

Magic Squares for Daily Life

JACQUES SESIANO

Introduction

The science of magic squares in Islamic civilization is in keeping with the history of Islamic mathematics. It appeared in the ninth century, developed over the tenth and eleventh, and began to decline in the thirteenth. From then on, magic squares (originally called *wafq al-a'dād*, that is, 'harmonious disposition of the numbers') became increasingly put to magic purposes, due to the supposed capacities of some of them to create good or evil according to the wish of the user. Now this user did not have to be familiar with the general methods for constructing magic squares; all he needed was the knowledge of isolated squares and their properties. That is why the accompanying texts rarely if ever give the slightest hint as to how the squares depicted had been obtained. Europe came to know magic squares through such texts; this gave rise to their modern name and the first attempts to reconstitute their construction. In these magic texts translated into Latin, the magic squares of the first seven possible orders, from $n = 3$ to $n = 9$ (where n is the number of cells on each side), which are filled with the sequence of natural numbers, are associated with the seven planets then known. Therefore the magic attributes of these squares depended upon those of the planets and their efficacy upon the position, favourable or unfavourable, of the planets.

An example of such a text translated (or adapted) from an Arabic one of Moorish origin, occurs in MS Vienna, Österreichische Nationalbibliothek, cvp 5239, fols 147^{va}–149^{ra}, copied in the fourteenth century, an excellent reproduction of which is found in K. Nowotny's reprint of Cornelius Agrippa's *De occulta philosophia* (see his Appendix IX, pp. 665–6). This text, which we shall transcribe and translate here, has suffered from the carelessness of its copyist or his predecessors, so we have occasionally had to

modify the text, not only to correct some minor errors but also to remove repetitions or irrelevant glosses incorporated into the extant copy. (Angle brackets indicate the few additions of our own.) The correction was occasionally made easier by examining analogous passages occurring in later Latin writings, such as two works by the Jesuit Athanasius Kircher, who mentions such squares and their applications as examples of what he calls, depreciatingly, Saracenic and Hebrew cabbala (the amulets he found were often engraved with Hebrew characters). The translation of this Latin text will be followed by an explanation of the methods used to construct these squares.

Latin text

Incipiunt figure 7 planetarum.

Et scias quod in istis 7 figuris occultaverunt philosophi et sapientes antiqui 7 Dei nomina propter hoc quod nullus posset ea pronuntiare indigne, quia forte multi nescientes cum illis mala plurima perpetrarent. Unde videndum est de virtutibus earum.

Figura Saturni est quadrata, et sunt 3 multiplicata per 3, et sunt in quolibet latere 15 (Fig. 1). Cum volueris operari per eam ad mulierem laborantem in partu, scribe eam in panno novo de bombace in die Saturni et in hora eius cum Saturnus fuerit festinus cursu et auctus numero (148^{rb}) et lumine, et Luna sit velox, et sit Saturnus in bona dispositione cum Luna, et ligabis eam ad coxam mulieris dextram<; tunc parturiet felicissime>. Et si scripseris eam in lamina plumbi Saturno retrogrado, vel diminuto lumine et numero, et in mala dispositione ad Lunam, vel sub radiis vel sub stationario, et posueris eam in edificio novo vel plantatione nova, nunquam proficiet illud edificium et homines a plantatione fugient. Et si posueris eam in sede alicuius prelati, cito ab ea deponetur. Et si scripseris eam in lamina plumbi, Saturno in bono statu, et suffumigabis eam cum aluma, et portaveris tecum, non timebis regem neque personam aliquam que tibi nocere velit, et quod cuperis ab eis ferendum tibi observabis.

Figura Iovis quadrata est, et sunt 4^{or} multiplicata per 4<, et sunt in quolibet latere 34> (Fig. 2). Cum volueris operari per eam, facies laminam argenteam in die et in hora Iovis, et sit Iupiter in bono statu; et in dicta lamina sculpes figuram et suffumigabis eam cum ligno aloes et cum ambra, et portabis tecum. Et diligent te

qui viderunt te et impetrabis ab eis quod quesieris. Et si posueris eam in repositoio mercatoris, mercatura eius augmentabitur. Et si posueris eam in columbario vel ubi sunt apes, congregabuntur ibi columbe et apes. Et si quis portaverit eam qui sit infortunatus, fortunabitur, de bono in melius proficiet. Et si posueris eam in sede prelati, prelatio eius durabitur et suos inimicos non timebit, sed proficiet apud eos.

