Preface

This book, the third in a series presenting annotated English translations of the major Arabic and Latin mediaeval editions of, and commentaries on, Euclid's *Elements of Geometry*, contains the first translation into a modern language of the Commentary on Book I of Euclid's *Elements* attributed to Albertus Magnus, the Universal Doctor. In due course, I will publish the translation of Books II-IV, the remaining extant portion of the Commentary. The translation is made from the critical edition of the Latin text of Book I published by Paul M. J. E. Tummers in 1984 [**130**]. Chapter II contains notes on the translation. In Chapter III, the reader will find a critical examination of the mathematical content of the commentaries thusfar translated in the series, namely, that of al-Nayrizi [**95**], its Latin version by Gerard of Cremona [**96**], and that of Albert. Gerard's version of the Commentary of al-Nayrizi is conveniently distinguished from the original Arabic work by referring to the former as the *Commentary of Anaritius*, the name by which the Persian al-Nayrizi was known in the Latin West. The Commentary of Anaritius was one of two identified main sources for the Commentary of Albert; the other identified source, the Vatican-Bonn ("V-B") edition of Euclid [**32**], will, it is hoped, be the subject of a later volume in this series.

The Dominican Scholastic philosopher Albertus Magnus (1193?-1280) acquired the title *Doctor Universalis* from the breadth of his learning, which astonished his contemporaries. His fame earned him a place in the *Divine Comedy*, where his most famous student, Thomas Aquinas, introduces him to Dante:

> Questi che m'è a destra più vicino
> Frate e maëstro fummi, ed esso Alberto
> È di Cologna, e io Thomas d'Aquino. (*Paradiso*, X, 97-99)
>
> He who is nearest to me on the right
> Was my colleague and teacher, namely, Albert
> Of Cologne, and I am Thomas Aquinas.

The program of the scholastic philosophers was to use the deductive method of mathematics to demonstrate by reason the existence of Deity and to describe His attributes, to prove the immortality of the soul, to assert free will, and in general to establish thereby the truth of the Catholic religion. Their first axiom was, that this was possible. Even Russell, who considered theology nothing more than organized ignorance, could nevertheless respect the Mediaeval mastery of logic:

> The mediaeval outlook of educated men had a logical unity which has now been lost. We may take Thomas Aquinas as the authoritative exponent of the creed which science was compelled to attack. He maintained – and his view is still that of the Roman Catholic Church – that some of the fundamental truths of the Christian religion could be proved by the unaided reason, without the help of revelation. Among these was the existence of an omnipotent and benevolent Creator. From His omnipotence and benevolence followed that He would not leave His creatures without knowledge of His decrees, to the extent that might be necessary to obey His will. There must therefore be a Divine revelation, which, obviously, is contained in the Bible and the decisions of the Church. This point being established, the rest of what we need to know can be inferred from the Scriptures and the pronouncements of oecumenical Councils. The whole argument proceeds deductively from premisses formerly accepted by almost the whole population of Christian countries, and if the argument is, to the modern reader, at times faulty, its fallacies were not apparent to the majority of learned contemporaries
>
> Now logical unity is at once a strength and a weakness. It is a strength because it insures that whoever accepts one stage of the argument must accept all later stages; it is a weakness because whoever rejects any of the later stages must also reject some, at least, of the earlier stages. The Church, in its conflict with science, exhibited both the strength and the weakness resulting from the logical coherence of its dogmas [**110**, 12-13].

Within this program, it was the special enterprise of Albertus Magnus to interpret the works of Aristotle for his Mediaeval contemporaries, insofar as the writings of that philosopher could be known through the veil of Latin translations of Arabic translations of the Greek texts, which at that time covered the hearts of the learned. In so doing, he made a major contribution to culture, for Aristotle was interested in the natural world, and investigation into his activities tended to promote the rebirth of knowledge, which was accomplished in the period called the *Renaissance*. The labors of people like Albertus Magnus were appreciated by Macaulay, who noticed them in one of the most famous chapters in literature:

> Whatever reproach may, at a later period, have been justly thrown on the indolence and luxury of religious orders, it was surely good that, in an age of ignorance and violence, there should be quiet cloisters and gardens, in which the arts of peace could be safely cultivated, in which gentle and contemplative natures could find an asylum, in which one brother could employ himself in transcribing the *Aeneid* of Virgil, and another in meditating the *Analytics* of Aristotle, in which he who had a genius for art might illuminate a martyrology or carve a crucifix, and in which he who had a turn for natural philosophy might make experiments on the properties of plants and minerals [**98**, I, 9].

Albertus Magnus is one of those personalities who are appreciated even by those with a critical attitude towards the Catholic religion. For example, White, cofounder and first president of Cornell University, a declared enemy of dogmatic theology, had a sympathetic opinion of him:

> First among these was Albert of Bollstädt, better known as Albert the Great, the most renowned scholar of his time. Fettered though he was by the methods sanctioned in the Church, dark as was all about him, he had conceived better methods and aims; his eye pierced the mists of scholasticism; he saw the light, and sought to draw the world toward it. He stands among the great pioneers of physical and natural science; he aided in giving foundations to botany and chemistry, he rose above his time, and struck a heavy blow at those who opposed the possibility of human life on opposite sides of the earth; he noted the influence of mountains, seas, and forests upon races and products, so that Humboldt justly finds in his works the germs of physical geography as a comprehensive science [**138**, I, 377].

The most recent biography of Albert, written on the occasion of the seven hundredth anniversary of his death, is by Weisheipl [**137**].

