.2.2.5) in the Alfonsine *Libro del astrolabio llano* and *Libro del cuadrante para rectificar*. These tables were also useful for *muwaqqits* and al-Ḥabbāk's *Natā'ij al-afkār* (2.0) contains a solar table for year 1514, based on Muḥyī al-Dīn al-Maghribī's *Tāj al-azyāj*.⁷⁷⁵

Further evidence is to be found in the almanac compiled by *magister* Petrus Gileberth/ Pere Gilbert (d. 1362) and his disciple *magister* Dalmacius Planes/ Dalmau ses Planes (d. 1383) in 1360–66 (7.6.3.1), for King Pedro IV of Aragon. A long passage from the introduction to this work is extant and was edited by Thorndike⁷⁷⁶ and Millás.⁷⁷⁷ The lost tables contained a set of solar, lunar and planetary ephemerides for each year between 1361 ("a 1360 anno Domini exclusive") and 1433 ("inclusive"). The positions were tropical⁷⁷⁸ and had been

773 Pedersen, 2002, II, pp. 542–6.

774 Pedersen, 2002, IV, pp. 1607–1611.

775 Saidi, 2013, pp. 52–55.

776 Thorndike, 1937.

777 Millás, 1962, pp. 86–93.

778 The prologue to the canons of the Barcelona Tables state that the tables of the almanac gave positions of the celestial bodies in the ninth sphere (Millás, 1962, pp. 124 and 142). I do not believe one should accept the remark found in the explicit of the introduction

calculated for the meridian of Barcelona. According to this prologue, the tables contained conjunctions and oppositions of the Sun and the Moon, with the day, hour and minute of the corresponding syzygy, the true longitude of the Sun, the Moon, the five planets and the ascending node, the centre and anomaly of the Moon and the planets, as well as the lunar and planetary latitudes for each day during the aforementioned period.

7.7.3 *Perpetual Almanacs*

7.7.3.1 Ibn al-Zarqālluh's *Almanac*

The earliest extant Andalusī perpetual almanac was compiled by the Toledan astronomer Ibn al-Zarqālluh (d. 1100) using a previous source ascribed to an otherwise unknown Awmātiyūs, who is probably the same Armāniyūs mentioned by the fifteenth-century Moroccan astronomer al-Baqqār in his work on astrometeorology *Kitāb al-amṭār wa l-as'ār*.⁷⁷⁹ He has been identified with the Neoplatonic philosopher and astronomer Ammonius (b. c. 440–d. between 517 and 526), although, as we will see, the original almanac was probably written at an earlier date, and Neugebauer rejected this identification because of Ammonius' strict adherence to Ptolemy's methods.⁷⁸⁰ Ibn al-Zarqālluh's almanac is extant in Arabic, in a summarised Latin translation of the canons (where Awmātiyūs is named Humeniz) and in a Castilian Alfonsine version of the canons.⁷⁸¹

The goal-years used in Ibn al-Zarqālluh's work are Ptolemaic, with the sole exception of the cycle of Jupiter (83 solar years during which the planet makes 76 synodical revolutions and seven sidereal ones). The solar tables, based on the standard 4-year cycle, were analysed by Toomer,⁷⁸² who established that the solar maximum equation is $1;52,50^\circ$ and the solar apogee is $79;30^\circ$. $1;52,50^\circ$ is not far from the $1;52,42^\circ$ attributed to Ibn al-Zarqālluh by Bernard of Virdun (6.3.3.2) or from the $1;52,44^\circ$ of Ibn al-Kammād's *Muqtabas zīj* (6.3.3.5). As for the position of the solar apogee, Toomer has proved that it derives from the sidereal apogee of the *Toledan Tables* ($77;50^\circ$ for the beginning of the Hijra) (7.2.4.2.2.4) with an increase of $1;40^\circ$ which corresponds to the motion of the solar apogee between 622 and 1087 AD, with a velocity of $1^\circ/279$ years.

to the almanac in which I read: "Quibus 5 planetis est addenda etiam motus axi octave spere". The parameter given in this passage ($0;0,28,14^\circ/\text{month}$) corresponds to the mean motion of the Head of Aries of the Barcelona Tables, which should not be added to the sidereal position of a planet: see Millás, 1962, p. 93 and the first column of the table in p. 190.

779 Guesmi & Samsó, 2018, pp. 58–9, 206 (section [36]), 69, 211 (section [52]).

780 Neugebauer, 1975, pp. 1031, 1037.

781 The Arabic canons and the tables were edited by Millás, 1943–50, pp. 75–134.

782 Toomer, 1969, pp. 320–2.

The planetary tables were studied by Marion Boutelle⁷⁸³ who noted that the planetary models and their parameters derive from Ptolemy. The planetary apogees obtained by Boutelle can be compared to those in the *Almagest* (XI, chapters 1, 5, 7) for Ptolemy’s own time (c. 120–140 AD):

TABLE 7.47 Planetary apogees

Planet	Almanac	Almagest	Difference
Saturn	235°	233	2°
Jupiter	162	161	1
Mars	117	115;30	1;30
Venus	87	55	32
Mercury	210	190	20

As we have already seen (7.2.1.2.1.3.3), Ptolemy’s value of Mercury’s apogee was inaccurate by about 30° in his own time, and the value we find in the *Almanac* (190°) seems to be an attempt to correct this error. Besides, Venus’s apogee could be related to Ibn al-Zarqālluh’s determination of the position of the solar apogee c. 1075 (85;49°, see 6.3.2), although the use of a tropical parameter seems peculiar in a set of sidereal tables. Whatever the case, these two positions are not useful here for the comparison between Ptolemy’s apogees and those of the *Almanac*.

On the other hand, the differences in longitude corresponding to Saturn, Jupiter and Mars seem to imply that the original almanac was compiled between 100 and 200 years after Ptolemy, in the third or fourth century of the CE, assuming that Ptolemaic precession (1°/100 years) had been used to compute the position of the apogees.

Ibn al-Zarqālluh probably knew the original almanac in an Arabic translation, and Swerdlow suggested that the initial entries of the solar and planetary tables correspond to the time in which the almanac acquired its present format. These entries correspond, more or less, to 1 September 994, as we can check by comparing the longitudes for this date in the almanac with those appearing in Stahlman and Gingerich’s tables:⁷⁸⁴

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Boutelle, 1967. This article should be read together with Noel Swerdlow’s remarks in *Mathematical reviews* 41.4 (1971), no. 5149.

784

Stahlman & Gingerich, 1963.

TABLE 7.48 Positions 1.9.994

Planet	Almanac	St./Ging.	Difference
Sun	156;55°	164°	-7;5°
Saturn	342	342	0
Jupiter	122	118	+4
Mars	162	162	0
Venus	149	150	-1
Mercury	172	185	-13

It seems likely, therefore, that an Arabic translation or adaptation of the Greek original was made towards the end of the tenth century. Besides this, when Ibn al-Zarqālluh prepared his final recension of the almanac, he was well aware of the errors any user would make because, for example, he would be unable to correct in precession the positions of the apogees. This is why Ibn al-Zarqālluh adopted a new epoch (beginning of year 1400 of Alexander's era, 1 September 1088) and indicated the necessary corrections that should be introduced in the planetary positions for this year, as well as some simple rules to adjust them for dates before and after 1088. It is interesting to compare the corrected solar and planetary longitudes of the almanac with the tropical positions obtained for midday of the same date (1.9.1088) using the *Toledan Tables*, by adding 7;18° (the value of trepidation for year 1088, calculated with the tables of the *Liber de motu*, see 6.2.1) to the sidereal longitudes:

TABLE 7.49 Positions 1.9.1088

Planet	Almanac	Toledan	Difference
Sun	157;14°	164;53°	-7;39°
Saturn	57;30°	63;54	-6;14
Jupiter	91;30	101;4	-9;34
Mars	151;30	159;36	-8;6
Venus	115;30	124;23	-8;53
Mercury	167	175;51	-8;51

The obvious conclusion is that the almanac gives sidereal positions, as Swerdlow suggests; this supports the idea of a set of tables whose purpose is to

facilitate the task of practising astrologers. The differences given in table 7.49 are values comprised between $6;14^{\circ}$ and $9;34^{\circ}$ and the solar $7;39^{\circ}$ is not far from the value $7;18^{\circ}$, calculated for 1088 with the trepidation tables of the *Liber de motu*.

As I remarked in 7.2.1.2.1.2, Ibn al-Zarqālluh was not aware of the existence of a lunar cycle equivalent to the planetary goal-year periods and, in his almanac, he tries to simplify the computation of lunar longitudes using the Metonic cycle of 19 years to obtain the mean lunar motion in longitude, and a cycle of 180 years for the mean motion in anomaly. To this he adds a set of standard equations and explains the possibility of using an Indian approximate method, based on a lunar model with only one equation.

The problem attracted the attention of many Iberian astronomers who compiled tables giving the day and hour of mean or true syzygies that could be used for calendaric purposes and could also be applied in medical astrology. In 3.1.2.2.3, I mentioned the *Kalendarium* of Raymundus Bancalis (who was in Barcelona in 1326–7), which contains a table of mean luni-solar conjunctions for the period 1311–1386, based on *Toledan* parameters,⁷⁸⁵ as well as the *Lunari* compiled by Bernat de Granollachs (fl. 1418–1487) using Bonjorn's tables (7.4.1.5).⁷⁸⁶ Another instance is found in Cresques Abraham's *Catalan Atlas* (1375?) where we find a calendaric wheel giving poorly computed 'true' conjunctions of the Sun and the Moon, based on the 19-year cycle, probably for the period 1368–1386.⁷⁸⁷ As we have already seen (7.4.1.5), the final solution which allowed the use of almanac cycles for the computation of lunar position was reached towards the middle of the fourteenth century: both Ibn 'Azzūz al-Qusanṭīnī (c. 1340) and Jacob b. David Yomtov (= Bonjorn) of Perpignan (c. 1361), used a new lunar cycle of 31 Egyptian years, 9 days, 23 hours, 34 minutes and 11 seconds, equivalent to 383.5 synodic months (767 consecutive syzygies), which solved the problem and was later used in Zacut's *Almanach*.

7.7.3.2 The Almanac of Jacob b. Makhir b. Tibbon (ca. 1236–ca. 1305) and the So-Called "Almanac of Tortosa" (1307)

In the thirteenth century a very popular almanac was produced in the Languedoc by Jacob b. Makhir b. Tibbon, also called Profeit Tibbon or Profatius Iudeus, who was active in Montpellier. This scholar belonged to an important family of scientists and translators of scientific texts from Arabic into Hebrew. I mention him here for two reasons: 1) the Tibbon family had migrated to

⁷⁸⁵ Nothaft, 2015.

⁷⁸⁶ Chabás & Roca, 1985, pp. 57–59; Chabás & Roca, 1998.

⁷⁸⁷ Samsó, 2005.

southern France from the Iberian Peninsula, and their translations were based on Arabic texts brought from the other side of the Pyrenees, and 2) Montpellier was under the dominion of the Crown of Aragon during the thirteenth and the first half of the fourteenth century.

Ibn Tibbon's *Almanac* was originally written in Hebrew and translated into Latin, probably with the collaboration of the author.⁷⁸⁸ Both texts have been preserved in several manuscripts. The prologue refers to the *Toledan Tables* and mentions Ibn al-Zarqālluh as well as a certain Armenius or Ammonius (see 7.7.3.1), a disciple of "King" Ptolemy, who compiled a similar almanac some 600 years before Ibn al-Zarqālluh. The cycles used are the same as the ones used by Ibn al-Zarqālluh and include the characteristic Zarqāllian 83 years for the cycle of Jupiter. The planetary positions tabulated begin on 1 March, 1300 and, as established by Toomer⁷⁸⁹ and confirmed by Goldstein and Chabás,⁷⁹⁰ they were computed with the *Toledan Tables*. Interestingly, these positions are tropical, which implies that the Toledan longitudes are corrected in precession. This correction was made using the trepidation tables preserved in the *Liber de motu* and in the *Toledan Tables* (6.2.1), as the canons explain that the corrections to be applied to the planetary positions after the end of the first planetary cycle are only valid during the period of time in which the values of the equation of the eighth sphere are increasing, and state that this period will end ca. 1600 CE. This agrees with the trepidation tables of the *Liber de motu*, according to which the equation will change its algebraic sign ca. 1600. The lunar tables are more or less standard, although certain improvements have been made in order to facilitate the computation: the *Almanac* contains tables of the true lunar anomaly for the period between March 1300 and December 1323, and a displaced double argument table for the complete lunar equation, which can also be found in the *zīj* of Joseph ibn Waqār (1359–60, see 7.6.4.2).

Soon after Jacob b. Makhir, we find a tradition of almanacs in the Iberian Peninsula which compute sidereal positions. The first text of this kind has the *incipit* : *Incipit almanach perpetuum translatum de arabico in latinum annis Xristi 1307 imperfectis*.⁷⁹¹ This work became very popular and, on addition to the Latin version for 1307 calculated for the meridian of Tortosa, we have a second Latin version, with epoch 1391, a Catalan translation of the canons (without tables) which mentions 1306,⁷⁹² another Castilian translation also of

788 Latin text edited in Boffito & Melzi d'Eril, 1908.

789 Toomer, 1973.

790 Goldstein & Chabás, 2019.

791 Millás, 1949 a: edition of the canons in pp. 394–7.

792 Edited in Millás & Faraudo, 1948.

the canons (no tables) with references to 1306 and 1357, a Hebrew translation (only tables) also for epoch 1307, and, finally, a Portuguese version (canons and tables), whose tables were edited by Rico y Sinobas, with his own translation of the Portuguese headings of the tables in which he tried to imitate thirteenth-century Castilian and claimed that they corresponded to the original Castilian *Alfonsine Tables*.⁷⁹³ Later, in the same fourteenth century, the Archdeacon of Écija, Ferrand Martines, compiled a new version of the almanac of 1307, to which he added four chapters to the canons as well as some new tables of diverse origins, such as a table of the length of daylight for Seville (latitude 37;43° according to the text, although a better fit is obtained with a latitude of 37;15°, as in Ibn al-Kammād's geographical table). In this almanac, the epoch used is 1391.⁷⁹⁴

All these different versions use the same cycles as in Ibn al-Zarqālluh's almanac, as well as a cycle of 76 years for the mean lunar motion in longitude, another of 86 years for its mean motion in anomaly, and a third one of 93 years for the mean motion of the lunar node. The positions given in the table are sidereal: the Latin almanac for 1307 states that to obtain the solar tropical longitude 11;44,48° should be added to the almanac's value,⁷⁹⁵ while precession in the Portuguese recension amounts to 10;40° for 1321.⁷⁹⁶ The canons of the Catalan version mention a mean solar motion of 0;59,8,9,27,15°, while 0;59,8,9,21° appears in the canons of the Portuguese translation. I believe that 0;59,8,9,27,15° contains a copying mistake in the fourth sexagesimal fraction and that it should be 0;59,8,9,21,15°, which is the daily mean solar motion in Ibn al-Kammād's *Muqtabas zīj* and, as we have already seen (7.2.5.3.2), it corresponds to an anomalistic year of 365;15,36,35 days. The mean lunar motion in longitude (13;10,34,52,47° or 13;10,34,52,39°) and in anomaly (13;3,53,56,4° or 13;3,53,56,21°) are very near Ibn al-Kammād's parameters (13;10,34,52,46° and 13;3,53,56,19°). A lunar equation table with a maximum of 4;56° (an Indian parameter appearing in al-Khwarizmī's zīj, as well as in many other tables in our tradition) appears in the almanac of 1307, the Portuguese translation and the recension of Ferrand Martines. Finally, the positions of the apogees in the almanac of 1307 derive from al-Khwārizmī, with the exception of those of the Sun (79;6°) and Saturn (260;55°).

793 Rico, 1863–67, IV, pp. 187 ff.: 84 unnumbered pages in which the numbers try to imitate a certain medieval style.

794 Chabás, 1996.

795 Millás, 1949 a, pp. 389–90.

796 Millás, 1949 a, p. 392.

7.7.3.3 Almanacs in the Maghrib

In 7.4.1.5 we have already seen that Ibn ‘Azzūz al-Qusanṭīnī gives an in-depth analysis of the goal-year cycles used in Ibn al-Zarqālluh’s almanac, to which he adds a lunar cycle, later applied by Jacob b. David Yomtov (= Bonjorn) of Perpignan (c. 1361) to compute syzygies and eclipses. Apart from this evidence, and the fact that the word *al-manākh* appears in the title of three, apparently, lost works by Ibn al-Bannā’ (*al-Manākh fī ru’yat al-ahilla*, *al-Manākh li-ta’ dīl al-kawākib* and *al-Manākh* or *al-Minhāj fī tarkīb al-azyāj*),⁷⁹⁷ the best evidence of the existence of this kind of astronomical tables is found in ms. Hyderabad 298, which contains the anonymous Tunisian recension of the *zīj* of Ibn Ishāq.⁷⁹⁸

In this manuscript, tables 224–226 contain positions, approximated to one degree, of Saturn, Jupiter and Mars for the first day of each month (beginning 1 October 1581 of Alexander’s era/ 1269 AD) and for the standard cycles already used by Ibn al-Zarqālluh (59 years for Saturn, 83 for Jupiter and 79 for Mars). Tables 224–226 were clearly included in the original Tunisian recension, because they are mentioned in chapter 35/24 of the canons, in an astrological context; their main purpose is to facilitate the task of calculating planetary conjunctions (Saturn-Jupiter, or Saturn-Mars). The headings of these tables seem to imply that they derive from another work named *al-Qānūn*,⁷⁹⁹ a word which appears often in Ibn al-Zarqālluh’s almanac.⁸⁰⁰ A derivation from another source is also supported by the fact that the planetary positions are sidereal but do not seem to have been calculated with Ibn Ishāq’s parameters or with Ibn al-Zarqālluh’s almanac. It is possible that *qānūn* was a technical term used to describe this kind of table based on goal-year cycles, because in the two sources mentioned we do not find the word almanac, which is documented in al-Andalus and the Maghrib from the twelfth century onwards.⁸⁰¹

797 Jabbār & Aballāgh, 2001, pp. 119 and 198.

798 Mestres, 1996, pp. 429–30 and 1999, pp. 124–127 (of the commentary) and 198–199 of the Arabic edition.

799 *Jadwal min jadāwil al-qānūn li-ma’rifat mawāḍi‘ al-thalātha al-kawākib al-‘ubwiyya bi-taqrīb* (One of the tables of the *Qānūn* to obtain the approximate positions of the three superior planets).

800 Millás, 1943–50, pp. 76, 79, 80, 82. The reference on page 82 may be significant: *Bāb ma’rifat islāḥ al-taqrīb fī ḥarakat al-kawākib al-khamsa wa l-shams wa l-qamar li-man arāda l-‘amal bi-hādhā l-qānūn* (Chapter on how to correct approximately the motion of the five planets, the Sun and the Moon by somebody who wants to use this *qānūn*). A similar expression appears in 35/24 of the canons of the Hyderabad manuscript: *wa-qad waḍa‘tu bi-hādhā l-zīj qānūnan bi-adwār al-kawākib al-thalātha al-‘ubwiyya* (I have included in this *zīj* a canon based on the cycles of the three superior planets).

801 Steinschneider, 1888; Renaud, 1947; Dozy, 1967, II, p. 734. The word almanac seems to appear, for the first time, in Abraham ibn ‘Ezra’s *De rationibus tabularum* (7.7.2).

I give as an example the planetary positions for the first year (1 October 1269–1 September 1270), adding the difference between table values in parentheses, and my recomputation based on Ibn Ishāq’s *zīj* and for the meridian of Toledo.⁸⁰²

TABLE 7.50 Positions of the superior planets

Date	Saturn	Jupiter	Mars
1.10.1269	115° (−2°)	175° (−1°)	219° (id.)
1.11	117 (−2)	182 (−4)	241 (−1°)
1.12	116 (−3)	187 (+2)	263 (−2)
1.1.1270	115 (−2)	190 (+1)	286 (−3)
1.2	113 (−1)	192 (+2)	311 (−3)
1.3	110 (−2)	190 (+1)	334 (−1)
1.4	110 (−2)	187 (+2)	357 (−2)
1.5	111 (−2)	183 (+1)	19 (−2)
1.6.	113 (−2)	183 (+3)	41 (−3)
1.7	117 (−2)	182 (id)	62 (−2)
1.8	120 (−3)	186 (id)	82 (−3)
1.9	124 (−3)	192 (+1)	106 (+2)

7.7.3.4 Zacut’s Almanacs

7.7.3.4.1 *The Hibbur*

I have already referred to the important figure of Abraham Zacut (1452⁸⁰³–ca. 1515) in 3.1.2.2.3, 6.2.7.4 and 7.6.4, where I dealt with his contributions to astrology, his treatment of precession, and his *zīj*es. In this section I will try to say something about his two almanacs entitled *ha-Hibbur ha-gadol* (“The great composition”), and *Almanach Perpetuum*, whose tables have been thoroughly analysed by Goldstein and Chabás⁸⁰⁴ and I have very little to add in this respect.

802 I am using a computer programme (ishaqsid) prepared by Josep Casulleras.
803 It has been assumed that Zacut was born on 12 August 1452, at 3 PM, a date that appears in the *Hibbur* (Cantera, *Zacut* pp. 68, 160, 260–1, Chabás & Goldstein, 2000, p. 6) in an example of the computation of the *taḥwīl* (*revolutio*) of a nativity horoscope 25 years later, which implies that the author was writing this part of the book ca. 1452 + 25 = 1477.
804 Chabás & Goldstein, 2000.

The Hibbur is a most interesting work, extant in several Hebrew manuscripts, written by Zacut ca. 1478.⁸⁰⁵ It seems, therefore, that Zacut wrote this book when he was young. The canons were translated into Castilian in 1481 by Juan de Salaya⁸⁰⁶ with the collaboration of the author.⁸⁰⁷ Manuscript Heb. 14 of the Madrid Academia de la Historia contains the tables of the *Hibbur* with the headings in Latin and notes in Castilian.⁸⁰⁸

The main authorities quoted in the *Hibbur* are Jewish: King Alfonso X's collaborators Yehudah ben Mosheh and Ishāq b. Sīd, Maimonides, Abraham ibn 'Ezra, Jacob ben Tibbon, Judah ben Asher II, Isaac Israeli, Levi ben Gerson (1288–1344), Jacob Puel (Jacob ben David Bonjorn), Ḥayyim of Briviesca (fl. in the second half of the fourteenth century) and Judah ben Verga. Most of these authors have already been mentioned in 7.6.4.

