A Survey of Rules for Computing the True Daily Motion of the Planets in India

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This is a survey of rules for computing true daily motion of the planets from mean daily motion appearing in Indian astronomical treatises (*siddhāntas* and *karaṇas*). They are arranged in chronological order.

1 *Notation*

$\bar{\alpha}$: manda anomaly
$\tilde{\alpha}$: śīghra anomaly
$\bar{v}$: mean daily motion of the planet
$\tilde{v}$: manda-corrected daily motion
v: true daily motion ($= \tilde{v}$ for the sun and the moon)
v_A: daily motion of manda apogee
v_S: daily motion of śīghra apogee
$\bar{v}_\alpha$: daily motion of manda anomaly ($= \bar{v} - v_A$)
$\tilde{v}_\alpha$: daily motion of śīghra anomaly ($= v_S - \tilde{v}$)
v_α: daily motion of corrected anomaly
ε: correction to mean daily motion
R: radius of the standard circle
C: circumference of the standard circle
r: radius of an epicycle
c: circumference of an epicycle
H: "hypotenuse", true geocentric distance
I: interval of a Rsine table (usually $225'$)
$J[n]$: nth tabulated Rsine
$\Delta J[n]$: nth Rsine-difference ($J[n] - J[n-1]$)
ΔJ_α: Rsine-difference corresponding to an anomaly α

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2 *Abbreviation of Sanskrit Texts*

PS: *Pañcasiddhāntikā* of Varāhamihira (composed ca. 550)
BSS: *Brāhmasphuṭasiddhānta* of Brahmagupta (composed 628)
MBh: *Mahābhāskarīya* of Bhāskara I (composed before 629)
LBh: *Laghubhāskarīya* of Bhāskara I (composed after MBh)
KhKh: *Khaṇḍakhādyaka* of Brahmagupta (epoch 665)
SDV: *Śiṣyadhīvṛddhidatantra* of Lalla (composed ca. 750?)
SS: *Sūryasiddhānta* (composed ca. 800)
VS: *Vaṭeśvarasiddhānta* of Vaṭeśvara (composed 904)
LMM: *Laghumānasa* of Mañjula (Muñjāla)[1] (composed ca. 932)
SSE: *Siddhāntaśekhara* of Śrīpati (composed ca. 1050)
MS: *Mahāsiddhānta* of Āryabhaṭa II (composed ca. 950–1000?)
SSI: *Siddhāntaśiromaṇi* of Bhāskara II (composed 1150)
SOS: *Somasiddhānta* (composed before 1400)
KA: *Karaṇāmṛta* of Citrabhānu (composed ca. 1530)

3 *Introduction*

We can find two types of rules in Indian astronomical texts for calculating true daily motion from mean daily motion.
Type 1: convert $\bar{v}$ or $\bar{v}_\alpha$ directly into v or v_α. In the type 1 rules, the true geocentric distance of the planet (*karṇa*, H) is usually used. For example H for p_1 in figure 1 is Op_1 and the true daily motion of p_1 (v), or of anomaly (v_α) is calculated:

$$v = \bar{v} \cdot \frac{R}{H}, \quad v_\alpha = \bar{v}_\alpha \cdot \frac{R}{H}.$$

This rule is sometimes called *karṇabhukti.*
Type 2: calculate the difference between true and mean daily motion ($\varepsilon = |v_\alpha - \bar{v}_\alpha| = |v - \bar{v}|$ in figure 1) and add or subtract it from $\bar{v}$. Most of the type 2 rules use Rsine-differences for calculating ε.
First of all, the Rsine-difference including the mean anomaly of planet $\bar{p}1$ (ΔJ_α) is found from a Rsine table in this way:
when

$$J[n-1] \leq \mathrm{Sin}\,\bar{\alpha} < J[n],$$

[1] About the name of the author, see Shukla 1990, p. 1.

