CHAPTER 11

Hypothetical Syllogistic

Aristotle's logic, whose reception in Avicenna and his heirs we have been charting in the previous chapters, is a *predicative* logic (or "categorical" logic: as we have noted, *kategorein* means "to predicate"). That is, it studies propositions and their combinations, where the propositions in question have the form "P is said of S," where P is a predicate and S a subject. One can analyze a wide range of arguments in this kind of logic, including of course the arguments that satisfy Aristotle's requirements for demonstration as laid out in the *Posterior Analytics*. But many valid arguments have a structure that cannot be captured as categorical syllogisms. Perhaps the most obvious are *modus ponens:*

If A, then B; and A; therefore B

and *modus tollens*:

If A, then B; but not-B; therefore not-A

Here the first premises are *hypothetical*. One can also concatenate hypothetical propositions to produce an argument that looks perfectly valid:

If A, then B; and if B, then C; therefore if A, then C

Hypothetical arguments belong to what is called "propositional" logic, which was developed in antiquity by the Stoics. This sort of logic studied other argument forms too, for example disjunctive syllogisms (Not both A and B; but A; therefore B).

Aristotle already made some remarks that might suggest a reduction of hypothetical arguments to categorical ones [T1].[1] And later Aristotelians did try to incorporate such inferences.[2] For instance members of the Baghdad school recognized the validity of *modus ponens*, *modus tollens*, and disjunctive syllogisms. But Avicenna went further than any of his predecessors by systemat-

1 See further D. Ebrey, "Why Are There No Conditionals in Aristotle's Logic?" *Journal of the History of Philosophy* 53 (2015), 185–205.

2 In fact, not just Aristotelians: see S. Bobzien, "Peripatetic Hypothetical Syllogistic in Galen," *Rhizai: Journal for Ancient Philosophy and Science* 1 (2004), 57–102.

ically combining predicative and propositional logic within a single system.[3] He supposed that Aristotle had written a work on the hypothetical syllogism that had not survived [T3], a suggestion dismissed as implausible by Abū al-Barakāt [T8].[4] That left Avicenna to figure out things largely for himself, as he was keenly aware. He critiqued an earlier scholar, apparently al-Fārābī, for his inadequate treatment of hypotheticals [T2].[5] (For both this and the supposedly lost work of Aristotle, see also [T25].) As Avicenna says in his criticism of al-Fārābī, there was both need and opportunity to unify the treatment of hypothetical and categorical propositions, because the same quantifications can be applied to both. For instance just as one can say "Always, P is said of S" or "Necessarily, P is said of S," so one can say that "If A, then B" holds always, or holds necessarily. He observed that in general, the combinations of quantified propositions carry over from categorical to propositional logic [T4].

Avicenna made two further distinctions that allowed him to generate a taxonomy of hypothetical syllogisms. First, a contrast between two types of arguments: combinatorial (*iqtirānī*) and reiterative (*istithnā'ī*) syllogisms [T5]. The difference here is that the latter involve "reiterating" or repeating an element from the first premise in the second premise, whereas the former do not. For example *modus ponens* is reiterative. Second, a contrast between two kinds of hypothetical premises: conditional (*muttaṣila*: If P then Q) and disjunctive (*munfaṣila*: either P or Q). Remembering that there are also categorical propositions to consider, there are five ways to combine types of proposition to get "mixed" syllogisms:

Conditional-conditional
Disjunctive-disjunctive
Conditional-disjunctive
Categorical-conditional
Categorical-disjunctive

These five types are laid out by Bahmanyār [T6] and discussed in detail by al-Kātibī [T20] [T21], who also breaks them down into sub-types where appro-

3 N. Shehaby, *The Propositional Logic of Avicenna* (Dordrecht: 1973).

4 For traces of this supposedly lost book, see Di Vincenzo, "The Missing Logic: Traces of a Lost Book On Hypothetical Syllogistic in Avicenna's *Risāla Mūğaza fī Uṣūl al-Manṭiq*," *Arabic Sciences and Philosophy* 33 (2023), 55–81.

5 For arguments that this is a reference to al-Fārābī, see T. Street, "'The Eminent Later Scholar' in Avicenna's Book of the Syllogism," *Arabic Sciences and Philosophy* 11 (2001), 205–218. Note that the author is identified as al-Fārābī in [T24] below.

priate. This was the easy part. The difficult part would be to work through the formidable number of possible combinations—bearing in mind both quantification and the option of mixed syllogisms—to determine which are productive and which sterile. Since doing this completely would be a truly mammoth task, al-Sāwī undertakes to treat only the more "natural" or intuitive productive syllogisms falling under all five kinds [T7]. Al-Rāzī provides a parallel discussion [T9]; see also the options laid out by al-Abharī [T13].

As with other areas of post-Avicennan logic, al-Khūnajī's *Kashf al-asrār* raises important questions bearing on hypothetical syllogisms. We can begin with [T10, T11], where al-Khūnajī doubts that one can infer from an exclusive disjunction:

Either A or B (but not both)

the following conditional:

If A, then not-B

He argues that this inference is good only if the disjunction is not "coincidental," which introduces us to another important contrast. Our texts regularly distinguish between the implicative (or "non-coincidental": *luzūmī*) and the coincidental (*ittifāqī*),[6] where the former involve a necessary connection and the latter do not. Thus, as al-Khūnajī observes in [T10], "Either two is a quality or fire is hot" is a true proposition. Yet one cannot truly infer from it that, if two *were* a quality, then fire would stop being hot. Al-Urmawī considers combinations of coincidental and non-coincidental premises and says that the only combination that is not worth considering is where two coincidental premises are joined together [T23].

That example of two being a quality brings us to the issue of impossible assumptions, which are of interest to al-Khūnajī throughout his logic. Avicenna had already introduced the problem in his *Qiyās* (240),[7] considering the conditional proposition "If five is even, then five is a number." He allows that such impossible antecedents may be introduced for the sake of dialectical disputation. For instance when he is refuting proponents of the void he says things like, "If void exists, then it is an extension," a proposition with an impossible

6 On this terminology see K. El-Rouayheb, "Impossible Antecedents and Their Consequences: Some Thirteenth-Century Arabic Discussions," *History and Philosophy of Logic* 30 (2009), 109–125, at 211.

7 For the passage and further discussion see El-Rouayheb, "Impossible Antecedents," 213.

antecedent since void cannot exist. One might then go on to show that the consequent is false, so as to refute the proponents of void using *modus tollens* (cf. [T18] for the example). But impossible assumptions would, for Avicenna, play no role in positive science. Al-Khūnajī ignores the issue about dialectical use, and becomes interested in what would follow from an impossibility just as a matter of logic [T12].

The same issue attracted the attention of al-Abharī, whose remarks on this topic are quoted and discussed by al-Ṭūsī [T18, T19]. Al-Abharī was worried about problems of the following sort. Suppose we have the conditional:

(P1) If five is even, then five is a number

This looks true, though we might capture the intuition underlying it by rephrasing in counterfactual terms ("if five were an even number, then five would still be a number": put in this way, it even looks like a tautology). Yet we can combine it with the following, even more plausibly true conditional:

(P2) If five is a number, then five is odd

Applying the argument scheme If A then B, and if B then C, we get the absurd conclusion:

If five is even, then five is odd

Something must have gone wrong here, since we shouldn't be able to infer a contradiction from true premises. Al-Abharī's diagnosis is that the first conditional (P1) is true as a matter of conceptual truth (again the terminology is that it holds as "non-coincidental," *luzūmī*), whereas the second conditional (P2) is true only as a matter of fact (*fī nafs al-amr*). If the antecedent in (P1) were in fact true, then (P2) would stop being true. This solution is accepted by e.g. al-Urmawī at the end of [T23]. And al-Ṭūsī observes at the end of [T18] that the solution could be applied to hypothetical arguments involving disjunctions, too. But in [T19] he traces the problem to a different source: one should not be trying to make any inferences involving an impossible assumption like "five is even." He makes the same point in [T17]: an absurdity does not imply contradictory propositions, as al-Abharī had claimed, but in fact implies nothing at all (as opposed to saying that it implies *anything whatsoever*, an idea found in medieval Latin logic). Al-Ḥillī makes the same point [T27], and replies to al-Abharī's solution in [T28]. Al-Samarqandī says, by contrast, that we need to make inferences involving impossible assumptions (he gives the example of

assuming that the void exists in order to do a *reductio* argument against the possibility of void). So he proposes that conditionals with impossible antecedents do not convert to particular affirmative conditionals: for example, "if P and not P, then P" does not convert as "sometimes, if P, then P and not P."[8]

Now, even where there are no impossible antecedents involved, some logicians worried that valid-looking arguments made up entirely of hypotheticals may not actually be productive, because it may be that the conditional inference in one premise is blocked once one assumes the antecedent in the other premise. This may be the case when the antecedent in one premise is incompatible with the consequent of the other. Consider:

> Whenever the wolf population increases, few hunters shoot deer
> Whenever few hunters shoot deer, the deer population increases
> Whenever the wolf population increases, the deer population increases

The premises seem both true, but the conclusion does not. The problem here may be one of context-sensitivity: an increasing wolf population, leads, all things being equal, to a decreasing deer population, so that the first antecedent blocks the second consequent.

Al-Abharī, for example [T14, T15], argues that these kinds of wholly hypothetical syllogisms are not productive in the same way as their categorical counterparts. Instead of concluding in a conditional, he thinks that the result of such a syllogism will merely be a disjunction [T16]. Al-Ṭūsi responds to this in [T19], arguing that the syllogism will be valid as long as the premises hold in exactly the same way; if they do not then there is only an apparently shared middle term. Ibn Wāṣil [T24] rejected al-Abharī's position, arguing that the quantifier "whenever" ranges over all possible situations with which the antecedents are compatible.[9]

Finally, we mention that another problem was raised concerning syllogisms made up only of hypothetical premises. On this kind of syllogism and the question of complete or incomplete middles, see al-Kātibī's discussion in [T22]. An "incomplete middle" occurs when e.g. the consequent of one premise is "A is C" and the antecedent of the other premise is "A is B," so that we have partial overlap between the categorical propositions that have been made part of a hypothetical inference.