Apparently, like many of his predecessors, Zacut was mainly concerned at the beginning with the study of astronomical problems related to the Jewish calendar,⁸⁰⁹ as well as other aspects of solar and lunar motion: positions of the two luminaries, computation of eclipses,⁸¹⁰ and the visibility of the crescent Moon, for which he follows Maimonides (2.3.4.1).⁸¹¹ This branch of interests includes his computation of a parallax table for the latitude of Salamanca,⁸¹² and his explanation of a rule of thumb to obtain the parallax for other latitudes.

805 1478 appears as the *annus presens* in a passage of the *Hibbur* (Cantera, 1931, pp. 93, 192).

806 On this translator, see Cantera, 1931, pp. 384–389. In this section I am mainly using Salaya's Castilian translation (Cantera, 1931, pp. 151–236) and Cantera's Castilian translation of a good part of the Hebrew canons (Cantera, 1931, pp. 239–369).

807 The explicit of this translation says: "Explicit Deo gratias fuit iste libellus de lingua hebraica in lingua uulgaris ispanica anno domini 1481 per magistrum de selaya ipso abraam cecuth interprete et perfecta fuit hec escriptura anno predicto in nonis decembris". Cantera, 1931, p. 236. A reference to Zacut's work for this translation seems to appear in the testament of Gonzalo de Vivero, Bishop of Salamanca, whom Zacut may have served as an astrologer. In it the bishop orders that the Jew Abraham's notebooks should be kept in a volume, together with other books of his [Abraham's or the bishop's?], which are most useful in order to understand the tables of this Jew: Cantera, 1931, pp. 76, 86.

808 Chabás & Goldstein, 2000, p. 53.

809 As well as other chronological problems also related to the Christian calendar: see Chabás & Goldstein, 2000, pp. 56–59, 76–81, 84–85.

810 Cantera, 1931, pp. 173–183, 271–280. On the corresponding tables see Chabás & Goldstein, 2000, pp. 62–63.

811 "Has de saber que nuestra intencion no era sino endereçar los logares de los luminaires para saber quando se uera la luna e para ligeramente saber quando seran los eclipses": Cantera, 1931, pp. 96, 154, 249.

812 He considers it unnecessary to compute parallax tables for other latitudes: "e non me fagas question porque non hize tablas de diuersitat de aspeçto para los otros climas ... por que los que hazen almanach non han nezesario hazer tales tablas para sus lugares porque en los libros de astrologia hallaran tablas para todos los climas" (Cantera, 1931, p. 173).

He is aware (and strongly critical) of Jacob b. Tibbon's almanac based on the *Toledan Tables*. Besides, Jacob b. Tibbon was unable to compute lunar longitudes with almanac techniques, because he was unaware of the existence of an adequate lunar cycle. Zacut used Jacob Puel's most useful lunar cycle of 31 Julian years + 2 days, equivalent to 11,325 days and to 383;29,32 mean lunar months or 766;59 mean syzygies.⁸¹³

Most of the tables of the *Hibbur* are concerned, however, with the computation of solar, lunar and planetary positions. These tables give the longitude of these celestial bodies using the same cycles as Ibn al-Zarqālluh, with the sole exception of Mercury, for which a cycle of 125 years is used. The epoch is 1473 and the positions are given for the meridian of Salamanca. Besides the tables giving true longitudes, the *Hibbur* also contains mean motion tables of the centre and anomaly for each planet, which are used as arguments to calculate their latitudes. As acknowledged by Zacut, the whole set of solar, lunar and planetary tables have been computed using the Alfonsine mean motion and equation tables and as a result give tropical positions.⁸¹⁴ He seems to have the most favourable opinion of the quality of the *Alfonsine Tables*, for which he has only one criticism: the use of trepidation tables for the computation of precession. In fact, Zacut attributes this error to Alfonso's use of trepidation to justify the changes in the velocity of the motion in precession,⁸¹⁵ which are the main cause of the adoption of trepidation by Andalusi and Maghribi astronomers.

Obviously, here Zacut is following the tradition adopted by many Jewish Iberian astronomers since the twelfth century (see 7.6.4.1), which uses tropical longitudes even for astrological purposes. As a matter of fact, Zacut's *Hibbur* often insists on the astrological applications of his tables and explains carefully how to divide the ecliptic into the twelve houses of the horoscope, using the "standard" method (3..2.3.1.1.1) and how to use the table for "domification", which gives the longitude of houses 1–6 for noon in Salamanca, and for each day of the solar year beginning in March.⁸¹⁶ It is easy to check that the longitude of midheaven ($\lambda_{10} = \lambda_4 + 180^\circ$) is the same as the solar midday longitude found in

813 Cantera, 1931, pp. 152, 243. On this topic see 7.4.1.5 and Chabás, 1992.

814 "E el comienzo en que está la cuenta en la cantidad de los tiempos de los mouimientos medios e los sus lugares e de la cantidad de la equaçion van cymentada a la opinion del sabio rey don Alonso": Cantera, 1931, pp. 152, 243.

815 "Has de saber que en esto erraron grandes honbres e sabios e metiolos en este hierro Açarquel e aun trabó desta opinion en parte el Rey don Alfonso: aunque segun las equaçiones que el haze sale en alguna manera al camino de la uerdad, ca el diçe que sienpre la 8ª ua adelante saluo que por el açeso e receso de la cabeça de Aries a las ueçes haze a la spera que no ande tanto: a las ueçes que ande mas que lo que ha de andar": Cantera, 1931, pp. 193, 297.

816 Cantera, 1931, pp. 189–190, 290–292.

the solar tables,⁸¹⁷ and, consequently, the longitudes of the beginnings of the houses are tropical, in open contradiction to Andalusī and Maghribī practice.

Despite this abandonment of the tradition just mentioned, we can easily find a few examples showing that the Western-Islamic astronomical background still exerted a certain influence. The canons of the *Ḥibbur* describe, for example, a table of ascensional differences⁸¹⁸ (7.2.1.2.1.4); precession 0° was attained in 563 AD (see 6.2.7.4) and, eight years later, on 28 March 571 there was a Saturn-Jupiter conjunction in Scorpio 7;39°, with Venus at the beginning of Capricorn.⁸¹⁹ This conjunction is known as *Qirān al-Milla al-Islāmiyya* (conjunction of the Islamic religion) and it announced the birth of the Prophet Muḥammad, Scorpio being the sign of the Arabs and Venus having an important place in the corresponding horoscope (see 3.2.2.2.2.5.1 and 3.2.5.2). I have not been able to identify the source used by Zacut, and it is clear that he is not using his own tables or the Alfonsine ones to compute the conjunction.

Another interesting item is related to the positions of the solar and planetary apogees:⁸²⁰

TABLE 7.60 Apogees in the *Ḥibbur*

	<i>Ḥibbur</i>	Alfonsine Tables	Diff.
Sun	90;56 ^{oa}	75;59°	14;57°
Saturn	252;54	237;58	14;56
Jupiter	173;7	158;11	14;56
Mars	134;42	119;46	14;56
Venus	90;56	75;59	14;57
Mercury	210;10	195;14	14;56

a The recomputation of the solar table in the *Almanach Perpetuum* yields a position of the solar apogee of 90;55,36°: Chabás & Goldstein, 2000, pp. 103–104.

Zacut’s apogees obviously derive from the *Alfonsine Tables*, but the difference in longitude (14;56°) exceeds the value of precession for 1478 (13;52°),

817 The table in the *Ḥibbur* is the same as the table printed in the *Almanach Perpetuum*. In this section I am using the facsimile edition (Lisbon, 1986, with an introduction by Luis de Albuquerque) of the volume printed in Leiria, 1496: see pp. 109–114. Cf. Chabás & Goldstein, 2000, pp. 100–102.

818 Cantera, 1931, pp. 119–192, 292–293; Chabás & Goldstein, 2000, pp. 63–64.

819 Cantera, 1931, p. 303.

820 Chabás & Goldstein, 2000, pp. 68–75.

established by Zacut himself (6.2.7.4), by $1;4^{\circ}$. Assuming a difference in time between Alfonso x and Zacut of 226 years (1478–1252, Zacut's date for the *Alfonsine Tables*) or 222 years (1478–1256, date of the *Libros de la ochava es-pera*), the implied motion of the apogees is $0;0,17^{\circ}$ /year, equivalent to 1° in 212 years. This seems to be a new parameter, not attested in the previous tradition (see 6.3.2), and different from the one which underpins the table for the motion of the apogees for all planets except the Moon.⁸²¹ In this table the entry for 30 days is $0;0,1,1,5,55,36^{\circ}$, from which I obtain $0;0,2,2,11,51,12^{\circ}$ /day and 1° in $290;22,14$ years, which are values very near those of Ibn al-Kammād (6.3.2).

These are not the only parameters for the motion of the apogees in the *Hibbur*, since in the canons we find a reference to an increase of $1'$ every two years.⁸²² In this case we seem to have a rounding of Levi ben Gerson's parameter which, as we have seen (7.6.4.5), was also used by Jacob ben David Bonjorn ($0;0,28,46^{\circ}$ /year or $1^{\circ}/125$ years). A similar value ($1^{\circ}/136$ years or $0;0,26,28^{\circ}$ /year) also appears in one of the Hebrew manuscripts of the *Hibbur*.⁸²³ It seems, therefore, that Zacut is following here the Western Islamic tradition which applies to all planets the motion of the solar apogee, although he is inconsistent in his choice of the corresponding parameter.

We can also find some echoes of Ibn al-Kammād in the values given for the maximum lunar latitude: in chapter 4 of the *Hibbur*, Zacut ascribes to "some recent scholars" a maximum value of $4;29^{\circ}$ which is considered to be "the exact latitude".⁸²⁴ This parameter appears in Yaḥyā ibn Abī Maṣṣūr's *zīj* (ca. 830) as well as in Ibn al-Kammād's *Muqtabas*,⁸²⁵ and in the Barcelona Tables, while $4;30^{\circ}$ is a common value of Indian origin, used by al-Khwārizmī (among others) and in more recent times by Levi ben Gerson,⁸²⁶ Jacob Poel⁸²⁷ and Judah ben Asher. A second value for the inclination of the lunar orbit is also found in the aforementioned chapter 4: $4;45^{\circ}$ according to other authors. Interestingly, Zacut's *Almanach Perpetuum* contains two tables of lunar latitude: one of them is standard and is computed for the Ptolemaic maximum (5°), while in the second one, intended for the computation of eclipses, the argument ranges from $-6;0^{\circ}$ to $17;0^{\circ}$, and from 163° to 186° , while the entries reach a maximum of $4;29^{\circ}$.⁸²⁸

821 Chabás & Goldstein, 2000, pp. 83–84.

822 Cantera, 1931, pp. 204, 325.

823 Cantera, 1931, p. 300, fn. 304.

824 Chabás & Goldstein, 2000, pp. 122–123; Cantera, 1931, pp. 168–169, 266.

825 Chabás & Goldstein, 1994, pp. 20, 22.

826 See for example Goldstein, 1974, p. 123.

827 Chabás, 1992, pp. 136, 143.

828 Chabás & Goldstein, 2000, pp. 130–131; *Almanach Perpetuum* (ed. Leiria, 1496) pp. 221, 223.

Finally in this regard I should mention Zacut's use of the meridian of water as prime meridian. As we have seen, this was common practice among Iberian Jewish astronomers (7.6.4.2) and our author argues that some authors measured geographical longitudes from a western island (one of the Canary islands, used by Ptolemy) and did not use the true West ("el propio ocidente", "el uerdadero ocidente"), located 17;20° (or 17;30° in another passage) west of the aforementioned island, and states that there are errors in some geographical tables because they use two different prime meridians.⁸²⁹ He says that he is going to use the true West in his table⁸³⁰ in which we find that the longitude of Toledo is 28;30° and that of Salamanca is 25;46°.

7.7.3.4.2 *The Almanach Perpetuum*

A new version of the tables of the *Ḥibbur* was printed in 1496 in Leiria (Portugal) by the printer Samuel Ortas, with the headings of the tables in Latin. These tables were accompanied by canons, entirely independent of those of the *Ḥibbur*, in a set of copies written in Latin (structured in 12 chapters), and in another set in Castilian (23 chapters). The Latin explicit of the tables states that the author of the tables was Abraham Zacut, astronomer of King Manuel I of Portugal (r. 1495–1521) but the canons were translated from Hebrew into Latin by Joseph Vizinho, disciple of the author of the tables ("cum canonibus traductis a lingua ebrayca in latinum per magistrum Joseph Vizinum discipulum eius actoris").⁸³¹ A second explicit, at the end of the Castilian canons, repeats the same idea:

Aqui se acaba la reçela [Ar. *risāla*] et las tablas tresladadas de abrayco en latin et de latin en noestro vulgar romance per mestre juseppe vezino deçipolo del actor de las tablas⁸³²

These explicits led many scholars to believe that the canons of the *Almanach Perpetuum* were a translation of those of the *Ḥibbur*, until this hypothesis was clearly rejected after a comparison of the two sources. As stated by Chabás and Goldstein, "the *Ḥibbur* and the *Almanach Perpetuum* are distinct works".⁸³³ Another inconsistency found in the printed Latin edition (not in the Castilian

829 Cantera, 1931, pp. 187, 286–287.

830 Cantera, 1931, pp. 363–369. This table contains the coordinates of 92 cities and it is found in the *Ḥibbur* and in the Latin version of the *Almanach Perpetuum*. See Chabás & Goldstein, 2000, pp. 64–65, 155.

831 Facsimile Lisbon, 1986, p. 411.

832 Facsimile Lisbon, 1986, p. 100.

833 Chabás & Goldstein, 2000, pp. 97–98.

one) is the dedication to the Bishop of Salamanca Gonzalo de Vivero (d. 1480); Chabás and Goldstein have shown that this dedication is an almost literal copy of another one found in Regiomontanus' *Tabulae directionum*, written in Esztergom in 1467 and printed in Augsburg in 1490, in which the dedicatee is the Hungarian archbishop János Vitéz (d. 1472).⁸³⁴ According to these scholars the dedication was probably added by Vizinho; this seems likely, because Zacut had probably left Salamanca as a result of the death of Gonzalo de Vivero in 1480, and found a new patron in Juan de Zúñiga y Pimentel in Gata, whom he served as an astrologer from at least 1486 onwards.⁸³⁵ He was obviously well aware of the death of the Bishop of Salamanca and a dedication to his deceased patron would have been unnecessary.

A different problem is that of establishing whether Zacut took part in the compilation of the canons of the *Almanach*. Chabás and Goldstein conclude that "we see no persuasive evidence that Zacut was involved in the edition of the latter" [*Almanach*],⁸³⁶ something which is obviously true, although a small difficulty remains: why would a Portuguese Jew like Vizinho publish a Castilian version of the canons in Portugal, instead of a Portuguese one?⁸³⁷ Zacut was in Portugal between ca. 1492 and ca. 1496 and, according to both explicit, he was Vizinho's master. Could he have been the author of the Castilian canons? Did a Hebrew version exist? In my opinion, the question remains open.

The canons of the *Almanach Perpetuum* have a practical character (simple instructions for the use of the tables) and in this respect are less interesting, from an astronomical point of view, than those of the *Hibbur*. Besides this, they lack the information contained in the *Hibbur* related to the Jewish calendar, although they explain how to compute the conjunctions and oppositions of the Sun and the Moon. Their main concern is the computation of the positions of the two luminaries and the five planets in longitude and latitude. To this, at the end of the tables the *Almanach* adds a calendar of the Christian religious festivities throughout the year, a table of Sunday letters and a table allowing to compute the movable feast days, taken from Regiomontanus and probably added by Vizinho. All this leads Chabás and Goldstein to believe that "the *Almanach Perpetuum* was intended for a Christian audience, whereas the

834 Chabás & Goldstein, 2000, pp. 90–95.

835 Villaseñor, 2013.

836 Chabás & Goldstein, 2000, p. 98.

837 Chabás & Goldstein, 2000, p. 96.

Ḥibbur for a Jewish readership”.⁸³⁸ This explains the success of the *Almanach* in Europe⁸³⁹ and, as we will see, in the Arab world as well.

As for the survival of an Andalusī-Maghribī tradition in the *Almanach*, to what I said regarding the *Ḥibbur* I can only add a remark on the table of 56 stars,⁸⁴⁰ which gives their magnitude, zodiacal sign, longitude, latitude, position north or south of the ecliptic and astrologically associated planets, but no names allowing their identification. However, the stars have been identified and their longitudes show an increase of $6;38^\circ$ on the longitudes of the star catalogue of the *Almagest*. This increase is the same we have found in the star tables of Ibn al-Kammād and Ibn al-Bannāʾ, which, in my opinion, were calculated for an epoch in which the value of precession was 0° .⁸⁴¹ An almost identical list of 61 stars appears in the *Ḥibbur* but, in this case, the longitudes are tropical (see 6.2.7.4). Both lists seem to derive from the table of 60 stars, with sidereal longitudes, compiled by Judah ben Asher II.⁸⁴²

7.7.3.4.2.1 Arabic Versions of the *Almanach Perpetuum*

7.7.3.4.2.1.1 *The Two Translations of Ms. Escorial 996*

The earliest Arabic translation of the *Almanach Perpetuum* seems to have been made by Mūsā Jālīnūs in 911/1505–6 and was dedicated to a certain ‘Abd al-Raḥmān who was *qāḍī bi l-ʿaskar*, a judge in the Ottoman army. This latter figure may have been the Kazasker Abdurrahman Mueyyezade (d. 1516). As for Mūsā Jālīnūs, he has also been identified with the Jewish physician and scientist Moses Judah Galeano,⁸⁴³ who spent most of his life in lands of the Ottoman Empire and worked at the court of the Sultan Bayazid II (1481–1512). Between 1497 and 1502 he was in Venice and, as he was knowledgeable of the astronomical works of Ibn al-Shāṭir, he has been considered one of the possible links for the transmission of the works of this Eastern astronomer to

838 Chabás & Goldstein, 2000, p. 98.

839 The *Almanach* was the object of several printed editions: Venice, Johannes Lucilius Santritter, 1498; Venice, Petrus Liechtenstein, 1502; Venice, Lucas Antonius Iunta, 1525; Venice, Petrus Liechtenstein, 1528, annotated by Alfonso de Córdoba.

840 Facsimile Lisbon, 1986, pp. 400–401.

841 Chabás & Goldstein, 2000, pp. 145–150. See also 6.2.7.4.

842 This is mentioned in the canons of the *Ḥibbur* (Cantera, 1931, pp. 194, 304). According to Zacut himself, he compiled first a table of 60 stars (of the first and second magnitudes and a few belonging to the third magnitude) although he adds that, at the end of the *Ḥibbur*, the reader will find a complete star catalogue of 1022 stars corrected in longitude for Zacut’s own time (Cantera, 1931, pp. 192, 295–296). This star catalogue does not seem to be extant.

843 On this important scientific personality, see Langermann, 2007; Langermann, 2009; Morrison, 2011; Morrison, 2016.

Renaissance Europe. He has also been identified with the maker of the spherical astrolabe of the Oxford Museum for the History of Science (4.1.2). During his stay in Venice he may have had access to one of the printed editions of the *Almanach* made in that city (Venice, 1498 or 1502).

Mūsā's translation has been preserved in ms. Escorial 996, first identified with an abridged translation of the *Almanach* by Vernet⁸⁴⁴ and recently edited and studied by María José Parra.⁸⁴⁵ In this manuscript, Mūsā appears with a *nisba* that is not easy to read, but interpreted by Vernet and Parra as al-Yatrawī (derived from the Greek *yatrós*, physician); King suggests a reading of al-Tirawī, from Tira, a town some 70 km. south-east of Izmir, in Turkey, which may have been our author's birth-place.

Parra's analysis of this work shows that Mūsā's translation of the canons is a summarised version that is nearer the *Ḥibbur* than the *Almanach*.⁸⁴⁶ It deals with solar and lunar positions, syzygies, the position of the ascending node, argument of the lunar latitude, positions and centrum of Mercury, Venus, Mars and Saturn, revolutions of the years (*taḥwīlāt*), conjunctions of Saturn and Jupiter, solar and lunar eclipses, division of the houses of the horoscope, latitude of Saturn and Venus, and corrections to apply to the positions of the five planets after the first cycle of each one. These canons are contained between fols. 1v and 12v, where we find a colophon dating the translation in 912/1506–7 and stating that the text is a translation from a “Frankish” language into Arabic (*wa qad nuqila hādhihi l-zījāt [sic]... min lisān al-ifranjīyya ilā l-'arabīyya*).⁸⁴⁷ This date agrees with the examples given in the text: a lunar eclipse which took place on 14 August 1505 (Oppolzer no. 4190), and the longitude of Mercury and Venus for this same day.

In folios 13r–18v a new text begins which contains six chapters (on conjunctions and oppositions, solar and lunar eclipses, revolution of the nativities, animodar and lunar crescent visibility) which seem to be literal translations of the Castilian version of the *Almanach* in the Leiria edition.⁸⁴⁸ Lunar crescent visibility does not appear in the *Almanach*, although, as we have seen, it is the topic of chapter 10 of the *Ḥibbur*. The numerical tables which follow are the

844 Vernet, 1950.

845 Parra, 2013.

846 Parra (2013, p. 85) notes that Mūsā uses some planetary cycles for Saturn (30 years), Mars (32 years) and Mercury (20 years) which differ from those of the *Almanach*, but appear in mss. of the *Ḥibbur*: cf. Chabás & Goldstein, 2000, pp. 68–74.

847 A similar expression appears at the beginning of the text (fol. 1r): *Kitāb al-Zīj al-mutarjam bi l-'arabīyya min al-ifranjīyya*.

848 These chapters are unrelated to al-Ḥajārī's translation, on which see below.

same as those in one of the Latin editions of the *Almanach*, and they include the star tables without providing the names of the stars.

As a conclusion, following Parra, we can say that the Escorial manuscript contains two versions of Zacut's *Almanach*: the first, dated in 1506–7 and due to Mūsā Jālīnūs, and the second one compiled at an unknown date by an unknown translator. Neither one seems to have had reached a wide readership.

