Ibn al-Haytham and Eudoxus: The Revival of Homocentric Modeling in Islam

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David Pingree has written the book (actually many) on the transmission of science between cultures. So it seems appropriate to present him with a Persian text for his seventieth birthday, a text describing an astronomical model whose roots go back to fourth-century B.C. Athens, one that managed to get criticized in second-century A.D. Alexandria, that somehow influenced an eleventh-century Cairene transplanted from Basra, that was picked up in thirteenth-century Iran, and then made its way—through channels unknown—to Europe. A convoluted and still puzzling tale to warm the heart of the master!

1 *Introduction*

Ibn al-Haytham (965–ca. 1040) wrote a number of works on astronomy.[1] Historically he was best known for his *Maqāla fī hay'at al-ʿālam* (Treatise on the Configuration of the World), which gained for him considerable renown in both Islamdom and in Europe.[2] In particular, Islamic sources often accorded him the role of having taken Ptolemy's imaginary circles and turning them into solid, spherical bodies. Though this is, at best, dubious history, it did provide a nice story for Islamic astronomers that con-

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[1] Sabra [1972], 197–9, 205–8; Sezgin [1978], 251–61.

[2] Langermann [1990] provides an edition and translation along with a discussion of the influence of the work.

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trasted their work with that of their Alexandrian predecessor.[3] It also provided a backdrop for another of Ibn al-Haytham's works, namely his *al-Shukūk ʿalā Baṭlamyūs* (Doubts About Ptolemy).[4] In it, he criticized Ptolemy for certain models (for example those employing the equant) that allowed irregular motions to occur in the heavens.

It is often stated that Ibn al-Haytham (as well as other early critics of Ptolemy) 'did not propose new models to replace those to which they were objecting...' (Saliba [1994], 21) but this turns out not to be the case. A number of years ago, A. I. Sabra [1979] published Ibn al-Haytham's reply to an anonymous critic of a work of his called *Maqāla fī ḥarakat al-iltifāf*, in which Ibn al-Haytham proposed a model to account for the latitudinal motions of the planetary epicycles. This work was evidently written before the *Doubts About Ptolemy*, which is promised at the end of his reply (Sabra [1979], 398 = 207 (Arabic numeration)). Unfortunately, the *Maqāla fī ḥarakat al-iltifāf* itself seems to be irretrievably lost. Its contents, however, can be reconstructed, thanks to a number of later accounts, the most extensive of which turns out to be in Persian. Naṣīr al-Dīn al-Ṭūsī (1201–1274) wrote this exposition in a supplement that he appended to his *Risālah-i Muʿīniyya*, written in 1235 while he was part of the court of the Ismāʿīlī governor of Qā'in in eastern Iran. This Appendix, variously entitled *Ḥall-i mushkilāt-i Muʿīniyya, Sharḥ-i Muʿīniyya*, or *Dhayl-i Muʿīniyya* ('The Solution of Difficulties', 'A Commentary' or 'An Appendix' for the *Muʿīniyya*), is important for a number of reasons, not the least of which is that it is the first instance in which Ṭūsī presents the rectilinear version of his famous 'couple' by which he used an oscillation on a straight line to deal with a number of difficulties in Ptolemaic planetary theory. But it is clear that he had not yet figured out how to produce a curvilinear oscillation on a great circle arc (which he later used in the *Tadhkira* to deal with such problems as Ptolemy's latitude theory), so in the Appendix he instead presented Ibn al-Haytham's model from the *Maqāla fī ḥarakat al-iltifāf* as one possible way to

[3] See, for example, Ibn al-Akfānī (d. 1348), *Irshād al-Qāṣid*, in Witkam [1989], 57–8 (= 408–407), who is probably following astronomers such as al-Khiraqī (d. 1138–9) ; cf. Ragep [1993], 1: 30–3.

[4] The work has been edited by A. I. Sabra and N. Shehaby [1971]. An English translation is in D. L. Voss [1985].

deal with the problem (Ragep [1993], 1: 65-70 and Ragep [2000]). It is this chapter of the Appendix that is presented below in a Persian edition and English translation.

That Ṭūsī was not satisfied with Ibn al-Haytham's proposal was made abundantly clear in his *al-Tadhkira fī ʿilm al-hayʾa*, where he notes that such a model, by retaining Ptolemy's small circles, will produce motion in longitude as well as latitude, thus altering the correct positions of the apex and perigee. He also was dissatisfied with the fact that motion on the small circles is uniform about a point other than the center of the circle, thus making the motion analogous to the irregular motion of the epicycle center on the deferent due to its uniformity with respect to the equant (Ragep [1993], 1: 214-7, 2: 450-2). It is interesting that Ṭūsī indicates similar dissatisfaction with Ibn al-Haytham's model in the *Risālah-i Muʿīniyya*, where he says that 'even with this postulation the irregularity is not ordered, and in addition several other corruptions come into being. But this is not the place to explain them' (Ragep [2000], 125). Nevertheless, he presents Ibn al-Haytham's model without criticism of any sort in the Appendix, thus clearly signaling that, despite having devised and introduced the rectilinear couple in the Appendix, he has yet to produce a model of his own (i.e. the curvilinear couple of the *Tadhkira*) to account for Ptolemy's latitudinal motions.