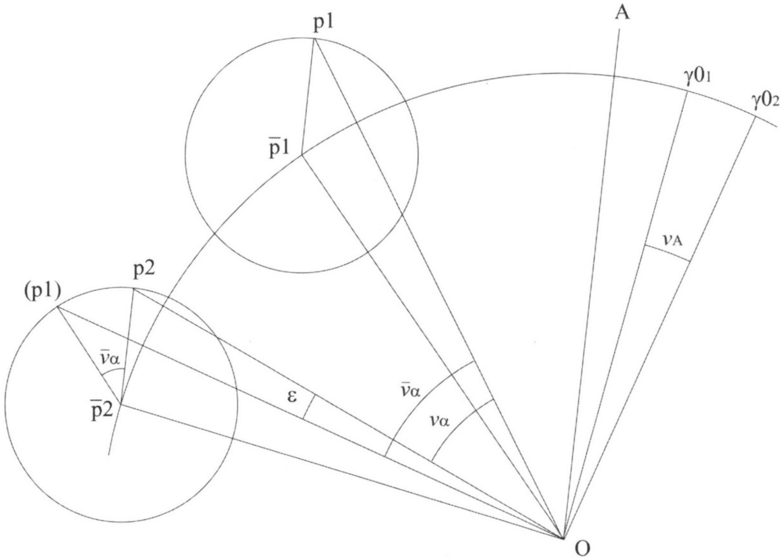

Figure 1: General

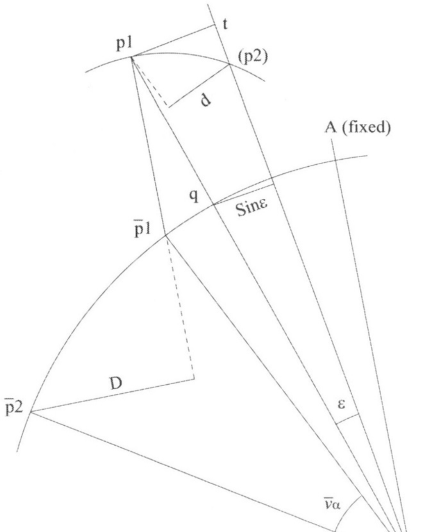

Figure 2: jīvabhukti

$$\Delta J_\alpha = \Delta J[n] = J[n] - J[n-1]. \tag{1}$$

Then, in figure 2, an Rsine-difference corresponding to $\bar{v}_\alpha$ (D) is calculated by a proportion from the interval of a Rsine table (I) and ΔJ_α:

$$I : \Delta J_\alpha = \bar{v}_\alpha : D.$$

Then D is converted into d by another proportion using the circumference of the standard circle (C) and that of an epicycle (c):

$$D : C = d : c.$$

Assuming $d = \operatorname{Sin}\varepsilon$, it is converted into arc ε by the third proportion with the first tabulated Rsine ($J[1]$):

$$J[1] : I = \operatorname{Sin}\varepsilon : \varepsilon.$$

Combining these three proportions we get:

$$\varepsilon = \bar{v}_\alpha \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{c}{C}. \tag{2}$$

This rule is called *jīvabhukti* because it uses Rsines (*jīva*). The problem of *jīvabhukti* is that the correction ε does not change continuously; it remains the same while $\operatorname{Sin}\alpha_1$, $\operatorname{Sin}\alpha_2$, $\operatorname{Sin}\alpha_3 \ldots$ are between the same Rsines. Bhāskara I criticizes this in LBh II, 14 and some later astronomers provide inprovements (SS II, 50–51; LMM II, 4cd; MS III, 15–16ab; SSI II, 37).

4 *Pañcasiddhāntikā*

4.1 *PS IX, 12–13: Type 2*

$$\bar{v}_\alpha = \bar{v} - v_A,$$

$$\operatorname{Sin}\varepsilon = \bar{v}_\alpha \frac{\Delta J_\alpha}{225} \cdot \frac{c}{360},$$

$$v = \bar{v} \pm \varepsilon.$$

360 is the circumference of the standard circle expressed in degrees and 225 is the interval of the Rsine table in minutes. Varāhamihira does not mention how to convert $\operatorname{Sin}\varepsilon$ into ε here[2].

[2] Formula (2) of Neugebauer and Pingree 1970-71, part II p. 71 does not correctly reflect what the text says.

4.2 *PS IX, 14cd: Type 1*

$$H = \bar{v}\frac{R}{v}.$$[3]

5 *Brāhmasphuṭasiddhānta*

5.1 *BSS II, 41–42ab: Type 2*

$$\varepsilon = \bar{v}_\alpha \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{c}{360}.$$

5.2 *BSS II, 42cd–44: Type 1*

$$v = v_S - (v_S - \bar{v}) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}.$$

This rule uses crude approximations (See Ikeyama 2003a).

6 *Mahābhāskarīya and Laghubhāskarīya*

6.1 *MBh IV, 14–17; LBh II, 9–13: Type 2*

6.1.1 *MBh IV, 14; LBh II, 9–10*

$$v = \bar{v} \pm \bar{v}\frac{\Delta J_\alpha}{I} \cdot \frac{c'}{80}.$$

Bhāskara clearly mentions that the mean daily motion is divided by 'the arc', I, i.e., 225 minutes. This means that the process for converting $\mathrm{Sin}\,\varepsilon$ into ε is omitted, but this does not lead to a problem in practice because the first Rsine ($J[1]$) is also 225 in MBh whose R is 3438.

2/9 of the circumferences of the epicycles (c') are used here which are given in MBh VII, 13–16 and LBh I, 19–22 as in the *Āryabhatīya* I, 8–9.

In the Ārya school $v_A = 0$ (and therefore $\bar{v} = \bar{v}_\alpha$) except in the case of the moon, so that $\bar{v}$ is used instead of $\bar{v}_\alpha$ in this formula.

