1. Formal Necessity

1.1 Inferential Necessity

We often use modal language to express the necessity in logical inferring, for example: “All of Anna’s sons are married. Peter[6] is a son of Anna. Therefore, necessarily, Peter is married.” Anyone who speaks like this obviously does not want to say that being married is an essential property of Peter, nor do they want to say that it was a natural necessity for him to get married. They also do not want to assume (regarding the first premise) that all of Anna’s sons are *necessarily* married. It is therefore completely inconceivable for anyone who speaks like this to mean that there is any necessity in the matter, i.e., a *material necessity*. They simply want to express the coerciveness of their logical inference. This necessity is often called “logical” or “*necessitas consequentiae*”. I would like to call it “inferential”, because this better expresses the place it occupies within the framework of the system developed here. It will become clear why.

It seems that this is the necessity we mean when we speak of necessity in everyday epistemic contexts – let us call this “epistemic necessity” (which is different from “doxastic modality”, see below). When Sherlock Holmes says that *necessarily* the gardener was the murderer, he does not mean to say that the gardener was forced to murder by the laws of nature, or even by logical-metaphysical laws, that is, that he commits the murder in question in all possible worlds. What Holmes means is that, on the basis of the premises known to him, and with the rational capacities he has, he necessarily *comes to the conclusion* that the gardener was the murderer.

Epistemic necessity in this sense – i.e., as the expressions “necessarily” and “possibly” (or “must” and “can”) are used in epistemic contexts – is to be distinguished from so-called “doxastic modality” in discussions of modal logic.[7] Doxastic modality does *not* speak of necessity and possibility in the narrower sense (although these words are often used to express

6 The use of “Peter” in my examples is, among other things, a reference to Jean-Paul Sartre.

7 Which is in turn distinct from talk of epistemic modality in linguistic discourse.

it, as are the words "must" and "can"), but instead understands the necessity operator (loosely speaking) as "one knows that …" (or "it seems that …") and the possibility operator as "one does not know (for sure) that not …". Under this interpretation of modality, one can, of course, model the logic of exactly this way of speaking (that is, of "one knows that …" and "one does not know that not …"). But this does not model our modal talk in examples such as "necessarily the gardener was the murderer", because we cannot always replace one way of speaking with the other. For example, we know that Heidelberg is on the Neckar; but we do not say in this context: "*Necessarily* Heidelberg is located on the Neckar." So, when we say that necessarily the gardener was the murderer, that does not mean simply that we know that the gardener was the murderer, but rather that we have come to this knowledge by valid inference.

Doxastic modality can be modeled in PWS as follows: We form the set of all possible worlds that are consistent with what "one" knows (whoever "one" may be), or what a particular person knows. Then what is the case in all these worlds is doxastically necessary (with respect to "us" or the person in question) and what is the case in at least one of them is doxastically possible. Consequently, this results in the speech practice just criticized: If one knows that Heidelberg is on the Neckar, then *necessarily* Heidelberg is on the Neckar, because this is the case in all the possible worlds that have been selected.

However, this misunderstanding clarifies a point that will be important later: If we select a set of possible worlds under the criterion of the existence of a certain state of affairs (or a state of affairs that includes this state of affairs as a partial state of affairs) within them, then this state of affairs is trivially "necessary" in the sense of the quantification over this selection. This means that such "necessity" can have no meaning at all, because according to this schema, any (possible) state of affairs is necessary. The same applies, of course, if we make a selection of possible worlds under the criterion of the truth of the premises of a certain inference in it – for example, if we select all worlds in which it is true that all of Anna's sons are married, *and* in which Peter is a son of Anna, then trivially, the conclusion also is "necessarily" true regarding these worlds, namely that Peter is married. But this "necessity" expresses nothing at all, neither logical necessity nor inferential, epistemic, or any other necessity. It is simply a consequence of the fact that with regard to the selected worlds – trivially – the premises are necessarily true, and that if all the premises are true, then necessarily the conclusion is not only true, but necessarily true (that is, according to the so-called axiom schema K: If, necessarily: A implies B, then necessarily-A implies necessarily-B).

This is important because in what follows I want to present a type of necessity that, formulated in PWS, in fact has to do with making a selection of possible worlds according to the criterion of the existence of a certain state of affairs in them. Such an approach only makes sense if the necessity that we are talking about is not already given by this selection alone, but instead only arises from it together with something else, typically a *nomologically* necessary premise.

Colloquial epistemic necessity (which is different from doxastic necessity) can be called by this name – in contrast to other forms of inferential necessity, if one wants to name it specifically at all – because it is bracketed by the clause "as far as we know …". But if we look more closely, this clause does not actually bracket the modality in question, but rather the premises (as nonmodal, i.e., as purely actual affirmations) or the inferential transition from these to the conclusion.[8] This bracketing, consequently, is also inherited by the conclusion. However, the fact that we speak of necessity and possibility in the context of what we know has nothing to do with "as far as we know …" as such, but rather with the fact that *within* such epistemic contexts we draw logical conclusions.[9] For example, if a witness directly observed the gardener committing the murder, then they will not say: "Necessarily the gardener is the murderer",

8 The subjective limitation of our knowledge may not only lie in the content we know (because we are not omniscient), but also in our inferential capabilities: we are not (yet) able to understand all the implications of what we (already) know. This latter is very evident in mathematics: we have not yet found all proofs that there are that are based on the axioms we already know (or have established, depending on one's metamathematical beliefs).

9 On the contrary, we colloquially cancel the formula "as far as we know …" precisely when we have reached a clear conclusion. We say: "As far as we know, either the maid or the lawyer or the gardener could have been the murderer." Obviously this only applies "as far as we know", because *in fact* it was exactly one of the three, and the other two cannot have been the murderer. However, once we have come to the conclusion, through a process of eliminating possibilities, that the gardener must have been the murderer, we typically no longer say: "as far as we know …", but simply affirm: "The gardener must have been the murderer."

but will simply say: "The gardener is the murderer."[10] They have no reason to speak modally because they have not arrived at their knowledge through an inference. Of course, the formula "as far as we know …" has to do with the use of modal language in that, in cases like the one presented, we rely on inferential procedures precisely because our direct knowledge of the facts is limited. But the modal character of the relevant statements still stems from the process or the pragmatics of inference. If our knowledge of a fact is limited, but we have not arrived at it through inference, then we use the formula "as far as I know …" without modal expressions: "As far as I know, Peter is married to Mary." So I heard from a friend a long time ago. This information may have been unreliable; Peter and Mary may have since divorced, etc.