The model itself is Ibn al-Haytham's attempt to give a physical representation to Ptolemy's 'small circles', which the latter introduced in *Almagest*, XIII.2 to account for latitudinal variations in the epicycles, the so-called deviation and slant (Toomer [1984], 599-600). What is of great interest is that this model turns out to be essentially the same as one that is attributed to Eudoxus of Cnidus (fourth century B.C.). In particular, Ibn al-Haytham proposes to produce the small circles by means of two homocentric spheres that have different axes of rotation, each rotating at the same speed but with opposite angular rotations (see Figure C1). One rotating sphere (viz. KL), of course, could cause a given point to describe the needed circle, but in such a case the entire epicycle would also rotate, thus seriously disrupting the position of the planet. The second sphere (MN), which is contained inside the first and has an axis that always goes through the apex and perigee of the epicycle, would return the epicycle to its correct position. At least this is the theory. In fact, these two orbs of Ibn al-Haytham correspond to orbs three and four of Eudoxus's planetary models, i.e. the ones that are meant to account for the retrograde motion by means of the 'hippopede' (Figure C2).[5]

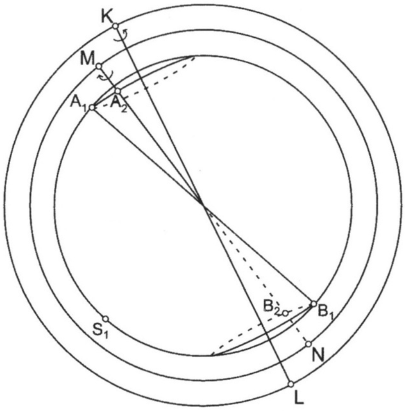

Figure C1

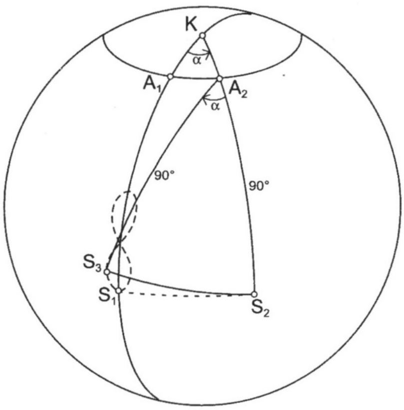

Figure C2

But this consequence of the Eudoxan model plays no role in Ibn al-Haytham's theory and indeed the hippopedal motion of the mean distance S between the epicyclic apex A and perigee B, if acknowledged, would have been an unwelcome complication of an already complex theory, since Ibn al-Haytham's goal is to have sphere MN return all points—other than the apex and perigee—of the epicycle, including S, to the eccentric plane.

[5] For a lucid account of the Eudoxan system, see Heath [1913], 190–211.

Has Ibn al-Haytham been somehow influenced by Eudoxus, most likely through the intermediation of Aristotle who presented Eudoxan planetary theory in Book XII, Chap. 8 of his *Metaphysics*? Ibn al-Haytham himself in his reply to a critic of his original *Iltifāf* treatise states that there is a generic affiliation between Aristotle's *iltifāf* and the *iltifāf* motion that he discusses in his own treatise. But the two are distinctly different, according to Ibn al-Haytham, since 'the latter is not the former, it resembles it only. The proof of this is that the mathematicians do not use it nor mention it, i.e. that which is referred to by Aristotle, since they do not need it and Aristotle does not employ the motion of the epicycle orb, nor does he [even] have a word for it' (Sabra [1979], 401=204 (Arabic numeration)). It is interesting that Ibn al-Haytham does not cite the obvious connection between his model and that of Eudoxus's hippopedal model but instead merely refers to 'Aristotle's' solar model that employs orbs with different axes for the daily motion, ecliptic motion and latitudinal motion (Sabra [1979], 403–402=202–203 (Arabic numeration)). Thus the 'generic' affiliation seems to be that homocentric orbs with different axes are employed both by himself and Aristotle to achieve an *iltifāf* motion. But for Aristotle, this is an 'accidental' result and the end product is a spiral motion with endpoints at the solstices whereas Ibn al-Haytham's *iltifāf* is based on 'specific bodies' that are designed to produce a circular motion on the epicycle orb (Sabra [1979], 401=204 (Arabic numeration)). From this discussion, it is clear that Ibn al-Haytham has not explicitly connected what he is doing with Eudoxus's planetary orbs three and four; there remains, though, the indirect influence from Aristotle of using connected homocentric spheres with different axes to produce a desired result. In the *Doubts* Ibn al-Haytham criticizes Ptolemy's abandonment of the 'small circle' latitude model in the *Almagest* in favor of a two-sphere arrangement in the *Planetary Hypotheses*.[6] But it is curious that he refrains from referring to his own treatise on *Iltifāf*, which undoubtedly was written earlier; this tends to underscore that the *Doubts* is a work of criticism rather than constructive engage-

[6] For an account of the latitude model of the *Planetary Hypotheses*, see Neugebauer [1975], 2: 908–9, 924–5. For the Arabic text of Ibn al-Haytham's criticisms, see Sabra and Shehaby [1971], 43–58; Voss [1985] provides an English translation, 61–78 and discussion, 147–71.

ment.