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8 Our thanks to an anonymous referee for this point.

9 See K. El-Rouayheb, "Mixed Conditional-Categorical Syllogisms from Avicenna to Urmawi," *History and Philosophy of Logic* 43 (2022), 232–250, as well as the introduction to K. El-Rouayheb, *Ibn Wasil: Commentary on the Jumal on Logic by Khunaji*. (Boston: 2022).

Texts from: Aristotle, Avicenna, Bahmanyār, al-Sāwī, Abū l-Barakāt, al-Rāzī, al-Khūnajī, al-Abharī, al-Ṭūsī, al-Kātibī, al-Urmawī, Ibn Wāṣil, al-Samarqandī, al-Ḥillī

[T1] Aristotle, *Prior Analytics*, 47ª22–41 [trans. Striker, mod.]

[*Aristotle on reducing hypothetical arguments to categorical arguments*]

In some arguments it is easy to see what is missing, but others escape our notice and appear to be proper syllogisms because something necessary results from the assumptions. For example, if one had assumed that a substance will not be destroyed when a non-substance is destroyed, and that when the constituents of a thing are destroyed, what consists of them is also destroyed—when these things have been laid down, it is necessary indeed that the part of a substance should be a substance, but this has not been deduced from the assumptions; some premises are missing. Again, *if what is a man is necessarily an animal, and what is an animal, a substance, then what is a man is necessarily a substance*. But this has not yet been deduced, for the premises are not related as we said. We are deceived in these cases because something necessary results from what is laid down, and the syllogism too is something necessary. But necessity extends beyond the syllogism, for while every syllogism is necessary, not everything that is necessary is a syllogism. Thus, if something results from certain assumptions, one should not try to reduce it right away, but first find the two premises, then divide them into their terms, and take as middle term the one that is said in both premises; for that the middle term occurs in both premises is necessary in all the figures.

[T2] Avicenna, *Shifāʾ, Manṭiq, Qiyās*, 356.7–357.15 [trans. Di Vincenzo, mod.]

[*Avicenna on the faulty hypothetical syllogistic of an anonymous author*]

We have presented the concise discussion of hypothetical propositions, and the syllogisms made from them, included [in this book]. Now, in our region (*bilād*) we had become acquainted with a lengthy, extended book on this subject, but then lost track of it during our travels, having to move around to attend to our affairs. It might still be in that region where we were. After we figured out this part of the science for just about eighteen years, we came across a book on hypotheticals ascribed to an eminent later [scholar] (*fāḍil al-mutaʾakhkhirīn*). However, it appears to be spurious, as it is neither clear nor reliable. It does not

get to the bottom of the topic, nor does it achieve what it sets out to do. It is defective in its definition of the hypothetical proposition itself, in many things it carries forward from [the discussion of] syllogisms, and when it comes to the types of proof for their productivity and sterility, as well as in the enumeration of moods and figures. The student should under no circumstances turn to that [book], for it is deviant and misleading. For its author does not understand affirmation and negation in hypotheticals, nor how universal, particular, or unquantified forms work, nor how contradiction works for hypotheticals, nor their conversion or subalternation. Rather [357] he just adds all this to the parts of conditionals, and uses the converse of a hypothetical in one manner only. Nor does he understand any of the connective [syllogisms]: he deals with them only as combinations of conditionals and disjunctions, rather than postponing them all [to a dedicated discussion], and reckons them as more numerous than necessary, because he treats a number of things falling under a single judgment as being like things that should have different judgments. What one should do is to apply the universal judgment to them: it is enough then to multiply their types on account of the numerous changes made to the antecedent and the consequent, with different quantities and qualities. But the [universal] judgment does not change when those are changed. Moreover, he thought that many of the sterile syllogisms are in fact productive, especially the first one he enumerates[10] in the book. And he considered many of these sterile syllogisms to be evidently productive, and that these syllogisms need no proof of their perfection. Conversely, he thought that many of the productive syllogisms are sterile. Then too, he did not succeed in showing that many of the productive syllogisms can be shown to be productive, but followed [proof] procedures that are not valid. And he thought that in a disjunction, the consequent is specified by the antecedent, so that a number of compositions are now in one figure, now in another, because in one of the two the shared element comes first, whereas in the other it comes last. Moreover, he thought that many connective syllogisms, in which there is no middle term given as such, are simple syllogisms, because something follows from them thanks to there being a single middle term potentially. But this applies only to the case of compound syllogisms, because a simple syllogism must have a single middle term in itself without fail. These things and similar ones are some aspects in which [the author] in this book is simply wrong.

10 Reading *yā'* for *bā'* at the beginning of the verb.

[T3] Avicenna, *al-Mūjaz fī l-manṭiq* [trans. Di Vincenzo, mod.], 78[11]

[*Avicenna on hypothetical syllogistic*]

One of those who intended to provide an account of hypotheticals believed that the conditional [premise] is affirmative, while the disjunctive is negative, but he made a huge mistake, for the negative removes the affirmative. Then if someone says: "If this is the case, then that is the case," whoever denies and refutes [this claim] does not oppose it by a disjunctive. Rather, he says: "Not: if this is the case, then that is the case", thus removing the implication. [...] Besides not adding, with regard to hypothetical [propositions] and the syllogisms that are formed of them, anything but prolixity to what people [already] had, that man [also] made a huge mistake concerning things similar to those I have mentioned. The reason for this is that people have lost a work by the Philosopher [viz. Aristotle] on hypothetical [propositions]. As for us, we have already discovered, concerning the states of their propositions and the composition of syllogisms out of them [i.e., out of hypotheticals], what we hope to have thoroughly examined in a long book we have composed; here, however, we seek brevity.

[T4] Avicenna, *Ishārāt, Manṭiq*, 157.3–159.2 [trans. Inati, mod.]

[*the combinatorial hypothetical syllogism*]

We will mention some of these [syllogisms], leaving aside the ones that are not grasped by us naturally, after having treated all of them sufficiently in *al-Shifā'* and elsewhere. We say that among the conditional combinatorial syllogisms, three figures may be formed, as with the figures of the categorical syllogisms. They have in common either a consequent or an antecedent, and they differ in a consequent or an antecedent, just as the figures of the categorical syllogisms have in common a subject or predicate, and differ in a subject or predicate. Now, the rules (*aḥkām*) governing these figures are the same as those governing the categorical ones. [158] An element may be shared between a categorical proposition and a disjunctive one, as when you say "Two is a number, and every number is either even or odd." From what has preceded it is easy to infer the rules for this. Also a disjunctive proposition and a categorical one may share an element, as when you say the following: "Let A be either B, C, or D; and every

11 The paragraph is edited (page 78) and translated (79) in Di Vincenzo, S. (2023). The Missing Logic: Traces of A Lost Book on Hypothetical Syllogistic in Avicenna's *Risāla mūǧaza fī uṣūl al-manṭiq*. *Arabic Sciences and Philosophy*, *33*(1), 55–81.

B, C, and D is E; therefore, every A is E." From what has preceded, it is easy to infer the rules for this too.

Further, a conjunction may be formed of a connective conditional proposition and a categorical one. Of this the most natural case is where the categorical proposition shares the consequent with the affirmative conditional, in one of the ways in which the categorical propositions share the predicate. But then the conclusion is a connective [i.e. a conditional], whose antecedent is the same as that one [i.e. the antecedent in the conditional premise], and whose consequent is the conclusion of the composition of the consequent which was conjoined in the categorical proposition. For instance: "If A is B, then every C is D, and every D is E," from which it follows that "If A is B, then every C is E." It is up to you to enumerate the rest of the divisions from what you already know. This combination may also be of two conditional propositions, each of which shares the consequent of the other, if [159] that consequent is also a connective; and its syllogism is the same as that one. A complete discussion of conditional connective syllogisms is not appropriate in a summary like this.

[T5] Avicenna, *Ishārāt, Manṭiq,* 159.12–160.16 [trans. Inati, mod.]

[*the reiterative hypothetical syllogism*]

A reiterative conditional syllogism is either:

(a) One in which there is a conditional, and what is repeated is either the same as its antecedent, giving the consequent itself as a conclusion. For example: "If the sun is up, then the stars are hidden; but the sun is up; therefore, the stars are hidden." Or the contradictory of its consequent, thus giving the contradictory of the antecedent as a conclusion. For example: [160] "… but the stars are not hidden," yielding the conclusion "therefore, the sun is not out." Nothing other than this follows as a conclusion.

(b) One in which there is a strict disjunctive proposition [i.e. an exclusive disjunction], and what is repeated is whichever part of the disjunction, yielding the contradictory of the other parts as a conclusion. For example: "This number is either perfect, excessive, or deficient; but it is perfect." Thus, the conclusion is the contradictory of the remaining parts [i.e. "therefore the number is not excessive or deficient"]. Or, what is repeated may be the contradictory of whichever part of the disjunction, yielding what remains—be that one or more parts—as a conclusion. For example: "… but it is not perfect, therefore,

it is either excessive or deficient." And so on, until the repeatable elements are exhausted, and one option will remain [e.g. if one added that the number is not excessive, it would remain only that it is deficient].

(c) One in which there is a non-strict disjunctive proposition, in which both disjuncts may be true [but not jointly false]. Repeating the contradictory of [one disjunct] renders the other [disjunct] itself as the conclusion. For example: "Either he is in water or he is not drowned; but he is drowned; therefore, he is in water" or alternatively "… he is not in water; therefore, he is not drowned." Another example: "either this is not an animal, or this is not a plant; but it is an animal; therefore, it is not a plant," or alternatively "… it is a plant; therefore, it is not an animal." Or the disjunctive proposition might be of the type in which neither disjunct may be true, but it permits the negation of all the parts together. Some have called this the "incomplete disjunction" or "conflicting." Repeating a part of it yields only the contradictory of the rest. For example if you say: "Either this is an animal, or this is a tree," in answer to someone who says, "This is both an animal and a tree."[12]

[T6] Bahmanyār, *Taḥṣīl*, 138.7–140.9

[***the types of hypothetical syllogisms***]

On the number of combinatorial syllogisms productive of conditionals [*as their conclusion*]*, and of combinatorial syllogisms consisting of conditionals and categoricals that are productive of categoricals* [*as their conclusion*].