If "it is necessary that …" in its inferential sense means: "it unequivocally follows from a valid inference that …", then, in their inferential senses, "it is not necessary/it may not be the case that …" mean: "it does not clearly follow from a valid inference that …", and "it is possible that …" means: "it does not follow from a valid inference that not …". In the case of epistemic possibility, this fact is still bracketed by "as far as we know …". "The Goldbach conjecture is possibly true" in its epistemic sense means: "As far as we know, it is not the case that it unequivocally follows from a valid inference that the Goldbach conjecture is not true",[11] or more simply: "We do not know of any proof that the Goldbach conjecture is false." That is, we in fact talk about the *necessity of the inference*, which *as far as we know* is not given, precisely in the sense of inferential possibility.

It seems like epistemic possibility is also what we have in mind when we talk about *theoretical* possibility, as in the following example: If an agnostic admits the possibility of God's existence, then (if they are consistent) they do not admit the *material* possibility of God's existence, i.e., God's existence in some possible world. Rather, they only admit its

10 In a sense, in this context the expression "The gardener was the murderer" is stronger than the expression "Necessarily the gardener was the murderer". The eyewitness's testimony carries more weight than the detective's conclusion.

11 In this case, the constraint "as far as we know …" does not bracket the premises, but rather the *whole* of the premises and the possible conclusion: we do not know of any inferential link from the premises (which we may very well know) to the falsity of the Goldbach conjecture. However, this bracketing does not affect the mechanics of inferential necessity.

theoretical possibility. Because if God exists, this must be a necessarily existent being. It cannot be the case that God exists only in *some* possible worlds (eventually including our own) and not in others. (This could only be the case, if at all, with regard to nonuniversal, nonabsolute gods, maybe in the context of some form of polytheism.) However, as usually understood (i.e., leaving aside questions of modal access and other problems, which, intuitively, should have no bearing on this question), what is *possibly* necessary, is *in fact* necessary, because, in PWS, "x possibly exists necessarily" means: "In some world there exists x, which exists in all worlds." Therefore, when we talk in ordinary language of the possibility of God's existence, we typically talk about a form of possibility which has nothing to do with possible worlds. And this is exactly what characterizes inferential possibility. If we say: "Possibly God exists", then this means that there is no proof of God's nonexistence. Notice that this sentence can be stated in an epistemic context, meaning: "We do not know of any proof of God's nonexistence." However, it may also be the case that there is, *in fact*, no proof of God's existence, and no proof of God's nonexistence either; at least that is what some philosophers like Immanuel Kant have argued. Of course, this is a strong thesis to defend (perhaps even stronger than the thesis of God's existence, or of God's nonexistence): "There is, *in fact*, no proof of either the existence or the nonexistence of God." But, as far as we know, this thesis is (again) theoretically possible: There is (*pace* Kant) no proof of the provability or unprovability of God's existence or nonexistence, at least none that we know to be valid with sufficient certainty.

Epistemic necessity and possibility look subjective, because they have to do with our knowledge or lack thereof. This is correct insofar as this necessity is not an objective one. When Holmes says: "Necessarily the gardener was the murderer", it is not *factually* necessary that the gardener committed the murder. (Traditionally this difference is formulated as that between *necessitas consequentiae*, which I have already briefly mentioned, and *necessitas consequentis*. More on this below.) This nonobjectivity or "irreality" of epistemic necessity becomes even clearer in the case of statements of possibility that later turn out to be false: "As far as we know, the lawyer could also have committed the murder", says Holmes. He then finds out that the lawyer was already on a ship bound for South Africa at the time in question. "The lawyer could not have been the murderer", he concludes. Of course, only the epistemic modal situation has changed, not the factual situation.

But if it is true that epistemic necessity essentially represents inferential necessity, then this *necessity* is itself not subject-dependent at all. Only the epistemic bracketing of the premises or of the inferential nexus (i.e., one's knowledge of that nexus) is subjective, and this is then inherited by the conclusion or, in the case of epistemic possibility, one's lack of knowledge of a conclusion to its negation. But inferential necessity does not concern premises and a conclusion as such, but rather the passage from the former to the latter. However, the necessity in the steps of the inference is entirely objective. It has nothing to do with any subjective practice as such (and of course not with any empirical mental conditions or mental processes – inferential necessity is not some kind of "mental necessity"), even if we typically know it from our subjective practice of reasoning on the basis of premises that are known to us. Inferential necessity concerns the "transition as such" from the premises to the conclusion. As such, and as will be shown below, inferential necessity is the fundamental form of necessity – at least, viewed from the perspective of abstraction, or from the point of view of formality. All other types of necessity must presuppose it, from the perspective of abstraction.

1.2 Axiomatic necessity

We often use the word "necessary" to express the way in which the ultimate, general principles that we presuppose in our reasoning are valid. We may call this necessity "axiomatic" (if we understand the term "axiom" broadly). For example, we say that everything is necessarily identical with itself. And we say that bodies with mass necessarily attract each other. Now, there are various contexts in which we use modal expressions: logic, metaphysics, physics, morality, the legislation of a particular country, the rules of a particular game, and so on. With regard to all of these nexuses we speak, *de re*, of specific axiomatic necessity.

In the context of PWS, the *original* meaning of axiomatic necessity is not intramundane necessity (i.e., material necessity), but world-determining or "world-selecting" necessity. It tells us what kind of world we are in – or what kind of country we are in, what kind of game we are playing, etc. If we understand statements that are intended to indicate axiomatic necessity in the sense of intramundane necessity, then they become tautologically empty – just as empty as the statement: "Heidelberg is necessarily located on the Neckar", where this statement is based on the selection of those possible worlds in which Heidelberg is located on the Neckar. Correspondingly, when we say: "All massive bodies that are together in space necessarily attract one another", we obviously do not mean to say that this is the case in all possible worlds, i.e., in all logically possible worlds, because the laws of nature (as far as we know) are not logically necessary. So in order to express what we want to say in PWS, we must specify: "In all nomologically possible worlds …", that is: "In all worlds in which the same laws of nature are valid as in our world, all massive bodies that are together in space attract one another." However, nomologically possible worlds are defined, among other things, by the fact that the law of gravity is valid in them. It is therefore unsurprising that no states of affairs which contradict this law can be found in such a world.[12] The statement that the law of gravity is valid in all nomologically possible worlds is trivially analytic: it explicates the term "nomologically possible world".[13]

This perhaps becomes even clearer when we consider axiomatic necessity in the context of games. We can say, e.g., that in a game of chess, the rooks necessarily move exclusively orthogonally. Now, the players can change this rule and agree that in their next game the towers can also

12 I do not want to go into the question of the possibility of miracles here. My short answer would be that if miracles are possible in a world, then this is part of its nomological setup. In this case not only the laws of nature are valid, but also, e.g., the law that God can act against the laws of nature.