What is the significance of Ibn al-Haytham's *Iltifāf*? Ultimately, it was a minor work that nevertheless possesses a certain historical significance. For one thing, it is one of several works by early (pre-thirteenth century) Islamic astronomers that proposed new models to deal with a variety of ills they felt they had inherited from the Ancients.[7] It also evidently played an important role in the thinking of Naṣīr al-Dīn al-Ṭūsī , who not only summarized it for his Persian readers but also would later give it a certain prominence by transforming it in the *Tadhkira* into his own curvilinear version of the Ṭūsī couple.[8]

Finally Ibn al-Haytham's text marks an important step in the revival of homocentric modeling in the Middle Ages. The latter is best known in Islam through the work of the twelfth-century Andalusian Nūr al-Dīn al-Biṭrūjī, who sought to abandon the eccentrics and epicycles of Ptolemaic astronomy in favor of a purer, homocentric system in which all the orbs were geocentric. Whether this was inspired by the Eudoxan system has been a matter of some controversy.[9] Perhaps it would help to distinguish between a homocentric model and a homocentric system. An astronomical model (*aṣl* in Arabic; *hypóthesis* in Greek) is a device that can be used for a particular purpose within a larger system (Arabic: *hay'a*). Thus one may speak of an eccentric or epicyclic model but not of an eccentric or epicyclic system. On the other hand, one may refer to either a homocentric model or a homocentric system. The latter would include the systems of Eudoxus and al-Biṭrūjī. One might say that Biṭrūjī's *system* is doubtless inspired by an Aristotelian view that all motion in the heavens must be about a single center. Since Biṭrūjī also knows the Eudoxan system through Aristotle, it does not take much of a leap to conclude, as has E. S. Kennedy, that Biṭrūjī's system owes much to fourth-century B.C. Greek astronomy. On the other

[7] Examples include the various early theories of trepidation and Jūzjānī's eleventh-century attempt to deal with the equant; cf. Ragep [1993], 2: 452.

[8] It is of great interest that Ṭūsī presents his model as a transformation of Ibn al-Haytham's; see Ragep [1993], 1: 216–17, 2: 452–5.

[9] See, for example, Kennedy [1973] and Goldstein [1964] and [1971]. The former held that Eudoxus was the ultimate source of Biṭrūjī's astronomical system, whereas the latter insisted upon an origin in the various Islamic trepidation theories.

hand, this does not necessarily imply that Biṭrūjī knew or understood all the details of Eudoxan astronomy per se; as was the case with Ibn al-Haytham, he could have simply understood that the systems described in Aristotle's *Metaphysics* were homocentric and based upon a series of embedded, interconnected orbs with different axes of rotation. As B. Goldstein has noted, there could have been other sources that Biṭrūjī drew upon for the details of his *models*, and he pointed to several theories of trepidation that employ homocentric models. But as we now know, there are other astronomers before Biṭrūjī, such as Ibn al-Haytham, who were using homocentric modeling for purposes other than trepidation. And we know that this aspect of Ibn al-Haytham's astronomical corpus was influential both in Islam and the Latin West.[10]

2 *Edition and Translation of Chapter 5 of Ṭūsī's Ḥall-i mushkilāt-i Muʿīniyya.*

The edition is based upon 3 manuscripts:

1) F Istanbul, Fatih 5302/4, ff. 204a-205a (722 H./1322 A.D.)
2) K Oxford Or. 208 (Bodl.), ff. 145b-148a (n.d.)
3) M Tehran, Malik 3503, pp. 14–17 (658 H./1260 A.D.); published in facsimile in al-Ṭūsī 1956

Abbreviations used in Apparatus:

F_{ab} = above in MS F
F_{cr} = crossed out in MS F
F_{mr} = in margin of MS F
F_{rp} = repeated in MS F

The Persian text is part of a collaborative effort by Professor Wheeler Thackston, Ms. Sally Ragep and myself to establish the entire *Risālah-i Muʿīniyya* and its *Ḥall*. Thackston has established a preliminary edition of both works based upon MSS F and M, and S. Ragep has been collating this preliminary edition with another nine manuscripts. It has been my responsibility to then establish a 'final' apparatus, text and commentary. In the case of this particular chapter, only the three listed manuscripts

[10] For evidence of the influence of his homocentric models in the Latin West, see Mancha [1990].

of the eleven we have been using contained all of Chapter 5 of the *Ḥall*.

The English translation generally follows the procedures and vocabulary used in my translation of Ṭūsī's *Tadhkira* (Ragep [1993]). This represents a revised version of an earlier translation done a number of years ago by Prof. Thackston and myself. But, as with the edition, all remaining shortcomings are the responsibility of the present author alone.

Glosses are keyed to the translation and are given as footnotes. Only problematic readings or interpretations (such as the mix-up in the order of the orbs) are discussed; a more detailed commentary on Ibn al-Haytham's model and its relation to that of Ṭūsī can be found in Ragep [1993], 2: 450-5.

Figures 1 and 2 have been established from the three manuscripts, which exhibit minor variations. Figure 2 follows the medieval convention of rendering a 3-dimensional effect by 'flipping' the circles horizontally onto the page. (See Ragep [1993], 1: 218-9 for another example.) Figure 2c is an attempt to present the diagram using a more modern perspective.