Albertus Magnus conceived and carried out the plan of commenting on all human knowledge by beginning with the natural sciences, proceeding to mathematics, and finishing in philosophy and theology. Thus, in his writings in the natural sciences, he refers to mathematics as the object of future activity:

> Primo complebimus, Deo adiuvante, scientiam naturalem, et deinde loquemur de mathematicis omnibus, et intentionem finiemus in scientia divina (*Physica*, Book I, Treatise 1, Chapter i).
>
> With God's help, we shall first complete natural science, and then we shall talk about all of mathematics, and we shall finish our program in divine science.
>
> Longum esset demonstrare, sed in geometria hoc docebitur et in astronomia, Domino concedente (*Op. cit.,* I, 2, i).
>
> It would be long to prove, but, God willing, this will be taught in geometry and astronomy.
>
> Haec autem omnia supponenda sunt, probanda autem in libris de visu in Perspectivis, quae scientia compleri non potest, nisi primum consideremus ea quae pertinent ad geometriam (*De Sensu et Sensato*, Treatise 1, Chapter 14).
>
> All these things, however, must be supposed for now; they are to be proven, though, in the books on sight in *Perspective*, which science cannot be completed unless we shall first consider those things that pertain to geometry.

In his compositions on philosophy and theology, however, Albert refers to his work on geometry as a thing of the past, for he accepted the idea of Plato, that one could not competently study philosophy without having first mastered mathematics.

> Naturalibus et doctrinalibus iam, quantum licuit, scientiis elucidatis, iam ad veram philosophiae sapientiam accedamus (*Metaphysica*, Book I, Treatise 1, Chapter i).
>
> Now that the natural and mathematical sciences have been elucidated as much as was possible, let us proceed to the true wisdom of philosophy.
>
> Hoc autem iam a nobis in geometricis est demonstratum (*Ibid.*, I, 2, x).
>
> For this [*sc.* that the diameter and side of a square are incommensurable] has already been proven by us in the geometrical [works].
>
> Sicut in XV et XVI tertii geometriae nostrae demonstratum est (*Ibid.*, III, 2, iii).
>
> Just as has been proven in the fifteenth and sixteenth [propositions] of the third [book] of our *Geometry* [*sc.* namely, that a tangent line to a circle intersects it at only one point].
>
> Sicut nos in I nostrae geometriae ostendimus (*Ibid.*, V, 3, i).
>
> As we showed in the first [book] of our *Geometry* [*sc.*, that two straight lines do not enclose a surface].

Since the *Physics* was written about 1250, and the *Metaphysics* in the period 1262-1263, these quotations establish that Albertus Magnus wrote on Euclid's *Elements* sometime around 1250-1262. This fact is also alluded to in the old catalogues [**60**, 163]:

> [Scripsit] expositionem Euclidis, perspectivae et almagesti (Henry of Herford, mid fourteenth century).
>
> [He wrote] an exposition of Euclid, of Perspective, and of the *Almagest.*
>
> Item exposuit Euclidem, perspectivam, almagestum, et quosdam alios mathematicos (Catalogue of the Cistercian Abbey of Stams in the Tyrol).

> Furthermore, he explained Euclid, perspective, the *Almagest*, and certain other mathematicians.

The fact that such a celebrity as Albertus Magnus interrupted his religious routine and violated the tyranny of custom in order to write a book about Euclid's *Elements* shows that he was wiser than his generation, an achievement that cannot reasonably be demanded of anyone.

The question now arises: Has this work survived? There is but one candidate, Manuscript 80/45 in the Dominican Convent in Vienna, which contains, on folio pages 105r-145r, an exposition of the first four books of Euclid's *Elements of Geometry*, introduced by a rubricator with the heading *Priumus Euclidis cum commento Alberti*, the First [Book] of Euclid with the Commentary of Albert. In the catalogue drawn up in 1513, this manuscript is referred to as *29 Albertus Magnus, Super Euclidem* (29, Albertus Magnus, On Euclid); the catalogue of 1758 reads: *Item Euclidis Geometria libri 4 cum commento Alberti, verisimillime Magni, si verum est quendam Albertum auctorem esse huius commentarii* (Item, Euclid's *Geometry*, four books, with the comments of Albert, most likely the Great Albert, if indeed a certain Albert is the author of this Commentary). Geyer [**60**], Ostlender [**104**], Hofmann [**77**], Tummers [**128**], Hossfeld [**78**], Ineichen [**79**], and Anzulewicz [**3**] all agree, not only that this manuscript presents the Commentary on Euclid's *Elements* written by Albertus Magnus, but that it is either an autograph of the Universal Doctor, written by his very hand, or a manuscript written by a scribe upon dictation from Albert; Busard [**32**, 23] and Folkerts [**56**, 16, 48], on paleographic grounds, do not agree, for they believe that the handwriting of the manuscript is of the fourteenth century.

My own first impression immediately upon reading the preface to the Commentary was that the author of this work was a most cultured and highly educated fellow and an experienced writer, because his Latin is that of a learned veteran, who composes in a style beyond that of the commonality of scribblers, whose command of the language, not based upon wide reading in the works of the best authors, is not adequate to insure that their compositions, at least as regards diction, attain a higher level than that of mediocrity. One has only to compare the style of Albert's preface with that which precedes the "Adelard III" version of the *Elements* (see pages xxi-xxv below) to observe that the former is the production of a man of parts, and the latter of an intellectually limited smatterer.