On the other hand, it seems that Mūsā was probably quite unfamiliar with Iberian and Maghribī culture. He does not use the standard Andalusī and Maghribī names of the months of the Julian year, but pure transliterations of the Latin names. It is also surprising that the name of Salamanca, whose meridian is used systematically in Zacut's tables, was probably entirely unknown to him: throughout his canons he repeats that the tables are calculated for the meridian of Toledo, although the tables are exactly the same as in the printed version of the *Almanach*. In spite of this, Mūsā Jālīnūs seems to have correctly understood the use of the tables of the *Almanach*, is aware that the lunar cycle used by Zacut corresponds to a period of 31 Julian years and two days, and had information on the *Alfonsine Tables*: he states that “the Christian king of the West called Alfonso” (*al-malik al-naṣrānī al-maghribī al-musammā bi-Alfunsh*) compiled two small tables which allow the easy computation of the ascendant of any anniversary (*taḥwīl*). He is clearly referring to the tables edited under number 11 in Poulle's classification.⁸⁴⁹

7.7.3.4.2.1.2 *Al-Ḥajarī's Translation*

The Maghribī tradition is entirely different, and dates from one century later. The *Almanach* was introduced in the Maghrib via the translation by a Morisco exile called Aḥmad b. Qāsim b. Aḥmad b. al-Faqīh Qāsim b. al-Shaykh al-Ḥajarī al-Andalusī (1570–after 1640) who left Spain in 1599 and settled in Marrākush where, after 1608, he became the secretary and Castilian interpreter of Sultan Mawlāy Zaydān al-Manṣūr (d. 1627) and of his two sons, Sultans Abū Marwān ‘Abd al-Malik b. Zaydān (1627–1630) and al-Walid b. Zaydān (1630–1635).⁸⁵⁰ We know that, during his stay in Marrākush, he studied astrology with the *faqīh* Aḥmad al-Maṣyūb/ Maṣ‘ūb al-Fāsī (d. 1613).⁸⁵¹ We do not know the date of his translation of the *Almanach* although it has been suggested that he must have been working on it around 1624 due to the existence of an autograph letter

849 Poulle, 1984, p. 130.

850 Koningsveld, 1997; Manūnī, 1963–64.

851 Koningsveld, 1997, pp. 190–192; Lamrabet, 2013, M312, pp. 228–229.

addressed by him to the famous Arabist Jacob Golius in which he asks him about certain astronomical terms.⁸⁵²

Whatever the case, al-Ḥajārī's translation is extant in several manuscripts: Parra's critical edition⁸⁵³ is based on six of them: two of them produced in Yemen,⁸⁵⁴ and the other four of Maghribī origin.⁸⁵⁵ The existence of two Yemeni manuscripts is the first indication of the diffusion of Zacut's *Almanach* in the Mashriq.⁸⁵⁶ As a matter of fact there is plenty of evidence of the use of the Maghribī ms. Cairo DM 1081 in Cairo in the nineteenth century and possibly also in Yemen. The Yemeni connection of ms. Milan Ambrosiana 338, copied by a Turkish scholar in 1675 with the help of a Maghribī copyist (part of the tables are written by a Maghribī hand) is plain to see.

Unlike Mūsā Jālīnūs' summary of Zacut's *Almanach*, al-Ḥajārī's translation is an attempt to produce a faithful translation of the almanac. It seems obvious that his Arabic translation was based on the Castilian text printed in Leiria, for two reasons: on the one hand, a Morisco living in the beginning of the seventeenth century was, no doubt, knowledgeable of Castilian but it would be quite unusual for him to know Latin; on the other, manuscripts Ḥassaniyya 8184, 1433, and Cairo DM 1081 contain a translation of the colophon of the Castilian canons of the *Almanach Perpetuum*⁸⁵⁷ which does not appear in the Latin version.⁸⁵⁸ In spite of this, as Parra has shown,⁸⁵⁹ the Arabic manuscripts of al-Ḥajārī's translation contain tables which are missing in the Castilian Leiria edition. It seems, therefore, that either al-Ḥajārī himself or later copyists of his work had access to one of the Latin editions.⁸⁶⁰

852 Koningsveld, 1997, pp. 43–44.

853 Parra, 2013, pp. 306–418.

854 Ms. Milan Ambrosiana 338 (C82), described by Griffini, 1916; ms. Ar, Vatican 963, described by Levi della Vida, 1935, pp. 101–102, apparently a copy of the former, although both manuscripts contain certain materials derived from different sources.

855 Ms. Rabat Hassaniyya 1433 (Khaṭṭābī, 1983, pp. 286–287, no. 345); ms. Rabat Hassaniyya 8184 (Khaṭṭābī, *Fahāris*, pp. 284–286, no. 344); ms. Rabat Hassaniyya 1331 (Khaṭṭābī, 1983, p. 258, no. 309, p. 406, no. 494, pp. 231–232, no. 271); Cairo NL DM 1081 (King, 1981–86 vol. I, pp. 139–141; King, 1986 b, no. F31, F33, F52, pp. 140 and 143).

856 Samsó, 2002–3.

857 “Aqui se acaba la reçela [Ar. *risāla*] et las tablas tresladadas de abrayco en latin et de latin en noestro vulgar romançe per mestre juseppe vezino deçipolo del actor de las tablas”.

858 Samsó, 2007 a.

859 Parra, 2013, p. 44.

860 I know of, at least, one example of the diffusion of the Latin editions of the *Almanach Perpetuum* in the Maghrib: ms. Rabat National Library D2014 contains Aghbāl's canons (see below) followed by a copy of the edition of the *Almanach* printed in Venice 1502 (15 July) by Petrus Liechtenstein. The Latin titles and subtitles as well as the arguments

7.7.3.4.2.1.3 *Zacut's Almanach Perpetuum in the Maghrib in the Seventeenth and Eighteenth Centuries*

The diffusion of the almanac in the Maghrib is documented not only by the extant Maghribī manuscripts containing al-Ḥajārī's translations, but also by a certain number of recensions of this text dated in the seventeenth and eighteenth centuries. The existence of these recensions may be due to the fact that al-Ḥajārī's Arabic – understandably in an author who spent the first 30 years of his life in Spain at the end of the sixteenth century and the beginning of the seventeenth – was by no means faultless and he did not seem to have a thorough knowledge of Arabic technical vocabulary. Indeed, his readers had difficulty in understanding some of the procedures they needed to use to perform certain computations. Evidence of this can be found in ms. Cairo DM 1081, which contains a revised version of al-Ḥajārī's translation as stated in the colophon of the canons:⁸⁶¹

This is the end of the text as it was found, glory to God. The written canons were extremely corrupt. The text from which we copied was almost illegible and we could not have read it without God's help, let Him be exalted. We contented ourselves with a simple copy of the original when there was no other solution, but we made our best efforts in the transcription and we tried to establish the correct meaning. When we had no other alternative, we just copied the text as it was. Our Lord provides [us with everything] and knows [everything].⁸⁶²

This correction, undertaken by the copyist of the Cairo manuscript, was probably insufficient; at the end of chapter 18 (on the determination of the latitude of Mars), an interlinear note in Maghribī script, apparently different from that of the copyist, says: "This chapter is almost incomprehensible because it is extremely complicated and cannot be applied for a calculation, but God knows best".⁸⁶³ Similar remarks appear in other parts of the same manuscript.

I am aware of the following Maghribī recensions of al-Ḥajārī's translation:⁸⁶⁴

of the tables (names of the months, for example) are translated into Arabic between the lines and Arabic (and Hebrew) notes and comments are added to the printed text.

861 See the edited Arabic text in Parra, 2013, pp. 417–8.

862 Lamrabet, 2013, no. M344, pp. 235–237.

863 Parra, 2013, p. 398, fn. 1772.

864 Samsó, 2004.

Seventeenth century:

- ‘Abd Allāh Aṣṇāk al-Marrākushī (fl. ca. 1655) seems to be the author of the *Risāla ‘arabiyya* mentioned by ‘Abd al-Raḥmān al-Fāsī (see below) and extant in ms. Hassaniyya 1331 (pp. 53–61).⁸⁶⁵ Some materials of this source also appear in ms. Cairo 1081 (pp. 1–2). It contains a useful collection of worked examples for the computation of syzygies and the longitudes of Saturn, Mars, Venus and Mercury. The dates used by the author correspond to 1652, 1656 and 1657, which is further proof of the early interest Maghribī astronomers had in al-Ḥajarī’s translation. It is possible that Aṣṇāk’s text was written in Marrākush, for the time difference between this locality and Salamanca is given as 17 minutes.⁸⁶⁶ He is one of the few authors (amongst those examined here) who avoids using Zacut’s tables for the computation of the ascendant and the division of the houses, which were calculated for Salamanca and should not be used for other localities. Instead of using these tables, Aṣṇāk explains the dual longitude method (3.2.3.1.3) and gives an example in which the latitude implied is around 33°, a value which seems to fit Fez better than Marrakesh.

In ms. Hassaniyya 1331, Aṣṇāk’s text is followed by an abridgement of al-Ḥajarī’s translation (pp. 61–65), which may be a part of his *risāla*, although this is not certain. A colophon on p. 65 explains the history of al-Ḥajarī’s translation and it is followed, on the same page, by a chronological note in which it is stated that, if we add 585 to the Hijra years we will obtain the CE years: this rule would obviously be correct for ca. 1205 H./1790 CE. This ms. was, then, used in the second half of the eighteenth century and it contains (p. 67) twelve verses of an *urjūza* on the use of Zacut’s *zīj* by the *shaykh* Sīdī ‘Abd al-‘Azīz al-Wazkānī, who can probably be identified as Abū ‘Abd Allāh Muḥammad b. ‘Abd al-‘Azīz b. ‘Abd al-Salām al-Wazzānī al-Wazkānī (or his father) who was alive in 1183/1769.⁸⁶⁷

- ‘Abd al-Raḥmān al-Fāsī (Fez, 1631–1685), author of the *Tuḥfat al-muḥtāj fi ‘ilm al-ta’dīl wa l-azyāj*, extant in ms. Cairo DM 1081 (pp. 20–28),⁸⁶⁸ and Hassaniyya 1433 (fols. 1r–4v). In this work, ‘Abd al-Raḥmān al-Fāsī says that he read the *Risāla Zakūtiyya* in a manuscript annotated by ‘Abd Allāh b. ‘Abd al-Qādir Abī l-Shaykh al-Lakhmī (fl. 1668), a *muwaqqit* in al-Qaṣr al-Kabīr. Our author often quotes ‘Abd Allāh b. ‘Abd al-Qādir’s notes and he calls him

865 Khaṭṭābī, 1983, pp. 231–2, no. 271.

866 This time correction corresponds to a longitude of Marrakesh of 21;20° (Kennedy & Kennedy, 1987, pp. 117–8), for which the calculated value would be 17;44^m.

867 King, 1986 b, no. F58, p. 144; Lamrabet, 2013, M379, p. 242.

868 King, 1986 b, p. 140, no. F31.

ṣāhib al-ṭurar (“the author of the notes”). A second source, also used by ‘Abd al-Raḥmān al-Fāsī, is a *risāla* on the same topic written by the aforementioned *faqīh* and *mu’addil* Sīdī ‘Abd Allāh Aṣṇāk al-Marrākushī. Al-Fāsī decided to assemble together in a single text the materials he had gathered in both sources and this is why the colophon of both manuscripts states that this is the end of the *Risālat al-risālatayn al-madḥkūratayn* (“The epistle [based] on the two aforementioned epistles”). To this, he added instructions for calculating the Christian date from the date in the Muslim calendar. For that purpose he begins with a long quotation from Ibn al-Bannā’s *Minḥāj* (1256–1321, see 7.3.2.2)⁸⁶⁹ which explains the procedure for calculating the *rūmī* date with the era of Alexander and, from here (using an equivalence given by the *ṣāhib al-ṭurar*) the corresponding Julian date from CE. This is followed by an example of conversion of 23 Shawwāl 1113/11 March (Julian) 1702 CE for which al-Fāsī (?) uses a direct procedure.

Eighteenth century:

Zacut’s tradition continued in the Maghrib in the eighteenth century. A well-known astronomer, Abū l-Rabī’ Sulaymān b. Aḥmad al-Fishtālī al-Fāsī (d. 1793)⁸⁷⁰ compiled a *Risālat al-anwār fī l-ta’dīl bi l-adwār* (not seen), in which the expression *ta’dīl bi l-adwār* (computation of true longitudes using cycles) seems to refer to the use of Zacut’s tables.

- An unidentified *faqīh* and *mu’addil* Sīdī ‘Abd al-Karīm, known as Aghbāl, must have lived sometime between the second half of the seventeenth century⁸⁷¹ and 1797, by which time he is known to have died.⁸⁷² Aghbāl’s work is extant in ms. Rabat NL D2014 (pp. 1–55) and it has a descriptive title: *Risāla fī kayfiyyat al-ta’dīl bi l-zīj alladhī waḍa’a-hu Ibrāhīm al-Yahūdī al-ma’rūf bi-Azkūt li-ta’dīl al-kawākib al-sayyāra ...* (“Epistle on how to compute, with the *zīj* calculated by the Jew Abraham, known as Azkūt, to obtain the true longitudes of the planets ...”). It seems that this character was active in the north of Morocco because he states (p. 1) that Zacut’s *zīj* had been calculated for his own city, Salamanca, at the extreme limits of the Christian lands (*bi-aqṣā bilād al-Rūm*), whose longitude is 25;46° and latitude 41;19°. According to the author, this longitude is similar to that of Ceuta, Tanger,

⁸⁶⁹ Vernet, 1952, pp. 19–20.

⁸⁷⁰ Brockelmann, *GALS* II, 709; Renaud, 1932, p. 183, no. 543; Calvo, 1994; Lamrabet, 2013, M403, pp. 244–5.

⁸⁷¹ According to Muḥammad al-Mu’tī (see below), Aghbāl wrote his *risāla* using Aṣṇāk’s *Risāla ‘arabiyya*, which seems to be dated between 1650 and 1660. Lamrabet (2013, no M579, p. 273) mentions Aghbāl and dates him after the fifteenth century.

⁸⁷² Date of Muḥammad al-Mu’tī’s *Kanz al-asrār* (see below) who quotes this source.

Aṣīla, al-Qaṣr [al-Kabīr] or Fez, for which he gives a longitude of $24;52^\circ$, not documented in other sources. In spite of this, he gives (p. 14) a time correction for the computation of lunar longitude of 17^m which is the value given by Aṣnāk probably for Marrākush.

Aghbāl wrote his *risāla* after reading many other works of the same kind, all of which are excessively brief. His *risāla* is not concise but it contains a certain number of errors which are difficult to understand: for example, on p. 3 he says that Zacut's *zīj* was calculated for Alexander's era, from which he obtained the Christian era. This is followed by an incomplete rule to perform this kind of operation. He also has difficulty (pp. 19–21) in understanding the function of the two tables of lunar latitude in Zacut's *Almanach* (7.7.3.4.1) and confuses the argument of latitude with the lunar anomaly (in Arabic the word *ḥiṣṣa* can refer to either) (pp. 21–24). He also refers to tables to calculate the longitude of Mercury structured in Arabic (?) and '*ajamī* months and to tables of right ascension, included in Zacut's *zīj*, computed from Aries 0° : the tables of right ascension in the *Almanach* have their usual origin (Capricorn 0°). Far more interesting is the fact that he mentions a table for the velocity of Mars which reaches a maximum of $0;39^\circ/\text{day}$ (p. 40). No table of this kind appears in the *Almanach* and, probably, Aghbāl's source derives from the Toledan and Alfonsine tradition or from the *zīj* of Ibn 'Azzūz al-Qusanṭīnī (7.4.1.6), which compute two different functions whose arguments are the true centre and the true anomaly of the planet. In the case of Mars, the maximum value of the first function is $0;38,40^\circ$ in the Alfonsine tradition and $0;38,30^\circ$ in Ibn 'Azzūz, which agree with the $0;39^\circ$ mentioned by Aghbāl.

- Muḥammad al-Mu'ṭī b. Aḥmad al-Ṭayyib b. Muḥammad Marīn al-Najjār al-Ribāṭī (d. 1808)⁸⁷³ is the author of the *Kanz al-asrār wa-fayḍ al-anwār fī ta'dīl al-nayyirayn wa l-khamsa al-mutaḥayyira bi l-adwār* ("Treasure of secrets and abundance of lights on the computation of the true positions of the Sun, Moon, and five planets using cycles") written in 1211/1797. This work has been preserved in ms. Rabat NL 1468K (pp. 278–303) and it was finished in the *zāwīya* of Wazzān, to the north-west of Fez and near al-Qaṣr al-Kabīr, an important institution in eighteenth-century Morocco.⁸⁷⁴ This points, again, to another area of diffusion of al-Ḥajārī's translation in addition to Fez and Marrākush. We have already seen that 'Abd Allāh al-Lakhmī was a *muwaqqit* in al-Qaṣr al-Kabīr (in the seventeenth century) and that 'Abd al-'Azīz al-Wazkānī was also called al-Wazzānī. In this *risāla*, Muḥammad

⁸⁷³ Lamrabet, 2013, M412, p. 246.

⁸⁷⁴ Lourido, 1978.

al-Muʿṭī mentions (p. 84) a list of astronomers who had written works on Zacut's tables. Among them we find an unidentified *risāla* written by the *imām* al-H.mānī al-Andalusī, which is (according to Muḥammad al-Muʿṭī) the best of the whole collection. Three other sources quoted are already known to us: ʿAbd al-Raḥmān al-Fāsī, ʿAbd Allāh b. ʿAbd al-Qādir al-Lakhmī and ʿAbd al-Karīm Aghbāl.

The *Kanz al-asrār* is a fairly long text (18 chapters) which gives a competent account of the use of the tables of the *Almanach*, introducing some corrections of Zacut's canons. Apart from the sources mentioned above, he also quotes Aṣnāk's *Risāla ʿarabiyya* (pp. 89, 102), Ibn Muʿādh's *Tabulae Jahen* (pp. 101–2) on the limits of solar eclipses for the latitude of Jaén (7.2.2) and for another unspecified latitude (probably Damascus) which he takes from an otherwise unknown *Yasāra mukhtaṣara min Zīj ʿAlāʾ al-Dīn ibn al-Shāṭir al-Dimashqī* (p. 103), a summary of Ibn al-Shāṭir's *zīj* (7.5.2).

Like many other authors, Muḥammad al-Muʿṭī begins by posing the chronological problem (pp. 84–7) and deals with the Christian era and its relation to that of Alexander. He gives a correct rule for date conversion and uses Wednesday 22 April 1797, three hours and thirty minutes after midday, as an example for the correction of the hour as a result of the difference in geographical longitude.

The *risāla* ends with a rule for lunar crescent visibility (2.3.3), based on the Indian condition of visibility (the difference in setting times between the Sun and the Moon should be 12°, at least). There seems to be a mistake in the formulation of the extant text because, instead of applying a correction of 2/3 (a standard value which corresponds roughly to the tangent of the latitude of Baghdad but also to that of Fez) of the lunar latitude to the lunar longitude in order to obtain a modified lunar longitude, this correction is applied to the difference in descensions between the Moon and the Sun, which does not seem to make much sense.

7.8 Conclusions

This long chapter has tried to offer a panoramic vision of the history of astronomical tables (*zīj*es) in al-Andalus, the Maghrib and in the Christian kingdoms of the Iberian Peninsula. In it we have seen that two astronomical schools of thought lived together, called the *Sindhind* and the *Mumtaḥan* from at least the tenth century onwards. These two schools are represented by the two main sources of the Eastern Islamic input in Andalusī and Maghribī astronomy: al-Khwārizmī (fl. ca. 830) and al-Battānī (d. 929). Al-Khwārizmī's *zīj*

was introduced at the beginning of the ninth century, and the *Sindhind* tradition remained in place until, at least, the end of the fourteenth. This school was mainly characterised by the computation of sidereal longitudes, which could be transformed into tropical by using trepidation tables that seem to have appeared in the eleventh century. Only two *zīj*es belonging to this tradition remained faithful to Indian models for the computation of planetary equations: al-Khwārizmī's *zīj* mentioned above (7.2.1.2.1) and, in the fourteenth century, the versified *zīj* of Abū l-Ḥasan 'Alī al-Qusanṭīnī (7.2.3). Previously, in the eleventh century, the *Tabulae Jahan* of Ibn Mu'ādh al-Jayyānī (7.2.2) used Khwārizmī's parameters, but computed the solar, lunar and planetary equations using Ptolemaic methods instead of Indian and Iranian ones.

In the rest of the *zīj*es belonging to the *Sindhind* school, the Ptolemaic methods and parameters were introduced in the computation of equations. Equation tables derived from Ptolemy's *Handy Tables* in most cases. These equation tables reached al-Andalus through al-Battānī's *zīj* which had arrived in this country in the second half of the tenth century, at least, since we know that they were studied by Maslama al-Majrīṭī (d. 1007), who might have been responsible for the introduction of Ptolemaic materials through his recension of al-Khwārizmī's *zīj*.

The *Sindhind* school was the object of important changes in the eleventh century due to the research done by Ibn al-Zarqālluh (d. 1100) who is, no doubt, the most important astronomer of this tradition, and one of the very few who made systematic solar and lunar observations over a long period of time, although not a single description of one of these observations seems to be extant. I have analysed his contributions in chapter 6: trepidation theory, motion of the solar apogee, solar model with variable eccentricity and a small correction of Ptolemy's lunar model. Zarqāllian theories reached the Maghrib in the first half of the thirteenth century, due to the connections between the astronomer Ibn Ishāq al-Tūnisī (fl. 1193–1222) and an unnamed Sicilian Jew who served the Norman king William II (1154–1166) probably as an astrologer. This was the beginning of the development of a Zarqāllian school of Maghribī *zīj*es in the thirteenth and fourteenth centuries (7.3), represented by the unfinished *zīj* of Ibn Ishāq, and those of his followers: the author of the anonymous Tunisian recension (fl. 1266–1281), Ibn al-Bannā' (1256–1321) and Ibn al-Raqqām (d. 1315).

Sidereal astronomy survived in al-Andalus and the Maghrib mainly for astrological reasons. The Andalusi-Maghribī tradition favoured casting sidereal horoscopes,⁸⁷⁵ and the details given in Ibn 'Azzūz al-Qusanṭīnī's computations

875 A possible exception is Ibn Abī l-Rijāl (3.2.2) who may have computed tropical horoscopes. This is my own guess due to the fact that, in his treatment of the revolution of

of his horoscopes related to the battle of El Salado (1340) show us how a Western Islamic horoscope was calculated (3.1.1.8). In spite of this (7.5), in the second half of the fourteenth century, an important change took place in Maghribī astronomy which implied the abandonment of the Zarqāllian tradition and its replacement by Eastern *zīj*es like those of Muḥyī al-Dīn al-Maghribī (7.5.1), Ibn al-Shāṭir (7.5.2) and, much later, Ulugh Beg (7.5.3). This was mainly due to the incapacity of Zarqāllian *zīj*es to predict planetary positions in agreement with observations. It is also clear that this change entailed the use of *zīj*es giving directly tropical planetary positions. In spite of this, Maghribī astrologers continued to use Zarqāllian sidereal *zīj*es until a later date.