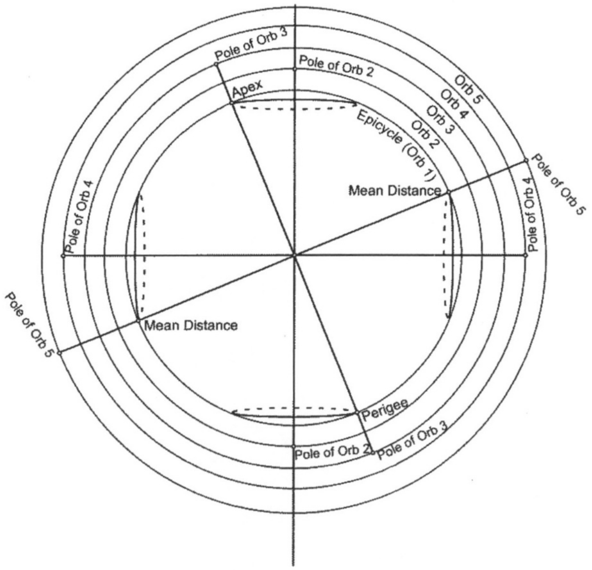

Figure 2C

حلّ مشکلات معینیه در هیئه لنصیر الدین طوسی

فصل ۵

در هیأت افلاك تداویر سیارگان[1]
بر مذهب ابو علی بن الهیثم[2]

این ابو علی[3] از مبرزان علم ریاضی بوذه است و هیأت افلاك بر وجه تجسیم بیشتر مستفاد از سخن اوست . و اورا رسالهٔ است در بیان افلاك تداویر کواکب بر وجهی که این حرکات مختلف از آن صادر شوذ . می‌گویذ هر یکی از[4] کواکب علوی سه فلك تدویر دارند بیکدیگر محیط .

فلك اول که در میان دو فلك دیگر باشد ، فلکی بوذ مصمت که کوکب[5] بر یك طرف او باشد . و آن کره متحرك بوذ بحرکت خاصهٔ کوکب . و منطقهٔ او توهم کنیم که از سطح منطقهٔ خارج مرکز خارج بوذ و با او مقاطع[6] در دو[7] بعد اوسط . پس قطری که بدو بعد اوسط بگذرذ در سطح منطقهٔ خارج بوذ ، و قطری که بذروه و حضیض[8] بگذرذ یك نیمه در جهتی و دیگر نیمه در جهتی دیگر[9]. و چون خطی از مرکز مایل[10] بمرکز تدویر کشند و اخراج کنند تا بر تدویر بگذرذ ، هر آینه با این قطر[11] که بذروه و حضیض گذشته است بر مرکز تدویر تقاطع کنذ . و چون این خط در سطح فلك خارج باشد ، بعد میان این خط و قطر تدویر در ذروه و حضیض بقدر میل ذروه و حضیض باشد برین صورت :[12]

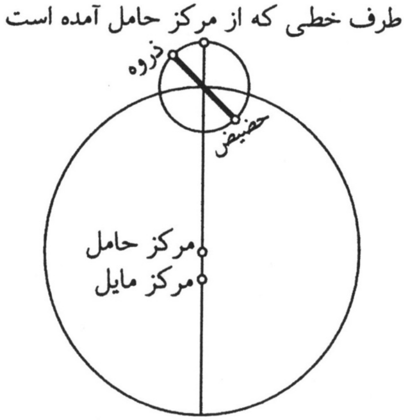

Figure 1A

پس فلك دوم توهم كنيم كه بذين فلك محيط باشد و مركز هر دو فلك[13] يكى بوذ . و اين فلك[13] بر دو قطب ازين خط كه[14] از مركز مايل آمذه است ، متحرك باشد بحركتى شبيه بحركت مركز تدوير بر[15] محيط مركز معدل مسير . و لامحالة چون اين فلك حركت كند ، و فلك اول را با خوذ ببرذ ، ذروه و حضيض را دو[16] مدار حادث شوذ كه مركز هر يكى از آن دو مدار بر خطى بوذ كه از مركز مايل آمذه است . و آن دو دايرهٔ خرد[17] بوذ كه سطح هر يكى با سطح فلك خارج متقاطع بوذ بر زواياى[18] قايمه ، مانند هدفى كه قطرش بر محيط سپرى نهند . پس دو موضع ازين دايره[19] در سطح فلك حامل بوذ . و چون ذروه و حضيض بر محيط اين دايره[19] حركت كنند ، هرگاه كه بذين دو نقطه رسند در سطح فلك خارج باشند . و چون بر منتصف هر دو نقطه باشند در غايت ميل باشند از سطح خارج .

لیکن[20] ازین حرکت فسادی لازم آیذ . و آن چنان بوذ که چون همه تدویر بذان حرکت متحرك بوذ ، قطری که بدو بعد اوسط گذشته باشد از سطح حامل خارج شوذ و دوری[21] بکند که در اثنای آن نصف شرقی از تدویر غربی شوذ و نصف غربی شرقی . پس از جهت دفع این فساد ، فلکی[22] دیگر توهم کنیم که آن فلك سوم باشد محیط باین هر دو فلك چنانکه مرکزش مرکز هر دو فلك بوذ[23] ، و دو قطب او بر دو طرف قطر فلك تدویر بوذ که مارّ باشد بذروه و حضیض[24] ، و حرکت او در خلاف جهت حرکت فلك دوم بوذ ، و هم بمقدار آن حرکت تا آن قدر که منطقهٔ فلك تدویر بحرکت فلك دوم از وضع خوذ زایل شوذ ، این فلك اورا با وضع خوذ برد[25] و قطر بعد اوسط همیشه در سطح فلك خارج بماند ، اما دور[26] ذروه و حضیض بر مدارهای[27] مذکور بر قرار بماند از جهت آن که قطبهای[28] این فلك بر دو طرف قطر فلك تدویر است و قطبهای[29] فلك دوم غیر این[30] دو قطب است ، و میان هر دو قطبی[31] ازین چهار قطب[32] بقدر نصف قطر مدار ذروه یا حضیض[33] . پس ازین حرکات لازم آیذ که نصف ذروه دایما در جهتی بوذ و نصف حضیض در جهتی دیگر مخالف جهت اول ، و در هر[34] دوری از ادوار دو نوبت[35] منطقهٔ تدویر در سطح منطقهٔ خارج آیذ و بگذرذ بر[36] وجهی که جهات متبادل شوذ .