MS Wien Dominikanerkloster 80/45 contains two performances; on folio pages 1-104 one finds the work of Peter of Alvernia, a personality of the second half of the thirteenth century, *Super libros Metheorum Aristotelis* (About Aristotle's Books on Meteors), written in a fourteenth century hand in two columns, and on folio pages 105r-145r there is the aforementioned Commentary of Albert. (The front side of a sheet is referred to as *recto* and denoted by *r*; the back side is referred to as *verso* and denoted by *v*.) The Euclid Commentary is in one column, 27 cm. by 19 cm.; it consists of three quires with six double sheets (folio pages 105-116, 117-128, and 129-140), and one quire with three double sheets (folio pages 141-146). Folio page 145v has some concentric circles drawn on it and an early owner's annotation; folio pages 146r, and 146v are blank [**60**, 167]. The manuscript is available on microfilm from the Hill Monastic Manuscript Library of St. John's Abbey and University, Collegeville, Minnesota 56321 (1-320-363-3993), which has 251 of the manuscripts of the library of the Dominikanerkloster available in its microfilm holdings; an inventory list is found in Julian G. Plante's *A Checklist of Manuscripts*, volume I, part 2. Microfilms cost $50 per reel, plus $10 library fee and shipping and handling; printouts are .50 per page, plus a minimum of $25 an hour service fee and shipping and handling. The address of their website is http://www.hmml.org/intro/inquiry.html. They have assigned "Part Identification Number" 9271 to the Vienna manuscript; the Commentary of Albert has "Text ID" 32264. The Dominikanerkloster was founded in 1226, ten years after the foundation of the Dominican order; its library contains volumes mostly by Dominican authors.

The Albertus Magnus Institute of Cologne was founded in 1931 by Karl Josef Cardinal Schulte to produce a critical edition of Albert's works; it has since moved to Bonn. The Prelate and Doctor of Theology Bernhard Geyer (1880-1974), Professor of Dogmatics, of the History of Dogma, and of Patrology at the University of Bonn, and first Director of the Institute, inaugurated the period of modern research into the mathematical activities of Albertus Magnus with a paper, written in 1944 but not published until 1958, in which he argued that it was very probable (*sehr wahrscheinlich*) that the Vienna manuscript Dominikanerkloster 80/45 was an autograph manuscript of Albertus Magnus [**60**]. (In 1960 he somewhat modified his enthusiasm by writing, in his introduction to the Cologne edition of Albert's paraphrase of Aristotle's *Metaphysics*, that the situation with regard to Albert's authorship of the Vienna manuscript was best described by the seventy-eighth line of the *Ars Poetica* [**2**, 16/1, xix]: *Adhuc sub iudice lis est*; the jury is still out.) He indicated several peculiarities of the Euclid Commentary that prevented him from making the ascription with certainty; of these, the three most important were:

1) The Euclid Commentary in MS Wien Dominikanerkloster 80/45 is written in a stylish, festive, artistic script, whereas Albert's other autographs are written in a hurried cursive.

2) In the Euclid Commentary, the author frequently makes use of the enclitic *–que* for *and*, but Albertus Magnus was not a "-que person"; this usage is foreign to him.

3) In the Euclid Commentary, the symbol ÷ is commonly used for *est* (is), a usage not found in the recognized autographs of Albertus Magnus, except in a fragment of his *Commentary on the Sentences* (circa 1249) found in Uppsala. Albertus Magnus usually writes *ē* for *est.*

Dirk Struik considered this paper of Geyer worthy of being cited among the references of his *Concise History of Mathematics* (fourth edition, page 91).

Ostlender [**104**] and Hofmann [**77**] accepted the Vienna manuscript as an autograph of Albertus Magnus; indeed, when a manuscript ascribed to an *Albert* is found in a Dominican convent, it is the great Albert that immediately comes to mind. Tummers, however, though he agreed that the Commentary was a work of Albertus Magnus, argued that the palaeographic difficulties raised by the manuscript (see the preceding paragraph) made it more likely that the manuscript was a copy and not an autograph [**128**]; he later analyzed the one hundred six citations of Euclid and the *Elements of Geometry* that appear in the undisputed works of Albertus Magnus and concluded that this evidence established as a well grounded hypothesis the assumption that Albert the Great was the author of the Commentary [**130**, II, 192-223]. We may list the following considerations that led him to this conclusion:

1) The passage in the Metaphysics, V, 3, I, quoted on page iii above, namely, that Albert proved the postulate, that two straight lines do not enclose a surface, is evidence that our Commentrary is by Albertus Magnus, because, as far as Tummers knows, the only Latin expositions of Euclid where such a "proof" is offered are our Commentary and one of its two major sources, Gerard's translation of al-Nayrizi ("Anaritius").

2) The philosophical prologue to our Commentary, since it is not taken from a source (as is most of the rest of the Commentary), must be examined to see if its diction, style, and content are in harmony with those of the undisputed works of Albertus Magnus. Tummers finds that they are. He refers to the many parallel passages accumulated by Msgr. Geyer (see Chapter II, pages 148-152 below) and further writes:

> Many points of view expressed in our prologue are similar to those found in Albertus Magnus, e. g., the relation between mathematical and physical objects, the way in which a point generates a line and a line a surface, the notion of point as the principle of a line, and so forth. Further, both the prologue and Albertus Magnus agree that mathematics concerns forms existing in moveable matter, but conceived without matter in its

> "ratio diffinitiva"; that imagination plays a crucial role in the constitution of mathematical objects; that mathematics deserves the real name of "science" because it belongs to human imagination, which conceives the "ratio diffinitiva" of mathematical being without the variation of sensible matter [**128**, 497-498].

3) In the discussion following the definition of a straight line (page 10 below), Albert ascribes a definition of a straight line to Aristotle which is found nowhere else (not even in Aristotle) except in three undisputed works of Albertus Magnus, namely, *De Lineis Indivisibilibus* 3, *De Predicamentis* 3, vii and 4, v, and *De Caelo* I, 1, x.

4) The author of our Commentary reports (see page 9 below) that the Platonists call a plane surface either a shaving (*rasura*) or a plate (*lamina*). Both names and the same remark are found elsewhere only in the *De Caelo* of Albertus Magnus, III, 1, ii.

5) The mistake which the author of our Commentary makes below (page 20), when he confuses the isosceles and equilateral triangles, is the same error as that made by Albertus Magnus in *De Lineis Indivisibilibus* [**1**, 3, 471].