Figura Martis infortunati significat bellum et graverias, et est figura quadrata, et sunt 5 multiplicata per 5, et sunt in quolibet latere 65 (Fig. 3). Cum volueris operari per eam, accipe laminam cupri in die et hora Martis cum Mars fuerit diminutus numero vel lumine, vel infortunatus et retrogradus (148^{ra}) vel in aliquo malo statu, et sculpes ipsam in lamina; et suffumigabis ipsam cum stercoribus murium vel murilegorum; et si posueris eam in edificio novo, non complebitur. Et si posueris eam in sede prelati, deteriorabitur cottidie et infortunabitur. Et si posueris eam in operatorio mercatoris, totum destruetur. Et si feceris predictam figuram (cum) nomine duorum mercatorum et subterraveris eam in domo unius illorum, cadet inter eos odium et inimicitia. Et si forte timorem de rege vel de aliquo magnato habueris, vel de inimicis, vel ad iudicium vel ante iudicem intrare volueris, sculpe hanc figuram ut supra dictum est, et sit Mars fortunatus, directus, auctus numero et lumine, et suffumiga eam cum dragma 1 de lapide corneola; et ponatur predicta lamina in panno serico rubeo, et portabis eam tecum; tunc vinces in iudicio et in bello adversarios tuos, et fugient a facie tua et timebunt te et reverebuntur. Et si posueris (eam) super crus mulieris, continuo fluxum sanguinis patietur. Et si scripseris eam in pergamenio die et hora Martis et suffumigabis eam cum aristologia et eam posueris in loco apum, fugient omnes apes.

Figura Solis appropriata est regibus et principibus huius mundi. Et est figura quadrata, et sunt 6 multiplicata per 6, et (est) figura durationis totius regni, et sunt in quolibet latere 111 (Fig. 4). Cum volueris operari per eam, vide cum Sol fuerit in exaltatione sua, in 15 gradu Arietis, in mense Aprili; accipe 6 dragmas auri et fac inde laminam rotundam in die Solis et formabis in ea figuram Solis, et suffumiga eam cum croco, et lava eam cum aqua rosacea in qua fuerit muscus et ambra, et involves eam in panno serico croceo, et tenebis eam tecum continue; et eris fortunatus

in omnibus rebus et in petitionibus, et impetrabis ab omnibus quicquid quesieris, et ponet Deus benedictionem in manibus tuis (et) in omnibus rebus (tuis). In hoc est magnum secretum.

Figura Veneris est quadrata, et sunt 7 multiplicata per 7, et sunt in quolibet latere 175 (Fig. 5). Et est fortuna propria in facto mulierum et in venustate et de illis. Cum volueris operari per eam, vide quod sit Venus in Piscibus, quia ibi est exaltatio sua, vel in Thauro vel in Libra in quibus est domus eius, et quod sit fortunata, aucta lumine, velox cursu, directa; tunc accipe 7 dragmas argenti puri in die et hora Veneris, et fac laminam et sculpe in ea predictam figuram, et suffumiga eam cum ligno aloes et ambra et mastice, et involve eam in panno serico albo, et porta tecum; et omnes mulieres que viderunt (te) diliget te et videbis mirabilia. Et si vir vel mulier tardet se multum ad uxorandum, portet secum hanc figuram, et terminabitur eius negotium de maritatione et finem laudabilem consequetur. Et si uxor tua vel quelibet alia persona habet te odio, lava predictam laminam cum aqua pluviali, vel fontis vel roris, et da ei bibere, et diliget te et patietur quod quesieris. Et si coqueris camomillam cum aqua et laveris predictam figuram in aqua illa, et asperseris illam aquam in loco ubi est discordia vel in locis iudiciorum (148^{va}), cessabit omne malum et multiplicabitur omne bonum et sapientia. Et si asperseris aquam istam in locis ubi sunt mercature vel bestie, homines in illis mercaturis lucrabuntur, et multiplicabuntur et proficient illa que dicta sunt. Et si posueris figuram istam in columbario vel (ubi) sunt apes, multiplicabuntur ille res. Et si posueris eam in lecto, habundabis in coitu et diliget te uxor tua plurimum, et erit cito pregnans.