و چنانکه مرکز تدویر محیط مایل را[37] بحرکتی قطع می کنذ[38] بنسبت با مرکز مایل غیر متشابه است بنسبت[39] با

مرکز معدل مسیر متشابه تا در دو ربع که در نصف اوج افتد بطئ باشد[40] و در دو ربع دیگر سریع . همچنان[41] ذروه و حضیض این دو مداررا قطع کنند[42] بحرکتی که بنسبت با مرکز مدار[43] غیر متشابه بوذ و بنسبت با نقطهٔ دیگر غیر مرکز[44] مدار و داخل مدار که بجای مرکز معدل مسیر باشد متشابه بوذ[45] تا سیر ذروه بر محیط این مدار در دو ربع بطئ بوذ و در دو ربع سریع . و هم چنین سیر حضیض نا[46] متشابهست تا سیر مرکز محفوظ بوذ .

و این دو دایرهٔ خرد آنست[47] که صاحب منتهی الادراك در اثنای[48] اثبات اجسامی که مبادی حرکات باشند ایراد[49] کرده است و بر آن اقتصار[50] نموذه . و هر چند باول اثبات این دو دایره بطلمیوس کرده است در مجسطی ، اما چون بطلمیوس در همه احوال بر دوایر اقتصار[51] کرده است ، این باب مناسب دیگر ابواب باشذ . و کسی که در دیگر مواضع اثبات اجسام کند و اینجا بر[52] ایراد دوایر اقتصار کند ، شرط مناسبت رعایت نکرده باشد .

و اما در دو کوکب[53] سفلی همین دو فلك زاید[54] بر فلك تدویر جهت میل ذروه و حضیض اثبات می کند . [55]
و جهت حرکت انحراف دو فلك دیگر اثبات می‌کند . فلك اول ، که فلك چهارم بوذ ، از افلاك تداویر ایشان محیط[55] بوذ[56] بذان سه فلك . و دو قطب این فلك[57] دو نقطه بوذ از خطی که بمرکز تدویر بگذرذ در سطح فلك حامل و با خطی که از مرکز مایل آمذه باشد بر زوایای[58] قایمه متقاطع بوذ . و چون[59] فلك حرکت کند قطری‌را[60]

که مارّ بوذ بدو بعد اوسط لامحالة بر حوالی این دو قطب حرکت بایذ کرد . پس حرکت انحراف حادث شوذ ، الا آنکه چون همه منطقهٔ تدویر[61] حرکت کند ، ذروه و حضیض از وضع خوذ زایل شود و ذروه بجای[62] حضیض آیذ و حضیض بجای[63] ذروه . پس فلکی پنجم برین چهار فلك محیط بوذ که دو قطبش دو طرف خطی بوذ که بدو بعد اوسط گذشته باشد و حرکت او مخالف حرکت فلك چهارم و مساوی او تا آنچه از موضع خود[64] زایل شوذ با وضع اول آیذ . و دو بعد اوسطرا از[65] حرکت[66] فلك چهارم دو مدار لازم آیذ ، و آن دو دایرهٔ خرد بوذ که با سطح[67] فلك حامل بر زوایای[68] قایمه متقاطع باشذ ، مانند هدفی که بر سپری نهند ، چنانکه[69] هر دو محیط بر نقطهٔ مماس باشند و سطح با سطح متقاطع بر زوایای قایمه[70] . و حرکت این دو بعد بر محیط این دو دایره مختلف در یك نیمه سریع و دیگر نیمه بطئ ،[71] مانند[72] سیر مرکز بر فلك[73] مایل . و چون ذروه در مدار خوذ در غایت میل بوذ از سطح مایل ، بعد اوسط در سطح مایل بوذ . و چون بعد اوسط از[74] سطح مایل در غایت میل بوذ ، ذروه در سطح مایل بوذ تا این دو عرض بر سبیل تبادل لازم آیذ .

و صورت افلاك تداویر این دو کوکب[75] بر حسب[76] آنچه بر سطح توان کشید اینست . و صورت افلاك کواکب علوی هم از اینجا معلوم شوذ چون بر سه فلك[77] اقتصار[78] کنند .

اینست بیان این مقالت ، و الله اعلم بالصواب[79] .