This change took place earlier in the Christian kingdoms of the Iberian Peninsula and it seems to be due to a choice made by Jewish astronomers, beginning in the twelfth century with Abraham bar Ḥiyya and Abraham ibn ʿEzra (7.6.1) and culminating, in the thirteenth century, with the *Alfonsine Tables* (7.6.2). Several Jewish astronomers of the fourteenth and fifteenth centuries (7.6.4) continued to practise tropical astronomy, although some of them were still submitted to the influence of the *Sindhind* tradition, represented mainly by Ibn al-Kammād (7.2.5, first half of the twelfth century).

In spite of this radical change in attitude which affected both astronomy and astrology, it seems clear, as we have seen, that astronomers who compiled tropical *zīj*es were still under the influence of the Zarqāllian school. The most surprising case is represented by the *Alfonsine Tables* in which the tropical mean motions derive from Ibn Ishāq's sidereal *zīj*, corrected for precession (7.6.2.2.2.3).

This story ends with Zacut's almanacs, derived from the *Alfonsine Tables*, with the addition of the influence of Jewish astronomers who worked in southern France (Jacob b. David Bonjorn and Levi b. Gerson, for example). The fact that his *Almanach Perpetuum* was translated (apparently three times) into Arabic and had a great diffusion from the Maghrib to Yemen, between the sixteenth and the nineteenth centuries, implies an indirect introduction of Alfonsine astronomy in the Arab world. The success of Zacut's *Almanach* was probably due to the fact that the computation of the longitudes of the Sun, Moon and planets was much easier than with a standard *zīj*.

Who would have used the Arabic almanac? Perhaps it was used by *muwaqqits*, who could be interested in the solar and lunar tables, and this could explain why Muḥammad al-Muʿṭi (d. 1808) deals with lunar crescent visibility,

the months (*taḥwīl al-shuhūr*, 3.2.2.2.2.3), the length of his "astrological months" implies a tropical year of 365^d 5;47,3^h. This could be explained by considering that this author only uses Eastern Islamic sources.

a question ignored in the *Almanach Perpetuum*, although it is analysed in Zacut's *Ḥibbur*. Similar references to lunar visibility can be found in mss. Cairo DM 1081 (pp. 5–6) and Ambrosiana 338 (fol. 1r).

Another possibility is that the almanac was used by astrologers who agreed to cast tropical horoscopes. It is evident that Zacut's *Almanach* could be used for astrological purposes, as it contained tables for the computation of the ascendant and the division of the houses (as well as tables to calculate the animodar), although Maghribī astronomers were well aware that the tables for the division of the houses could only be used for the latitude of Salamanca. In spite of this, I am not aware of the existence of any tropical horoscopes calculated with this almanac, and ms. Vatican 963 (fol. 29r) has a marginal note on the projection of rays and the computation of a sidereal ascendant using the *ta'dīl ḥarakat ra's al-Ḥamal* (equation of the motion of the head of Aries) in a passage in which Ibn Ishāq is also mentioned. This implies that sidereal astrology was still in use. In spite of this lack of evidence, it seems clear that Zacut's almanac was an excellent instrument which greatly simplified the computation of yearly ephemerides in the lands of Eastern Islam, and could be used by astrologers in localities in which tropical astrology was standard.

Bibliography

Acronyms

AHDLMA	<i>Archives d'histoire doctrinale et littéraire du Moyen Âge</i>
AHES	<i>Archive for History of Exact Sciences</i>
AIHS	<i>Archives Internationales d'Histoire des Sciences</i>
ASPh	<i>Arabic Sciences and Philosophy</i>
BA	Jorge Lirola & José Miguel Puerta (eds.), <i>Enciclopedia de la Cultura Andalusí. Biblioteca de al-Andalus</i> . 8 vols. Fundación Ibn Tufayl, Almería, 2004–12.
BEA	Thomas Hockey (ed.) <i>Biographical Encyclopedia of Astronomers</i> . 2 vols, Springer, New York, 2007.
DSB	Charles C. Gillispie (ed.), <i>Dictionary of Scientific Biography</i> . 16 vols., Charles Scribner's Sons, New York, 1970–80.
EI2	<i>Encyclopaedia of Islam. Second edition</i> . 13 vols., Brill, Leiden, 1960–2005.
EI3	<i>Encyclopaedia of Islam. Third edition</i> . Brill, Leiden, 2007– ...
EOBA	<i>Estudios Onomástico-Biográficos de al-Andalus</i> . CSIC, Madrid.
HATA	Historia de los autores y transmisores andalusíes. Data bank on Andalusí authors accessible in http://kohepocu.cchs.csic.es .
JHA	<i>Journal for the History of Astronomy</i>
JHAS	<i>Journal for the History of Arabic Science</i>
JNES	<i>Journal of Near Eastern Studies</i>
JAOS	<i>Journal of the American Oriental Society</i>
NL	National Library
RIEEI	<i>Revista del Instituto Egipcio de Estudios Islámicos en Madrid</i>
ZGAIW	<i>Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften</i>

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Index of Parameters and Numerical Values

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0;36,59,29,25,27,33	801	1;3	415, 487
0;36,59,29,27,29	862	1;3,5	487
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1;47,27	434	2;8	835
1;47,51	636, 675-6, 769, 804	2;10	434, 833, 835, 839, 849-51, 856
1;49,7	675, 766, 779, 804	2;10,46,40	850
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1;54,15	675, 766	2;37	451
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1;56	793	2;46,4	486
1;56,13	675-6	2;48,48	486
1;56,16	779	2;51,23	486
1;57,16	674	2;51,26	486, 757
1;58	662, 734, 747, 757, 850-1	2;52	827
1;58,2	675, 747	2;53,4	623
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1;59,10	662-3, 675, 734, 826, 851, 870	3;2	734, 780, 782, 826-7
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2;1	662, 780	3;5,2,51,2,17	802
2;2,6	674	3;6	757
2;4	757	3;6,24,7,19	741, 802
2;4,5	674	3;6,24,7,20	802
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2;4,45	675	3;6,24,7,42,40,49	718, 772, 802, 845
2;4,46	662	3;6,24,7,42,40,52	845
2;5	490	3;6,24,7,45,13	728
		3;6,24,7,54,5	862
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3;25	486, 734, 757,	5:53	734, 787
	781	5:57	757, 839, 849,
	645		856
3;34,35	600	6;0	459, 486, 696,
3;51	600, 803		734, 780, 781,
3;51,11	600, 623, 803		782
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3;53,55	592, 594	6;13	236, 780, 787,
3;54	616		826
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4;5	626	6;37	236, 787
4;7,57	598, 602, 615,	6;38	734, 747
4;7,58	624, 626, 763	6;48	747
	621, 763	6;49	747
4;8	686, 758, 833	6;50	490
4;18	582-3, 598	7	235
4;18,43	763	7;18	485
4;19,26	595	7;40	236, 734
4;22	687, 750, 891	7;48	833, 852
4;29	687, 693, 865	7;54	852
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	849-50, 856,	8;36,45	712
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	865, 891	9;0	235, 645
5	734, 826	9;46,54	592
5;1	713	9;53,10	596-7
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5;6	759	9;59	615-7, 626, 650
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5;9,41	757, 835	10;15	586
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5;15	849		781, 782
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5;41			

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10;52,34	713	13;10,35,1,8,30,4	825
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11;24	734, 826-7	14;55	731
11;25	780, 782	15;7	731
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	781	17;0	346
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11;35	734, 787	17;8	647-8, 846, 854
12;0	235, 490	18;0	346, 431, 436
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12;30	490	19;2	734
12;40	451	20;0	346
13;0	459	20;30	759
13;3.53.54.51.29	862	20;58,21	759
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13;3.53.56.4	885	22;0	696
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	800, 885		641, 733
13;3.53.56.21	885	23;31	249, 640
13;3.53.57.30.21.4.13	845	23;32,12	441
13;3.56.58.49.54.44.11	694	23;32,29	837
13;8	826-7	23;32,30	97, 172, 627,
13;9	734, 780, 782,		629, 640, 793,
	827		837
13;10.34.52.39	885	23;32,40	77, 90, 97, 98
13;10.34.52.40.30	870	23;33	97, 179, 332,
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	885		592, 599, 612,

	615, 624, 629,	29;30,44,3,47,49,25,36	754
	645, 687, 699,	29;31,50,0,27,46,40	826
	726, 740, 811,	29;31,50,5,43,22	701
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23;33,1	629	29;31,50,7,44,35	847
23;33,8	507, 629, 740	29;31,50,8,6,49,32	825
23;33,30	629, 699, 726	29;31,50,8,20	48, 825
23;34	97	36;45	734
23;35	79, 97, 249, 260,	38;53,53	486
	261, 368, 384,	39;0	696
	423, 433, 546,	39;30	486, 734, 757,
	702, 865		781
23;36	97, 433	40;26,58	713
23;43	599, 612, 615,	40;31	696
	616	41;9	780, 826
23;50	73, 75	41;10	236
23;50,27	629	41;30	696
23;51	94, 167, 368,	41;55	826
	433, 585, 691,	42;2,57	486
	698, 739, 852-3	42;56,9	864
23;51,20	73,852	42;57	864
23;52,30	627	43;10	486, 734, 757,
23;53	599, 734		781, 849
24;0	99, 332, 355, 433	43;46,51	849
24;3	236	44;0	696, 849
24d 5;21,11h	761	44;28,34	773
24d 8;24h	190	44;29,24	773
24;13,22,57	761	44;30	773, 803, 847
26;0	505	44;30,43	773
27d 5;30h	298	44;49	734
27d 5;32h	298	45;2	803
27d 7;42h	297	45;12	732
27d 7;43,5h	297	45;21	741-2
27d 7,43,12h	297	45;22,53	803
27d 7;44h	297	45;28,37	732, 773, 803
27d 8h	297	45;59	236, 734, 780,
27d 8;2h	297		826
28d 2;19,51h	760	46;0,3	694
28d 2,17,30h	241	47;11	696
28d 2;18h	190	47;15	734
29d 12;2,30h	825	47;22,6	713
29d 12,44,0,11,6,40h	826	47;41,17,16	847
29d 12,44,3,2,58,51h	847	47;52	236
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29;30,6,15	825	62;0	639

65;30	657	83;40,53	665, 803
66;0	639, 651, 657,	84;30	418
	873	84;40	665, 803
66;30	358	85;0	418
67;0	873	85;49	418, 462,
68;0	873		657, 741, 775,
68;30	873		849, 875, 881
68;53,10	694	85;50	657
70;15	627	86;0	462, 657
72;0	639	87;0	418, 881
73;18,22	773	87;4,37,20	872
73;22,23	773	87;4,38	872
73;25,29	773, 803, 847	87;18,55,53,30,45	846
73;26,14,31	847	87;18,55,55	846
73;29,6	773	87;18,55,55,30	846
73;46	732	87;21	661, 824-5, 846
73;46,18	732, 773, 803	88;0	793
73;54,57	803	88;40,6	849
74;1	741-2, 803	88;48	190
75;59	890	90;0	490, 875-6
75;59,21	848	90;56	890
76;44,17	774, 775, 848	92;14	711
76;45, 21	741-2, 775,	92;20,45	799, 800
	874-6	92;20,55	728
	441	92;20,56,40,12	728, 760
77;13,35	717	92;21,0,0,2	739
77;19	732, 733, 774,	92;21,45	661, 729, 760,
77;50	880		800
	694, 711, 732,	92;22	438
77;55	733	92;24	344, 438, 661,
79	874		728, 800
79;6	885	92;25,34	871
79;25	864	92;33,22,30	800
79;25,55	864	93;2	345
79;30	864	93;2,15	345, 711
79;32,45	875	93;30	743, 800
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81;0	849	94;56	711
81;15	694, 733	100;0	873, 881
81;36,9	864	107;21,27,42,28	847
81;57	864	10[7];38,34	773
82;0	849	107;55,35	773
82;11,38	849	108;8,4	803
82;14	733	108;8,39	732, 773, 803,
82;17	833		847
82;39	657	108;11	741-2, 803
82;45	657	108;23,54	773
83;25,48	694	111;3	864
83;40,31	665, 803	113;18,58	773

113;20,14	773	157;44,17	774, 776, 848
113;20,30	741-2, 803	158;10,58,58	848
113;21,14	773, 803, 847	158;11	890
113;21,56	803	158;21,30	741-2, 876
113;22,23	773	159;42,45	774, 777
113;41,11	732, 773, 803	159;43	717, 774
114;5,29	732	161;0	849, 881
114;52,2,0,51	847	162;0	881
115;30	881	161;3	864
115;30,30	741-2, 803	161;53,9	864
115;32,2	803	161;57	864
115;51,15	732, 773, 803	161;58,20	849
116;15	732	162	849
116;17,9	773	164;28	732, 733
116;17,13	773	164;30	732, 733, 774
116;17,15	773, 803, 847	172;0	876
116;17,17	773	172;32	694, 733
117;0	881	173;7	890
117;45,17	694	190;0	881
117;58,37	694	193;24	717
118;21,0,3	847	195;13,31,58	848
119;0	849	195;14	890
119;14,11	849	197;30	733, 774
119;41,0	875	198;21,0	875
119;41,30	741-2, 876	198;21,30	741-2, 876
119;46	890	198 ;24,17	774, 776, 848
119;46,11,58	848	201;8,54	875
119;54,17	774, 776, 848	201;28	733
120;2,20	773	210;0	881
120;19,21	773	210;10	890
120;32,32	773, 803, 847	210;25,15	694
120;34,42	741-2, 803	210;35,31	773
120;39,5	803	210;36,13	773
120;47,54	773	210;36,54	773
120;58,18	732, 773, 803	210;37,25	773, 803, 843
121;22,35	732	211;7,57	803
121;36,9	864	211;24,59	732, 773, 803
121;37	864	211;27	741-2, 803
121;40	849	211;52	732
121;51	733, 774	212;0	876
122;1,16,23,53	847	212;42,3,32	847
122 ;12,45	774, 777	224;54	694, 733
122;13	717, 774, 864	233;0	881
122;23	864	233;20,35,21	847
122;28,54	875	233;30	803
126;58	733	233;45,42	732
128;24	694, 733	234;9,55	732, 773, 803
133;0	876	234;19,30	773
134;42	890	234;19,37	773, 803, 847

234;19,41	773	365d 5;49,24h	661, 824
235;0	881	365d 5;50,22,56,18h	863
237;44,17	774, 776, 848	365d 5;54h	190, 821
237;57,40,58	848	365d 5;55h	190, 821
237;58	890	365d 5;55,12h	191, 821
238;38,30	741-2, 875-6	365d 6;0,46,48h	864
239;42,45	774, 777	365d 6;0,46,48h	864
239;43	717, 747	365d 6;0,47,48h	864
240;5	733, 774	365d 6;0,48	864
241;23	864	365d 6;1h	864
241;25,24	875	365d 6;9,23,46,40,48h	760
244;28	733	365d 6;9,24,5h	739
244;55	694, 733	365d 6;9,26,46,41,56	743
252;0	876	365d 6;9,27h	661, 760
252;54	890	365d 6;9,36h	661
260;55	885	365d 6;14,23,24h	743
274;58	745	365d 6;14,24,12h	743
277;12,55	600	365d 6;14,31,49,40,48h	743
278;50,28	659	365d 6;14,31,50,8,48h	743
279;0	658, 874, 880	365d 6;14,38h	743
280;0	874	365d 6;18,3h	760
289;9,23	659	365;14,26,0	311
290;0	658, 874	365;14,26,2,30	825
299;0	657-8	365;14,30,46,13	872
330;10,49	694, 773	365;14,32,29,22,32	825
330;10,55	773	365;14,32,36,24,33,16	825
330;11	773, 803, 847	365;14,33,9,57,4,26,5	846
330;11,5	773	365;14,33,9,57,27,21,36	645
330;19	741-2, 803	365;14,33,9,59,20,7,30	846
330;20,18	303	365;14,33,10	645
331;39,37	732, 773, 803	365;14,33,30	661, 824
331;43,9,52	847	365;14,35,57,21	863
332;4	732	365;14,45,47	311
354;8,48,45,33,53,7,12	754	365;14,48	311, 362
354;22,0,5,33,20	826	365;14,48,33,35	311
354;22,1,40	48	365;14,48,33,35,47,22	49
365d 5;46,25h	825	365;14,48,45	311
365d 5;47,30h	241, 729	365;14,52,26	311
365d 5;48,18,29h	872	365;14,58,45	311
365d 5;48,18,31h	872	365;15,1,57	864
365d 5;48,59,45,0,47,52h	825	365;15,1,59,30	864
365d 5;49h	846	365;15,2	864
365d 5;49,2,33,49,18,30h	825	365;15,2,30	864
365d 5;49,15,43h	846	365;15,23,27,33	438
365d 5;49,15,43,41,15h	846	365;15,23,29	438
365d 5;49,15,43,42h	846	365;15,23,29,26,42	760
365d 5;49,15,58,49,46,26h	846	365;15,23,29,26,45	743
365d 5;49,15,59,34,3h	846	365;15,23,37,30	661, 729, 760

365;15,23,40	438	365;15,36,19,35,22	743
365;15,24	438, 661, 729	365;15,36,34,51,38	662
365;15,24,15,40	871	365;15,36,35	742–4, 885
365;15,30,22,15	712	365;15,36,35,0,8	662, 744, 754
365;15,30,22,30	168	365;15,45,7,30	760
365;15,35,58,30	743	11,325	805, 889
365;15,36,0,30	743	7,000	645, 857
365;15,36,19,34,12	743	49,000	645, 857

Index of Names and Subjects

- ‘Abbās b. Fīrnās (d. 887) 23, 24, 124, 154–5, 352, 683
- ‘Abbās b. Nāṣih (d. after 844) 6, 7, 23, 220, 683, 709
- Abbon abbot of Fleury (fl. 988–1004) 398
- ‘Abd Allāh b. ‘Abd al-Qādir Abi l-Shaykh al-Lakhmī (fl. 1668) 899–901
- ‘Abd Allāh b. Aḥmad al-Ṭulayṭulī (fl. 1066) 161, 215–218
- ‘Abd Allāh Aṣnāk al-Marrākushī (fl. ca. 1655) 272, 899, 900–1
- ‘Abd Allāh ibn Buluqqīn, king of Granada (r. 1073–1090) 164, 167–9
- ‘Abd Allāh Ḥusayn Quṣ’a b. Muḥammad b. Ḥusayn al-Ḥanafī al-Tūnisī (fl. 1680) 817
- ‘Abd Allāh b. Masrūr (first half of the 10th c.) 244
- ‘Abd Allāh b. Muḥammad al-Tamīmī al-Kātib, governor of Ifrīqiya (974–987) 251–2
- ‘Abd Allāh al-Naṣrānī (9th c.) 823
- ‘Abd Allāh al-Ṣanhājī al-Dādisī, *muwaqqit* in Marrakesh (?) 816
- ‘Abd al-Malik ibn Ḥabīb (d. ca. 853) *Kitāb fī l-nujūm* 3, 50, 85, 147, 158
- ‘Abd al-Malik b. Zaydān, Abū Marwān, sultan of Morocco (1627–30) 896
- ‘Abd al-Mu‘min, Almohad Caliph (1133–1163) 9
- ‘Abd al-Raḥmān I Umayyad amīr of Córdoba (756–788) 131
- ‘Abd al-Raḥmān II Umayyad amīr of Córdoba (821–852) 2, 7, 134, 154–6, 158, 352, 683
- ‘Abd al-Raḥmān al-Lujā‘ī al-Fāsī, Abū Zayd (d. 1371) 18, 127
- ‘Abd al-Raḥmān III al-Nāṣir Umayyad Caliph of Córdoba (912–961) 6, 141, 158, 220
- ‘Abd al-Raḥmān al-Fāsī (1631–1685) 899–900
- ‘Abd al-Raḥmān b. Rustum (778–788) 7
- ‘Abd al-Raḥmān al-Ṣūfī (903–998) 335, 561, 564, 642, 647–8, 654, 687, 822–5
- ‘Abd al-Wād dynasty in Tilimsān (1236–1550) 10
- ‘Abd al-Wahhāb b. Rustum (788–824) 7
- Abraham ibn Dāwūd (fl. ca. 1148–1180) 189
- Abraham ibn ‘Ezra (ca. 1089–ca. 1167) 26, 27, 189, 191–197, 203, 209, 213, 235, 250, 255, 271, 279–80, 292, 294–296, 301, 303, 313, 321, 690, 832, 846, 856, 866–7, 886, 888, 904; recensions of his astrological works 191–193; Hebrew and Latin texts 191–193; travels 191; Arabic sources 193–4; factors that alter the results of nativity horoscopes 195; horoscopes 196–7; other translations of Ibn ‘Ezra’s works 197; *Liber de rationibus tabularum* 194–5, 642–3, 655, 661, 687, 689, 704, 821–2; on the use of the astrolabe 414–17; *Luḥot* 821–2; *Tabulae Pisanae* 687, 822; almanac 878, 880–902
- Abraham bar Ḥiyya (ca. 1070–1145) 27, 153, 186, 189–191, 690, 831–2, 856, 866–71, 904; *Yesodeh ha-tebunah u migdal ha-emunah* 189–190; *Sefer ha-‘ibbur* 189; *Ṣūrat ha-‘areṣ* 189, 567–8, 572–3, 704; *Megillat ha-megalleh* and astrological history 190, 304, 307–319; *Sefer tiqqun ha-tequfot min ha-luhot ha-‘eleh* 190, 821; *Sefer Heshbon mahalakhot ha-kokhavim* 820; *Luḥot ha-nasi’* 820–1
- Abraham Hebreo, scientific collaborator of Alfonso X (fl. 1264–77) 477, 555–6
- Abraham Zacut; see Zacut
- Abū ‘Abd Allāh al-Mu‘addib (? , before 1400) 75
- Abū ‘Alī al-Ḥasan al-Marrākushī : see Abū l-Ḥasan ‘Alī al-Marrākushī
- Abū Bakr al-Munajjim al-Khulānī (fl. ca. 1050–1090) 163
- Abū Bakr ibn Yūsuf (fl. 1208–18) 401
- Abū l-Barakāt al-Baghdādī (11th–12th c.) 540–1, 824
- Abū Ḥafṣ ‘Umar b. Ḥafṣ b. Abī Tammām (d. 928) 502–3
- Abū Ḥāmid al-Gharnāṭī (d. 1169): *al-Mu‘rib ‘an ba‘ḏ ‘ajā‘ib al-Maghrib* 86, 145, 146
- Abū Ḥammū Mūsā (1359–1389), sultan of Tilimsān 122
- Abū Ḥanīfa al-Dīnawārī (d. 895) 638, 767