Figura Mercurii est quadrata, et sunt 8 multiplicata per 8, et sunt in quolibet latere 260 (Fig. 6). Mercurius est in motu velox, vel in mutatione velox, per vim et naturam habet inclinationem et complexionem aliarum planetarum, et habet complexionem similem signorum, et sibi data est anima naturalis et virtutes eius et sapientia et philosophia et totum quadrivium et scriptura. Cumque volueris operari per eam, accipe 8 dragmas puri argenti in die et hora Mercurii, et inde fac laminam, et figurabis in ea figuram Mercurii quando Mercurius fuerit fortunatus et directus, velox in cursu; suffumiga eam cum ligno aloes et gariofolia et mastice. Et si non poteris habere argentum, scribe eam

in papiro citrino, et valebit tantum. Et porta eam tecum, et omnia que petieris impetrabis. Et si posueris eam in sede prelati, durabit illius prelatio, et multiplicabuntur bona illius. Et si imprimas et scribas istam figuram in annulo vel (148^{vb}) significaveris eam in scutella vitrea vel in vitro albo vel in pelvi ex latone in augmento lumine in die Mercurii mane secundum hoc in quinque primis diebus lunationis, vel septima die, et delaveris eam cum aqua fontis munda et bibes aquam istam per 3 dies, aufert a te omnem oblivionem et addisces de facili scientiam et omnia que volueris. Et si figuraveris eam in speculo calibeo et inspexit se in illo qui habet spasmus, curabitur, et ille qui est paralyticus, et ille qui amisit visum, vel intellectum per frequentiam, et e converso. Et si ieiunaveris per unum diem, vel melius per 3 dies continuos, quod <est ut> non comedas nisi panem cum melle vel cum uvis passis, et post 3 dies illos scripseris istam figuram in panno serico croceo vel citrino, et suffumiga eam cum ligno aloes sic dicendo 'Deus per virtutem istius figure ostende mihi talem rem', et ponas eam sub cussino vel sub capite tuo in nocte, tunc videbis in sompnis petitionem quam a Domino primitus postulasti.

Figura Lune est quadrata, et sunt 9 multiplicata per 9, et sunt in quolibet latere 369 (Fig. 7). Cum volueris per eam operari, accipe pergamenum virgineum et scribe in eo istam figuram in die et in hora Lune, et sit Luna in augmento, et scribe cum musco et croco distemperatis cum aqua rosacea, (149^a) et suffumiga eam cum semine cucumerum et citrulorum et camphora; et postea duplica cartulam et mitte eam in canna facta de argento, et porta tecum. Et valet ad petendum res nobiles et pretiosas et honores, et aufert omne malum; et qui habuerit eam secum in via, non timebit latronem nec aliquam rem pessimam. Et quando volueris quod inimicus tuus vel alter exeat civitatem, scribe istam figuram in pergamenis nigro ovis, et ex altera parte figuras figuram eius et scribas cum sanguine nigri galli decollati in die Lune quando Luna erit in diminutione; et cum illud feceris, mitte cartam illam in poculo novo parvulo et imple ipsum aqua, et dimitte ipsum per totam noctem ad stellas; sequenti die accipe urceolum illum, et dic 'Exeat etc., filius etc., ab ista civitate vel villa', et fiet quod volueris. Tunc effundas vel spergas aquam ex poculo contra 4 plagas mundi, et subterrabis poculum extra villam ubi volueris. Item si vis quod mulier nunquam nubat, scribe hanc

figuram in lamina plumbi in diminutione Lune in die Lune et ex altera parte figurabis figuram mulieris, et dic 'Virgo etc., filia etc., quod nunquam possit nubere', et subterrabis laminam in sepulcro ignoto. Et si mercimonia tua (vel bestias) vendere non possis, scribe (eam) in papiro Luna crescente et pone papirum plicatum in collo 1 bestie; (vel) scribe istam figuram in panno citrino in die et hora Lune cum croco distemperato cum aqua rosacea, et suffumigabis eam suffumigatione predicta, et duplicabis cartulam, et pone eam super res quas vendere velis vel in loco ubi sunt res; tunc cito ac prospere vendes eas.