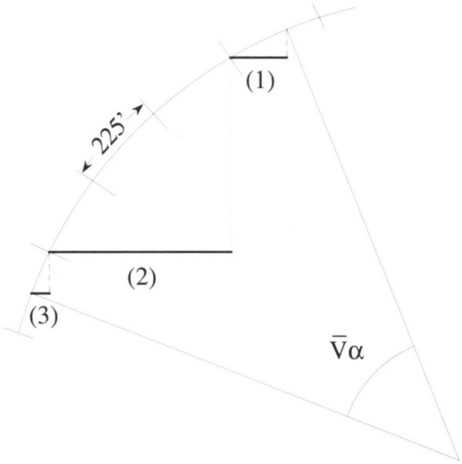

Figure 3: Moon Rule in MBh and LBh

6.1.2 *MBh IV, 15–17; LBh II, 11–13*

For the daily motion of the Moon Bhāskara gives a special rule to calculate D because the motion of the Moon is great and the proportion $D = \bar{v}_\alpha \cdot \Delta J_\alpha / I$, cannot be used (see figure 3). His rule is:

$$D = (1) + (2) + (3),$$

where (2) is the sum of tabulated Rsine-differences and (1) and (3) are computed by means of the proportion.

6.2 *MBh IV, 58–59: Type 2*

The same rule as in MBh IV, 14 and LBh II, 9–10 is used here for the other five star-planets for calculating both manda correction and śīghra correction:

$$\tilde{v} = \bar{v} \pm \bar{v} \frac{\Delta J_\alpha \cdot c'}{18000}; \; v = \tilde{v} \pm (v_S - \tilde{v}) \frac{\Delta J_\alpha \cdot c'}{18000} \cdot \frac{R}{H},$$

where

$$18000 = 225 \times 80.$$

In the case of śīghra correction, Bhāskara converts d into Sin ε by means of a proportion:

$$d : H = \operatorname{Sin} \varepsilon : R.$$

6.3 *MBh IV, 13; LBh II, 8: Type 1*

This rule is said to be only for the Sun and Moon.

$$v = \bar{v}\frac{R}{H}.$$

7 *Khaṇḍakhādyaka*

7.1 *KhKh I, 19–20: Type 2*

$$v_{\text{sun}} = \bar{v}_{\text{sun}} \pm \frac{\Delta J_\alpha}{15};\ v_{\text{moon}} = \bar{v}_{\text{moon}} \pm \frac{\Delta J_\alpha \cdot 7}{8}, \qquad \text{(I, 19)}$$

$$v = \bar{v} \pm \bar{v}_\alpha \frac{\Delta J_\alpha}{900}. \qquad \text{(I, 20)}$$

The first two formulae can be derived from the third when the value of the mean daily motion of anomaly of the sun (about 60′) and of the moon (about 784′) are substituted for $\bar{v}_\alpha$. In the third formula, 900 corresponds to $J[1] \cdot 360/c$. Therefore, if this formula is for all the planets, it follows that the circumferences of the manda epicycles of all the planets would be the same.

8 *Śiṣyadhīvṛddhida*

8.1 *SDV II, 15: Type 2*

$$v_{\text{sun}} = \bar{v}_{\text{sun}} \pm \frac{\Delta J_\alpha}{101};\ v_{\text{moon}} = \bar{v}_{\text{moon}} \pm \frac{\Delta J_\alpha \cdot 10}{33}.$$

These are the same rules as in SDV III, 11 but applied specifically to the sun and the moon; $J[1]$ of SDV is 225, c' of the sun is 3, and that of the moon is 7.

8.2 *SDV III, 11–13: Type 2*

Exactly the same rule as MBh IV 58–59:

$$\tilde{v} = \bar{v} \pm \bar{v}\frac{\Delta J_\alpha}{225} \cdot \frac{c'}{80}, \qquad \text{(III, 11)}$$

$$v = \tilde{v} \pm (v_S - \tilde{v})\frac{\Delta J_\alpha}{225} \cdot \frac{c'}{80} \cdot \frac{R}{H}. \qquad \text{(III, 12–13)}$$

8.3 *SDV III, 18–19: Type 1*

For śīghra correction Lalla uses the same rule as BSS II, 42cd–44:

$$\tilde{v} = \bar{v} \cdot \frac{R}{H}, \qquad \text{(III, 18ab)}$$

$$v = v_S - (v_S - \tilde{v}) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}. \qquad \text{(III, 18cd–19)}$$

9 *Sūryasiddhānta*

9.1 *SS II, 47–49: Type 2*

$$\tilde{v} = \bar{v} \pm \bar{v}\frac{\Delta J_\alpha}{225} \cdot \frac{c}{360},$$

in the case of the moon,

$$\tilde{v} = \bar{v} \pm (\bar{v}_{moon} - v_A)\frac{\Delta J_\alpha}{225} \cdot \frac{c}{360}.$$

This is the rule only for the manda correction. '225' is the first tabulated Rsine, not I. $\bar{v}$ is used instead of $\bar{v}_\alpha$ because the manda apogees of the planets except the moon are fixed in the *Sūryasiddhānta* as in the Ārya school.