Any such combination falls either (a) into two conditionals, or (b) two disjunctions, or (c) one categorical and one conditional sharing the antecedent, or (d) one categorical and one conditional sharing the consequent, or (e) one categorical and one disjunction, or (f) one conditional and one disjunction.

(a) Concerning the combination established between conditionals, its conclusions are conditional hypotheticals. But this is so only if they share amongst them a complete part—by which I mean an antecedent or a consequent—or an incomplete part—by which I mean part of an antecedent or part of a

12 Note that since Avicenna, hypothetical premises have been understood as quantified: in a conditional, the universal quantifier ranges over all possible situations, so that the conditional is true if the consequent follows from the antecedent in all cases in which the antecedent can possibly occur. A particular conditional is true if the consequent follows in some cases in which the antecedent can possibly occur.

consequent. But let us postpone the discussion of this last sort, because its properties are the same as those of the combination established between a conditional and a categorical. So let us begin with that.

(a1) If the shared element is a complete part, then it may be the antecedent in one of the two premises and the consequent of the other, for instance:

> Whenever A is B, then J is D
> Whenever J is D, then H is Z

This [139] resembles the first figure in categorical syllogisms, and the way it is productive, as to the moods and conditions for productivity, are just like in the first figure.

(a2) The shared element between them may also be in the consequent, like when you say:

> Whenever A is B, then J is D
> Not: if once H is Z, then J is D

This is like the second figure in categorical syllogisms, and the situation regarding its productivity is the same as with categorical syllogisms.

(a3) Or the shared element may be in the antecedent, like when you say:

> Whenever A is B, then J is D
> Whenever A is B, then H is Z

the moods of this are like the moods of the second figure in categorical syllogisms. And the form of the conclusion in the second figure, for example, is like this:

> Not: If A is once B, then H is Z

In the third figure it is

> If A is B, then H is Z.

(b) As to the combination established between disjunctions, this kind of combination cannot possibly have a complete part, but only part of the consequent

or the antecedent. In this case they are [formed] according to the following example:

> Either this number is even [i.e. a pair][13] or it is odd

Then we take "is a pair" as the middle term (it being an incomplete part), and put it together with the parts of the disjuncts in the second disjunction, and say:

> Every even [thing] is either a pair of even [things], a pair of odd [things], or a pair of even and odd [things]

And from this it follows:

> Therefore, every number is either odd or a pair, that is, of evens, of odds, or of an even and an odd

As for the productivity conditions, they are that the minor premise (which is like the first affirmative disjunction) [140] be either particular or universal, and that the shared element in it is affirmed, while the disjunction in the major premise is universal.

There are twelve combinations in which the minor premise necessitates something together with the major premise. For the major premise is either negative, and its parts are negated, or negative and its parts are affirmed, or negative and some of its parts are negative and some of its parts affirmed. Or else it is affirmative, with the same three options. This makes six possibilities, which are further divided into two divisions, namely that the major premise is particular and the minor premise universal, or the minor premise particular and the major premise universal, like the last example:

> Either A is B or A is J
> Never: J is D or J is H
> Therefore: Either A is B, or it is not the case that A is either D or H

13 It is difficult to render the Arabic here, because the word for even also means "a pair": *kull zauj zauj al-zauj aw zauj al-farḍ aw zauj al-zauj wa l-farḍ*.

[T7] Al-Sāwī, *Baṣā'ir*, 165.21–168.23

[*focus on the more intuitive hypotheticals*]

The [connective element in a hypothetical] combination either falls between two conditionals, or between two disjunctions, or between a categorical and a conditional—where the shared element may be the antecedent or the consequent—or between a categorical and a disjunction, or between a conditional and a disjunction. We don't deem it advisable to provide a thorough discussion of all these combinations, as some of them are rather unnatural (*baʿīd ʿan al-ṭabʿ*) and trying to prove their productivity can only be done with a lot of effort, and it is not fitting for summaries like this to present such tedious things. So let us focus on what lies closer to hand for someone of sound nature, in terms of its productivity. But if someone wishes to go through all the possible combinations and their productivity and sterility, let him seek out one of the books by the most eminent of the later logicians, who independently worked out most of their judgments and distinguished the productive from the sterile cases [166] as no one had done before. If God grants me a long enough life for it, I will dedicate an entire book to these combinations, both the familiar and the unfamiliar.

[*wholly hypothetical syllogisms composed of two conditionals*]

Concerning the combinations between conditionals, the productive ones among them are those that share between their premises a complete part, that is, in the antecedent or in the consequent. In this case the three figures composed from them are like the figures in categorical syllogisms, because either the shared element between them is the consequent in one of them and the antecedent in the other, and then it is the first figure; or the consequents of both premises together and then it is the second figure; or both antecedents, and in that case it is the third figure. One should also consider here the conditions for productivity of the categorical syllogisms where the minor premise is an affirmation and the major is universal, in the first figure; and where the major premise is universal and one of the two premises is negated, in the second figure; and where the minor premise is an affirmation and one of the two premises is universal, in the third figure; in all these cases the conclusion is a conditional. With two universal or two particular [premises], the first figure is productive; the second is not productive except with a negation; and the third is not productive unless its major premise is particular. All share the property that there is no syllogism if there are two particular quantified premises or two negations, or if the minor premise is negated and the major premise particular. […]

[***wholly hypothetical syllogisms composed of two disjunctions***]

[166.17] Concerning the combinations between disjunctions, a syllogism cannot be composed from strict disjunctions except when an incomplete part is involved, either in the consequent or the antecedent. Its natural form is like the form of the first figure. Its productivity conditions are that the minor premise (which can be particular or universal) is affirmative; that the shared element is affirmative; and that the major premise (which can be negated or affirmed) be universal. [...]

[***mixed categorical-conditional syllogisms***]

[166.23] Concerning the combinations established between a conditional and a categorical, they are more natural. Here an element is shared between the consequent of the conditional and the categorical (not between the latter and the antecedent). Furthermore, we put the categorical first, in the place of the major premise. From this three figures can be composed:

First: if they share an element in the predicate of the consequent and the subject of the categorical, then its productivity conditions are that, if the conditional is affirmative, the consequence must be affirmative and the categorical universal, as is the case in categorical syllogisms. The conclusion is a hypothetical whose antecedent is the antecedent of the conditional and whose consequent is that which would follow as a conclusion from the consequent and the categorical on their own. [...]

[167] *Second*: if they share an element in the predicates of the consequent and the categorical, then the productivity conditions are that if the conditional is affirmative, as is the case in the second figure of categorical syllogisms, the major premise is universal and the categorical or the consequent is negated. [...]

Third: if they share an element in the subjects of the consequent and the categorical, the productivity conditions are that if the connective conditional is affirmative as is the case in the third figure of categorical syllogisms, the consequent is affirmative and one of the two premises is universal. [...] if it is negated then we have three other figures that are less natural, so we won't mention them. [...]

[***mixed categorical-disjunctive syllogisms***]

[167.13] Concerning the combination between a disjunction and a categorical, if the categorical is the minor premise then this is more natural, and it follows the same pattern as in the first figure. This means that the categorical is affirmative and its predicate is the subject of the parts of the disjunction all together, where the disjunction is universal. […]

[***mixed conditional-disjunctive syllogisms***]

[168.10] Concerning the combination between a conditional and a disjunction, either this involves a complete part, in which case the conditional must be the minor premise and the disjunction the major premise, and the disjunction must be affirmative and [at least] one of the two premises has to be universal; if they are not both universals, the conclusion will not be universal. One may say that the conditional is productive and that the disjunction is productive. […] [168.15] Or [the combination may] involve an incomplete part, in which case it will naturally be that the predicate of the consequent is the subject in the parts of the disjunction, and the consequent will be a universal affirmative. The disjunction is productive on what remains of the consequent, and the conclusion will be a conditional of the disjunction of the consequent. For instance: "If this thing is multiple, then it has number; everything that has number is either even or odd," from which follows the conclusion, "if this thing is multiple, then it is either even or odd."

You should know that every possible combination between a categorical and a hypothetical may also be made between a conditional and that hypothetical so long as that part of the hypothetical is a conditional. Then what is affirmed is the element that is shared between this condition and premise and that part of the conditional that is either in the antecedent or the consequent. That's enough on the combinations of hypotheticals.

[T8] Abū al-Barakāt, *Muʿtabar*, vol. 1, 155.11–18 [trans. Street, mod.]

[***on Avicenna's and Aristotle's treatment of hypothetical syllogisms***]

Regarding syllogisms which are composed from hypothetical propositions, Aristotle only made mention of the reiterative ones (*al-istithnaʾiyyā*) in his book. What touches on connective [hypothetical] syllogisms, both pure, and mixed with categoricals, is clear from what he says, and the sound mind (*al-dhihn al-salīm*) will recognize them on the basis of what has been said. He

omitted mentioning them in his book, either due to the little benefit they have for the sciences (*li-qillat fā'idatihā fī l-ʿulūm*), such that he disliked the thought of dwelling on them (*fa-kariha l-taṭwīl bi-hā*); or because he relied on the fact that minds which have come to know the categoricals may conclude from them to [the valid connective syllogisms with hypotheticals as premises], so that you will recognize them from what you have come to know in the categoricals. Or [perhaps he omitted mention of them] for both [reasons]. A certain later scholar (*baʿḍ al-mutaʾakhkhirīn*) said that Aristotle had dedicated a book specifically to them, which had not been translated into Arabic. But this is baseless conjecture (*takhmīn lā ḥaqīqata lahu*). Had [Aristotle] wanted to mention them, why would he move them from here, their proper place? Anyway, there is not enough concerning them that would merit a separate book with separate principles and conclusions.