- Abū l-Ḥasan ‘Alī, Merinid sultan of Morocco (1331–1351) 8, 13, 173–4, 179, 797
 Abū l-Ḥasan ‘Alī al-Marrākushī: (late 13th c.)
 Kitāb al-mabādī’ wa l-ghāyāt 17, 104,
 334–5, 350–1, 403, 406, 407, 429–30,
 463, 464, 471, 489–90, 504, 577, 586,
 600, 612, 615–6, 655, 658, 704, 739, 743,
 748, 812, 828
 Abū ‘Inān, Merinid sultan (1348–1358) 14,
 122, 307
 Abū Ishāq al-Zajjāj (d. 923) 638
 Abū Ja’far al-Sulamī (d. 1346) *Tawassuṭ al-*
 manāzil fi l-shuhūr bi ma’rifat waqt al-fajr
 wa l-ṣuḥūr 87
 Abū Kursūm al-Yahūdī: see Jacob Corsuno
 Abū Ma’shar (ca. 786–886) 183–4, 194, 209,
 221, 233, 235, 240, 244, 275, 288, 302,
 304, 313, 318, 321, 767, 779, 850
 Abū Miqrā’/Muqrī’/Maqra’, Abū ‘Abd Allāh
 Muḥammad b. ‘Abd al-Ḥaqq al-Baṭṭiwi
 (fl. 1320) 16, 46, 47, 51, 52, 73, 75, 78, 86
 Abū l-Qāsim b. Sirāj (first half of the 15th c.)
 45
 Abū l-Qāsim b. Ḥasan al-Shaddād (fl. 1345)
 113
 Abū l-Qāsim al-Zahrāwī (ca. 936–ca. 1013)
 510
 Abū l-Ṣalt Umayya b. ‘Abd al-‘Azīz b. Abī
 l-Ṣalt (ca. 1068–1134) 403, 405–7, 414–5,
 471; equatorium 487–9, 498
 Abū Tashufīn I (1318–1337), sultan of Tilimsān
 122
 Abū Ya’qūb Yūsuf, Almohad caliph
 (r. 1163–1184) 169
 Abū Yūsuf al-Manṣūr, Almohad caliph
 (r. 1184–99) 751
 Abū Yūsuf Ya’qūb, Merinid sultan (1258–1286)
 143
 Abū l-Yusr al-Shaybānī (d. 911) 5
 Adelard of Bath (fl. 1116–1142) 27, 53,
 688–690
 al-Afḍal, minister of the Fatimid Caliph
 al-Musta’lī (1094–1101) 405, 498
 Aflaḥ b. Rustum (824–872) 7
 Aghbāl, Sidi ‘Abd al-Karīm (fl. ca. 1700–1797)
 900–1
 Aghlabi dynasty in the Maghrib (800–910) 5
Aḥkām al-ṣulub: see crosses
 Aḥmad b. Abī l-Ḥusayn, Kalbī emir of Sicily
 (d. 1036) 227–8, 251
 Aḥmad b. ‘Abd al-Raḥmān al-Dahmānī
 (fl. 1450) 106
 Aḥmad b. ‘Alī al-Sharafi (fl. 1328) 401
 Aḥmad b. Fāris al-Munajjim (fl. ca. 968–982),
 Mukhtaṣar fi l-anwā’ 50, 85–86, 142,
 159–160
 Aḥmad b. Jumhūr al-Judāmī (d. 1229) *Qaṣida*
 fi ma’rifat al-mutawassiṭ min al-manāzil fi
 waqt al-fajr 9, 87–93
 Aḥmad al-Maṣyūb/Maṣ’ūb al-Fāsī (d. 1613)
 896
 Aḥmad b. Muḥammad Bū Dayḍāḥ
 al-Qayrawānī (fl. before 1737) 818
 Aḥmad b. Muḥammad b. Hārūn al-Hadabī
 (fl. 1328) 401
 Aḥmad b. Muḥammad al-Naqqāsh (fl. 1080)
 400
 Aḥmad b. Muḥammad al-Yaḥṣubī al-Qurṭubī
 (12th c.) 9
 Aḥmad b. Muḥammad b. Yūsuf al-Anṣārī,
 astrologer of Muḥammad VI of Granada
 (1360–62) 12, 13
 Aḥmad b. Qāsim b. ‘Ammār al-Sūsī (fl. 1842)
 113
 Aḥmad b. ‘Umar al-Kabbī (fl. 1585) 402
 Aḥmad b. ‘Uthmān al-Aṣṭurlābī, Abū ‘Alī
 (first half of the 10th c.) 354, 373
 Aḥmad b. Yūsuf b. al-Dāya, Abū Ja’far
 (d. ca. 944) 181, 183
 Albertus Magnus (ca. 1193–1280) 543
 Alchandreana 20, 219–223
Alchandreus 219
 Alexander Macedo 219
 Alfonso I of Aragon (1104–1134) 21, 26, 690
 Alfonso IV of Portugal (1326–1367) 13, 38,
 206
 Alfonso V of Aragón (1416–1458) 207
 Alfonso V of Portugal (1438–1481) 213
 Alfonso VI of Castile (1065–1109) 21, 166,
 578
 Alfonso X of Castile (1252–1284) 23, 32, 33
 Alfonsine works 34–36, 122, 125–126, 794,
 871; Alfonsine collaborators 36, 198.
 Alfonsine Tables 35, 40, 42, 49, 54,
 198, 285, 303, 310, 348, 444–5, 554–5,
 679, 704, 725, 754, 814, 827–858, 865,

Alfonsine works (cont.)

867–71, 873, 885, 889, 896, 901, 904;
trepidation 642, 643–649, 857, 859,
874; lunar equation of anomaly 692,
779; solar, lunar and planetary
velocities 808, 834–5; Ibn al-
Zarqālluh's lunar correction 833–4;
Islamic context of the Castilian canons
835–6; *Tabulae resolutae* 40, 41, 830,
840; sexagesimal structure 840–4;
chronological tables 841–4;
connection with the Marāgha
observatory? 372, 828.

*Libro conplido en los iudizios de las
estrellas* 198, 228–254; horoscope of
the translation 198–200, 832, 856.

Libro de las cruces 556; see crosses.

Libro del saber de astrología 325, 556,
709, 827; *Libro de la açafeha* 262, 450,
556–7; *Libro de la alcora or de la fayçon
dell espera* 262, 281, 330–341; *Libro de
las armellas* 269, 281, 285, 287, 353,
368–72; *Libro del astrolabio llano* 137,
262, 271, 421, 432–440, 444–5, 851, 856,
879; *Libro del astrolabio redondo* 269,
271, 341–352, 377, 444–5; *Libro
del ataçyr* 269, 290; *Libro de las
estrellas fixas/IV Libros de la ochava
espera* 555, 563–4, 647, 854; *Libro de
la lámina universal* 262, 443–450;
*Libro primero de las láminas de los siete
planetas* 476–9; *Libro segundo de las
láminas de los siete planetas* 479–
487; *Libro del quadrante para
rectificar* 444–5, 851, 879; *Libro
del relojio del agua* 125–126, 348,
444–5; *Libro del relojio del argent
vivo* 125–126, 555; *Libro del relojio
de las candelas* 122; *Libro del relojio
de la piedra de la sombra* 114–5, 629,
837–9, 856.

Libro del quadrante sennero 474–5, 857
Translation of al-Battānī's canons 684,
856–7

Translation of Ibn al-Haytham's *Fī hay'at
al-ālam* 553–567; Alfonso X was not
very interested in cosmology 554–5;
a restructured translation 556–60;
references to other Alfonsine works

560–61; terminology 561–4; *figura
pinee/pineata/pineatum* 562–4;
wrong position of the equant
point in the model for the superior
planets 565; other errors 566–7.

Partidas 843–4

Setenario 555

Alfonso XI of Castile (1312–1350) 13, 179, 206

Alfonso de Córdoba (1458–ca. 1502) 42;

Lumen coeli 42; *Tabule astronomice*

Elisabeth Regine 42, 867, 869–70, 873

Alfonso Dinis (ca. 1300–1350) 38, 206, 208

‘Alī ibn Abī ‘Alī al-Qusanṭīnī, Abū l-Ḥasan
(fl. 1359–61) 12, 13, 56, 173, 619,

623, 624–6, 642, 660, 675, 714–720,

903; computation of planetary

longitudes 716; influence of Ibn

al-Zarqālluh and the school of Ibn

Isḥāq 717

‘Alī b. Aḥmad al-‘Imrānī (d. 955) *Kitāb al-
ikhtiyārāt* 180, 186

‘Alī b. Aḥmad al-Ṣanhājī al-Tilimsānī ibn
al-Faḥḥām, Abū l-Ḥasan (fl. 1357) 18,
122–3, 127–8

‘Alī b. Khalaf, Abū l-Ḥasan b. Aḥmar al-
Ṣaydalānī/al-Shajjār (11th c.): universal
plate 262, 285, 440–1, 443–450, 465, 466,
468; meridian stereographic projection
441–3; relative chronology of the three
universal astrolabes 460–2

‘Alī b. Muḥammad b. ‘Abd Allāh b. Faraj
(fl. 1608) 402

‘Alī b. Muḥammad al-Azdī (fl. 1544) 402

‘Alī ibn Mūsā al-Lakhmī al-Qarabaqī (d. 1440)
45

‘Alī ibn Riḍwān (998–1061) 208, 322

‘Alī b. Shadhān b. Mūsā b. ‘Abd Allāh al-
Ḥasanī (fl. 1317) 767

‘Alī b. Sulaymān al-Zahrāwī al-Ḥāsib
(fl. end of 10th c.), *Risāla fī ma‘rifat sa‘at
al-mashriq* 45

‘Alī b. Yūsuf b. Tashfin Almoravid caliph
(1106–1143) 8, 143

Alidades (triquetrum?) 324–5

Almanacs 37, 38, 39, 40, 153, 202, 726,
877–8, 880–902

Almanach of Tortosa 121, 884–5

Almohads (ca. 1147–1276) 7, 8, 9, 10, 169–171

Almoravids (ca. 1050–ca. 1147) 7, 8, 169–171

- Aluaten Sarracenus 219
 Alvarus Ovetensis 198
 Aly Abenragel: see 'Alī ibn Abī l-Rijāl
 Ammonius (ca. 440–ca. 520) 880, 884
 al-Andalus' role in the transmission of Arabic
 science to Europe 18–31
 Andalusian revolt against Ptolemy 516–544;
 criticisms of Ptolemy 525
 Andarzaghār b. Zādān Farrūkh 194
 Andrés Rodríguez of Zamora (fl. 1438) 39,
 209, 572
 Andrés de Li (fl. 1492ff) 207
 Animodar 153, 186, 213, 291–300, 722, 738;
 animodar of the ruling planet 293–4;
 animodar of the conception 294–300
 Anonymous Toledan astronomer of the 11th c.:
 Kitāb al-Istidrāk 498, 507–8; *Risālat al-*
 hay'a 506–7, 656
 Anonymous Tunisian recension of Ibn
 Ishāq's *zīj* (ca. 1266–81) 12, 53, 56, 57, 63,
 66, 67, 94, 95, 97, 98, 118, 137, 261, 272,
 280, 292–4, 297–8, 301–2, 304–306, 441,
 590, 611, 622–5, 627–8, 629–634, 638,
 655–6, 660, 663–5, 667, 669, 673–5,
 679, 688, 699, 709, 711, 715, 717–20,
 728–9, 735, 736, 738, 739, 740, 748, 750,
 752, 764–781, 786, 789–791, 793, 799,
 802–3, 804, 808, 813, 819, 837–9, 841,
 844–5, 847–8, 886–7, 903
 Antoninus, Roman Emperor (r. 137–161) 648
Anwā' 3, 9, 49–50, 74
 Apogees, motion 507, 578, 629–634,
 657–661, 733, 740–2, 775–8, 833, 848–9,
 864–5, 874–76, 890
 Apolonius' *Conics* (3rd c. BCE) 563
 Arc of light (*qaws al-nūr*) 66–67
 Argafalau Caldeus 219
 Argote de Molina, Gonzalo (1548–1596) 858
 'Arīb b. Sa'īd (d. 980) 3, 50, 294–5, 437, 506,
 684, 685
 Aristotle (385–323 BCE), *Physics* 526;
 Metaphysics, *De caelo* 530, 541
al-Arkand 23, 220, 683
 Armāniyūs 880
 Armillary ring (*ḥalqa*) 324–5
 Armillary sphere 174, 324, 352–372, 828
 Arrival of Eastern sources to al-Andalus
 21–23, 25–31, 42–43
 Ascalu Hismaelita 119
 Ascelin of Augsburg (fl. beginning of the
 11th c.) 382–3, 392, 398
 Ascensional difference: see Timekeeping
 al-Asfī al-Andalusī (fl. 1701) 148
aṣḥāb al-Mumtaḥan 823
 al-Ashrafi *zīj* 686
 Aṣnāk: see 'Abd Allāh Aṣnāk al-Marrākushī
 Astrolabe 18, 100; standard astrolabe 373–
 440; dhāt al-ṣafā'iḥ 354; munbaṭaḥ/
 mubaṭṭaḥ 354, 373; ḥūti 354, 373;
 zodiacal-calendar scale 373–4;
 Maslama and stereographic
 projection 374, 376; Ibn al-Ṣaffār
 and Ibn al-Samḥī treatises on the
 instrument 376–8; introduction in
 Catalonia (10th c.) 378–392; the “old
 corpus” of texts on the astrolabe and
 other instruments 378–9; Catalan
 origin of the collection 380–1;
 sources 381–7; is the “old corpus”
 the result of translations? 387–392;
 Carolingian astrolabe 18, 392–397;
 European diffusion of the “old
 corpus” 397–399; astrolabe makers
 in al-Andalus and the Maghrib 399–
 402, 411–12; treatises on the use of
 the astrolabe in al-Andalus and the
 Maghrib 402–410; astrolabes made
 in the Christian kingdoms of the
 Iberian Peninsula 410–13; Latin
 texts on the construction and use of
 the astrolabe 413–431; Alfonsine
 books on the construction and
 use of the instrument 432–440;
 linear astrolabe 471. See universal
 astrolabes
Astrolabii sententiae 18, 118
 Astrological geography 247–9
 Astrological history 153
 Astrological months 190, 241, 760–1
 Astrology between 950 and 103 1158–160
 Astromagical compilation in Catalan (end of
 the 14th c.) 205–6
 al-'Aṭṭār, author of a *zīj* (?) 816
 Autolycus of Pitane (fl. ca. 300 BCE) 330
 Avendauth: see Abraham ibn Dāwūd
 Awmātiyūs 880
 Azarquiel: see Ibn al-Zarqāllūh
 al-'Azīz, Fāṭimid caliph (975–996) 5

- Bādīs b. al-Manṣūr Zīrid king in Qayrawān (r. 996–1016) 169, 227
- Banū Mūsā b. Shākir (fl. 850–875) 687, 823–5
- Banū Waqār (14th c.) 39, 208, 868
- al-Baqqār, Abū ‘Abd Allāh (fl. 1411–18) 14, 15, 230, 292, 814; *Kitāb al-adwār fī tasyīr al-anwār* 14, 174, 175, 283–5, 304, 309, 634–6, 638–41, 817; *Kitāb al-amṭār wa l-as‘ār* 14, 165–6, 174, 214, 215–219, 249–50, 303, 319–323, 880; sources 319–20; weather prediction 320–1; oscillation of prices 322; eclipses 323
- Bartomeu de Tresbens (fl. 1361–1374) 37, 201, 203–5
- Batecombe: see William Batecombe
- al-Battānī 2, 54, 114, 136, 187, 188, 244, 248, 276, 282–5, 311–2, 314–7, 348, 369, 423, 444–5, 504, 514, 546, 568, 588–591, 593–5, 597–8, 601, 608–10, 637, 642–4, 648, 651, 662, 674, 675, 684–686, 688, 722–29, 732–4, 747, 753, 767, 776, 779, 786, 789, 804, 808, 820–1, 826–7, 833, 848–9, 851–3, 856–7, 865, 868–72, 903
- Bayazid II, Ottoman sultan (1481–1512) 894
- bayda*, astronomical instrument 18, 326–7, 354
- Bern de Prüm abbot of Reichenau (beginning of the 11th c.) 398
- Bernard of Verdun (ca. 1300) 495, 543, 655–7, 747, 851, 880
- Bernardo el Arábigo (ca. 1250–80) 827
- Bernat de Granollachs (fl. 1418–1487), *Lunari* 38, 207, 883
- Bianchini: see Giovanni Bianchini
- al-Birjandī (d. 1525) 586
- Birth of the Prophet 54, 300, 306, 318, 601, 890
- al-Birūnī (973–1048) 262, 264, 269, 288, 321–2, 429, 442–3, 452, 487, 511, 841, 877
- Bishr al-Isrā‘īlī 767
- al-Biṭrūjī (fl. before 1200) 9, 499, 516–17, 528–9, 530–544, 546, 553, 568–70, 575–6, 857; astronomical knowledge 524; spiral motion (*lawlab*) 529–30; motions in opposite directions 529; astronomical system 530–544; planetary order 531; use of eccentrics and epicycles 531–2; planetary models 532–3; *taqṣūr*, *incurtatio*, delay 532, 534–5, 547, 573; *istifā*, completion 535; motion of the fixed stars 532–8, 644; Eudoxian hippopede 536–8; transmission of motion 538–42; *shawq/tashāwuq/desiderium* 540–2, 547; diffusion of al-Biṭrūjī’s system 543–4
- Bonfils, Immanuel b. Jacob (fl. 1340–65) 869–70
- Bonjorn (= Yomṭob), Jacob b. David (fl. 1361) 38, 41, 203, 692, 805–6, 868–70, 873, 883, 886, 888, 891, 904
- Brāhmasphuṭasiddhānta* (7th c.) 601, 691
- Calendars 48–52
- Calendar of Córdoba 3, 51, 52, 73, 75, 83, 437, 506, 684, 702, 767
- Calo Calonymos (fl. 1531?) 544
- Candle clocks 6, 18, 121–23
- “Carolingian” astrolabe: see astrolabe
- Casting of rays: see projection of rays
- Celestial globe: *De horologio secundum alcoram* 326–7; celestial spheres in the Iberian Peninsula 328–30; Qustā b. Lūqā and Alfonso x on the celestial sphere 330–341; astrological uses of the celestial sphere 336–7
- Centre of the world (Qubbat al-arḍ) 247–8, 305, 314
- Charles v, king of France (1364–80) 858
- Chaucer, Geoffrey (1340?–1400) 720–21
- Clepsydras: ‘Abbās ibn Firnās 124; Gerbert of Aurillac 124; ms. Ripoll 124; Ibn Khalaf al-Murādī 124–125; al-Zarqāl 125; *Libro del relogio del agua* 125–126; *Libro del relogio del argent vivo* 125–126; Pedro IV of Aragón 126–127; Qarawiyyīn Mosque of Fez 17–18, 127, 143; of the *madrasa Bū ‘Ināniyya* in Fez 18, 127–128.
- Comets 206, 207
- Conjunctions: see planetary conjunctions
- Constantin, dean of Fleury (988–996) and abbot of Micy (1011–1021) 398
- Copernicus (1473–1543) 510, 544, 655
- Córdoba Royal Library 7
- Cresques Abraham (fl. 1375), *Catalan Atlas* 121, 411, 883

- Cresques Vivier of Avignon (d. 1391) 201
 Crosses, astrological system for simple predictions 15, 20, 35, 156, 161, 198, 214–219, 223
 Cycles, astrological 179; 240–1, 243–7, 274–6, 300, 307–8
- al-Ḍabbī (fl. ca. 788–ca. 852), *Urjūza* 154–6, 158, 214
 al-Dahmānī, Aḥmad b. ‘Abd al-Raḥmān (fl. 1450) 473
 Dalmacius Planes (d. 1383) 37, 201–202, 329, 649, 859, 879
 Dalmau ses Planes: see Dalmacius Planes
Dār al-ḥikma (Raqqāda) 5, 153
 David ben Yomṭob (14th c.) 203
dawr 244, 246–7, 275–6
De utilitatibus astrolabii 19
 Diego Ortiz de Calzadilla (fl. 1469–76) 40, 210, 212
 Diego de Torres (fl. 1480–87) 40, 41, 110
Dīnesis 430
 Dioscorides (ca. 40–ca 90 CE) 374
 Displaced equations 667–671, 781–82, 819–20, 831
 Distances and sizes of the celestial bodies 503, 547–553, 573–575
 Division of the houses of the horoscope 257–8, 905; Standard method 258–261, 346, 889; Prime vertical method 262–267; Equatorial method 267–270, 709; hour lines method 271, 346, 416–7, 437; dual longitude method 271–2, 304–5, 346, 899; split differences method 272–3
 Ḍiyā’ al-Dīn Muḥammad (fl. 1680) 464
 Dominicus Gundissalinus (fl. 1148–78) 32
 Dorotheos of Sidon (first c. CE) 194, 241
 Double argument tables 819, 831, 884
 Double epicycle 569
 Duarte, king of Portugal (1433–38) 212–3
 Dūnash ibn Tamīm, Abū Sahl (fl. 900–960) 6, 353–368, 575, 684; works 353–6, 373, interest in *hay’a* 353–4, 356, 499–503; armillary sphere 354–368; the inhabited world 356–7
- lunar 15.9.973 705–708; 8.12.1052 681; 3.3.1067 681; 7.10.1074 681; 30.1.1078 681; 5.8.1263 828–9; 21.11.1471 207; 6/7.11.1519 214.
 Egidius of Tebaldis 198, 199, 857
 El Salado, battle (1340) 13; horoscopes of the battle 173–179, 796–7
 Enrique IV of Castile (1454–1474) 210
 Enrique de Villena (ca. 1384–1434) 330; *Tratado de astrología* 39, 40, 71, 208, 549, 553, 572–75, 830. Distances and sizes of the planetary spheres 573–5
 Enoch 194
 Ephemerides 876–7, 878–80
 Epistles of the Brethren of Purity (Rasā’il Ikhwān al-Ṣafā’) 497
 Equation of time 603, 622, 761–3, 766, 793–5
 Equatoria 3, 42, 153, 458, 475–492
 Eras 52–55, 710, 754, 843–4
 Eudoxus of Knidos (390–337 BC) 532, 576
- Fahṣ Ṭarīf: see El Salado
 Fardariyya/Fardār/Ifrādiyya 240–1, 300–1, 321
 al-Farghānī (d. 861) 546, 549, 552, 568, 570, 690, 821–3
 Fātimids in the Maghrib (910–972) 5
 al-Fazārī (second half of the 8th c.) 437, 506, 683, 691, 850
 Fernando de Fontiveros (fl. 1476–ca. 1480) 40
 Fernando Gallego (1440–1507) 209
 Ferrand Martínez (fl. 1391) 38, 885
 Fishtālī, Abū l-Rabīʿ Sulaymān b. Aḥmad (d. 1794) 468, 900
Fitna, period of civil war after the fall of the Cordoban Caliphate 161
 Flood 54
 Franco of Poland (end of the 13th c.) 495
 Frederick II of Sicily (r. 1220–1250) 521
 Fulbert, Bishop of Chartres (1006–28) 398–9
- Gedalia b. Shlomo b. Yahyā (fl. 1428–ca. 1480) 212
 Gerard of Cremona (fl. 1157–1176) 22, 33, 437, 494, 508–9, 545, 709, 724–5
 Gerbert of Aurillac (945–1003) 120, 124, 221–222, 380, 397–8
- Eclipses: solar 9.9.1438 572; 29.7.1478 873; 16.3.1485 210–212, 214; 8.6.1518 214;