9.2 *SS II, 50–51: Type 2*

$$v = \tilde{v} \pm (v_S - \tilde{v}) \cdot \frac{|H - R|}{H}.$$

This is a *jīvabhukti* type rule but does not use Rsine-differences. In figure 2, $Oq = R$, $Op1 = H$, $p_1q = |H - R|$, and $\bar{p}_1p_1 = r$.

$$\text{arc}\,\bar{p}_1\bar{p}_2 : R = \text{arc}\,p_1(p_2) : r.$$

Assuming

$$\triangle p_1(p_2)t \propto \triangle p_1\bar{p}_1 q,$$

$$p_1(p_2)(\approx \operatorname{arc} p_1(p_2)) : p_1 t = r : |H - R|,$$

and also

$$H : p_1 t = R : \operatorname{Sin}\varepsilon,$$

combining these three proportions, we get this formula. See Burgess' explanation (Burgess 1860, pp. 91–92) and Sengupta's discussion in the reprint (Burgess 1860, reprint pp. xx–xxvi).

10 *Vaṭeśvarasiddhānta*

10.1 *VS II, i, 97–98: Type 2*

This is a rule for manda correction.

$$\tilde{v} = \bar{v} \pm \bar{v}_\alpha \frac{\Delta J_\alpha}{J[1]} \cdot \frac{c}{360}.$$

10.2 *VS II, i, 99: Type 1*

This is also the rule for manda correction only.

$$v = v_A + \bar{v}_\alpha \cdot \frac{R}{H}.$$

10.3 *VS II, i, 100: Type 2*

The same rule as VS II, i, 97-98 applied specifically to the sun and the moon. In VS, $J[1]$ is 56;15 (225/4), the circumference of the manda epicycle of the sun is 14°, and that of the moon is 31; 30°.

$$v_{\text{sun}} = \bar{v}_{\text{sun}} \pm \frac{\Delta J_\alpha \cdot 7}{172}; \; v_{\text{moon}} = \bar{v}_{\text{moon}} \pm \frac{\Delta J_\alpha \cdot 49}{40}.$$

10.4 *VS II, iii, 18: Type 1*

$$\tilde{v} = v_A + (\bar{v}_\alpha) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}; \; v = v_S - (\tilde{v}_\alpha) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}.$$

This is a BSS type rule but used for both manda and śīghra correction.

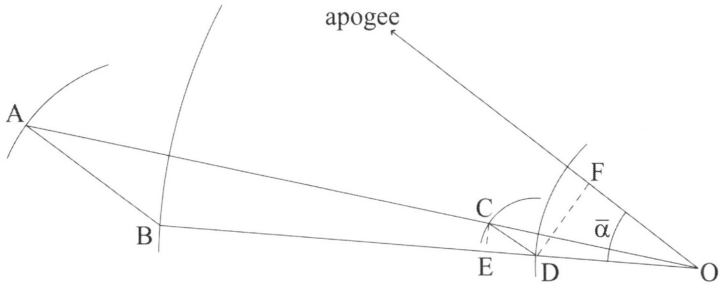

Figure 4: Laghumānasa

11 *Laghumānasa*

11.1 *LMM 14cd (II, 4cd): Type 2*

$$v = \bar{v} \pm \bar{v} \cdot \frac{\text{Cos}\,\bar{\alpha}}{d \pm \frac{\text{Cos}\,\bar{\alpha}}{2}},$$

where $\text{Cos}\,\bar{\alpha}$ is $\overline{OF}$ of figure 4 expressed in degrees and d, which are given in LMM 13 (II, 3), are equal to $R/\bar{r}$; R is $\overline{OD}$ in minutes and $\bar{r}$ is $\overline{AB}$, the radius given in degrees of a manda epicycle corresponding to $\bar{R}$ ($\overline{OB}$). $\bar{v}_\alpha = \bar{v}$ here which means the motions of apogees (v_A) are assumed to be 0.
Using superscripts $\circ$ and m which mean degrees and minutes respectively, we can express this formula like:

$$v^m = \bar{v}^m \pm \bar{v}^m \cdot \frac{\text{Cos}\,\bar{\alpha}^\circ}{\frac{R^m}{\bar{r}^\circ} \pm \frac{\text{Cos}\,\bar{\alpha}^\circ}{2}},$$