Expliciunt figure 7 planetarum.

Translation.

The figures of the seven planets.

You are to know that in these seven figures the ancient philosophers and scholars have hidden the seven names of God, the reason being that nobody might pronounce them unworthily; for many ignorant persons may do much harm with them. So let us consider their qualities.

• The figure of Saturn is square, three by three, with 15 on each side (Fig. 1).

2	9	4
7	5	3
6	1	8

Figure 1

– If you wish to use it for a woman in labour during parturition: write it on a new piece of cotton on the day and the hour of Saturn when Saturn is moving rapidly and increasing in number and brightness, while the Moon is rapid and Saturn is in a favourable situation to the Moon; then you will bind this figure to the woman's right hip; this will grant a most successful delivery.

– If you write it on a lead plate while Saturn is retrograding, or decreasing in brightness and number, and is unfavourably situated to the Moon, or under the rays or stationary, and you place it in a new building or a new plantation, the building will never serve any purpose and people will flee the plantation.

– If you place it in the see of some prelate he will soon be removed from it.

– If you write it on a lead plate while Saturn is propitious and you fumigate it with alum and carry it with you, you will not fear the king nor any person who wishes to harm you, and you will see them bringing you anything you desire.

● The figure of Jupiter is square, four by four, with 34 on each side (Fig. 2).

– If you wish to operate with it: make a silver plate in the day and the hour of Jupiter, provided Jupiter is propitious, and engrave upon it the figure; you will fumigate it with aloes wood and amber. When you carry it with you, people who see you will love you and you will obtain from them whatever you request.

– If you place it in the store-house of a merchant, his trade will increase.

– If you place it in a dovecote or in a hive, a flock of birds or a swarm of bees will gather there.

– If someone unlucky carries it, he will prosper and be always more successful.

– If you place it in the see of a prelate, he will enjoy a long prelature and will not fear his enemies, but be successful among them.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Figure 2

14	10	1	22	18
20	11	7	3	24
21	17	13	9	5
2	23	19	15	6
8	4	25	16	12

Figure 3

● The figure of Mars when unfavourable means war and exactions. It is a square figure, five by five, with 65 on each side (Fig. 3).

– If you wish to operate with it, take a copper plate in the day and hour of Mars when Mars is decreasing in number and brightness, or malefic and retrograding, or in any way unfavourable, and engrave the plate with this figure; and you will fumigate it with the excrement of mice or cats; if you place it in a new building, it will never be completed.

- If you place it in the see of a prelate, he will suffer daily harm and misfortune.
- If you place it in the shop of a merchant, it will be wholly destroyed.
- If you make this plate with the names of two merchants and bury it in the house of one of them, hatred and hostility will come between them.
- If you happen to fear the king or some powerful person, or enemies, or have to appear before a judge or a court of justice, engrave this figure as said above when Mars is favourable, in direct motion, increasing in number and brightness; fumigate it with one drachma of carnelian stone; if you put this plate in a piece of red silk and carry it with you, you will win in court and against your enemies in war, for they will flee at the sight of you, fear you and treat you with deference.
- If you place it upon the leg of a woman, she will suffer from a continuous blood flow.
- If you write it on parchment on the day and the hour of Mars and fumigate it with birthwort and place it in a hive, the bees will all fly away.
- The figure of the Sun applies specifically to the kings and the princes of this world. It is a square figure, six by six, indicating the duration of a dynasty, with 111 on each side (Fig. 4).

1	32	34	3	35	6
30	8	27	28	11	7
20	24	15	16	13	23
19	17	21	22	18	14
10	26	12	9	29	25
31	4	2	33	5	36

Figure 4

- If you wish to operate with it, look for when the Sun is in its exaltation, in the 15th degree of Aries, during the month of April. Take six drachmas of gold, make with it a circular plate on the day of the Sun, and cast in it the figure of the Sun. Fumigate it

with saffron, and wash it with dew water containing musk and amber. Wrap it up in a piece of saffron-coloured silk, and keep it with you continuously. You will be lucky in everything and in your requests, and you will obtain from all whatever you ask. God will place his benediction in your hands and in everything belonging to you. In this figure lies a great secret.