- Ghāyat al-ḥakīm/Picatrix* 158, 205, 220, 579, 767
- Ghiyām b. Rujār (William, son of Roger) = William II of Sicily (1154–1166) 632, 765, 903
- Giovanni Bianchini (d. after 1469) 647, 830
- Gonzalo García Gudiel: see Gonzalo Pérez
- Gonzalo Pérez, archbishop of Toledo (1280–1299) 32
- Gonzalo de Vivero, bishop of Salamanca (d. 1480) 888, 893
- Guillaume Lunell (fl. 1384–88) 201
- Ḥabash al-Ḥasib (d. 869) 248, 333, 359, 377–8, 420, 443, 466, 469, 588, 826
- al-Ḥabbāk al-Tilimsānī, Abū ‘Abd Allāh Muḥammad (fl. 1271–1286) 18, 46, 127, 143
- al-Ḥabbāk al-Tilimsānī, Abū ‘Abd Allāh Muḥammad b. Aḥmad b. Abi Yahyā (d. 1463) 403, 795, 796, 815; *Fajr al-anḥār* 46, 47
- al-Ḥabbāk, Abū ‘Abd Allāh b. ‘Abd Allāh b. Aḥmad (fl. 1514–1562), *Natā’ij al-afkār fī sharḥ Rawḍat al-azhār* 46, 47, 76, 78, 79, 80, 81, 84, 86, 139, 437, 636–9, 640, 771, 794, 816, 879
- Ḥabṭaq method: see double argument tables
- Ḥabūs ibn Mākzan (r. 1019–1038), sultan of Granada 160
- Ḥaḍari, star 142
- Ḥafsid dynasty in Tunis (1206–1574) 7, 10
- al-Ḥajārī, Aḥmad b. Qāsīm (1570–after 1640) 41, 55, 56, 896–7
- al-Ḥakam I, Umayyad amīr of Córdoba (796–822) 7, 154, 214, 220
- al-Ḥakam II al-Mustanṣir, Umayyad Caliph of Córdoba (961–976) 24, 50, 141, 142, 149, 684
- al-Ḥākīm bi amr Allāh, Abū ‘Alī al-Manṣūr (996–1021) 632
- al-Ḥākīm al-Mirrikh (fl. 1304) 640
- Ḥanash al-Ṣan‘ānī (fl. ca. 700) 130, 141
- al-Harawī, Abū ‘Abd Allāh (?) 637
- al-Ḥarrānī (fl. middle of the 9th c.) 155
- al-Ḥasan b. ‘Alī b. Khalaf al-Umawī (d. 1205–6), *Kitāb al-amwā’, Kitāb al-lu’lu’ al-manẓūm fī ma’rifat al-awqāt bi l-nujūm* 9, 44, 45, 50, 74, 85, 115, 142, 146, 324
- al-Ḥasan b. Khalil b. ‘Alī al-Karādīsī al-Ṭubnī (1420–1482), *Ashkāl al-wasā’iṭ fī rasm al-munḥarīfāt wa l-basā’iṭ* 115
- Ḥasday b. Shaprūt, Abū Yūsuf (d. ca. 970) 6, 353, 380, 500
- al-Ḥashimī, ‘Alī b. Sulaymān (second half of the 9th c.) 443, 823
- Ḥassān b. al-Nu‘mān al-Ghassānī (fl. ca. 703) 129
- haylāj* 238, 252, 274, 276–7
- Ḥayyim de Briviesca (fl. ca. 1350–1400) 888
- Hebrew translations of Arabic sources in the 13th–14th c. 22
- Henrique the Navigator, prince of Portugal (1394–1460) 213
- Henry I (1100–1135) King of England 690
- Hermann of Carinthia (fl. 1138–1143) 27, 32, 181–183, 185, 376, 413, 688
- Hermann Contractus (1013–1054) 115, 382–3, 392, 398, 420
- Hermann the German (fl. 1240–1256) 32, 33
- Hermes 194, 218, 244, 264, 267, 287, 291, 294
- Hibat Allāh’s astrolabe (1120–21) 479
- Hipparchus (ca. 190–ca. 120 BCE) 588–591, 593, 595, 597–8, 608–10, 614, 617, 658, 662, 673, 753
- Hishām I, Umayyad emir of Córdoba (788–796) 154
- Hour 70–128
- Hugo Sanctiellensis (fl. 1145) 26–27, 32, 181, 183, 184, 185, 186, 194, 690; *Liber imbrum ab antiquo Indorum astrologo nomine Iafar editus* 182
- Humeniz 880
- Ḥusayn b. ‘Alī (fl. 1309) 469
- Ḥusayn b. Muḥammad al-Hamdānī (fl. 968) 142
- Ibn ‘Abd Rabbihi (860–940) 158
- Ibn Abī l-Fayyād (986–1066) 130
- Ibn Abī l-Rijāl al-Qayrawānī, ‘Alī (d. after 1035): Biography 227–8.
- Kitāb al-bārī fī aḥkām al-nujūm* 6, 14, 35, 114, 153, 169, 151, 198, 203–6, 208, 228–254, 278, 279, 281, 321, 554, 767, 856, 903–4; sources 228–234, 241–2; *anwār al-kawākib* 234–5, maximum equations of anomaly 236; nativities 237; *tasyir* 237, 273; Seven ages of

- men and the corresponding ruling planets 239–40; cycles 240–1, 274–5; human activities suitable to apply the technique of elections 242–3; Saturn-Jupiter conjunctions and other cycles 243–247, 300, 304; astrological geography 247–9; astrometeorology 249–50; horoscopes 251–254.
- Other works 254–7; *Hall al-'aqad wa bayān al-raṣad* 255, 764; *Urjūza fī dalīl al-ra'd* 255; *Urjūza fī aḥkām al-nujūm* 255–7
- Ibn Abī l-Ṣalt: see Abū l-Ṣalt
- Ibn Abī l-Shukr al-Maghribī, Muḥyī al-Dīn (d.1283) 586, 637–8; *Tāj al-azyāj* 12, 14, 106, 640–1, 704, 814–6, 817, 879, 904
- Ibn al-Ādamī (first half of the 10th c.) 580, 585, 683
- Ibn al-'Assāl (fl. 1281) 544
- Ibn 'Āṣim (d. 1013) 50, 85
- Ibn 'Azzūz al-Qusanṭīnī, Abū l-Qāsim (d. 1354)
- Kitāb al-fuṣūl fī jam' al-uṣūl* 12, 13, 14, 15, 173, 175, 275, 277–8, 283, 303–4, 307, 796.
- al-Zīj al-Muwāfiq* 12, 58, 60, 65, 66, 71, 93, 95, 96, 97, 98, 174–5, 261, 288, 303, 600, 615–6, 623, 626, 634–5, 638, 651, 659–60, 663–5, 667, 671–3, 675, 679–80, 692, 699, 709, 712, 714, 747–9, 795–813, 871–2, 901, 903–4; correction of Ibn Ishāq's mean motion tables 796–8; armillary sphere 353, 638, 797, 798; goal year cycles for the planets and the Moon 804–807, 883, 886, 889.
- al-Zīj al-Kāmil* 796
- Risāla fī adwār al-nayyirayn* 796
- Madkhal al-ṣinā'a 'alā madhhab al-jamā'a* 173
- Maqālat al-Zuḥal* 173
- Ḥurūz fī sharḥ al-rumūz* 173, 797
- Ibn Bajja (1070?–1138) 8, 499, 508, 514, 516–17, 527, 542, 575; competence in astronomy 518–21; *Kalām fī l-hay'a* 525–6
- Ibn al-Bannā' al-Marrākushī, Abū l-'Abbās Aḥmad b. Muḥammad b. 'Uthmān al-Azdi (1256–1321) 12, 13, 15, 16, 61, 145, 402–3, 465–6, 504
- Kitāb al-anwā'* 15, 50, 52, 74
- Minhāj al-ṭālib fī ta'dīl al-kawākib* 12, 53, 57–58, 60, 67, 68, 69, 93, 95, 261, 280, 285, 590, 600, 623–5, 634–7, 641, 651, 655, 658, 660, 663–71, 673–5, 679–80, 699, 717–20, 747–8, 764–792, 794, 799, 808, 813–4, 836–7, 841, 844–5, 847, 850, 872, 886, 894, 900, 903; displaced equations 667–671, 781–82; new procedures for the computation of planetary equations 781–791, 851
- Ilm al-awqāt bi l-ḥisāb* 48, 76, 78
- al-Kalām 'alā l-tasyīrāt wa-maṭāriḥ al-shu'ā'āt* 275, 285
- Ibn Bargūth, 'Abd Allāh (d. 1056) 496, 587, 588, 590, 608–10, 753
- Ibn Bāṣoh, Aḥmad b. Ḥasan/Ḥusayn (d. 1310) 45, 111, 266–7, 269, 282, 288, 401
- Ibn Bāṣoh, Ḥasan/Ḥusayn b. Muḥammad (d. 1316) 45, 79, 111, 136, 279–80, 402; Plate for all latitudes 263, 268, 281, 282, 285, 286, 287, 288, 443, 449, 466–469; plate of sines 471
- Ibn Bāṣoh, Ibrāhīm b. 'Alī al-Andalusī (14th c.?) 468
- Ibn al-Bayṭār (?) 767, 819
- Ibn al-Damīna (?) 194
- Ibn Ghāzī b. al-Kās, Abū Bakr, minister of the Merinid sultans Abū Fāris (1366–72) and Abū Zayyān (1372–74) 14, 173
- Ibn Ḥabīb: see 'Abd al-Malik ibn Ḥabīb
- Ibn al-Hā'im, Abū Muḥammad 'Abd al-Ḥaqq al-Ghāfiqī (fl. 1205) *al-Zīj al-Kāmil fī l-Ta'ālīm* 10, 53, 94, 97, 580, 586, 612, 614, 617, 657, 659–60, 672, 681, 721, 722, 728, 751–764, 767, 769, 793–4, 799, 842; trepidation 619–622, 627, 652, 749, 754, 776; solar model 664–5, 667, 755, 757; anomalistic year 754, 759; sidereal year 759–60; lunar model 676–9, 756, 758, 778; criticism of Ibn al-Kammād 744, 749, 752–3, 764; planetary parameters 757; equation of time and trepidation 761–3; planetary velocities
- Ibn al-Ḥajjām al-Qayrawānī (876–957) *Kitāb al-mawāqit wa ma'rifat al-nujūm wa l-azmān* 3

- Ibn al-Ḥātim, Abū 'Alī al-Ḥasan (fl. 939), *De imaginibus caelestibus* 159, 220
- Ibn al-Haytham (d. after 1040) 532
Shukūk 'alā Baṭlamīyūs 8, 508, 517,
 519–20, 521, 527, 553, 567, 571, 575, 637
Qaṣīda fī tarḥīl al-nayyirayn/ Qaṣīda fī l-hay'a 85
Fī hay'at al-'ālam 477, 484, 488, 498, 517,
 521, 526, 553–567
- Ibn Ḥayyān, *Muqtabis* 133–134, 154, 156–8, 683
- Ibn Ḥazm *Risāla fī faḍl al-Andalus* 21, 31;
 nativity horoscope 159
- Ibn Hibintā (fl. ca. 850) 229, 233, 237, 247,
 249, 273, 278, 281, 290, 323, 701, 779, 850
- Ibn Hilāl al-Sabtī, Muḥammad (first half of
 the 14th c.) 292, 637, 639, 640, 767
- Ibn Hishām al-Lakhmī (d. 1181–2) 85–86
- Ibn 'Idhārī (ca. 1250–ca. 1312) 161
- Ibn Ishāq al-Tamīmī al-Tūnisī, Abū l-'Abbās
 Aḥmad b. 'Alī (fl. 1193–1222) 11, 39, 57,
 66, 68, 69, 70, 172, 174, 632, 636–7,
 639–40, 655, 661, 764–781, 793–4, 799,
 802–3, 839, 848, 865, 868, 871–2, 903;
 solar model with variable eccentricity
 766; see Anonymous Tunisian
 recension of Ibn Ishāq's *zīj*
- Ibn al-Jallāb (alife ca. 1069?) 496
- Ibn Jaydūr (?) 637
- Ibn al-Kammād, Abū Ja'far Aḥmad b. Yūsuf
 (fl. 1116) 8, 38, 39, 41, 42, 53, 93, 94, 95, 97,
 504, 639, 735–753, 767
Kitāb mafātīḥ al-asrār 169, 291–300, 767;
Muqtabas 261, 272, 288, 600, 610–17,
 626, 642, 650–1, 655, 659, 661–5, 667,
 674–5, 679, 686, 688, 704, 712, 728,
 735–7, 739–751, 776, 779, 799, 802–4,
 811–3, 819, 833, 842, 848, 850, 865–8,
 870–1, 874–5, 880, 885, 890–1, 894, 904;
 sidereal and anomalous years 743–
 5; star list 747–750;
al-Kawr 'alā l-dawr 291, 611–2, 672,
 737–8, 745–6;
al-Amad 'alā l-abad 571, 611–2, 738–9;
 trepidation theory 610–17, 618, 621,
 627, 634, 738, 812, 874
- Ibn Khalaf al-Murādī, Aḥmad or Muḥammad
 (ca. 1050?) 4, 18; *Kitāb al-asrār fī natā'ij*
al-afkār 4, 73, 115–116, 118, 124–125, 126,
 147
- Ibn Khaldūn (1332–82) 497–8, 632, 765, 768
- Ibn al-Khaṣīb al-Kūfī, Abū Bakr al-Ḥasan
 (alive in 845) 186
- Ibn al-Khaṭīb (1313–1374) 5, 12, 45, 168;
A'māl al-a'lām 161; *Iḥāṭa* 171–172
- Ibn al-Khayyāt, Yahyā b. Aḥmad (977–1055)
Lāmīyya 15, 164–166; *Risāla fī l-qirānāt*
al-nujūmīyya 164
- Ibn Lubbūn, Abū 'Isā 328
- Ibn Marāna al-Sabtī (fl. 1069) 7
- Ibn Marzūq (d. 1379) 13
- Ibn Mas'ūd b. Farmīja (fl. 1372–1394) 815;
Ḥuṣūl al-maqāṣid wa l-āmāl min al-ṭuruq
wa l-fawā'id allatī tu'lamu minhā mudad
al-wulāt wa l-'ummāl 14, 174; *Risāla*
kāfiyat al-sayb fī l-'amal bi l-jayb
 105–106, 473
- Ibn al-Mashshāṭ al-Asturlābī, Muḥammad b.
 Sa'īd al-Saraqustī (11th c.) 404
- Ibn Masrūr 690
- Ibn al-Mu'ādh: see Khalaf ibn al-Mu'ādh
- Ibn Mu'ādh al-Jayyānī, Abū 'Abd Allāh
 Muḥammad b. Ibrāhīm b. Muḥammad al-
 Sha'bānī (d. 1093) 4, 8, 118, 611, 735
Liber de crepusculis 835; height of the
 atmosphere 755
Kitāb majhūlāt qisī al-kura 26, 475, 492,
 622, 708–9, 762
Tabulae Jahan 53, 57, 61, 62, 63, 65,
 94, 95, 97, 137, 139, 261, 263, 290, 704,
 708–714, 767, 811, 813, 903; maximum
 equations 712–13, 779, 850; tables of
 apparent sizes of planets 713; star
 table 713–14; constant precession
 713–14; tables of solar, lunar and
 planetary velocities 714, 808–9.
Risāla fī maṭraḥ al-shu'ā'āt 263–4,
 267–8, 283, 290–1, 714;
- Ibn al-Muftī, Abū Zayd 'Abd al-Rahmān
 b. 'Umar b. Aḥmad al-Sūsī al-Jazūlī al-
 Bu'aqilī (d. 1611) 637, 639, 641, 771
- Ibn Mun'im (12th–13th c.), *Fiqh al-ḥisāb* 10,
 751
- Ibn al-Muthannā, Aḥmad ibn 'Abd al-Karīm
 (fl. ca. 875–925?) 688, 690–1, 724, 823,
 841

- Ibn al-Naṭṭāḥ, Muḥammad b. 'Alī b. Yaḥyā,
 Abū 'Amir (12th c.?) 130, 147, 150, 403,
 407–9, 415
- Ibn al-Qifṭī (1172–1248) 462–3
- Ibn Qunfudh al-Qusanṭīnī (1339–1407) 14,
 15, 161, 173, 174, 235, 255–6, 272, 273,
 276–7, 279, 304–306, 307, 636–7, 658,
 795
- Ibn Qutayba (d. 889) 638
- Ibn al-Raqqām, Abū 'Abd Allāh Muḥammad
 b. Ibrāhīm b. 'Alī b. Aḥmad/Muḥammad
 b. Yūsuf al-Mursī al-Andalusī al-Tūnisī al-
 Awsī (d. 1315) 12, 16, 17, 144, 402–3,
 492, 636, 663–7, 717, 764–781, 799, 813,
 837, 844–5, 868, 871–2, 903
- Risāla fī 'ilm al-ẓilāl* 17, 109–110, 139
- al-Zīj al-Qawīm* 12, 17, 54, 57, 58, 61, 62,
 63, 95, 97, 138–139, 144, 178, 261, 272,
 280, 619, 623–6, 634–5, 637, 642, 655,
 660, 675, 699, 740, 769–70, 793, 795,
 812, 836, 847–8
- al-Zīj al-Shāmīl* 12, 54, 57, 58, 60, 61, 65,
 89, 93, 95, 97, 138–139, 261, 268, 272–3,
 280, 285, 287, 619, 623, 626, 627–8,
 634–6, 655, 660, 667, 674–5, 679, 699,
 752, 758, 769, 775, 793, 794, 798, 804,
 808–9, 836, 841, 847–8
- al-Zīj al-Mustawfī* 12, 54, 58, 59, 63, 64,
 79, 93, 95, 96, 97, 138–139, 261, 272,
 280–1, 282, 283, 284–5, 286–7, 288, 291,
 305, 619, 623–5, 626, 627–8, 634–5, 655,
 660–1, 667, 674–5, 679–80, 699, 704,
 766, 770–1, 793, 794–5, 848
- Ibn al-Rassām (?) or Abū 'Abd al-'Azīz
 al-Rassām 637
- Ibn Rushd (1126–1198) 9, 31, 142, 149, 499,
 508, 514, 516–17, 525, 526, 538, 546, 575;
 competence in astronomy: *Mukhtaṣar*
al-Majisṭī 521–22, 526–8; trepidation
 522, 527; observation of Suhayl 522;
 number of movers 528; *Jāmi'* on
 the Metaphysics 528, 541; *Tafsīr*/
Sharḥ on the *Metaphysics* 529; spiral
 motion (*lawlab*) 529–30, motions in
 opposite directions 529, 573
- Ibn Rushd al-Jadd, Abū l-Walīd (d. 1126) 143,
 146
- Ibn al-Ṣaffār, Aḥmad (d. 1035) 53, 106, 110,
 116, 131, 136, 147, 160, 270, 344, 376–7,
 408–9, 415, 417–8, 420, 431, 436–7, 473,
 688, 689, 700, 703–5, 715, 767
- Ibn al-Samḥ, Abū l-Qāsim (d. 1035) 3, 53,
 136, 160, 262–4, 267, 271, 288, 344,
 376–8, 387, 408, 415, 424, 426, 431, 473,
 496, 688; equatorium 476–479, 480,
 487, 490, 492, 692
- Ibn al-Samīna (d. 927–8) 158
- Ibn al-Sarrāj (d. ca. 1329) 449, 465
- Ibn Sayyid, 'Abd al-Raḥmān (fl. 1063–1096)
 496
- Ibn Shaddād, Abū l-Qāsim (fl. 1345) 17
- Ibn al-Shamir / Shimr (fl. 821–852) 24,
 154–5, 683
- Ibn al-Shāṭir (d. 1375–6) 12, 465, 487, 544,
 640–1, 814, 816–7, 894, 904
- Ibn al-Shāṭṭ, Qāsim b. 'Abd Allāh b.
 Muḥammad al-Sabtī (1246–1323) 404,
 409–10
- Ibn Sinā (980–1037) 497, 527
- Ibn al-Tarjumān (fl. 1300) 640
- Ibn Ṭibughā/Ṭaybughā 'Alā' al-Dīn Abū
 l-Ḥasan 'Alī (fl. 1400) 465
- Ibn Ṭufayl (d. 1185/6) 516–17, 526, 531, 575
- Ibn Wāfid (d. 1074–5) 441
- Ibn Yūnus (d. 1009) 487, 627, 632, 819
- Ibn Yūnus, Abū l-Ḥasan 'Alī al-Balansī
 al-Ḥakīmī (fl. 1331) 638, 824–5
- Ibn al-Zarqālluh, Abū Ishāq Ibrāhīm ibn
 Yaḥyā al-Naqqāsh (d. 1100) 3, 8, 11, 30,
 31, 39, 418–9, 431–2, 473, 496, 504, 507,
 519, 532, 571, 576, 577–9, 611, 681–2, 687,
 721, 724–5, 728–9, 735, 740, 755, 811, 824,
 867, 871–2, 884, 903–4
- Almanac* 39, 57, 63, 67, 68, 69, 94, 95, 96,
 106, 347, 506, 689, 692–3, 699, 724, 727,
 740, 805, 853, 857, 880–3
- Ṣafīḥas zarqālīyya* and *shakkāziyya* 104,
 136, 268, 281, 285, 286, 287, 324–5,
 344, 440–1, 444, 450–66, 468,
 556; *ma'mūniyya* 451, 460;
 'abbādiyya 460; meridian
 stereographic projection 441–3;
 meridian orthographic projection
 452, 456–459; sine quadrant 452,
 470–1; circle of the Moon and the
 computation of the geocentric lunar
 distance 453, 459–60; transversal
 ruler 455–6, 459; relative chronology of