Combining a formula which is also found in MBh IV, 58–59 (section 6.2) and SDV III, 12–13 (section 8.2):

$$\varepsilon^m = \bar{v}^m \cdot \frac{\varDelta J_\alpha{}^\circ}{J[1]^\circ} \cdot \frac{\bar{r}^m}{\bar{R}^m} \cdot \frac{R^\circ}{H^\circ},$$

and the relationship Bhāskara II mentions (formula (3) in section 14.1 below):

$$\varDelta J_\alpha{}^\circ = \text{Cos}\,\bar{\alpha}^\circ \cdot \frac{J[1]^\circ}{R^\circ},$$

we get:

$$\varepsilon^m = \bar{v}^m \cdot \frac{\text{Cos}\,\bar{\alpha}^\circ \cdot \bar{r}^m}{\bar{R}^m \cdot H^\circ}.$$

Assuming

$$H^\circ(\overline{OC}) \approx \overline{OD} + \overline{DE} = R^\circ + \text{Cos}\,\bar{\alpha}^\circ \cdot \frac{\bar{r}^m}{\bar{R}^m},$$
$$\bar{R}^m \approx 3600,$$

we finally get:

$$\varepsilon^m = \bar{v}^m \cdot \frac{\text{Cos}\,\bar{\alpha}^\circ}{R^\circ \cdot \frac{\bar{R}^m}{\bar{r}^m} + \text{Cos}\,\bar{\alpha}^\circ}$$
$$= \bar{v}^m \cdot \frac{\text{Cos}\,\bar{\alpha}^\circ}{\frac{\bar{R}^m}{\bar{r}^\circ} + \text{Cos}\,\bar{\alpha}^\circ}.$$

It seems, as Majumdar points out (Majumdar 1951, p. 53), that Cos $\bar{\alpha}/2$ of Mañjula's formula should be Cos $\bar{\alpha}$.
This is a variation of the type 2 rule which does not use any Rsine-difference. Compare Bhāskara's formula (section 14.1) and see the explanations of Majumdar (Majumdar 1951, pp. 45–56) and of Shukla (Shukla 1990, pp. 125–131).

11.2 *LMM 17 (II, 7): Type 1*

$$v = v_S - (v_S - \tilde{v}) \cdot \frac{\text{vyāsa} - \frac{\text{śīghraphala}}{12}}{\text{śīghra divisor}}.$$

The 'śīghra divisors' are calculated in LMM 15–16ab (II, 5–6ab):

$$\text{śīghra divisor} = d_m \cdot \frac{r_m}{r_s} + \frac{\text{Sin}\,\tilde{\alpha}}{3} \pm \text{Cos}\,\tilde{\alpha},$$

where r_s is the radius of a śīghra epicycle given in degrees.
Up to now I have not found a satisfactory explanation for this formula. See Majumdar (Majumdar 1951, pp. 50–61) and Shukla (Shukla 1990, pp. 131–136).

12 *Siddhāntaśekhara*

12.1 *SSE III, 40–41ab: Type 2*

$$v = \bar{v} \pm \bar{v}_\alpha \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{c}{360}.$$

12.2 *SSE III, 42–43: Type 1*

$$v = v_S - (v_S - \tilde{v}_\alpha) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}.$$

This is the same rule as BSS II, 42cd–44.

13 *Mahāsiddhānta*

13.1 *MS III, 15cd–16ab: Type 2*

$$v = \bar{v} \pm \bar{v} \cdot \frac{\text{koṭiphala}}{R}.$$

This is the same rule as Bhāskara II (see section 14.1 below) if $\bar{v}_\alpha = \bar{v}$, i.e., v_A of the sun is assumed to be zero.

14 *Siddhāntaśiromaṇi*

14.1 *SSI II, 37: Type 2*

$$\tilde{v} = \bar{v} \pm \bar{v}_\alpha \cdot \frac{\text{koṭiphala}}{R}.$$

Bhāskara says in his auto-commentary that the difference of the true longitudes of the plenet in two successive days is 'crude motion (*sthūlā gatiḥ*)', and motion calculated by this rule is 'accurate and instantaneous (*sūkṣmā tātkārikī*)'.

Bhāskara assumes $\Delta J_\alpha \propto \mathrm{Cos}\,\bar{\alpha}$, so that the equation ε changes continuously. For finding the Rsine-difference corresponding to $\bar{\alpha}$, he does not use the traditional rule (formula (1) of the introduction) but calculates it with a proportion (Chaturvedi 1981, p. 120):

$$\Delta J_\alpha = \mathrm{Cos}\,\bar{\alpha} \cdot \frac{J[1]}{R}. \tag{3}$$

Then, substituting this 'instantaneous Rsine-difference' for ΔJ_α in the typical *jīvabhukti* (formula (2) of the introduction), we get:

$$\varepsilon = \bar{v}_\alpha \cdot \frac{\operatorname{Cos}\bar{\alpha}}{R} \cdot \frac{c}{C},$$

where $(\operatorname{Cos}\bar{\alpha} \cdot c)/C$ is the *koṭiphala.*

14.2 *SSI II, 39: Type 1*

$$v = v_S - \tilde{v}_\alpha \cdot \frac{\operatorname{Sin}(90 - \text{phalāṃśa})}{H}.$$