[T9] Al-Rāzī, *Manṭiq al-Mulakhkhaṣ*, 313.5–318.6

[***the five kinds of hypothetical syllogisms***]

First kind, composed of conditionals. The productive ones among [the first kind] are those which share a complete part. What is shared may be the consequent in the minor premise and the antecedent in the major, in which case it is the first figure. Or it may be the consequent in both [premises], in which case it is the second figure. Or it may be the antecedent in both [premises], in which case it is the third figure. Or it may be the antecedent in the minor and the consequent in the major premise, in which case it is the fourth figure. The conditions [for productivity] in consideration of these figures are exactly the same as the ones that were considered regarding categorical [syllogisms], so there is no point in enumerating them [again here]. You should know that these syllogisms are put to good use only when they have relations of implication, be they coincidental or not.

Second kind, composed of disjunctions. A syllogism cannot be composed from strict disjunctions, unless the shared term is an incomplete part. Its properties are the same as that of first figure syllogisms. Its productivity conditions are that the minor premise is affirmative and the shared part in it is likewise affirmative, and that the major premise is quantified as universal. For example:

> Either this number is even, or it is odd
> Every odd thing is either one or many
> Therefore: this number is either even, or one, or many

You know that this minor premise here may be quantified as universal or as particular, and the disjunction may be either affirmative or negative. Accordingly, it is either composed of affirmations or two negations or a mix between them.

Third kind, composed of a categorical and a conditional. If the shared element is with the antecedent and the categorical, then this is rather unnatural. If [315] it is with the consequent and the categorical, then the categorical is either the major premise, or the minor premise. If it is the major, then the conditional is either affirmative or negative. If it is affirmative, then the conclusion is a conditional whose antecedent is the same as the original antecedent, and whose consequent is the conclusion reached from the consequent of the conditional and the categorical. [...] [315.7] If it is negative, then the condition is that everything we express in a simple categorical as being affirmative, here has to be negative. For the negation of a negation is an affirmation and the conditions [for productivity] remain the same. An example of the first mood in the first figure is:

> Not: if once H is Z, then J is not D
> Every D is A
> Therefore: Not: if once H is Z, then not every J is A

Which is proved as follows: the conditional implies "whenever H is Z, then every J is D" and "every D is A," which concludes in "whenever H is Z, then every J is A." From this follows "Not: if once H is Z, then not every J is A." And you know how to extrapolate to other cases.

If the categorical is the minor premise, the above figures apply. If the conditional is affirmative, then the conditions between the categorical and the consequent are the same as have been mentioned. And if it is [316] negative, then the condition is that everything that we express in a plain categorical that is quantified as universal needs to be quantified as particular in the consequents that are being substituted here. This is because when the conditional is negative, it means a denial that if the antecedent [holds], then [so does] this particular. This requires the use of universal quantification, because withdrawing the particular presupposes that the universal obtains. And the remaining conditions are the same. An example for the first mood in the first figure is:

> Every J is B
> Not: if once H is Z, then not every B is A
> Therefore: Not: if once H is Z, then every A is J

This is shown by converting the conditional to the affirmative and then taking the implicand of the conclusion.

Fourth kind, composed of a categorical and a disjunction. If the categorical is the minor premise, then this is more natural, in accordance with the method for the first figure. Thus the categorical is affirmative and its predicate will be the subject of each of the parts of the disjunction, and the disjunction should be quantified as universal. For example:

> Every movable is a body
> Every body is either mineral, plant, or animal
> Therefore: every movable is either mineral, plant, or animal

[317] If the categorical is the major premise, then it consists of several propositions or a single one. If it is several propositions, then either they share a single predicate, or they do not. If they do, then its properties are the same as when we follow the method of the first figure. The disjunction and its parts must be affirmative and the categoricals quantified as universal; and the parts of the disjunction must be shared with the subject. For example:

> Every movable is either mineral, plant, or animal
> Every mineral, plant, or animal is a body
> Therefore: every movable is a body

If they do not share a single predicate, the conditions are still the same as we mentioned, but the conclusion is an exclusive disjunction. For it could be that all the predicates that are elements of the disjunction have a greater extension than [the conclusion] does. If the categorical is a single [proposition], then the conclusion is likewise an exclusive disjunction, because the predicate of that part could imply something with a greater extension than it, without negating its remaining elements.[14]

Fifth kind, composed of conditionals and disjunctions. In the strict disjunction the shared element is either a complete or an incomplete part. If it is [com-

14 For example:
> Every movable is either mineral, plant, or animal
> Every mineral, plant, or animal is animate or inanimate
> Therefore, every movable is either animate or inanimate

Here, the conclusion cannot be a universal affirmative categorical proposition like "Therefore, every movable is animate," because minerals are movable but not animate. Instead,

plete], then this is more natural, and the conditional is the minor premise while the disjunction is the major. And it is [318] affirmative with [at least] one of the two needing to be quantified as universal. Unless both premises are quantified as universal, neither will be the conclusion. Then, its conclusion may be turned into a conditional and a disjunction.

If it is [incomplete], then its properties are the same as those of the predicate of the consequent whose subject is among the parts of the disjunction and the universal consequent. The conclusion is in this case a conditional of the disjunction and the consequent.

This was a very abridged presentation of hypotheticals. A more elaborate treatment of them will appear, if God grants it, in the *Great Logic*.

[T10] Al-Khūnajī, *Kashf al-Asrār*, 410.6–411.10

[*doubts on the productivity of certain combinatorial hypothetical syllogisms*]

The following doubts concerning these syllogisms have been advanced.

First, that two affirmative disjunctions that are different in kind are not productive of a conditional, as you stated. *Proof*: given it is not incumbent on the debtor to give alms, it is true to say "Either it is incumbent on the rich or it is incumbent on the debtor," where it is ruled out that both [are true]. But it is also true to say "Either it is incumbent on the debtor, or it is not incumbent on both," where at least one [is true]. But then it follows that the following is also true: "Whatever is incumbent on both of them, is not incumbent on the rich." In this way, if it is incumbent on the debtor, then a disjunction is affirmed between the giving of alms not being incumbent on the debtor and its not being incumbent on the poor, where the disjunction is exclusive, so that the impossibility we mentioned follows. If the syllogism mentioned above were productive, then two contradictories would both be true, which is absurd.

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the conclusion is itself a disjunction. The same applies if one of the premises is a simple categorical proposition:

Every movable is either mineral, plant, or animal
Every animal is breathing
Therefore, every movable is either breathing, or a mineral, or a plant.

[411] *Second*, if it is incumbent on the debtor to give alms, then an exclusive disjunction is affirmed between the giving of alms that is incumbent on the debtor and its being incumbent on the poor; as is an exclusive disjunction between its being incumbent on the poor and the negation of its being incumbent of both. Hence [the syllogism] is productive of the conclusion "Whatever is denied of both is incumbent on the debtor." In this way, if it is incumbent on the debtor, then a disjunction, where at least one disjunct [is true], is affirmed between its not being incumbent on the rich and its not being incumbent on the debtor. Hence an impossibility follows, and the productivity leads to two contradictories jointly being true.

Third, if [giving] a quarter [of the tenth as alms] is incumbent on the debtor, then it is true to say: "Either a quarter is incumbent on the debtor, or a half," where at least one holds. This is not true unless it is true that "On whomever a quarter is not incumbent, on him a half is incumbent." In this way, if a quarter is not incumbent, then either a quarter is not incumbent, or a sixth is not incumbent. This, however, is not true.

[T11] Al-Khūnajī, *Kashf al-Asrār*, 413.9–414.2

[***reply to the doubts in T10***]

Given the assumptions that were made, the true disjunction is merely coincidental, and what is produced by the conditional is [only truly] opposed propositions. Coincidental premises do not yield conditionals [formed] with one of its extremes and the contradiction of the other. It may be true coincidentally [to say] "Always: either two is a quality or fire is hot," yet it does not follow that "Whenever two is a quality, fire is not hot" or "Whenever fire is not hot, then two is a quality." If it is claimed that given the assumptions, an oppositional proposition is implied, then we deny this, because what is known on these assumptions is that [the giving of alms] being incumbent is not jointly true for both. For its not being [414] incumbent on the debtor precludes its not being incumbent on the other, but its being incumbent on one of the two does not require its not being incumbent on the other.

[T12] Al-Khūnajī, *Kashf al-Asrār*, 371.3–13 [trans. El-Rouayheb, mod.]

[*rejection of Avicenna's solution to mixed conditional-categorical syllogisms*]

The Master [Avicenna] brought forth a question concerning this division [of the syllogism], namely that the categorical premise may be true in itself but not true along with supposing the antecedent [of the conditional premise] [...]

He answered this in two ways: *first*, that we may specify that we are speaking of cases in which they are both true and are not incompatible, by which he means that we suppose that we are speaking of cases in which there is no incompatibility between the categorical [premise] and the antecedent [of the conditional premise], rather they are both true. *Second*, to deny the impossibility of the consequent [of the conditional conclusion] on the basis that the impossible may follow from the impossible existence of the void. [...]

The first of the two responses mentioned by the Master calls for further inquiry, in my eyes. As for the second, it is a refutation of a specific counterexample and does not dispel the objection to the syllogism.

[T13] Al-Abharī, *Īsāghūjī*, 25.1–15

[*the combinatorial hypothetical syllogism*]

The combinatorial syllogism is either such that it is composed of two sentences as has been said, or of two conditionals, for example: "if the sun is up, then it is day; whenever it is day, then the world is lit"; this concludes in "if the sun is up, then the world is lit." Or it is such that it is composed of two disjunctions, for example: "every number is either even or odd; every even thing is either divisible into even things or divisible into odd things," which concludes in: "every number is either odd or even and divisible into even things or even and divisible into odd things." Or it is such that it is composed of a categorical and a conditional, for example: "Whenever this is a human, then it is an animal: every animal is a body," which concludes in "whenever this is a human, then it is a body." Or it may be such that it is composed of a categorical and a disjunction, for example: "every number is either even or odd; every even thing is divisible into equal parts," which concludes in "every number is either odd, or divisible into equal parts." Or it may be such that it is composed of a conditional and a disjunction, for example: "whenever this is a human, then it is an animal;

every animal is either white or black," which concludes in "whenever this is a human it is either white or black."