- Ibn al-Zarqālluh, Abū Ishāq Ibrāhīm (cont.)
 the three universal astrolabes 460–2;
 transmission of the *ṣafīḥas* to the
 Mashriq and the Maghrib 462–466,
 682; armillary sphere 353, 368–372;
al-Ṣafīḥa al-zījīyya (equatorium) 479–487;
 identification of Mercury's deferent
 with an ellipse 483–5, 563;
On the solar year 521, 578, 654–667,
 772, 825, 828; motion of the solar and
 planetary apogees 578, 657–661,
 733, 776–7, 833, 848, 874; three kinds
 of solar year (tropical, sidereal and
 anomalistic) 661–2, 846; solar model
 with variable eccentricity 662–667,
 672, 674, 737, 739, 747, 778, 833, 850–1
On the motion of the fixed stars 329, 521,
 524, 534, 568, 578, 583, 585, 586–610,
 612, 614, 615, 617, 618, 622, 623, 639,
 642–4, 749, 762, 857; cyclical variation
 of the obliquity of the ecliptic 603–
 7, 627, 641, 762, 832, 837
 Invalidity of Ptolemy's method to
 determine Mercury's apogee 578;
 Modified lunar model 578–9, 676–681,
 811, 833–4
 See Toledan Tables
- Ibn al-Zarqiyāl: see Ibn al-Zarqālluh
- Ibn Zurayq al-Khayrī/Jizī, Muḥammad b. 'Alī
 b. Ibrāhīm (d. 1400) 816
- al-Zayyāt, Ishāq b. al-Ḥasan b. Abī l-Ḥusayn
 (fl. 975–1025) 709
- Ibrāhīm II, Aghlabī amīr in Tunis (875–902)
 5, 153–4
- Ibrāhīm b. 'Abd al-Karīm (fl. 1066) 400
- Ibrāhīm b. 'Alī al-Andalusī al-Marrākushī
 (fl. 1727) 344
- Ibrāhīm b. Muḥammad al-Balawī (fl. 1686)
 469
- Ibrāhīm b. Muḥammad b. Futūḥ/Fattūḥ al-
 'Uqaylī al-Gharnāṭī (d. 1464) 403
- Ibrāhīm b. Muḥammad b. al-Raqqām
 (fl. 1320) 402
- Ibrāhīm b. Sa'īd al-Sahlī (fl. 1067–1090):
 celestial globes 328–9; astrolabes
 377, 400
- Ibrāhīm ibn al-Sahlī (fl. 1086) 400
- Ibrāhīm b. Sinān (907–946) 469, 580, 585, 599
- 'Imād al-Dawla, king of Zaragoza (1110–1130)
 26
- Incident horizon (*al-ufq al-ḥādith*) 262
- Intihā'* 244, 246–7, 274–275, 302–3
- Íñigo López de Mendoza, marquis of
 Santillana (1398–1458) 572
- Isaac b. Abraham b. 'Ezra (fl. 1143) 541, 824
- Isaac b. al-Ḥadīb (fl. 1370–96) 868, 870–1;
 equatorium 491–2
- Isaac Israeli (fl. 1310–30), *Yesod 'Olam* 38–
 39, 544, 577, 656, 720, 828, 868, 870–1,
 874, 888; *Sha'ar ha-shamayim* 874–5
- Ishāq b. Sid/ Rabiṣag aben Cayut (13th c.)
 36, 38, 39, 119, 125, 269, 281, 344, 368,
 443–4, 474–5, 828, 856, 868, 888
- Ishāq b. Sulaymān al-Isrā'īlī (ca. 858–955) 353
- Isidore of Seville (c. 560–636) 473, 877
- Ismā'īl b. Aḥmad al-Qurtubī, Abū l-Walid (?)
 463
- Ismā'īl al-Ṭallā' (d. ca. 900) 5, 153–4
- Ismā'īl II of the Naṣrid dynasty of Granada
 (1359–60) 12
- Ismā'īl b. al-Qā'im, al-Manṣūr, Fatimid Caliph
 (946–953) 353, 356, 500
- al-Istijī, Abū Marwān (fl. 1068) 54, 163–4,
 230, 274, 276–7, 279, 283, 286–7, 291,
 293, 301–2, 578, 580, 585, 617–9, 722
- Iudicia Almansoris* 181
- Jābir ibn Aflaḥ (first half of the 12th c.):
 chronology and works 508; *Iṣlāḥ
 al-Majistū* 8, 26, 475, 499, 509–16,
 521, 524, 527, 536, 762; observational
 instrument 492–5; title and
 recensions 509; aim of the
Iṣlāḥ 510; criticism of the
Almagest 511–15; planetary
 order 514–15, 531
- Jacob Anatoli (ca. 1194–ca. 1256) 521
- Jacob Corsuno (fl. 1376–1380) 37–38, 201,
 202, 636–7, 649, 860–1, 866–7
- Jacob b. David Bonjorn/Yomṭob: see Bonjorn
- Jacob Golius (1596–1667) 897
- Jacob b. Makhir b. Tibbon (ca. 1236–ca. 1305)
 450, 494, 554, 883–4, 888, 889
- Jacob Puel: see Bonjorn
- al-Jādirī, Abū Zayd 'Abd al-Raḥmān b.
 Muḥammad (ca. 1375–1416) *Rawḍat*

- al-azhār fī 'ilm waqt al-layl wa l-nahār;
Iqtitāf al-anwār min Rawḍat al-azhār* 16,
47, 48, 51, 52, 54, 73, 75, 76, 77, 78, 79–82,
84, 86, 139, 437, 636, 638–9, 771, 815
- al-Jaghminī (fl. 1200–20) 499
- Jamāl al-Dīn al-Māridīnī (d. 1406) 464–5
- al-Jazzār (or al-Ḥarrār), 'Alī b. Ibrāhīm
(fl. 1328) 449, 469–70
- João I, king of Portugal (1385–1433) 212
- João II, king of Portugal (d. 1495) 212
- Johan Daspa 331
- Johannes de Bonie (fl. 1464) 105
- Johannes David Toletanus (fl. after 1187) 170
- Johannes de Dumpno (fl. 1262) 611, 672, 736,
737, 745
- Johannes de Halebeke/Harlebeke (fl. 1303)
335
- Johannes Hispalensis (fl. 1135–1143) 181–186,
198, 207, 209, 300, 377, 417, 426, 431,
642–3
- Johannes Hispanus (d. 1215) 33, 479, 642–3,
721, 724
- John, archbishop of Toledo (1152–1166) 32
- John of Gorze (fl. 953) 380
- John of Lignères (fl. 1320–35) 644, 808, 829,
831, 834–5, 840, 855, 871
- John of London (fl. 1246) 418, 426
- John of Murs (fl. 1310–48) 647, 829, 831, 840,
850, 855
- John Pecham (ca. 1230–92) 543
- John Philopponus (6th c.) 539
- John of Saxony (fl. 1321–27) 829–31, 840, 855
- John Vimond (fl. 1320) 644, 829, 833, 840,
848, 851, 855–6
- José Vizinho (fl. 1496) 41, 55, 892
- Josef Franco, son of Gedalia (fl. 1411) 208
- Joseph ibn Naḥmias (ca. 1400) 544,
568–570, 576
- Joseph b. Yehudah b. Sham'un (d. 1226)
508–9
- Joseph Hispanus 397
- Joseph Sapiens 222
- Juan I, king of Aragón (1387–1396) 200, 205,
410; astrologers 201
- Juan Gil de Castiello or de Burgos (fl. 1350–52)
39, 208, 615, 867, 870
- Juan Gil de Zamora (13th c.) 547–9, 551
- Juan González of Burgos, translator (end of
the 13th c.) 32
- Juan de Salaya or Selaya (fl. 1464–81) 40, 41,
212, 651–3, 888
- Juan de Zúñiga y Pimentel (1459–1504) 214,
893
- Judah b. Asher II of Burgos (d. 1391) 39, 651,
828, 833, 868, 871, 874, 888, 891, 894
- Judah b. Matqa (13th c.) 544
- Judah b. Verga (fl. 1455–80) 41, 570, 867,
869–73, 875, 888; observations 871–2;
Huqot Shamayim 41, 42
- al-Kaff al-Khaḍīb 360, 504
- Kaliyuga* 54
- Kalpa* 308
- Kanz al-Yawāqūt* (early 14th c.) 738
- Karbāšo/Karbāša, Abū l-Ḥasan 'Alī b. Māmī
al-Ḥanafī (fl. before 1750) 818
- al-Kāshī (d. 1436) 464, 485, 487
- al-Kātib al-Andalusī: see 'Arīb b. Sa'īd
- al-Kawm al-Rishi, Shihāb al-Dīn Aḥmad b.
Ghulām Allāh (d. 1432) 816
- Kazasker Abdurrahman Mueyyezade (d. 1516)
894
- Khalaf ibn al-Mu'adh (10th c.) 19, 373–375,
389, 390, 393, 399, 412
- Khālīd b. 'Abd al-Malik al-Marwarrūdhī
(fl. 800–850) 687, 823–4
- al-Khamā'irī, Muḥammad ibn Fattūh
(fl. 1216–1236) 104, 401, 465
- Khandakhadyaka* 779, 850
- al-Khāzin, Abū Ja'far (d. ca. 961–971) 476,
479, 480, 580, 599
- al-Khāzinī (12th c.) 335
- al-Khayyāt, Abū 'Alī (fl. ca. 854), *Kitāb al-
mawālīd* 181, 185, 194, 767
- al-Khwārizmī (fl. 830) 19, 137, 381; *Sindhind
zīj* 2, 7, 27, 29, 39, 52, 53, 56, 64–65, 73,
156–8, 160, 165–6, 168, 170–1, 196, 253–4,
259, 260, 290, 300, 312, 437, 440, 506,
683, 684, 685–6, 688–708, 712, 724–7,
732–735, 742, 750–1, 767, 776, 779, 811–3,
819, 841, 850, 854, 866–8, 878, 885, 890,
903; solar and lunar tables 691–2;
planetary tables and computation
of longitudes 693–697; method of
declinations 691–2; method of sines
696; ascensional differences 698–
700, 740; Maslama's intervention and
later Iberian additions 700–702

- al-Kindī (d. ca. 873) 185, 194, 232–3, 249, 321, 356–7, 363
 Kūshyār b. Labbān (ca. 970–1030) 321–2
kylisis 530
- Latin astrological tradition 2, 15, 154
 Levi ben Gerson (1288–1344) 476, 532, 544, 553, 869–70, 875, 888, 891, 904
Liber de motu octave spere 578–586, 587, 612, 617, 618, 621, 626, 627, 642–4, 648, 729, 882, 884
 Libraries in al-Andalus 21, 23–30, 43; Royal library in Córdoba 23–25, 28, 44, 159; Library of King al-Mu'taman in Zaragoza 25–27, 193; Libraries in Toledo 27–30
 Lot (*sahm*, pl. *sihām*) 321
 Lunar cycle in perpetual almanacs 38, 41, 805–7, 883
 Lunar month: see crescent visibility
Lunaria 207
 Lupitus Barchinonensis: (fl. 975–995) 18, 32, 222, 380, 397
- Macrobius (end of the 4th c. CE) 546, 550–3
 Madīnat al-Zahrā' 24, 139–140
 Madrasa Ya'qūbiyya (= Madrasat al-Ṣaffārīn) in Fez 143
magistri probationum 823, 826
 Mahdiyya, founded 921 5, 354, 355, 358, 360, 361, 366
 Maimonides (1135–1204) 57, 60, 62, 116, 189, 311, 508–9, 516, 526, 527, 544, 575, 836, 888
 Mālik b. Anas (d. 795–6) 3
 Mālik b. Wuhayb (1061/2 – 1130/1) 8, 143
 al-Ma'mūn, Abbasid Caliph (813–33) 504, 507, 546–7, 588, 656–7, 684, 687, 823, 828
 al-Ma'mūn b. Dhī l-Nūn, king of Toledo (1043–1075) 164, 328, 443, 450, 460, 462, 577
 Manuel I, king of Portugal (r. 1495–1521) 212, 858
 al-Manṣūr, Ismā'il b. al-Qā'im: see Ismā'il b. al-Qā'im
 al-Manṣūr ibn Abī 'Āmir, powerful governor in Córdoba (981–1002) 24, 159
- al-Maqqarī (1577–1632) 141
 al-Marghīthī/Mīraghtī, Abū 'Abd Allāh Sayyidī Muḥammad b. Sa'īd (d. 1678), *Al-Muqni' fī ikhtisār 'ilm Abi Muqri'*; *al-Mumti' fī sharḥ al-Muqni'* 46, 47, 51, 75, 86
 Marinid dynasty in Morocco (ca. 1269–1465) 7
 Mark of Toledo (fl. 1193–1216) 27, 33
 Martianus Capella (360–428 CE) 221, 546, 548–9, 553
 Martin I, king of Aragón (1396–1410) 206, 859
 Martín Gil, abbot of Santa María de Moreruela (fl. 1260–67) 545
 Marwān b. Ghazwān (9th c.) 154–5, 158
 Māshā'allāh (d. ca. 815) 184, 194, 202, 209, 221, 241, 312–3, 318, 779, 850; pseudo-Māshā'allāh on the construction and use of the astrolabe 288, 413, 417–436; influence of Maslama 417–8, 421–8, 431, 433
 Maslama b. Aḥmad al-Majrīṭī (d. 1007–8) 2, 19, 27, 29, 53, 159, 161, 164, 258, 260, 288, 290, 413, 417, 496, 503, 544, 591, 684, 903; on stereographic projection 374, 376, 429; revision of al-Khwārizmī's *zīj* 683, 688–708; star table and its influence on the "old corpus" 382–7, 425–6, 451, 504, 588, 608–10; influence on pseudo-Māshā'allāh treatises on the astrolabe 417–8, 421–428, 431, 433; precession 504
 Maslama b. al-Qāsim al-Qurṭubī, Abū l-Qāsim (d. 964), *Ghāyat al-ḥakīm/Picatrix* 158, 205, 497, 579
 al-Maṣmūdī, 'Ubayd Allāh Abū 'Alī Ṣāliḥ ibn Abī Ṣāliḥ (fl. 1314) *Kitāb al-qibla* 145, 149
 Mathematical astrology 152–3, 257–291
 al-Māwāsī, Abū l-'Abbās Aḥmad b. Muḥammad b. 'Īsā (d. 1505) 139, 145
 Menelaos (fl. 98 CE) 330, 510, 654
 Mercury: identification of Mercury's deferent with an ellipse 483–5, 563; apogee in al-Khwārizmī-Maslama's *zīj* 695
 Meridian stereographic projection 441–3
 Meridian of water 689, 703–708, 709, 725, 740, 870, 892

- Method of declinations 691–2
 Method of sines 696
 Michael, bishop of Tarazona (1119–1151) 26, 32
 Michael Scott (fl. 1217–27) 32, 33, 531, 539, 540, 543, 544
Minkān in Granada, Fez and Tlemcen 122–123
 Miqāt: see timekeeping
 Miró Bonfill, bishop of Gerona (d. 984) 21, 32, 221, 397
 al-Mittijī, Abū ‘Alī (d. ca. 1150), *Kitāb dalā’il al-qibla* 129, 144–145, 147, 151
 al-Mizzī, Muḥammad b. Aḥmad (b. 1291, alife 1334) 468–9, 640, 816
 Months, astrological: see astrological months
 Moon: crescent visibility 55–70, 367, 836, 888; lunar mansions 46, 47, 85–93, 156–8, 332, 411; computation of the geocentric lunar distance 453, 459–60; Ibn al-Zarqālluh’s modified lunar model 578–9, 676–681; Khwārizmī’s *zīj* 692; Ibn al-Zarqālluh’s *Almanac* 692–3, 727, 883; lunar cycle 805–7
 Moses b. Abraham of Nîmes (fl. 1460) 869
 Moses Farissol Botarel of Avignon (late 15th c.) 869, 873
 Mosheh b. Shemuel b. Yehudah b. Tibbon (fl. 1259) 494, 531, 544
 Mosheh, collaborator of Alfonso x 331
 Mosheh Sefardi: see Petrus Alfonsi
Mu’addil 409; see *muwaqqit*
 Mu’ayyad al-Dīn al-‘Urḍī (d. 1266) 371
 Muḥammad I, Umayyad amir of Córdoba (852–886) 24, 158, 683
 Muḥammad II of Granada (1273–1302) 769
 Muḥammad V of Granada (1354–59 and 1362–91) 13, 122, 171; nativity horoscope 171–173
 Muḥammad VI of Granada (1360–62) 13
 Muḥammad b. ‘Abd Allāh al-‘Adhrā’ 155, 158
 Muḥammad b. ‘Abd Allāh b. Muḥammad, Abū ‘Abd al-Raḥmān: see al-‘Utāqī
 Muḥammad b. ‘Abd al-‘Azīz b. ‘Abd al-Salām al-Wazzānī al-Wazkānī (fl. 1769) 899, 901
 Muḥammad b. Abī Jum’a (end 10th c.) 159
 Muḥammad b. Aḥmad al-Baṭṭūṭī (fl. 1730) 106, 474
 Muḥammad b. ‘Alī al-Anṣarī (d. 1361) 403
 Muḥammad b. ‘Alī Ḥajjāj (fl. 1597) 106, 473–4
 Muḥammad b. ‘Alī al-Siddīnī (fl. 1317) 127
 Muḥammad b. ‘Alī b. Yahyā al-Ṣanhājī al-Naṭṭāḥ (fl. 1317) 127
 Muḥammad al-‘Arabī, Abū ‘Abd Allāh (fl. 1346–48) 18, 127
 Muḥammad b. Faraj (fl. 1476) 402
 Muḥammad al-Fihri, minister of Ismā‘īl II of Granada (fl. 1359) 12
 Muḥammad b. al-Ḥasan (d. 1022) 251
 Muḥammad b. al-Ḥusayn, Abū I-Ḥasan, probably governor of Mahdiyya (first half of the 10th c.) 352–3
 Muḥammad ibn Kunāsa (d. 824) 638
 Muḥammad b. Muḥammad b. Hudhayl (fl. 1253) 465
 Muḥammad al-Mu‘ṭī b. Aḥmad al-Ṭayyib b. Muḥammad Marīn al-Najjār al-Ribāṭī (d. 1808) 900–2, 904
 Muḥammad al-Qālī, Abū ‘Abd Allāh (early 18th c.) 818
 Muḥammad b. Qāsim al-Qurṭubī (fl. 1320) 401
 Muḥammad al-Ṣabbān (fl. 1073–1082) 266, 400
 Muḥammad b. al-Ṣaffār (fl. 1026–9) 399
 Muḥammad b. al-Sahlī (fl. 1090) 400
 Muḥammad al-Sanūsī (d. 1490) 637
 Muḥammad al-Ṣinhājī, Abū ‘Abd Allāh (fl. 1317) 18
 Muḥammad b. Sulaymān b. al-Hannāt (d. 1045) 163
 Muḥammad b. al-Ṭayyib b. al-Makkī al-Ajbarī al-Ḥasanī (fl. 1799) 817
 Muḥammad b. Yūsuf b. Ḥātim (fl. 1240) 401
 Muḥammad b. Yūsuf al-Laythī, al-Ishbīlī al-Sabtī, Abū Bakr (13th c.), *Urjūza fī tawassuṭ al-manāzil ‘inda ṭulū’ al-shams* 87
 Muḥammad b. Yūsuf b. ‘Umar b. Shu‘ayb al-Ḥasanī al-Sanūsī al-Tilimsānī (1428–90) 403
 Muḥammad b. Zawāl (fl. 1482) 402
 Muḥyī al-Dīn al-Maghribī: see Ibn Abī I-Shukr