13 By "in the trivial sense", I want to say that the tautology is "direct", as in the case of "All bachelors are unmarried" or "All triangles have three angles": the predicate concept is directly contained in the definition of the subject concept. This specification is necessary because many believe that the propositions of mathematics and logic are analytic throughout. But, of course, many of those propositions are analytic in a nontrivial sense, because it is not directly contained in the definition of a triangle, e.g., that the sum of its angles (in Euclidean space) is always 180°. Of course, it may be disputed what a definition directly contains, and what it indirectly contains or implies – and whether triangles cannot also be defined directly by the sum of their angles. But it seems to me that this distinction is sufficiently clear for our current argumentative context.

move diagonally. But then, we will say, they are no longer playing chess, because the game of chess is constituted by its rules of play.

Sentences that speak of necessity in this sense are nontrivial in *definitions*: In a chess manual, the rule as to how rooks are allowed to move is nontrivial. Sentences like "The law of gravity is valid in all nomologically possible worlds" are not definitions, but they are not trivial in the corresponding typical linguistic contexts because – unlike in games – we do not know the laws of nature from the outset, nor can we find them in game instructions. Rather, we must find out about them. We do not yet know all that the concept of a "nomologically possible world" encompasses.[14] Physics has discovered a lot about this but is far from having concluded its research on the laws of nature. Such sentences are nontrivial precisely when they are understood as explicating the concept of a "nomologically possible world", that is, as a (partial) explication of what such a world is. They are *then* tautologically trivial (but by no means superfluous, as we will see) if they are understood as statements about intramundanely necessary states of affairs, because in order to explicate this necessity we must use a concept that already contains the determination of these states of necessity: the concept of (consistent) nomological determination.

The Systematic Indispensability of Formal Necessity

Due to the triviality of axiomatic necessity on the one hand, and the apparent subjectivity of inferential necessity on the other hand, as well as the formality of both, it may seem sensible at first glance not to

14 We can therefore call the concept that we (currently) have of nomologically possible worlds, or of the (total) laws of nature, a "preliminary concept". We typically form a preliminary concept through indexical reference: "the concept of the totality of those natural laws that apply in *this world of ours*"; "the natural kind of animal that is crawling around on the ground *in front of me*". Of course, we do not *know* these concepts yet (we are not yet "acquainted" with them), but they are clearly "fixed" through the respective indexical reference. Evidently, many of the concepts that we use every day are preliminary concepts in our (current) usage of them, i.e., we do not yet know them completely and do not know what (exactly) they do and do not include or imply.

consider these two as forms of necessity in the strict sense, because they do not represent necessity at all with regard to “the matter” or “things in themselves” – i.e., they represent no material necessity. In the case of what I called “axiomatic necessity”, talking about necessity can even lead to misunderstandings, because we can certainly discuss the objective, material necessity of natural laws, be it of individual laws or the laws as a whole – e.g., whether it is metaphysically or even logically necessary that the law of gravity is valid. (In this case the *material necessity of the law of gravity itself* is discussed, with regard to *all* possible worlds.) Therefore it may seem appropriate, at first glance, to reduce talk of necessity to talk of validity (or truth) in the two cases described. In our previous example it is *true* (as far as we know) that the gardener was the murderer; and the laws of nature are *valid* in our world (which in PWS is equivalent to: our world is one of the nomologically possible worlds).

As long as this reduction is supposed to be a purely terminological decision, there is of course nothing wrong with it. But the *validity* of logic, and of the laws of nature, is directly connected with the *necessity* that we unquestionably understand as such, that is, with intramundane, material necessity. The validity of logic and natural laws manifests as a necessity *within* our world. The so-called Necessitation Rule expresses this connection when understood broadly. Narrowly understood, this rule states that if a statement *p* is logically derivable, then *necessarily p* is also logically derivable. Broadly understood, it states this connection in relation not only to logical derivability, but also to derivability from axioms in the sense presented (whereby the rule trivially also necessitates the axioms themselves).

However, this rule is required because special laws such as the laws of nature often function as premises for conclusions about the necessity of individual states of affairs within the world, that is, according to the axiom schema K: “$\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$”, which underlies all so-called normal systems of modal logic. But if these premises were not necessary, then we could not infer necessity from them. However, we do this all the time, every day. We say, for example, that when an atom splits, it will necessarily emit a certain amount of radiation. We do not just want to say that it will *actually* (in the actual world) do this, because, purely theoretically, it could actually emit radiation by chance. Nor do we just want to say that it will *actually always* do this, because that too could be a matter of coincidence. (Perhaps it is always the case in our world, but not in all nomologically possible worlds.) What we want to say is that an atom *necessarily* emits radiation when it splits. And we could not do so (according to K) unless all the premises that we implicitly draw on in our conclusion were (materially) necessary, that is, the premises stating the laws of nuclear force and those stating the relevant essential properties of the atom.

We must therefore insist that the principles that determine the world, such as the laws of nature, have material necessity *within the world* (we must therefore adhere to the Necessitation Rule) – as is quite inevitable when the corresponding world is formally determined by this principle. But we cannot *reduce* the validity of such axioms to material necessity, because then it loses its meaning. (It would then only be "necessary" in the sense in which Heidelberg is "necessarily" on the Neckar in our above example.) The fact that axiomatic necessity and therefore *a fortiori* also inferential necessity is presupposed by material necessity, and cannot be reduced to it, can be seen as follows: The set of possible worlds in which a certain natural law is valid is not identical with the set of possible worlds in which no state of affairs occurs that contradicts this law of nature, because there are possible worlds in which a certain law of nature is not valid, but in which, as it happens, no states of affairs occur that violate this law. Otherwise, talk of particular laws such as the laws of nature would be completely trivial, because it could then only mean that in all worlds in which no states of affairs occur that do not conform to the law in question, no states of affairs occur that do not conform to that law. (The necessity of natural laws would thus reduce completely to "Heidelberg necessity".) According to this latter schema, we can form the set B of all worlds in which there is no cow that is not called Berta, and then say that in all B-possible worlds it is necessary that all cows are named Berta. The selection of those possible worlds that constitute the set of nomologically possible worlds (or other types of possible worlds) cannot be determined by the (nonlawlike) contents of this world. It must instead be determined by the *validity* or *formal necessity* of the relevant laws. (The selection cannot be determined extensionally. It must be determined intensionally.) This validity is formulated via the use of modal speech according to §1.2 – but not by formulating the necessity of the laws of nature as intramundane necessity.