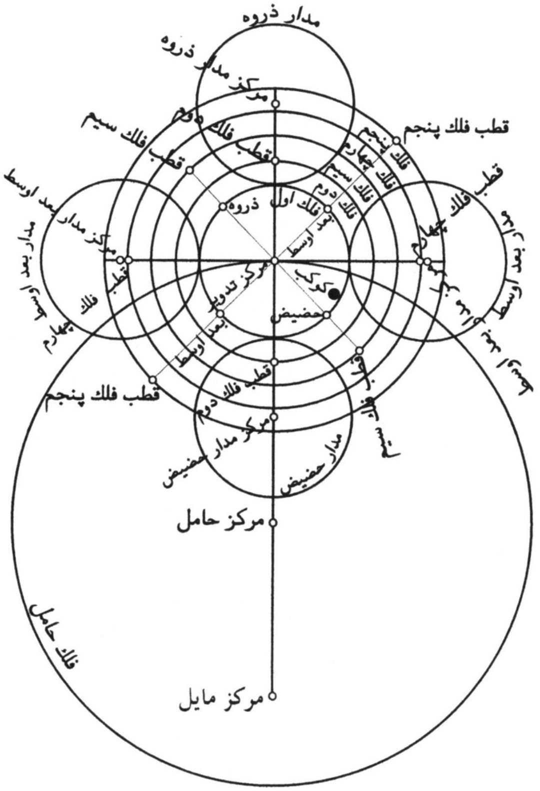
مدار ذروه
مركز مدار ذروه
قطب فلك پنجم
فلك سيم
فلك دوم
فلك اول
ذروه
بعد اوسط
مدار بعد اوسط
مركز مدار بعد اوسط
مركز تدوير
كوكب
حضيض
قطب فلك پنجم
قطب فلك دوم
مركز مدار حضيض
مدار حضيض
مركز حامل
فلك حامل
مركز مايل

Figure 2a

Text Variants

1 سیارگان] ستارکان : F . 2 الهیثم] +رحمه الله : F . 3 ابو علی]
مرد : F . 4 از] ازین : K . 5 که کوکب] که ~~فلك~~ کوکب : K .
6 مقاطع] K_{mr} , K_{ab} = مطالع : K_{cr} . 7 در دو] + بروج :
M_{cr} . 8 و حضیض] حضیض: K . 9 دیگر] + افتد : K . 10
مایل] حامل : K . 11 قطر] قدر : F . 12 این] آن : K . 13 یکی بوذ
و این فلك] –F . 14 که] –M . 15 بر] –K . 16 دو] هر دو : M .
17 خرد] F_{mr} . 18 زوایای] زوایا : M . 19 در سطح ... محیط این
دایره] –M . 20 لیکن] لکن : M . 21 و دوری] دوری : K .
22 فلکی] فلك : K, M . 23 بوذ] –F . 24 باشد] باشند : M . 25 برد]
–M = اورد : K . 26 دور] در : F . 27 مدارهای] مدارها : M . 28
قطبهای] قطبها : M . 29 قطبهای] قطبها : M . 30 این] ان : K . 31
قطبی] قطب : K . 32 قطب] قطر : M . 33 یا حضیض] تا
حضیض : F = با حضیض : M = + باشد : K . 34 در هر] در هم
: M . 35 دو نوبت] و نوبت : M . 36 بر] و بر : M .
37 مایل‌را] حامل‌را : K . 38 کنذ] + که : F, K . 39 بنسبت] و
بنسبت : F, K . 40 بطئ باشد] بطیست : M . 41 همچنان] و
همچنان : K . 42 کنند] کند : K . 43 مدار] K_{rp} . 44 مرکز]
مر~~و~~کز : M . 45 بوذ] باشد : F . 46 نا] تا : F . 47 آنست]
است : F . 48 اثنای] اثنا : F . 49 ایراد] این او : M . 50 اقتصار]
اقتضا : M . 51 اقتصار] اختصار : M . 52 بر] –K . 53 دو
کوکب] کوکب : K . 54 زاید] M_{rp} . 55 و جهت ...ایشان
محیط] و بجهت میل طرف قطر بعد اوسط همجنین فلکی چهارم
توهم کنند که محیط : K . 56 بوذ] –F . 57 این فلك] این

~~دو~~ فلك : K . 58 زوايای] زوايا : K, M . 59 و چون] جون : K
= + اين : F . 60 قطری‌را] قطری : F, K . 61 تدوير] + نيز : F .
62 بجای] بحالی : M . 63 بجای] بحالی : M . 64 خود] –M . 65 را از]
راز : M . 66 حركت] M_{ab} . 67 با سطح] با ~~دو~~ سطح : F .
68 زوايای] زوايا : M . 69 چنانكه] F_{rp} . 70 زوايای قايمه] زوايا :
M . 71 و ديگر] و در ديكر : K = ديكر : M . 72 مانند] باشد :
F . 73 بر فلك] –K . 74 از] در : K, M . 75 دو كوكب]
كواكب : F . 76 حسب] سبيل : F . 77 فلك] + تدوير : F .
78 اقتصار] اختصار : K . 79 بالصواب] –F, –K .

The Solution of Difficulties in the Muʿīniyya [Treatise] on Astronomy by Naṣīr al-Dīn al-Ṭūsī

Chapter Five: 'On the Configuration of the Epicycle Orbs of the Wandering Planets According to the Theory of Abū ʿAlī b. al-Haytham'

This Abū ʿAlī was a prominent mathematician, and the configuration of the orbs as solid bodies is mostly taken from his work.[11] He has a treatise[12] explaining the orbs of the planets' epicycles in such a way that the various motions result from them. He states that each of the upper planets has three epicycle orbs, one enclosing another.