[T14] Al-Abharī, *Kashf al-ḥaqā'iq*, 155.7–156.3

[***on wholly hypothetical syllogisms with complete middles***]

First inquiry, regarding the widely accepted doctrines held by the Imām [al-Rāzī] and those before him. [Their view was] that if the middle term between the two premises is a complete part, then the number of productive moods in each figure are the same as in the categorical syllogisms, and their conclusions are conditionals. But this calls for further inquiry. Because if we say

> Whenever A is B, then J is D
> Whenever J is D, then H is Z

then the major premise signifies that the major term [i.e. H is Z] is implied by the middle term [i.e. J is D] in actual fact (*fī nafs al-amr*), while the minor premise signifies that the middle term is true when the minor term [i.e. A is B] is *assumed* to be true.[15] So it may be that this true implication in actual fact no longer applies, once the minor term is assumed to hold. This being so, we deny that the implication of the major term for the minor follows from the truth of the two premises. It only follows if the implication between the middle term and the major term remains valid once that assumption is made. So then, why have you said that it remains valid on this assumption? The upshot of this argument is that whenever J is [156] D, then H is Z in actual fact; but J is D on the assumption that A is B, so that H is Z only on that assumption, too. It is well known that this [combination] is not productive. If you simply say that the implication is affirmed on this assumption, then we deny this: it needs a proof.

[T15] Al-Abharī, *Kashf al-ḥaqā'iq*, 156.5–157.12

[***on wholly hypothetical syllogisms with incomplete middles***]

Second inquiry, regarding what was said about [wholly hypothetical syllogisms] in which both conditionals have an incomplete middle term. Al-Kashshī said:

15 That is, the minor premise's being true requires that either the consequent is true, or both the antecedent and consequent are false. But if both are false then no implication goes through.

the middle term may be shared (a) between the two consequents, or (b) between the antecedent of the minor premise and the consequent of the major premise, or (c) vice-versa, or (d) between both antecedents.

(a) *Concerning the first case*, the number of productive moods in each figure is twice the number [of productive moods] in categorical syllogisms, because the minor premise may be quantified either as universal or particular. He also claimed that its conclusions are evident. The first mood in the first figure is thus:

> Whenever A is B, then every J is D
> Whenever H is Z, then every D is T
> Therefore: Whenever A is B, if H is Z, then every J is T

You can do the same for all remaining moods and figures.

But this calls for further inquiry. *For we say*: we don't concede that the implication in the major premise will hold on the assumption that A is B remains valid, so that it will imply the truth of the above conclusion [cf. the objection in T14 above].

(b) *Concerning the second case*, he claimed that it is productive, because it may be reduced to the first case by converting the minor premise. The number of productive moods in each figure is thus the same as in the first case. The first mood in the first figure is thus:

> Whenever J is D, then A is B
> Whenever H is Z, then D is T
> Therefore: If A is B, then: if H is Z, then every J is T

[157] But we deny that the converse of the conditional is implied. And even on the assumption that the converse is implied, we deny that it is productive. The rejection is based on [the same argument that] you know already.

(c) *Concerning the third case*, he claimed that it is productive by converting the major premise. But this is utter nonsense! For if the major premise is converted, it comes to be quantified as particular, and as such can no longer serve as a major premise. [...]

[157.9] (d) *Concerning the fourth case*, it is not possible to use conversion for it. If we say:

Whenever every J is D, then A is B
Whenever every D is T, then H is Z

and then we convert the two premises, then they become:

If once A is B, then every J is D
If once H is Z, then every D is T

But they are both particular and thus they are not productive.

[T16] Al-Abharī, *Kashf al-ḥaqā'iq*, 158.20–162.18

[*Abharī's own position on the productivity of wholly hypothetical syllogisms*]

Third inquiry, on the productivity conditions of non-coincidental conditionals, according to our own opinion.

The middle term of two premises is either (a) complete or (b) incomplete [in both], or (c) complete in one and incomplete in the other.

[159] (a) *Concerning the first case*, the productive moods are the first mood in the first figure and the first mood in the fourth figure. The conclusion in the first is an exclusive disjunction composed of the contradictory of the antecedent [as it appears] in the minor premise and the consequent as it appears in the major premise. For example:

Whenever A is B, then J is D
Whenever J is D, then H is Z

always concludes either to:

A is not B, or H is Z

[where the disjuncts are exclusive], because this follows from the minor premise, or to:

A is not B, or J is D

[where the disjuncts are again exclusive]. Here the exclusive [disjunction] together with the major premise, if A is not B, leads to what was sought. If J is D, it follows that H is Z, and thus leads to what was sought.

The conclusion in the fourth figure is an exclusive disjunction composed of the consequent as it appears in the minor premise, and the contradictory of the antecedent [as it appears] in the major premise. This is proven by conversion.

(b) *Concerning the second case*, here the two premises inevitably have to be affirmative and quantified as universal, and the [middle term] is shared between (b1) the two consequents, or (b2) the two antecedents, or (b3) the consequent of the minor premise and the antecedent of the major premise, or (b4) vice-versa.

(b1) As for the first [sub-]case, a condition for its productivity is that it must contain the two consequents in each figure according to the composition productive in that figure. The conclusion is an exclusive disjunction, and the number of productive moods in each figure is the same as in categorical syllogisms. The first mood in the first figure is:

> Whenever A is B, then every J is D
> Whenever H is Z, then every D is T

which always concludes in:

> A is not B, or H is not Z, or every J is T

This is because the minor premise implies either "A is not B" or "every J is D," and the major premise implies either "H is not Z" or "every D is T"—each of which are understood as exclusive disjunctions—and they produce the intended conclusion, because [160] the first disjunction being true, if A is not B, this leads to what was sought. If every J is D, then this being true together with the major premise, if H is not Z, leads to what was sought as well. If every D is T, and the following is true:

> Every J is D
> Every D is T
> Therefore: every J is T

then this leads to what was sought as well. You can do the same for the remaining moods and figures.

(b2) As for the second [sub-]case, a condition for its productivity is that it contains the two contradictories of the two premises in each figure according to the composition productive in that figure. The conclusion is an exclusive disjunction composed of the two consequents and the composition of the contradictories of both premises. The number of productive moods in each figure is the same as in categorical syllogisms. The first mood in the first figure is:

> Whenever not every J is D, then A is B
> Whenever not every D is T, then H is Z

which always concludes in

> Either A is B, or H is Z and every J is D

This is because from the minor premise it follows that either every J is D, or A is B. From the major premise it follows that either every D is T, or H is Z. Both are productive of the intended conclusion. You can do the same for the remaining moods and figures.

(b3) As for the third [sub-]case, a condition for its productivity is that it contains the consequent of the minor premise together with the contradictory of the antecedent in the major premise in each figure according to the composition productive in that figure. The conclusion is an exclusive disjunction composed of the contradictory of the antecedent [as it appears in the] minor premise and the contradictory of the antecedent [as it appears in the] major premise. The number of productive moods in each figure is the same as in categorical syllogisms. The first mood in the first figure is [161]:

> Whenever A is B, then every J is D
> Whenever not every D is T, then H is Z

which always concludes in

> Either A is not B, or H is Z, or every J is T

You can do the same for the remaining moods and figures.

(b4) As for the fourth [sub-]case, a condition for its productivity is that it contains the contradictory of the antecedent in the minor premise together with the consequent as it appears in the major premise in each figure according to the composition productive in that figure. The conclusion is an exclusive disjunction composed of the consequent as it appears in the minor premise and the contradictory of the antecedent in the major premise. The conclusion is then a composition of the contradictory of the antecedent in the minor premise and the consequent in the major premise. The number of productive moods in each figure is the same as in categorical syllogisms. The first mood in the first figure is:

> Whenever not every J is D, then A is B
> Whenever every H is Z, then every D is T

which always concludes in

> Either A is B, or H is not Z, or every J is T

You can do the same for the remaining moods and figures. [...]

[162.8] (c) *Concerning the third case*, the productive moods are the first mood in the first figure and the first mood in the fourth figure, and the complete part is either in the major or in the minor premise. If it is [in the major premise], then the conclusion is an exclusive disjunction composed of the contradictory of the antecedent in the minor premise and the conclusion that obtains from the consequent in the minor premise together with the major premise. For example:

> Whenever A is B, then: whenever J is D, then H is Z
> Whenever H is Z, then J is T

which always concludes in

> Either A is not B, or: J is not D or J is T

You can do the same for the first mood in the fourth figure.

Whereas in the second case [where the complete part is in the minor premise], the conclusion is an exclusive disjunction composed of the contradictory of the antecedent in the major premise and the conclusion obtaining from the consequent of the major premise together with the minor premise. For example:

Whenever A is B, then J is D
Whenever H is Z, then: whenever J is D, then J is T

always concludes in:

Either H is not Z or: A is not B or J is T

You can do the same for the first mood in the fourth figure.

[T17] Al-Ṭūsī, *Taʿdīl*, 170.3–171.12

[***critique of al-Abharī's claim that, given that anything follows from an impossible assumption, the consequence of a negation cannot be said to imply the negation of a consequence***]

[*Al-Abharī*] *says*: it is not clear to the mind that the implication of a negation implies the negation of an implication, because the antecedent of a conditional might be an absurdity, and an absurdity may imply two contradictories. Nor is it clear to the mind that the implication of an affirmation implies the implication of a negation, for the same reason.