If a set of worlds is constituted on the basis of axiomatic necessity, then – as I have said – we can of course also talk about the intramundane occurrence of the correspondent lawlike state of affairs regarding these worlds – for example about the fact that in all nomologically possible worlds, all massive bodies attract each other. Although this is trivial in itself, it can make sense in the context of an inference. Nevertheless, there remains a fundamental difference as to whether we talk about the necessary *existence of this state of affairs* within the world (that is, about its occurrence in all nomologically possible worlds), or instead about the *validity of the law of gravity* for all nomologically possible worlds – that is, whether we are talking about material or formal necessity. On the other hand, however, it seems sensible to speak of *necessity* in ordinary language in both cases, because logical and nomological validity are directly connected to intramundane necessity via the Necessitation Rule. Loosely speaking, the talk of formal necessity is justified if we read the Necessitation Rule backwards, that is, in such a way that inferential or axiomatic validity is characterized by the fact that it is necessary within the world. However, it seems that neither inferential nor axiomatic validity can be characterized (in this respect) in any other way than by resorting to the Necessitation Rule. More precisely, inferential or axiomatic validity is characterized by *outright* universality (whereby outright universality can be restricted, e.g., to all nomologically possible worlds). In theory, we can consider actuality without modality, e.g., we can discuss nonmodal logic. Then the outright universality of this validity is of course not represented modally, but only as actual. But as soon as we expand our nonmodal logic into a modal logic, this validity *qua* outright universal automatically also extends to this expansion; so to speak, to the space of determination that is opened up by this expansion. And this is exactly what the Necessitation Rule expresses. Logical space is the space of the possible (in PWS: of all possible worlds). The validity of logic in the entire logical space (which the Necessitation Rule articulates) is therefore trivial, but it still characterizes logic decisively: as that which has trivial or *eo ipso* necessary validity. (An analogous point applies to natural laws and other axioms when restricted to the appropriate selection of possible worlds.) And that is why it makes sense to articulate the (specific type of) validity of logic – including the validity of specific axiomatic systems – as formal *necessity*.

We can also say that this usage of language expresses, so to speak, the "necessity" of the Necessitation Rule itself: "It must be the case that this rule applies" – formal necessity must precipitate materially. Consequently, if formal *validity* precipitates materially as *necessity*, then it must also be articulated *formally* as necessity or necessary validity. Because if (in any theory) the space of the modal is opened up, then the connection between validity and the scope of validity is no longer directly given, because there are now different possible scopes (in PWS: different possible worlds) in which something may or may not apply. If there are *different* possible scopes of validity, then this *eo ipso* opens up the (theoretical) possibility of limited validity: of validity not in relation to all possible scopes of validity (in PWS: all possible worlds), but only to some. Therefore it is of particular significance when some specific validity, in fact, applies to all possible scopes of validity, and this particular significance is articulated by formal necessity. Therefore it makes sense to say that logic is *necessarily valid*, and not just that it is valid (or just *universally* valid).

But it also makes sense to speak of necessity with regard to validity that does not simply exist with regard to *all* possible scopes of validity (i.e., logical and mathematical validity), but is rather a case of merely axiomatic validity (such as that of the laws of nature), because we also speak of validity in relation to limited scopes within the world we talk about – for example, about the validity of a country's laws or of the rules of a game. The *necessity* in, e.g., "nomological-axiomatic necessity" states that the validity of the axioms in question refers to our world as a whole, namely it constitutes this world: the laws of nature determine, among other things, what kind of world our world is (as such). This means that we cannot exit the scope of this validity, unlike the scope of the laws of a specific country or the rules of a specific game, because we would have to exit our world to do so.

This relationship can then be further generalized or relativized: It *always* makes sense to speak of axiomatic necessity when the area of validity that it constitutes (possibly) gives room for limited, particular validity (that is, within this area) – and not only when this former area represents a world, or our world. For example, a certain field of music is constituted by the validity of the diatonic-chromatic-enharmonic tonal system. If a musician plays a note outside of this system within this field, then he is playing a wrong note. But within this field there are many subfields (classical music, jazz, pop, etc.), each of which has additional, specific rules. To put it the other way around: When we speak of particular validity, we implicitly always presuppose general validity, that is, the validity that constitutes the space of our speech, that is, of (the givenness of) what we talk about. Because without such general validity (and hence without any grammar) we could not speak at all, and we could speak about nothing at all (for it is impossible to speak about something that is completely disordered). This *possibility-conditioning* validity fundamentally and always has a different status than the validity that may show up as a particular one within a specific field, or the validity that we may be talking *about* within this field. As I have said, this distinction is entirely a relative one: What is possibility-conditioning validity in a certain context may simply be particular validity in another, more comprehensive context. But this relativity does not abolish the fundamentality of the distinction as such, because whenever we speak of validity, we must already assume validity, which makes our speaking possible in the first place.[15]

That is, intramundane, material necessity always presupposes formal or transcendental[16] necessity because it can only articulate itself on this basis. But conversely, formal necessity derives its meaning *as* necessity solely from material necessity. Without this it does not represent necessity, but merely validity. But this validity remains empty as long as it is not validity *for* something. And in this "being valid *for*", validity presents itself as necessity. Therefore it is no contradiction if, although formal necessity is presupposed by material necessity as a condition of its possibility, the latter (or even a particular form of the latter) may represent the original type of necessity in the ontological sense (as I would like to argue myself, although not in this book). The formal necessity of logic and world-constitutive laws originally, purely on its own, does not represent necessity at all, but rather empty validity.

15 This is why we must ultimately base axiomatic formal necessity on inferential formal necessity: without any given (or pragmatically functional) determination and order we cannot even formulate axioms.

16 Here in the "generalized" sense, that is, not only in the sense of a "condition of the possibility of our cognition", but simply in the sense of a "condition of (any) possibility (at all)".

Since, according to the Necessitation Rule, the laws of logic and nature are *also* intramundane states of affairs, one could perhaps come up with the idea of considering them *solely* as intramundane states of affairs, for the sake of theoretical economy. Obviously there are laws that exist only intramundanely, such as game rules or legal systems. So why should the laws of nature, or even the laws of logic, not also be states of affairs that obtain only *in* the world, or in the worlds? For logic, the absurdity of such mundanization is quite obvious. It would turn out that logic is valid in our world just because it exists in our world. Of course, a given thing in *one* world cannot, as such, have any meaning for other worlds, except for being part of a *possible* world (which, from their perspective, is counterfactual). Such validity of logic in our world could not mean that *all* possible worlds are logically possible worlds, or that logical space is the space of all possible worlds. Of course, we could select the "logically possible worlds" according to the content criterion of the existence of the state of affairs of the validity of logic in them, and then quantify over them in order to speak of logical necessity. But then worlds could *also* be possible in which logic is not valid. Conversely, the fact that logic applies in our world would be completely coincidental. There would then be worlds in which this state of affairs exists and others in which it does not exist. Ours happens to be one of the former. However, the problem then arises that we cannot know for sure whether our world, in fact, is one of the former. We could only say: "As far as we have observed so far, the laws of logic are not violated in our world." As with the conformity to natural laws discussed above, it could now also be the case that our world only behaves in accordance with logic by chance, so that the validity of logic would not be a state of affairs in it at all. At the very least, this does not correspond to our usual intuitions about the validity of logic. Logic, as most people would intuitively say, is not just factually valid, it is necessarily valid. It is simply not possible for there to be a world in which logic is not valid.