• The figure of Venus is square, seven by seven, with 175 on each side (Fig. 5). It is particularly favourable in all matters concerning women, and brings luck in love.

4	35	10	41	16	47	22
29	11	42	17	48	23	5
12	36	18	49	24	6	30
37	19	43	25	7	31	13
20	44	26	1	32	14	38
45	27	2	33	8	39	21
28	3	34	9	40	15	46

Figure 5

– If you wish to operate with it, look for Venus to be in Pisces, because it has its exaltation there, or in Taurus or Libra, because its houses are there, and for Venus to be favourable, increasing in brightness, with a rapid and direct motion. Then take seven drachmas of pure silver on the day and the hour of Venus, make a plate and engrave in it the above figure. Fumigate it with aloes wood, amber and resin. Wrap it up in a piece of white silk, and carry it with you. All the women who see you will love you and you will see wonderful things.

– If a man or a woman is slow to marry, he should carry with him this plate; a marriage will be contracted and witness a happy end.

– If your wife or some other person hates you, wash the above plate with rain water, or water from a spring or the dew, and give it to them to drink; they will love you and let you have your own way.

– If you boil camomile with water and wash the above figure in this water, then sprinkle with it a place where there is discord, or a court of justice, any evil will cease, and good and wisdom increase.

– If you sprinkle this water in markets or where there are farm animals, men will make profit with their merchandise, they will increase and thrive.

– If you place this plate in a dovecote or a hive, they will multiply.

– If you place it in a bed, you will enjoy abundance of coitus, your wife will love you and become rapidly pregnant.

• The figure of Mercury is square, eight by eight, with 260 on each side (Fig. 6). Mercury has a rapid motion, changes rapidly, by its own power and nature has the tendency and the temperament of the other planets, and has a temperament similar to that of the signs; to it was given the natural soul and its virtues, and wisdom, philosophy, the whole quadrivium and writing.

– If you wish to operate with this figure, take eight drachmas of pure silver on the day and the hour of Mercury. Make a plate of it, where you will represent the figure of Mercury when Mercury is favourable and direct, rapid in motion. Fumigate it with aloes wood, clove and resin. But if you cannot have silver, write the figure on lemon-yellow paper; it will have the same effect. Carry it with you and you will obtain whatever you wish.

8	7	59	60	61	62	2	1
49	15	54	12	53	51	10	16
41	42	22	21	20	19	47	48
32	34	35	29	28	38	39	25
40	26	27	37	36	30	31	33
17	18	46	45	44	43	23	24
9	55	14	52	13	11	50	56
64	63	3	4	5	6	58	57

Figure 6

– If you place it in the see of a prelate, his prelature will last, and his goods will multiply.

– If you impress and mark this figure on a ring, or represent it on a glass dish, or on white glass, or in a brass cauldron, on the day of Mercury in the morning, when Mercury is increasing in brightness, and this during the first five days of the lunation, or the seventh day, and wash it off with clean spring water, and drink this water for three days, it will protect you from any forgetfulness, and you will learn with ease science or whatever you wish.

– If you represent it on a mirror of steel and someone having spasms looks at himself in it, he will be cured, so also whoever is paralysed, or whose sight or mind often fails, or conversely.

– If you fast for one day, or, better, for three consecutive days without eating anything but bread with honey or raisins, and write after these three days this figure on a piece of saffron or lemon-coloured silk, then fumigate it with aloes wood saying thus ‘God, by the power of this figure, show me such a thing’, and place it under your pillow or your head for the night, then you will see in your dreams what you had requested from God before.

• The figure of the Moon is square, nine by nine, with 369 on each side (Fig. 7).