I say: if it were really a problem (*law qadaḥa*) that an absurd antecedent may imply two contradictories—in light of the fact that what follows from a negation may imply the negation of what follows from an affirmation, given that it both implies this and leads to whatever follows from the negation at the same time, and furthermore, it implies it insofar as it leads to whatever follows from the affirmation—then this problem would already arise for whatever jointly follows from a negation, along with what follows from the affirmation. For the implication of two opposed [propositions] is the reason (*ʿilla*) why, when they are taken together, something both follows and does not follow. The problem lies with the false result (*maʿlūl*), not with the reason that leads to its falsehood, which can in no way obtain.[16]

16 The point of this rather convoluted passage is as follows. Al-Abharī argued that "if not-p, then q" does not imply "not: if p, then q" because p might be impossible, in which case it could imply both q and not-q. For al-Ṭūsi this is confused: an absurdity does not imply anything taken on its own. But once someone admits the absurdity then they can be forced to admit both of two contrary propositions.

The point stands even though in a given example, it is possible for something both to follow and not to follow in two different ways, so that you get two possible implications. For instance "animal" implies that acts of sensation (*iḥsās*) proceed from it, insofar as [animal] is sensitive, but it does not imply that voluntary motion proceeds from it in the same way. [Rather,] it implies that voluntary motion proceeds only insofar as [the animal] is moving *voluntarily*, which is not how acts of sensation proceed from it. So both [moving and not moving] may follow from [animal], or not follow, in two different ways.

If this can happen with possible [things], then obviously no such problem need arise in the way mentioned as a point of inquiry. Rather, what must be considered here is implication itself, as being capable of jointly implying contradictories. So what he ought to have said is: "it is not clear to the mind that something in itself implies anything *at all*," because the antecedent of a conditional may be impossible, and the impossible may imply two contradictories, so that the issue of implication would be blocked altogether.

As for what *is* clear regarding the following of two contradictories from something absurd, and the impossibility of clearly [understanding] that it leads to both [171] opposed propositions, the solution is to accept the premises but to say that the conclusion must not be accepted.

You need to realize that the absurd (*maḥāl*) as such, or rather the nonexistent (*maʿdūm*) as such, does not imply anything; nor can one judge that it implies anything. It is however possible to judge that it does *not* imply anything. Just as a subject, when it is nonexistent, cannot bear an affirmative judgment, whereas one can deny anything of it; a subject must be existent, in order that an affirmative judgment may be passed on it. Or at least it must be taken to be some thing that is actually present (*thābit*) in the mind, without being considered as [extramentally] existent or nonexistent. In the same way, the antecedent that implies something must be taken either as actually existing [extramentally] or as present in the mind without its being considered as being extramentally real (*thubūt fī l-khārijī*), either as possible or as impossible. This, then, is the notion of a stipulation (*farḍ*) or an assumption (*taqdīr*).

So if something is stipulated as being present [in the mind], one must judge what is implied by it and not make problems about the implication itself on the grounds that it implies [different things] on two sides. If what follows is absurd, this does not show that nothing is implied by it: rather, it shows only that what follows is nonexistent or just absurd.

[T18] Al-Ṭūsī, *Taʿdīl*, 211.5–213.7

[*response to al-Abharī on mixed categorical-conditional syllogisms with counterfactual assumptions*]

[*Al-Abharī*] *says*: if the conditional is non-coincidental (*luzūmiyya*), then from [the syllogism] no conditional conclusion obtains. For when we say: "Whenever A is B, then every J is D; and every D is H" then the conditional[17] signifies the implication from A being B to every J being D. The categorical premise signifies that every D is H in actual fact (*fī nafs al-amr*), but it might be that the categorical does not remain [true] on this assumption, so [the syllogism] does not produce the conclusion "whenever A is B, then every J is H."

I say: this objection is advanced by the Master [Avicenna] in the *Shifāʾ*, and from the same point of view (*ʿibāra*).[18] *He said*: "one needn't object to these moods and what is like them," and then went on to say, "it may be that the categorical premise is true in itself, but that it is not true on the assumption of the antecedent [in the conditional], in which case there is no syllogism. For example: 'Whenever void is existent, there is a self-subsistent extension,' but then one says: 'every extension is not self-subsistent,' or, 'nothing that subsists in itself is an extension.' So the true categorical premise is, in potentiality, the contradictory of the consequent. But one may answer in two ways. One is for us to count [only] the [conditional] connection in which both are true together. The other is [to say] that what follows from the two premises is true, but if the void is existent, then it follows that the extension is not an extension; and the implication is absurd. If the consequent [is true], then the categorical is not true." Thus far the passage from the Master.

He adduces the first reply with regard to the true *matter* [of the syllogism]. For perfect and true syllogisms are those that are important in the sciences. Whereas this combination here [sc. the one raised in al-Abharī's objection] is not like that, except in cases where the antecedent is not absurd, and the parts are not contradictories. The second reply he adduces with regard to the *form* of the syllogism. He explains that it is a syllogism, and specifies its type by saying that it falls under *reductio* syllogisms and what is like them. His words indicate that he does not make the major premise [212] true by assuming the antecedent

17 Reading *muttaṣila*.

18 Avicenna, *Shifāʾ, Manṭiq, Qiyās*, 326.3 ff.

as a condition for the validity of the syllogism. This is what has been transmitted; it is not a proof in the rational sciences, but may help our minds see what is going on (*istishhād*).

Also, whoever says that the productivity of hypothetical syllogisms in this case is to be considered [the same as in] a perfect syllogism, and who means by this that the conclusion obviously follows, with no need of proof for its productivity, this [person] has to deal with the same problem (*qadḥ*) that arises with first principles.

Resuming, then: a [hypothetical] syllogism is like something that is knitted together from two premises that are posited, without their positing being dependent on any [actual] thing. Just as the composition of two categorical premises is knitted together from two premises that are posited as dependent on two propositions, so [there may be] a composition of two hypothetical premises that share an incomplete part, or one of the two things posited [may be] dependent on a proposition, the other not, as is the case in mixed conditional-categorical [syllogisms]. They are no different in this respect, because the conclusion in the first form is posited without being dependent on a proposition, but rather insofar as it follows from the two premises of the syllogism absolutely. And in the second form it is posited as dependent on two propositions, which are in turn dependent on two premises. And in the third form it is dependent on a single proposition that itself is again dependent on its conditional premise. The dependence of whatever is posited is just like implication and association (*muṣāḥaba*), or like opposition and negation. What it depends on is in the first case another posit, and in the second case a negation. In this sense, what they share is in the same domain (*miḍmār*).

This being so, there is a difference between on the one hand, talking about (a1) the productivity of a posit that depends on negating a proposition, as opposed to (a2) a posit that depends on positing a proposition; and on the other hand, (b) something that yields no productivity at all. For the two [sc. (a1) and (a2)] share in being posits, not in the sense that they are judged to occur absolutely, but rather, that they occur as following on the negation or positing of some other proposition. For example "either five is odd or it is even," or "if five is not odd, then it is even." Here, we don't judge that evenness occurs for five absolutely. Rather, we judge that its occurrence cannot occur jointly with its being odd, or that it ensues from its not being odd. Changing the particles [indicating] disjunction or conditionality does not change the situation as to whether the occurrence depends on something else. Therefore, the discussion [213] of

the productivity of one of the two, as opposed to the other, can be undertaken only after going over that.

As for what the author of the book [i.e. al-Abharī] mentioned concerning the categorical [premise], which is the major premise in his example, namely that the situation may not remain the same once it is posited as true, on the assumption that a corollary of the shared part occurs, which in turn depends on positing the shared part—this is not something specific to conditionals as opposed to disjunctions. For if we add to the two abovementioned propositions the following: "every even thing is divisible into equal parts" as the major premise of the syllogism, then this is true in actual fact (*fī nafs al-amr*), but it does not remain true on the assumption that five is not odd, or on the assumption that it is even.[19]

[T19] Al-Ṭūsī, *Taʿdīl*, 213.22–215.19

[***on wholly hypothetical syllogisms***]

[*Al-Abharī*] *says*: when two premises are non-coincidental conditionals, then from [the syllogism] no conditional conclusion obtains. For when we say: [214] "whenever A is B, then every J is D; whenever J is D, then H is Z," the major premise signifies the implication from the middle term to the minor term in actual fact (*fī nafs al-amr*). The minor premise signifies that the middle term holds true on the assumption that the minor term holds true, too; but this does not imply that the major term is true of the minor term, and they only produce a conclusion that is a disjunction.

I say: the Master [Avicenna] mentioned this doubt in the *Shifāʾ* by giving an example, namely:[20]

> Whenever two is odd, it is a number
> Whenever two is a number, it is even
> Therefore, whenever two is odd, it is even

19 In the envisioned example, the minor premise is "every five is either odd or even." The point then would be that although the stated major and this minor premise are both true, one can reach the conclusion that five is divisible into equal parts only if one assumes that it is even (or not odd).

20 Avicenna, *Shifāʾ, Manṭiq, Qiyās*, 296.14 ff.

But this is a contradiction, and producing a contradictory conclusion from two true premises is impossible. Hence, this combination is not productive.

While answering [this doubt], he stated that this contradiction does follow [from the premises], so it is necessary for someone who accepts the premises to accept the conclusion, in light of this implication. But [the conclusion] is not true in actual fact (*fī nafs al-amr*), because the two premises are not [true in actual fact].

The author of the book [i.e. al-Abharī] explains this point as follows:

"It may be that the implication in the major premise, once one assumes that [the implication in the] minor premise no longer holds, does not remain intact either. For the judgment that 'two is even' is no longer true on the assumption that two is odd. If this requires the composition to be sterile, then similar categorical compositions would have to be sterile, too; we can exemplify such a composition with an example like:

> Everything that is both two and odd is two
> Everything that is two is even
> Therefore: everything that is two and odd is even

The falsity of the conclusion in the categorical is not due to an invalid composition, given that what is composed here is a perfect syllogism that is self-evidently [valid]. And just as the conclusion's following [from the premises in this categorical syllogism] is immediately known to the mind, so it is in our case, I mean in the case of a composition consisting of two conditionals. The problem here lies in the matter [of the syllogism]: it is putting two contradictories together as a subject term [in the categorical version]. So likewise in our case it is putting them together in the antecedent. Just as someone who accepts the syllogism composed of the two categorical premises is necessarily forced to accept the conclusion in that case, so one may capture the impossibility in the [categorical] case by saying that the subject containing two contradictories is necessarily non-existent. Hence, no affirmative judgment of it is true, and it is not true in actual fact (*fī nafs al-amr*). Likewise, the impossibility may be captured in the [conditional] example by saying that the antecedent[21] of the minor premise, on account of its containing two contradictories, does

21 Reading *muqaddim*.

not imply anything at all, inasmuch as it is contradictory and necessarily [215] non-existent. When we stipulate that it does exist, though, it may imply a contradiction, either on its own or when occurring in composition with something else."