However, this way of putting things implies a presupposition problem: If logic determines what is (logically) necessary, possible, impossible, and nonnecessary, then what determines the necessity of logic itself? To claim that logic itself is *logically* necessary is obviously circular. The answer is: logic represents the minimal requirement for determining necessity and possibility, or for the semantics of modal concepts. Without logic there is no differentiation of *possibility at all*: the negation of possibility (or necessity, etc., depending on what you want to use as your basis for modal theory); i.e., not-possible, possibly-not and not-possible-not, could no longer be distinguished from one another, because then there would simply be no more restriction on the possible. But this would make the concept of possibility meaningless. Without modality, however, semantics itself is no longer possible, because semantics requires that there is not just bivalence (e.g., what is blue is not not blue), but also quadrivalence (e.g., what is blue cannot also be yellow, but can also be square, warm, etc.). Beyond logical space there is *nothing*, or nothing definite. There can only be *something* in logical space. This is why the validity of logic cannot *just* be an intramundane fact – although, as I have said, it *also* is.

For nonlogical axiomatic necessity, the requirement of formality is not so easy to see. At first glance there seems to be no argument against the idea that the laws of nature are merely states of affairs *in* the world. To get clear about this, let us look at the example of games again. Games are *constituted* by their rules of play. Anyone who disobeys these rules will no longer play the game in question. But some games apparently have no rules. Other games allow rule violations ("cheating"); others allow their rules to be changed or reinvented within the game. Hence, apparently, we could conceive of a game in which everything is allowed, including establishing rules *in* the game. In such a game, the rules would be immanent, but not constitutive. Let us think about what such a game would look like. The rules of a game, broadly understood, include all its restrictions, including regulations about when the game begins, when it ends, and who plays. Participation in a game does not have to be based on consent. Children may, for example, let their father take part in their game without asking him, and without his knowing: he is the evil monster that you must hit with the water balloon without being discovered. So what would a game be without any restrictions? It would be the global game: everyone and everything would participate in it in any way they wanted, from time immemorial and throughout the future (including establishing new rules, or even playing other subgames within the game). Individual players could try to impose limitations on the game by introducing new rules, for example regarding time or space, or regarding its participants. But because there are no rules for introducing new rules, any other player could refuse to accept these rules and introduce other rules themselves.

This means that as a whole, this game is no longer distinguishable from the world; it is no longer possible to say specifically what is valid in this game and what is not, because there are no rules for introducing rules or breaking rules. But this means that this game loses the meaning of being a game in any distinct sense. So at least *some* constitutive rules (in the broadest sense) are required for something to be a game in any distinct sense, even if these rules themselves may allow for *immanent* rules (including the creation of such rules within the game).

Now let us consider what a *world* would look like that had no constitutive limitations. It may well have intramundane restrictions, but there would be no restrictions on the introduction, removal, scope and limitation of the scope of such restrictions. So, for example, it would be allowed to introduce a universal restriction (a universal rule), but it would also be allowed to introduce a "universe" in which this restriction does not apply. In particular, there would be no restrictions on spatial and temporal dimensions in this world. There would also be no restrictions on introducing restrictions on certain dimensions. So, for example, there could be three specific spatial dimensions within this world, together with a temporal dimension, which would be internally subject to certain natural laws and contain certain objects, but which would not have any connection with anything in the other dimensions. But this means that this completely unrestricted world would contain unlimited worlds within it, that is, all possible worlds.[17] It would therefore be identical to the space of possible worlds. But with this, it would no longer be a world.

In order for a single world not to be all possible worlds, we must demand that everything in it stands within a definite (determinate) nexus (an indefinite or indeterminate nexus would be as good as none). This may (theoretically) include the possibility that there are *particular* nexuses in a world that arise and disappear (as in the emergence of natural laws). But if this coming into being and passing away would not

17 Of course, such a world would be inconsistent if understood in the normal sense of "world" – just as a game is inconsistent when one person is allowed to introduce the rule: "All people must wear blue hats", and, at the same time, another person may introduce the rule: "No person is allowed to wear any hat at all." But this only shows that such a "game" or "world" is not a game or world in any normal sense at all, where "normal" implies the norm of consistency.

stand – or, more precisely, would not occur – within *some* definite (determinate) nexus, then it would mean as much as the coming into being of a new world, or the passing away of a given world. But the determination (or objective definition) of a general nexus is simply a law or a rule.[18] It is therefore constitutive of worlds that they are determined by formal or axiomatic necessity. One quickly realizes that logic does not suffice to guarantee a definite nexus *within* a possible world. These nexuses, being particular, must be regulated in a particular way. In other words, logical connections alone are not sufficient for possible worlds to differ from one another. However, in order for worlds to be distinguished by nexuses *within* them, there must be something that regulates these nexuses and makes them comparable, so that one can then say: "This and that does not behave in this world as it does in that other world." But this thing that gives meaning to specific nexuses in a world is (specific) axiomatic necessity. A world *must* therefore be subject to certain nomological laws: these are constitutive for something to be a world. A world without such laws has no internal nexus whatsoever, and dissolves into indeterminacy.[19]

18 Space and time alone are not sufficient for the inner nexus of a world (i.e., insufficient to "hold a world together"). It is disputable whether we can individuate space and time at all, but if we assume that this is possible, then different worlds can be located within the same space and the same temporality (temporal extension), as long as things in different worlds have no effect on one another. Moreover, it is outright impossible to distinguish the purely abstract space and the purely abstract temporality of one world from that of another (i.e., to individuate them from each other). This is different in the case of a world with relativistic spacetime, as in the general theory of relativity, because relativistic spacetime can be locally curved, and its local curvatures may differ from those of the spacetimes of other worlds. But the questions of where and when and to what extent local curvature occurs in a specific world is only decided by the natural laws that prevail in that world. Hence, even in this case, we need natural laws beyond pure geometry in order to constitute the inner nexus of a world (and its difference from those of other worlds).

19 If this does not immediately make sense to someone, then the following illustration may perhaps help: At first sight, one may think that mere logical possibility suffices for there to be, e.g., a certain body with a certain mass in a certain possible world, and a certain other body with a different mass in the same world. But having mass is completely undetermined as long as it is not determined what difference it makes, e.g., what difference the relationship of different masses to one another makes within a world (e.g., that they attract one another, e.g., in proportion to the ratio of their masses, or something like that). But this is only determined by the

The Irreducibility of Inferential Necessity to Axiomatic Necessity

We have discussed why talking about formal necessity makes sense, and why it does not make sense to reduce modal linguistic practice to talking about material necessity. But now it may seem as if, *within* the field of formal necessity, inferential necessity can still be reduced to axiomatic necessity, that is, to the *axioms* of logic. The problem, however, is that these axioms also have to be applied in turn, and general principles are applied through inference. Let us say, e.g., that "A ↔ ¬¬A" is an axiom of our propositional logic. If we want to apply this to "Socrates is a philosopher", then we must "see" that this expression can be substituted for "A" in that formula. Thus we implicitly "infer":

> A ↔ ¬¬A
> "Socrates is a philosopher" can be substituted for "A"
> ∴ Socrates is a philosopher ↔ Socrates is not not a philosopher.