The question of whether the Vienna manuscript was an autograph of Albertus Magnus was next taken up by Paul Hossfeld [**78**], who, in addition to those eccentricities cited by Geyer (see page viii above), noted the following additional peculiarities that prevented him from declaring the manuscript an autograph of the Universal Doctor:

1) In the Euclid Commentary, the word *etiam* is usually written *etiā*, but Albertus Magnus generally writes *et* with a horizontal line above it for that word.

2) In the Euclid Commentary, *igitur* is often written *ig*^{ur}, whereas Albertus Magnus customarily writes *igit*^{ur}.

For all these reasons, Hossfeld denied that the manuscript was written by the hand of Albertus Magnus, but, since it showed all the signs of an autograph, he proposed that it had been dictated by Albertus Magnus to a scribe. (This was a common method of composition for him; that Albertus Magnus frequently wrote by dictation to an amanuensis is more than once mentioned by Weisheipl [**137**, 37, 40].) The fact that it is an autograph he established by examining all the portions of text that had been crossed out during the writing:

> I can only account for the state of affairs in these passages thus, that the writer thinks productively; therefore, the author is the writer, or the productively thinking author is dictating to a scribe whom he lets cross things out occasionally and then gets on with the dictation, or whom he makes read the last passage aloud once more, in order then to correct it [**78**, 118-119 (translated from the German)].

The fancy script Hossfeld explained by suggesting that the manuscript was written while Albertus Magnus was Prince Bishop of Ratisbon (1260-1262), a position which called for an unusual embellishment of any work produced by such an authority. Hossfeld's hypotheses were accepted by Tummers [**130**], Ineichen [**79**], and Anzulewicz [**3**].

Busard [**32**, 23], however, and Folkerts [**56**, 16, 48], doubted that the script of MS Wien Dominikanerkloster 80/45, folio pages 105r-145r, was of the thirteenth century, and so did not accept the ascription of our commentary to Albertus Magnus. Since all agree that our manuscript is an autograph (if only through the medium of dictation), it follows that if the handwriting is that of the fourteenth century, then the work cannot be by Albertus Magnus. There is no published account listing the reasons which favor the rejection of the hypothesis, that Albertus Magnus is the author; it is, though, to be noted that the latest catalogue of the Library of the Dominikanerkloster, Felix Czeike's *Verzeichnis der Handschriften des Dominikanerkonventes in Wien bis zum Ende des 16. Jahrhunderts* (1952 typescript, available from the Hill Monastic Manuscript Library mentioned on page vii above), assigns the whole manuscript to the fourteenth century. Since the findings of palaeography, in such a debate as this, are as decisive as the conclusions of DNA testing in the biological arena, it was necessary to ask an expert professional palaeographer to study the manuscript and to date it as closely as possible in accordance with scientific palaeographic principles, and thereby decide the matter as only a specialist of that sort can. I therefore approached Dr. Michael I. Allen of the University of Chicago, supplied him with the prints of the Vienna manuscript, and asked him for his professional opinion as to the date of the document. Dr. Allen's reply, in a personal communication of October 15, 2001, was immediate and conclusive:

> The script (perhaps by several hands) is a clear, informal book hand (*textualis currens*) of a German sort, not later than the middle of the thirteenth century. I would say that it is of the late second quarter of the thirteenth century. That is my conclusion based on looking through everything. As for details, the German origin is indicated by the lightning-bolt shape of an abbreviation that is sometimes, but not uniformly, used for *est*. (The form is a give-away marker for Germany.) The script is not very late, because the bows of various letters (*b, o, d, p, e, etc.*) virtually never touch in the space-saving unions increasingly characteristic of copy-work as the thirteenth century advanced. This textbook is also not heavily abbreviated, although one would very much expect that in the jargon-heavy idiom of this sort of text; the situation, again, speaks for an early, rather than a later, dating. The 7-shaped Tironian abbreviation for *et* is also still uncrossed, as also starts to appear after the mid-century north of the Alps… There is [thus] no doubt about the basic region and date. Do not mind that the leaves are now part of a composite volume with later materials [the work of Peter of Alvernia mentioned on page vii above]. That sort of binding was a space- and resource-saver, and means nothing for the handwritten text…I am quite certain of the things that I observed. I was, in fact, rather surprised to have to conclude that the manuscript is so early.

In accordance with the principle of Augustine, *Roma locuta est, causa finita est*, I therefore accept the hypothesis of Tummers and Hossfeld, that the Euclid Commentary was dictated by Albertus Magnus himself about the year 1250.

We have remarked in the very first paragraph that our manuscript of the Commentary is not complete; it breaks off after an incomplete treatment of IV 16. I say *incomplete*, because:

1) We miss the usual and therefore expected concluding statement (see, for example, page 147 below) that Book IV is finished.

2) The last addition which al-Nayrizi made to this proposition [**131**, 124] is missing. (The preceding additions Albert puts after IV 15.)

3) The edition of Campanus had many other additions to IV 16, none of which are found in the text of Albert.

Tummers thinks that the Commentary really does end at this point, though Albert may very well have intended to finish it at some future date:

> It is difficult to decide if Book IV ends thus, and even more difficult to decide if Albert went on to comment on the remaining books of Euclid. Apart from the question of whether Book IV itself is complete, we can report that the text ends at the bottom of folio page 145r, and that folio page 145v has only a number of concentric circles and a declaration by an [early] owner, that four leaves have been cut out of the last quire, and that Albert, at the ends of Books I, II, and III, has always clearly announced that the book is finished, whereas here the like statement is missing. This leads then to the hypothesis, that the text of Albert really ends here, and that Albert never got further than here, but this does not prove that he did not have the intention of finishing it later [**130**, 50, translated from Dutch].