Now that this has been laid out, *we say*: the answer to what the author of the book [i.e., al-Abharī] mentioned, namely that the implication in the major premise may not remain valid once the antecedent of the minor premise is assumed to be the case, is as follows. If the middle term in the minor premise (I mean the consequent implied by the antecedent of the minor premise) occurs just as it occurs in the major premise, as following from the major term, then nothing changes with the implication upon assuming the implication of the minor premise. For on this assumption, what is implied follows precisely in the way required by the second implication, and the second implication on this assumption is in its meaning a part of what follows (*juz' min al-lāzim*). It would be absurd that there should remain a *part* of what follows and is required [due to the implication], when [*all of*] what follows is eliminated, on the assumption of that from which it follows. If however they occur in different ways, then the composition is no longer a syllogism because there is no middle term that repeated in it as just the same [in both premises]. The invalidity derives from this, not from what the author of the book said in his consideration of the aforementioned example.

The "two" that is mentioned in the minor premise of the composition is the "two" of which it is true that it is odd, not the "two" whose nature it is to be "two-ness" in actual fact (*fī nafs al-amr*), and which cannot be odd. Now, what follows from this [first, counterfactual] "two" is oddness, or that oddness is true of it, by which I mean that the [oddness] which is mentioned in the consequent of the minor premise can only mean that which holds true of [two] insofar as it is odd. [By contrast,] the "two" that is mentioned in the *major* premise is such that this very same meaning is false for it. If you admit [the major premise nonetheless], the false conclusion has to follow. If on the other hand [the "two" in the major premise] is such as it is in actual fact (*fī nafs al-amr*), then the major is true. But then the conclusion does not follow, because the "two" that occurs in the position of the middle term is repeated in the two mentioned premises only as an utterance, but not as [having the same] meaning.

[T20] Al-Kātibī, *Shamsiyya*, 227.23–229.10 [trans. Sprenger, mod.]

[*the five kinds of hypothetical syllogisms*]

The first kind is composed of connective premises. The norm (*maṭbūʿ*) for this class is a syllogism in which the two premises have a complete part in common; [syllogisms of this kind] are classed under the four figures. For if the middle term [i.e. the part shared in common] is the consequent in the minor premise and the antecedent in the major, we have the first figure. If it is the consequent in both, we have the second. If it is the antecedent in both, we have the third figure. If it is the antecedent in [228] the minor premise and the consequent in the major, we have the fourth figure. The conditions of arriving at conclusions, the number of moods and the quantity and quality of the conclusion of every figure are exactly the same as in the categorical [syllogisms]. Example of the first mood: "whenever A is B, C is D, and whenever C is D, E is Z, thus whenever A is B, E is Z."

The second kind is composed of two disjunctive premises. The norm for this class is a syllogism in which the two premises do *not* have a complete part in common, as in: "always, either every A is B or every C is D; and always, either every D is E, or every D is Z; thus, either every A is B or every C is E or every D is Z." [...]

[228.11] *The third kind* is composed of a categorical and a connective premise. The norm of this class is a syllogism in which the categorical proposition is the major and has a term in common with the consequent of the connective [minor]. The conclusion of the syllogism is a connective proposition, the antecedent of which is the antecedent of the connective premise, and the consequent is the conclusion of the composition between the consequent [in the minor] and the categorical [premise]. For example, "whenever A is B, C is D; furthermore, D is E; therefore, whenever A is B, every C is E." [...]

[228.16] *The fourth kind* is composed of a categorical and a disjunctive premise, and it is of two types.

(a) The number of categorical propositions may be the same as the number of the parts of the disjunction, such that each categorical proposition has one term in common with the parts of the disjunction. The composition may then be either identical with that in the conclusion, or it may be different. Example of a case in which the composition is identical: "every C is either B or D or E; every B is T; every D is T; and every E is T; therefore every C is T," because

the parts of the disjunction [B, D, E] are true of that term of the categorical premises [T] which is shared with the disjunctive premise. Example in which there is a difference of composition in the conclusion: "every C is either B or D or E; every B is G; every D is T; and every E is Z; therefore every C is either G or T or Z," for the reasons just mentioned.

(b) If there are fewer categorical propositions than there are parts of the disjunction, then let there be a categorical proposition that has just one part, and a disjunctive one that has two parts, where the categorical proposition shares a term in common with the latter. Like this: "either, every A is T, or every C is B; but every B is D; hence it follows that either every A is T, or every C is D." [...]

[229] *The fifth kind* is composed of a connective and a disjunctive proposition, and the two premises have either a complete part in common or an incomplete part. Either way, the norm is that the connective proposition forms the minor and the disjunctive an affirmative major. Example of the first case [with a complete part in common]: "whenever A is B, then C is D; always, either every C is D, or every E is Z, but not both; therefore always, either A is B or E is Z, but not both. [...]" [229.8] Example of the second case [with an incomplete part in common]: "whenever A is B, every C is D, and always either every D is E, or D is Z, but not both; therefore, whenever A is B, either every C or E, or D is Z."

[T21] Al-Kātibī, *Jāmiʿ al-daqā'iq*, fol. 82r3–9

[***combinatorial hypothetical syllogisms***]

Know that hypothetical propositions may be studied as an end [in themselves] and in their own right, or insofar as they are premises for other hypothetical or categorical propositions which are studied in themselves and for the former reason. The categorical syllogisms are deduced from the hypothetical syllogisms, as you will learn.

Now that you understand this, *we say*: the combinatorial hypothetical syllogisms are of five types. For what is meant by combinatorial syllogisms is that which is composed either purely from hypothetical premises, or from those and from categorical premises. The types are by necessity [five], as we mentioned, because they may be either composed of two conditionals, or of two disjunctions, or of a categorical and a conditional, or of a categorical and a disjunction, or of a conditional and a disjunction. The four figures all apply to each of them, as will be verified. The discourse about them necessarily falls into five investigations.

[T22] Al-Kātibī, *Munaṣṣaṣ*, fol. 133^{r}1–11

[*criticism of al-Rāzī's remarks concerning the productivity of wholly hypothetical syllogisms only if the premises share a complete part*]

He [*al-Rāzī*] *said*: "Third Division on Hypothetical Syllogisms, in five parts. The first part deals with a composition of two conditional premises. The conclusion produced from them is whatever is the shared element in the complete part," etc.

I say: by "hypothetical syllogisms" are meant [syllogisms] composed of hypothetical premises alone, or of those and of categorical premises. Necessarily, there are five parts [in this division]. The first deals with compositions with two conditionals. His statement "the conclusion produced from them is whatever is the shared element in the complete part," is rejected. For the shared element produces a conclusion regardless whether it is in the *complete* part of the premises (that is, within one of the two terms in each of the two [premises]) or in an *incomplete* part of them (that is, is within one of the two terms in each of the two [premises]), or in a complete part in just one of the premises and in an incomplete part in the other (that is, in one of the terms in one of the premises, while being one of the terms in the other).

The four figures all apply in all three cases. As for the first case, if the shared element in it is the consequent in the minor premise and is the antecedent in the major premise, then this is the first figure. For example:

> Whenever A is B, then J is D
> Whenever J is D, then H is Z
> Therefore: whenever A is B, then H is Z

[…] [133^{r}15] And the productivity conditions, the number of moods, and the conclusions in quality and quantity are the same as in categorical syllogisms, without any difference.

[T23] Al-Urmawī, *Maṭāliʿ*, 81.15–83.6

[*the productivity conditions of wholly hypothetical syllogisms with mixed coincidental and non-coincidental conditional premises*]

If the middle term is a complete part in each of the [premises], then the four figures all apply, because if the middle term is the consequent of the minor

premise and the antecedent of the major premise, then this is the first figure. If it is the other way around, then it is the fourth figure. If it is the consequent in both, then it is the second figure, and if it is the antecedent in both, it is the third figure. [82] The productivity conditions, number of moods, modalities of the conclusion, and proofs for productivity in cases that are not self-evident, are in each figure the same as in the categorical syllogisms, regardless whether the syllogism is composed from non-coincidental (*luzūmī*) conditionals or of coincidental (*ittifāqī*) ones, as far as the account of its syllogistic implication is concerned. As for those [syllogisms] that are mixes between coincidental and non-coincidental conditionals, we need to draw some distinctions.

We say then that a negative conclusion requires the middle term to be the consequent in an affirmative, non-coincidental [conditional premise], whereas an affirmative conclusion requires it to be the antecedent in [such a premise]. Furthermore, [for affirmative conclusions the middle term] has to be (a) the consequent for the minor term in a coincidental [premise]; otherwise, it has to be either (b) the antecedent for the major term in it; or (c) the coincidental premise has to be specific (*khāṣṣa*).[22]

(a) Concerning the first, this is because if there is no correspondence between what implies (*malzūm*) and the way things are, it does not follow that there is no correspondence between what is implied (*lāzim*) and the way things are. Rather, from the lack of correspondence in what is implied, it follows that there is no correspondence with what implies.[23]

22 In cases where the truth of the coincidental consequent is necessary and its antecedent could be either true or false, the conditional is called either general (*ʿāmma*) or specific (*khāṣṣa*). It is called "general" if the truth of the consequent permits stipulating the antecedent; it is called "specific" if the truth of the consequent permits the truth of the antecedent. While a true *general* coincidental conditional may consist of a true antecedent and a true consequent, or of a false antecedent and a true consequent, a true *specific* coincidental conditional can only consist of an antecedent and a consequent that are both true. Cf. Q. al-Taḥtānī, *Lawāmiʿ al-asrār fī Sharḥ Maṭāliʿ al-anwār*, ed. A. Raḥmānī (Tehran: 2014), vol. 2, 435–436.