- al-Mu'izz b. Bādīs, Zīrid king in Qayrawān (r. 1016–1062) 169, 227
- Mujāhid, king of Denia (ca. 1010–1045) 160
- Mumtaḥan zīj* 51, 363, 684, 685–8, 692, 722, 735, 767, 820
- Muntakhab zīj* 617, 618, 619, 625, 712
- al-Muqri', Abū l-Ḥasan (fl. 1384) 44, 76, 78
- al-Muqtadir ibn Hūd, king of Zaragoza (1046–1081) 25, 164
- Mural quadrant (*libna*) 641
- Mūsā Jalinūs/ Moses ben Judah Galeano (fl. 1500) 41, 342, 894–6
- Mūsā b. Nawbakht (d. ca. 1000) 318, 767
- Mūsā ibn Nuṣayr (640–717) 55–56, 141, 154, 156
- Muṣaḥḥaḥ zīj* 722–3, 730
- Muslim b. Aḥmad al-Laythī, Abū 'Ubayda (d. 907) 131–132
- al-Musta'in, Abū Sālīm Ibrāhīm, Merinid sultan (1359–1361) 13, 173, 715
- al-Musta'in, Sulaymān, Umayyad Caliph of Córdoba (r. 1009, 1013–1016) 164
- al-Musta'in II, king of Zaragoza (1085–1110) 517
- al-Musta'li b. al-Mustanṣir, Fatimid Caliph (1094–1101) 405, 498
- al-Mustanṣir, king of Zaragoza (1130–1146) 26
- al-Mu'taḍid b. 'Abbād (1042–1069) 462
- al-Mu'taman, Yūsuf ibn Hūd, king of Zaragoza (1081–85) 25–27, 164, 517
- al-Mu'tamid b. 'Abbād (1069–1091), king of Seville 163, 450–1, 460–1, 480, 571
- al-mutaqaddam* 276
- muwaqqit* 14, 16, 18, 44, 45, 106, 409, 466, 715
- Muẓaffar b. 'Abd Allāh (fl. 1394) 815
- al-Muẓaffar, 'Abd al-Malik (975–1008), governor of Córdoba: nativity horoscope 159
- al-Nafawī, Abū Zayd 'Abd al-Raḥmān al-Jānātī (late 16th c.) 637
- Najm al-Dīn al-Miṣrī (fl. 1325–40) 449–50, 463, 465, 468
- al-Nāṣir, Abū 'Abd Allāh, Almohad Caliph (1199–1213) 10, 751
- Naṣir al-Dīn al-Ṭūsī (1201–1274) 499, 569, 614
- Naṣr, Abū l-Faṭḥ, *ḥājiḥ* of 'Abd al-Raḥmān II (d. 851) 156–8
- Naṣrid dynasty in Granada (1232–1492) 10
- Naṣṭūlus (fl. 890–930) 374, 405
- al-Nayrizī, al-Faḍl b. Hātim (d. ca. 922) 823–4
- naw'* 49
- Negro family 212
- Nicholas, Byzantine monk in Córdoba (ca. 950) 374
- Nicholaus of Heybech (fl. ca. 1400) 871
- Nicholaus Polonius/ Nicolas Polonio (fl. 1460–64), *Tabulae resolutae* 40, 41, 830, 869
- Nocturnal 119–121
- Nuḥūs*: unfortunate indicators in a horoscope 157
- Obliquity of the ecliptic 603, 605–7, 612–14, 619–622, 627–34, 640–1
- Observations 638–40, 656, 681, 828–9, 859–60, 871–3
- Old Catalan corpus of texts on the astrolabe and other astronomical instruments: see astrolabe
- Opening of the door (*fath al-bāb*) 249–50, 321
- Oliva, abbot of Ripoll (1008–46) 398
- Ortiz de Zúñiga (fl. 1677) 827
- Patronage of translations 32–38, 43
- Pedro IV, king of Aragón (1336–1387) 37–38, 126–127, 200, 329, 410–11, 642, 649; astrologers 201; Almanac (Ephemerides for 1361–1433) 202, 859–860, 879–80; Tables of Barcelona 858–867, 880; star tables 859–60; trepidation 649, 859; solar apogee 658
- Pedro, Prince of Portugal (1392–1449) 213
- Pedro González Pérez (d. 1267) see Petrus Gallecus
- Pere Gilbert: see Petrus Gileberth
- Pero Fernández (14th c.) 38, 207
- Peter the Venerable, abbot of Cluny (d. 1156) 32
- Petrus Alfonsi (ca. 1062–after 1110) 27, 53, 189, 688, 690
- Petrus Gallecus, bishop of Cartagena (1250–1267) *Summa de astronomia* 40, 545–553, 567, 575; sources: Ibn Rushd and al-Biṭrūjī 546–7; distances and sizes of the celestial bodies 547–552

Petrus Gileberth (d. 1362) 37, 201, 329, 649, 859, 879
 Petrus Raimundus 411
 Petrus de Reggio 199, 857–8
 Peuerbach, Georg (1423–1461) 485
Picatrix: see *Ghāyat al-ḥakīm*
 Planetary conjunctions 9, 15, 157, 214, 300, 307–319, 320; Saturn and Jupiter in 571 162, 245, 304–6, 317–9, 322, 890; in 610 323; in 809 162; in 848 246; in 1007 160, 162; in 1067 161, 310; in 1107 518; in 1226 314–317 in 1305 175, in 1345 176, 310, 797; in 1484 111; Saturn and Mars in 1033 166; in 1108 518–9; in 1228 320; five planets in Libra (1186) 169–171
 Planetary cycles (goal-years) 804–5, 807, 877–8, 880, 883
 Planetary retrogradation 231
 Planetary velocities 714, 806–810, 871
 Planets: in al-Khwārizmī-Maslama's *zīj* 693–7
 Plato (ca 427–347 BCE), *Timaeus* 530
 Plato of Tivoli (fl. 1133–1145) 27, 180–1, 183, 186, 187, 188, 377, 820
 Prediction of the end of the Cordoba Caliphate 156
 Precession 0° 581–2, 600–1, 748–9, 872, 874, 894
 Precession and trepidation 29, 504–5, 546, 570, 579–654, 713–14, 812; decline and fall of trepidation in the Maghrib 636–641; id. in the Iberian Peninsula 872–874, 889
 Profeit Tibbon: see Jacob b. Makhir b. Tibbon
 Profiat Duran (d. 1415) 570
 Projection of rays 15, 283; direct method on the ecliptic 283–5; right ascension method 286; oblique ascension method 286–7; single position semicircle method 287; single hour line method 287–9, 813; four position circles method 289–91; seven hour lines method 291
 Propatius Iudeus: see Jacob b. Makhir b. Tibbon
 Prorogation: see *tasyīr*

Prosdocimo de Beldomandi of Padua (fl. 1424) 647
 Ptolemy (2nd c. CE) and pseudo-Ptolemy 501; *Almagest* 2, 8, 28, 54, 194, 311–2, 329, 333–4, 362, 369, 371, 492–3, 496, 499, 510, 511–5, 519–23, 524, 527, 545–6, 588–591, 593–5, 597–8, 608–10, 614, 617, 642, 647–8, 654, 662, 749–50, 753, 821, 826, 852–4, 865, 868, 877, 881; *Planisphaerium* 18, 421, 429, 444, 460; Geography 28, 356–9; Optics 28; *Qānūn/ Handy Tables* 54, 28, 357, 501–2, 747, 779, 786, 787, 804, 827, 850–1, 865, 903; *Tetrabiblos (Quadripartitum)* 28, 181, 202, 208, 279, 287, 293, 554, 833, 849; *Hypotheses* 362, 498, 502, 521, 524, 549, 551–2, 573; *Kitāb al-Thamara (Karpós, Centiloquium)* 8, 181, 183, 193, 202, 294.
 al-Qā'im [ibn] al-Mahdi, Fatimid Caliph (934–946) 356
 al-Qabīṣī (fl. ca. 950) *Introductorius* 182, 186, 207, 249–50, 321
 Qāḍī Ḥasan al-Makkī (17th c.) 817
 al-Qādir b. Dhī l-Nūn, Yaḥyā 11, king of Toledo (1075–85) 166, 328, 460, 578
 Qāḍīzāda al-Rūmī (15th c.) 464
 al-Qalaṣādī (ca. 1412–1486) 11, 483
Qalb al-asad (Regulus) 329, 362, 587–591, 608–10, 627, 634, 651, 731, 753
al-Qānūn 23, 683, 886
 Qāsim b. Muṭarrif al-Qaṭṭān (fl. ca. 950) 5, 6, 73, 78, 85, 116, 121, 498, 502–6, 575; stars 504; precession of the equinoxes 504–5; names of planets and zodiacal signs 505
qaṭ', astrological cut or interruption in the sequence of events 159
qāṭi' pl. *qawāṭi'* 238, 276
 Qayrawān, mosque of Sidi 'Uqba 113, 128–130
Qibla 16, 45, 128–151, 352, 410, 439
 Qayrawān, Mosque of Sidi 'Uqba 128–130; Zaragoza 130–131; Córdoba 130–135, 408–10; mosques in Granada 144, 410

- Qibla indicators 147–151: rising point of Suhayl 130, 131, 134, 142–143, 149–150; rising point of Qalb al-'Aqrab 148; rising Sun in the winter solstice 129–130, 147–148; rising Sun in the summer solstice 131; rising Sun in the equinoxes 148–149; midpoint of the South-Eastern quadrant of the horizon 150; South 150; rising point of other stars 151
- Legal criteria: *taqlid*, *ijtihād*, *jiha*, *samt* 144–147
- Orientation of mosques following the street-plan 129, 131; orientation in parallel to the Ka'ba 131, 132
- Astronomical solutions of the *qibla* problem 135–139; method of the *zījes* 137–139, 473, 709–10
- Astronomers and the orientation of mosques 139–144; Madinat al-Zahrā' 139–140
- qisma 244, 246–7, 275, 321
- Quadrants 18, 100–108, 324, 470–75; *zarqālī* and *shakkāzī* quadrants 464–5; almucantar quadrant 473–4; myrtle quadrant 474; cuadrante sennero 474–475; see Timekeeping
- al-Qurashī, Abū Bakr 'Ubayd Allāh (d. 1052), *Ta'lif fi awqāt al-ṣalawāt 'alā madhāhib al-'ulamā'* 44; *Taḥṣirat al-mubtadi' wa tadhkirat al-muntahī fi ma'rifat al-awqāt bi l-ḥisāb min ghayr āla wa-lā kitāb* 44
- Qustā b. Lūqā (fl. second half of the 9th c.) *De differentia spiritus et animae* 181–2; treatise on the celestial sphere 330–341
- Qutb al-Dīn al-Shīrazī (d. 1311) 514
- al-Quwaydis, Abū Ishāq b. Lubd (d. 1062) 496
- Rabī' b. Zayd (fl. 955)/ Recemundus 3, 380, 684
- Radolphus of Liège (fl. 1025) 390
- Ragimbaldus of Köln (fl. 1025) 390
- Ramar/ Renardi/Remaur/Raymar Ohem (fl. 1392ff) 201
- Ramon Llull: (ca. 1232–1316) 20, 36–37, *Tractatus novus de astronomia/ De nova astronomia* 20, 37, 200, 223–227; *Liber principiorum medicine* 120–121; *De nova geometria* 120–121
- raqīb* 49
- Raymond Bancal (fl. 1326) 207, 883
- Raymond of La Sauvetat, archbishop of Toledo (1125–1152) 32
- Raymundus Bancelis: see Raymon Bancal
- Raymundus Lullius: see Ramon Llull
- al-Rāzī, Abū Zakariyā' (d. 925) 194
- Recemundus: see Rabī' b. Zayd
- Regiomontanus (1436–1476) 510, 544, 655, 893
- Reinhold, Erasmus (1511–43) 485
- Revolution of the months (*taḥwīl al-shuhūr*) 241
- Revolution of the years (*taḥwīl al-sinīn*) 243–7
- al-Riqūṭī al-Mursī, Muḥammad b. Aḥmad (fl. 1266) 827–8
- Risālat al-hay'a* by an anonymous Toledan astronomer of the 11th c. 370
- Risāla fi awqāt al-sana* 50, 75
- Robert of Chester (fl. 1140) 824
- Robert Grosseteste (ca. 1175–1253) 543
- Robert of Ketton (fl. 1141–1157) 27, 32, 18, 185, 721
- Robert Losinga, bishop of Hereford (12th c.) 689–90
- Rodrigo of Vassurto (fl. 1487–1504) 40, 210
- Roger Bacon (ca. 1214–ca. 1292) 543
- Roger of Howden (end 12th c.) 170
- Roger of Wendover (13th c.) 170
- al-Rudānī. Muḥammad b. Muḥammad b. Sulaymān (1627–83), *al-Nāqī'a 'ala al-ālat al-jāmī'a* 343–4, 346, 351
- Rudolf of Bruges (fl. 1144) 430
- al-Rukānī 767
- Rustum, dynasty (778–909) 7
- Rūṭa (Rueda del Jalón) 26
- Sahl b. Bishr (fl. 821–850) 185, 194, 209, 235
- Sahl b. 'Uthmān b. Abī Ḥabīb (fl. 968) 142
- Ṣā'id al-Andalusī (1029–70) 3, 21, 23, 28, 163, 681, 721–23, 735, 839; *Iṣlāḥ ḥarakāt al-nujūm* 29, 580; *Ṭabaqāt al-umam* 21, 23, 130, 163, 461, 496, 577, 579–80, 610, 618, 622, 689, 691
- Sa'id b. Khazrūn (d. 1037–8) 228

- Salamanca University: chair of Astrology
(ca. 1460–1504) 40, 209–10
- Samuel ben Jehudah of Marseilles (fl. 1335)
494
- Samuel ben Tibbon (ca. 1165–1232) 544
- Sancho of Aragón, archbishop of Toledo
(1266–1275) 32
- Sanjaq Dār al-Tūnisi, Muḥammad al-Sharīf
(fl. late 17th c.), *Zij al-Sharīf* 642, 817–820
- al-Sanjufīnī (fl. 1366) 629, 794, 837
- al-Saqaṭī (fl. ca. 1200–1225) 153
- Scorpio 20°: period of rain 249, 320–1
- Seasons of the year 51
- Secants table 96
- Second accession (*al-iqbāl al-thānī/ iqbāl al-
‘amūd fī l-quṭr*) 603, 622, 626, 761–3
- Seniofredus: see Lupitus Barchinonensis
- Sententiae astrolabii* 18
- Seven ages of men and the corresponding
ruling planets 239–40
- al-Sijzī (ca. 950–ca. 1025) 429–30
- al-Simāk al-A’zal* 651
- Sindhind* 23, 51, 683, 709
- Size of the Mediterranean 703–708
- Size of the Universe 40, 547–552, 573–575
- Solomon Franco (fl. 1375) 39, 652, 736, 868,
870, 874
- Spherical astrolabe 341–352
- Stephen of Antioch/ of Pisa 553–4, 564–5
- Sukūt al-Barghawāṭī (11th c.) 7
- Sulaymān b. Yahyā al-Munajjim 295–6
- Sun: Ibn al-Zarqālluh’s book on the solar year
654–667; solar and planetary apogees
507, 629–634, 657–661, 733; Ibn
al-Zarqālluh’s solar model with
variable eccentricity 661–667, 737,
739; sunspots 514, 518, 521; in al-
Khwarizmi’s *zīj* 691–2
- Sundials 17, 18, 109–119, 324; sundial tables
115; cylindrical dials 115, 324;
conical dials (*mukḥala*) 115, 324;
balāṭa 115–119; monumental sundial
in the Alfonsine *Libro del reloj del
Palacio de las horas* 119; see Ibn
al-Raqqām
- Tables of Barcelona 38, 201, 202–3, 261,
615, 636, 642, 686, 804, 812, 858–867,
891; trepidation 649–50, 749, 859,
866, 880; solar apogee 658, 660; star
tables 859–60; combination of mean
motions and equations 861, 866;
tropical or sidereal tables? 861, 863,
866; revolutio anni 863–4
- Tabulae resolutae*: see Nicholas Polonius
- al-Tājūrī (d. 1582), *Tanbih al-ghāfilin ‘an qiblat
al-ṣaḥāba wa l-tābi’in* 145, 146
- Taqwīm* 340, 876–7, 878–80
- Ta’āsīs* 250, 321
- Tasyīr* 15, 179, 237, 274–283; ascensions,
right, oblique and mixed 238–9, 279;
astrological cycles 274–276; direct
method on the ecliptic 276–9; on the
equator: hour line method 279–281,
346; position semicircle method 281–2;
distance method 282–3
- al-Tawzarī, ‘Umar b. ‘Abd al-Raḥmān
(m. 1454) 106, 473
- Thābit b. Qurra (d. 901) 330, 363, 457, 579,
585, 587, 588, 589, 590, 591, 608–10,
662, 673, 687, 823–4
- Theodosius of Tripoli (first c. CE) 330
- Theon of Alexandria (ca. 335–405) 524,
529–30, 642, 644, 727
- Thietmar of Magdeburg (d. 1018) 120
- Thomas Aquinas (ca. 1225–1274) 543
- Thomas of Bologna (fl. 1385–88) 201
- Thurayyā* 121–122
- al-Tibb al-Qasṭālī al-Malūkī* 39, 570–1
- Tiberius, Roman Emperor (r. 14–37) 648
- Ṭibughā/Ṭaybugha, ‘Alā’ al-Dīn (m. 1394) 465
- Tilimsān/ Tlemcen: mosques of Sidi Ḥalwī
and al-Manṣūra 113
- Timekeeping 3, 9, 15, 16, 17, 44–128, 406,
504, 506; ascensional difference 94–
95, 698–700, 740, 811, 890; astrolabe
100; crescent visibility 55–70, 702,
709–10, 715, 769–70, 836, 902, 904–5;
daylight tables 96–99, 726–27, 885;
folk-astronomy 73–93; hour 95–96;
instruments for telling the time
100–128: *quadrans novus* 106,
quadrans vetus 107–108, *quadrans
vetustissimus* 100–103, sexagenarium
105, 471–2, sine quadrant 104–6, 452,
472–4; lunar mansions for telling the

Timekeeping (cont.)

time during the night 85–93; prayer tables 17; prayer times 71–73, 113–114; *zuhr* & *ʿaṣr* prayers 79–82; *fajr* & *maghrib* prayers 83–93, 835; shadow schemes 73; tables for timekeeping in the Maghrib: 71; *zīj*es 93–99. See sundials, nocturnal, candle clocks, clepsydras

Toledan Tables 3, 39, 53, 56, 57, 60, 61, 62, 63, 94, 95, 97, 163, 170, 171, 260, 263, 290, 368, 437, 506, 514, 545, 577, 580, 642, 644, 662, 675, 687–8, 692, 698–9, 701, 704, 715, 719–734, 735, 742, 747, 750, 767, 776, 779, 798–803, 829, 831, 833, 835–6, 841, 842, 844, 848–9, 851–4, 856, 865, 868, 870–1, 879, 882, 884, 889, 901; origin 721–24; sets of canons 724–26; numerical tables calculated for Toledo 726–27; star tables 730–31

Transits of Mercury and Venus on top of the Sun 514, 518

Transits (astrological) 712–13, 738

Translations in the first half of the 12th c. 180–6; works selected 182

Trepidation theory: see precession and trepidation

Triplicities 160

Tropical versus sidereal astronomy and astrology 255, 317, 636–642, 644, 649–50, 814–821, 823–4, 831–2, 839, 844–7, 855–6, 861, 863–9, 879, 882–5, 889, 902–5

Tunisian recension: see Anonymous Tunisian recension of Ibn Ishāq's *zīj*

al-ʿUdhri (1003–1085) 130

Ulrich of Strasbourg (d. ca. 1278) 543

Ulugh Beg (1393–1449) 12, 631, 814, 817–20, 904

ʿUmar b. Farrukhān al-Ṭabarī (d. ca. 815) 184, 300, 321, 703

Universal astrolabes 440–70

al-ʿUtaqī al-Furriyānī al-Ifriqī, Muḥammad b. ʿAbd Allāh b. Muḥammad, Abū ʿAbd al-Raḥmān (d. 955) *Kitāb fī l-nujūm wa-aḥkāmī-hā* 5

ʿUthmān b. ʿAbd Allāh al-Ṣaffār (fl. 1300) 401

ʿUthmān al-Sayqal (d. 941) 5, 153

Validity period of a *zīj* 672, 746–7, 755, 760

Venus: apogee does not coincide with the solar apogee 695

Vettius Valens (2nd c. C.E.) 194, 241, 289, 295–6, 299

Walcher of Malvern (fl. 1092) 120, 690

al-Walid b. Zaydān, sultan of Morocco (1630–35) 896

Wālis al-Miṣr: see Vettius Valens

al-Wansharisī 139, 141–142

al-Wāsiṭī (d. 1045–6?) 496

William II of Sicily (1154–1166): see Ghiyām b. Rujār

William of Auvergne (ca. 1185–1249) 543

William Batecombe (fl. 1348) 840

Yaḥyā b. Abī Maṣṣūr (fl. ca. 830) 332, 487, 546, 588, 636, 684, 686–7, 740, 750, 758, 778, 823, 826, 891

Yaḥyā al-Ghazāl (773–864) 154–8

Yaʿqov ben Abī Abraham Yizḥāq al-Corsuno: see Jacob Corsuno

Yaʿqūb al-Manṣūr, Almohad Caliph (1184–1199) 8

Yaʿqūb b. Ṭāriq (second half of the 8th c.) 247, 683, 750

Yehudah b. Mosheh/ Yehuda fi de Mose fi de Mosca (13th c.) 36, 38, 198, 331, 828, 856, 888

Yehudah b. Salomon ha-Kohen (fl. 1247) 544

Yiḥaq ibn Barukh ibn al-Baliyya (1034–1093) 163

Yomṭob: see Bonjorn

Yosef b. Ishāq b. Mosheh b. Waqār (fl. 1357–1396), *Sefer Luḥot* 39, 804, 839, 867–8, 871, 874, 884

Yosef ha-Parsi (fl. 1439–1444): equatorium 492

Yūnus al-Barghawātī (r. 842–884) 6

Yūsuf I of Granada (1333–1354) 13, 179

Yūsuf Hallāj (15th c.) 463–4

Yūsuf b. Tashfin (d. 1106–7), leader of the Almoravids 7, 166

Yūsuf ibn ʿUmar ibn Abī Thalla (d. 1043) 163

Zacut, Abraham (1452–1515) 212, 867–9, 870–73, 904–5; observations 872–3; *ha-Hibbur ha-gadol* 38, 40, 210, 213,

- 647, 883, 887–92; precession 650–54, 873; motion of the apogees 889–90; meridian of water 891; *Almanach perpetuum* 38, 41, 42, 55, 213, 261, 293, 295–6, 890–1, 892–4; star tables 894; Arabic versions of the *Almanach Perpetuum* 894–902; other tables 869–71; *Tratado breve en las ynfluencias del cielo* 213–4
- Zaradusht 237, 252, 283
- al-Zarqāl, Abū l-Qāsim b. ‘Abd al-Raḥmān 125, 578
- al-Zarqālī: see Ibn al-Zarqālluh
- Zaydān al-Manṣūr (d. 1627), sultan of Morocco 896
- al-Zayyāt, Ishāq b. al-Ḥasan b. Abī l-Ḥusayn (fl. 975–1025) 709
- al-Zīj* 23, 683
- Zīj al-Shāh* 696, 779, 850
- Zīrid dynasty in the Maghrib (972–1152) 6
- Ziryāb 155
- Ziyād ibn ‘Abd Allāh al-Anṣārī (d. 1085) 45
- Ziyādat Allāh b. Ibrāhīm, emir of Ifrīqiya (fl. 836) 129
- Zodiacal-calendar scale 373–4, 390–1, 397, 406–8, 452
- al-Zubayr b. Aḥmad b. Ibrāhīm b. al-Zubayr al-Thaqafi (d. after 1300) 402