37	78	29	70	21	62	13	54	5
6	38	79	30	71	22	63	14	46
47	7	39	80	31	72	23	55	15
16	48	8	40	81	32	64	24	56
57	17	49	9	41	73	33	65	25
26	58	18	50	1	42	74	34	66
67	27	59	10	51	2	43	75	35
36	68	19	60	11	52	3	44	76
77	28	69	20	61	12	53	4	45

Figure 7

— If you wish to operate with it, take a blank parchment, and write on it this figure on the day and the hour of the Moon while the Moon is increasing; you have to write with musk and saffron tempered with dew water. Fumigate it with cucumber and pumpkin seeds and camphor. Next, fold over this scrap and place it in a cane made of silver, and carry it with you. It will have the power to direct you towards noble or precious objects and honours, and it will remove any evil; whoever has it with him on his way will not fear any robber or misadventure.

— If you wish your enemy, or whoever, to leave the town, write this figure on a black sheep's skin and represent on the other side the person's picture, writing with the blood of a black cock beheaded on the day of the Moon when the Moon is decreasing; this being done, place it in a small new cup, fill this with water, and leave it for the whole night under the stars; on the next day, take this vessel and say: 'May X, son of Y, leave this town or village'. Then you must pour out or sprinkle the water from the cup towards the four cardinal directions, and you will bury the cup at any place outside the locality. What you wish will happen.

— Now if you wish a certain woman never to marry, write this figure on a lead plate when the Moon is decreasing on the day of the Moon, and represent on the other side the woman's picture, and say: 'May the virgin X, daughter of Y, never marry'. Bury the plate in an anonymous grave.

— If you cannot sell your merchandise or your farm animals, write this figure on paper when the Moon is increasing. Put the folded paper on the neck of one of the animals. Or write this figure on a piece of lemon-coloured cloth on the day and hour of the Moon, with saffron tempered with dew water, and fumigate it as indicated above. You will then fold this piece of paper and place it on the things you want to sell, or in the room where they are. You will sell them soon and with profit.

Construction of these squares

The general methods for constructing magic squares do not depend upon their size but upon the parity of their order: odd (3, 5, 7, ...), evenly-even (4, 8, 12, ...—thus even and divisible by 4), or oddly-even (6, 10, 14, ...—thus even but divisible by 2 only). A construction is possible for any order greater than 2. If the

squares are filled in with the sequence of natural numbers starting with 1, the magic sum, which is to be found in each row, each column and each of the two diagonals, is given by the formula

$$M_n = \frac{n(n^2 + 1)}{2}$$

where n represents the order of the considered square.

(a) Odd order squares.

Two general methods are seen to have been used in the construction of the squares presented.

First method (Fig. 8, here for $n = 7$). Put 1 in the cell below the central cell in the blank square. Then proceed with the next natural numbers from cell to cell diagonally downwards. When a side of the square is reached, continue by writing the next number on the opposite side, in the cell where we would be if the square were reproduced all around itself. After placing a quantity of numbers equal to the order n , we shall arrive at a cell already occupied. Move then two cells downwards vertically, remaining thus in the column where a number was last written. Resume diagonally from here: this will place the second set of n numbers. Do likewise for the following sets. It was by using such a method—the most common way to construct magic squares of odd order—that two squares appearing in our text were constructed (see Fig. 5 and 7; the diagonal movement is in the first case from right to left).

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

Figure 8

			1			
		15	9	3		
	29	23	17	11	5	
43	37	31	25	19	13	7
	45	39	33	27	21	
		47	41	35		
			49			

Figure 9

26	20	14	1	44	38	32
34	28	15	9	3	46	40
42	29	23	17	11	5	48
43	37	31	25	19	13	7
2	45	39	33	27	21	8
10	4	47	41	35	22	16
18	12	6	49	36	30	24

Figure 10

Second method. Imagine that the blank square contains a rhombus, the corners of which meet its sides in the middle. Fill the cells within this rhombus with the sequence of odd numbers taken in succession. The rhombus will then contain all the odd numbers to be placed in the square (Fig. 9). Write in the first even numbers opposite the first odd numbers, to form the hypotenuse of one of the two lower triangles (Fig. 10). Next, fill the hypotenuse of the other lower triangle by adding 1 to each of the odd numbers in the cells vertically above. Finish by adding successively to each of the two sets of even numbers already placed a quantity equal to the order of the square plus 1, and write the results in the sequence of cells below (moving to the top of the square when

the lower side of the square is reached). Such is the method used for the square of order 5 found in our Latin text (Fig. 3).