23 Consider for instance "if Socrates is in the room, someone is in the room." It doesn't follow from the falsehood of the antecedent that the consequent is false (someone else could be in the room), but from the falsehood of the consequent does follow the falsehood of the antecedent (if no one is in the room, then Socrates is not there).

(b) Concerning the second, this is because it does not follow from the correspondence of what implies that what is implied has such correspondence, but it does follow from the correspondence of what is implied that what implies has such correspondence.

(c) And when the coincidental [conditional] is specific, then it necessitates the correspondence of what implies. Whereas its being general necessitates it, if the middle term is the consequent for the minor term, as in the first figure, or if it is the antecedent, as in the third figure. If it does not necessitate this, but necessitates the truth of the major term and its not being denied of the minor term, then it is denied of what implies, which was the middle term. But this is a contradiction. The conclusion follows the coincidental premise in quality and in generality and specificity. Otherwise, if it is general and is the major premise in the second figure or the minor in the fourth, then the conclusion is specific. You know about the necessity of the universal non-coincidental [conditional], and you must learn now that the truth of the consequent is not sufficient in the general non-coincidental [conditional]; it also needs the antecedent not to be denied. If the syllogism is composed of two coincidental conditionals, it is of no use, because knowledge of it stops with the knowledge of the major term, which when it is known is known together with everything that is the case. But it is not considered in coincidental scenarios, except those that involve scenarios that are in fact the case. As an inquiry into [wholly] coincidental [syllogisms] does not turn out to be very useful, we will not talk [about them] in what follows, except in [the section] on what implies something (*fī l-malzūmāt*).[24]

24 To illustrate Urmawī's point, consider the following example:

Whenever the void is non-existent, there is extension (non-coincidental universal conditional)

Whenever there is extension, man is rational (specific coincidental conditional)

Therefore: Whenever the void is non-existent, man is rational (specific coincidental conditional)

In hypothetical syllogisms with coincidental conditionals as premises, depending on whether the conclusion is negative or affirmative, certain productivity conditions (a)–(c) must be met. If the conclusion is affirmative, the middle term must be the consequent for the minor term in the coincidental premise, and (a), (b), or (c). If not (a) and (b), i.e. the middle term is neither the consequent of the minor term in a coincidental premise nor the antecedent for the major term in it, then the coincidental has to be specific, i.e. it has to be such that both antecedent and consequent are true to ensure productivity. Were it only a general coincidental, the antecedent could be false and thus "there is extension" would no longer imply "man is rational."

[*problem of counterfactual hypotheticals*]

[83] The Master [Avicenna] raised a doubt concerning the first figure with two non-coincidental premises: whereas it is true to say "whenever two is odd, it is a number; and whenever it is a number, it is even" it is false to say "whenever two is odd, it is even." The reply to this is that the major premise, if taken as coincidental, cannot possibly be productive, while if it is non-coincidental, it cannot possibly be true. For [two's] being even does not follow in all scenarios in which it is a number, as explained previously, and in general, being an odd number [does not follow either].

[T24] Ibn Wāṣil, *Sharḥ al-Jumal*, 157.7–158.5 [trans. El-Rouayheb, mod.]

[*rejection of al-Abharī's arguments showing wholly hypothetical syllogisms to be unproductive*]

Athīr al-Dīn [al-Abharī] and other eminent scholars rejected the productivity of syllogisms composed of two connective conditionals, saying: If we say, "Whenever A is B then J is D" and "Whenever J is D then H is Z" and we suppose these to be true, we do not concede that it follows from this that "Whenever A is B then H is Z." For it may be that "A is B" is impossible and that by supposing it to be true the major premise—which is true in fact—ceases to be true.

Response: the major premise means that whenever J is D, in all situations and states that are not incompatible with "J is D", then H is Z. It is known that "A is B" is one of the items that are not incompatible with "J is D," for "A is B" implies "J is D" and what implies is not incompatible in any way with what is implied. So "J is D" implies "H is Z" also on the supposition that "A is B." Thus, whenever "A is B" is supposed to be true, it is true that what is implied by it implies "H is Z." So it is necessary that the major premise is true whenever "A is B" is supposed to be true.

You should know that when I heard this objection [by al-Abharī] it struck me as very strong, and I became unsure of the productivity of two conditionals, and I mentioned this in my book *Hidāyat al-albāb ila tamyīz al-khaṭaʾ wa l-ṣawāb*. But then, when I considered this more closely, my soul lit upon the [above] response, and my doubts concerning the productivity of two conditionals ceased.

[T25] Al-Samarqandī, *Qisṭās*, 385.19–387.12

[***on the history of texts devoted to hypothetical syllogisms***]

Hypothetical propositions may be empirical (*fiṭrī*), and they may be sought through demonstration. Since deducing hypotheticals from categoricals is a difficult matter, the logician must study hypothetical syllogisms. [387] The [first] teacher [i.e. Aristotle] did not mention them in his instruction (*fī l-taʿlīm*) [on logic]. Some have claimed that he left them out simply because knowledge of them can be based on knowledge of categorical syllogisms. But this is nonsense, because there are many distinctions and differences regarding the properties (*aḥkām*) of each of them. The Master [Avicenna] said [see T3] that perhaps the teacher [Aristotle] did discuss them, but [these discussions] were not translated into Arabic. The author of *al-Kitāb al-Muʿtabar* [i.e. Abū al-Barakāt, see T8] disputes this, on the grounds that if he had discussed them, then this would have been translated into Arabic. But this isn't necessarily so (*ghayr lāzim*).

The Master [Avicenna] said in the *Shifāʾ* [see T2]: "after studying this topic over a period of eighteen years, and having worked it out, we came across a book on hypotheticals ascribed to the eminent al-Fārābī. However, it appears to be spurious, because it was of very little clarity and full of errors and weaknesses concerning the proofs; [the author] made sterile what is productive and what is productive sterile."[25]

I say: perhaps the teacher [Aristotle] only omitted them because they are not certain in terms of their productivity, as we will explain later, if God the exalted wishes.

[T26] Al-Samarqandī, *Qisṭās*, 389.8–19

[***on hypotheticals with impossible antecedents***]

The Master [Avicenna] raised a doubt about the first figure with two non-coincidental conditionals by saying that the following is true: "whenever two is odd, it is a number; whenever two is a number, it is even"; yet "whenever two is odd, it is even" [i.e. the conclusion of these two premises] is false. To resolve

25 Note that this text departs from the wording of [T2], though it is more or less the same in meaning.

this doubt, the Master said that the minor premise is false, because two being odd is an impossibility. This is why it can imply something absurd. [...]

[389.15] But this calls for further inquiry. For this would raise a problem for all hypotheticals whose antecedents are impossibilities, even though he openly states them to be true in many places, and in general where he said that the following is true: "whenever something is void, then it is a dimension." It is more appropriate to rule out the truth of the major universal premise, since it would imply that [two] is even in all cases where it is a number; for there are amongst all the cases some where it is odd, and this case is one that would involve denying that it is even.

[T27] Al-Ḥillī, *Marāṣid*, 280.11–281.7

[*a defense of the later logicians' position on hypotheticals with impossible antecedents*]

Most of the later logicians reject the productivity of [wholly hypothetical syllogisms] with two conditionals, because the major premise signifies the implication between the middle term and the major term in actual fact (*fī nafs al-amr*) and the minor premise requires the truth of the middle term on account of its antecedent. So that it may be that the implication in the major premise no longer remains valid once one assumes the antecedent in the minor premise. For example:

> Whenever two is odd, it is a number
> Whenever two is a number, it is even

Yet it is false that:

> Whenever two is odd, it is even

But this is wrong. For the antecedent of the major premise implies its consequent on any assumption and in any case, and in all of these [281] cases, the antecedent of the minor premise yields the conclusion immediately.

But the major premise in the example is false. The judgment that "two is even" is not true on the assumption that two is odd, and here the implication between [two's] being a number and [its] being even is false. But if one accepts the truth of the two premises, then the conclusion does follow; the falsity of the con-

clusion together with the falsity of some of the premises does not make the inference invalid. Its just like saying, with categorical propositions:

> Everything that is two and is odd is two
> Every two is even

Yet it is false that

> Everything that is two and is odd, is even.

The root [of the problem] is that when the antecedent of the minor premise contains contradictories, then it is necessarily null and void (*kāna wājib al-ʿadam*). Nothing follows from an absurdity; but once it is stipulated, then it may imply an absurdity.

[T28] Al-Ḥillī, *Asrār*, 141.5–142.1

[***hypotheticals with impossible antecedents***]

You should know that if the two premises [of a syllogism] are non-coincidental [conditionals], then so is the conclusion.

A question arises here: the syllogism is only productive if the major premise is true on the assumption of the antecedent of the minor premise. When we say, "whenever A is B, then J is D; and whenever J is D, then H is Z," then J being D implies H being Z in actual fact (*fī nafs al-amr*), not on the assumption of the antecedent in the minor premise. But the conclusion obtains only in the second case. Yet this cannot be, as it is true to say, "whenever this color is black and white, then it is black; whenever it is black, it is not white." [Which would yield the absurd conclusion, "whenever this color is black and white, it is not white."]

Response: if the middle term occurs in both premises with the same meaning, then the conclusion follows necessarily. Otherwise, there is no shared element in the way you posed the example. For "black" is taken in the major premise in the sense of being contrary to the major term [sc. "white"], but in the minor premise in the sense of being conjoined to it. Hence, the implication of the major premise is no longer valid once the minor term is assumed. But when the middle term is the same in all respects, then the conclusion does follow.

Question: it is true to say "whenever two is odd, then it is a number; whenever it is a number, it is even," even though the conclusion [sc. "whenever two is odd, it is even"] is false.

Reply: if the claim (*taʿn*) in the major premise is taken to be non-coincidental, then one of the things stipulated [by it] is that the number [142] is odd, in which case it is ruled out that evenness is implied. But if it is taken to be coincidental, then the conclusion will not follow.