Of course, we could formulate a rule that decrees in which case a linguistic expression can be substituted for "A": "Whenever a linguistic expression in English has the form xyz, it can be substituted for 'A' in 'A ↔ ¬¬A'." However, again, this rule would have to be applied. That such application is an additional step may become clearer from the fact that we can go wrong in this step. For simplicity, let us consider a mathematical example: Someone may take the rule "x × y = y × x", and apply it to "5 × 3 + 4", in the following way: "5 × 3 + 4 = 3 + 4 × 5", on the grounds that 3 + 4 = 7, and "7" can be substituted for "y". However, "5 × 3 + 4" is evidently not a case of "x × y".

General rules apply to singular cases. The application of rules cannot ultimately consist of applying a rule, namely the rule for applying rules, because then we would end up in an infinite regress of application

special laws that affect mass. Logic by itself determines that what has mass does not have no mass (and similar general, formal nexuses). However, by itself, it is not sufficient to determine the particular differences that are made by "having mass". The mass laws required for this may be different in different worlds. But without *any* such laws in a world, having mass in the respective world is meaningless.

rules: there would have to be a rule for how the application rule for rules applies, etc.[20] Hence, since an axiomatic system, by definition, is based on general rules, it must presuppose the coerciveness of the application of these rules. It cannot incorporate this coerciveness into the system by incorporating general rules for the application of its general rules – at least it cannot do so completely. Hence, for any axiomatic system, there always "remains" an inferential coerciveness or inferential necessity which it cannot account for. Therefore inferential necessity cannot be reduced to axiomatic necessity.

We can also argue starting from Tarski's theorem: According to this, formal systems that are sufficiently explanatory (to express the basic arithmetic of natural numbers) are incapable of deciding or defining their own concept of provability.[21] If we metaphorically take the axioms of a formal theory to be certain places (e.g., in the space of discourse), and take proofs to be paths to other places (which stand for theorems), then this not only means that there are places to which the path network (of the respective axiomatic system) does not provide access. It also means that it is undecided or undefined (in the respective axiomatic system) what a path even is.[22] But this means that the coerciveness or necessity of the transition from one place to the other cannot (ultimately) be

20 That is why it does not help to locate the application rule in a metalanguage. For the metalanguage requires, in turn, metalinguistic rules (relative to itself). We would end up in an infinite regress of metalanguages. But that is not all: The application rule would concern the very relationship between the rule and the regulated. So, considered in terms of semantic ascent, it would concern precisely the relationship between the metalanguage and the object language, that is, it would regulate this relationship. Hence we would not only need further metalanguages, but also an additional language that is, so to speak, orthogonal to the semantic ascent, i.e., whose *object* is semantic ascent. But then we would need a language that regulates the relationship of *this* language to the relationship between the metalanguage and the object language, etc.

21 Cf. Beeh 2003, 97.

22 Admittedly, one can conceive of a "platonic" theory of metalogic, where Logic as such is given as a system (together with all its inferential paths) independently of any axiomatic system. In this case, this Logic beyond axiomatic systems would be the logic of inferential necessity – which is different from axiomatic necessity, i.e., the distinction I want to make would still hold. The difference from what I have suggested is only that, in this case, inferential necessity would not be dynamic, but, once again, systematically fixed (as a network of given paths).

a derived one (i.e., one that can itself be proven), and must therefore be original. (This is also immediately obvious: The definition or decidability of the concept of provability implies that if a proposition p is given, then it can be decided whether p is provable. But this would require a proof or, to use our visual metaphor, a path from p to its provability. But if provability defines what a "path" is, then the givenness of the path *to* provability already presupposes this provability – or else, it presupposes a *different* concept of provability, typically in a metalanguage – which in turn is undecided or undefined.)

The originality and irreducibility of inferential necessity is, so to speak, the positive flip side of Tarski's theorem: provability is undecided or undefined for sufficiently explanatory theories within those theories. However, it is *evident* that we do move from one proposition to another in the act of inference. (Again, it is not defined or definable what "evident" means, but we can at least *pragmatically* argue that Tarski's theorem is itself a result of a transition in the act of proving, that is, that it is at least evident to us that we (can) move from one proposition to another in the act of inferencing, insofar as we accept this theorem. And if we do not want to accept it, then we must *prove* that it does not hold.) Therefore this transition must *originally* (or "underivedly") be coercive (and not due to the prior givenness of a path), and this is precisely what inferential necessity is.

From a pragmatic perspective, this means that we must attribute to ourselves as subjects the ability to apply rules or concepts *originally*, that is, without the "how" of the application being defined in turn. We *implicitly* master the application of rules and concepts, and we (ultimately or originally) cannot master them in any other way. Nobody could have taught us how to apply rules at all: this must be part of our basic transcendental equipment (or "natural" equipment, depending on one's philosophical preferences), so to speak.[23]

For the *matter itself*, however, this means that we cannot avoid attributing original necessity to the (act of) application of principles or the (act of) derivation from them – that is, attributing to them a necessity that is not derived from principles – because that very necessity would be

23 Cf. I. Kant, KdrV B 172.

required for the derivation.[24] For this reason, the name "*necessitas consequentiae*" seems unsuitable for inferential necessity, because in philosophical usage it is not just used for the (pragmatic, operational) progression in logical reasoning, but also or even primarily for the *principle* of implication; that is, insofar as it is logically valid and hence necessarily valid. *Necessitas consequentiae* is given, we say, when the implication "A → B" represents a tautology, as in: "A → ¬¬A". We can, in fact, squeeze any transition to the conclusion in logical inference into the schema "Necessarily: A → B", for example: "*Necessarily: if:* ((if Peter is in Heidelberg, then he is not not in Heidelberg) and (Peter is in Heidelberg)), *then:* Peter is not not in Heidelberg"; where the entire expression in brackets after "*if:*" stands for "A". But by representing the transition in this way, namely as a necessary or "strict" or logical implication, we no longer represent it as the genuine coerciveness of the inference as such, but rather as a *principle* of this transition (or an application of this principle), which is abstracted from the process of inferring.