If the text of our Commentary ends on 145r, if the concentric circles and owner's annotation are all that we find on 145v, if 146r and 146v are blank, and if what would have been 147, 148, 149, and 150 have been cut out, then this last quire originally had only five folded (i. e., double) sheets, unlike the ones before it, which had six.

The question then arises, if Albertus Magnus left the Commentary in this unfinished state, why does he refer to it later on (see the quotations on page iv above) in ways that are most naturally interpreted as indicating that the work was completed? *Naturalibus et doctrinalibus iam, quantum licuit, scientiis elucidatis* (Now that the natural and mathematical sciences have been elucidated as much as was possible) is not the usual way of referring to a book of which less than the first third is completed, and the remark *Hoc autem iam a nobis in geometricis est demonstrandum* (For this has already been proven by us in the geometric [works]) refers to X 9 or to the spurious X 117. It is possible to attempt to extenuate the force of these quotations. In the first passage just quoted, is *quantum licuit* (as much as I could) a reference to the incomplete condition of the Commentary on Euclid's *Geometry*? Furthermore, Tummers does not find the reference to X 9 proof that Albertus Magnus had actually gotten that far in his Commentary:

> Does this mean, then, that Albertus Magnus commented on Book X of Euclid? Not necessarily. As already noted, Albertus Magnus here gives no exact reference to book or proposition. It could refer to other geometric works of Albertus Magnus. In fact, he does discuss Book X of Euclid and incommensurability in his authentic *De lineis indivisibilibus*. But the possibility remains that Albertus Magnus did in fact comment on the whole of Euclid's *Elements* [**128**, 497].

We next consider the sources that Albert used in composing his Commentary, and the manner in which he employed them.

Albertus Magnus, in his capacity as provincial of the Dominican province of *Teutonia* (1254-1257), visited the thirty-nine houses of the order under his supervision from Antwerp to Riga, from Vienna to Hamburg, all on foot. For other reasons during his long life, he made more journeys, even into France and Italy. His method of traveling is described by Weisheipl:

> It was Albert's custom while traveling – always by foot – first to visit the chapel of the religious house where he intended to stay the night, to thank God for the safe journey,

> then immediately to visit the library to see whether there were any books there that he had not seen. Often his candle burned late into the night as he copied long passages of interest to him that could be used later. Hence, Albert frequently cites titles of books and gives direct quotations from works now lost [**137**, 34].

Albert's concern to rescue manuscripts of secular learning was particularly appreciated by Milman, an author who adopted a most critical attitude to the scholastic enterprise:

> He [*sc.* Albertus Magnus] severely reproved the Monks, almost universally sunk in ignorance and idleness; he rescued many precious manuscripts which in their ignorance they had left buried in dust, or in their fanaticism cast aside as profane [**100**, VIII, 258].

Now when we look into our commentary on the *Elements*, we see that its author indeed cites the names of various authorities in the course of his work, often as the sources of information that he is relating. The list has already been compiled by Tummers [**130**, I, 64-71]: Adelard, "Aganyz" (i. e., Agapius), al-Farabi (but see note L 33 below), "Anaritius" (al-Nayrizi), Apollonius, "an Arab", Archimedes, Aristippus, Aristotle, Avicenna, Boethius, Diodorus of Alexandria, Euclid, a certain "Hermydes", Heron of Alexandria, Plato, the Platonists, Posidonius, Ptolemy, the Pythagoreans, Simplicius, and Thabit bin Qurra. This multitude of learned references is another necessary, though not sufficient, condition for the work to be by Albertus Magnus.

We next inquire, What was the situation in Western Europe with respect to the transmission of Euclid's *Elements of Geometry* at the time when our commentary was written? In those days, the learned world conducted its business in Latin, and it is not much of an exaggeration to say that neither Albertus Magnus nor any other high authority knew more Greek than *Kyrie eleison* and other such formulas, which they found in the Roman liturgy. The learned therefore generally worked from texts which had been translated into Latin from Arabic, which was the intermediary between Greek antiquity and the High Middle Ages. The Latin editions of Euclid's *Elements* by the following mathematicians (if such they may be called) were available in various manuscripts to those who were able to find them:

1) Boethius: This version, by the last of the Romans (470?-524), was made directly from the Greek, but contained no proofs except those of the first three propositions. Nothing survives of this work beyond Book IV, so it may not have extended any futher than that point.

2) Adelard of Bath: This translation, from the Arabic, was made around 1120.

3) Hermann of Carinthia: This translation from the Arabic, inferior to that by Adelard, was made around 1140.

4) Robert of Chester: This edition, made around 1150, was not a translation, but a compilation made from the previous three versions. There were no proofs, just hints on how to go about composing them. Before Busard's critical edition of 1968 [**24**], it was mistakenly believed that Robert's version was also by Adelard of Bath, whence it was called "Adelard II" by Clagett [**36**] (and by everyone else thereafter) to distinguish it from the translation listed as #2 above.

5) Gerard of Cremona: He translated the *Elements* from Latin sometime before his death in 1187. The translations by Adelard, Hermann, and Gerard were made from Arabic editions of the *Elements* which descended from the activity of al-Hajjaj, Ishaq, and Thabit; for the history of these translators, see [**95**, 20-23].

6) "Mélange" versions: Various late twelfth century compilers concocted mélange editions of the *Elements* by borrowing now from Boethius, now from Robert of Chester, and adding directions on how to write the proofs.

7) "Adelard III": An unknown author of the end of the twelfth century took the Robert of Chester edition mentioned above, removed the directions for the proofs, and added his own full demonstrations, together with commentary. This was long considered a third version by Adelard, until Busard proved this assumption false in 1983 [**27**].