Both these methods were devised towards the end of the 10th century or at the beginning of the 11th in the Islamic world (Sesiano, *Traité médiéval*, pp. 32–40). The second received particular attention because of its separation of the numbers by parity. As a matter of fact, these two methods are closer than would seem at first sight. A relationship is already suggested by the presence of the same numbers in the rows and columns of two same-order squares thus constructed (comp. Fig. 8 & 10). An auxiliary device used for their construction in Arabic texts also points to such a relationship (Sesiano, HV I & III). Consider within the blank square a rhombus as before; but this time, make it a figure with the same number of cells as the square under consideration by connecting the opposite points of intersection of the rhombus's sides with the lines separating the rows and columns of the square (Fig. 11).

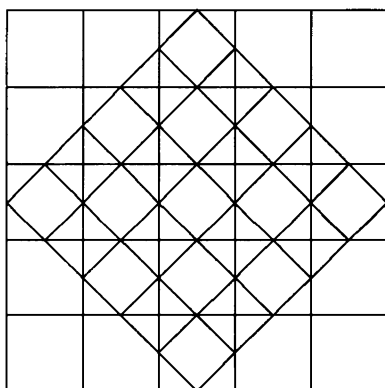


Figure 11

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Figure 12

Then fill the square with the numbers in their natural sequence (Fig. 12). Move those in the four corner triangles towards the opposite sides of the rhombus (Fig. 13). They will thus find their place in the still empty cells of the rhombus, which will now be a magic square as if it had been constructed by the first method.

Suppose now you start with the natural sequence in the cells of the rhombus (Fig. 14). All the odd numbers then fall within cells of the square, leaving the even numbers on intersections. Considering then the rhombus to be divided in four triangles

meeting at its centre, move the even numbers in each into the empty corner triangles opposite (Fig. 15). This will make the proposed square magic, and arranged as if it had been constructed by the second method.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Figure 13

		1		
	6	2		
	11	7	3	
16	12	8	4	
21	17	13	9	5
22	18	14	10	
	23	19	15	
	24	20		
		25		

Figure 14

14	10	1		
	6	2		
20	11	7	3	
16	12	8	4	
21	17	13	9	5
22	18	14	10	
	23	19	15	
	24	20		
		25		

Figure 15

(b) Evenly-even order squares.

At this point, the reader may be somewhat disappointed to discover that the evenly-even order squares in our text (Fig. 2 and 6) were not constructed according to a general method — that is, a method applicable mechanically to any order of the same class — as was the case above. Nevertheless, the principles underlying their constructions *are* general. This may come as a surprise to anyone who already has some knowledge of magic squares, because the general methods of construction for evenly-even squares are so straightforward. But these methods, which are so easy to

apply, were not so easily invented; and did not appear until some time early in the 11th century, whereas the general principles for constructing squares of even order were already known around the mid 10th century: they are used by Abū'l-Wafā' for individual squares (Sesiano, 'Traité d'Abū'l-Wafā'', pp. 133-135) and are quite clearly stated by Ibn al-Haytham (Sesiano, HV I). This makes the squares in our text all the more interesting, for they are one of the few remnants surviving from this short interval between the discovery of the principles and the development of mechanically applicable methods.

8	7	6	5	4	3	2	1
16	15	14	13	12	11	10	9
24	23	22	21	20	19	18	17
32	31	30	29	28	27	26	25
40	39	38	37	36	35	34	33
48	47	46	45	44	43	42	41
56	55	54	53	52	51	50	49
64	63	62	61	60	59	58	57

Figure 16

Consider first, as we did with odd order squares, an even natural square, say of order 8 (Fig. 16, filled in from right to left, according to the Arabic way). Elementary mathematical considerations show that the excess in any lower row over the intended magic sum equals the deficit in the symmetrically located upper row, and the same holds for pairs of symmetrically located columns. Since the difference between corresponding elements in two such rows or columns is constant, we may obtain the magic sum in either case by merely interchanging half of their elements. Great care must be taken, however, in the process: it can only apply to elements in their original (natural) location; and elements in the main diagonals may not be moved as freely as the others, for our above consideration concerned the equalization of

rows and columns only. Ideally, the diagonal elements should not be touched at all: the diagonals of the natural square already give the magic sum. This is why the easy methods found later will leave them untouched. But this is not the case for one of the squares of our Latin text (Fig. 2). They have thus been constructed individually, but on the basis of the general principles.