24 Interestingly, Wittgenstein formulates this idea clearly in the *Tractatus* (I omit the paragraph separations): "4.121 The sentence cannot represent the logical form; it reflects itself in it. What reflects *itself* in language cannot be represented. What expresses *itself* in language, we cannot express through language. The sentence shows the logical form of reality. It exhibits this form. 4.1212 What *can* be shown *cannot* be said. 6.522 However, there is the unspeakable. The unspeakable *shows* itself, it is the mystical." In my opinion, Wittgenstein slightly oversteps the mark with this: What reflects itself in a sentence, what a sentence exhibits, or what "shows itself" in it (in the sense that something shows "itself" in a mirror), is what *the sentence itself* cannot say. But *another* sentence can certainly talk about this and can at least try to say it. However, it is true that this other sentence must then already have a logical structure, which it can only show and not state, and so on *ad infinitum*. Hence the inevitability of inferential necessity remains, but it can certainly be made explicit, although never completely and definitively (see below). The logical structure of the world (and thus also of sentences, since they belong to the world as facts) is, in my opinion, nothing mystical – although that is of course the inevitable conclusion given the premises of the *Tractatus*.

Two Pragmatic Understandings of Logic

Inferential necessity is necessity *simpliciter*, in that we must always presuppose it in all our inferring and applying. Logic (as an enterprise) can be seen as an attempt to explicate this necessity-*simpliciter*.[25] However, this explicating is done by formulating this necessity axiomatically, because we cannot articulate it discursively otherwise.[26] This then often leads to a misunderstanding about what Logic (as an enterprise) actually represents. Because if Logic represents the explication of inferential necessity, then it is the explication not of an axiomatic system, but of what makes axiomatic systems possible in the first place, that is, what makes applications of and derivations from general principles possible (subjectively formulated: the foundation of the use of rules and concepts *as such*). We can therefore distinguish between two different operational or pragmatic understandings of Logic *qua* enterprise: Logic as an attempt to explicate inferential necessity; and Logic as an attempt to freely construct formal axiomatic systems that may then fulfill certain functions. The latter may also be called "logistics" (in contrast to the former) if you want. These two enterprises do not differ in the formal appearance of their results. We cannot tell from the sheer shape of a logical-axiomatic system whether it is

---

25 This, then, explains why it makes sense to search for *Logic* with a capital "L", and not just for one of many possible logics, that is, to construct coherent formal axiomatic systems. *Logic* is (or would be) the appropriate explication of the inferential necessity which is assumed in all application and deduction. (Cf. Wittgenstein, *Tractatus* 6.1223: "It now becomes clear why one often felt as if the 'logical truths' had to be *'requested'* by us: namely, we can request them insofar as we can request a sufficient notation.") However, it may (or must) be the case that this explication will always remain imprecise and incomplete, which is why there is definitely a certain scope for ambiguity and therefore also for plurality in this undertaking. There may be, on some points, a dispute about what the correct Logic is, and in some cases, there may be no directly obvious solution to that dispute. In principle, however, this "correct" logic is what is called "classical logic".

26 Logic in general articulates the original synthetic unity (which is, at the same time, *originally synthetic*) of inferring (as an original act). Formal, axiomatic logic analytically explicates this unity. (This was already stated by Kant, KdrV B 134, annotation, albeit within the framework of his transcendental idealism which, in my opinion, is not theoretically sustainable.) However, the analytic explication of synthetic unity necessarily always remains incomplete (or, alternatively, gives rise to inconsistencies).

intended to represent a free construction or the explication of inferential necessity. But the difference in comprehension between the two undertakings is fundamental. In particular, the latter project is not allowed to assume from the outset (1) that inferential necessity can be adequately represented axiomatically at all, or (2) that such an explication can be complete and/or consistent, or (3) that it can be clearly decided which of the – possibly – different explications is the "correct" one, i.e., that one formal logical-axiomatic system could be distinguished from all the others according to internal, formal criteria. But this means that, conversely, if it turns out that all of this or at least some of this should *not* be the case, then that in no way speaks against inferential necessity or against its determinacy, consistency and coerciveness.

That is, the fact that axiomatic systems can only represent attempts to explicate inferential necessity, but not the latter itself, explains why there can be incompleteness and uncertainty in this explication and why there may be different ways of explicating it. Finally, this also explains the plurality of logics that emerged in the 20th century: If Logic is just understood as an axiomatic system and no longer as an explication of a necessity that underlies all axiomatics, then we can of course design "logical" systems as we wish – as long as they are consistent and reasonably complete. (However, in order to apply these two criteria, we must in turn assume inferential necessity.) There may be a hierarchy of axiomatic systems, e.g., a logical axiomatic system may be the basis of a nomological axiomatic system. But this hierarchical priority *within* the field of axiomatic necessity is only a relative one; only a priority within the same type of necessity, namely the axiomatic one. *Inferential* necessity, on the other hand, categorically conditions axiomatic necessity; it is presupposed by it *as such*.

Nonlogical Axiomatic Necessity

Axiomatic necessity – that is, the "authentic" one, which does not represent the explication of inferential necessity – is, in contrast to inferential necessity, not a necessity-*simpliciter*, but a particular one – or better: one that introduces particular necessity into reality (or into logical space). Because, as we have seen, the laws of nature, and even more so other laws such as the rules of chess, are not necessary *simpliciter* – it is easily conceivable that counterfactual natural laws prevail in a (logically possible) world different from ours. However, it can be observed that philosophers, especially modal logicians, are reluctant to deal with such particular nexuses of necessity and prefer to stick to logical necessity (which they generally understand as axiomatic). But in our everyday speech we rarely talk about logical necessity; we are mostly interested in necessities and possibilities that are given under particular laws, be these the laws of nature or others. Furthermore, as I have said, the fundamental difference does not lie *within* the field of axiomatic necessity, between logical-axiomatic and non-logical-axiomatic necessity, because this difference could only be a relative one. The character of the necessity of logic as *simpliciter* or absolute, and thus its difference from the particular necessity of laws of nature and other nexuses, lies in the fact that the former is (originally, in itself) not an axiomatic necessity, but *inferential* necessity, which underlies the axiomatic necessity as a condition of possibility. *Within* the field of axiomatic necessity considered on its own, there is no reason to "absolutely" privilege a particular axiomatic system by attributing necessity in the material sense to this one system alone. (This is quite unavoidably the case and therefore also applies to Logic, *if* Logic is taken to be a purely axiomatic system. Then logical pluralism is trivially correct.) For even if this one system is considered fundamental to the others, its necessity is not fundamentally different from that of the other axiomatic systems. Only the contrast between inferential and axiomatic necessity can manifest this fundamental difference. But as soon as we make the former explicit (as "logical" axiomatic necessity), we inevitably transfer it into the field of axiomatic systems and thereby relativize it.