The first orb, which is inside the two other orbs, is a complete solid orb on one side of which is the planet.[13] That sphere moves with the proper motion of the planet. Let us conceive that its equator is in a plane different from that of the eccentric equator, intersecting the latter at the two mean distances. Now the diameter passing through the two mean distances is in the plane of the eccentric equator and the diameter passing through the epicyclic apex and perigee will be in one half in one direction and in the other half in the other direction. If a line is drawn from the center of the inclined [orb] to the center of the epicycle and extended until it reaches the epicycle, then it necessarily intersects the diameter passing through the apex and the perigee at the center of the epicycle. Since this line is in the plane of the eccentric orb, the distance between this line and the diameter of the epicycle [passing] through the apex and perigee is equal to the inclination of the apex and the perigee according to this illustration.[14]

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[11] As is common among Islamic astronomers and encyclopedists starting at least as early as al-Khiraqī (d. 1138-9), Ṭūsī reiterates the view that Ibn al-Haytham is responsible for putting forward the basic solid configuration (*hay'a*) of the celestial orbs. No doubt this is due to Ibn al-Haytham's *Maqāla fī hay'at al-ʿālam* (Treatise on the Configuration of the World). That Ptolemy had attempted this in the *Planetary Hypotheses*, a work known to Ibn al-Haytham and Ṭūsī, seems not to have made much of an impression (cf. Ragep [1993], 1: 30-3 and the introduction above).

[12] This is Ibn al-Haytham's work whose title was most likely *Maqāla fī ḥarakat al-iltifāf*; see Sezgin [1978], 260 (no. 25) and Sabra [1979], 390.

[13] This first orb, of course, is the 'original' epicycle.

[14] Apparently the apex and perigee in Fig. 1 are meant to be above and below the plane of the paper at the maximal inclination of the epicyclic

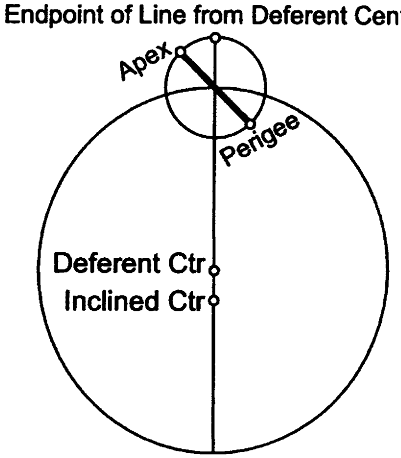

Figure 1B

Let us now conceive the second orb as enclosing this [first] orb, and each of these two orbs shares the same center. This orb, which is on two poles on the line coming from the center of the inclined orb, moves with a motion like that of the epicycle center on the circumference of the equant center [circle].[15] There is no doubt that when this orb moves and carries the first orb with it, then [both] the apex and the perigee describe two circuits each of whose centers is on the line which comes from the center of the inclined [orb]. And these two small circles are such that each of their planes is perpendicular to the plane of the eccentric orb, like a stud whose diameter is on the circumference of a shield. Now two places on [each] circle are in the plane of the deferent orb. When the apex and perigee move along the circumference of [each] circle, they will be in the plane of the eccentric orb whenever they reach these two points. At the mid-point between these two points, they are at the maximum inclination from the plane of the eccentric.

There follows, however, from this motion a distortion, namely that since the entire epicycle moves with this motion, the diameter passing through the two mean distances goes out of the plane

'deviation' (cf. Ragep [1993], 1: 190–3).

[15] This is somewhat misleading. In fact Orb 2 moves the apex and perigee with a motion correlated with the actual irregular motion of the epicycle center on the deferent, not with the idealized uniform motion on the imaginary equant circle. Cf. note 17 below.

of the deferent. As it moves around, the eastern half of the epicycle becomes western and the western half becomes eastern. Then in order to rectify this distortion, let us conceive another orb, which is the third orb enclosing these [previous] two orbs, in such a way that its center is the center of both orbs. Its two poles are at the two end-points of the diameter of the epicycle orb that passes through the apex and perigee. Its motion is in the opposite direction of the second orb's motion but equal to it, so that by however much the equator of the epicyclic orb is displaced from its proper place by the motion of the second orb, this orb brings it back to its proper place, and the diameter of the mean distance always remains in the plane of the eccentric orb. However the revolutions of the apex and the perigee on the above-mentioned circuits remain fixed inasmuch as the poles of this orb are at the two end-points of the diameter of the epicycle orb. The poles of the second orb are different from these two poles. The distance between each two of these four poles is equal to the radius of the circuit of the apex or of the perigee. Therefore from these motions it follows that the apex half is always in one direction and the perigee half is in another direction opposite that of the first. In every revolution the equator of the epicycle passes twice through the plane of the eccentric equator in such a way that the directions interchange.[16]

As the epicycle center traverses the circumference of the inclined [orb] with a motion that is nonuniform with respect to the inclined center, it is uniform with respect to the equant center, so that in the two quadrants that fall in the apogee half it is slower while in the other two quadrants it is faster. Similarly, the apex and the perigee traverse [their] two circuits with a motion that is nonuniform with respect to the circuit's center but uniform with respect to a point other than the circuit's center that is within the circuit in a position [corresponding to that of] the

[16] There is a serious error in the order of the orbs. For the model to actually work, Orb 3 should be contained inside Orb 2 and not the reverse, as is presented here and in Figure 2; otherwise the apex and perigee will not remain on the small circle and the diameter connecting them will not stay aligned with the poles of Orb 3. Whether this was a careless error due to Ibn al-Haytham or Ṭūsī is not clear. Ṭūsī does correct the mistake, without comment, in the *Tadhkira* where he places Orb 3 between Orb 2 and the epicycle (Ragep [1993], 1: 214–5 and 358, Fig. C23).

equant center, so that the movement of the apex on the circumference of this circuit is slower in two quadrants and faster in [the other] two quadrants. And likewise the movement of the perigee is nonuniform so that the motion of center is preserved.[17]