This is an improvement of Brahmagupta's rule in BSS II 42cd–44 (section 5.2 above). According to Bhāskara's auto-commentary (Chaturvedi 1981, pp. 121–123), when formula (3) is combined with the rule of Brahmagupta:

$$v = v_S - (v_S - \tilde{v}) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}$$

a formula is obtained:

$$v = v_S - (v_S - \tilde{v}) \cdot \frac{\operatorname{Cos}\tilde{\alpha}}{H}$$

and this is the formula mentioned here. Therefore, the word *phalāṃśa*, literally means 'degrees of the result' should be understood as śīghra anomaly $\tilde{\alpha}$.

15 *Somasiddhānta*

15.1 *SOS II, 29: Type 1*

$$v = v_S \pm \tilde{v}_\alpha \cdot \frac{R}{H}.$$

16 *Karaṇāmṛta*

16.1 *KA I, 28cd–29ab: Type 2*

$$\tilde{v} = \bar{v} \pm \bar{v} \cdot \frac{\operatorname{Cos}\bar{\alpha}}{\text{divisor}}.$$

Divisors are given in I, 25–27. KA 28cd is very close to LMM II, 4cd (section 11.1 above). The anonymous commentator interprets the word *koṭi* as *koṭiphala*, $(\operatorname{Cos}\bar{\alpha}\cdot c)/C$ (Namboodiri 1975, p. 14).

16.2 *KA I, 36–37: Type 1*

$$v = v_S - (v_S - \bar{v}) \cdot \frac{\text{śīghravyāsa} - \frac{\text{doḥphala}}{14}}{\text{śīghra divisor}}.$$

This rule seems to be the same as that of LMM II, 7 (see section 11.2 above) except that the divisor of *doḥphala* is 14 (*śakvarī*), not 12.

Source Text

PS IX 12–13
saptakalā vitryaṃśāś candroccasyendubhuktir anayonā |
kendrasya parijñeyā sphuṭabhuktiś cānayā kāryā || 12
kendrāntarajyāguṇitā tithivargeṇoddhṛtā ca pariṇāmya |
tatkārmukaṃ kṣayacayau bhuktau mṛgakarkaṭādyeṣu || 13
(Neugebauer and Pingree 1970-71, part I pp. 92, 94)

PS IX 14cd
vyāsārdhahatā bhuktiḥ sphuṭabhuktihṛtā sphuṭaḥ karṇaḥ || 14cd
(Neugebauer and Pingree 1970-71, part I p. 94)

BSS II, 41–44
grahamandakendrabhuktir jyāntaraguṇitādyajīvayā bhaktā |
labdhaṃ sphuṭaparidhiguṇaṃ bhagaṇāṃśahṛtaṃ phalakalābhiḥ || 41
mṛgakarkyādāv ūnādhikā svamadhyamagatiḥ sphuṭārkendvoḥ |
śīghragatiṃ mandaphalasphuṭabhuktyūnāṃ kujādīnām || 42
śīghraphalabhogyajīvāsaṃguṇitām ādyajīvayā vibhajet |
phalaguṇitaṃ vyāsārdhaṃ vibhājayec chīghrakarṇena || 43
labdhonā śīghragatiḥ sphuṭabhuktir bhavati labdham adhikaṃ cet |
śīghragateḥ śīghragatiṃ labdhāt saṃśodhya vakragatiḥ || 44
(Ikeyama 2003a)

MBh IV, 13–17
viṣkambhārdhahatā bhuktiḥ sūryācandramasoḥ sadā |
svāviśeṣeṇa karṇena sphuṭabhuktir avāpyate || 13
antyajīvāthavā bhuktyā guṇitā dhanuṣā hṛtā |
svaparidhyāhate ’śītyā labdhe hīnādhike sphuṭā || 14
antyajīvādhanuḥkhaṇḍaṃ kendrabhogād viśodhayet |
tadviśodhya makheḥ śeṣaṃ pātyate ’viṣame tataḥ || 15
uccabhuktivihīnāyā bhukteḥ śītāṃśumālinaḥ |
utkramajyā krame grāhyā kramajyā cotkramasthite || 16
ādyantayoḥ phalaṃ yuktvā guṇayoś cānupātataḥ |
tatphalena vihīnādhyā bhuktiḥ sphuṭatarā hi sā || 17
(Shukla 1960, text section p. 23)