There is therefore no good reason – contrary to the reduction of inferential necessity to axiomatic necessity considered above – to limit modal speech within the field of formal necessity to inferential necessity and to withhold the title of "necessity" from axiomatic necessity. This can also be seen from the fact that limiting our modal speech solely to logical necessity would greatly impoverish our (possible) knowledge, because with regard to the future, and to a large extent also with regard to the past, we can only gain knowledge through inferences on the basis of particular *axiomatic* necessity – as when we say that when an atom splits, it will necessarily emit radiation, that is, due to its essential properties and the laws of nature.

The Necessity of Theorems

If we combine axiomatic and inferential necessity, then we arrive at the necessity of the theorems that can be derived from the axioms – regardless of whether these axioms are logical ones, namely attempts to axiomatically articulate inferential necessity, or axioms of a different type, such as nomological ones. The necessity of theorems follows directly from inferential and axiomatic necessity, that is, it also represents *formal* necessity. Hence, if one wishes, one can count this as a third type of formal necessity; although since this type follows directly from the first two, it is not of much interest.

In formal logic, this necessity is expressed by the aforementioned Necessitation Rule in its narrower sense: If *p* is a theorem, then *necessarily-p* is also a theorem. It is crucial for modal theory that the necessity of its theorems does not have to be based on material necessity, i.e., on the content of possible worlds, but can simply be decided formally. To put it colloquially: We can simply test whether *p* is a theorem without considering the question of modality, *and then*, if the result is positive, assign necessity to *p*. This necessity then (primarily) indicates simply that *p* is a theorem, and nothing more – just as the axiomatic necessity of *q* (primarily) indicates nothing more than that *q* is an axiom – albeit *in view of* material necessity (see above). We can, so to speak, take theorems with us from nonmodal logic into modal logic (that is, by attributing necessity to them). For our discussion this will mean: We can *without further ado* (*or further input*) take theorems (as well as axioms) from the field of formal necessity over to the field of material necessity; they "automatically" precipitate materially.

This means, however, that *formal* necessity, including the necessity of theorems, is unoriginal (not genuine) insofar as, viewed with regard to the sphere of the formal itself, it just says that the axioms are axioms and the theorems are theorems. In the field of the formal itself, this state of affairs has no modal meaning whatsoever. (In other words, the modal formulation contributes nothing to the axioms and theorems in this field. It does not make them "even more irrefutable" or anything else. The modal formulation of the validity of axioms and theorems makes sense solely in view of material necessity.) This also applies to the axioms and theorems of *modal* logic: they speak of modality in terms of content, but as such, *qua* axioms and theorems, they are not *necessarily* given, but are simply *given*. As I have said, axioms and theorems get their modal meaning from the Necessitation Rule. Specifically, this means that if a "space of possibilities" is opened up (in thought or in reality) under certain axioms, then these axioms and the theorems derived from them are reflected in this space of possibilities as necessarily given *within* this space. The Necessitation Rule, so to speak, articulates this reflection or this "precipitation" or substantiation of formal necessity as material. Since a space of possibilities that is not subject to *any* axiomatic determinations is completely inarticulate and therefore decrepit – that is, since a space of possibilities can only be sensibly opened up (or can itself open up) under an axiomatic system, the Necessitation Rule is constitutive of modality. On the other hand, in a certain way, this weakens its meaning. This rule does not award the axioms and theorems anything that they would not have had without it, so that modality would be (logically) prior to axioms and theorems and *then* attributed to them, in a (logically) secondary step. No, modality (in the genuine sense) only opens up under axioms, and the Necessitation Rule, so to speak, only expresses the right that the respective axioms and theorems have from the outset over the respective space of possibilities, precisely because this space has been constituted under their specifications. But this means that formal necessity takes its meaning from material necessity. If there were no material necessity, then it would be meaningless to say that, for example, logic is *necessarily* valid.

This has consequences for the theoretical possibilities for ontology, which I want to outline briefly. The respective space of possibilities and, consequently, the corresponding modal ontology, do not have to answer for modeling the specific necessity of the axioms and theorems. We do not have to make any additional ontological arrangements for axioms and theorems to be reflected in material necessity. It suffices that there is *formal* (i.e., immaterial, i.e., ontologically speaking, *nonentitative*) necessity, that is, inferential and axiomatic necessity. Specifically, this means that, with regard to the latter, it suffices (ontologically speaking) that there are, for example, natural laws. We do not have to introduce any other entities into our ontology besides these (such as special structures of the world, or merely possible worlds or things) in order to ontologically guarantee the necessity of theorems.

Specifically, this means that the logic of *formal* necessity is the so-called modal logical system "S5"[27], that is, the maximal normal modal logic that is not identical to predicate logic (i.e., in which *p* is not equivalent to necessarily-*p*). But formal necessity is independent of further ontological conditions (apart from the validity of the axioms themselves), i.e., for example, it does not need a system of possible worlds in order for it to be given. Therefore it is not ontologically necessary to introduce such a system of possible worlds (or any other equivalent ontological conditions). In order for there to be inferential and axiomatic necessity, it is *not* necessary to develop an ontological model that corresponds to the logic of this necessity (i.e., the system S5). It is therefore not necessary for the logical space of possible worlds to exist in an ontological sense. Of course, we can develop corresponding *semantic* models to articulate formal necessity materially. But we need not burden ontology with such models. (Consequently, in my opinion, we should not do so. But this would be a different discussion.)

Modal pragmatics therefore reveals a negative or "unlimiting" determination of the modal ontologies that are theoretically compatible with it: such modal ontologies do not have to accept all logically possible worlds as real. As I have said, this is completely compatible with the fact that the logic of formal necessity is adequately described by the system S5 – to which (logically), in possible world semantics, quantification over all logically possible worlds corresponds. It is therefore conceivable that a modal ontology should only accept a specific selection of possible worlds as real – e.g., the nomologically possible worlds. And finally, it is theoretically possible (i.e., it is compatible with formal necessity) to conceive of a model of modal ontology in which only *one* possible world is real, namely the actual one, but which, e.g., expands modally into the future. This could be completely sufficient to ensure that possibility and necessity can be spoken of meaningfully and truthfully in reality – including

27 Cf. e.g. Uwe Meixner 2008, 20–26.

the insurance that *axiomatic* and *inferential* possibility and necessity can be spoken of meaningfully and truthfully. Our talk of logical possibility does not require logical space (including all the possible worlds in it) to be ontologically given; for it is possible to design an ontology which, under its modal aspect, is limited to just a "few" or even just one real world (in the broadest sense), and which nevertheless supports meaningful and truthful talk of logical necessity, and of formal necessity in general. Obviously, I emphasize this point because I myself defend such an ontology. However, of course, modal pragmatics does not commit us to such an ontology; nor does it even clearly suggest it. It only opens up the theoretical possibility, the possibility of a sufficiently clear, distinct and consistent formulation of such an ontology.