These two small circles are those that the author of *Muntahā al-idrāk* introduced when positing the bodies that are the principles of motion, and he limited himself to that. Even though these two circles were first posited by Ptolemy in the *Almagest*, this chapter is consistent with other chapters inasmuch as Ptolemy limited himself in all cases to circles. [However] someone who in other places posits bodies but here limits himself to circles is not observing the condition of consistency.[18]

Moreover for the two lower planets, he [Ibn al-Haytham] likewise posits two orbs in addition to the epicycle orb to account for the inclination of the apex and the perigee; to account for the motion of the slant, he posits two other orbs. The first orb, which is the fourth orb of their [i.e. Ibn al-Haytham's] orbs for the epicycles, encloses the other three orbs. The two poles of this orb are two points on a line passing through the epicycle center in the plane of the deferent orb and intersecting at right angles the line that comes from the center of the inclined [orb]. When

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[17] Though Ptolemy claims that the small circle 'revolves with uniform motion, with a period equal to that of the motion in longitude' (Toomer [1984], 599), it is actually, as Ṭūsī notes here, nonuniform since the motion must be correlated with the motion of the epicycle center on the deferent, which is uniform with respect to the equant center but nonuniform with respect to the actual deferent center. Ṭūsī himself presented this in the *Tadhkira* as part of his criticism of Ptolemy's model (and by implication Ibn al-Haytham's) since this results in the apex and perigee moving nonuniformly on the small circle (Ragep [1993], 1: 212–7). One might well interpret what he says in the *Muʿīniyya* itself as also being a criticism of this aspect of Ibn al-Haytham's model: 'Yet even with this postulation the irregularity is not ordered, and in addition several other corruptions come into being' (Ragep [2000], 125). But here he presents Ibn al-Haytham's model without explicit criticism.

[18] The author in question is Shams al-Dīn abū Bakr Muḥammad b. Aḥmad al-Khiraqī (d. 533 H./1138–9), who, in addition to the *Muntahā al-idrāk fī taqāsīm al-aflāk*, wrote *al-Tabṣira fī ʿilm al-hay'a*, both of which were very influential in the development of the *hay'a* (mathematical cosmography) tradition in Islam; see Ragep [1993], 1: 33, 36. Note the criticism of Khiraqī's inconsistency in presenting circles in some places but physical bodies in others; at least, according to Ṭūsī, Ptolemy was consistent in always using circles in the *Almagest*. (This represents a rather rare apology for the much maligned Alexandrian astronomer.)

the orb moves, the diameter, which passes through the two mean distances, must necessarily move around these two poles. Thus there results the motion of the slant except that since the entire epicyclic equator moves, the apex and the perigee will become displaced from their proper places; the apex goes to the perigee's place and the perigee to the place of the apex. Therefore a fifth orb encloses these four orbs; its two poles are the two end-points of the line that passes through the two mean distances. Its motion is opposite and equal to the motion of the fourth orb so that whatever is displaced from its proper place will return to its original position.[19] Now two circuits for the two mean distances result from the motion of the fourth orb, and these are the two small circles that intersect the plane of the deferent at right angles, like a stud on a shield, such that each of the two circumferences are tangent at a point and one plane intersects the other at right angles. The motion of these two [mean] distances on the circumference of these two circles varies in each half, being faster in one half, slower in the other, corresponding to the motion of center on the inclined orb. When the apex on its own circuit is at the maximum inclination from the inclined plane, the mean distance is in the inclined plane. And when the mean distance is at the maximum inclination from the inclined plane, the apex is in the inclined plane. Thus it follows that these two latitudes are inverses of one another.

The illustration of the orbs of the epicycles of these two planets, to the extent they can be drawn in a plane, is as follows. The illustration of the orbs of the upper planets may also be known from [the following] since they are limited to three orbs.

This is the exposition of this treatise, and God is the Knower of Truth.

[19] The same criticism regarding the ordering of Orbs 2 and 3 also applies here to Orbs 4 and 5, which should be reversed. See note 16 above.

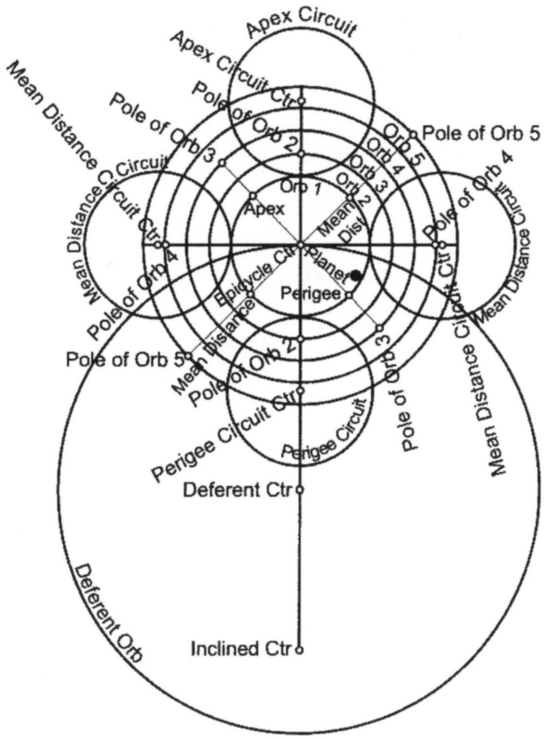

Figure 2B

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