LBh II, 8–13
vyāsārdhasaṅguṇā bhuktir madhyā karṇena labhyate |

sphuṭabhuktiḥ sahasrāṃśo śītāṃśor apy ayaṃ vidhiḥ || 8
antyamaurvīhatāṃ bhuktiṃ madhyamāṃ dhanuṣā haret |
labdhaṃ svavṛttasaṃkṣuṇṇaṃ chitvāśītyā viśodhayet || 9
makarādisthite kendre karkaṭādau tu yojayet |
madhyabhuktau sahasrāṃśoḥ sphuṭabhuktir udāhṛtā || 10
utkramakramato grāhyāḥ padayor ojayugmayoḥ |
vartamānaguṇād indoḥ kendrabhukteḥ kalāvaśāt || 11
ādyantadhanuṣor jñeyaṃ phalaṃ trairāśikakramāt |
gatagantavyadhanuṣī kendrabhukter viśodhayet || 12
ittham āptaguṇaṃ hatvā vṛttenāśītisaṃhṛtam |
prāgvat kṣayodayāv indor madhye bhoge sphuṭo mataḥ || 13
(Shukla 1963, text section pp. 5–6)

MBh IV, 58–59
mandāntyajīvāguṇitāṃ svabhuktiṃ bhūyaḥ svavṛttena hatāṃ vibhajya |
rāśeḥ kalābhir daśatāḍitābhir bhuktau dhanarṇaṃ padayuktito 'rdham || 58
śīghroccabhuktes tad apāsya śeṣāt kendrāntyajīvāvidhinā yad āptam |
trijyāhataṃ karṇavibhaktabhedaṃ nyāsena śīghrasya dhanarṇam iṣṭam || 59
(Shukla 1960, text section p. 27)

KhKh I, 19–20
pañcadaśakena vibhajed bhānumato bhogyamānakaṃ piṇḍam |
śaśino 'gaguṇaṃ vasubhiḥ kṣayadhanadhanahānayaḥ svagatau || 19
gatibhogyakhaṇḍakavadhāl labdhaṃ navabhiḥ śatai ravīnduphalam |
prāgvac chukrādīnāṃ kṣayadhanadhanahānayaḥ svagatau || 20
(Sengupta 1941, pp. 22–23)

SDV II, 15
svabhogyakhaṇḍaṃ kṣitikhendubhir hṛtaṃ
raver vidhor digguṇitaṃ suroddhṛtam |
tadūnayukte bhavataḥ sphuṭe gatī
kramāt svakendre mṛgakarkaṭādike || 15
(Chatterjee 1981, part 1 p. 34)

SDV III, 11–13
jyākhaṇḍakena guṇitā mṛdukendrajena
bhuktir grahasya śarayugmayamair vibhaktā |
kṣuṇṇā sphuṭena guṇakena hṛtā khanāgair
liptā gateḥ phalam ṛṇaṃ dhanam uktavac ca || 11
tadvarjitā svacalatuṅgagatiḥ svabhogya-
khaṇḍāhatā śarayamākṣihṛtā hatā ca |
svena sphuṭena guṇakena khanāgabhaktā
trijyāhatā śrutihṛtāśuphalaṃ gateḥ syāt || 12
mandasphuṭā grahagatiḥ sphuṭatām upaiti
yuktonitā virahitā sahitām unā ca |
śīghrābhidhānanijakendrapadakrameṇa
vakrā gatir bhavati ced ṛṇato viśuddhā || 13
(Chatterjee 1981, part 1 p. 46)

SDV III, 18–19
trijyāhatā grahagatir mṛdukarṇabhaktā

mandasphuṭā bhavati tadrahitāśubhuktiḥ |
trijyāhatā svacalakarṇahṛtāśucāpa-
bhogyajjyayā viguṇitā vihṛtādimaurvyā || 18
labdhaṃ tyajet svacalatuṅgagateḥ sadaiva
śeṣaṃ sphuṭā bhavati ca grahabhuktir evam |
labdhaṃ bhaved yad adhikaṃ calatuṅgabhukter
vyastaṃ bhunakti khacaraḥ prativāsaraṃ tat || 19
(Chatterjee 1981, part 1 p. 49)

SS II, 47–51
svamandabhuktisaṃśuddhā madhyabhuktir naśāpateḥ |
dorjyāntarādikaṃ kṛtvā bhuktāv ṛṇadhanaṃ bhavet || 47
grahabhukteḥ phalaṃ kāryaṃ grahavan mandakarmaṇi |
dorjyāntaraguṇā bhuktis tattvanetroddhṛtā punaḥ || 48
svamandaparidhikṣuṇṇā bhagaṇāṃśoddhṛtāḥ kalāḥ |
karkyādau tu dhanaṃ tatra makarādāv ṛṇaṃ smṛtam || 49
mandasphuṭīkṛtāṃ bhuktiṃ projjhya śīghroccabhuktitaḥ |
taccheṣaṃ vivareṇātha hanyāt trijyāntyakarṇayoḥ || 50
calakarṇahṛtaṃ bhuktau karṇe trijyādhike dhanam |
ṛṇam ūne 'dhike projjhya śeṣaṃ vakragatir bhavet || 51
(Chaudhary 1987, pp. 77–80)