Let us represent the interchanges with particular symbols: | will designate a vertical interchange between opposite rows in the natural square, — will designate a horizontal interchange between opposite columns, and X a diagonal interchange. To have the required number of interchanges, we must thus find, for a square of order n , bearing in mind that X performs the interchange for both row and column, altogether $\frac{n}{2}$ signs X and | in each row (the — being irrelevant here since the numbers remain in the same row); likewise, we must find altogether $\frac{n}{2}$ signs X and — in each column. In our cases, the arrangement is as shown in Fig. 17 and 18, the dots indicating where the corresponding numbers keep their original place.

× |

•			•
×	—	—	×
×	—	—	×
•			•

— •

Figure 17

— •

•	•	×	×	×	×	•	•
×	•		—	×		•	—
×	×	•	•	•	•	×	×
•	×	×	•	•	×	×	•
•	×	×	•	•	×	×	•
×	×	•	•	•	•	×	×
×	•		—	×		•	—
•	•	×	×	×	×	•	•

×

Figure 18

Once this stage is reached, completing the magic square is an easy matter, for there is no need to perform the individual interchanges: it comes to the same thing if we count the cells from each corner and replace the relevant sign with the number reached. To that purpose, we choose a corner for the dots, count the cells and write in the numbers concerned. We do the same

from the opposite corner, but this time for the X. For the —, we start from the corner horizontally aligned with the corner of the dots, but for the | from the corner vertically aligned with that of the dots. Doing this with the above two figures produces the magic squares of the Latin manuscript (Fig. 2 and 6).

(c) The square of oddly-even order.

We are now left with the square of order 6 (Fig. 4). The comment will be brief: obviously, it has not been constructed according to the general principles of interchanges. On the one hand, this is not surprising since a general method for the oddly-even order had to await the end of the 11th century (Sesiano, HV III). On the other hand, constructions using the interchanges had, as for the evenly-even order, appeared earlier. Thus, a square of order 6 constructed by interchanges is already found in the treatise by Abū'l-Wafā'. Furthermore, the three types of interchange are seen in another square of order 6 found in Spain; it is found in a text, attributed to the astronomer al-Zarqālī, of a content similar to that of our Latin text (reproduction p. 660 of Nowotny's edition). This is the square described in our figures 19 and 20.

•	×	—	×	×	•
—	•	×	×	•	×
×	—	•	•	×	×
	—	•	•	×	
—	•			•	×
•		—	×		•

Figure 19

6	32	3	34	35	1
7	11	27	28	8	30
19	14	16	15	23	24
18	20	22	21	17	13
25	29	10	9	26	12
36	5	33	4	2	31

Figure 20

Bibliography

- Agrippa, Cornelius, *De occulta philosophia*, ed. K. A. Nowotny, Graz, 1967.
- Kircher, Athanasius, *Arithmologia, sive de abditis numerorum mysteriis*, Rome, 1665.
- . *Œdipus ægyptiacus, hoc est universalis hieroglyphicæ doctrinæ instauratio*, Rome, 1652-4.
- Sesiano, HV I: Jacques Sesiano, 'Herstellungsverfahren magischer Quadrate aus islamischer Zeit (I)', *Sudhoffs Archiv*, 64, 1980, pp. 187-96.
- . HV III: Jacques Sesiano, 'Herstellungsverfahren magischer Quadrate aus islamischer Zeit (III)', *Sudhoffs Archiv*, 79, 1995, pp. 193-226.
- . *Traité médiéval*: Jacques Sesiano, *Un traité médiéval sur les carrés magiques*, Lausanne, 1996.
- . 'Traité d'Abū'l-Wafā': Jacques Sesiano, 'Le traité d'Abū'l-Wafā' sur les carrés magiques', *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften*, 12, 1998, pp. 121-244.