8) "V-B": In the first half of the thirteenth century, an anonymous editor took the Robert of Chester version of the *Elements*, changed the formulations of some of the enunciations (for example, of Propositions 4, 5, 25, and 26 of Book I), and replaced the proof sketches with his own full arguments. This adaptation of Robert's edition is called the V-B edition by Tummers [**130**] after the two major manuscript witnesses, one in the Vatican and one in Bonn. In the Vatican manuscript, a later hand added the Boethian translation of the statements of I 2, 3, 6, 7, 9, 10 as alternative enunciations at the end of the proofs.

All these versions were driven off the stage by the version of Campanus of Novara, a contemporary of Albertus Magnus; that our commentary shows no dependence on or knowledge of the edition of Campanus is one of the arguments in favor of its having been composed before that standard work became established. For a full account of the history of the transmission of Euclid's *Elements* in the Middle Ages, see Chapter I of [**95**] and the Preface of [**96**], the two earlier volumes in this series. In addition to the eight versions of the *Elements* listed above, there was also available in the time of Albertus Magnus a Latin translation, by Gerard of Cremona, of the Commentary of al-Nayrizi on the *Elements*, the only Arabic commentary on that work to be translated into Latin. For a translation of the Arabic text of that commentary, see [**95**], and for a translation of the Latin version made by Gerard of Cremona, see [**96**]; one main difference between the two texts is that the Arabic version has al-Nayrizi's edition of Euclid's *Elements* incorporated into it, while the Latin version omits this. To distinguish between the two versions, it is convenient to refer to the Latin edition as the

Commentary of *Anaritius*, the Latin metamorphosis of the name al-Nayrizi, and to reserve the Arabic name for the original text.

Albert himself cites in various places the manuscripts that he was using in writing his commentary, and it is possible to identify which of the categories mentioned above that most belong to:

In his presentation of Definitions 6 and 18 and of Postulate 5, Albert says that his source is the *translatio ex Greco* ("the translation from the Greek"); upon examining the statements of these two definitions and of the postulate, and, indeed, of all the definitions, postulates, axioms, and the enunciations of the propositions (but not their proofs), we see that these are the same, except for occasional trivial variants, as those of the V-B edition mentioned as #8 above; it is therefore this V-B version of the *Elements* that Albert is using for those portions of his commentary and which he mistakenly refers to as a translation from the Greek. I will publish a translation of this version into English in a later volume of this series.

When Albert comments upon his fifth postulate, that two straight lines do not enclose a plane region, when he explains what sort of proposition is called an *inventum* (a "discovery" or "finding"), and when he presents alternative formulations of definitions 3, 5, 6, and 18 of Book I and 7 of Book III, he says that his source is the *translatio ex arabico* (the translation from the Arabic) or "the Arabs". Upon investigation, we see that this is nothing other than Gerard of Cremona's translation of the Arabic Commentary on Euclid's *Elements* by al-Nayrizi ("Anaritius" to the Latins), which I have translated into English in [**96**].

At the end of the proofs of three propositions (I 6, 9, and 10), Albert gives alternative formulations of the enunciations which he says that he has copied from *alia translatio*, from another translation. We observe that these alternative enunciations are those of Boethius, and are contained in the Vatican manuscript of the V-B version. That Vatican manuscript also contains Boethian alternate formulations of the enunciations of I 2, 3, and 7, but Albert does not give these. Thus his "other translation" refers to the later additions made to the manuscript of the V-B tradition that he was using.

In his discussion of how the fifth proposition of the first book is proven, namely, that the angles below the base are proven equal first, and then the angles that are above the base, Albert notes that not only is this Euclid's way, but that it is also the way found in the *commenta Boethii et Adelardi*, that is, in the commentaries of Boethius and Adelard. Since Boethius and Adelard wrote translations and not commentaries, and since the translation of the *Elements* by Boethius has no proofs after those of I 1, 2, and 3, it is not clear what sources Albert means by this phrase. Tummers suggests as an attractive hypothesis that by the *Commentary of Boethius* Albert means the V-B version of the *Elements*, and by the *Commentary of Adelard*, he means some additional manuscript in the Adelard tradition, whence Albert took those passages which we find nowhere else except in his production and in the version of the *Elements* by Campanus.

Where did Albert get the proofs of the propositions in his Commentary? The Commentary of "Anaritius", which was his source for the additional theorems, had full proofs, but Albert did not copy these over word for word. Rather, he rewrote them, all the while adhering to the main outline, introducing many mistakes as he did so. It is always clear, though, that he is copying; for example, if we compare the diagrams for the first addition to Proposition 1 in both Anaritius and Albert, we notice that the seven letters A, B, G, D, E, Z, T are the same in each case, and each one of them is put at the same place, which is not explainable as a coincidence, since Albert, in the Euclidean proofs, uses the Latin order of letters, A, B, C, D, E,… The proof, though, is the same; Albert has simply rewritten that of Anaritius, without making any mistakes in this instance.

When we look at Albert's proofs for the Euclidean propositions, we notice that these are Euclid's, but reformulated by Albert in his capacity as editor and sometimes ruined as a result; see, for example, the proofs of Propositions I 11 and 16. We recognize the influence of the V-B demonstrations in many places, but we also notice that Albert does not imitate the V-B style of composition, which is full of idiosyncrasies that are typical of writers who, not having read the best authors, compose in a low style obnoxious to the learned in any discipline. Among the peculiarities of V-B (which are also evident in the Adelard III edition), we may cite:

1) Addressing the reader in the second person singular

So, for example, frequently, especially after the completion of a construction, we meet with expressions like *Quo facto, videbis*…("Having done this, you will see…"). This was the inevitable result of V-B's dependence on the Robert of Chester version, which started the habit of addressing the reader in the second person singular in its proof sketches.