VS II, i, 97–100
mandatuṅgagativarjitā gatiḥ kendrabhuktir iha khecarasya sā |
dorguṇāntarahatādyajīvayā bhājitā svamṛduvṛttasaṃguṇā || 97
bhagaṇāṃśahṛtā phalaṃ gatau nijakendre makarādike kṣayaḥ |
dhanam indugṛhādike sphuṭā śravaṇāgre khalu vā gatis tadā || 98
nijakendragatiḥ samāhatā tribhamaurvyā mṛdukarṇabhājitā |
svamṛdūccagatiḥ phalānvitā grahabhuktis tv athavā parisphuṭā || 99
bhujabhojyaguṇāntaraṃ raveḥ svaranighnaṃ dvisvarendubhājitam |
śaśino 'ṅkajalāhataṃ hṛtaṃ khakṛtair bhuktiphalaṃ kalādi vā || 100
(Shukla 1986, part I pp. 102–103)

VS II, iii, 18
nijaphalabhojyajyāghnā kendragatiś cādyajīvayā bhaktā |
trijyāghnā karṇahṛtā labdhonayutā svaśīghramandagatiḥ || 18
(Shukla 1986, prat I, p. 115)

LMM 14cd (II, 4cd)
koṭir gatighnā cchedāptaṃ vyastaṃ gatikalāḥ phalam || 4
(Āpaṭe 1952, p. 10)

LMM 17 (II, 7)
vyāsaṃ śīghraphalārkāṃśabhāgonaṃ grahaśīghrayoḥ |
gatyantaraghnaṃ chedāptaṃ tyaktvā śīghragater gatiḥ || 7
(Āpaṭe 1952, p. 11)

SSE III, 40–43
mandakendragatir arkacandrayor jyāntareṇa guṇitā hṛtādyayā |
jīvayā svapariṇāhatāḍitā kharturāmavihṛtā gateḥ phalam ||40
ādyaturyapadayos tadūnitā madhyayos tadadhikā sphuṭā gatiḥ |
yātayeyadinajagrahāntaraṃ varttamānadivase gatiḥ sphuṭā ||41

cañcalakendragatiḥ phalabhogyajyāguṇitādyaguṇena vibhaktā |
vyāsadalaghnaphalaṃ śrutibhaktaṃ tadrahitāśugatiḥ sphuṭabhuktiḥ || 42
syād avanītanayādikhagānāṃ śīghragateḥ phalam abhyadhikaṃ cet |
tatphalato 'pi viśodhaya śeṣaṃ vakragatir bhavati dyucarāṇām || 43
(Miśra 1932/1947, pp. 172–175)

MS III 15–16ab
mandaphalaṃ kendravaśāt svarṇaṃ sūrye sphuṭo bhavati |
koṭiphalaghnī bhuktir gajyābhaktā kalādiphalam || 15
bhuktau karkimṛgādye kendre svarṇaṃ bhavet spaṣṭā | 16ab
(S. Dvivedī 1910, pp. 58–59)

SSI II, 36cd–39
dināntaraspaṣṭakhagāntaraṃ syād gatiḥ sphuṭā tatsamayāntarāle || 36
koṭīphalaghnī mṛdukendrabhuktis trijyoddhṛtā karkimṛgādikendre |
tayā yutonā grahamadhyabhuktis tātkālikī mandaparisphuṭā syāt || 37
samīpatithyantasamīpacālanaṃ vidhos tu tatkālajayaiva yujyate |
sudūrasaṃcālanam ādyayā yataḥ pratikṣaṇaṃ sā na samā mahatvataḥ || 38
phalāṃśakhāṅkāntaraśiñjinīghnī drākkendrabhuktiḥ śrutihṛdviśodhyā |
svaśīghrabhukte sphuṭakheṭabhuktiḥ śeṣaṃ ca vakrā viparītaśuddhau || 39
(Chaturvedi 1981, pp. 119–121)

SOS II, 29
śīghrakendragatis trijyākṣuṇā karṇoddhṛtā ṛṇam |
śīghroccabhukteḥ syād bhuktir vakrabhuktir viparyaye || 29
(V. P. Dvivedī 1912, p. 10)

KA I, 28cd–29ab
koṭir gatighnī chedāptā vyastaṃ gatikalāphalam || 28cd
tatsaṃskṛtā madhyagatir mandasphuṭagatir bhavet | 29ab
(Namboodiri 1975, p. 14)

KA I, 36–37
śīghravyāso doḥphalataḥ śakvaryaṃśavivarjitaḥ |
sa mandasphuṭaśīghroccagatibhedasamāhataḥ || 36
śīghracchedahṛto labdhaṃ śodhyaṃ śīghragatir gatiḥ |
śodhyāt tyakte tūccabhoge vakrabhuktis tu śiṣyate || 37
(Namboodiri 1975, p. 17)

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