2) Clumsy use of letters on diagrams

V-B does not use the conventional three-letter method of referring to an angle; he identifies an angle merely by the letter at its vertex. When there is more than one angle at a vertex, this leads to confusion, as in the proof of Proposition 47, where C can mean any one of three angles; sometimes V-B uses some circumlocution, which may not suffice to remove the confusion, such as "the total angle" or "the partial angle". V-B often has unnecessary letters on its diagrams, letters that are not used, for example, F and G in the proof of I 11. Sometimes V-B has no letters at all on the pictures and makes no reference to any letters in the proof, such as in I 1, 3, 4, and 15. V-B sometimes chooses an eccentric sequence of letters, often omitting D and E and including favorite letters like M and R. As a result of this incompetence, V-B uses twice as many letters as are necessary in the proof of I 33 and 34, and in I 44 uses E, N, and P twice. At the other extreme, in I 47, V-B puts no letter at all at an intersection where one is needed. Albert expressly rejects this defective method of referring to angles and says that using three letters is the "philosophic way"; see his "proof" of Postulate 3 on pages 25-27 below.

3) Outlandish diction

V-B says "the triangle enclosed by the lines A, B, C" instead of "triangle ABC", and "that which is AC" instead of just "AC".

4) Elementary instruction for the beginner

V-B draws pictures or gives examples to explain a definition or a construction; see the special pictures in the proofs of I 9 and 15 and the examples of what are and what are not alternate interior angles in I 27.

In some cases, however, it is clear that Albert's proof relies on the argument that he found in V-B. For example, in Proposition 10, there are many similarities of diction shared by V-B and Albert. In Proposition 12, Albert uses Proposition 4 in the proof, something that otherwise only V-B does; furthermore, V-B gives here an alternate proof without the circle, which Albert misinterprets as a proof of a special case. In Proposition 34, V-B and Albert both omit the corollary, that the diameter of a parallelogram bisects it; as a result, they must both prove this later on, at Proposition 37.

We may now summarize Albert's general method in composing his Commentary. He copied the Definitions, Postulates, Axioms, and the statements of the Propositions word for word from a manuscript in the V-B tradition. He composed the proofs for the propositions after consulting V-B and some other sources, from which other sources he learned a much higher standard of composition than that which he had found in V-B. Nevertheless, although he could master the mathematical style of writing, he did not succeed in achieving any technical distinction, and his proofs are error-ridden; one clear source of his mistakes is his tendency to work from the often faulty pictures that he found in his sources, and to allow his proofs to degenerate into statements on how to draw these pictures. For the additional matter, he relied on Gerard of Cremona's translation of the Commentary of al-Nayrizi ("Anaritius"); here too he does not copy his source literally, but rewrites the proofs as he thinks best. He then added a preface of his own composition as well as critical remarks at every opportunity.

As a result of the attention given to mathematics by people like Albertus Magnus, it was possible, in the generation after his death, for the greatest poet between Vergil and Milton to end his poem by comparing his inability to describe the manifestation of Deity to the loss of the geometer on how to begin to attempt the solution of the problem of squaring the circle:

> Qual è 'l geometra che tutto s'affige
> Per misurar lo cerchio, e non ritrova
> Pensando quell principio ond' elli indige,
> Tal era io a quella vista nova. (*Paradiso*, XXXIII, 133-136)

> Who versed in geometric lore, would fain
> Measure the circle; and, though pondering long
> And deeply, that beginning, which he needs,
> Finds not: e'en such was I (Cary's translation).

The magnificent edifice of the Holy Church promoted, for a thousand years, the diminution of ferocity, the creation of beauty, and the preservation of knowledge within the walls of the *una, sancta, catholica et apostolica ecclesia* in a way that cannot be understood by the general reader any more.

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APPENDIX

The Preface to the Adelard III Version of the *Elements* Contained in Oxford, Bodleian Library, Digby MS 174 and Venice, Biblioteca Nazionale Marciana, fondo antico 332

The application of geometry, as of the other skills, antedated its theoretical development. The application of it developed among the Egyptians. When, every year, the overflowing Nile wiped out the boundaries of their fields, they introduced certain specific ways of reckoning whereby they might restore their fields to their former extent. And so, philosophers, revising and approving these ways, took up the subject of measuring when the occasion arose. But some, on account of the difficulty of the matter, fell short of perfection, others (like Pythagoras, Theraxtosthenes, and Architas) made only a beginning, and still others (like Plato and Aristotle and Alexander) made further progress, until, finally, Euclid, by adding on a comprehensive view and logical order, fashioned a perfected art. He edited and put his stamp on the monument.

And so, at the beginning of this instruction, we propose first to consider the following, namely, what the art itself is, what are its parts, what its name, what the reason for its name, what its genus, what its master, what its function, what its instrument, what its intention, what its matter, what its usefulness or goal, what its mode, what its order, and what its title. This perfected art is the science of immobile magnitude, contemplated in the context of reasonable figures. *Of immobile magnitude* means of quantity considered without motion, to distinguish it from astronomy, which considers mobile magnitude, that is, the course of the firmament in motion. *In the context of reasonable figures* is added to rule out *helmurifas* and the figures of irrational things to which geometry pays no attention. [See page 168 below, note L 93.]

The parts of this art are distinguished according to the parts of its matter, namely, according to line, surface, solid, and number, or, according to the fifteen distinctions made by the author himself, made, he says, and called according to the distinction of principles, and called by the mode of doing.

The name, indeed, is *geometry*, from *ge*, which means *earth*, and *metros*, which is *measure*. The reason for this name is, as has been said, from its first appearance, that is, from the measuring of the fields.

The genus is considered according to three aspects, according to the effect, according to the content, and according to the natural supposition. The genus according to the effect is mensuration. For, it is of that genus, that is, effect, that renders its master an examiner and a measurer. According to its contents, of course, its genus is mathematics. Let us go into this even further. Science is divided into wisdom and eloquence, eloquence into grammar, dialectics, and rhetoric, wisdom into the theoretical and the practical, the practical into ethics, mechanics, and practical liberal things, ethics into personal morality, economics, and politics. Mechanics is divided into spinning, forging, agriculture, navigation, hunting, medicine, and the theater. Practical liberal things are divided according to the liberal arts; each one has its own application. The theoretical is divided into theology, physics, and mathematics, mathematics into arithmetic, geometry, and astronomy. There we have it – mathematics contains geometry insofar as genus is concerned. Finally, the genus according to the natural supposition is magnitude, about whose species there will be more to say.

The master, furthermore, is as much a demonstrator as a practitioner. The duty of the demonstrator is to explain the theorems in accordance with the art of interpreting the discipline, and for this, seven things are necessary: the proposition, the example, the disposition, the reasoning, the conclusion, the ratiocination, and the disproof of a case. Now the proposition is the enunciation of the theorem. The example is the substitution of a certain figure. The disposition is the application of other figures to the example. The reasoning is the bringing in of first principles or of conclusions from them, that is, the argument. The conclusion is the inferring of the proposition. The ratiocination is the weaving together of the reasoning and the conclusion, that is, the argumentation. The disproof of a case is when a falsifier insists that things are not as the geometer claims, but otherwise.

The function of the practitioner is to measure. Measuring, moreover, is the assigning of a certain quantity. The instrument of the demonstrator, moreover, is a rod and a table with sand. The instruments of the practitioner, however, are geometrical measures, namely, a pole with a palm, a finger's breadth, a step, a pace, and an ell. To these six, add two with stade, mile. Or, according to Gerbert, inch, palm, dodrantal, foot, tile, cubit, step, pace, pole or tenfooter, *actus minimus*, clime, gate, *actus quadratus* or *agrippennus*, acre, century, stade, mile, league. Pay attention now. A finger's breadth consists of four grains of barley placed in a line; this geometrical finger's breadth is to be distinguished from the others, which are various and of various quantities. An inch contains one and a third finger's breadths. The palm contains four finger's breadths or three inches. The dodrantal is twelve finger's breadths, or nine inches, or three palms. A foot is sixteen finger's breadths, and so on. For, once the total measures have been given, it is easy to figure out the partial ones, and conversely. The tile, so called from the lack of inclination of the side, has one foot in width and one and eleven-twelfths feet in length. The cubit has one foot and a half. The step has two cubits. The pace has one step and two-thirds. A pole has two paces. An *actus minimus* is a surface four feet wide and one hundred forty feet long. A clime too is a surface, forty feet by forty feet. A gate is eighty long by thirty wide. An *actus quadratus* has twelve poles on all sides. An acre is two agrippenes in length (twenty-three poles), on the side, twelve poles, a quarter of which is called a table, which has seventy-two short poles. A century contains two hundred acres; it is also customary for the length, sometimes, to be reduced as one pleases, and sometimes the width, provided that the total of the contents is made whole again in the modification. A stade is one hundred twenty-five paces. A mile is eight stades. A league is one and a half miles.

One must know that all measures drawn against [i. e., multiplied by] unity pertain to length, but if drawn against themselves [i. e., squared] to a surface, and if drawn against themselves twice [i. e., cubed], to a solid; for example, just as four grains of barley make a linear foot, so four again make a surface [i. e., a square] foot, and four again a solid [i. e., cubic] foot, and so on with respect to the remaining linear measures.

The intention of the author is to facilitate the measurement of rational figures. The matter, furthermore, consists of the rational figures, divided into four species: into numbers, into lines, medial, binomial, and residual (just as they are called rational as being subject to the reckoning of this art, so [others are called] irrational, that is, not named by any geometrical measure according to any number, namely, neither one-footed nor two-footed), into surfaces, [namely,] circle, triangle, polygon, and into bodies, [namely,] sphere, square [cube?] cylinder, pyramid, and polyhedron.

The utility or end is, the exemplification of measuring science and of demonstrative necessity. Its way of doing is this: It does by proving. A proof, moreover, is an argumentation, arguing from first and true things to what can be concluded from them. For so is the proposed art weaved together, that the things that follow necessarily happen as a result of those that have gone before, or from the first principles, one after another. For demonstrative science is that which teaches how to prove and itself proves, like the posterior analytics, and that which proves and does not teach how to prove, like geometry.

There is a double reckoning of order in this work, namely, the order of the parts of the matter itself, according to which one must begin with lines, proceed to the surface and to number, and finally treat the solid. The second kind of order is that imposed by the logic of the proof, according to which the things that precede cause the things that follow, according to which a proposition that pertains to a line draws credibility from those that pertain to a surface or a solid. The author of a proof, moreover, necessarily acquiesces in the order.

The title is this: Here begins the first book of Euclid the philosopher on the geometrical art, or, Here begins the geometrical art containing four hundred sixty-four propositions and proposed things [i. e., problems] composed in Arabic by Euclid and taken over into Latin by Adelard of Bath. The propositions are stated in the indicative, the proposed things in the infinitive [mood]. Furthermore, the propositions propose that something is or is not, the proposed things, on the other hand, that something must be done, or not. The rest is clear. One must stick to the letter, one after another, beginning with the principles. These, moreover, are divided into axioms, postulates, and conceptions. *Axiom* means *dignity*. [See note L 127 on page 173 below.] For, it explains the definitions of things. The postulates are those things which, when conceded by hypothesis, nothing unsuitable follows. The conceptions are those things which occur spontaneously to the human intelligence, in which one must not demand "Why?" Nor are all the principles written down. They must be accounted principles that are similar to what are obvious facts, as, Whatever is greater than something equal to a greater thing, is greater than something that is equal to a smaller thing. One gets to know principles without an exposition. [And indeed, the two manuscripts containing this introduction proceed directly to the propositions, omitting the lists of definitions, postulates, and axioms.]